Computer Algebra Independent Integration Tests

Summer 2023 edition

1-Algebraic-functions/1.2-Trinomial-products/1.2.1-Quadratic/37-1.2.1.6-g+h-x-^m-a+b-x+c-x^2-^p-d+e-x+f-x^2-^q

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Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [143]. This is test number [37].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (143)	0.00 (0)
Mathematica	100.00 (143)	0.00 (0)
Maple	97.90 (140)	2.10 (3)
Fricas	58.04 (83)	41.96 (60)
Giac	37.06 (53)	62.94 (90)
Mupad	13.29 (19)	86.71 (124)
Maxima	10.49 (15)	89.51 (128)
Sympy	8.39 (12)	91.61 (131)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

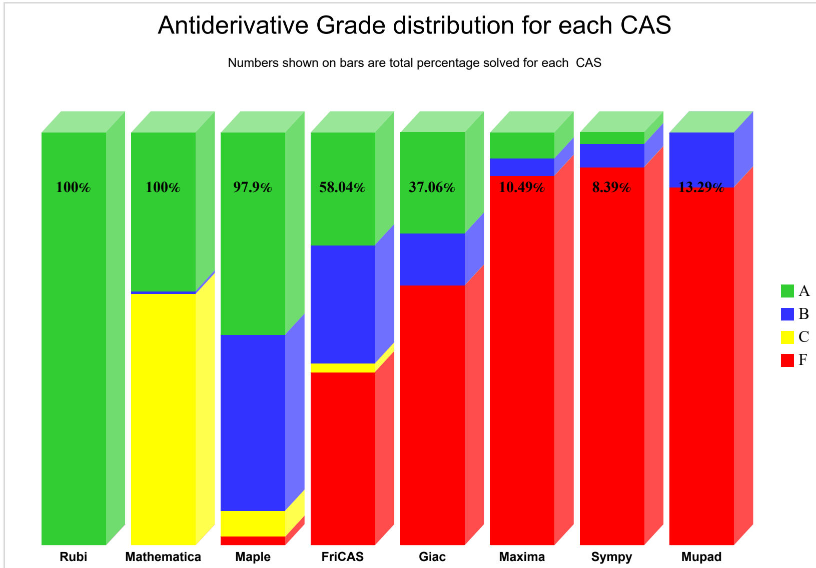
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

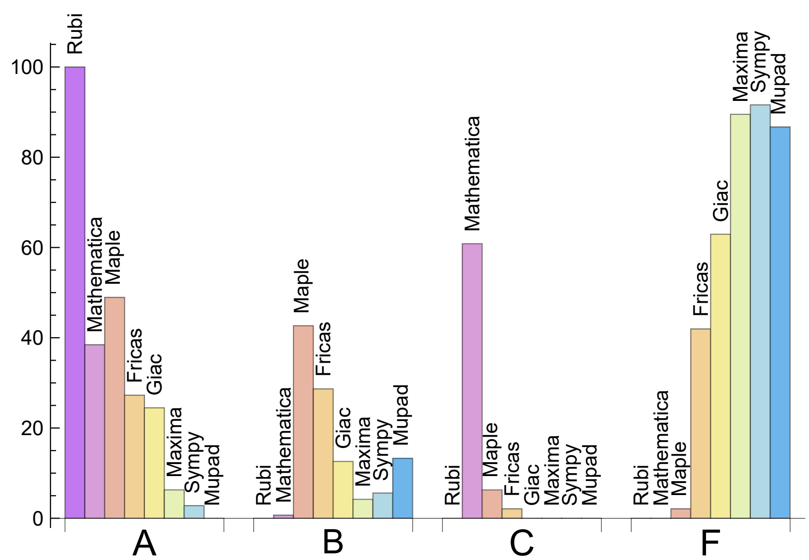
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.000	0.000	0.000	0.000
Maple	48.951	42.657	6.294	2.098
Mathematica	38.462	0.699	60.839	0.000
Fricas	27.273	28.671	2.098	41.958
Giac	24.476	12.587	0.000	62.937
Maxima	6.294	4.196	0.000	89.510
Sympy	2.797	5.594	0.000	91.608
Mupad	0.000	13.287	0.000	86.713

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Maple	3	66.67	33.33	0.00
Fricas	60	1.67	98.33	0.00
Giac	90	8.89	15.56	75.56
Mupad	124	0.00	100.00	0.00
Maxima	128	52.34	0.00	47.66
Sympy	131	82.44	17.56	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.28
Mathematica	0.81
Rubi	1.16
Maple	1.74
Giac	8.89
Sympy	9.53
Mupad	17.25
Fricas	23.82

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Rubi	333.90	1.00	302.00	1.00
Mathematica	361.78	1.04	264.00	0.89
Maxima	388.27	2.32	220.00	1.11
Sympy	738.42	3.40	214.00	2.84
Giac	967.21	5.89	165.00	1.34
Fricas	3703.05	11.25	435.00	2.34
Mupad	7822.16	9.68	187.00	1.25
Maple	49720.61	103.14	533.00	1.56

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

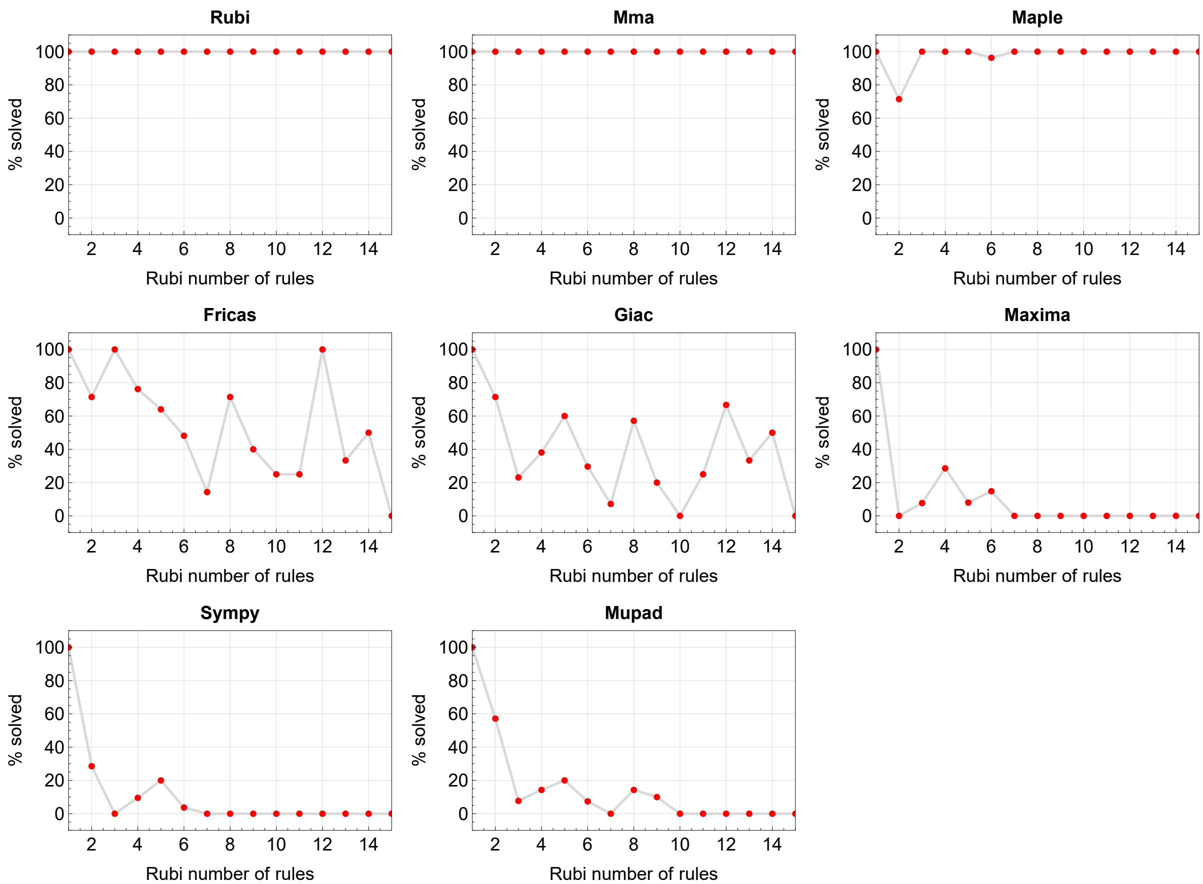


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

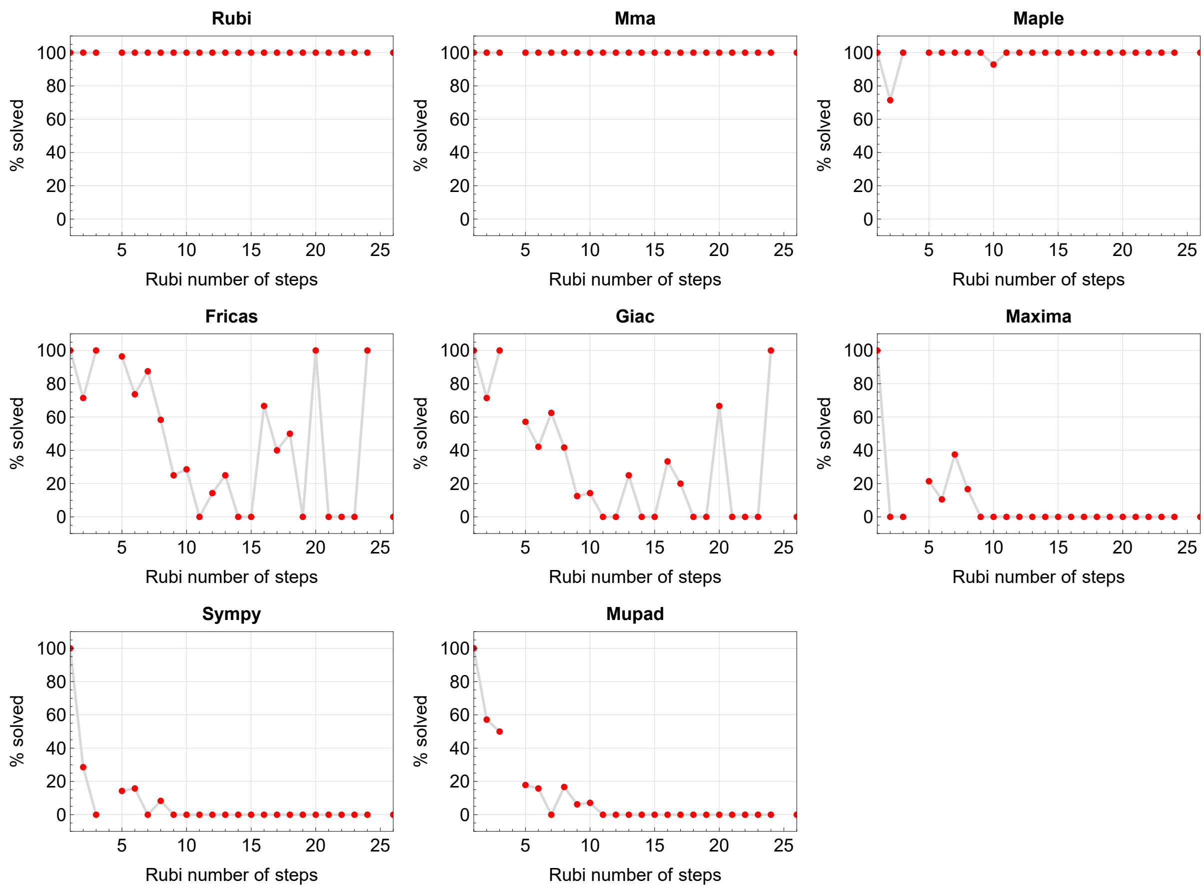


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

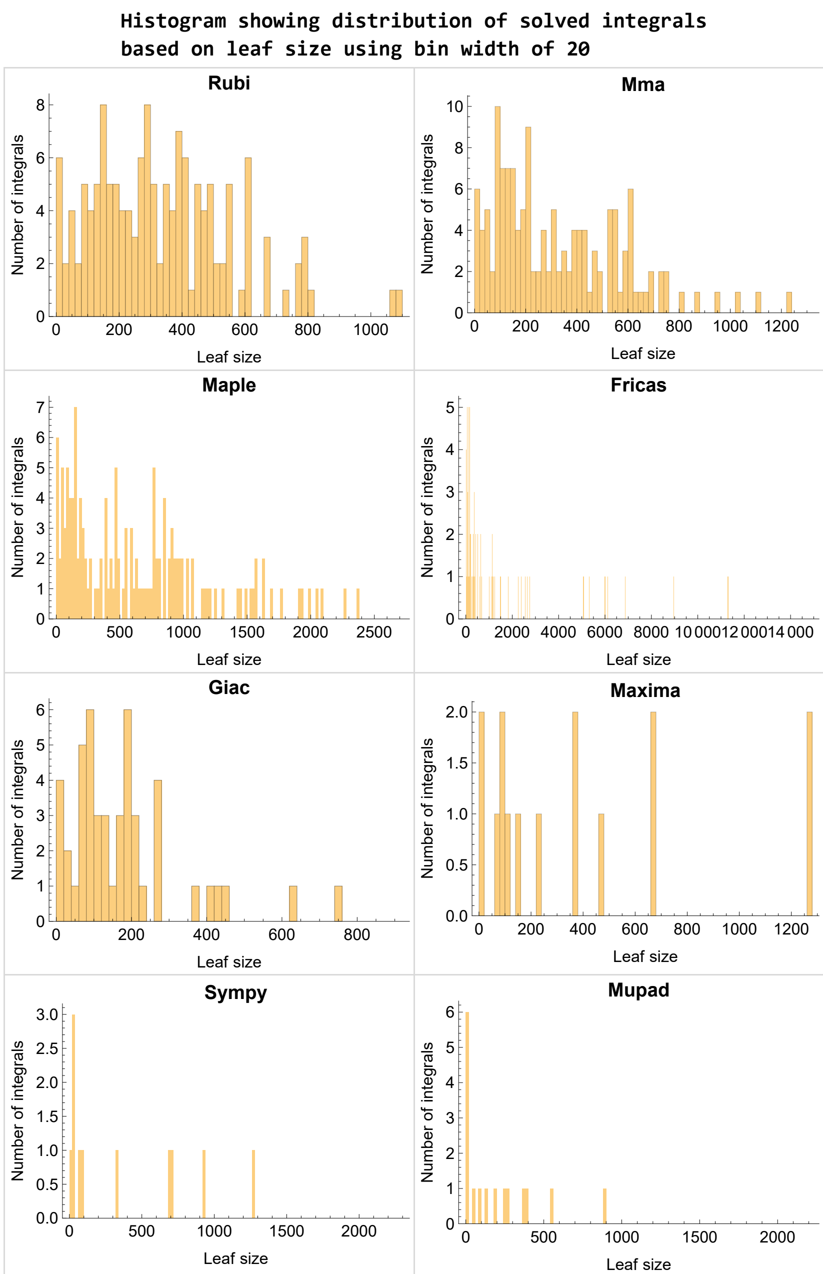


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

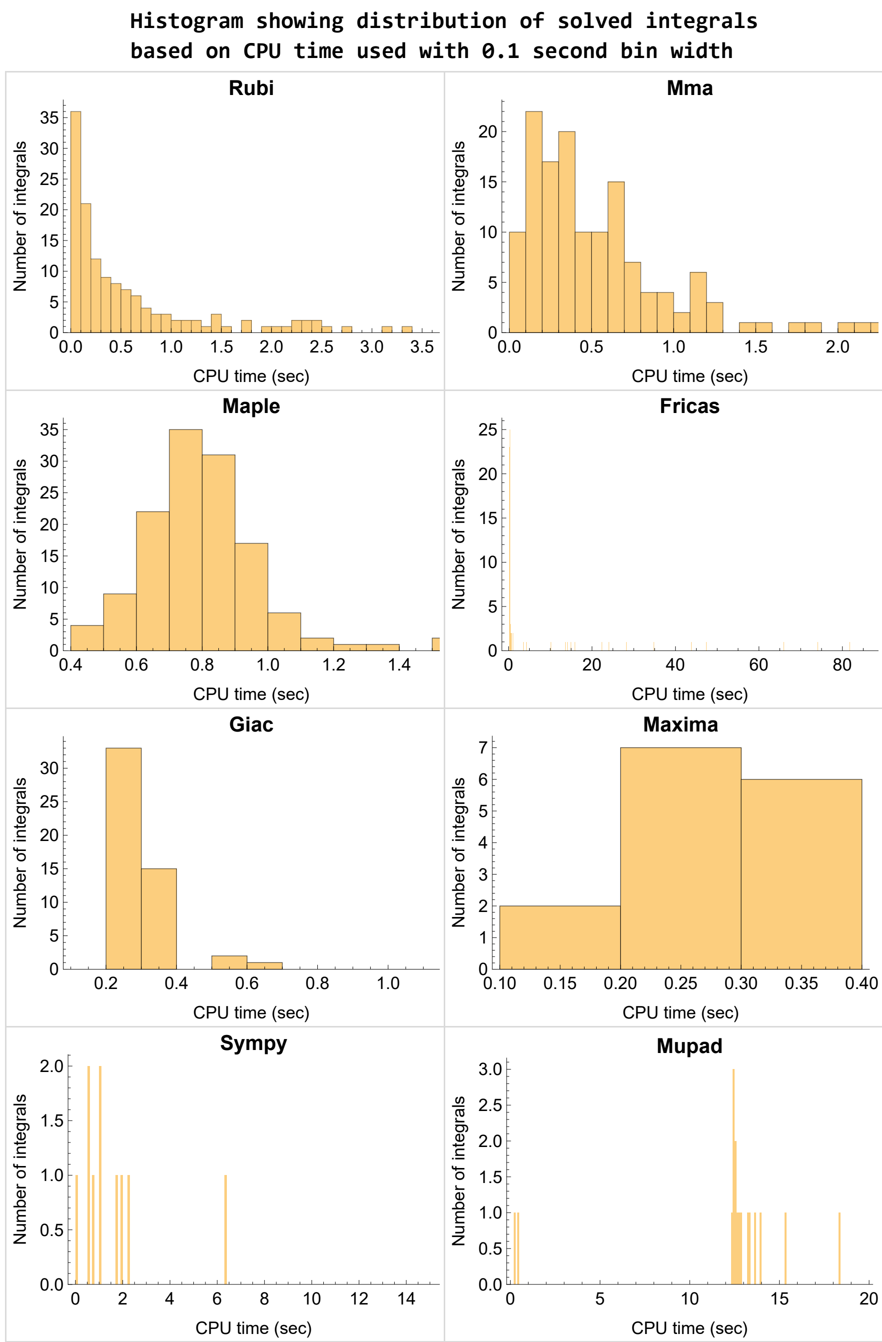


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

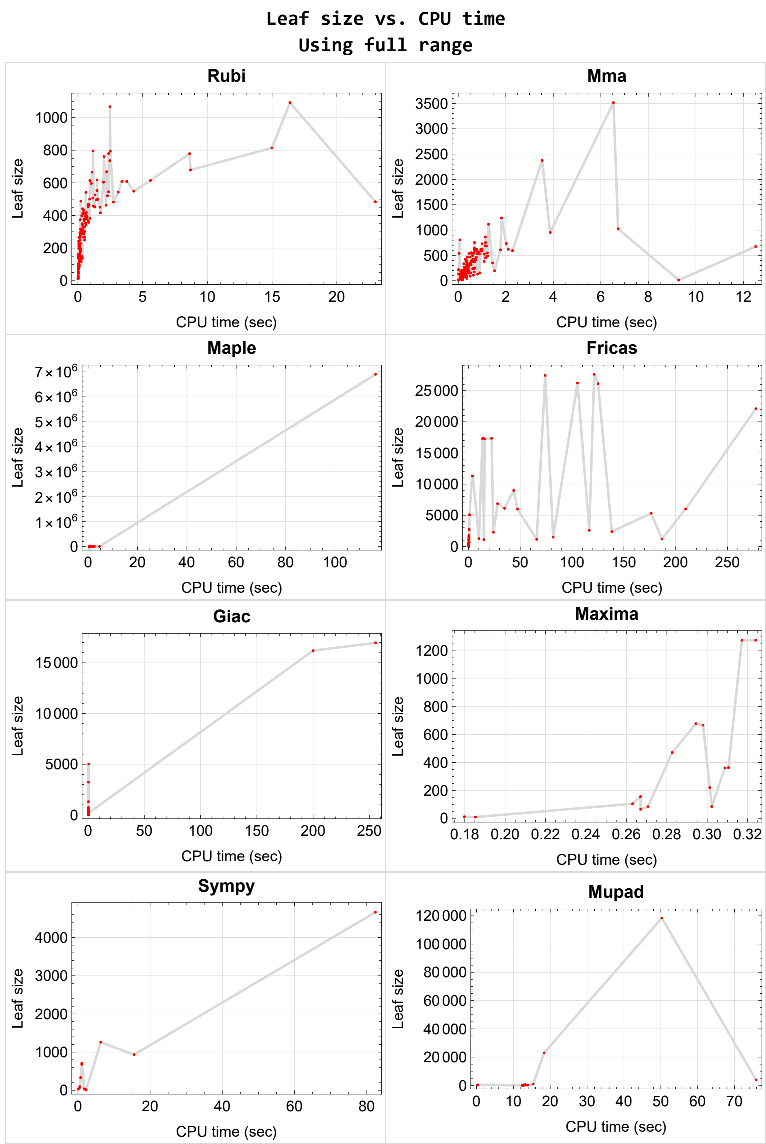


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {19, 20}

Mathematica {}

Maple {11, 12, 27, 30, 40, 41, 42, 43, 49, 125}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
   r"""
   Return the tree size of this expression.
   """
   if expr not in SR:
      # deal with lists, tuples, vectors
      return 1 + sum(tree_size(a) for a in expr)
   expr = SR(expr)
```

```
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

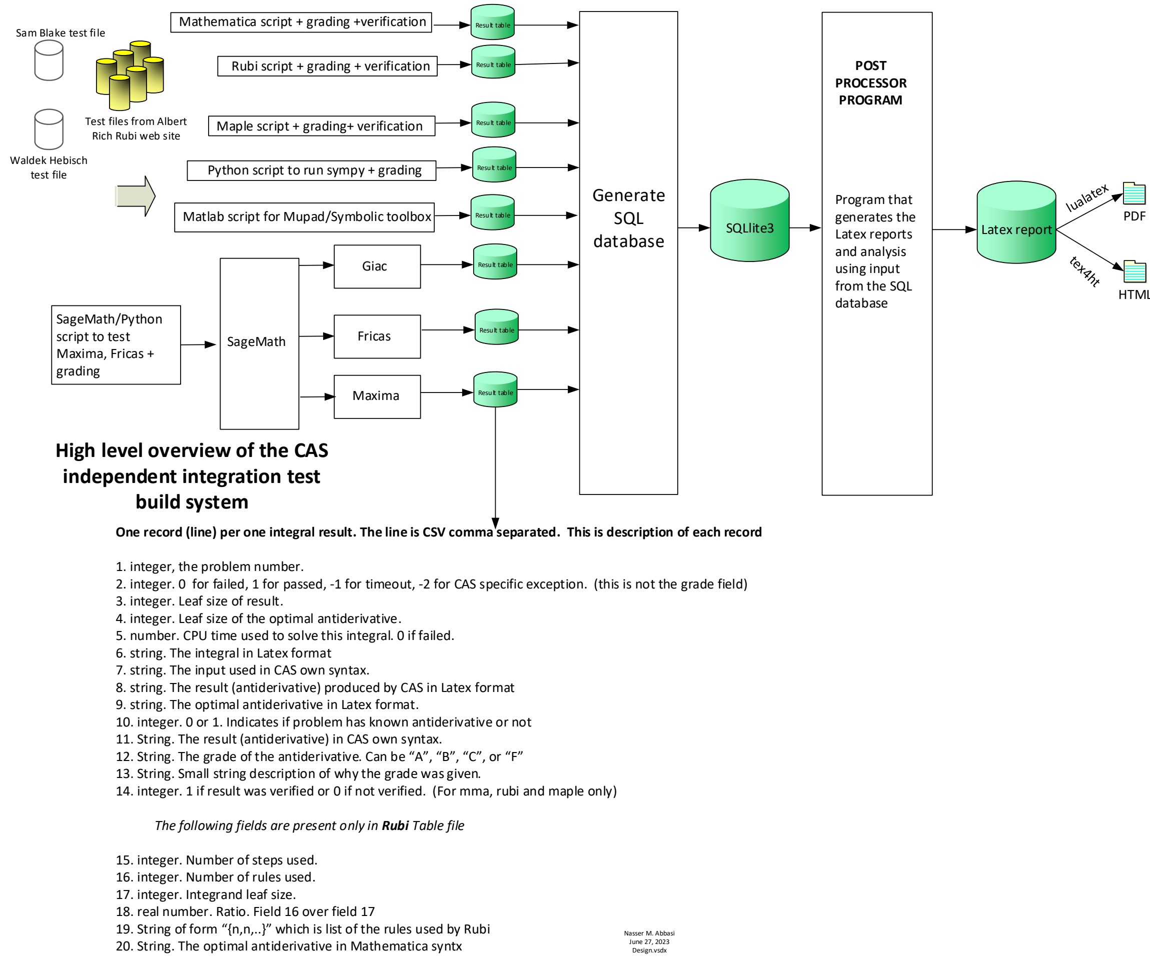
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 9, 10, 13, 14, 15, 16, 17, 18, 28, 29, 30, 31, 32, 33, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 91, 92, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143 }

B grade { 24 }

C grade { 6, 7, 8, 11, 12, 19, 20, 21, 22, 23, 25, 26, 27, 34, 35, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade $\{ 1, 2, 3, 4, 10, 13, 14, 15, 17, 18, 22, 25, 26, 28, 29, 31, 32, 33, 34, 36, 37, 38, 39, 44, 45, 46, 47, 48, 50, 51, 57, 58, 62, 63, 70, 77, 78, 83, 84, 85, 89, 90, 91, 92, 94, 95, 96, 99, 100, 101, 109, 113, 114, 119, 120, 126, 127, 128, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141 \}$

B grade $\{ 5, 6, 7, 8, 9, 12, 19, 20, 21, 23, 24, 35, 52, 53, 54, 55, 56, 59, 60, 61, 64, 65, 66, 67, 68, 69, 71, 72, 73, 74, 75, 76, 79, 80, 81, 82, 86, 87, 88, 93, 97, 98, 102, 103, 104, 105, 106, 107, 108, 110, 111, 112, 115, 116, 117, 118, 121, 122, 123, 124, 125 \}$

C grade $\{ 11, 27, 30, 40, 41, 42, 43, 49, 129 \}$

F normal fail $\{ 142, 143 \}$

F(-1) timedout fail $\{ 16 \}$

F(-2) exception fail $\{ \}$

Fricas

A grade $\{ 1, 2, 3, 10, 12, 13, 14, 33, 36, 38, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 91, 92, 126, 127, 128, 129, 130, 131, 132, 133, 134, 136, 137, 138, 139, 140, 141 \}$

B grade $\{ 7, 17, 18, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 37, 54, 55, 56, 66, 67, 71, 72, 73, 74, 79, 80, 81, 82, 83, 97, 98, 99, 100, 103, 104, 105, 106, 116, 117, 135 \}$

C grade $\{ 11, 34, 93 \}$

F normal fail $\{ 39 \}$

F(-1) timedout fail $\{ 4, 5, 6, 8, 9, 15, 16, 19, 20, 35, 52, 53, 57, 58, 59, 60, 61, 62, 63, 64, 65, 68, 69, 70, 75, 76, 77, 78, 84, 85, 86, 87, 88, 89, 90, 94, 95, 96, 101, 102, 107, 108, 109, 110, 111, 112, 113, 114, 115, 118, 119, 120, 121, 122, 123, 124, 125, 142, 143 \}$

F(-2) exception fail $\{ \}$

Maxima

A grade $\{ 1, 2, 3, 10, 92, 133, 134, 138, 139 \}$

B grade $\{ 25, 26, 27, 28, 29, 30 \}$

C grade $\{ \}$

F normal fail $\{ 11, 12, 22, 24, 31, 32, 33, 34, 35, 36, 37, 38, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 55, 56, 57, 61, 62, 63, 68, 69, 70, 75, 76, 81, 82, 83, 88, 89, 90, 93, 99, 100, 101, 107, 108, 112, 113, 118, 119, 120, 125, 126, 127, 128, 129, 130, 131, 132, 135, 136, 137, 140, 141, 142, 143 \}$

F(-1) timedout fail $\{ \}$

F(-2) exception fail $\{ 4, 5, 6, 7, 8, 9, 13, 14, 15, 16, 17, 18, 19, 20, 21, 23, 39, 52, 53, 54, 58, 59, 60, 64, 65, 66, 67, 71, 72, 73, 74, 77, 78, 79, 80, 84, 85, 86, 87, 91, 94, 95, 96, 97, 98, 102, 103, 104, 105, 106, 109, 110, 111, 114, 115, 116, 117, 121, 122, 123, 124 \}$

Giac

A grade { 1, 2, 3, 4, 10, 13, 14, 15, 17, 18, 28, 29, 30, 36, 40, 41, 42, 44, 45, 46, 47, 48, 50, 92, 126, 127, 129, 131, 133, 134, 137, 138, 139, 140, 141 }

B grade { 5, 11, 16, 25, 26, 31, 32, 33, 34, 35, 37, 38, 51, 128, 130, 132, 135, 136 }

C grade { }

F normal fail { 12, 39, 81, 99, 107, 125, 142, 143 }

F(-1) timedout fail { 24, 27, 66, 67, 68, 70, 75, 113, 116, 117, 118, 119, 120, 121 }

F(-2) exception fail { 6, 7, 8, 9, 19, 20, 21, 22, 23, 43, 49, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 69, 71, 72, 73, 74, 76, 77, 78, 79, 80, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 93, 94, 95, 96, 97, 98, 100, 101, 102, 103, 104, 105, 106, 108, 109, 110, 111, 112, 114, 115, 122, 123, 124 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 13, 14, 16, 17, 18, 31, 33, 36, 133, 134, 138, 139, 140, 141 }

C grade { }

F normal fail { }

F(-1) timedout fail { 6, 7, 8, 9, 10, 11, 12, 15, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 34, 35, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 135, 136, 137, 142, 143 }

F(-2) exception fail { }

Sympy

A grade { 33, 133, 134, 139 }

B grade { 1, 2, 13, 14, 17, 18, 31, 138 }

C grade { }

F normal fail { 6, 7, 10, 11, 12, 19, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 34, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 59, 60, 61, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 107, 108, 109, 110, 111, 112, 114, 115, 116, 117, 118, 119, 120, 125, 126, 127, 128, 129, 130, 131, 132, 135, 136, 137, 140, 141, 142, 143 }

F(-1) timedout fail { 3, 4, 5, 8, 9, 15, 16, 20, 35, 58, 62, 63, 84, 102, 103, 104, 105, 106, 113, 121, 122, 123, 124 }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name **`Mathematica`** was abbreviated to **`MMA`**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	86	84	84	200	333	87	97
N.S.	1	1.00	0.91	0.89	0.89	2.13	3.54	0.93	1.03
time (sec)	N/A	0.075	0.093	0.645	0.302	0.304	0.754	0.261	12.889

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	228	204	235	220	500	933	263	253
N.S.	1	1.00	0.89	1.03	0.96	2.19	4.09	1.15	1.11
time (sec)	N/A	0.198	0.152	0.734	0.301	0.288	15.556	0.258	0.265

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	441	441	422	592	471	1014	0	623	552
N.S.	1	1.00	0.96	1.34	1.07	2.30	0.00	1.41	1.25
time (sec)	N/A	0.370	0.303	0.754	0.283	0.306	0.000	0.261	13.203

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	274	274	212	239	0	0	0	266	3888
N.S.	1	1.00	0.77	0.87	0.00	0.00	0.00	0.97	14.19
time (sec)	N/A	0.173	0.219	0.950	0.000	0.000	0.000	0.259	75.857

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	596	596	523	1254	0	0	0	1313	23006
N.S.	1	1.00	0.88	2.10	0.00	0.00	0.00	2.20	38.60
time (sec)	N/A	1.037	1.100	1.549	0.000	0.000	0.000	0.276	18.332

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	331	331	619	541	0	0	0	0	0
N.S.	1	1.00	1.87	1.63	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.387	0.947	0.819	0.000	0.000	0.000	0.000	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	249	249	219	389	0	6113	0	0	0
N.S.	1	1.00	0.88	1.56	0.00	24.55	0.00	0.00	0.00
time (sec)	N/A	0.167	0.385	0.786	0.000	34.857	0.000	0.000	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	F(-1)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	381	380	753	934	0	0	0	0	0
N.S.	1	1.00	1.98	2.45	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.536	1.151	0.786	0.000	0.000	0.000	0.000	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	797	796	674	1768	0	0	0	0	0
N.S.	1	1.00	0.85	2.22	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.169	12.518	0.753	0.000	0.000	0.000	0.000	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	37	46	65	46	0	48	0
N.S.	1	1.00	0.79	0.98	1.38	0.98	0.00	1.02	0.00
time (sec)	N/A	0.021	0.112	0.639	0.267	0.268	0.000	0.279	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	117	117	106	246	0	93	0	457	0
N.S.	1	1.00	0.91	2.10	0.00	0.79	0.00	3.91	0.00
time (sec)	N/A	0.114	0.127	2.078	0.000	0.414	0.000	0.340	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	484	484	210	6870946	0	307	0	0	0
N.S.	1	1.00	0.43	14196.17	0.00	0.63	0.00	0.00	0.00
time (sec)	N/A	22.979	0.363	116.444	0.000	0.368	0.000	0.000	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	182	175	190	0	583	1260	189	273
N.S.	1	0.99	0.95	1.03	0.00	3.17	6.85	1.03	1.48
time (sec)	N/A	0.219	0.104	0.846	0.000	0.312	6.326	0.264	13.661

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	542	542	535	800	0	1837	4663	743	893
N.S.	1	1.00	0.99	1.48	0.00	3.39	8.60	1.37	1.65
time (sec)	N/A	0.620	0.453	0.925	0.000	0.418	82.394	0.275	15.368

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	406	398	267	384	0	0	0	405	0
N.S.	1	0.98	0.66	0.95	0.00	0.00	0.00	1.00	0.00
time (sec)	N/A	0.295	0.242	1.569	0.000	0.000	0.000	0.275	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	F(-2)	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1075	1067	952	0	0	0	0	3236	118429
N.S.	1	0.99	0.89	0.00	0.00	0.00	0.00	3.01	110.17
time (sec)	N/A	2.476	3.869	180.000	0.000	0.000	0.000	0.332	50.268

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	131	141	0	1150	709	219	395
N.S.	1	1.00	0.94	1.01	0.00	8.21	5.06	1.56	2.82
time (sec)	N/A	0.082	0.087	0.902	0.000	0.347	1.085	0.275	0.422

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	131	141	0	1130	680	207	375
N.S.	1	1.00	0.94	1.01	0.00	8.07	4.86	1.48	2.68
time (sec)	N/A	0.060	0.018	0.950	0.000	0.341	1.083	0.269	12.763

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	617	615	1115	1176	0	0	0	0	0
N.S.	1	1.00	1.81	1.91	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	5.606	1.276	0.994	0.000	0.000	0.000	0.000	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	F(-1)	F(-1)	F(-2)	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1092	1092	3516	2278	0	0	0	0	0
N.S.	1	1.00	3.22	2.09	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	16.384	6.532	1.081	0.000	0.000	0.000	0.000	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	416	416	278	805	0	22103	0	0	0
N.S.	1	1.00	0.67	1.94	0.00	53.13	0.00	0.00	0.00
time (sec)	N/A	1.743	0.435	1.011	0.000	277.457	0.000	0.000	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	780	780	218	425	0	6861	0	0	0
N.S.	1	1.00	0.28	0.54	0.00	8.80	0.00	0.00	0.00
time (sec)	N/A	2.375	0.373	0.773	0.000	28.212	0.000	0.000	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	302	302	195	639	0	8977	0	0	0
N.S.	1	1.00	0.65	2.12	0.00	29.73	0.00	0.00	0.00
time (sec)	N/A	0.512	0.378	0.773	0.000	43.725	0.000	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	349	337	0	1515	0	0	0
N.S.	1	1.00	3.46	3.34	0.00	15.00	0.00	0.00	0.00
time (sec)	N/A	0.086	1.447	0.575	0.000	0.375	0.000	0.000	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	149	176	361	309	0	16965	0
N.S.	1	1.00	1.07	1.27	2.60	2.22	0.00	122.05	0.00
time (sec)	N/A	0.140	0.188	2.369	0.309	0.294	0.000	255.510	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	137	195	678	361	0	16200	0
N.S.	1	1.00	0.83	1.17	4.08	2.17	0.00	97.59	0.00
time (sec)	N/A	0.143	0.330	2.395	0.294	0.308	0.000	199.977	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	B	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	193	193	183	484	1276	439	0	0	0
N.S.	1	1.00	0.95	2.51	6.61	2.27	0.00	0.00	0.00
time (sec)	N/A	0.170	0.448	2.271	0.324	0.307	0.000	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	109	186	363	245	0	93	0
N.S.	1	1.00	0.72	1.23	2.40	1.62	0.00	0.62	0.00
time (sec)	N/A	0.152	0.636	2.316	0.311	0.308	0.000	0.314	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	130	209	668	365	0	112	0
N.S.	1	1.00	0.75	1.20	3.84	2.10	0.00	0.64	0.00
time (sec)	N/A	0.168	0.817	2.460	0.298	0.323	0.000	0.314	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	197	197	154	471	1276	435	0	121	0
N.S.	1	1.00	0.78	2.39	6.48	2.21	0.00	0.61	0.00
time (sec)	N/A	0.192	0.908	2.517	0.317	0.299	0.000	0.319	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	0	49	32	31	13
N.S.	1	1.00	1.00	0.93	0.00	3.27	2.13	2.07	0.87
time (sec)	N/A	0.016	0.149	0.835	0.000	0.301	1.958	0.283	12.559

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	55	40	0	106	0	108	0
N.S.	1	1.00	1.25	0.91	0.00	2.41	0.00	2.45	0.00
time (sec)	N/A	0.034	0.200	0.823	0.000	0.323	0.000	0.285	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	20	0	34	39	39	19
N.S.	1	1.00	1.00	0.83	0.00	1.42	1.62	1.62	0.79
time (sec)	N/A	0.011	0.193	1.776	0.000	0.287	1.740	0.276	12.551

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	92	45	0	203	0	133	0
N.S.	1	1.00	1.64	0.80	0.00	3.62	0.00	2.38	0.00
time (sec)	N/A	0.031	0.118	0.986	0.000	0.301	0.000	0.299	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	249	249	2374	1430	0	0	0	5022	0
N.S.	1	1.00	9.53	5.74	0.00	0.00	0.00	20.17	0.00
time (sec)	N/A	0.523	3.530	1.378	0.000	0.000	0.000	0.532	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	46	48	0	85	0	81	49
N.S.	1	1.00	0.96	1.00	0.00	1.77	0.00	1.69	1.02
time (sec)	N/A	0.029	0.361	1.046	0.000	0.515	0.000	0.300	12.421

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	18	0	56	0	98	0
N.S.	1	1.00	1.00	1.06	0.00	3.29	0.00	5.76	0.00
time (sec)	N/A	0.013	0.153	0.819	0.000	0.312	0.000	0.280	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	51	123	0	132	0	163	0
N.S.	1	1.00	0.59	1.43	0.00	1.53	0.00	1.90	0.00
time (sec)	N/A	0.110	0.181	0.820	0.000	0.291	0.000	0.303	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	108	121	0	0	0	0	0
N.S.	1	1.00	0.79	0.89	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.067	0.233	0.896	0.000	0.000	0.000	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	212	212	107	103	0	175	0	117	0
N.S.	1	1.00	0.50	0.49	0.00	0.83	0.00	0.55	0.00
time (sec)	N/A	0.076	0.167	0.454	0.000	0.284	0.000	0.291	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	161	161	95	83	0	157	0	98	0
N.S.	1	1.00	0.59	0.52	0.00	0.98	0.00	0.61	0.00
time (sec)	N/A	0.041	0.128	0.510	0.000	0.287	0.000	0.276	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	148	148	84	65	0	128	0	79	0
N.S.	1	1.00	0.57	0.44	0.00	0.86	0.00	0.53	0.00
time (sec)	N/A	0.033	0.140	0.507	0.000	0.308	0.000	0.286	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	160	160	118	92	0	341	0	0	0
N.S.	1	1.00	0.74	0.58	0.00	2.13	0.00	0.00	0.00
time (sec)	N/A	0.074	0.170	0.487	0.000	0.298	0.000	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	109	112	0	333	0	126	0
N.S.	1	1.00	0.70	0.72	0.00	2.13	0.00	0.81	0.00
time (sec)	N/A	0.070	0.180	0.503	0.000	0.294	0.000	0.288	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	124	107	0	377	0	199	0
N.S.	1	1.00	0.77	0.66	0.00	2.34	0.00	1.24	0.00
time (sec)	N/A	0.074	0.247	0.451	0.000	0.283	0.000	0.290	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	317	317	236	266	0	517	0	368	0
N.S.	1	1.00	0.74	0.84	0.00	1.63	0.00	1.16	0.00
time (sec)	N/A	0.202	0.607	0.673	0.000	0.354	0.000	0.314	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	176	199	0	391	0	265	0
N.S.	1	1.00	0.78	0.88	0.00	1.72	0.00	1.17	0.00
time (sec)	N/A	0.082	0.418	0.674	0.000	0.323	0.000	0.286	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	172	146	0	287	0	180	0
N.S.	1	1.00	0.87	0.74	0.00	1.45	0.00	0.91	0.00
time (sec)	N/A	0.069	0.671	0.667	0.000	0.328	0.000	0.502	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	211	211	149	215	0	651	0	0	0
N.S.	1	1.00	0.71	1.02	0.00	3.09	0.00	0.00	0.00
time (sec)	N/A	0.110	0.440	0.699	0.000	0.779	0.000	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	151	201	0	647	0	203	0
N.S.	1	1.00	0.75	1.00	0.00	3.20	0.00	1.00	0.00
time (sec)	N/A	0.098	0.452	0.591	0.000	0.450	0.000	0.658	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	155	157	0	693	0	434	0
N.S.	1	1.00	0.72	0.73	0.00	3.22	0.00	2.02	0.00
time (sec)	N/A	0.110	0.530	0.608	0.000	0.527	0.000	0.339	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	452	452	615	842	0	0	0	0	0
N.S.	1	1.00	1.36	1.86	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.299	0.691	0.816	0.000	0.000	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	395	395	379	769	0	0	0	0	0
N.S.	1	1.00	0.96	1.95	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.598	0.339	0.793	0.000	0.000	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	298	298	264	1212	0	2384	0	0	0
N.S.	1	1.00	0.89	4.07	0.00	8.00	0.00	0.00	0.00
time (sec)	N/A	0.216	0.288	0.740	0.000	138.398	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	358	358	310	1316	0	2266	0	0	0
N.S.	1	1.00	0.87	3.68	0.00	6.33	0.00	0.00	0.00
time (sec)	N/A	0.836	0.363	0.680	0.000	24.096	0.000	0.000	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	382	382	467	776	0	5324	0	0	0
N.S.	1	1.00	1.22	2.03	0.00	13.94	0.00	0.00	0.00
time (sec)	N/A	0.920	0.384	0.759	0.000	176.365	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	507	507	533	789	0	0	0	0	0
N.S.	1	1.00	1.05	1.56	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.156	0.792	0.840	0.000	0.000	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-1)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	795	795	1239	1183	0	0	0	0	0
N.S.	1	1.00	1.56	1.49	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.502	1.822	0.655	0.000	0.000	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	553	553	755	1039	0	0	0	0	0
N.S.	1	1.00	1.37	1.88	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.467	0.683	0.756	0.000	0.000	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	484	482	542	886	0	0	0	0	0
N.S.	1	1.00	1.12	1.83	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.743	0.552	0.776	0.000	0.000	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	496	496	552	2379	0	0	0	0	0
N.S.	1	1.00	1.11	4.80	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.432	0.547	0.652	0.000	0.000	0.000	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	604	604	497	957	0	0	0	0	0
N.S.	1	1.00	0.82	1.58	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.977	0.598	0.783	0.000	0.000	0.000	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	668	668	617	911	0	0	0	0	0
N.S.	1	1.00	0.92	1.36	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.223	0.783	0.701	0.000	0.000	0.000	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	380	380	312	713	0	0	0	0	0
N.S.	1	1.00	0.82	1.88	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.679	0.356	0.783	0.000	0.000	0.000	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	344	344	261	663	0	0	0	0	0
N.S.	1	1.00	0.76	1.93	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.334	0.356	0.744	0.000	0.000	0.000	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	294	294	156	622	0	5085	0	0	0
N.S.	1	1.00	0.53	2.12	0.00	17.30	0.00	0.00	0.00
time (sec)	N/A	0.153	0.283	0.737	0.000	1.029	0.000	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	266	131	589	0	5073	0	0	0
N.S.	1	1.00	0.49	2.21	0.00	19.07	0.00	0.00	0.00
time (sec)	N/A	0.083	0.283	0.634	0.000	1.030	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	330	330	236	681	0	0	0	0	0
N.S.	1	1.00	0.72	2.06	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.498	0.314	0.674	0.000	0.000	0.000	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	367	367	298	733	0	0	0	0	0
N.S.	1	1.00	0.81	2.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.787	0.394	0.812	0.000	0.000	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	457	457	422	777	0	0	0	0	0
N.S.	1	1.00	0.92	1.70	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.205	0.612	0.853	0.000	0.000	0.000	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	499	499	401	1576	0	27621	0	0	0
N.S.	1	1.00	0.80	3.16	0.00	55.35	0.00	0.00	0.00
time (sec)	N/A	1.532	0.590	0.645	0.000	121.373	0.000	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	410	410	385	1525	0	26116	0	0	0
N.S.	1	1.00	0.94	3.72	0.00	63.70	0.00	0.00	0.00
time (sec)	N/A	0.381	0.546	0.654	0.000	125.127	0.000	0.000	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	411	411	330	1490	0	26234	0	0	0
N.S.	1	1.00	0.80	3.63	0.00	63.83	0.00	0.00	0.00
time (sec)	N/A	0.505	0.499	0.748	0.000	105.280	0.000	0.000	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	416	416	346	1457	0	27447	0	0	0
N.S.	1	1.00	0.83	3.50	0.00	65.98	0.00	0.00	0.00
time (sec)	N/A	0.383	0.503	0.788	0.000	74.110	0.000	0.000	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	526	526	542	1565	0	0	0	0	0
N.S.	1	1.00	1.03	2.98	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.321	0.820	0.679	0.000	0.000	0.000	0.000	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	618	618	684	1639	0	0	0	0	0
N.S.	1	1.00	1.11	2.65	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.474	1.186	0.792	0.000	0.000	0.000	0.000	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	392	392	409	525	0	0	0	0	0
N.S.	1	1.00	1.04	1.34	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.621	0.689	0.905	0.000	0.000	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	316	316	319	483	0	0	0	0	0
N.S.	1	1.00	1.01	1.53	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.317	0.541	0.724	0.000	0.000	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	282	282	349	460	0	1192	0	0	0
N.S.	1	1.00	1.24	1.63	0.00	4.23	0.00	0.00	0.00
time (sec)	N/A	0.206	0.370	0.747	0.000	186.758	0.000	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	266	389	772	0	1139	0	0	0
N.S.	1	1.00	1.46	2.90	0.00	4.28	0.00	0.00	0.00
time (sec)	N/A	0.161	0.355	0.678	0.000	65.905	0.000	0.000	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	267	267	334	850	0	1253	0	0	0
N.S.	1	1.00	1.25	3.18	0.00	4.69	0.00	0.00	0.00
time (sec)	N/A	0.522	0.235	0.730	0.000	10.140	0.000	0.000	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	286	286	288	464	0	1094	0	0	0
N.S.	1	1.00	1.01	1.62	0.00	3.83	0.00	0.00	0.00
time (sec)	N/A	0.471	0.390	0.902	0.000	14.916	0.000	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	353	353	381	479	0	1485	0	0	0
N.S.	1	1.00	1.08	1.36	0.00	4.21	0.00	0.00	0.00
time (sec)	N/A	0.590	0.604	0.961	0.000	81.785	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-1)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	501	501	734	758	0	0	0	0	0
N.S.	1	1.00	1.47	1.51	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.913	2.019	0.802	0.000	0.000	0.000	0.000	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	417	417	608	648	0	0	0	0	0
N.S.	1	1.00	1.46	1.55	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.651	1.774	0.763	0.000	0.000	0.000	0.000	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	349	349	619	602	0	0	0	0	0
N.S.	1	1.00	1.77	1.72	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.327	0.985	0.734	0.000	0.000	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	315	315	729	545	0	0	0	0	0
N.S.	1	1.00	2.31	1.73	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.353	1.001	0.844	0.000	0.000	0.000	0.000	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	469	469	601	1639	0	0	0	0	0
N.S.	1	1.00	1.28	3.49	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.812	0.755	0.894	0.000	0.000	0.000	0.000	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	463	463	541	585	0	0	0	0	0
N.S.	1	1.00	1.17	1.26	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.785	0.649	0.850	0.000	0.000	0.000	0.000	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	614	614	587	549	0	0	0	0	0
N.S.	1	1.00	0.96	0.89	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.925	0.976	0.915	0.000	0.000	0.000	0.000	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	196	308	0	2579	0	0	0
N.S.	1	1.00	1.04	1.63	0.00	13.65	0.00	0.00	0.00
time (sec)	N/A	0.186	1.529	0.712	0.000	116.634	0.000	0.000	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	57	102	83	70	0	73	0
N.S.	1	1.00	0.76	1.36	1.11	0.93	0.00	0.97	0.00
time (sec)	N/A	0.036	0.089	0.694	0.271	0.311	0.000	0.279	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	121	267	0	132	0	0	0
N.S.	1	1.00	0.93	2.05	0.00	1.02	0.00	0.00	0.00
time (sec)	N/A	0.110	0.110	4.599	0.000	0.327	0.000	0.000	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	369	369	252	463	0	0	0	0	0
N.S.	1	1.00	0.68	1.25	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.493	0.682	0.841	0.000	0.000	0.000	0.000	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	287	287	210	409	0	0	0	0	0
N.S.	1	1.00	0.73	1.43	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.408	0.347	0.743	0.000	0.000	0.000	0.000	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	266	194	399	0	0	0	0	0
N.S.	1	1.00	0.73	1.50	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.126	0.263	0.800	0.000	0.000	0.000	0.000	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	220	149	354	0	2753	0	0	0
N.S.	1	1.00	0.68	1.61	0.00	12.51	0.00	0.00	0.00
time (sec)	N/A	0.111	0.203	0.822	0.000	0.648	0.000	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	220	160	358	0	2641	0	0	0
N.S.	1	1.00	0.73	1.63	0.00	12.00	0.00	0.00	0.00
time (sec)	N/A	0.078	0.234	0.786	0.000	0.717	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	267	267	193	391	0	5995	0	0	0
N.S.	1	1.00	0.72	1.46	0.00	22.45	0.00	0.00	0.00
time (sec)	N/A	0.435	0.229	0.764	0.000	47.358	0.000	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	291	291	216	426	0	6018	0	0	0
N.S.	1	1.00	0.74	1.46	0.00	20.68	0.00	0.00	0.00
time (sec)	N/A	0.414	0.443	0.809	0.000	209.662	0.000	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	376	376	241	457	0	0	0	0	0
N.S.	1	1.00	0.64	1.22	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.494	0.672	0.866	0.000	0.000	0.000	0.000	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	F(-1)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	466	466	453	1064	0	0	0	0	0
N.S.	1	1.00	0.97	2.28	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.861	1.149	0.827	0.000	0.000	0.000	0.000	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	B	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	341	341	410	960	0	17339	0	0	0
N.S.	1	1.00	1.20	2.82	0.00	50.85	0.00	0.00	0.00
time (sec)	N/A	0.631	0.743	0.871	0.000	22.372	0.000	0.000	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	B	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	297	297	371	946	0	17285	0	0	0
N.S.	1	1.00	1.25	3.19	0.00	58.20	0.00	0.00	0.00
time (sec)	N/A	0.289	0.684	0.770	0.000	13.619	0.000	0.000	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	B	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	299	299	407	899	0	17258	0	0	0
N.S.	1	1.00	1.36	3.01	0.00	57.72	0.00	0.00	0.00
time (sec)	N/A	0.238	0.722	0.684	0.000	15.820	0.000	0.000	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	B	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	310	310	464	903	0	17397	0	0	0
N.S.	1	1.00	1.50	2.91	0.00	56.12	0.00	0.00	0.00
time (sec)	N/A	0.270	0.758	0.769	0.000	14.101	0.000	0.000	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	394	394	486	990	0	0	0	0	0
N.S.	1	1.00	1.23	2.51	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.703	1.233	0.747	0.000	0.000	0.000	0.000	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	454	454	620	1065	0	0	0	0	0
N.S.	1	1.00	1.37	2.35	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.790	2.103	0.948	0.000	0.000	0.000	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	761	761	864	1148	0	0	0	0	0
N.S.	1	1.00	1.14	1.51	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.017	1.142	0.915	0.000	0.000	0.000	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	549	549	642	1022	0	0	0	0	0
N.S.	1	1.00	1.17	1.86	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	4.314	0.678	0.907	0.000	0.000	0.000	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	431	431	539	1547	0	0	0	0	0
N.S.	1	1.00	1.25	3.59	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.450	0.035	0.935	0.000	0.000	0.000	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	523	521	467	1691	0	0	0	0	0
N.S.	1	1.00	0.89	3.23	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.286	0.502	1.048	0.000	0.000	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	736	736	582	976	0	0	0	0	0
N.S.	1	1.00	0.79	1.33	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.465	0.690	0.952	0.000	0.000	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	545	545	423	914	0	0	0	0	0
N.S.	1	1.00	0.78	1.68	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.373	0.712	1.073	0.000	0.000	0.000	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	463	463	318	844	0	0	0	0	0
N.S.	1	1.00	0.69	1.82	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.178	0.436	0.878	0.000	0.000	0.000	0.000	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	402	402	204	794	0	11311	0	0	0
N.S.	1	1.00	0.51	1.98	0.00	28.14	0.00	0.00	0.00
time (sec)	N/A	0.621	0.364	0.898	0.000	3.566	0.000	0.000	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	374	374	218	761	0	11287	0	0	0
N.S.	1	1.00	0.58	2.03	0.00	30.18	0.00	0.00	0.00
time (sec)	N/A	0.182	0.014	0.826	0.000	4.272	0.000	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	451	451	319	859	0	0	0	0	0
N.S.	1	1.00	0.71	1.90	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.705	0.455	0.888	0.000	0.000	0.000	0.000	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	543	543	423	927	0	0	0	0	0
N.S.	1	1.00	0.78	1.71	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.112	0.676	1.216	0.000	0.000	0.000	0.000	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	679	679	547	991	0	0	0	0	0
N.S.	1	1.00	0.81	1.46	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	8.697	1.224	1.151	0.000	0.000	0.000	0.000	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	F(-1)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	779	779	690	2083	0	0	0	0	0
N.S.	1	1.00	0.89	2.67	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	8.628	1.129	0.996	0.000	0.000	0.000	0.000	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	F(-1)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	609	609	534	1992	0	0	0	0	0
N.S.	1	1.00	0.88	3.27	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.779	0.896	0.859	0.000	0.000	0.000	0.000	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	F(-1)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	609	609	570	1939	0	0	0	0	0
N.S.	1	1.00	0.94	3.18	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.397	0.849	0.857	0.000	0.000	0.000	0.000	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	F(-1)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	666	666	806	1906	0	0	0	0	0
N.S.	1	1.00	1.21	2.86	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.090	0.070	1.086	0.000	0.000	0.000	0.000	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	816	814	1025	2059	0	0	0	0	0
N.S.	1	1.00	1.26	2.52	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	14.984	6.728	1.156	0.000	0.000	0.000	0.000	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	99	155	0	178	0	188	0
N.S.	1	1.00	0.71	1.11	0.00	1.27	0.00	1.34	0.00
time (sec)	N/A	0.340	0.225	0.976	0.000	0.280	0.000	0.295	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	94	144	0	175	0	185	0
N.S.	1	1.00	0.82	1.25	0.00	1.52	0.00	1.61	0.00
time (sec)	N/A	0.253	0.179	0.819	0.000	0.331	0.000	0.297	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	77	130	0	161	0	171	0
N.S.	1	1.00	0.79	1.33	0.00	1.64	0.00	1.74	0.00
time (sec)	N/A	0.107	0.156	0.727	0.000	0.287	0.000	0.293	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	69	33	81	0	50	0	68	0
N.S.	1	1.01	0.49	1.19	0.00	0.74	0.00	1.00	0.00
time (sec)	N/A	0.036	0.112	0.688	0.000	0.313	0.000	0.288	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	54	121	0	132	0	165	0
N.S.	1	1.00	0.57	1.27	0.00	1.39	0.00	1.74	0.00
time (sec)	N/A	0.062	0.153	0.684	0.000	0.304	0.000	0.287	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	90	152	0	170	0	199	0
N.S.	1	1.00	0.69	1.17	0.00	1.31	0.00	1.53	0.00
time (sec)	N/A	0.258	0.174	0.768	0.000	0.295	0.000	0.292	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	113	169	0	194	0	269	0
N.S.	1	1.00	0.75	1.12	0.00	1.28	0.00	1.78	0.00
time (sec)	N/A	0.288	0.219	0.831	0.000	0.299	0.000	0.306	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	89	80	155	88	95	85	187
N.S.	1	1.00	0.60	0.54	1.04	0.59	0.64	0.57	1.26
time (sec)	N/A	0.061	0.501	0.658	0.267	0.275	0.531	0.304	13.916

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	74	65	104	73	76	70	136
N.S.	1	1.00	0.72	0.63	1.01	0.71	0.74	0.68	1.32
time (sec)	N/A	0.027	0.272	0.529	0.263	0.288	0.519	0.312	13.332

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	30	32	0	53	0	63	0
N.S.	1	1.00	1.07	1.14	0.00	1.89	0.00	2.25	0.00
time (sec)	N/A	0.029	0.139	0.636	0.000	0.317	0.000	0.312	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	80	57	0	126	0	159	0
N.S.	1	1.00	0.95	0.68	0.00	1.50	0.00	1.89	0.00
time (sec)	N/A	0.046	0.263	0.770	0.000	0.269	0.000	0.323	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	95	72	0	186	0	232	0
N.S.	1	1.00	0.68	0.52	0.00	1.34	0.00	1.67	0.00
time (sec)	N/A	0.075	0.357	0.808	0.000	0.269	0.000	0.318	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	13	12	11	11	31	11	15
N.S.	1	1.00	0.87	0.80	0.73	0.73	2.07	0.73	1.00
time (sec)	N/A	0.003	9.279	0.552	0.180	0.281	0.084	0.278	12.694

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	13	10	9	11	10	11	13
N.S.	1	1.00	0.81	0.62	0.56	0.69	0.62	0.69	0.81
time (sec)	N/A	0.004	0.001	0.520	0.185	0.275	2.276	0.277	12.354

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	13	14	0	11	0	11	15
N.S.	1	1.00	0.87	0.93	0.00	0.73	0.00	0.73	1.00
time (sec)	N/A	0.017	0.004	0.512	0.000	0.270	0.000	0.303	12.442

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	13	14	0	11	0	11	13
N.S.	1	1.00	0.87	0.93	0.00	0.73	0.00	0.73	0.87
time (sec)	N/A	0.031	0.003	0.451	0.000	0.262	0.000	0.270	12.488

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	242	242	381	0	0	0	0	0	0
N.S.	1	1.00	1.57	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.056	1.088	0.000	0.000	0.000	0.000	0.000	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	488	488	593	0	0	0	0	0	0
N.S.	1	1.00	1.22	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.229	2.284	0.000	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [63] had the largest ratio of [.5555999999999999983]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	5	4	1.00	25	0.160
2	A	5	4	1.00	27	0.148
3	A	5	4	1.00	27	0.148
4	A	8	8	1.00	27	0.296
5	A	9	9	1.00	27	0.333
6	A	9	6	1.00	30	0.200
7	A	5	3	1.00	30	0.100
8	A	6	4	1.00	30	0.133
9	A	7	5	1.00	30	0.167
10	A	5	4	1.00	23	0.174
11	A	5	4	1.00	23	0.174
12	A	5	4	1.00	30	0.133
13	A	6	5	0.99	28	0.179
14	A	6	5	1.00	30	0.167
15	A	9	5	0.98	30	0.167
16	A	10	6	0.99	30	0.200
17	A	5	5	1.00	34	0.147
18	A	5	5	1.00	34	0.147
19	A	9	6	1.00	32	0.188
20	A	10	7	1.00	32	0.219
21	A	5	3	1.00	32	0.094
22	A	5	3	1.00	29	0.103
23	A	5	3	1.00	29	0.103
24	A	6	6	1.00	26	0.231

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
25	A	5	4	1.00	30	0.133
26	A	7	6	1.00	30	0.200
27	A	7	6	1.00	30	0.200
28	A	5	3	1.00	30	0.100
29	A	6	4	1.00	30	0.133
30	A	7	5	1.00	30	0.167
31	A	2	2	1.00	26	0.077
32	A	5	5	1.00	26	0.192
33	A	2	2	1.00	24	0.083
34	A	5	5	1.00	20	0.250
35	A	6	5	1.00	36	0.139
36	A	2	2	1.00	36	0.056
37	A	2	2	1.00	32	0.062
38	A	13	9	1.00	32	0.281
39	A	5	5	1.00	38	0.132
40	A	6	6	1.00	35	0.171
41	A	5	5	1.00	33	0.152
42	A	5	5	1.00	32	0.156
43	A	8	8	1.00	35	0.229
44	A	8	8	1.00	35	0.229
45	A	8	8	1.00	35	0.229
46	A	6	6	1.00	38	0.158
47	A	5	5	1.00	36	0.139
48	A	5	5	1.00	35	0.143
49	A	7	6	1.00	38	0.158
50	A	7	6	1.00	38	0.158
51	A	7	6	1.00	38	0.158
52	A	9	6	1.00	27	0.222
53	A	9	6	1.00	25	0.240
54	A	8	5	1.00	24	0.208
55	A	12	9	1.00	27	0.333
56	A	18	12	1.00	27	0.444
57	A	22	13	1.00	27	0.482
58	A	10	7	1.00	27	0.259
59	A	10	7	1.00	25	0.280

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
60	A	9	6	1.00	24	0.250
61	A	17	11	1.00	27	0.407
62	A	21	14	1.00	27	0.518
63	A	26	15	1.00	27	0.556
64	A	10	6	1.00	27	0.222
65	A	8	5	1.00	27	0.185
66	A	5	3	1.00	25	0.120
67	A	5	3	1.00	24	0.125
68	A	10	7	1.00	27	0.259
69	A	11	8	1.00	27	0.296
70	A	15	9	1.00	27	0.333
71	A	10	7	1.00	27	0.259
72	A	6	4	1.00	27	0.148
73	A	6	4	1.00	25	0.160
74	A	6	4	1.00	24	0.167
75	A	12	9	1.00	27	0.333
76	A	14	11	1.00	27	0.407
77	A	15	9	1.00	28	0.321
78	A	9	6	1.00	28	0.214
79	A	9	6	1.00	26	0.231
80	A	8	5	1.00	25	0.200
81	A	17	9	1.00	28	0.321
82	A	16	8	1.00	28	0.286
83	A	20	10	1.00	28	0.357
84	A	17	10	1.00	28	0.357
85	A	10	7	1.00	28	0.250
86	A	10	7	1.00	26	0.269
87	A	9	6	1.00	25	0.240
88	A	19	11	1.00	28	0.393
89	A	18	10	1.00	28	0.357
90	A	26	13	1.00	28	0.464
91	A	9	6	1.00	24	0.250
92	A	8	6	1.00	22	0.273
93	A	10	9	1.00	17	0.529
94	A	13	7	1.00	28	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
95	A	10	6	1.00	28	0.214
96	A	8	5	1.00	28	0.179
97	A	5	3	1.00	26	0.115
98	A	5	3	1.00	25	0.120
99	A	9	4	1.00	28	0.143
100	A	10	5	1.00	28	0.179
101	A	13	6	1.00	28	0.214
102	A	13	9	1.00	28	0.321
103	A	9	6	1.00	28	0.214
104	A	6	4	1.00	28	0.143
105	A	6	4	1.00	26	0.154
106	A	6	4	1.00	25	0.160
107	A	12	7	1.00	28	0.250
108	A	12	7	1.00	28	0.250
109	A	9	6	1.00	30	0.200
110	A	9	6	1.00	28	0.214
111	A	8	5	1.00	27	0.185
112	A	17	9	1.00	30	0.300
113	A	23	10	1.00	30	0.333
114	A	12	6	1.00	30	0.200
115	A	8	5	1.00	30	0.167
116	A	5	3	1.00	28	0.107
117	A	5	3	1.00	27	0.111
118	A	9	4	1.00	30	0.133
119	A	12	5	1.00	30	0.167
120	A	16	7	1.00	30	0.233
121	A	10	7	1.00	30	0.233
122	A	6	4	1.00	30	0.133
123	A	6	4	1.00	28	0.143
124	A	6	4	1.00	27	0.148
125	A	12	7	1.00	30	0.233
126	A	24	14	1.00	30	0.467
127	A	20	13	1.00	30	0.433
128	A	16	12	1.00	30	0.400
129	A	6	4	1.01	28	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
130	A	10	8	1.00	27	0.296
131	A	17	11	1.00	30	0.367
132	A	20	12	1.00	30	0.400
133	A	8	6	1.00	34	0.176
134	A	6	5	1.00	30	0.167
135	A	3	3	1.00	34	0.088
136	A	5	5	1.00	34	0.147
137	A	7	7	1.00	34	0.206
138	A	1	1	1.00	17	0.059
139	A	1	1	1.00	15	0.067
140	A	2	2	1.00	23	0.087
141	A	3	3	1.00	21	0.143
142	A	2	2	1.00	40	0.050
143	A	2	2	1.00	104	0.019

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int \frac{(A+Bx)(a+bx+cx^2)}{d+fx^2} dx$

Optimal result

Integrand size = 25, antiderivative size = 94

$$\int \frac{(A+Bx)(a+bx+cx^2)}{d+fx^2} dx = \frac{(bB+Ac)x}{f} + \frac{Bcx^2}{2f} - \frac{(bBd+Acd-aAf)\arctan\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)}{\sqrt{d}f^{3/2}} - \frac{(Bcd-Abf-aBf)\log(d+fx^2)}{2f^2}$$

```
[Out] (A*c+B*b)*x/f+1/2*B*c*x^2/f-1/2*(-A*b*f-B*a*f+B*c*d)*ln(f*x^2+d)/f^2-(-A*a*
f+A*c*d+B*b*d)*arctan(x*f^(1/2)/d^(1/2))/f^(3/2)/d^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1643, 649, 211, 266}

$$\int \frac{(A+Bx)(a+bx+cx^2)}{d+fx^2} dx = -\frac{\arctan\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)(-aAf+Acd+bBd)}{\sqrt{d}f^{3/2}} - \frac{\log(d+fx^2)(-aBf-Abf+Bcd)}{2f^2} + \frac{x(Ac+bB)}{f} + \frac{Bcx^2}{2f}$$

```
[In] Int[((A + B*x)*(a + b*x + c*x^2))/(d + f*x^2),x]
```

```
[Out] ((b*B + A*c)*x)/f + (B*c*x^2)/(2*f) - ((b*B*d + A*c*d - a*A*f)*ArcTan[(Sqrt
[f]*x)/Sqrt[d]])/(Sqrt[d]*f^(3/2)) - ((B*c*d - A*b*f - a*B*f)*Log[d + f*x^2
])/(2*f^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{bB+Ac}{f}+\frac{Bcx}{f}-\frac{bBd+Acd-aAf+(Bcd-Abf-aBf)x}{f\left(d+fx^2\right)}\right)dx \\
&= \frac{(bB+Ac)x}{f}+\frac{Bcx^2}{2f}-\frac{\int \frac{bBd+Acd-aAf+(Bcd-Abf-aBf)x}{d+fx^2}\,dx}{f} \\
&= \frac{(bB+Ac)x}{f}+\frac{Bcx^2}{2f}-\frac{(bBd+Acd-aAf)\int \frac{1}{d+fx^2}\,dx}{f}-\frac{(Bcd-Abf-aBf)\int \frac{x}{d+fx^2}\,dx}{f} \\
&= \frac{(bB+Ac)x}{f}+\frac{Bcx^2}{2f}-\frac{(bBd+Acd-aAf)\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)}{\sqrt{d}f^{3/2}} \\
&\quad -\frac{(Bcd-Abf-aBf)\log\left(d+fx^2\right)}{2f^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.91

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{d+fx^2}\,dx = \frac{fx(2bB+2Ac+Bcx)-\frac{2\sqrt{f}(bBd+Acd-aAf)\arctan\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)}{\sqrt{d}}+(-Bcd+Abf+aBf)\log\left(d+fx^2\right)}{2f^2}$$

```
[In] Integrate[((A + B*x)*(a + b*x + c*x^2))/(d + f*x^2),x]
```

```
[Out] (f*x*(2*b*B + 2*A*c + B*c*x) - (2*Sqrt[f]*(b*B*d + A*c*d - a*A*f)*ArcTan[(S
qrt[f]*x)/Sqrt[d]])/Sqrt[d] + (-(B*c*d) + A*b*f + a*B*f)*Log[d + f*x^2])/(2
*f^2)
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.89

method	result
default	$\frac{\frac{1}{2}Bc\,x^2+Acx+Bbx}{f}+\frac{\frac{(Abf+Baf-Bcd)\ln\left(f\,x^2+d\right)}{2f}+\frac{(Aaf-Acd-Bbd)\arctan\left(\frac{fx}{\sqrt{df}}\right)}{\sqrt{df}}}{f}$
risch	$\frac{Bc\,x^2}{2f}+\frac{Acx}{f}+\frac{Bbx}{f}+\frac{\ln\left(Aafd-Ac\,d^2-Bb\,d^2-\sqrt{-df(Aaf-Acd-Bbd)^2}\,x\right)Ab}{2f}+\frac{\ln\left(Aafd-Ac\,d^2-Bb\,d^2-\sqrt{-df(Aaf-Acd}\right.}{2f}$

```
[In] int((B*x+A)*(c*x^2+b*x+a)/(f*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/2*B*c*x^2+A*c*x+B*b*x)+1/f*(1/2*(A*b*f+B*a*f-B*c*d)/f*ln(f*x^2+d)+(A
*a*f-A*c*d-B*b*d)/(d*f)^(1/2)*arctan(f*x/(d*f)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 200, normalized size of antiderivative = 2.13

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{d+fx^2}\,dx = \left[\frac{Bcdfx^2+2\,(Bb+Ac)dfx-(Aaf-(Bb+Ac)d)\sqrt{-df}\log\left(\frac{fx^2-2\,\sqrt{-df}x-d}{fx^2+d}\right)-\left(Bcd^2-(Ba+Ab)df\right)\log}{2\,df^2}\right.$$

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)/(f*x^2+d),x, algorithm="fricas")
```

```
[Out] [1/2*(B*c*d*f*x^2 + 2*(B*b + A*c)*d*f*x - (A*a*f - (B*b + A*c)*d)*sqrt(-d*f
)*log((f*x^2 - 2*sqrt(-d*f)*x - d)/(f*x^2 + d)) - (B*c*d^2 - (B*a + A*b)*d*
f)*log(f*x^2 + d))/(d*f^2), 1/2*(B*c*d*f*x^2 + 2*(B*b + A*c)*d*f*x + 2*(A*a
*f - (B*b + A*c)*d)*sqrt(d*f)*arctan(sqrt(d*f)*x/d) - (B*c*d^2 - (B*a + A*b
)*d*f)*log(f*x^2 + d))/(d*f^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 333 vs. $2(90) = 180$.

Time = 0.75 (sec) , antiderivative size = 333, normalized size of antiderivative = 3.54

$$\int \frac{(A+Bx)(a+bx+cx^2)}{d+fx^2}\,dx = \frac{Bcx^2}{2f} + x\left(\frac{Ac}{f}+\frac{Bb}{f}\right) + \left(\frac{Abf+Baf-Bcd}{2f^2} - \frac{\sqrt{-df^5}(Aaf-Acd-Bbd)}{2df^4}\right)\log\left(x + \frac{-Abdf-Badf+Bcd^2+2df^2\left(\frac{Abf+Baf-Bcd}{2f^2} - \frac{\sqrt{-df^5}(Aaf-Acd-}{2df^4}\right.}{Aaf^2-Acdf-Bbdf}\right. + \left(\frac{Abf+Baf-Bcd}{2f^2} + \frac{\sqrt{-df^5}(Aaf-Acd-Bbd)}{2df^4}\right)\log\left(x + \frac{-Abdf-Badf+Bcd^2+2df^2\left(\frac{Abf+Baf-Bcd}{2f^2} + \frac{\sqrt{-df^5}(Aaf-Acd-}{2df^4}\right.}{Aaf^2-Acdf-Bbdf}\right.$$

```
[In] integrate((B*x+A)*(c*x**2+b*x+a)/(f*x**2+d),x)
```

```
[Out] B*c*x**2/(2*f) + x*(A*c/f + B*b/f) + ((A*b*f + B*a*f - B*c*d)/(2*f**2) - sq
rt(-d*f**5)*(A*a*f - A*c*d - B*b*d)/(2*d*f**4))*log(x + (-A*b*d*f - B*a*d*f
 + B*c*d**2 + 2*d*f**2*((A*b*f + B*a*f - B*c*d)/(2*f**2) - sqrt(-d*f**5)*(A
*a*f - A*c*d - B*b*d)/(2*d*f**4)))/(A*a*f**2 - A*c*d*f - B*b*d*f)) + ((A*b*
f + B*a*f - B*c*d)/(2*f**2) + sqrt(-d*f**5)*(A*a*f - A*c*d - B*b*d)/(2*d*f*
*4))*log(x + (-A*b*d*f - B*a*d*f + B*c*d**2 + 2*d*f**2*((A*b*f + B*a*f - B*
c*d)/(2*f**2) + sqrt(-d*f**5)*(A*a*f - A*c*d - B*b*d)/(2*d*f**4)))/(A*a*f**
2 - A*c*d*f - B*b*d*f))
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.89

$$\int \frac{(A+Bx)(a+bx+cx^2)}{d+fx^2}\,dx = \frac{(Aaf-(Bb+Ac)d)\arctan\left(\frac{fx}{\sqrt{df}}\right)}{\sqrt{df}\,f} + \frac{Bcx^2+2\,(Bb+Ac)x}{2\,f} - \frac{(Bcd-(Ba+Ab)f)\log\left(fx^2+d\right)}{2\,f^2}$$

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)/(f*x^2+d),x, algorithm="maxima")
```

```
[Out] (A*a*f - (B*b + A*c)*d)*arctan(f*x/sqrt(d*f))/(sqrt(d*f)*f) + 1/2*(B*c*x^2
+ 2*(B*b + A*c)*x)/f - 1/2*(B*c*d - (B*a + A*b)*f)*log(f*x^2 + d)/f^2
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{d+fx^2}\,dx = -\frac{(Bbd+Acd-Aaf)\arctan\left(\frac{fx}{\sqrt{df}}\right)}{\sqrt{df}f} - \frac{(Bcd-Baf-Abf)\log\left(fx^2+d\right)}{2f^2} + \frac{Bcfx^2+2Bbfx+2Acfx}{2f^2}$$

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)/(f*x^2+d),x, algorithm="giac")
```

```
[Out] -(B*b*d + A*c*d - A*a*f)*arctan(f*x/sqrt(d*f))/(sqrt(d*f)*f) - 1/2*(B*c*d -
 B*a*f - A*b*f)*log(f*x^2 + d)/f^2 + 1/2*(B*c*f*x^2 + 2*B*b*f*x + 2*A*c*f*x
)/f^2
```

Mupad [B] (verification not implemented)

Time = 12.89 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.03

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{d+fx^2}\,dx = \frac{x\,(A\,c+B\,b)}{f} - \frac{\mathrm{atan}\left(\frac{\sqrt{f}\,x}{\sqrt{d}}\right)\,(A\,c\,d-A\,a\,f+B\,b\,d)}{\sqrt{d}\,f^{3/2}} + \frac{B\,c\,x^2}{2\,f} + \frac{\ln\left(f\,x^2+d\right)\,\left(4\,A\,b\,d\,f^3+4\,B\,a\,d\,f^3-4\,B\,c\,d^2\,f^2\right)}{8\,d\,f^4}$$

```
[In] int(((A + B*x)*(a + b*x + c*x^2))/(d + f*x^2),x)
```

```
[Out] (x*(A*c + B*b))/f - (atan((f^(1/2)*x)/d^(1/2))*(A*c*d - A*a*f + B*b*d))/(d^
(1/2)*f^(3/2)) + (B*c*x^2)/(2*f) + (log(d + f*x^2)*(4*A*b*d*f^3 + 4*B*a*d*f
^3 - 4*B*c*d^2*f^2))/(8*d*f^4)
```

3.2 $\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{d+fx^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 228

$$
\begin{aligned}
\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{d+fx^2}\,dx
&= \frac{(Ab^2f-Ac(cd-2af)-bB(2cd-2af))\,x}{f^2}+\frac{(2Abcf-B(c^2d-b^2f-2acf))\,x^2}{2f^2} \\
&\quad+\frac{c(2bB+Ac)x^3}{3f}+\frac{Bc^2x^4}{4f}-\frac{(Ab^2df-2bBd(cd-af)-A(cd-af)^2)\arctan\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)}{\sqrt{d}f^{5/2}} \\
&\quad-\frac{(2Abf(cd-af)-B(c^2d^2-2acdf-f(b^2d-a^2f)))\log\left(d+fx^2\right)}{2f^3}
\end{aligned}
$$

```
[Out] (A*b^2*f-A*c*(-2*a*f+c*d)-b*B*(-2*a*f+2*c*d))*x/f^2+1/2*(2*A*b*c*f-B*(-2*a*
c*f-b^2*f+c^2*d))*x^2/f^2+1/3*c*(A*c+2*B*b)*x^3/f+1/4*B*c^2*x^4/f-1/2*(2*A*
b*f*(-a*f+c*d)-B*(c^2*d^2-2*a*c*d*f-f*(-a^2*f+b^2*d)))*ln(f*x^2+d)/f^3-(A*b
^2*d*f-2*b*B*d*(-a*f+c*d)-A*(-a*f+c*d)^2)*arctan(x*f^(1/2)/d^(1/2))/f^(5/2)
/d^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used

$= \{1026, 649, 211, 266\}$

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{d+fx^2}\,dx$$
$$= -\frac{\log\left(d+fx^2\right)\left(2Abf(cd-af)-B(-f(b^2d-a^2f)-2acdf+c^2d^2)\right)}{2f^3}$$
$$-\frac{\arctan\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)\left(-A(cd-af)^2-2bBd(cd-af)+Ab^2df\right)}{\sqrt{d}f^{5/2}}$$
$$+\frac{x^2(2Abcf-B(-2acf+b^2(-f)+c^2d))}{2f^2}$$
$$+\frac{x(-Ac(cd-2af)-bB(2cd-2af)+Ab^2f)}{f^2}+\frac{cx^3(Ac+2bB)}{3f}+\frac{Bc^2x^4}{4f}$$

```
[In] Int[((A + B*x)*(a + b*x + c*x^2)^2)/(d + f*x^2),x]
```

```
[Out] ((A*b^2*f - A*c*(c*d - 2*a*f) - b*B*(2*c*d - 2*a*f))*x)/f^2 + ((2*A*b*c*f -
 B*(c^2*d - b^2*f - 2*a*c*f))*x^2)/(2*f^2) + (c*(2*b*B + A*c)*x^3)/(3*f) +
(B*c^2*x^4)/(4*f) - ((A*b^2*d*f - 2*b*B*d*(c*d - a*f) - A*(c*d - a*f)^2)*Ar
cTan[(Sqrt[f]*x)/Sqrt[d]])/(Sqrt[d]*f^(5/2)) - ((2*A*b*f*(c*d - a*f) - B*(c
^2*d^2 - 2*a*c*d*f - f*(b^2*d - a^2*f)))*Log[d + f*x^2])/(2*f^3)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1026

```
Int[((g_.) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + c*x^2)^p*(d + e*x +
 f*x^2)^q*(g + h*x), x], x] /; FreeQ[{a, c, d, e, f, g, h}, x] && NeQ[e^2 -
 4*d*f, 0] && IntegersQ[p, q] && (GtQ[p, 0] || GtQ[q, 0])
```

Rubi steps

$$\text{integral} = \int \left(\frac{Ab^2f - Ac(cd-2af) - bB(2cd-2af)}{f^2} + \frac{(2Abcf - B(c^2d-b^2f-2acf))\,x}{f^2} + \frac{c(2bB+Ac)x^2}{f} + \frac{Bc^2x^3}{f} + \frac{-Ab^2df + 2bBd(cd-af) + A(cd-af)^2 - (2Abf(cd-af) - B(c^2d^2-2acdf-f(b^2d-a^2f)))\,x}{f^2\,(d+fx^2)}\right) dx$$

$$= \frac{(Ab^2f - Ac(cd-2af) - bB(2cd-2af))\,x}{f^2} + \frac{(2Abcf - B(c^2d-b^2f-2acf))\,x^2}{2f^2} + \frac{c(2bB+Ac)x^3}{3f} + \frac{Bc^2x^4}{4f} + \frac{\int \frac{-Ab^2df+2bBd(cd-af)+A(cd-af)^2-(2Abf(cd-af)-B(c^2d^2-2acdf-f(b^2d-a^2f)))x}{d+fx^2}\,dx}{f^2}$$

$$= \frac{(Ab^2f - Ac(cd-2af) - bB(2cd-2af))\,x}{f^2} + \frac{(2Abcf - B(c^2d-b^2f-2acf))\,x^2}{2f^2} + \frac{c(2bB+Ac)x^3}{3f} + \frac{Bc^2x^4}{4f} - \frac{(Ab^2df - 2bBd(cd-af) - A(cd-af)^2)\int \frac{1}{d+fx^2}\,dx}{f^2} - \frac{(2Abf(cd-af) - B(c^2d^2-2acdf-f(b^2d-a^2f)))\int \frac{x}{d+fx^2}\,dx}{f^2}$$

$$= \frac{(Ab^2f - Ac(cd-2af) - bB(2cd-2af))\,x}{f^2} + \frac{(2Abcf - B(c^2d-b^2f-2acf))\,x^2}{2f^2} + \frac{c(2bB+Ac)x^3}{3f} + \frac{Bc^2x^4}{4f} - \frac{(Ab^2df - 2bBd(cd-af) - A(cd-af)^2)\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)}{\sqrt{d}f^{5/2}} - \frac{(2Abf(cd-af) - B(c^2d^2-2acdf-f(b^2d-a^2f)))\log(d+fx^2)}{2f^3}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.89

$$\int \frac{(A+Bx)\,(a+bx+cx^2)^2}{d+fx^2}\,dx = \frac{(-Ab^2df + 2bBd(cd-af) + A(cd-af)^2)\arctan\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)}{\sqrt{d}f^{5/2}} + \frac{fx(12Abcfx + 6b^2f(2A+Bx) + 3Bcx(-2cd+4af+cfx^2) + 4Ac(-3cd+6af+cfx^2) + 4bB(-6cd}{12f^3}$$

```
[In] Integrate[((A + B*x)*(a + b*x + c*x^2)^2)/(d + f*x^2),x]
```

[Out] ((-(A*b^2*d*f) + 2*b*B*d*(c*d - a*f) + A*(c*d - a*f)^2)*ArcTan[(Sqrt[f]*x)/
Sqrt[d]])/(Sqrt[d]*f^(5/2)) + (f*x*(12*A*b*c*f*x + 6*b^2*f*(2*A + B*x) + 3*
B*c*x*(-2*c*d + 4*a*f + c*f*x^2) + 4*A*c*(-3*c*d + 6*a*f + c*f*x^2) + 4*b*B
*(-6*c*d + 6*a*f + 2*c*f*x^2)) + 6*(2*A*b*f*(-(c*d) + a*f) + B*(c^2*d^2 - b
^2*d*f - 2*a*c*d*f + a^2*f^2))*Log[d + f*x^2])/(12*f^3)

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.03

method	result
default	$\frac{\frac{1}{4}B c^2x^4f+\frac{1}{3}A c^2f x^3+\frac{2}{3}Bbcf x^3+Abcf x^2+Bacf x^2+\frac{1}{2}B b^2f x^2-\frac{1}{2}B c^2d x^2+2Aacfx+A b^2fx-A c^2dx+2Babfx-2Bbcdx}{f^2}+\frac{(2A\ldots}{\ldots}$
risch	Expression too large to display

[In] int((B*x+A)*(c*x^2+b*x+a)^2/(f*x^2+d),x,method=_RETURNVERBOSE)

[Out] 1/f^2*(1/4*B*c^2*x^4*f+1/3*A*c^2*f*x^3+2/3*B*b*c*f*x^3+A*b*c*f*x^2+B*a*c*f*
x^2+1/2*B*b^2*f*x^2-1/2*B*c^2*d*x^2+2*A*a*c*f*x+A*b^2*f*x-A*c^2*d*x+2*B*a*b
*f*x-2*B*b*c*d*x)+1/f^2*(1/2*(2*A*a*b*f^2-2*A*b*c*d*f+B*a^2*f^2-2*B*a*c*d*f
-B*b^2*d*f+B*c^2*d^2)/f*ln(f*x^2+d)+(A*a^2*f^2-2*A*a*c*d*f-A*b^2*d*f+A*c^2*
d^2-2*B*a*b*d*f+2*B*b*c*d^2)/(d*f)^(1/2)*arctan(f*x/(d*f)^(1/2)))

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 500, normalized size of antiderivative = 2.19

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{d+fx^2}\,dx$$
$$=\left[\frac{3\,Bc^2df^2x^4+4\,(2\,Bbc+Ac^2)df^2x^3-6\,(Bc^2d^2f-(Bb^2+2\,(Ba+Ab)c)df^2)x^2-6\,(Aa^2f^2+(2\,Bbc+A\ldots}{\ldots}\right.$$

[In] integrate((B*x+A)*(c*x^2+b*x+a)^2/(f*x^2+d),x, algorithm="fricas")

[Out] [1/12*(3*B*c^2*d*f^2*x^4 + 4*(2*B*b*c + A*c^2)*d*f^2*x^3 - 6*(B*c^2*d^2*f -
 (B*b^2 + 2*(B*a + A*b)*c)*d*f^2)*x^2 - 6*(A*a^2*f^2 + (2*B*b*c + A*c^2)*d^
2 - (2*B*a*b + A*b^2 + 2*A*a*c)*d*f)*sqrt(-d*f)*log((f*x^2 - 2*sqrt(-d*f)*x
 - d)/(f*x^2 + d)) - 12*((2*B*b*c + A*c^2)*d^2*f - (2*B*a*b + A*b^2 + 2*A*a
*c)*d*f^2)*x + 6*(B*c^2*d^3 - (B*b^2 + 2*(B*a + A*b)*c)*d^2*f + (B*a^2 + 2*
A*a*b)*d*f^2)*log(f*x^2 + d))/(d*f^3), 1/12*(3*B*c^2*d*f^2*x^4 + 4*(2*B*b*c
 + A*c^2)*d*f^2*x^3 - 6*(B*c^2*d^2*f - (B*b^2 + 2*(B*a + A*b)*c)*d*f^2)*x^2
 + 12*(A*a^2*f^2 + (2*B*b*c + A*c^2)*d^2 - (2*B*a*b + A*b^2 + 2*A*a*c)*d*f)

```
*sqrt(d*f)*arctan(sqrt(d*f)*x/d) - 12*((2*B*b*c + A*c^2)*d^2*f - (2*B*a*b +
 A*b^2 + 2*A*a*c)*d*f^2)*x + 6*(B*c^2*d^3 - (B*b^2 + 2*(B*a + A*b)*c)*d^2*f
 + (B*a^2 + 2*A*a*b)*d*f^2)*log(f*x^2 + d))/(d*f^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 933 vs. 2(209) = 418.

Time = 15.56 (sec) , antiderivative size = 933, normalized size of antiderivative = 4.09

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{d+fx^2}\,dx = \frac{Bc^2x^4}{4f} + x^3\left(\frac{Ac^2}{3f} + \frac{2Bbc}{3f}\right) + x^2\left(\frac{Abc}{f} + \frac{Bac}{f} + \frac{Bb^2}{2f} - \frac{Bc^2d}{2f^2}\right) + x\left(\frac{2Aac}{f} + \frac{Ab^2}{f} - \frac{Ac^2d}{f^2} + \frac{2Bab}{f} - \frac{2Bbcd}{f^2}\right) + \left(\frac{2Aabf^2 - 2Abcdf + Ba^2f^2 - 2Bacdf - Bb^2df + Bc^2d^2}{2f^3} - \frac{\sqrt{-df^7}\left(Aa^2f^2 - 2Aacdf - Ab^2df + Ac^2d^2 - 2Babdf + 2Bbcd^2\right)}{2df^6}\right)\log\left(x + \frac{-2Aabdf^2 + 2Abcd^2f - B\ldots}{\ldots}\right) + \left(\frac{2Aabf^2 - 2Abcdf + Ba^2f^2 - 2Bacdf - Bb^2df + Bc^2d^2}{2f^3} + \frac{\sqrt{-df^7}\left(Aa^2f^2 - 2Aacdf - Ab^2df + Ac^2d^2 - 2Babdf + 2Bbcd^2\right)}{2df^6}\right)\log\left(x + \frac{-2Aabdf^2 + 2Abcd^2f - B\ldots}{\ldots}\right)$$

```
[In] integrate((B*x+A)*(c*x**2+b*x+a)**2/(f*x**2+d),x)
```

```
[Out] B*c**2*x**4/(4*f) + x**3*(A*c**2/(3*f) + 2*B*b*c/(3*f)) + x**2*(A*b*c/f + B
*a*c/f + B*b**2/(2*f) - B*c**2*d/(2*f**2)) + x*(2*A*a*c/f + A*b**2/f - A*c*
*2*d/f**2 + 2*B*a*b/f - 2*B*b*c*d/f**2) + ((2*A*a*b*f**2 - 2*A*b*c*d*f + B*
a**2*f**2 - 2*B*a*c*d*f - B*b**2*d*f + B*c**2*d**2)/(2*f**3) - sqrt(-d*f**7
)*(A*a**2*f**2 - 2*A*a*c*d*f - A*b**2*d*f + A*c**2*d**2 - 2*B*a*b*d*f + 2*B
*b*c*d**2)/(2*d*f**6))*log(x + (-2*A*a*b*d*f**2 + 2*A*b*c*d**2*f - B*a**2*d
*f**2 + 2*B*a*c*d**2*f + B*b**2*d**2*f - B*c**2*d**3 + 2*d*f**3*((2*A*a*b*f
**2 - 2*A*b*c*d*f + B*a**2*f**2 - 2*B*a*c*d*f - B*b**2*d*f + B*c**2*d**2)/(
2*f**3) - sqrt(-d*f**7)*(A*a**2*f**2 - 2*A*a*c*d*f - A*b**2*d*f + A*c**2*d*
*2 - 2*B*a*b*d*f + 2*B*b*c*d**2)/(2*d*f**6)))/(A*a**2*f**3 - 2*A*a*c*d*f**2
 - A*b**2*d*f**2 + A*c**2*d**2*f - 2*B*a*b*d*f**2 + 2*B*b*c*d**2*f)) + ((2*
A*a*b*f**2 - 2*A*b*c*d*f + B*a**2*f**2 - 2*B*a*c*d*f - B*b**2*d*f + B*c**2*
d**2)/(2*f**3) + sqrt(-d*f**7)*(A*a**2*f**2 - 2*A*a*c*d*f - A*b**2*d*f + A*
c**2*d**2 - 2*B*a*b*d*f + 2*B*b*c*d**2)/(2*d*f**6))*log(x + (-2*A*a*b*d*f**
2 + 2*A*b*c*d**2*f - B*a**2*d*f**2 + 2*B*a*c*d**2*f + B*b**2*d**2*f - B*c**
2*d**3 + 2*d*f**3*((2*A*a*b*f**2 - 2*A*b*c*d*f + B*a**2*f**2 - 2*B*a*c*d*f
- B*b**2*d*f + B*c**2*d**2)/(2*f**3) + sqrt(-d*f**7)*(A*a**2*f**2 - 2*A*a*c
```

```
*d*f - A*b**2*d*f + A*c**2*d**2 - 2*B*a*b*d*f + 2*B*b*c*d**2)/(2*d*f**6)))/
(A*a**2*f**3 - 2*A*a*c*d*f**2 - A*b**2*d*f**2 + A*c**2*d**2*f - 2*B*a*b*d*f
**2 + 2*B*b*c*d**2*f))
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.96

$$\int \frac{(A+Bx)(a+bx+cx^2)^2}{d+fx^2}\,dx$$
$$= \frac{(Aa^2f^2+(2\,Bbc+Ac^2)d^2-(2\,Bab+Ab^2+2\,Aac)df)\arctan\left(\frac{fx}{\sqrt{df}}\right)}{\sqrt{df}\,f^2}$$
$$+ \frac{3\,Bc^2fx^4+4\,(2\,Bbc+Ac^2)fx^3-6\,(Bc^2d-(Bb^2+2\,(Ba+Ab)c)f)x^2-12\,((2\,Bbc+Ac^2)d-(2\,Bab}{12\,f^2}$$
$$+ \frac{(Bc^2d^2-(Bb^2+2\,(Ba+Ab)c)df+(Ba^2+2\,Aab)f^2)\log(fx^2+d)}{2\,f^3}$$

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)^2/(f*x^2+d),x, algorithm="maxima")
```

```
[Out] (A*a^2*f^2 + (2*B*b*c + A*c^2)*d^2 - (2*B*a*b + A*b^2 + 2*A*a*c)*d*f)*arcta
n(f*x/sqrt(d*f))/(sqrt(d*f)*f^2) + 1/12*(3*B*c^2*f*x^4 + 4*(2*B*b*c + A*c^2
)*f*x^3 - 6*(B*c^2*d - (B*b^2 + 2*(B*a + A*b)*c)*f)*x^2 - 12*((2*B*b*c + A*
c^2)*d - (2*B*a*b + A*b^2 + 2*A*a*c)*f)*x)/f^2 + 1/2*(B*c^2*d^2 - (B*b^2 +
2*(B*a + A*b)*c)*d*f + (B*a^2 + 2*A*a*b)*f^2)*log(f*x^2 + d)/f^3
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.15

$$\int \frac{(A+Bx)(a+bx+cx^2)^2}{d+fx^2}\,dx$$
$$= \frac{(2\,Bbcd^2+Ac^2d^2-2\,Babdf-Ab^2df-2\,Aacdf+Aa^2f^2)\arctan\left(\frac{fx}{\sqrt{df}}\right)}{\sqrt{df}\,f^2}$$
$$+ \frac{(Bc^2d^2-Bb^2df-2\,Bacdf-2\,Abcdf+Ba^2f^2+2\,Aabf^2)\log(fx^2+d)}{2\,f^3}$$
$$+ \frac{3\,Bc^2f^3x^4+8\,Bbcf^3x^3+4\,Ac^2f^3x^3-6\,Bc^2df^2x^2+6\,Bb^2f^3x^2+12\,Bacf^3x^2+12\,Abcf^3x^2-24\,Bbcdf^2}{12\,f^4}$$

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)^2/(f*x^2+d),x, algorithm="giac")
```

```
[Out] (2*B*b*c*d^2 + A*c^2*d^2 - 2*B*a*b*d*f - A*b^2*d*f - 2*A*a*c*d*f + A*a^2*f^
2)*arctan(f*x/sqrt(d*f))/(sqrt(d*f)*f^2) + 1/2*(B*c^2*d^2 - B*b^2*d*f - 2*B
*a*c*d*f - 2*A*b*c*d*f + B*a^2*f^2 + 2*A*a*b*f^2)*log(f*x^2 + d)/f^3 + 1/12
*(3*B*c^2*f^3*x^4 + 8*B*b*c*f^3*x^3 + 4*A*c^2*f^3*x^3 - 6*B*c^2*d*f^2*x^2 +
 6*B*b^2*f^3*x^2 + 12*B*a*c*f^3*x^2 + 12*A*b*c*f^3*x^2 - 24*B*b*c*d*f^2*x -
 12*A*c^2*d*f^2*x + 24*B*a*b*f^3*x + 12*A*b^2*f^3*x + 24*A*a*c*f^3*x)/f^4
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.11

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{d+fx^2}\,dx = x\left(\frac{A\,b^2+2\,B\,a\,b+2\,A\,a\,c}{f}-\frac{d\,(A\,c^2+2\,B\,b\,c)}{f^2}\right) + x^2\left(\frac{B\,b^2+2\,A\,c\,b+2\,B\,a\,c}{2\,f}-\frac{B\,c^2\,d}{2\,f^2}\right)+\frac{x^3\,(A\,c^2+2\,B\,b\,c)}{3\,f}+\frac{B\,c^2\,x^4}{4\,f} + \frac{\mathrm{atan}\left(\frac{\sqrt{f}\,x}{\sqrt{d}}\right)\,\left(A\,a^2\,f^2-2\,B\,a\,b\,d\,f-2\,A\,a\,c\,d\,f-A\,b^2\,d\,f+2\,B\,b\,c\,d^2+A\,c^2\,d^2\right)}{\sqrt{d}\,f^{5/2}} + \frac{\ln\left(f\,x^2+d\right)\,\left(4\,B\,a^2\,d\,f^5+8\,A\,a\,b\,d\,f^5-8\,B\,a\,c\,d^2\,f^4-4\,B\,b^2\,d^2\,f^4-8\,A\,b\,c\,d^2\,f^4+4\,B\,c^2\,d^3\,f^3\right)}{8\,d\,f^6}$$

```
[In] int(((A + B*x)*(a + b*x + c*x^2)^2)/(d + f*x^2),x)
```

```
[Out] x*((A*b^2 + 2*A*a*c + 2*B*a*b)/f - (d*(A*c^2 + 2*B*b*c))/f^2) + x^2*((B*b^2
 + 2*A*b*c + 2*B*a*c)/(2*f) - (B*c^2*d)/(2*f^2)) + (x^3*(A*c^2 + 2*B*b*c))/
(3*f) + (B*c^2*x^4)/(4*f) + (atan((f^(1/2)*x)/d^(1/2))*(A*a^2*f^2 + A*c^2*d
^2 + 2*B*b*c*d^2 - A*b^2*d*f - 2*A*a*c*d*f - 2*B*a*b*d*f))/(d^(1/2)*f^(5/2)
) + (log(d + f*x^2)*(4*B*a^2*d*f^5 - 4*B*b^2*d^2*f^4 + 4*B*c^2*d^3*f^3 + 8*
A*a*b*d*f^5 - 8*A*b*c*d^2*f^4 - 8*B*a*c*d^2*f^4))/(8*d*f^6)
```

3.3 $\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{d+fx^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 441

$$\begin{aligned}
&\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{d+fx^2}\,dx \\
&= -\frac{(b^3Bdf+3Ab^2f(cd-af)-3bB(cd-af)^2-Ac(c^2d^2-3acdf+3a^2f^2))\,x}{f^3} \\
&\quad -\frac{(Abf(3c^2d-b^2f-6acf)-B(c^3d^2-3ac^2df+3ab^2f^2-3cf(b^2d-a^2f)))\,x^2}{2f^3} \\
&\quad +\frac{(b^3Bf+3Ab^2cf-Ac^2(cd-3af)-3bBc(cd-2af))\,x^3}{3f^2} \\
&\quad +\frac{c(3Abcf-B(c^2d-3b^2f-3acf))\,x^4}{4f^2}+\frac{c^2(3bB+Ac)x^5}{5f}+\frac{Bc^3x^6}{6f} \\
&\quad +\frac{(b^3Bd^2f+3Ab^2df(cd-af)-3bBd(cd-af)^2-A(cd-af)^3)\arctan\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)}{\sqrt{d}f^{7/2}} \\
&\quad +\frac{(Abf(3c^2d^2-6acdf-f(b^2d-3a^2f))-B(cd-af)\left(c^2d^2-2acdf-f(3b^2d-a^2f)\right))\log\left(d+fx^2\right)}{2f^4}
\end{aligned}$$

```
[Out] -(b^3*B*d*f+3*A*b^2*f*(-a*f+c*d)-3*b*B*(-a*f+c*d)^2-A*c*(3*a^2*f^2-3*a*c*d*
f+c^2*d^2))*x/f^3-1/2*(A*b*f*(-6*a*c*f-b^2*f+3*c^2*d)-B*(c^3*d^2-3*a*c^2*d*
f+3*a*b^2*f^2-3*c*f*(-a^2*f+b^2*d)))*x^2/f^3+1/3*(b^3*B*f+3*A*b^2*c*f-A*c^2
*(-3*a*f+c*d)-3*b*B*c*(-2*a*f+c*d))*x^3/f^2+1/4*c*(3*A*b*c*f-B*(-3*a*c*f-3*
b^2*f+c^2*d))*x^4/f^2+1/5*c^2*(A*c+3*B*b)*x^5/f+1/6*B*c^3*x^6/f+1/2*(A*b*f*
(3*c^2*d^2-6*a*c*d*f-f*(-3*a^2*f+b^2*d))-B*(-a*f+c*d)*(c^2*d^2-2*a*c*d*f-f*
(-a^2*f+3*b^2*d)))*ln(f*x^2+d)/f^4+(b^3*B*d^2*f+3*A*b^2*d*f*(-a*f+c*d)-3*b*
B*d*(-a*f+c*d)^2-A*(-a*f+c*d)^3)*arctan(x*f^(1/2)/d^(1/2))/f^(7/2)/d^(1/2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 441, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1026, 649, 211, 266}

$$\int \frac{(A+Bx)(a+bx+cx^2)^3}{d+fx^2}\,dx$$
$$=\frac{\log(d+fx^2)\left(Abf(-f(b^2d-3a^2f)-6acdf+3c^2d^2)-B(cd-af)(-f(3b^2d-a^2f)-2acdf+c^2d^2)\right)}{2f^4}$$
$$-\frac{x^2(Abf(-6acf+b^2(-f)+3c^2d)-B(-3cf(b^2d-a^2f)+3ab^2f^2-3ac^2df+c^3d^2))}{2f^3}$$
$$-\frac{x(-Ac(3a^2f^2-3acdf+c^2d^2)+3Ab^2f(cd-af)-3bB(cd-af)^2+b^3Bdf)}{f^3}$$
$$+\frac{\arctan\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)(3Ab^2df(cd-af)-A(cd-af)^3-3bBd(cd-af)^2+b^3Bd^2f)}{\sqrt{d}f^{7/2}}$$
$$+\frac{cx^4(3Abcf-B(-3acf-3b^2f+c^2d))}{4f^2}$$
$$+\frac{x^3(-Ac^2(cd-3af)-3bBc(cd-2af)+3Ab^2cf+b^3Bf)}{3f^2}+\frac{c^2x^5(Ac+3bB)}{5f}+\frac{Bc^3x^6}{6f}$$

```
[In] Int[((A + B*x)*(a + b*x + c*x^2)^3)/(d + f*x^2),x]
```

```
[Out] -(((b^3*B*d*f + 3*A*b^2*f*(c*d - a*f) - 3*b*B*(c*d - a*f)^2 - A*c*(c^2*d^2
 - 3*a*c*d*f + 3*a^2*f^2))*x)/f^3) - ((A*b*f*(3*c^2*d - b^2*f - 6*a*c*f) - B
*(c^3*d^2 - 3*a*c^2*d*f + 3*a*b^2*f^2 - 3*c*f*(b^2*d - a^2*f)))*x^2)/(2*f^3
) + ((b^3*B*f + 3*A*b^2*c*f - A*c^2*(c*d - 3*a*f) - 3*b*B*c*(c*d - 2*a*f))*
x^3)/(3*f^2) + (c*(3*A*b*c*f - B*(c^2*d - 3*b^2*f - 3*a*c*f))*x^4)/(4*f^2)
+ (c^2*(3*b*B + A*c)*x^5)/(5*f) + (B*c^3*x^6)/(6*f) + ((b^3*B*d^2*f + 3*A*b
^2*d*f*(c*d - a*f) - 3*b*B*d*(c*d - a*f)^2 - A*(c*d - a*f)^3)*ArcTan[(Sqrt[
f]*x)/Sqrt[d]])/(Sqrt[d]*f^(7/2)) + ((A*b*f*(3*c^2*d^2 - 6*a*c*d*f - f*(b^2
*d - 3*a^2*f)) - B*(c*d - a*f)*(c^2*d^2 - 2*a*c*d*f - f*(3*b^2*d - a^2*f)))
*Log[d + f*x^2])/(2*f^4)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1026

```
Int[((g_.) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + c*x^2)^p*(d + e*x +
 f*x^2)^q*(g + h*x), x], x] /; FreeQ[{a, c, d, e, f, g, h}, x] && NeQ[e^2 -
 4*d*f, 0] && IntegersQ[p, q] && (GtQ[p, 0] || GtQ[q, 0])
```

Rubi steps

$$\text{integral} = \int \left(-\frac{b^3Bdf + 3Ab^2f(cd-af) - 3bB(cd-af)^2 - Ac(c^2d^2 - 3acdf + 3a^2f^2)}{f^3} - \frac{(Abf(3c^2d - b^2f - 6acf) - B(c^3d^2 - 3ac^2df + 3ab^2f^2 - 3cf(b^2d - a^2f)))\,x}{f^3} + \frac{(b^3Bf + 3Ab^2cf - Ac^2(cd - 3af) - 3bBc(cd - 2af))\,x^2}{f^2} + \frac{c(3Abcf - B(c^2d - 3b^2f - 3acf))\,x^3}{f^2} + \frac{c^2(3bB + Ac)x^4}{f} + \frac{Bc^3x^5}{f} - \frac{-b^3Bd^2f - 3Ab^2df(cd-af) + 3bBd(cd-af)^2 + A(cd-af)^3 - (Abf(3c^2d^2 - 6acdf - f(b^2d - 3a^2f))}{f^3\,(d + fx^2)}$$

$$= -\frac{(b^3Bdf + 3Ab^2f(cd-af) - 3bB(cd-af)^2 - Ac(c^2d^2 - 3acdf + 3a^2f^2))\,x}{f^3} - \frac{(Abf(3c^2d - b^2f - 6acf) - B(c^3d^2 - 3ac^2df + 3ab^2f^2 - 3cf(b^2d - a^2f)))\,x^2}{2f^3} + \frac{(b^3Bf + 3Ab^2cf - Ac^2(cd - 3af) - 3bBc(cd - 2af))\,x^3}{3f^2} + \frac{c(3Abcf - B(c^2d - 3b^2f - 3acf))\,x^4}{4f^2} + \frac{c^2(3bB + Ac)x^5}{5f} + \frac{Bc^3x^6}{6f} - \frac{\int \frac{-b^3Bd^2f - 3Ab^2df(cd-af) + 3bBd(cd-af)^2 + A(cd-af)^3 - (Abf(3c^2d^2 - 6acdf - f(b^2d - 3a^2f)) - B(cd-af)(c^2d^2 - 2acdf - f(3b^2d}{d+fx^2}}{f^3}$$

$$= -\frac{(b^3Bdf + 3Ab^2f(cd - af) - 3bB(cd - af)^2 - Ac(c^2d^2 - 3acdf + 3a^2f^2))\,x}{f^3}$$
$$- \frac{(Abf(3c^2d - b^2f - 6acf) - B(c^3d^2 - 3ac^2df + 3ab^2f^2 - 3cf(b^2d - a^2f)))\,x^2}{2f^3}$$
$$+ \frac{(b^3Bf + 3Ab^2cf - Ac^2(cd - 3af) - 3bBc(cd - 2af))\,x^3}{3f^2}$$
$$+ \frac{c(3Abcf - B(c^2d - 3b^2f - 3acf))\,x^4}{4f^2} + \frac{c^2(3bB + Ac)x^5}{5f} + \frac{Bc^3x^6}{6f}$$
$$+ \frac{(b^3Bd^2f + 3Ab^2df(cd - af) - 3bBd(cd - af)^2 - A(cd - af)^3)\int \frac{1}{d+fx^2}\,dx}{f^3}$$
$$+ \frac{(Abf(3c^2d^2 - 6acdf - f(b^2d - 3a^2f)) - B(cd - af)\,(c^2d^2 - 2acdf - f(3b^2d - a^2f)))\int \frac{x}{d+fx^2}\,d}{f^3}$$

$$= -\frac{(b^3Bdf + 3Ab^2f(cd - af) - 3bB(cd - af)^2 - Ac(c^2d^2 - 3acdf + 3a^2f^2))\,x}{f^3}$$
$$- \frac{(Abf(3c^2d - b^2f - 6acf) - B(c^3d^2 - 3ac^2df + 3ab^2f^2 - 3cf(b^2d - a^2f)))\,x^2}{2f^3}$$
$$+ \frac{(b^3Bf + 3Ab^2cf - Ac^2(cd - 3af) - 3bBc(cd - 2af))\,x^3}{3f^2}$$
$$+ \frac{c(3Abcf - B(c^2d - 3b^2f - 3acf))\,x^4}{4f^2} + \frac{c^2(3bB + Ac)x^5}{5f} + \frac{Bc^3x^6}{6f}$$
$$+ \frac{(b^3Bd^2f + 3Ab^2df(cd - af) - 3bBd(cd - af)^2 - A(cd - af)^3)\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)}{\sqrt{d}f^{7/2}}$$
$$+ \frac{(Abf(3c^2d^2 - 6acdf - f(b^2d - 3a^2f)) - B(cd - af)\,(c^2d^2 - 2acdf - f(3b^2d - a^2f)))\log(d +}{2f^4}$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 422, normalized size of antiderivative = 0.96

$$\int \frac{(A + Bx)\,(a + bx + cx^2)^3}{d + fx^2}\,dx$$
$$= \frac{(b^3Bd^2f + 3Ab^2df(cd - af) - 3bBd(cd - af)^2 - A(cd - af)^3)\arctan\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)}{\sqrt{d}f^{7/2}}$$
$$+ \frac{fx(10b^3f(-6Bd + 3Afx + 2Bfx^2) + 15b^2f(3Bx(-2cd + 2af + cfx^2) + 4A(-3cd + 3af + cfx^2)) +}{}$$

```
[In] Integrate[((A + B*x)*(a + b*x + c*x^2)^3)/(d + f*x^2),x]
```

```
[Out] ((b^3*B*d^2*f + 3*A*b^2*d*f*(c*d - a*f) - 3*b*B*d*(c*d - a*f)^2 - A*(c*d -
a*f)^3)*ArcTan[(Sqrt[f]*x)/Sqrt[d]])/(Sqrt[d]*f^(7/2)) + (f*x*(10*b^3*f*(-6
*B*d + 3*A*f*x + 2*B*f*x^2) + 15*b^2*f*(3*B*x*(-2*c*d + 2*a*f + c*f*x^2) +
```

```
4*A*(-3*c*d + 3*a*f + c*f*x^2)) + 3*b*(15*A*c*f*x*(-2*c*d + 4*a*f + c*f*x^2
) + 4*B*(15*a^2*f^2 + 10*a*c*f*(-3*d + f*x^2) + c^2*(15*d^2 - 5*d*f*x^2 + 3
*f^2*x^4))) + c*(5*B*x*(18*a^2*f^2 + 9*a*c*f*(-2*d + f*x^2) + c^2*(6*d^2 -
3*d*f*x^2 + 2*f^2*x^4)) + 4*A*(45*a^2*f^2 + 15*a*c*f*(-3*d + f*x^2) + c^2*(
15*d^2 - 5*d*f*x^2 + 3*f^2*x^4)))) - 30*(A*b*f*(-3*c^2*d^2 + b^2*d*f + 6*a*
c*d*f - 3*a^2*f^2) + B*(c*d - a*f)*(c^2*d^2 - 3*b^2*d*f - 2*a*c*d*f + a^2*f
^2))*Log[d + f*x^2])/(60*f^4)
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 592, normalized size of antiderivative = 1.34

method	result
default	$\frac{A b^2 c f^2 x^3 + A a c^2 f^2 x^3 - 3 A a c^2 d f x - 3 A b^2 c d f x - \frac{3}{2} B b^2 c d f x^2 - \frac{3}{2} B a c^2 d f x^2 - \frac{3}{2} A b c^2 d f x^2 + 3 A a b c f^2 x^2 - B b c^2 d f x^3 + 2 B a b c f^2 x^3 + A c}{}$
risch	Expression too large to display

```
[In] int((B*x+A)*(c*x^2+b*x+a)^3/(f*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f^3*(A*b^2*c*f^2*x^3+A*a*c^2*f^2*x^3-3*A*a*c^2*d*f*x-3*A*b^2*c*d*f*x-3/2*
B*b^2*c*d*f*x^2-3/2*B*a*c^2*d*f*x^2-3/2*A*b*c^2*d*f*x^2+3*A*a*b*c*f^2*x^2-B
*b*c^2*d*f*x^3+2*B*a*b*c*f^2*x^3+A*c^3*d^2*x+1/2*B*c^3*d^2*x^2+1/2*A*b^3*f^
2*x^2+1/3*B*b^3*f^2*x^3+1/6*B*c^3*x^6*f^2+1/5*A*c^3*f^2*x^5-6*B*a*b*c*d*f*x
+3/5*B*b*c^2*f^2*x^5+3/4*A*b*c^2*f^2*x^4+3/4*B*a*c^2*f^2*x^4+3/4*B*b^2*c*f^
2*x^4-1/4*B*c^3*d*f*x^4-1/3*A*c^3*d*f*x^3+3/2*B*a^2*c*f^2*x^2+3/2*B*a*b^2*f
^2*x^2+3*A*a^2*c*f^2*x+3*A*a*b^2*f^2*x+3*B*a^2*b*f^2*x-b^3*B*d*f*x+3*B*b*c^
2*d^2*x)+1/f^3*(1/2*(3*A*a^2*b*f^3-6*A*a*b*c*d*f^2-A*b^3*d*f^2+3*A*b*c^2*d^
2*f+B*a^3*f^3-3*B*a^2*c*d*f^2-3*B*a*b^2*d*f^2+3*B*a*c^2*d^2*f+3*B*b^2*c*d^2
*f-B*c^3*d^3)/f*ln(f*x^2+d)+(A*a^3*f^3-3*A*a^2*c*d*f^2-3*A*a*b^2*d*f^2+3*A*
a*c^2*d^2*f+3*A*b^2*c*d^2*f-A*c^3*d^3-3*B*a^2*b*d*f^2+6*B*a*b*c*d^2*f+B*b^3
*d^2*f-3*B*b*c^2*d^3)/(d*f)^(1/2)*arctan(f*x/(d*f)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 1014, normalized size of antiderivative = 2.30

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{d+fx^2}\,dx = \left[\frac{10\,Bc^3df^3x^6 + 12\,(3\,Bbc^2 + Ac^3)df^3x^5 - 15\,(Bc^3d^2f^2 - 3\,(Bb^2c + (Ba + Ab)c^2)df^3)x^4 - 20\,((3\,Bbc^2 + A}{}\right.$$

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)^3/(f*x^2+d),x, algorithm="fricas")
```

```
[Out] [1/60*(10*B*c^3*d*f^3*x^6 + 12*(3*B*b*c^2 + A*c^3)*d*f^3*x^5 - 15*(B*c^3*d^
2*f^2 - 3*(B*b^2*c + (B*a + A*b)*c^2)*d*f^3)*x^4 - 20*((3*B*b*c^2 + A*c^3)*
d^2*f^2 - (B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*d*f^3)*x^3 + 30*(B*c^
3*d^3*f - 3*(B*b^2*c + (B*a + A*b)*c^2)*d^2*f^2 + (3*B*a*b^2 + A*b^3 + 3*(B
*a^2 + 2*A*a*b)*c)*d*f^3)*x^2 - 30*(A*a^3*f^3 - (3*B*b*c^2 + A*c^3)*d^3 + (
B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*d^2*f - 3*(B*a^2*b + A*a*b^2 + A
*a^2*c)*d*f^2)*sqrt(-d*f)*log((f*x^2 - 2*sqrt(-d*f)*x - d)/(f*x^2 + d)) + 6
0*((3*B*b*c^2 + A*c^3)*d^3*f - (B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*
d^2*f^2 + 3*(B*a^2*b + A*a*b^2 + A*a^2*c)*d*f^3)*x - 30*(B*c^3*d^4 - 3*(B*b
^2*c + (B*a + A*b)*c^2)*d^3*f + (3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)
*d^2*f^2 - (B*a^3 + 3*A*a^2*b)*d*f^3)*log(f*x^2 + d))/(d*f^4), 1/60*(10*B*c
^3*d*f^3*x^6 + 12*(3*B*b*c^2 + A*c^3)*d*f^3*x^5 - 15*(B*c^3*d^2*f^2 - 3*(B*
b^2*c + (B*a + A*b)*c^2)*d*f^3)*x^4 - 20*((3*B*b*c^2 + A*c^3)*d^2*f^2 - (B*
b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*d*f^3)*x^3 + 30*(B*c^3*d^3*f - 3*(
B*b^2*c + (B*a + A*b)*c^2)*d^2*f^2 + (3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*
b)*c)*d*f^3)*x^2 + 60*(A*a^3*f^3 - (3*B*b*c^2 + A*c^3)*d^3 + (B*b^3 + 3*A*a
*c^2 + 3*(2*B*a*b + A*b^2)*c)*d^2*f - 3*(B*a^2*b + A*a*b^2 + A*a^2*c)*d*f^2
)*sqrt(d*f)*arctan(sqrt(d*f)*x/d) + 60*((3*B*b*c^2 + A*c^3)*d^3*f - (B*b^3
+ 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*d^2*f^2 + 3*(B*a^2*b + A*a*b^2 + A*a^2
*c)*d*f^3)*x - 30*(B*c^3*d^4 - 3*(B*b^2*c + (B*a + A*b)*c^2)*d^3*f + (3*B*a
*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*d^2*f^2 - (B*a^3 + 3*A*a^2*b)*d*f^3)*
log(f*x^2 + d))/(d*f^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A + Bx)\,(a + bx + cx^2)^3}{d + fx^2}\,dx = \text{Timed out}$$

```
[In] integrate((B*x+A)*(c*x**2+b*x+a)**3/(f*x**2+d),x)
```

```
[Out] Timed out
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 471, normalized size of antiderivative = 1.07

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{d+fx^2}\,dx$$
$$= \frac{(Aa^3f^3-(3\,Bbc^2+Ac^3)d^3+(Bb^3+3\,Aac^2+3\,(2\,Bab+Ab^2)c)d^2f-3\,(Ba^2b+Aab^2+Aa^2c)df^2)\arctan}{\sqrt{df}\,f^3}$$
$$+\frac{10\,Bc^3f^2x^6+12\,(3\,Bbc^2+Ac^3)f^2x^5-15\,(Bc^3df-3\,(Bb^2c+(Ba+Ab)c^2)f^2)x^4-20\,((3\,Bbc^2+Ac^3)d}{}$$
$$-\frac{(Bc^3d^3-3\,(Bb^2c+(Ba+Ab)c^2)d^2f+(3\,Bab^2+Ab^3+3\,(Ba^2+2\,Aab)c)df^2-(Ba^3+3\,Aa^2b)f^3)\log}{2\,f^4}$$

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)^3/(f*x^2+d),x, algorithm="maxima")
```

```
[Out] (A*a^3*f^3 - (3*B*b*c^2 + A*c^3)*d^3 + (B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*
b^2)*c)*d^2*f - 3*(B*a^2*b + A*a*b^2 + A*a^2*c)*d*f^2)*arctan(f*x/sqrt(d*f)
)/(sqrt(d*f)*f^3) + 1/60*(10*B*c^3*f^2*x^6 + 12*(3*B*b*c^2 + A*c^3)*f^2*x^5
 - 15*(B*c^3*d*f - 3*(B*b^2*c + (B*a + A*b)*c^2)*f^2)*x^4 - 20*((3*B*b*c^2
+ A*c^3)*d*f - (B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*f^2)*x^3 + 30*(B
*c^3*d^2 - 3*(B*b^2*c + (B*a + A*b)*c^2)*d*f + (3*B*a*b^2 + A*b^3 + 3*(B*a^
2 + 2*A*a*b)*c)*f^2)*x^2 + 60*((3*B*b*c^2 + A*c^3)*d^2 - (B*b^3 + 3*A*a*c^2
 + 3*(2*B*a*b + A*b^2)*c)*d*f + 3*(B*a^2*b + A*a*b^2 + A*a^2*c)*f^2)*x)/f^3
 - 1/2*(B*c^3*d^3 - 3*(B*b^2*c + (B*a + A*b)*c^2)*d^2*f + (3*B*a*b^2 + A*b^
3 + 3*(B*a^2 + 2*A*a*b)*c)*d*f^2 - (B*a^3 + 3*A*a^2*b)*f^3)*log(f*x^2 + d)/
f^4
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 623, normalized size of antiderivative = 1.41

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{d+fx^2}\,dx =$$
$$-\frac{(3\,Bbc^2d^3+Ac^3d^3-Bb^3d^2f-6\,Babcd^2f-3\,Ab^2cd^2f-3\,Aac^2d^2f+3\,Ba^2bdf^2+3\,Aab^2df^2+3\,Aa^2cd}{\sqrt{df}\,f^3}$$
$$-\frac{(Bc^3d^3-3\,Bb^2cd^2f-3\,Bac^2d^2f-3\,Abc^2d^2f+3\,Bab^2df^2+Ab^3df^2+3\,Ba^2cdf^2+6\,Aabcdf^2-Ba^3f^3}{2\,f^4}$$
$$+\frac{10\,Bc^3f^5x^6+36\,Bbc^2f^5x^5+12\,Ac^3f^5x^5-15\,Bc^3df^4x^4+45\,Bb^2cf^5x^4+45\,Bac^2f^5x^4+45\,Abc^2f^5x^4-}{}$$

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)^3/(f*x^2+d),x, algorithm="giac")
```

```
[Out] -(3*B*b*c^2*d^3 + A*c^3*d^3 - B*b^3*d^2*f - 6*B*a*b*c*d^2*f - 3*A*b^2*c*d^2
*f - 3*A*a*c^2*d^2*f + 3*B*a^2*b*d*f^2 + 3*A*a*b^2*d*f^2 + 3*A*a^2*c*d*f^2
- A*a^3*f^3)*arctan(f*x/sqrt(d*f))/(sqrt(d*f)*f^3) - 1/2*(B*c^3*d^3 - 3*B*b
^2*c*d^2*f - 3*B*a*c^2*d^2*f - 3*A*b*c^2*d^2*f + 3*B*a*b^2*d*f^2 + A*b^3*d*
f^2 + 3*B*a^2*c*d*f^2 + 6*A*a*b*c*d*f^2 - B*a^3*f^3 - 3*A*a^2*b*f^3)*log(f*
x^2 + d)/f^4 + 1/60*(10*B*c^3*f^5*x^6 + 36*B*b*c^2*f^5*x^5 + 12*A*c^3*f^5*x
^5 - 15*B*c^3*d*f^4*x^4 + 45*B*b^2*c*f^5*x^4 + 45*B*a*c^2*f^5*x^4 + 45*A*b*
c^2*f^5*x^4 - 60*B*b*c^2*d*f^4*x^3 - 20*A*c^3*d*f^4*x^3 + 20*B*b^3*f^5*x^3
+ 120*B*a*b*c*f^5*x^3 + 60*A*b^2*c*f^5*x^3 + 60*A*a*c^2*f^5*x^3 + 30*B*c^3*
d^2*f^3*x^2 - 90*B*b^2*c*d*f^4*x^2 - 90*B*a*c^2*d*f^4*x^2 - 90*A*b*c^2*d*f^
4*x^2 + 90*B*a*b^2*f^5*x^2 + 30*A*b^3*f^5*x^2 + 90*B*a^2*c*f^5*x^2 + 180*A*
a*b*c*f^5*x^2 + 180*B*b*c^2*d^2*f^3*x + 60*A*c^3*d^2*f^3*x - 60*B*b^3*d*f^4
*x - 360*B*a*b*c*d*f^4*x - 180*A*b^2*c*d*f^4*x - 180*A*a*c^2*d*f^4*x + 180*
B*a^2*b*f^5*x + 180*A*a*b^2*f^5*x + 180*A*a^2*c*f^5*x)/f^6
```

Mupad [B] (verification not implemented)

Time = 13.20 (sec) , antiderivative size = 552, normalized size of antiderivative = 1.25

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{d+fx^2}\,dx$$
$$= x^2\left(\frac{3\,B\,c\,a^2+3\,B\,a\,b^2+6\,A\,c\,a\,b+A\,b^3}{2\,f}-\frac{d\left(\frac{3\,B\,b^2\,c+3\,A\,b\,c^2+3\,B\,a\,c^2}{f}-\frac{B\,c^3\,d}{f^2}\right)}{2\,f}\right)$$
$$+x\left(\frac{3\,B\,a^2\,b+3\,A\,c\,a^2+3\,A\,a\,b^2}{f}-\frac{d\left(\frac{B\,b^3+3\,A\,b^2\,c+6\,B\,a\,b\,c+3\,A\,a\,c^2}{f}-\frac{d\,\left(A\,c^3+3\,B\,b\,c^2\right)}{f^2}\right)}{f}\right)$$
$$+x^3\left(\frac{B\,b^3+3\,A\,b^2\,c+6\,B\,a\,b\,c+3\,A\,a\,c^2}{3\,f}-\frac{d\,\left(A\,c^3+3\,B\,b\,c^2\right)}{3\,f^2}\right)$$
$$+x^4\left(\frac{3\,B\,b^2\,c+3\,A\,b\,c^2+3\,B\,a\,c^2}{4\,f}-\frac{B\,c^3\,d}{4\,f^2}\right)+\frac{x^5\,\left(A\,c^3+3\,B\,b\,c^2\right)}{5\,f}+\frac{B\,c^3\,x^6}{6\,f}$$
$$+\frac{\ln\left(f\,x^2+d\right)\,\left(4\,B\,a^3\,d\,f^7+12\,A\,a^2\,b\,d\,f^7-12\,B\,a^2\,c\,d^2\,f^6-12\,B\,a\,b^2\,d^2\,f^6-24\,A\,a\,b\,c\,d^2\,f^6+12\,B\,a\,c^2\right.}{8\,d\,f^8}$$
$$+\frac{\mathrm{atan}\left(\frac{\sqrt{f}\,x}{\sqrt{d}}\right)\,\left(A\,a^3\,f^3-3\,B\,a^2\,b\,d\,f^2-3\,A\,a^2\,c\,d\,f^2-3\,A\,a\,b^2\,d\,f^2+6\,B\,a\,b\,c\,d^2\,f+3\,A\,a\,c^2\,d^2\,f+B\,b^3\right.}{\sqrt{d}\,f^{7/2}}$$

```
[In] int(((A + B*x)*(a + b*x + c*x^2)^3)/(d + f*x^2),x)
```

```
[Out] x^2*((A*b^3 + 3*B*a*b^2 + 3*B*a^2*c + 6*A*a*b*c)/(2*f) - (d*((3*A*b*c^2 + 3
*B*a*c^2 + 3*B*b^2*c)/f - (B*c^3*d)/f^2))/(2*f)) + x*((3*A*a*b^2 + 3*A*a^2*
c + 3*B*a^2*b)/f - (d*((B*b^3 + 3*A*a*c^2 + 3*A*b^2*c + 6*B*a*b*c)/f - (d*(
```

```
A*c^3 + 3*B*b*c^2))/f^2))/f) + x^3*((B*b^3 + 3*A*a*c^2 + 3*A*b^2*c + 6*B*a*
b*c)/(3*f) - (d*(A*c^3 + 3*B*b*c^2))/(3*f^2)) + x^4*((3*A*b*c^2 + 3*B*a*c^2
 + 3*B*b^2*c)/(4*f) - (B*c^3*d)/(4*f^2)) + (x^5*(A*c^3 + 3*B*b*c^2))/(5*f)
+ (B*c^3*x^6)/(6*f) + (log(d + f*x^2)*(4*B*a^3*d*f^7 - 4*A*b^3*d^2*f^6 - 4*
B*c^3*d^4*f^4 - 12*B*a*b^2*d^2*f^6 + 12*A*b*c^2*d^3*f^5 + 12*B*a*c^2*d^3*f^
5 - 12*B*a^2*c*d^2*f^6 + 12*B*b^2*c*d^3*f^5 + 12*A*a^2*b*d*f^7 - 24*A*a*b*c
*d^2*f^6))/(8*d*f^8) + (atan((f^(1/2)*x)/d^(1/2))*(A*a^3*f^3 - A*c^3*d^3 -
3*B*b*c^2*d^3 + B*b^3*d^2*f - 3*A*a*b^2*d*f^2 + 3*A*a*c^2*d^2*f - 3*A*a^2*c
*d*f^2 - 3*B*a^2*b*d*f^2 + 3*A*b^2*c*d^2*f + 6*B*a*b*c*d^2*f))/(d^(1/2)*f^(
7/2))
```

3.4 $\int \frac{A+Bx}{(a+bx+cx^2)(d+fx^2)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 274

$$\begin{aligned}\int \frac{A+Bx}{(a+bx+cx^2)(d+fx^2)}\,dx &= \frac{\sqrt{f}(bBd-Acd+aAf)\arctan\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)}{\sqrt{d}\,(c^2d^2-2acdf+f(b^2d+a^2f))} \\ &\quad-\frac{(Ab^2f+2Ac(cd-af)-bB(cd+af))\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\,(c^2d^2-2acdf+f(b^2d+a^2f))} \\ &\quad+\frac{(Bcd+Abf-aBf)\log(a+bx+cx^2)}{2(c^2d^2-2acdf+f(b^2d+a^2f))}-\frac{(Bcd+Abf-aBf)\log(d+fx^2)}{2(c^2d^2-2acdf+f(b^2d+a^2f))}\end{aligned}$$

```
[Out] 1/2*(A*b*f-B*a*f+B*c*d)*ln(c*x^2+b*x+a)/(c^2*d^2-2*a*c*d*f+f*(a^2*f+b^2*d))
-1/2*(A*b*f-B*a*f+B*c*d)*ln(f*x^2+d)/(c^2*d^2-2*a*c*d*f+f*(a^2*f+b^2*d))-(A
*b^2*f+2*A*c*(-a*f+c*d)-b*B*(a*f+c*d))*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2)
)/(c^2*d^2-2*a*c*d*f+f*(a^2*f+b^2*d))/(-4*a*c+b^2)^(1/2)+(A*a*f-A*c*d+B*b*d
)*arctan(x*f^(1/2)/d^(1/2))*f^(1/2)/(c^2*d^2-2*a*c*d*f+f*(a^2*f+b^2*d))/d^(
1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {1037, 648, 632, 212, 642, 649, 211, 266}

$$\int \frac{A+Bx}{(a+bx+cx^2)(d+fx^2)}\,dx$$
$$=\frac{\sqrt{f}\arctan\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)(aAf-Acd+bBd)}{\sqrt{d}\,(f(a^2f+b^2d)-2acdf+c^2d^2)}-\frac{\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)(2Ac(cd-af)-bB(af+cd)+Ab^2f)}{\sqrt{b^2-4ac}\,(f(a^2f+b^2d)-2acdf+c^2d^2)}+\frac{\log(a+bx+cx^2)(-aBf+Abf+Bcd)}{2(f(a^2f+b^2d)-2acdf+c^2d^2)}-\frac{\log(d+fx^2)(-aBf+Abf+Bcd)}{2(f(a^2f+b^2d)-2acdf+c^2d^2)}$$

```
[In] Int[(A + B*x)/((a + b*x + c*x^2)*(d + f*x^2)),x]
```

```
[Out] (Sqrt[f]*(b*B*d - A*c*d + a*A*f)*ArcTan[(Sqrt[f]*x)/Sqrt[d]])/(Sqrt[d]*(c^2
*d^2 - 2*a*c*d*f + f*(b^2*d + a^2*f))) - ((A*b^2*f + 2*A*c*(c*d - a*f) - b*
B*(c*d + a*f))*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*(
c^2*d^2 - 2*a*c*d*f + f*(b^2*d + a^2*f))) + ((B*c*d + A*b*f - a*B*f)*Log[a
+ b*x + c*x^2])/(2*(c^2*d^2 - 2*a*c*d*f + f*(b^2*d + a^2*f))) - ((B*c*d + A
*b*f - a*B*f)*Log[d + f*x^2])/(2*(c^2*d^2 - 2*a*c*d*f + f*(b^2*d + a^2*f)))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1037

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_) + (f_.)*
(x_)^2)), x_Symbol] :> With[{q = Simplify[c^2*d^2 + b^2*d*f - 2*a*c*d*f + a
^2*f^2]}, Dist[1/q, Int[Simp[g*c^2*d + g*b^2*f - a*b*h*f - a*g*c*f + c*(h*c
*d + g*b*f - a*h*f)*x, x]/(a + b*x + c*x^2), x], x] + Dist[1/q, Int[Simp[b*
h*d*f - g*c*d*f + a*g*f^2 - f*(h*c*d + g*b*f - a*h*f)*x, x]/(d + f*x^2), x]
, x] /; NeQ[q, 0]] /; FreeQ[{a, b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0
]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\int \frac{-abBf+A(c^2d+b^2f-acf)+c(Bcd+Abf-aBf)x}{a+bx+cx^2}\,dx}{c^2d^2-2acdf+f\left(b^2d+a^2f\right)}+\frac{\int \frac{f(bBd-Acd+aAf)-f(Bcd+Abf-aBf)x}{d+fx^2}\,dx}{c^2d^2-2acdf+f\left(b^2d+a^2f\right)}\\
&= \frac{(f(bBd-Acd+aAf))\int \frac{1}{d+fx^2}\,dx}{c^2d^2-2acdf+f\left(b^2d+a^2f\right)}+\frac{(Bcd+Abf-aBf)\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2\left(c^2d^2-2acdf+f\left(b^2d+a^2f\right)\right)}\\
&\quad-\frac{(f(Bcd+Abf-aBf))\int \frac{x}{d+fx^2}\,dx}{c^2d^2-2acdf+f\left(b^2d+a^2f\right)}\\
&\quad+\frac{\left(Ab^2f+2Ac(cd-af)-bB(cd+af)\right)\int \frac{1}{a+bx+cx^2}\,dx}{2\left(c^2d^2-2acdf+f\left(b^2d+a^2f\right)\right)}
\end{aligned}
$$

$$= \frac{\sqrt{f}(bBd - Acd + aAf)\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)}{\sqrt{d}\left(c^2d^2 - 2acdf + f\left(b^2d + a^2f\right)\right)} + \frac{(Bcd + Abf - aBf)\log\left(a + bx + cx^2\right)}{2\left(c^2d^2 - 2acdf + f\left(b^2d + a^2f\right)\right)} - \frac{(Bcd + Abf - aBf)\log\left(d + fx^2\right)}{2\left(c^2d^2 - 2acdf + f\left(b^2d + a^2f\right)\right)} - \frac{\left(Ab^2f + 2Ac(cd - af) - bB(cd + af)\right)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b + 2cx\right)}{c^2d^2 - 2acdf + f\left(b^2d + a^2f\right)}$$

$$= \frac{\sqrt{f}(bBd - Acd + aAf)\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)}{\sqrt{d}\left(c^2d^2 - 2acdf + f\left(b^2d + a^2f\right)\right)} - \frac{\left(Ab^2f + 2Ac(cd - af) - bB(cd + af)\right)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(c^2d^2 - 2acdf + f\left(b^2d + a^2f\right)\right)} + \frac{(Bcd + Abf - aBf)\log\left(a + bx + cx^2\right)}{2\left(c^2d^2 - 2acdf + f\left(b^2d + a^2f\right)\right)} - \frac{(Bcd + Abf - aBf)\log\left(d + fx^2\right)}{2\left(c^2d^2 - 2acdf + f\left(b^2d + a^2f\right)\right)}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.77

$$\int \frac{A + Bx}{(a + bx + cx^2)(d + fx^2)}\,dx = \frac{2\sqrt{-b^2 + 4ac}\sqrt{f}(bBd - Acd + aAf)\arctan\left(\frac{\sqrt{f}x}{\sqrt{d}}\right) + \sqrt{d}\Big(2\left(Ab^2f + 2Ac(cd - af) - bB(cd + af)\right)\arctan}{2\sqrt{-b^2 + 4ac}\sqrt{d}\left(c^2d^2 - 2acdf + \right.}$$

```
[In] Integrate[(A + B*x)/((a + b*x + c*x^2)*(d + f*x^2)),x]
```

```
[Out] (2*Sqrt[-b^2 + 4*a*c]*Sqrt[f]*(b*B*d - A*c*d + a*A*f)*ArcTan[(Sqrt[f]*x)/Sq
rt[d]] + Sqrt[d]*(2*(A*b^2*f + 2*A*c*(c*d - a*f) - b*B*(c*d + a*f))*ArcTan[
(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]] + Sqrt[-b^2 + 4*a*c]*(B*c*d + A*b*f - a*B*f
)*(-Log[d + f*x^2] + Log[a + x*(b + c*x)])))/(2*Sqrt[-b^2 + 4*a*c]*Sqrt[d]*
(c^2*d^2 - 2*a*c*d*f + f*(b^2*d + a^2*f)))
```

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.87

method	result
default	$\frac{f\left(\frac{(-Abf+Baf-Bcd)\ln\left(f\,x^2+d\right)}{2f}+\frac{(Aaf-Acd+Bbd)\arctan\left(\frac{fx}{\sqrt{df}}\right)}{\sqrt{df}}\right)}{a^2f^2-2acdf+b^2df+c^2d^2} + \frac{\frac{\left(Abcf-Bacf+B\,c^2d\right)\ln\left(c\,x^2+bx+a\right)}{2c}+\frac{2\left(-Aacf+A\,b^2f+A\,c^2d-Bc\right.}{}}{a^2f^2-2acdf+b^2df+c}$
risch	Expression too large to display

```
[In] int((B*x+A)/(c*x^2+b*x+a)/(f*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] f/(a^2*f^2-2*a*c*d*f+b^2*d*f+c^2*d^2)*(1/2*(-A*b*f+B*a*f-B*c*d)/f*ln(f*x^2+
d)+(A*a*f-A*c*d+B*b*d)/(d*f)^(1/2)*arctan(f*x/(d*f)^(1/2)))+1/(a^2*f^2-2*a*
c*d*f+b^2*d*f+c^2*d^2)*(1/2*(A*b*c*f-B*a*c*f+B*c^2*d)/c*ln(c*x^2+b*x+a)+2*(
-A*a*c*f+A*b^2*f+A*c^2*d-B*a*b*f-1/2*(A*b*c*f-B*a*c*f+B*c^2*d)*b/c)/(4*a*c-
b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(a+bx+cx^2)(d+fx^2)}\,dx = \text{Timed out}$$

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)/(f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(a+bx+cx^2)(d+fx^2)}\,dx = \text{Timed out}$$

```
[In] integrate((B*x+A)/(c*x**2+b*x+a)/(f*x**2+d),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx}{(a+bx+cx^2)(d+fx^2)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)/(f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.97

$$\int \frac{A+Bx}{(a+bx+cx^2)(d+fx^2)}\,dx = \frac{(Bcd-Baf+Abf)\log(cx^2+bx+a)}{2(c^2d^2+b^2df-2acdf+a^2f^2)} - \frac{(Bcd-Baf+Abf)\log(fx^2+d)}{2(c^2d^2+b^2df-2acdf+a^2f^2)} + \frac{(Bbdf-Acdf+Aaf^2)\arctan\left(\frac{fx}{\sqrt{df}}\right)}{(c^2d^2+b^2df-2acdf+a^2f^2)\sqrt{df}} - \frac{(Bbcd-2Ac^2d+Babf-Ab^2f+2Aacf)\arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{(c^2d^2+b^2df-2acdf+a^2f^2)\sqrt{-b^2+4ac}}$$

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)/(f*x^2+d),x, algorithm="giac")
```

```
[Out] 1/2*(B*c*d - B*a*f + A*b*f)*log(c*x^2 + b*x + a)/(c^2*d^2 + b^2*d*f - 2*a*c
*d*f + a^2*f^2) - 1/2*(B*c*d - B*a*f + A*b*f)*log(f*x^2 + d)/(c^2*d^2 + b^2
*d*f - 2*a*c*d*f + a^2*f^2) + (B*b*d*f - A*c*d*f + A*a*f^2)*arctan(f*x/sqrt
(d*f))/((c^2*d^2 + b^2*d*f - 2*a*c*d*f + a^2*f^2)*sqrt(d*f)) - (B*b*c*d - 2
*A*c^2*d + B*a*b*f - A*b^2*f + 2*A*a*c*f)*arctan((2*c*x + b)/sqrt(-b^2 + 4*
a*c))/((c^2*d^2 + b^2*d*f - 2*a*c*d*f + a^2*f^2)*sqrt(-b^2 + 4*a*c))
```

Mupad [B] (verification not implemented)

Time = 75.86 (sec) , antiderivative size = 3888, normalized size of antiderivative = 14.19

$$\int \frac{A+Bx}{(a+bx+cx^2)(d+fx^2)}\,dx = \text{Too large to display}$$

```
[In] int((A + B*x)/((d + f*x^2)*(a + b*x + c*x^2)),x)
```

```
[Out] (log(B^3*c^2*f^2*x + (((B*c*d^2)/2 + (A*b*d*f)/2 - (B*a*d*f)/2 - (A*a*f*(-d
*f)^(1/2))/2 + (A*c*d*(-d*f)^(1/2))/2 - (B*b*d*(-d*f)^(1/2))/2)*((((B*c*d^2
)/2 + (A*b*d*f)/2 - (B*a*d*f)/2 - (A*a*f*(-d*f)^(1/2))/2 + (A*c*d*(-d*f)^(1
/2))/2 - (B*b*d*(-d*f)^(1/2))/2)*(4*A*a^2*c^2*f^4 + 4*A*c^4*d^2*f^2 - c*f^2
*x*(3*A*b^3*f^2 + 4*B*c^3*d^2 - B*a*b^2*f^2 + 4*B*a^2*c*f^2 - 12*A*a*b*c*f^
2 + 12*A*b*c^2*d*f - 8*B*a*c^2*d*f - 3*B*b^2*c*d*f) - 3*A*b^2*c^2*d*f^3 - 4
*B*b*c^3*d^2*f^2 - A*a*b^2*c*f^4 - 8*A*a*c^3*d*f^3 + B*b^3*c*d*f^3 + (2*c*f
^2*((B*c*d^2)/2 + (A*b*d*f)/2 - (B*a*d*f)/2 - (A*a*f*(-d*f)^(1/2))/2 + (A*c
*d*(-d*f)^(1/2))/2 - (B*b*d*(-d*f)^(1/2))/2)*(2*b*c^3*d^3 + 4*c^4*d^3*x - a
^2*b^2*f^3*x + 4*a*b^3*d*f^2 - 2*b^3*c*d^2*f + 4*a^3*c*f^3*x + 3*b^4*d*f^2*
x + 12*a*b*c^2*d^2*f - 14*a^2*b*c*d*f^2 - 4*a*c^3*d^2*f*x - 4*a^2*c^2*d*f^2
```

```
*x + 3*b^2*c^2*d^2*f*x - 10*a*b^2*c*d*f^2*x))/(d*(a^2*f^2 + c^2*d^2 + b^2*d
*f - 2*a*c*d*f)) + 4*B*a*b*c^2*d*f^3))/(d*(a^2*f^2 + c^2*d^2 + b^2*d*f - 2*
a*c*d*f)) - c*f^2*x*(2*B^2*c^2*d - 4*A^2*c^2*f - B^2*b^2*f + 2*B^2*a*c*f +
2*A*B*b*c*f) + A^2*b*c^2*f^3 + B^2*b*c^2*d*f^2 - 4*A*B*a*c^2*f^3 + A*B*b^2*
c*f^3 - 4*A*B*c^3*d*f^2))/(d*(a^2*f^2 + c^2*d^2 + b^2*d*f - 2*a*c*d*f)) + A
*B^2*c^2*f^2)*(f*((B*a*d)/2 - (A*b*d)/2 + (A*a*(-d*f)^(1/2))/2) - (B*c*d^2)
/2 - (A*c*d*(-d*f)^(1/2))/2 + (B*b*d*(-d*f)^(1/2))/2))/(c^2*d^3 + a^2*d*f^2
 + b^2*d^2*f - 2*a*c*d^2*f) - (log(B^3*c^2*f^2*x + (((B*c*d^2)/2 + (A*b*d*f
)/2 - (B*a*d*f)/2 + (A*a*f*(-d*f)^(1/2))/2 - (A*c*d*(-d*f)^(1/2))/2 + (B*b*
d*(-d*f)^(1/2))/2)*((((B*c*d^2)/2 + (A*b*d*f)/2 - (B*a*d*f)/2 + (A*a*f*(-d*
f)^(1/2))/2 - (A*c*d*(-d*f)^(1/2))/2 + (B*b*d*(-d*f)^(1/2))/2)*(4*A*a^2*c^2
*f^4 + 4*A*c^4*d^2*f^2 - c*f^2*x*(3*A*b^3*f^2 + 4*B*c^3*d^2 - B*a*b^2*f^2 +
 4*B*a^2*c*f^2 - 12*A*a*b*c*f^2 + 12*A*b*c^2*d*f - 8*B*a*c^2*d*f - 3*B*b^2*
c*d*f) - 3*A*b^2*c^2*d*f^3 - 4*B*b*c^3*d^2*f^2 - A*a*b^2*c*f^4 - 8*A*a*c^3*
d*f^3 + B*b^3*c*d*f^3 + (2*c*f^2*((B*c*d^2)/2 + (A*b*d*f)/2 - (B*a*d*f)/2 +
 (A*a*f*(-d*f)^(1/2))/2 - (A*c*d*(-d*f)^(1/2))/2 + (B*b*d*(-d*f)^(1/2))/2)*
(2*b*c^3*d^3 + 4*c^4*d^3*x - a^2*b^2*f^3*x + 4*a*b^3*d*f^2 - 2*b^3*c*d^2*f
+ 4*a^3*c*f^3*x + 3*b^4*d*f^2*x + 12*a*b*c^2*d^2*f - 14*a^2*b*c*d*f^2 - 4*a
*c^3*d^2*f*x - 4*a^2*c^2*d*f^2*x + 3*b^2*c^2*d^2*f*x - 10*a*b^2*c*d*f^2*x))
/(d*(a^2*f^2 + c^2*d^2 + b^2*d*f - 2*a*c*d*f)) + 4*B*a*b*c^2*d*f^3))/(d*(a^
2*f^2 + c^2*d^2 + b^2*d*f - 2*a*c*d*f)) - c*f^2*x*(2*B^2*c^2*d - 4*A^2*c^2*
f - B^2*b^2*f + 2*B^2*a*c*f + 2*A*B*b*c*f) + A^2*b*c^2*f^3 + B^2*b*c^2*d*f^
2 - 4*A*B*a*c^2*f^3 + A*B*b^2*c*f^3 - 4*A*B*c^3*d*f^2))/(d*(a^2*f^2 + c^2*d
^2 + b^2*d*f - 2*a*c*d*f)) + A*B^2*c^2*f^2)*(f*((A*b*d)/2 - (B*a*d)/2 + (A*
a*(-d*f)^(1/2))/2) + (B*c*d^2)/2 - (A*c*d*(-d*f)^(1/2))/2 + (B*b*d*(-d*f)^(
1/2))/2))/(c^2*d^3 + a^2*d*f^2 + b^2*d^2*f - 2*a*c*d^2*f) - (log(B^3*c^2*f^
2*x + A*B^2*c^2*f^2 - ((((A*f*(b^2 - 4*a*c)^(3/2) + 2*A*b*f*(4*a*c - b^2) -
 2*B*a*f*(4*a*c - b^2) + 2*B*c*d*(4*a*c - b^2) + 4*A*c^2*d*(b^2 - 4*a*c)^(1
/2) + A*b^2*f*(b^2 - 4*a*c)^(1/2) - 2*B*a*b*f*(b^2 - 4*a*c)^(1/2) - 2*B*b*c
*d*(b^2 - 4*a*c)^(1/2))*(c*f^2*x*(3*A*b^3*f^2 + 4*B*c^3*d^2 - B*a*b^2*f^2 +
 4*B*a^2*c*f^2 - 12*A*a*b*c*f^2 + 12*A*b*c^2*d*f - 8*B*a*c^2*d*f - 3*B*b^2*
c*d*f) - 4*A*c^4*d^2*f^2 - 4*A*a^2*c^2*f^4 + 3*A*b^2*c^2*d*f^3 + 4*B*b*c^3*
d^2*f^2 + A*a*b^2*c*f^4 + 8*A*a*c^3*d*f^3 - B*b^3*c*d*f^3 - 4*B*a*b*c^2*d*f
^3 + (c*f^2*(A*f*(b^2 - 4*a*c)^(3/2) + 2*A*b*f*(4*a*c - b^2) - 2*B*a*f*(4*a
*c - b^2) + 2*B*c*d*(4*a*c - b^2) + 4*A*c^2*d*(b^2 - 4*a*c)^(1/2) + A*b^2*f
*(b^2 - 4*a*c)^(1/2) - 2*B*a*b*f*(b^2 - 4*a*c)^(1/2) - 2*B*b*c*d*(b^2 - 4*a
*c)^(1/2))*(2*b*c^3*d^3 + 4*c^4*d^3*x - a^2*b^2*f^3*x + 4*a*b^3*d*f^2 - 2*b
^3*c*d^2*f + 4*a^3*c*f^3*x + 3*b^4*d*f^2*x + 12*a*b*c^2*d^2*f - 14*a^2*b*c*
d*f^2 - 4*a*c^3*d^2*f*x - 4*a^2*c^2*d*f^2*x + 3*b^2*c^2*d^2*f*x - 10*a*b^2*
c*d*f^2*x))/(2*(4*a*c - b^2)*(a^2*f^2 + c^2*d^2 + b^2*d*f - 2*a*c*d*f))))/(
4*(4*a*c - b^2)*(a^2*f^2 + c^2*d^2 + b^2*d*f - 2*a*c*d*f)) - c*f^2*x*(2*B^2
*c^2*d - 4*A^2*c^2*f - B^2*b^2*f + 2*B^2*a*c*f + 2*A*B*b*c*f) + A^2*b*c^2*f
^3 + B^2*b*c^2*d*f^2 - 4*A*B*a*c^2*f^3 + A*B*b^2*c*f^3 - 4*A*B*c^3*d*f^2)*(
A*f*(b^2 - 4*a*c)^(3/2) + 2*A*b*f*(4*a*c - b^2) - 2*B*a*f*(4*a*c - b^2) + 2
*B*c*d*(4*a*c - b^2) + 4*A*c^2*d*(b^2 - 4*a*c)^(1/2) + A*b^2*f*(b^2 - 4*a*c
```

```
)^(1/2) - 2*B*a*b*f*(b^2 - 4*a*c)^(1/2) - 2*B*b*c*d*(b^2 - 4*a*c)^(1/2)))/(
4*(4*a*c - b^2)*(a^2*f^2 + c^2*d^2 + b^2*d*f - 2*a*c*d*f)))*(A*f*(b^2 - 4*a
*c)^(3/2) + 2*A*b*f*(4*a*c - b^2) - 2*B*a*f*(4*a*c - b^2) + 2*B*c*d*(4*a*c
- b^2) + 4*A*c^2*d*(b^2 - 4*a*c)^(1/2) + A*b^2*f*(b^2 - 4*a*c)^(1/2) - 2*B*
a*b*f*(b^2 - 4*a*c)^(1/2) - 2*B*b*c*d*(b^2 - 4*a*c)^(1/2)))/(b^2*(4*a^2*f^2
 + 4*c^2*d^2 + 4*b^2*d*f - 24*a*c*d*f) - 4*a*c*(4*a^2*f^2 + 4*c^2*d^2 - 8*a
*c*d*f)) + (log(B^3*c^2*f^2*x + A*B^2*c^2*f^2 + ((((A*f*(b^2 - 4*a*c)^(3/2)
 - 2*A*b*f*(4*a*c - b^2) + 2*B*a*f*(4*a*c - b^2) - 2*B*c*d*(4*a*c - b^2) +
4*A*c^2*d*(b^2 - 4*a*c)^(1/2) + A*b^2*f*(b^2 - 4*a*c)^(1/2) - 2*B*a*b*f*(b^
2 - 4*a*c)^(1/2) - 2*B*b*c*d*(b^2 - 4*a*c)^(1/2))*(4*A*a^2*c^2*f^4 + 4*A*c^
4*d^2*f^2 - c*f^2*x*(3*A*b^3*f^2 + 4*B*c^3*d^2 - B*a*b^2*f^2 + 4*B*a^2*c*f^
2 - 12*A*a*b*c*f^2 + 12*A*b*c^2*d*f - 8*B*a*c^2*d*f - 3*B*b^2*c*d*f) - 3*A*
b^2*c^2*d*f^3 - 4*B*b*c^3*d^2*f^2 - A*a*b^2*c*f^4 - 8*A*a*c^3*d*f^3 + B*b^3
*c*d*f^3 + 4*B*a*b*c^2*d*f^3 + (c*f^2*(A*f*(b^2 - 4*a*c)^(3/2) - 2*A*b*f*(4
*a*c - b^2) + 2*B*a*f*(4*a*c - b^2) - 2*B*c*d*(4*a*c - b^2) + 4*A*c^2*d*(b^
2 - 4*a*c)^(1/2) + A*b^2*f*(b^2 - 4*a*c)^(1/2) - 2*B*a*b*f*(b^2 - 4*a*c)^(1
/2) - 2*B*b*c*d*(b^2 - 4*a*c)^(1/2))*(2*b*c^3*d^3 + 4*c^4*d^3*x - a^2*b^2*f
^3*x + 4*a*b^3*d*f^2 - 2*b^3*c*d^2*f + 4*a^3*c*f^3*x + 3*b^4*d*f^2*x + 12*a
*b*c^2*d^2*f - 14*a^2*b*c*d*f^2 - 4*a*c^3*d^2*f*x - 4*a^2*c^2*d*f^2*x + 3*b
^2*c^2*d^2*f*x - 10*a*b^2*c*d*f^2*x))/(2*(4*a*c - b^2)*(a^2*f^2 + c^2*d^2 +
 b^2*d*f - 2*a*c*d*f))))/(4*(4*a*c - b^2)*(a^2*f^2 + c^2*d^2 + b^2*d*f - 2*
a*c*d*f)) - c*f^2*x*(2*B^2*c^2*d - 4*A^2*c^2*f - B^2*b^2*f + 2*B^2*a*c*f +
2*A*B*b*c*f) + A^2*b*c^2*f^3 + B^2*b*c^2*d*f^2 - 4*A*B*a*c^2*f^3 + A*B*b^2*
c*f^3 - 4*A*B*c^3*d*f^2)*(A*f*(b^2 - 4*a*c)^(3/2) - 2*A*b*f*(4*a*c - b^2) +
 2*B*a*f*(4*a*c - b^2) - 2*B*c*d*(4*a*c - b^2) + 4*A*c^2*d*(b^2 - 4*a*c)^(1
/2) + A*b^2*f*(b^2 - 4*a*c)^(1/2) - 2*B*a*b*f*(b^2 - 4*a*c)^(1/2) - 2*B*b*c
*d*(b^2 - 4*a*c)^(1/2)))/(4*(4*a*c - b^2)*(a^2*f^2 + c^2*d^2 + b^2*d*f - 2*
a*c*d*f)))*(A*f*(b^2 - 4*a*c)^(3/2) - 2*A*b*f*(4*a*c - b^2) + 2*B*a*f*(4*a*
c - b^2) - 2*B*c*d*(4*a*c - b^2) + 4*A*c^2*d*(b^2 - 4*a*c)^(1/2) + A*b^2*f*
(b^2 - 4*a*c)^(1/2) - 2*B*a*b*f*(b^2 - 4*a*c)^(1/2) - 2*B*b*c*d*(b^2 - 4*a*
c)^(1/2)))/(b^2*(4*a^2*f^2 + 4*c^2*d^2 + 4*b^2*d*f - 24*a*c*d*f) - 4*a*c*(4
*a^2*f^2 + 4*c^2*d^2 - 8*a*c*d*f))
```

3.5 $\int \frac{A+Bx}{(a+bx+cx^2)^2(d+fx^2)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 596

$$\int \frac{A+Bx}{(a+bx+cx^2)^2(d+fx^2)}\,dx$$
$$= \frac{Abc(cd+af)-(Ab-aB)\left(2c^2d+b^2f-2acf\right)-c\left(Ab^2f+2Ac(cd-af)-bB(cd+af)\right)x}{(b^2-4ac)\left(b^2df+(cd-af)^2\right)(a+bx+cx^2)}$$
$$-\frac{f^{3/2}\left(Ab^2df+2bBd(cd-af)-A(cd-af)^2\right)\arctan\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)}{\sqrt{d}\left(c^2d^2-2acdf+f\left(b^2d+a^2f\right)\right)^2}$$
$$-\frac{\left(b^5Bdf^2-2Ab^4f^2(cd-af)-4Ac^2(cd-3af)(cd-af)^2+b^3Bf\left(5c^2d^2-4acdf-a^2f^2\right)-4Ab^2cf(2c}{(b^2-4ac)^{3/2}\left(c^2d^2-2acdf+f\right.}$$
$$-\frac{f\left(2Abf(cd-af)+B\left(c^2d^2-2acdf-f\left(b^2d-a^2f\right)\right)\right)\log\left(a+bx+cx^2\right)}{2\left(c^2d^2-2acdf+f\left(b^2d+a^2f\right)\right)^2}$$
$$+\frac{f\left(2Abf(cd-af)+B\left(c^2d^2-2acdf-f\left(b^2d-a^2f\right)\right)\right)\log\left(d+fx^2\right)}{2\left(c^2d^2-2acdf+f\left(b^2d+a^2f\right)\right)^2}$$

```
[Out] (A*b*c*(a*f+c*d)-(A*b-B*a)*(-2*a*c*f+b^2*f+2*c^2*d)-c*(A*b^2*f+2*A*c*(-a*f+
c*d)-b*B*(a*f+c*d))*x)/(-4*a*c+b^2)/(b^2*d*f+(-a*f+c*d)^2)/(c*x^2+b*x+a)-(b
^5*B*d*f^2-2*A*b^4*f^2*(-a*f+c*d)-4*A*c^2*(-3*a*f+c*d)*(-a*f+c*d)^2+b^3*B*f
*(-a^2*f^2-4*a*c*d*f+5*c^2*d^2)-4*A*b^2*c*f*(3*a^2*f^2-3*a*c*d*f+2*c^2*d^2)
+2*b*B*c*(3*a^3*f^3+3*a^2*c*d*f^2-7*a*c^2*d^2*f+c^3*d^3))*arctanh((2*c*x+b)
/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)/(c^2*d^2-2*a*c*d*f+f*(a^2*f+b^2*d))
^2-1/2*f*(2*A*b*f*(-a*f+c*d)+B*(c^2*d^2-2*a*c*d*f-f*(-a^2*f+b^2*d)))*ln(c*x
^2+b*x+a)/(c^2*d^2-2*a*c*d*f+f*(a^2*f+b^2*d))^2+1/2*f*(2*A*b*f*(-a*f+c*d)+B
*(c^2*d^2-2*a*c*d*f-f*(-a^2*f+b^2*d)))*ln(f*x^2+d)/(c^2*d^2-2*a*c*d*f+f*(a^
2*f+b^2*d))^2-f^(3/2)*(A*b^2*d*f+2*b*B*d*(-a*f+c*d)-A*(-a*f+c*d)^2)*arctan(
x*f^(1/2)/d^(1/2))/(c^2*d^2-2*a*c*d*f+f*(a^2*f+b^2*d))^2/d^(1/2)
```

Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 596, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1032, 1088, 648, 632, 212, 642, 649, 211, 266}

$$\int \frac{A+Bx}{(a+bx+cx^2)^2(d+fx^2)}\,dx$$
$$= -\frac{f^{3/2}\arctan\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)(-A(cd-af)^2+2bBd(cd-af)+Ab^2df)}{\sqrt{d}\left(f\left(a^2f+b^2d\right)-2acdf+c^2d^2\right)^2}$$
$$-\frac{f\log\left(a+bx+cx^2\right)(B(-f(b^2d-a^2f)-2acdf+c^2d^2)+2Abf(cd-af))}{2\left(f\left(a^2f+b^2d\right)-2acdf+c^2d^2\right)^2}$$
$$+\frac{f\log\left(d+fx^2\right)(B(-f(b^2d-a^2f)-2acdf+c^2d^2)+2Abf(cd-af))}{2\left(f\left(a^2f+b^2d\right)-2acdf+c^2d^2\right)^2}$$
$$-\frac{\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)(-4Ab^2cf(3a^2f^2-3acdf+2c^2d^2)+b^3Bf(-a^2f^2-4acdf+5c^2d^2)+2bBc(3a^3f^3+}{(b^2-4ac)^{3/2}\left(f\left(a^2f+b^2d\right)-2\right.}$$
$$+\frac{-(Ab-aB)(-2acf+b^2f+2c^2d)-cx(2Ac(cd-af)-bB(af+cd)+Ab^2f)+Abc(af+cd)}{(b^2-4ac)(a+bx+cx^2)((cd-af)^2+b^2df)}$$

```
[In] Int[(A + B*x)/((a + b*x + c*x^2)^2*(d + f*x^2)),x]
```

```
[Out] (A*b*c*(c*d + a*f) - (A*b - a*B)*(2*c^2*d + b^2*f - 2*a*c*f) - c*(A*b^2*f +
 2*A*c*(c*d - a*f) - b*B*(c*d + a*f))*x)/((b^2 - 4*a*c)*(b^2*d*f + (c*d - a
*f)^2)*(a + b*x + c*x^2)) - (f^(3/2)*(A*b^2*d*f + 2*b*B*d*(c*d - a*f) - A*(
c*d - a*f)^2)*ArcTan[(Sqrt[f]*x)/Sqrt[d]])/(Sqrt[d]*(c^2*d^2 - 2*a*c*d*f +
f*(b^2*d + a^2*f))^2) - ((b^5*B*d*f^2 - 2*A*b^4*f^2*(c*d - a*f) - 4*A*c^2*(
c*d - 3*a*f)*(c*d - a*f)^2 + b^3*B*f*(5*c^2*d^2 - 4*a*c*d*f - a^2*f^2) - 4*
A*b^2*c*f*(2*c^2*d^2 - 3*a*c*d*f + 3*a^2*f^2) + 2*b*B*c*(c^3*d^3 - 7*a*c^2*
d^2*f + 3*a^2*c*d*f^2 + 3*a^3*f^3))*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])
/((b^2 - 4*a*c)^(3/2)*(c^2*d^2 - 2*a*c*d*f + f*(b^2*d + a^2*f))^2) - (f*(2*
A*b*f*(c*d - a*f) + B*(c^2*d^2 - 2*a*c*d*f - f*(b^2*d - a^2*f)))*Log[a + b*
x + c*x^2])/(2*(c^2*d^2 - 2*a*c*d*f + f*(b^2*d + a^2*f))^2) + (f*(2*A*b*f*(
c*d - a*f) + B*(c^2*d^2 - 2*a*c*d*f - f*(b^2*d - a^2*f)))*Log[d + f*x^2])/(
2*(c^2*d^2 - 2*a*c*d*f + f*(b^2*d + a^2*f))^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x + c*x^2)^(p + 1)*((d + f*x^2)^
(q + 1)/((b^2 - 4*a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)))*((g*c)*((-b)*(c*
d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f)) + c*(g*(2*c^2*d + b^2
*f - c*(2*a*f)) - h*(b*c*d + a*b*f))*x), x] + Dist[1/((b^2 - 4*a*c)*(b^2*d*
f + (c*d - a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Si
mp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d)*((-b)*f))*(p + 1) + (b^2*(g*f) - b*
(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(a*f*(p + 1) - c*d*(p + 2)) - (2*f*(
(g*c)*((-b)*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f)))*(p +
q + 2) - (b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(b*f*(p + 1)
))*x - c*f*(b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(2*p + 2*q
 + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*
```

```
a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] &&  !( !IntegerQ[p
] && ILtQ[q, -1])
```

Rule 1088

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*((d_) + (f_.)*(x_)^2)), x_Symbol] :> With[{q = c^2*d^2 + b^2*d*f - 2*a*c*d
*f + a^2*f^2}, Dist[1/q, Int[(A*c^2*d - a*c*C*d + A*b^2*f - a*b*B*f - a*A*c
*f + a^2*C*f + c*(B*c*d - b*C*d + A*b*f - a*B*f)*x)/(a + b*x + c*x^2), x],
x] + Dist[1/q, Int[(c*C*d^2 + b*B*d*f - A*c*d*f - a*C*d*f + a*A*f^2 - f*(B*
c*d - b*C*d + A*b*f - a*B*f)*x)/(d + f*x^2), x], x] /; NeQ[q, 0]] /; FreeQ[
{a, b, c, d, f, A, B, C}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

integral

$$= \frac{Abc(cd+af)-(Ab-aB)\left(2c^2d+b^2f-2acf\right)-c\left(Ab^2f+2Ac(cd-af)-bB(cd+af)\right)x}{\left(b^2-4ac\right)\left(b^2df+(cd-af)^2\right)\left(a+bx+cx^2\right)} - \frac{\int \frac{-\left(\left(bB-2Ac\right)\left(b^2df+(cd-af)^2\right)\right)-af\left(Ab^2f+2Ac(cd-af)-bB(cd+af)\right)+\left(b^2-4ac\right)f(Bcd+Abf-aBf)x+cf\left(Ab^2f+2Ac(cd-af)-bB(cd+a\cdots\right.}{\left(a+bx+cx^2\right)\left(d+fx^2\right)}}{\left(b^2-4ac\right)\left(b^2df+(cd-af)^2\right)}$$

$$= \frac{Abc(cd+af)-(Ab-aB)\left(2c^2d+b^2f-2acf\right)-c\left(Ab^2f+2Ac(cd-af)-bB(cd+af)\right)x}{\left(b^2-4ac\right)\left(b^2df+(cd-af)^2\right)\left(a+bx+cx^2\right)} - \int \frac{-ab\left(b^2-4ac\right)f^2(Bcd+Abf-aBf)-ac^2df\left(Ab^2f+2Ac(cd-af)-bB(cd+af)\right)+a^2cf^2\left(Ab^2f+2Ac(cd-af)-bB(cd+af)\right)+c^2d\left(-\left(\cdots\right.\right.}{\cdots} - \int \frac{b\left(b^2-4ac\right)df^2(Bcd+Abf-aBf)+c^2d^2f\left(Ab^2f+2Ac(cd-af)-bB(cd+af)\right)-acdf^2\left(Ab^2f+2Ac(cd-af)-bB(cd+af)\right)-cdf\left(-\left(\left(bB\cdots\right.\right.\right.}{\cdots}$$

$$= \frac{Abc(cd+af)-(Ab-aB)\left(2c^2d+b^2f-2acf\right)-c\left(Ab^2f+2Ac(cd-af)-bB(cd+af)\right)x}{\left(b^2-4ac\right)\left(b^2df+(cd-af)^2\right)\left(a+bx+cx^2\right)} - \frac{\left(f^2\left(Ab^2df+2bBd(cd-af)-A(cd-af)^2\right)\right)\int \frac{1}{d+fx^2}\,dx}{\left(c^2d^2-2acdf+f\left(b^2d+a^2f\right)\right)^2} + \frac{\left(b^5Bdf^2-2Ab^4f^2(cd-af)-4Ac^2(cd-3af)(cd-af)^2+b^3Bf\left(5c^2d^2-4acdf-a^2f^2\right)-4Ab^2\cdots\right.}{2\left(b^2-4ac\right)\left(c^2d^2-2acdf-\cdots\right.} - \frac{\left(f\left(2Abf(cd-af)+B\left(c^2d^2-2acdf-f\left(b^2d-a^2f\right)\right)\right)\right)\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2\left(c^2d^2-2acdf+f\left(b^2d+a^2f\right)\right)^2} + \frac{\left(f^2\left(2Abf(cd-af)+B\left(c^2d^2-2acdf-f\left(b^2d-a^2f\right)\right)\right)\right)\int \frac{x}{d+fx^2}\,dx}{\left(c^2d^2-2acdf+f\left(b^2d+a^2f\right)\right)^2}$$

$$= \frac{Abc(cd+af)-(Ab-aB)\left(2c^2d+b^2f-2acf\right)-c(Ab^2f+2Ac(cd-af)-bB(cd+af))\,x}{(b^2-4ac)\left(b^2df+(cd-af)^2\right)(a+bx+cx^2)}$$
$$- \frac{f^{3/2}(Ab^2df+2bBd(cd-af)-A(cd-af)^2)\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)}{\sqrt{d}\left(c^2d^2-2acdf+f\left(b^2d+a^2f\right)\right)^2}$$
$$- \frac{f(2Abf(cd-af)+B(c^2d^2-2acdf-f(b^2d-a^2f)))\log\left(a+bx+cx^2\right)}{2\left(c^2d^2-2acdf+f\left(b^2d+a^2f\right)\right)^2}$$
$$+ \frac{f(2Abf(cd-af)+B(c^2d^2-2acdf-f(b^2d-a^2f)))\log\left(d+fx^2\right)}{2\left(c^2d^2-2acdf+f\left(b^2d+a^2f\right)\right)^2}$$
$$- \frac{(b^5Bdf^2-2Ab^4f^2(cd-af)-4Ac^2(cd-3af)(cd-af)^2+b^3Bf(5c^2d^2-4acdf-a^2f^2)-4A}{(b^2-4ac)\left(c^2d^2\right.}$$

$$= \frac{Abc(cd+af)-(Ab-aB)\left(2c^2d+b^2f-2acf\right)-c(Ab^2f+2Ac(cd-af)-bB(cd+af))\,x}{(b^2-4ac)\left(b^2df+(cd-af)^2\right)(a+bx+cx^2)}$$
$$- \frac{f^{3/2}(Ab^2df+2bBd(cd-af)-A(cd-af)^2)\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)}{\sqrt{d}\left(c^2d^2-2acdf+f\left(b^2d+a^2f\right)\right)^2}$$
$$- \frac{(b^5Bdf^2-2Ab^4f^2(cd-af)-4Ac^2(cd-3af)(cd-af)^2+b^3Bf(5c^2d^2-4acdf-a^2f^2)-4A}{(b^2-4ac)^{3/2}\left(c^2d^2-2ac\right.}$$
$$- \frac{f(2Abf(cd-af)+B(c^2d^2-2acdf-f(b^2d-a^2f)))\log\left(a+bx+cx^2\right)}{2\left(c^2d^2-2acdf+f\left(b^2d+a^2f\right)\right)^2}$$
$$+ \frac{f(2Abf(cd-af)+B(c^2d^2-2acdf-f(b^2d-a^2f)))\log\left(d+fx^2\right)}{2\left(c^2d^2-2acdf+f\left(b^2d+a^2f\right)\right)^2}$$

Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 523, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx}{\left(a+bx+cx^2\right)^2\left(d+fx^2\right)}\,dx$$
$$= \frac{-\frac{2\left(c^2d^2-2acdf+f\left(b^2d+a^2f\right)\right)\left(A\left(b^3f+bc(cd-3af)+b^2cfx+2c^2(cd-af)x\right)+B\left(2a^2cf-bc^2dx-a\left(2c^2d+b^2f+bcfx\right)\right)\right)}{(b^2-4ac)(a+x(b+cx))}+\frac{2f^{3/2}\left(-Ab^2df+A(c\right.}{}}{}$$

```
[In] Integrate[(A + B*x)/((a + b*x + c*x^2)^2*(d + f*x^2)),x]
```

```
[Out] ((-2*(c^2*d^2 - 2*a*c*d*f + f*(b^2*d + a^2*f))*(A*(b^3*f + b*c*(c*d - 3*a*f
) + b^2*c*f*x + 2*c^2*(c*d - a*f)*x) + B*(2*a^2*c*f - b*c^2*d*x - a*(2*c^2*
d + b^2*f + b*c*f*x))))/((b^2 - 4*a*c)*(a + x*(b + c*x))) + (2*f^(3/2)*(-(A
*b^2*d*f) + A*(c*d - a*f)^2 + 2*b*B*d*(-(c*d) + a*f))*ArcTan[(Sqrt[f]*x)/Sq
rt[d]])/Sqrt[d] - (2*(b^5*B*d*f^2 - 4*A*c^2*(c*d - 3*a*f)*(c*d - a*f)^2 + 2
```

```
*A*b^4*f^2*(-(c*d) + a*f) - b^3*B*f*(-5*c^2*d^2 + 4*a*c*d*f + a^2*f^2) - 4*
A*b^2*c*f*(2*c^2*d^2 - 3*a*c*d*f + 3*a^2*f^2) + 2*b*B*c*(c^3*d^3 - 7*a*c^2*
d^2*f + 3*a^2*c*d*f^2 + 3*a^3*f^3))*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])
/(-b^2 + 4*a*c)^(3/2) + f*(2*A*b*f*(c*d - a*f) + B*(c^2*d^2 - 2*a*c*d*f + f
*(-(b^2*d) + a^2*f)))*Log[d + f*x^2] + f*(2*A*b*f*(-(c*d) + a*f) + B*(-(c^2
*d^2) + 2*a*c*d*f + f*(b^2*d - a^2*f)))*Log[a + x*(b + c*x)])/(2*(c^2*d^2 -
 2*a*c*d*f + f*(b^2*d + a^2*f))^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1253 vs. 2(580) = 1160.

Time = 1.55 (sec) , antiderivative size = 1254, normalized size of antiderivative = 2.10

method	result	size
default	Expression too large to display	1254
risch	Expression too large to display	3364134

```
[In] int((B*x+A)/(c*x^2+b*x+a)^2/(f*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] f^2/(a^4*f^4-4*a^3*c*d*f^3+2*a^2*b^2*d*f^3+6*a^2*c^2*d^2*f^2-4*a*b^2*c*d^2*
f^2-4*a*c^3*d^3*f+b^4*d^2*f^2+2*b^2*c^2*d^3*f+c^4*d^4)*(1/2*(-2*A*a*b*f^2+2
*A*b*c*d*f+B*a^2*f^2-2*B*a*c*d*f-B*b^2*d*f+B*c^2*d^2)/f*ln(f*x^2+d)+(A*a^2*
f^2-2*A*a*c*d*f-A*b^2*d*f+A*c^2*d^2+2*B*a*b*d*f-2*B*b*c*d^2)/(d*f)^(1/2)*ar
ctan(f*x/(d*f)^(1/2)))-1/(a^4*f^4-4*a^3*c*d*f^3+2*a^2*b^2*d*f^3+6*a^2*c^2*d
^2*f^2-4*a*b^2*c*d^2*f^2-4*a*c^3*d^3*f+b^4*d^2*f^2+2*b^2*c^2*d^3*f+c^4*d^4)
*((c*(2*A*a^3*c*f^3-A*a^2*b^2*f^3-6*A*a^2*c^2*d*f^2+4*A*a*b^2*c*d*f^2+6*A*a
*c^3*d^2*f-A*b^4*d*f^2-3*A*b^2*c^2*d^2*f-2*A*c^4*d^3+B*a^3*b*f^3-B*a^2*b*c*
d*f^2+B*a*b^3*d*f^2-B*a*b*c^2*d^2*f+B*b^3*c*d^2*f+B*b*c^3*d^3)/(4*a*c-b^2)*
x+(3*A*a^3*b*c*f^3-A*a^2*b^3*f^3-7*A*a^2*b*c^2*d*f^2+5*A*a*b^3*c*d*f^2+5*A*
a*b*c^3*d^2*f-A*b^5*d*f^2-2*A*b^3*c^2*d^2*f-A*b*c^4*d^3-2*B*a^4*c*f^3+B*a^3
*b^2*f^3+6*B*a^3*c^2*d*f^2-4*B*a^2*b^2*c*d*f^2-6*B*a^2*c^3*d^2*f+B*a*b^4*d*
f^2+3*B*a*b^2*c^2*d^2*f+2*B*a*c^4*d^3)/(4*a*c-b^2))/(c*x^2+b*x+a)+1/(4*a*c-
b^2)*(1/2*(-8*A*a^2*b*c^2*f^3+2*A*a*b^3*c*f^3+8*A*a*b*c^3*d*f^2-2*A*b^3*c^2
*d*f^2+4*B*a^3*c^2*f^3-B*a^2*b^2*c*f^3-8*B*a^2*c^3*d*f^2-2*B*a*b^2*c^2*d*f^
2+4*B*a*c^4*d^2*f+B*b^4*c*d*f^2-B*b^2*c^3*d^2*f)/c*ln(c*x^2+b*x+a)+2*(6*A*a
^3*c^2*f^3-10*A*a^2*b^2*c*f^3-14*A*a^2*c^3*d*f^2+2*A*a*b^4*f^3+10*A*a*b^2*c
^2*d*f^2+10*A*a*c^4*d^2*f-2*A*b^4*c*d*f^2-4*A*b^2*c^3*d^2*f-2*A*c^5*d^3+5*B
*a^3*b*c*f^3-B*a^2*b^3*f^3-B*a^2*b*c^2*d*f^2-3*B*a*b^3*c*d*f^2-5*B*a*b*c^3*
d^2*f+b^5*B*d*f^2+2*B*b^3*c^2*d^2*f+B*b*c^4*d^3-1/2*(-8*A*a^2*b*c^2*f^3+2*A
*a*b^3*c*f^3+8*A*a*b*c^3*d*f^2-2*A*b^3*c^2*d*f^2+4*B*a^3*c^2*f^3-B*a^2*b^2*
c*f^3-8*B*a^2*c^3*d*f^2-2*B*a*b^2*c^2*d*f^2+4*B*a*c^4*d^2*f+B*b^4*c*d*f^2-B
*b^2*c^3*d^2*f)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)))
)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(a+bx+cx^2)^2(d+fx^2)}\,dx = \text{Timed out}$$

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)^2/(f*x^2+d),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(a+bx+cx^2)^2(d+fx^2)}\,dx = \text{Timed out}$$

```
[In] integrate((B*x+A)/(c*x**2+b*x+a)**2/(f*x**2+d),x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx}{(a+bx+cx^2)^2(d+fx^2)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)^2/(f*x^2+d),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1313 vs. $2(579) = 1158$.

Time = 0.28 (sec) , antiderivative size = 1313, normalized size of antiderivative = 2.20

$$\int \frac{A+Bx}{(a+bx+cx^2)^2(d+fx^2)}\,dx = \text{Too large to display}$$

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)^2/(f*x^2+d),x, algorithm="giac")
```

```
[Out] -1/2*(B*c^2*d^2*f - B*b^2*d*f^2 - 2*B*a*c*d*f^2 + 2*A*b*c*d*f^2 + B*a^2*f^3
 - 2*A*a*b*f^3)*log(c*x^2 + b*x + a)/(c^4*d^4 + 2*b^2*c^2*d^3*f - 4*a*c^3*d
^3*f + b^4*d^2*f^2 - 4*a*b^2*c*d^2*f^2 + 6*a^2*c^2*d^2*f^2 + 2*a^2*b^2*d*f^
3 - 4*a^3*c*d*f^3 + a^4*f^4) + 1/2*(B*c^2*d^2*f - B*b^2*d*f^2 - 2*B*a*c*d*f
^2 + 2*A*b*c*d*f^2 + B*a^2*f^3 - 2*A*a*b*f^3)*log(f*x^2 + d)/(c^4*d^4 + 2*b
^2*c^2*d^3*f - 4*a*c^3*d^3*f + b^4*d^2*f^2 - 4*a*b^2*c*d^2*f^2 + 6*a^2*c^2*
d^2*f^2 + 2*a^2*b^2*d*f^3 - 4*a^3*c*d*f^3 + a^4*f^4) - (2*B*b*c*d^2*f^2 - A
*c^2*d^2*f^2 - 2*B*a*b*d*f^3 + A*b^2*d*f^3 + 2*A*a*c*d*f^3 - A*a^2*f^4)*arc
tan(f*x/sqrt(d*f))/((c^4*d^4 + 2*b^2*c^2*d^3*f - 4*a*c^3*d^3*f + b^4*d^2*f^
2 - 4*a*b^2*c*d^2*f^2 + 6*a^2*c^2*d^2*f^2 + 2*a^2*b^2*d*f^3 - 4*a^3*c*d*f^3
 + a^4*f^4)*sqrt(d*f)) + (2*B*b*c^4*d^3 - 4*A*c^5*d^3 + 5*B*b^3*c^2*d^2*f -
 14*B*a*b*c^3*d^2*f - 8*A*b^2*c^3*d^2*f + 20*A*a*c^4*d^2*f + B*b^5*d*f^2 -
4*B*a*b^3*c*d*f^2 - 2*A*b^4*c*d*f^2 + 6*B*a^2*b*c^2*d*f^2 + 12*A*a*b^2*c^2*
d*f^2 - 28*A*a^2*c^3*d*f^2 - B*a^2*b^3*f^3 + 2*A*a*b^4*f^3 + 6*B*a^3*b*c*f^
3 - 12*A*a^2*b^2*c*f^3 + 12*A*a^3*c^2*f^3)*arctan((2*c*x + b)/sqrt(-b^2 + 4
*a*c))/((b^2*c^4*d^4 - 4*a*c^5*d^4 + 2*b^4*c^2*d^3*f - 12*a*b^2*c^3*d^3*f +
 16*a^2*c^4*d^3*f + b^6*d^2*f^2 - 8*a*b^4*c*d^2*f^2 + 22*a^2*b^2*c^2*d^2*f^
2 - 24*a^3*c^3*d^2*f^2 + 2*a^2*b^4*d*f^3 - 12*a^3*b^2*c*d*f^3 + 16*a^4*c^2*
d*f^3 + a^4*b^2*f^4 - 4*a^5*c*f^4)*sqrt(-b^2 + 4*a*c)) + (2*B*a*c^4*d^3 - A
*b*c^4*d^3 + 3*B*a*b^2*c^2*d^2*f - 2*A*b^3*c^2*d^2*f - 6*B*a^2*c^3*d^2*f +
5*A*a*b*c^3*d^2*f + B*a*b^4*d*f^2 - A*b^5*d*f^2 - 4*B*a^2*b^2*c*d*f^2 + 5*A
*a*b^3*c*d*f^2 + 6*B*a^3*c^2*d*f^2 - 7*A*a^2*b*c^2*d*f^2 + B*a^3*b^2*f^3 -
A*a^2*b^3*f^3 - 2*B*a^4*c*f^3 + 3*A*a^3*b*c*f^3 + (B*b*c^4*d^3 - 2*A*c^5*d^
3 + B*b^3*c^2*d^2*f - B*a*b*c^3*d^2*f - 3*A*b^2*c^3*d^2*f + 6*A*a*c^4*d^2*f
 + B*a*b^3*c*d*f^2 - A*b^4*c*d*f^2 - B*a^2*b*c^2*d*f^2 + 4*A*a*b^2*c^2*d*f^
2 - 6*A*a^2*c^3*d*f^2 + B*a^3*b*c*f^3 - A*a^2*b^2*c*f^3 + 2*A*a^3*c^2*f^3)*
x)/((c^2*d^2 + b^2*d*f - 2*a*c*d*f + a^2*f^2)^2*(c*x^2 + b*x + a)*(b^2 - 4*
a*c))
```

Mupad [B] (verification not implemented)

Time = 18.33 (sec) , antiderivative size = 23006, normalized size of antiderivative = 38.60

$$\int \frac{A+Bx}{(a+bx+cx^2)^2\,(d+fx^2)}\,dx = \text{Too large to display}$$

```
[In] int((A + B*x)/((d + f*x^2)*(a + b*x + c*x^2)^2),x)
```

```
[Out] ((A*b^3*f + A*b*c^2*d - 2*B*a*c^2*d - B*a*b^2*f + 2*B*a^2*c*f - 3*A*a*b*c*f
)/((4*a*c - b^2)*(a^2*f^2 + c^2*d^2 + b^2*d*f - 2*a*c*d*f)) - (x*(2*A*a*c^2
*f - 2*A*c^3*d + B*b*c^2*d - A*b^2*c*f + B*a*b*c*f))/((4*a*c - b^2)*(a^2*f^
2 + c^2*d^2 + b^2*d*f - 2*a*c*d*f)))/(a + b*x + c*x^2) + symsum(log((x*(4*A
^3*b^3*c^4*f^6 + 16*B^3*a^3*c^4*f^6 - 3*B^3*a^2*b^2*c^3*f^6 + B^3*b^2*c^5*d
^2*f^4 - 16*A^3*a*b*c^5*f^6 + 20*A^2*B*a^2*c^5*f^6 - 3*A^2*B*b^4*c^3*f^6 +
4*A^2*B*c^7*d^2*f^4 - 16*B^3*a^2*c^5*d*f^5 + 6*B^3*a*b^2*c^4*d*f^5 - 24*A^2
```

*B*a*c^6*d*f^5 + 6*A*B^2*a*b^3*c^3*f^6 - 28*A*B^2*a^2*b*c^4*f^6 + 8*A^2*B*a
*b^2*c^4*f^6 - 4*A*B^2*b*c^6*d^2*f^4 - 6*A*B^2*b^3*c^4*d*f^5 + 8*A^2*B*b^2*
c^5*d*f^5 + 16*A*B^2*a*b*c^5*d*f^5))/(16*a^2*c^6*d^4 + a^4*b^4*f^4 + b^4*c^
4*d^4 + 16*a^6*c^2*f^4 + b^8*d^2*f^2 - 8*a*b^2*c^5*d^4 - 8*a^5*b^2*c*f^4 +
2*a^2*b^6*d*f^3 - 64*a^3*c^5*d^3*f - 64*a^5*c^3*d*f^3 + 2*b^6*c^2*d^3*f + 9
6*a^4*c^4*d^2*f^2 + 54*a^2*b^4*c^2*d^2*f^2 - 112*a^3*b^2*c^3*d^2*f^2 - 20*a
*b^4*c^3*d^3*f - 12*a*b^6*c*d^2*f^2 - 20*a^3*b^4*c*d*f^3 + 64*a^2*b^2*c^4*d
^3*f + 64*a^4*b^2*c^2*d*f^3) - root(2560*a^3*b^2*c^9*d^8*f*z^4 - 1152*a^2*b
^4*c^8*d^8*f*z^4 + 384*a^5*b^8*c*d^3*f^6*z^4 + 384*a*b^8*c^5*d^7*f^2*z^4 +
288*a^3*b^10*c*d^4*f^5*z^4 + 288*a*b^10*c^3*d^6*f^3*z^4 + 224*a^7*b^6*c*d^2
*f^7*z^4 - 192*a^10*b^2*c^2*d*f^8*z^4 + 224*a*b^6*c^7*d^8*f*z^4 + 80*a*b^12
*c*d^5*f^4*z^4 + 48*a^9*b^4*c*d*f^8*z^4 - 33920*a^6*b^2*c^6*d^5*f^4*z^4 + 2
7936*a^5*b^4*c^5*d^5*f^4*z^4 + 26112*a^7*b^2*c^5*d^4*f^5*z^4 + 26112*a^5*b^
2*c^7*d^6*f^3*z^4 - 20352*a^6*b^4*c^4*d^4*f^5*z^4 - 20352*a^4*b^4*c^6*d^6*f
^3*z^4 - 13080*a^4*b^6*c^4*d^5*f^4*z^4 - 11520*a^8*b^2*c^4*d^3*f^6*z^4 - 11
520*a^4*b^2*c^8*d^7*f^2*z^4 + 8736*a^5*b^6*c^3*d^4*f^5*z^4 + 8736*a^3*b^6*c
^5*d^6*f^3*z^4 + 7488*a^7*b^4*c^3*d^3*f^6*z^4 + 7488*a^3*b^4*c^7*d^7*f^2*z^
4 + 3840*a^3*b^8*c^3*d^5*f^4*z^4 + 2560*a^9*b^2*c^3*d^2*f^7*z^4 - 2416*a^6*
b^6*c^2*d^3*f^6*z^4 - 2416*a^2*b^6*c^6*d^7*f^2*z^4 - 2160*a^4*b^8*c^2*d^4*f
^5*z^4 - 2160*a^2*b^8*c^4*d^6*f^3*z^4 - 1152*a^8*b^4*c^2*d^2*f^7*z^4 - 720*
a^2*b^10*c^2*d^5*f^4*z^4 - 16*b^8*c^6*d^8*f*z^4 - 2048*a^4*c^10*d^8*f*z^4 +
256*a^11*c^3*d*f^8*z^4 - 4*a^8*b^6*d*f^8*z^4 + 48*a*b^4*c^9*d^9*z^4 - 24*b
^10*c^4*d^7*f^2*z^4 - 16*b^12*c^2*d^6*f^3*z^4 + 17920*a^7*c^7*d^5*f^4*z^4 -
14336*a^8*c^6*d^4*f^5*z^4 - 14336*a^6*c^8*d^6*f^3*z^4 + 7168*a^9*c^5*d^3*f
^6*z^4 + 7168*a^5*c^9*d^7*f^2*z^4 - 2048*a^10*c^4*d^2*f^7*z^4 - 24*a^4*b^10
*d^3*f^6*z^4 - 16*a^6*b^8*d^2*f^7*z^4 - 16*a^2*b^12*d^4*f^5*z^4 - 192*a^2*b
^2*c^10*d^9*z^4 - 4*b^14*d^5*f^4*z^4 - 4*b^6*c^8*d^9*z^4 + 256*a^3*c^11*d^9
*z^4 + 912*A*B*a^6*b*c^3*d*f^6*z^2 + 192*A*B*a^4*b^5*c*d*f^6*z^2 + 920*A*B*
a^4*b^3*c^3*d^2*f^5*z^2 - 480*A*B*a^2*b^5*c^3*d^3*f^4*z^2 - 336*A*B*a^2*b^3
*c^5*d^4*f^3*z^2 - 272*A*B*a^3*b^3*c^4*d^3*f^4*z^2 + 240*A*B*a^3*b^5*c^2*d^
2*f^5*z^2 + 192*A*B*a*b*c^8*d^6*f*z^2 - 2496*A*B*a^5*b*c^4*d^2*f^5*z^2 + 18
72*A*B*a^4*b*c^5*d^3*f^4*z^2 - 744*A*B*a^5*b^3*c^2*d*f^6*z^2 - 720*A*B*a^2*
b*c^7*d^5*f^2*z^2 + 504*A*B*a*b^3*c^6*d^5*f^2*z^2 + 256*A*B*a^3*b*c^6*d^4*f
^3*z^2 + 168*A*B*a*b^7*c^2*d^3*f^4*z^2 - 144*A*B*a^2*b^7*c*d^2*f^5*z^2 + 14
4*A*B*a*b^5*c^4*d^4*f^3*z^2 - 56*B^2*a*b^2*c^7*d^6*f*z^2 - 36*B^2*a^5*b^4*c
*d*f^6*z^2 - 16*B^2*a*b^8*c*d^3*f^4*z^2 - 164*A^2*a^3*b^6*c*d*f^6*z^2 - 16*
A^2*a*b^8*c*d^2*f^5*z^2 - 96*A*B*b^5*c^5*d^5*f^2*z^2 - 24*A*B*b^7*c^3*d^4*f
^3*z^2 - 580*B^2*a^4*b^2*c^4*d^3*f^4*z^2 + 536*B^2*a^3*b^4*c^3*d^3*f^4*z^2
- 348*B^2*a^4*b^4*c^2*d^2*f^5*z^2 + 316*B^2*a^2*b^2*c^6*d^5*f^2*z^2 + 200*B
^2*a^5*b^2*c^3*d^2*f^5*z^2 - 120*B^2*a^2*b^4*c^4*d^4*f^3*z^2 - 66*B^2*a^2*b
^6*c^2*d^3*f^4*z^2 - 16*B^2*a^3*b^2*c^5*d^4*f^3*z^2 + 1952*A^2*a^4*b^2*c^4*
d^2*f^5*z^2 - 1792*A^2*a^3*b^2*c^5*d^3*f^4*z^2 - 1272*A^2*a^3*b^4*c^3*d^2*f
^5*z^2 + 976*A^2*a^2*b^2*c^6*d^4*f^3*z^2 + 960*A^2*a^2*b^4*c^4*d^3*f^4*z^2
+ 282*A^2*a^2*b^6*c^2*d^2*f^5*z^2 - 72*A*B*b^3*c^7*d^6*f*z^2 - 16*A*B*b^9*c
*d^3*f^4*z^2 - 16*A*B*a^3*b^7*d*f^6*z^2 + 16*A*B*a*b^9*d^2*f^5*z^2 - 180*B^

2*a*b^4*c^5*d^5*f^2*z^2 + 132*B^2*a^6*b^2*c^2*d*f^6*z^2 + 108*B^2*a^3*b^6*c
*d^2*f^5*z^2 + 20*B^2*a*b^6*c^3*d^4*f^3*z^2 - 736*A^2*a^5*b^2*c^3*d*f^6*z^2
 + 624*A^2*a^4*b^4*c^2*d*f^6*z^2 - 416*A^2*a*b^2*c^7*d^5*f^2*z^2 - 276*A^2*
a*b^4*c^5*d^4*f^3*z^2 - 196*A^2*a*b^6*c^3*d^3*f^4*z^2 + 31*B^2*b^6*c^4*d^5*
f^2*z^2 + 2*B^2*b^8*c^2*d^4*f^3*z^2 - 768*B^2*a^5*c^5*d^3*f^4*z^2 + 512*B^2
*a^6*c^4*d^2*f^5*z^2 + 512*B^2*a^4*c^6*d^4*f^3*z^2 - 128*B^2*a^3*c^7*d^5*f^
2*z^2 + 80*A^2*b^4*c^6*d^5*f^2*z^2 + 31*A^2*b^6*c^4*d^4*f^3*z^2 + 14*A^2*b^
8*c^2*d^3*f^4*z^2 - 1152*A^2*a^3*c^7*d^4*f^3*z^2 + 1008*A^2*a^4*c^6*d^3*f^4
*z^2 + 624*A^2*a^2*c^8*d^5*f^2*z^2 - 288*A^2*a^5*c^5*d^2*f^5*z^2 - 10*B^2*a
^2*b^8*d^2*f^5*z^2 - 48*A^2*a^6*b^2*c^2*f^7*z^2 - 16*A*B*b*c^9*d^7*z^2 + 20
*B^2*b^4*c^6*d^6*f*z^2 - 128*B^2*a^7*c^3*d*f^6*z^2 + 64*A^2*b^2*c^8*d^6*f*z
^2 - 112*A^2*a^6*c^4*d*f^6*z^2 + 3*B^2*a^4*b^6*d*f^6*z^2 + 14*A^2*a^2*b^8*d
*f^6*z^2 + 12*A^2*a^5*b^4*c*f^7*z^2 - 160*A^2*a*c^9*d^6*f*z^2 + 3*B^2*b^10*
d^3*f^4*z^2 - A^2*b^10*d^2*f^5*z^2 + 64*A^2*a^7*c^3*f^7*z^2 + 4*B^2*b^2*c^8
*d^7*z^2 - A^2*a^4*b^6*f^7*z^2 + 16*A^2*c^10*d^7*z^2 - 160*A*B^2*a*b*c^6*d^
4*f^2*z + 112*A*B^2*a^4*b*c^3*d*f^5*z - 24*A*B^2*a^2*b^5*c*d*f^5*z + 480*A^
2*B*a^2*b^2*c^4*d^2*f^4*z - 176*A*B^2*a^2*b^3*c^3*d^2*f^4*z - 10*A^2*B*a*b^
6*c*d*f^5*z + 384*A*B^2*a^2*b*c^5*d^3*f^3*z - 352*A*B^2*a^3*b*c^4*d^2*f^4*z
 - 288*A^2*B*a*b^2*c^5*d^3*f^3*z - 160*A^2*B*a^3*b^2*c^3*d*f^5*z - 148*A^2*
B*a*b^4*c^3*d^2*f^4*z + 112*A*B^2*a*b^3*c^4*d^3*f^3*z + 72*A^2*B*a^2*b^4*c^
2*d*f^5*z + 72*A*B^2*a*b^5*c^2*d^2*f^4*z + 48*A*B^2*a^3*b^3*c^2*d*f^5*z + 4
8*B^3*a*b^2*c^5*d^4*f^2*z - 36*B^3*a^4*b^2*c^2*d*f^5*z - 4*B^3*a*b^4*c^3*d^
3*f^3*z - 480*A^3*a^2*b*c^5*d^2*f^4*z - 160*A^3*a^2*b^3*c^3*d*f^5*z + 128*A
^3*a*b^3*c^4*d^2*f^4*z + 112*A^2*B*b^4*c^4*d^3*f^3*z - 64*A*B^2*b^5*c^3*d^3
*f^3*z + 16*A^2*B*b^2*c^6*d^4*f^2*z + 16*A*B^2*b^3*c^5*d^4*f^2*z - A^2*B*b^
6*c^2*d^2*f^4*z + 448*A^2*B*a^3*c^5*d^2*f^4*z - 352*A^2*B*a^2*c^6*d^3*f^3*z
 - 48*A^2*B*a^4*b^2*c^2*f^6*z + 12*B^3*a^3*b^4*c*d*f^5*z - 10*B^3*a*b^6*c*d
^2*f^4*z + 416*A^3*a^3*b*c^4*d*f^5*z + 224*A^3*a*b*c^6*d^3*f^3*z + 24*A^3*a
*b^5*c^2*d*f^5*z - 2*A*B^2*b^7*c*d^2*f^4*z - 272*A^2*B*a^4*c^4*d*f^5*z + 12
8*A^2*B*a*c^7*d^4*f^2*z + 12*A^2*B*a^3*b^4*c*f^6*z - 120*B^3*a^2*b^2*c^4*d^
3*f^3*z + 112*B^3*a^3*b^2*c^3*d^2*f^4*z + 16*A*B^2*b*c^7*d^5*f*z + 2*A*B^2*
a*b^7*d*f^5*z - 2*A^3*b^7*c*d*f^5*z - 16*A^2*B*c^8*d^5*f*z + 11*B^3*b^6*c^2
*d^3*f^3*z - 8*B^3*b^4*c^4*d^4*f^2*z - 64*A^3*b^3*c^5*d^3*f^3*z + 96*A^3*a^
3*b^3*c^2*f^6*z - 4*B^3*b^2*c^6*d^5*f*z - 32*A^3*b*c^7*d^4*f^2*z - B^3*a^2*
b^6*d*f^5*z - 128*A^3*a^4*b*c^3*f^6*z - 24*A^3*a^2*b^5*c*f^6*z + 64*A^2*B*a
^5*c^3*f^6*z - A^2*B*a^2*b^6*f^6*z + A^2*B*b^8*d*f^5*z + 2*A^3*a*b^7*f^6*z
+ B^3*b^8*d^2*f^4*z + 32*A^3*B*a*b*c^4*d*f^4 - 18*A^2*B^2*a*b^2*c^3*d*f^4 +
 32*A*B^3*a*b*c^4*d^2*f^3 - 28*A*B^3*a^2*b*c^3*d*f^4 + 6*A*B^3*a*b^3*c^2*d*
f^4 - 10*A^3*B*b^3*c^3*d*f^4 - 4*A^3*B*b*c^5*d^2*f^3 - 4*A*B^3*b*c^5*d^3*f^
2 - 28*A^3*B*a^2*b*c^3*f^5 + 6*A^3*B*a*b^3*c^2*f^5 + 9*A^2*B^2*b^2*c^4*d^2*
f^3 - 3*A^2*B^2*a^2*b^2*c^2*f^5 - 10*B^4*a*b^2*c^3*d^2*f^3 - 3*B^4*a^2*b^2*
c^2*d*f^4 - 10*A*B^3*b^3*c^3*d^2*f^3 + 3*A^2*B^2*b^4*c^2*d*f^4 + 36*A^2*B^2
*a^2*c^4*d*f^4 - 24*A^2*B^2*a*c^5*d^2*f^3 + 4*A^2*B^2*c^6*d^3*f^2 + 16*A^2*
B^2*a^3*c^3*f^5 + 16*B^4*a^3*c^3*d*f^4 + 8*A^4*b^2*c^4*d*f^4 - 8*A^4*a*b^2*
c^3*f^5 - 24*A^4*a*c^5*d*f^4 + 3*B^4*b^4*c^2*d^2*f^3 + 4*A^4*c^6*d^2*f^3 +

36*A^4*a^2*c^4*f^5 + B^4*b^2*c^4*d^3*f^2, z, k)*(root(2560*a^3*b^2*c^9*d^8*
f*z^4 - 1152*a^2*b^4*c^8*d^8*f*z^4 + 384*a^5*b^8*c*d^3*f^6*z^4 + 384*a*b^8*
c^5*d^7*f^2*z^4 + 288*a^3*b^10*c*d^4*f^5*z^4 + 288*a*b^10*c^3*d^6*f^3*z^4 +
224*a^7*b^6*c*d^2*f^7*z^4 - 192*a^10*b^2*c^2*d*f^8*z^4 + 224*a*b^6*c^7*d^8
*f*z^4 + 80*a*b^12*c*d^5*f^4*z^4 + 48*a^9*b^4*c*d*f^8*z^4 - 33920*a^6*b^2*c
^6*d^5*f^4*z^4 + 27936*a^5*b^4*c^5*d^5*f^4*z^4 + 26112*a^7*b^2*c^5*d^4*f^5*
z^4 + 26112*a^5*b^2*c^7*d^6*f^3*z^4 - 20352*a^6*b^4*c^4*d^4*f^5*z^4 - 20352
*a^4*b^4*c^6*d^6*f^3*z^4 - 13080*a^4*b^6*c^4*d^5*f^4*z^4 - 11520*a^8*b^2*c^
4*d^3*f^6*z^4 - 11520*a^4*b^2*c^8*d^7*f^2*z^4 + 8736*a^5*b^6*c^3*d^4*f^5*z^
4 + 8736*a^3*b^6*c^5*d^6*f^3*z^4 + 7488*a^7*b^4*c^3*d^3*f^6*z^4 + 7488*a^3*
b^4*c^7*d^7*f^2*z^4 + 3840*a^3*b^8*c^3*d^5*f^4*z^4 + 2560*a^9*b^2*c^3*d^2*f
^7*z^4 - 2416*a^6*b^6*c^2*d^3*f^6*z^4 - 2416*a^2*b^6*c^6*d^7*f^2*z^4 - 2160
*a^4*b^8*c^2*d^4*f^5*z^4 - 2160*a^2*b^8*c^4*d^6*f^3*z^4 - 1152*a^8*b^4*c^2*
d^2*f^7*z^4 - 720*a^2*b^10*c^2*d^5*f^4*z^4 - 16*b^8*c^6*d^8*f*z^4 - 2048*a^
4*c^10*d^8*f*z^4 + 256*a^11*c^3*d*f^8*z^4 - 4*a^8*b^6*d*f^8*z^4 + 48*a*b^4*
c^9*d^9*z^4 - 24*b^10*c^4*d^7*f^2*z^4 - 16*b^12*c^2*d^6*f^3*z^4 + 17920*a^7
*c^7*d^5*f^4*z^4 - 14336*a^8*c^6*d^4*f^5*z^4 - 14336*a^6*c^8*d^6*f^3*z^4 +
7168*a^9*c^5*d^3*f^6*z^4 + 7168*a^5*c^9*d^7*f^2*z^4 - 2048*a^10*c^4*d^2*f^7
*z^4 - 24*a^4*b^10*d^3*f^6*z^4 - 16*a^6*b^8*d^2*f^7*z^4 - 16*a^2*b^12*d^4*f
^5*z^4 - 192*a^2*b^2*c^10*d^9*z^4 - 4*b^14*d^5*f^4*z^4 - 4*b^6*c^8*d^9*z^4
+ 256*a^3*c^11*d^9*z^4 + 912*A*B*a^6*b*c^3*d*f^6*z^2 + 192*A*B*a^4*b^5*c*d*
f^6*z^2 + 920*A*B*a^4*b^3*c^3*d^2*f^5*z^2 - 480*A*B*a^2*b^5*c^3*d^3*f^4*z^2
- 336*A*B*a^2*b^3*c^5*d^4*f^3*z^2 - 272*A*B*a^3*b^3*c^4*d^3*f^4*z^2 + 240*
A*B*a^3*b^5*c^2*d^2*f^5*z^2 + 192*A*B*a*b*c^8*d^6*f*z^2 - 2496*A*B*a^5*b*c^
4*d^2*f^5*z^2 + 1872*A*B*a^4*b*c^5*d^3*f^4*z^2 - 744*A*B*a^5*b^3*c^2*d*f^6*
z^2 - 720*A*B*a^2*b*c^7*d^5*f^2*z^2 + 504*A*B*a*b^3*c^6*d^5*f^2*z^2 + 256*A
*B*a^3*b*c^6*d^4*f^3*z^2 + 168*A*B*a*b^7*c^2*d^3*f^4*z^2 - 144*A*B*a^2*b^7*
c*d^2*f^5*z^2 + 144*A*B*a*b^5*c^4*d^4*f^3*z^2 - 56*B^2*a*b^2*c^7*d^6*f*z^2
- 36*B^2*a^5*b^4*c*d*f^6*z^2 - 16*B^2*a*b^8*c*d^3*f^4*z^2 - 164*A^2*a^3*b^6
*c*d*f^6*z^2 - 16*A^2*a*b^8*c*d^2*f^5*z^2 - 96*A*B*b^5*c^5*d^5*f^2*z^2 - 24
*A*B*b^7*c^3*d^4*f^3*z^2 - 580*B^2*a^4*b^2*c^4*d^3*f^4*z^2 + 536*B^2*a^3*b^
4*c^3*d^3*f^4*z^2 - 348*B^2*a^4*b^4*c^2*d^2*f^5*z^2 + 316*B^2*a^2*b^2*c^6*d
^5*f^2*z^2 + 200*B^2*a^5*b^2*c^3*d^2*f^5*z^2 - 120*B^2*a^2*b^4*c^4*d^4*f^3*
z^2 - 66*B^2*a^2*b^6*c^2*d^3*f^4*z^2 - 16*B^2*a^3*b^2*c^5*d^4*f^3*z^2 + 195
2*A^2*a^4*b^2*c^4*d^2*f^5*z^2 - 1792*A^2*a^3*b^2*c^5*d^3*f^4*z^2 - 1272*A^2
*a^3*b^4*c^3*d^2*f^5*z^2 + 976*A^2*a^2*b^2*c^6*d^4*f^3*z^2 + 960*A^2*a^2*b^
4*c^4*d^3*f^4*z^2 + 282*A^2*a^2*b^6*c^2*d^2*f^5*z^2 - 72*A*B*b^3*c^7*d^6*f*
z^2 - 16*A*B*b^9*c*d^3*f^4*z^2 - 16*A*B*a^3*b^7*d*f^6*z^2 + 16*A*B*a*b^9*d^
2*f^5*z^2 - 180*B^2*a*b^4*c^5*d^5*f^2*z^2 + 132*B^2*a^6*b^2*c^2*d*f^6*z^2 +
108*B^2*a^3*b^6*c*d^2*f^5*z^2 + 20*B^2*a*b^6*c^3*d^4*f^3*z^2 - 736*A^2*a^5
*b^2*c^3*d*f^6*z^2 + 624*A^2*a^4*b^4*c^2*d*f^6*z^2 - 416*A^2*a*b^2*c^7*d^5*
f^2*z^2 - 276*A^2*a*b^4*c^5*d^4*f^3*z^2 - 196*A^2*a*b^6*c^3*d^3*f^4*z^2 + 3
1*B^2*b^6*c^4*d^5*f^2*z^2 + 2*B^2*b^8*c^2*d^4*f^3*z^2 - 768*B^2*a^5*c^5*d^3
*f^4*z^2 + 512*B^2*a^6*c^4*d^2*f^5*z^2 + 512*B^2*a^4*c^6*d^4*f^3*z^2 - 128*
B^2*a^3*c^7*d^5*f^2*z^2 + 80*A^2*b^4*c^6*d^5*f^2*z^2 + 31*A^2*b^6*c^4*d^4*f

^3*z^2 + 14*A^2*b^8*c^2*d^3*f^4*z^2 - 1152*A^2*a^3*c^7*d^4*f^3*z^2 + 1008*A
^2*a^4*c^6*d^3*f^4*z^2 + 624*A^2*a^2*c^8*d^5*f^2*z^2 - 288*A^2*a^5*c^5*d^2*
f^5*z^2 - 10*B^2*a^2*b^8*d^2*f^5*z^2 - 48*A^2*a^6*b^2*c^2*f^7*z^2 - 16*A*B*
b*c^9*d^7*z^2 + 20*B^2*b^4*c^6*d^6*f*z^2 - 128*B^2*a^7*c^3*d*f^6*z^2 + 64*A
^2*b^2*c^8*d^6*f*z^2 - 112*A^2*a^6*c^4*d*f^6*z^2 + 3*B^2*a^4*b^6*d*f^6*z^2
+ 14*A^2*a^2*b^8*d*f^6*z^2 + 12*A^2*a^5*b^4*c*f^7*z^2 - 160*A^2*a*c^9*d^6*f
*z^2 + 3*B^2*b^10*d^3*f^4*z^2 - A^2*b^10*d^2*f^5*z^2 + 64*A^2*a^7*c^3*f^7*z
^2 + 4*B^2*b^2*c^8*d^7*z^2 - A^2*a^4*b^6*f^7*z^2 + 16*A^2*c^10*d^7*z^2 - 16
0*A*B^2*a*b*c^6*d^4*f^2*z + 112*A*B^2*a^4*b*c^3*d*f^5*z - 24*A*B^2*a^2*b^5*
c*d*f^5*z + 480*A^2*B*a^2*b^2*c^4*d^2*f^4*z - 176*A*B^2*a^2*b^3*c^3*d^2*f^4
*z - 10*A^2*B*a*b^6*c*d*f^5*z + 384*A*B^2*a^2*b*c^5*d^3*f^3*z - 352*A*B^2*a
^3*b*c^4*d^2*f^4*z - 288*A^2*B*a*b^2*c^5*d^3*f^3*z - 160*A^2*B*a^3*b^2*c^3*
d*f^5*z - 148*A^2*B*a*b^4*c^3*d^2*f^4*z + 112*A*B^2*a*b^3*c^4*d^3*f^3*z + 7
2*A^2*B*a^2*b^4*c^2*d*f^5*z + 72*A*B^2*a*b^5*c^2*d^2*f^4*z + 48*A*B^2*a^3*b
^3*c^2*d*f^5*z + 48*B^3*a*b^2*c^5*d^4*f^2*z - 36*B^3*a^4*b^2*c^2*d*f^5*z -
4*B^3*a*b^4*c^3*d^3*f^3*z - 480*A^3*a^2*b*c^5*d^2*f^4*z - 160*A^3*a^2*b^3*c
^3*d*f^5*z + 128*A^3*a*b^3*c^4*d^2*f^4*z + 112*A^2*B*b^4*c^4*d^3*f^3*z - 64
*A*B^2*b^5*c^3*d^3*f^3*z + 16*A^2*B*b^2*c^6*d^4*f^2*z + 16*A*B^2*b^3*c^5*d^
4*f^2*z - A^2*B*b^6*c^2*d^2*f^4*z + 448*A^2*B*a^3*c^5*d^2*f^4*z - 352*A^2*B
*a^2*c^6*d^3*f^3*z - 48*A^2*B*a^4*b^2*c^2*f^6*z + 12*B^3*a^3*b^4*c*d*f^5*z
- 10*B^3*a*b^6*c*d^2*f^4*z + 416*A^3*a^3*b*c^4*d*f^5*z + 224*A^3*a*b*c^6*d^
3*f^3*z + 24*A^3*a*b^5*c^2*d*f^5*z - 2*A*B^2*b^7*c*d^2*f^4*z - 272*A^2*B*a^
4*c^4*d*f^5*z + 128*A^2*B*a*c^7*d^4*f^2*z + 12*A^2*B*a^3*b^4*c*f^6*z - 120*
B^3*a^2*b^2*c^4*d^3*f^3*z + 112*B^3*a^3*b^2*c^3*d^2*f^4*z + 16*A*B^2*b*c^7*
d^5*f*z + 2*A*B^2*a*b^7*d*f^5*z - 2*A^3*b^7*c*d*f^5*z - 16*A^2*B*c^8*d^5*f*
z + 11*B^3*b^6*c^2*d^3*f^3*z - 8*B^3*b^4*c^4*d^4*f^2*z - 64*A^3*b^3*c^5*d^3
*f^3*z + 96*A^3*a^3*b^3*c^2*f^6*z - 4*B^3*b^2*c^6*d^5*f*z - 32*A^3*b*c^7*d^
4*f^2*z - B^3*a^2*b^6*d*f^5*z - 128*A^3*a^4*b*c^3*f^6*z - 24*A^3*a^2*b^5*c*
f^6*z + 64*A^2*B*a^5*c^3*f^6*z - A^2*B*a^2*b^6*f^6*z + A^2*B*b^8*d*f^5*z +
2*A^3*a*b^7*f^6*z + B^3*b^8*d^2*f^4*z + 32*A^3*B*a*b*c^4*d*f^4 - 18*A^2*B^2
*a*b^2*c^3*d*f^4 + 32*A*B^3*a*b*c^4*d^2*f^3 - 28*A*B^3*a^2*b*c^3*d*f^4 + 6*
A*B^3*a*b^3*c^2*d*f^4 - 10*A^3*B*b^3*c^3*d*f^4 - 4*A^3*B*b*c^5*d^2*f^3 - 4*
A*B^3*b*c^5*d^3*f^2 - 28*A^3*B*a^2*b*c^3*f^5 + 6*A^3*B*a*b^3*c^2*f^5 + 9*A^
2*B^2*b^2*c^4*d^2*f^3 - 3*A^2*B^2*a^2*b^2*c^2*f^5 - 10*B^4*a*b^2*c^3*d^2*f^
3 - 3*B^4*a^2*b^2*c^2*d*f^4 - 10*A*B^3*b^3*c^3*d^2*f^3 + 3*A^2*B^2*b^4*c^2*
d*f^4 + 36*A^2*B^2*a^2*c^4*d*f^4 - 24*A^2*B^2*a*c^5*d^2*f^3 + 4*A^2*B^2*c^6
*d^3*f^2 + 16*A^2*B^2*a^3*c^3*f^5 + 16*B^4*a^3*c^3*d*f^4 + 8*A^4*b^2*c^4*d*
f^4 - 8*A^4*a*b^2*c^3*f^5 - 24*A^4*a*c^5*d*f^4 + 3*B^4*b^4*c^2*d^2*f^3 + 4*
A^4*c^6*d^2*f^3 + 36*A^4*a^2*c^4*f^5 + B^4*b^2*c^4*d^3*f^2, z, k)*(root(256
0*a^3*b^2*c^9*d^8*f*z^4 - 1152*a^2*b^4*c^8*d^8*f*z^4 + 384*a^5*b^8*c*d^3*f^
6*z^4 + 384*a*b^8*c^5*d^7*f^2*z^4 + 288*a^3*b^10*c*d^4*f^5*z^4 + 288*a*b^10
*c^3*d^6*f^3*z^4 + 224*a^7*b^6*c*d^2*f^7*z^4 - 192*a^10*b^2*c^2*d*f^8*z^4 +
224*a*b^6*c^7*d^8*f*z^4 + 80*a*b^12*c*d^5*f^4*z^4 + 48*a^9*b^4*c*d*f^8*z^4
- 33920*a^6*b^2*c^6*d^5*f^4*z^4 + 27936*a^5*b^4*c^5*d^5*f^4*z^4 + 26112*a^
7*b^2*c^5*d^4*f^5*z^4 + 26112*a^5*b^2*c^7*d^6*f^3*z^4 - 20352*a^6*b^4*c^4*d

^4*f^5*z^4 - 20352*a^4*b^4*c^6*d^6*f^3*z^4 - 13080*a^4*b^6*c^4*d^5*f^4*z^4
- 11520*a^8*b^2*c^4*d^3*f^6*z^4 - 11520*a^4*b^2*c^8*d^7*f^2*z^4 + 8736*a^5*
b^6*c^3*d^4*f^5*z^4 + 8736*a^3*b^6*c^5*d^6*f^3*z^4 + 7488*a^7*b^4*c^3*d^3*f
^6*z^4 + 7488*a^3*b^4*c^7*d^7*f^2*z^4 + 3840*a^3*b^8*c^3*d^5*f^4*z^4 + 2560
*a^9*b^2*c^3*d^2*f^7*z^4 - 2416*a^6*b^6*c^2*d^3*f^6*z^4 - 2416*a^2*b^6*c^6*
d^7*f^2*z^4 - 2160*a^4*b^8*c^2*d^4*f^5*z^4 - 2160*a^2*b^8*c^4*d^6*f^3*z^4 -
1152*a^8*b^4*c^2*d^2*f^7*z^4 - 720*a^2*b^10*c^2*d^5*f^4*z^4 - 16*b^8*c^6*d
^8*f*z^4 - 2048*a^4*c^10*d^8*f*z^4 + 256*a^11*c^3*d*f^8*z^4 - 4*a^8*b^6*d*f
^8*z^4 + 48*a*b^4*c^9*d^9*z^4 - 24*b^10*c^4*d^7*f^2*z^4 - 16*b^12*c^2*d^6*f
^3*z^4 + 17920*a^7*c^7*d^5*f^4*z^4 - 14336*a^8*c^6*d^4*f^5*z^4 - 14336*a^6*
c^8*d^6*f^3*z^4 + 7168*a^9*c^5*d^3*f^6*z^4 + 7168*a^5*c^9*d^7*f^2*z^4 - 204
8*a^10*c^4*d^2*f^7*z^4 - 24*a^4*b^10*d^3*f^6*z^4 - 16*a^6*b^8*d^2*f^7*z^4 -
16*a^2*b^12*d^4*f^5*z^4 - 192*a^2*b^2*c^10*d^9*z^4 - 4*b^14*d^5*f^4*z^4 -
4*b^6*c^8*d^9*z^4 + 256*a^3*c^11*d^9*z^4 + 912*A*B*a^6*b*c^3*d*f^6*z^2 + 19
2*A*B*a^4*b^5*c*d*f^6*z^2 + 920*A*B*a^4*b^3*c^3*d^2*f^5*z^2 - 480*A*B*a^2*b
^5*c^3*d^3*f^4*z^2 - 336*A*B*a^2*b^3*c^5*d^4*f^3*z^2 - 272*A*B*a^3*b^3*c^4*
d^3*f^4*z^2 + 240*A*B*a^3*b^5*c^2*d^2*f^5*z^2 + 192*A*B*a*b*c^8*d^6*f*z^2 -
2496*A*B*a^5*b*c^4*d^2*f^5*z^2 + 1872*A*B*a^4*b*c^5*d^3*f^4*z^2 - 744*A*B*
a^5*b^3*c^2*d*f^6*z^2 - 720*A*B*a^2*b*c^7*d^5*f^2*z^2 + 504*A*B*a*b^3*c^6*d
^5*f^2*z^2 + 256*A*B*a^3*b*c^6*d^4*f^3*z^2 + 168*A*B*a*b^7*c^2*d^3*f^4*z^2
- 144*A*B*a^2*b^7*c*d^2*f^5*z^2 + 144*A*B*a*b^5*c^4*d^4*f^3*z^2 - 56*B^2*a*
b^2*c^7*d^6*f*z^2 - 36*B^2*a^5*b^4*c*d*f^6*z^2 - 16*B^2*a*b^8*c*d^3*f^4*z^2
- 164*A^2*a^3*b^6*c*d*f^6*z^2 - 16*A^2*a*b^8*c*d^2*f^5*z^2 - 96*A*B*b^5*c^
5*d^5*f^2*z^2 - 24*A*B*b^7*c^3*d^4*f^3*z^2 - 580*B^2*a^4*b^2*c^4*d^3*f^4*z^
2 + 536*B^2*a^3*b^4*c^3*d^3*f^4*z^2 - 348*B^2*a^4*b^4*c^2*d^2*f^5*z^2 + 316
*B^2*a^2*b^2*c^6*d^5*f^2*z^2 + 200*B^2*a^5*b^2*c^3*d^2*f^5*z^2 - 120*B^2*a^
2*b^4*c^4*d^4*f^3*z^2 - 66*B^2*a^2*b^6*c^2*d^3*f^4*z^2 - 16*B^2*a^3*b^2*c^5
*d^4*f^3*z^2 + 1952*A^2*a^4*b^2*c^4*d^2*f^5*z^2 - 1792*A^2*a^3*b^2*c^5*d^3*
f^4*z^2 - 1272*A^2*a^3*b^4*c^3*d^2*f^5*z^2 + 976*A^2*a^2*b^2*c^6*d^4*f^3*z^
2 + 960*A^2*a^2*b^4*c^4*d^3*f^4*z^2 + 282*A^2*a^2*b^6*c^2*d^2*f^5*z^2 - 72*
A*B*b^3*c^7*d^6*f*z^2 - 16*A*B*b^9*c*d^3*f^4*z^2 - 16*A*B*a^3*b^7*d*f^6*z^2
+ 16*A*B*a*b^9*d^2*f^5*z^2 - 180*B^2*a*b^4*c^5*d^5*f^2*z^2 + 132*B^2*a^6*b
^2*c^2*d*f^6*z^2 + 108*B^2*a^3*b^6*c*d^2*f^5*z^2 + 20*B^2*a*b^6*c^3*d^4*f^3
*z^2 - 736*A^2*a^5*b^2*c^3*d*f^6*z^2 + 624*A^2*a^4*b^4*c^2*d*f^6*z^2 - 416*
A^2*a*b^2*c^7*d^5*f^2*z^2 - 276*A^2*a*b^4*c^5*d^4*f^3*z^2 - 196*A^2*a*b^6*c
^3*d^3*f^4*z^2 + 31*B^2*b^6*c^4*d^5*f^2*z^2 + 2*B^2*b^8*c^2*d^4*f^3*z^2 - 7
68*B^2*a^5*c^5*d^3*f^4*z^2 + 512*B^2*a^6*c^4*d^2*f^5*z^2 + 512*B^2*a^4*c^6*
d^4*f^3*z^2 - 128*B^2*a^3*c^7*d^5*f^2*z^2 + 80*A^2*b^4*c^6*d^5*f^2*z^2 + 31
*A^2*b^6*c^4*d^4*f^3*z^2 + 14*A^2*b^8*c^2*d^3*f^4*z^2 - 1152*A^2*a^3*c^7*d^
4*f^3*z^2 + 1008*A^2*a^4*c^6*d^3*f^4*z^2 + 624*A^2*a^2*c^8*d^5*f^2*z^2 - 28
8*A^2*a^5*c^5*d^2*f^5*z^2 - 10*B^2*a^2*b^8*d^2*f^5*z^2 - 48*A^2*a^6*b^2*c^2
*f^7*z^2 - 16*A*B*b*c^9*d^7*z^2 + 20*B^2*b^4*c^6*d^6*f*z^2 - 128*B^2*a^7*c^
3*d*f^6*z^2 + 64*A^2*b^2*c^8*d^6*f*z^2 - 112*A^2*a^6*c^4*d*f^6*z^2 + 3*B^2*
a^4*b^6*d*f^6*z^2 + 14*A^2*a^2*b^8*d*f^6*z^2 + 12*A^2*a^5*b^4*c*f^7*z^2 - 1
60*A^2*a*c^9*d^6*f*z^2 + 3*B^2*b^10*d^3*f^4*z^2 - A^2*b^10*d^2*f^5*z^2 + 64

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*A^2*a^7*c^3*f^7*z^2 + 4*B^2*b^2*c^8*d^7*z^2 - A^2*a^4*b^6*f^7*z^2 + 16*A^2
*c^10*d^7*z^2 - 160*A*B^2*a*b*c^6*d^4*f^2*z + 112*A*B^2*a^4*b*c^3*d*f^5*z -
 24*A*B^2*a^2*b^5*c*d*f^5*z + 480*A^2*B*a^2*b^2*c^4*d^2*f^4*z - 176*A*B^2*a
^2*b^3*c^3*d^2*f^4*z - 10*A^2*B*a*b^6*c*d*f^5*z + 384*A*B^2*a^2*b*c^5*d^3*f
^3*z - 352*A*B^2*a^3*b*c^4*d^2*f^4*z - 288*A^2*B*a*b^2*c^5*d^3*f^3*z - 160*
A^2*B*a^3*b^2*c^3*d*f^5*z - 148*A^2*B*a*b^4*c^3*d^2*f^4*z + 112*A*B^2*a*b^3
*c^4*d^3*f^3*z + 72*A^2*B*a^2*b^4*c^2*d*f^5*z + 72*A*B^2*a*b^5*c^2*d^2*f^4*
z + 48*A*B^2*a^3*b^3*c^2*d*f^5*z + 48*B^3*a*b^2*c^5*d^4*f^2*z - 36*B^3*a^4*
b^2*c^2*d*f^5*z - 4*B^3*a*b^4*c^3*d^3*f^3*z - 480*A^3*a^2*b*c^5*d^2*f^4*z -
 160*A^3*a^2*b^3*c^3*d*f^5*z + 128*A^3*a*b^3*c^4*d^2*f^4*z + 112*A^2*B*b^4*
c^4*d^3*f^3*z - 64*A*B^2*b^5*c^3*d^3*f^3*z + 16*A^2*B*b^2*c^6*d^4*f^2*z + 1
6*A*B^2*b^3*c^5*d^4*f^2*z - A^2*B*b^6*c^2*d^2*f^4*z + 448*A^2*B*a^3*c^5*d^2
*f^4*z - 352*A^2*B*a^2*c^6*d^3*f^3*z - 48*A^2*B*a^4*b^2*c^2*f^6*z + 12*B^3*
a^3*b^4*c*d*f^5*z - 10*B^3*a*b^6*c*d^2*f^4*z + 416*A^3*a^3*b*c^4*d*f^5*z +
224*A^3*a*b*c^6*d^3*f^3*z + 24*A^3*a*b^5*c^2*d*f^5*z - 2*A*B^2*b^7*c*d^2*f^
4*z - 272*A^2*B*a^4*c^4*d*f^5*z + 128*A^2*B*a*c^7*d^4*f^2*z + 12*A^2*B*a^3*
b^4*c*f^6*z - 120*B^3*a^2*b^2*c^4*d^3*f^3*z + 112*B^3*a^3*b^2*c^3*d^2*f^4*z
 + 16*A*B^2*b*c^7*d^5*f*z + 2*A*B^2*a*b^7*d*f^5*z - 2*A^3*b^7*c*d*f^5*z - 1
6*A^2*B*c^8*d^5*f*z + 11*B^3*b^6*c^2*d^3*f^3*z - 8*B^3*b^4*c^4*d^4*f^2*z -
64*A^3*b^3*c^5*d^3*f^3*z + 96*A^3*a^3*b^3*c^2*f^6*z - 4*B^3*b^2*c^6*d^5*f*z
 - 32*A^3*b*c^7*d^4*f^2*z - B^3*a^2*b^6*d*f^5*z - 128*A^3*a^4*b*c^3*f^6*z -
 24*A^3*a^2*b^5*c*f^6*z + 64*A^2*B*a^5*c^3*f^6*z - A^2*B*a^2*b^6*f^6*z + A^
2*B*b^8*d*f^5*z + 2*A^3*a*b^7*f^6*z + B^3*b^8*d^2*f^4*z + 32*A^3*B*a*b*c^4*
d*f^4 - 18*A^2*B^2*a*b^2*c^3*d*f^4 + 32*A*B^3*a*b*c^4*d^2*f^3 - 28*A*B^3*a^
2*b*c^3*d*f^4 + 6*A*B^3*a*b^3*c^2*d*f^4 - 10*A^3*B*b^3*c^3*d*f^4 - 4*A^3*B*
b*c^5*d^2*f^3 - 4*A*B^3*b*c^5*d^3*f^2 - 28*A^3*B*a^2*b*c^3*f^5 + 6*A^3*B*a*
b^3*c^2*f^5 + 9*A^2*B^2*b^2*c^4*d^2*f^3 - 3*A^2*B^2*a^2*b^2*c^2*f^5 - 10*B^
4*a*b^2*c^3*d^2*f^3 - 3*B^4*a^2*b^2*c^2*d*f^4 - 10*A*B^3*b^3*c^3*d^2*f^3 +
3*A^2*B^2*b^4*c^2*d*f^4 + 36*A^2*B^2*a^2*c^4*d*f^4 - 24*A^2*B^2*a*c^5*d^2*f
^3 + 4*A^2*B^2*c^6*d^3*f^2 + 16*A^2*B^2*a^3*c^3*f^5 + 16*B^4*a^3*c^3*d*f^4
+ 8*A^4*b^2*c^4*d*f^4 - 8*A^4*a*b^2*c^3*f^5 - 24*A^4*a*c^5*d*f^4 + 3*B^4*b^
4*c^2*d^2*f^3 + 4*A^4*c^6*d^2*f^3 + 36*A^4*a^2*c^4*f^5 + B^4*b^2*c^4*d^3*f^
2, z, k)*((4*b^5*c^8*d^7*f^2 + 4*b^7*c^6*d^6*f^3 - 4*b^9*c^4*d^5*f^4 - 4*b^
11*c^2*d^4*f^5 - 612*a^2*b^5*c^6*d^5*f^4 - 712*a^2*b^7*c^4*d^4*f^5 - 132*a^
2*b^9*c^2*d^3*f^6 + 1696*a^3*b^3*c^7*d^5*f^4 + 2736*a^3*b^5*c^5*d^4*f^5 + 8
96*a^3*b^7*c^3*d^3*f^6 - 5120*a^4*b^3*c^6*d^4*f^5 - 3140*a^4*b^5*c^4*d^3*f^
6 - 220*a^4*b^7*c^2*d^2*f^7 + 5664*a^5*b^3*c^5*d^3*f^6 + 1128*a^5*b^5*c^3*d
^2*f^7 - 2560*a^6*b^3*c^4*d^2*f^7 + 8*a*b^11*c*d^3*f^6 + 8*a^5*b^7*c*d*f^8
- 448*a^8*b*c^4*d*f^8 - 32*a*b^3*c^9*d^7*f^2 - 24*a*b^5*c^7*d^6*f^3 + 88*a*
b^7*c^5*d^5*f^4 + 88*a*b^9*c^3*d^4*f^5 + 64*a^2*b*c^10*d^7*f^2 + 128*a^3*b*
c^9*d^6*f^3 + 16*a^3*b^9*c*d^2*f^7 - 1600*a^4*b*c^8*d^5*f^4 + 3840*a^5*b*c^
7*d^4*f^5 - 4160*a^6*b*c^6*d^3*f^6 - 92*a^6*b^5*c^2*d*f^8 + 2176*a^7*b*c^5*
d^2*f^7 + 352*a^7*b^3*c^3*d*f^8)/(16*a^2*c^6*d^4 + a^4*b^4*f^4 + b^4*c^4*d^
4 + 16*a^6*c^2*f^4 + b^8*d^2*f^2 - 8*a*b^2*c^5*d^4 - 8*a^5*b^2*c*f^4 + 2*a^
2*b^6*d*f^3 - 64*a^3*c^5*d^3*f - 64*a^5*c^3*d*f^3 + 2*b^6*c^2*d^3*f + 96*a^
```

4*c^4*d^2*f^2 + 54*a^2*b^4*c^2*d^2*f^2 - 112*a^3*b^2*c^3*d^2*f^2 - 20*a*b^4
*c^3*d^3*f - 12*a*b^6*c*d^2*f^2 - 20*a^3*b^4*c*d*f^3 + 64*a^2*b^2*c^4*d^3*f
 + 64*a^4*b^2*c^2*d*f^3) + (x*(128*a^9*c^4*f^9 - 2*a^6*b^6*c*f^9 - 640*a^8*
c^5*d*f^8 + 6*b^12*c*d^3*f^6 + 24*a^7*b^4*c^2*f^9 - 96*a^8*b^2*c^3*f^9 + 12
8*a^2*c^11*d^7*f^2 - 640*a^3*c^10*d^6*f^3 + 1152*a^4*c^9*d^5*f^4 - 640*a^5*
c^8*d^4*f^5 - 640*a^6*c^7*d^3*f^6 + 1152*a^7*c^6*d^2*f^7 + 8*b^4*c^9*d^7*f^
2 + 22*b^6*c^7*d^6*f^3 + 26*b^8*c^5*d^5*f^4 + 18*b^10*c^3*d^4*f^5 + 672*a^2
*b^2*c^9*d^6*f^3 + 1224*a^2*b^4*c^7*d^5*f^4 + 1202*a^2*b^6*c^5*d^4*f^5 + 56
4*a^2*b^8*c^3*d^3*f^6 - 2048*a^3*b^2*c^8*d^5*f^4 - 2744*a^3*b^4*c^6*d^4*f^5
 - 1736*a^3*b^6*c^4*d^3*f^6 - 128*a^3*b^8*c^2*d^2*f^7 + 2656*a^4*b^2*c^7*d^
4*f^5 + 2648*a^4*b^4*c^5*d^3*f^6 + 570*a^4*b^6*c^3*d^2*f^7 - 1344*a^5*b^2*c
^6*d^3*f^6 - 904*a^5*b^4*c^4*d^2*f^7 - 160*a^6*b^2*c^5*d^2*f^7 + 2*a^4*b^8*
c*d*f^8 - 64*a*b^2*c^10*d^7*f^2 - 216*a*b^4*c^8*d^6*f^3 - 300*a*b^6*c^6*d^5
*f^4 - 240*a*b^8*c^4*d^4*f^5 - 92*a*b^10*c^2*d^3*f^6 + 10*a^2*b^10*c*d^2*f^
7 - 12*a^5*b^6*c^2*d*f^8 - 40*a^6*b^4*c^3*d*f^8 + 384*a^7*b^2*c^4*d*f^8))/(
16*a^2*c^6*d^4 + a^4*b^4*f^4 + b^4*c^4*d^4 + 16*a^6*c^2*f^4 + b^8*d^2*f^2 -
 8*a*b^2*c^5*d^4 - 8*a^5*b^2*c*f^4 + 2*a^2*b^6*d*f^3 - 64*a^3*c^5*d^3*f - 6
4*a^5*c^3*d*f^3 + 2*b^6*c^2*d^3*f + 96*a^4*c^4*d^2*f^2 + 54*a^2*b^4*c^2*d^2
*f^2 - 112*a^3*b^2*c^3*d^2*f^2 - 20*a*b^4*c^3*d^3*f - 12*a*b^6*c*d^2*f^2 -
20*a^3*b^4*c*d*f^3 + 64*a^2*b^2*c^4*d^3*f + 64*a^4*b^2*c^2*d*f^3)) + (A*a^4
*b^6*c*f^8 - 64*A*a^7*c^4*f^8 - 32*A*a*c^10*d^6*f^2 + 352*A*a^6*c^5*d*f^7 +
 A*b^10*c*d^2*f^6 - 12*A*a^5*b^4*c^2*f^8 + 48*A*a^6*b^2*c^3*f^8 + 224*A*a^2
*c^9*d^5*f^3 - 640*A*a^3*c^8*d^4*f^4 + 960*A*a^4*c^7*d^3*f^5 - 800*A*a^5*c^
6*d^2*f^6 + 8*A*b^2*c^9*d^6*f^2 + 16*A*b^4*c^7*d^5*f^3 + A*b^6*c^5*d^4*f^4
 - 6*A*b^8*c^3*d^3*f^5 - 4*B*b^3*c^8*d^6*f^2 - 12*B*b^5*c^6*d^5*f^3 - 4*B*b^
7*c^4*d^4*f^4 + 4*B*b^9*c^2*d^3*f^5 - 120*A*a*b^2*c^8*d^5*f^3 - 60*A*a*b^4*
c^6*d^4*f^4 + 36*A*a*b^6*c^4*d^3*f^5 - 8*A*a*b^8*c^2*d^2*f^6 - 20*A*a^3*b^6
*c^2*d*f^7 + 80*A*a^4*b^4*c^3*d*f^7 - 216*A*a^5*b^2*c^4*d*f^7 + 92*B*a*b^3*
c^7*d^5*f^3 + 72*B*a*b^5*c^5*d^4*f^4 - 20*B*a*b^7*c^3*d^3*f^5 - 176*B*a^2*b
*c^8*d^5*f^3 + 544*B*a^3*b*c^7*d^4*f^4 - 736*B*a^4*b*c^6*d^3*f^5 - 4*B*a^4*
b^5*c^2*d*f^7 + 464*B*a^5*b*c^5*d^2*f^6 + 44*B*a^5*b^3*c^3*d*f^7 + 384*A*a^
2*b^2*c^7*d^4*f^4 + 32*A*a^2*b^4*c^5*d^3*f^5 + 14*A*a^2*b^6*c^3*d^2*f^6 - 5
60*A*a^3*b^2*c^6*d^3*f^5 - 56*A*a^3*b^4*c^4*d^2*f^6 + 456*A*a^4*b^2*c^5*d^2
*f^6 - 360*B*a^2*b^3*c^6*d^4*f^4 - 64*B*a^2*b^5*c^4*d^3*f^5 + 504*B*a^3*b^3
*c^5*d^3*f^5 + 40*B*a^3*b^5*c^3*d^2*f^6 - 276*B*a^4*b^3*c^4*d^2*f^6 + 2*A*a
^2*b^8*c*d*f^7 + 16*B*a*b*c^9*d^6*f^2 - 112*B*a^6*b*c^4*d*f^7)/(16*a^2*c^6*
d^4 + a^4*b^4*f^4 + b^4*c^4*d^4 + 16*a^6*c^2*f^4 + b^8*d^2*f^2 - 8*a*b^2*c^
5*d^4 - 8*a^5*b^2*c*f^4 + 2*a^2*b^6*d*f^3 - 64*a^3*c^5*d^3*f - 64*a^5*c^3*d
*f^3 + 2*b^6*c^2*d^3*f + 96*a^4*c^4*d^2*f^2 + 54*a^2*b^4*c^2*d^2*f^2 - 112*
a^3*b^2*c^3*d^2*f^2 - 20*a*b^4*c^3*d^3*f - 12*a*b^6*c*d^2*f^2 - 20*a^3*b^4*
c*d*f^3 + 64*a^2*b^2*c^4*d^3*f + 64*a^4*b^2*c^2*d*f^3) + (x*(64*B*a^7*c^4*f
^8 + 4*A*a^3*b^7*c*f^8 - 256*A*a^6*b*c^4*f^8 - B*a^4*b^6*c*f^8 - 320*B*a^6*
c^5*d*f^7 + 3*B*b^10*c*d^2*f^6 - 48*A*a^4*b^5*c^2*f^8 + 192*A*a^5*b^3*c^3*f
^8 + 12*B*a^5*b^4*c^2*f^8 - 48*B*a^6*b^2*c^3*f^8 - 16*A*b^3*c^8*d^5*f^3 - 4
8*A*b^5*c^6*d^4*f^4 - 36*A*b^7*c^4*d^3*f^5 - 4*A*b^9*c^2*d^2*f^6 - 64*B*a^2

*c^9*d^5*f^3 + 320*B*a^3*c^8*d^4*f^4 - 640*B*a^4*c^7*d^3*f^5 + 640*B*a^5*c^6*d^2*f^6 + 4*B*b^4*c^7*d^5*f^3 + 23*B*b^6*c^5*d^4*f^4 + 22*B*b^8*c^3*d^3*f^5 + 320*A*a*b^3*c^7*d^4*f^4 + 352*A*a*b^5*c^5*d^3*f^5 + 76*A*a*b^7*c^3*d^2*f^6 - 512*A*a^2*b*c^8*d^4*f^4 - 60*A*a^2*b^7*c^2*d*f^7 + 1408*A*a^3*b*c^7*d^3*f^5 + 352*A*a^3*b^5*c^3*d*f^7 - 1792*A*a^4*b*c^6*d^2*f^6 - 976*A*a^4*b^3*c^4*d*f^7 - 132*B*a*b^4*c^6*d^4*f^4 - 196*B*a*b^6*c^4*d^3*f^5 - 40*B*a*b^8*c^2*d^2*f^6 - 20*B*a^3*b^6*c^2*d*f^7 + 52*B*a^4*b^4*c^3*d*f^7 + 64*B*a^5*b^2*c^4*d*f^7 + 4*A*a*b^9*c*d*f^7 - 1184*A*a^2*b^3*c^6*d^3*f^5 - 544*A*a^2*b^5*c^4*d^2*f^6 + 1664*A*a^3*b^3*c^5*d^2*f^6 + 80*B*a^2*b^2*c^7*d^4*f^4 + 520*B*a^2*b^4*c^5*d^3*f^5 + 210*B*a^2*b^6*c^3*d^2*f^6 - 192*B*a^3*b^2*c^6*d^3*f^5 - 456*B*a^3*b^4*c^4*d^2*f^6 + 96*B*a^4*b^2*c^5*d^2*f^6 + 64*A*a*b*c^9*d^5*f^3 + 1088*A*a^5*b*c^5*d*f^7 + 2*B*a^2*b^8*c*d*f^7))/(16*a^2*c^6*d^4 + a^4*b^4*f^4 + b^4*c^4*d^4 + 16*a^6*c^2*f^4 + b^8*d^2*f^2 - 8*a*b^2*c^5*d^4 - 8*a^5*b^2*c*f^4 + 2*a^2*b^6*d*f^3 - 64*a^3*c^5*d^3*f - 64*a^5*c^3*d*f^3 + 2*b^6*c^2*d^3*f + 96*a^4*c^4*d^2*f^2 + 54*a^2*b^4*c^2*d^2*f^2 - 112*a^3*b^2*c^3*d^2*f^2 - 20*a*b^4*c^3*d^3*f - 12*a*b^6*c*d^2*f^2 - 20*a^3*b^4*c*d*f^3 + 64*a^2*b^2*c^4*d^3*f + 64*a^4*b^2*c^2*d*f^3)) + (13*A^2*a^2*b^5*c^2*f^7 - 56*A^2*a^3*b^3*c^3*f^7 + 16*A^2*b^3*c^6*d^3*f^4 + A^2*b^5*c^4*d^2*f^5 + 8*B^2*b^3*c^6*d^4*f^3 + 9*B^2*b^5*c^4*d^3*f^4 - 64*A*B*a^5*c^4*f^7 - A^2*a*b^7*c*f^7 + 80*A^2*a^4*b*c^4*f^7 + 16*A^2*b*c^8*d^4*f^3 + 2*A^2*b^7*c^2*d*f^6 - 48*A^2*a*b*c^7*d^3*f^4 - 22*A^2*a*b^5*c^3*d*f^6 - 48*A^2*a^3*b*c^5*d*f^6 - 16*B^2*a*b*c^7*d^4*f^3 - 64*B^2*a^4*b*c^4*d*f^6 - A*B*b^8*c*d*f^6 - 8*A^2*a*b^3*c^5*d^2*f^5 + 64*A^2*a^2*b^3*c^4*d*f^6 - 56*B^2*a*b^3*c^5*d^3*f^4 + 2*B^2*a*b^5*c^3*d^2*f^5 + 96*B^2*a^2*b*c^6*d^3*f^4 - 11*B^2*a^2*b^5*c^2*d*f^6 - 16*B^2*a^3*b*c^5*d^2*f^5 + 40*B^2*a^3*b^3*c^3*d*f^6 + A*B*a^2*b^6*c*f^7 + 32*A*B*a*c^8*d^4*f^3 + 32*A*B*a^4*c^5*d*f^6 + B^2*a*b^7*c*d*f^6 - 8*B^2*a^2*b^3*c^4*d^2*f^5 - 12*A*B*a^3*b^4*c^2*f^7 + 48*A*B*a^4*b^2*c^3*f^7 - 160*A*B*a^2*c^7*d^3*f^4 + 160*A*B*a^3*c^6*d^2*f^5 - 24*A*B*b^2*c^7*d^4*f^3 - 24*A*B*b^4*c^5*d^3*f^4 + A*B*b^6*c^3*d^2*f^5 + 120*A*B*a*b^2*c^6*d^3*f^4 - 4*A*B*a*b^4*c^4*d^2*f^5 - 24*A*B*a^2*b^4*c^3*d*f^6 + 8*A*B*a^3*b^2*c^4*d*f^6 - 24*A*B*a^2*b^2*c^5*d^2*f^5 + 10*A*B*a*b^6*c^2*d*f^6)/(16*a^2*c^6*d^4 + a^4*b^4*f^4 + b^4*c^4*d^4 + 16*a^6*c^2*f^4 + b^8*d^2*f^2 - 8*a*b^2*c^5*d^4 - 8*a^5*b^2*c*f^4 + 2*a^2*b^6*d*f^3 - 64*a^3*c^5*d^3*f - 64*a^5*c^3*d*f^3 + 2*b^6*c^2*d^3*f + 96*a^4*c^4*d^2*f^2 + 54*a^2*b^4*c^2*d^2*f^2 - 112*a^3*b^2*c^3*d^2*f^2 - 20*a*b^4*c^3*d^3*f - 12*a*b^6*c*d^2*f^2 - 20*a^3*b^4*c*d*f^3 + 64*a^2*b^2*c^4*d^3*f + 64*a^4*b^2*c^2*d*f^3) + (x*(104*A^2*a^4*c^5*f^7 - 32*B^2*a^5*c^4*f^7 + 8*A^2*c^9*d^4*f^3 + A^2*b^8*c*f^7 + 50*A^2*a^2*b^4*c^3*f^7 - 96*A^2*a^3*b^2*c^4*f^7 - 12*B^2*a^3*b^4*c^2*f^7 + 42*B^2*a^4*b^2*c^3*f^7 + 208*A^2*a^2*c^7*d^2*f^5 + 8*A^2*b^2*c^7*d^3*f^4 + 18*A^2*b^4*c^5*d^2*f^5 - 32*B^2*a^2*c^7*d^3*f^4 + 32*B^2*a^3*c^6*d^2*f^5 + 2*B^2*b^2*c^7*d^4*f^3 - 6*B^2*b^4*c^5*d^3*f^4 + 9*B^2*b^6*c^3*d^2*f^5 - 12*A^2*a*b^6*c^2*f^7 + B^2*a^2*b^6*c*f^7 - 64*A^2*a*c^8*d^3*f^4 - 256*A^2*a^3*c^6*d*f^6 + 2*A^2*b^6*c^3*d*f^6 + 32*B^2*a^4*c^5*d*f^6 - 36*A^2*a*b^4*c^4*d*f^6 - 2*B^2*a*b^6*c^2*d*f^6 - 2*A*B*a*b^7*c*f^7 - 144*A^2*a*b^2*c^6*d^2*f^5 + 168*A^2*a^2*b^2*c^5*d*f^6 + 24*B^2*a*b^2*c^6*d^3*f^4 - 64*B^2*a*b^4*c^4*d^2*f^5 + 2

6*B^2*a^2*b^4*c^3*d*f^6 - 88*B^2*a^3*b^2*c^4*d*f^6 + 72*A*B*a^4*b*c^4*f^7 -
8*A*B*b*c^8*d^4*f^3 + 2*A*B*b^7*c^2*d*f^6 + 84*B^2*a^2*b^2*c^5*d^2*f^5 + 2
4*A*B*a^2*b^5*c^2*f^7 - 84*A*B*a^3*b^3*c^3*f^7 + 4*A*B*b^3*c^6*d^3*f^4 - 20
*A*B*b^5*c^4*d^2*f^5 + 148*A*B*a*b^3*c^5*d^2*f^5 - 192*A*B*a^2*b*c^6*d^2*f^
5 - 4*A*B*a^2*b^3*c^4*d*f^6 + 16*A*B*a*b*c^7*d^3*f^4 - 12*A*B*a*b^5*c^3*d*f
^6 + 112*A*B*a^3*b*c^5*d*f^6))/(16*a^2*c^6*d^4 + a^4*b^4*f^4 + b^4*c^4*d^4
+ 16*a^6*c^2*f^4 + b^8*d^2*f^2 - 8*a*b^2*c^5*d^4 - 8*a^5*b^2*c*f^4 + 2*a^2*
b^6*d*f^3 - 64*a^3*c^5*d^3*f - 64*a^5*c^3*d*f^3 + 2*b^6*c^2*d^3*f + 96*a^4*
c^4*d^2*f^2 + 54*a^2*b^4*c^2*d^2*f^2 - 112*a^3*b^2*c^3*d^2*f^2 - 20*a*b^4*c
^3*d^3*f - 12*a*b^6*c*d^2*f^2 - 20*a^3*b^4*c*d*f^3 + 64*a^2*b^2*c^4*d^3*f +
64*a^4*b^2*c^2*d*f^3)) - (16*A^3*a*c^6*d*f^5 - 4*A^3*c^7*d^2*f^4 - B^3*b^3
*c^4*d^2*f^4 - 12*A^3*a^2*c^5*f^6 - 16*A*B^2*a^3*c^4*f^6 + 2*A^3*a*b^2*c^4*
f^6 - 6*A^3*b^2*c^5*d*f^5 + 4*B^3*a*b*c^5*d^2*f^4 + 3*B^3*a*b^3*c^3*d*f^5 -
12*B^3*a^2*b*c^4*d*f^5 + 3*A*B^2*a^2*b^2*c^3*f^6 + A*B^2*b^2*c^5*d^2*f^4 -
3*A^2*B*a*b^3*c^3*f^6 + 16*A^2*B*a^2*b*c^4*f^6 - 8*A*B^2*a*c^6*d^2*f^4 + 2
4*A*B^2*a^2*c^5*d*f^5 - 3*A*B^2*b^4*c^3*d*f^5 + 4*A^2*B*b*c^6*d^2*f^4 + 9*A
^2*B*b^3*c^4*d*f^5 + 4*A*B^2*a*b^2*c^4*d*f^5 - 28*A^2*B*a*b*c^5*d*f^5)/(16*
a^2*c^6*d^4 + a^4*b^4*f^4 + b^4*c^4*d^4 + 16*a^6*c^2*f^4 + b^8*d^2*f^2 - 8*
a*b^2*c^5*d^4 - 8*a^5*b^2*c*f^4 + 2*a^2*b^6*d*f^3 - 64*a^3*c^5*d^3*f - 64*a
^5*c^3*d*f^3 + 2*b^6*c^2*d^3*f + 96*a^4*c^4*d^2*f^2 + 54*a^2*b^4*c^2*d^2*f^
2 - 112*a^3*b^2*c^3*d^2*f^2 - 20*a*b^4*c^3*d^3*f - 12*a*b^6*c*d^2*f^2 - 20*
a^3*b^4*c*d*f^3 + 64*a^2*b^2*c^4*d^3*f + 64*a^4*b^2*c^2*d*f^3))*root(2560*a
^3*b^2*c^9*d^8*f*z^4 - 1152*a^2*b^4*c^8*d^8*f*z^4 + 384*a^5*b^8*c*d^3*f^6*z
^4 + 384*a*b^8*c^5*d^7*f^2*z^4 + 288*a^3*b^10*c*d^4*f^5*z^4 + 288*a*b^10*c^
3*d^6*f^3*z^4 + 224*a^7*b^6*c*d^2*f^7*z^4 - 192*a^10*b^2*c^2*d*f^8*z^4 + 22
4*a*b^6*c^7*d^8*f*z^4 + 80*a*b^12*c*d^5*f^4*z^4 + 48*a^9*b^4*c*d*f^8*z^4 -
33920*a^6*b^2*c^6*d^5*f^4*z^4 + 27936*a^5*b^4*c^5*d^5*f^4*z^4 + 26112*a^7*b
^2*c^5*d^4*f^5*z^4 + 26112*a^5*b^2*c^7*d^6*f^3*z^4 - 20352*a^6*b^4*c^4*d^4*
f^5*z^4 - 20352*a^4*b^4*c^6*d^6*f^3*z^4 - 13080*a^4*b^6*c^4*d^5*f^4*z^4 - 1
1520*a^8*b^2*c^4*d^3*f^6*z^4 - 11520*a^4*b^2*c^8*d^7*f^2*z^4 + 8736*a^5*b^6
*c^3*d^4*f^5*z^4 + 8736*a^3*b^6*c^5*d^6*f^3*z^4 + 7488*a^7*b^4*c^3*d^3*f^6*
z^4 + 7488*a^3*b^4*c^7*d^7*f^2*z^4 + 3840*a^3*b^8*c^3*d^5*f^4*z^4 + 2560*a^
9*b^2*c^3*d^2*f^7*z^4 - 2416*a^6*b^6*c^2*d^3*f^6*z^4 - 2416*a^2*b^6*c^6*d^7
*f^2*z^4 - 2160*a^4*b^8*c^2*d^4*f^5*z^4 - 2160*a^2*b^8*c^4*d^6*f^3*z^4 - 11
52*a^8*b^4*c^2*d^2*f^7*z^4 - 720*a^2*b^10*c^2*d^5*f^4*z^4 - 16*b^8*c^6*d^8*
f*z^4 - 2048*a^4*c^10*d^8*f*z^4 + 256*a^11*c^3*d*f^8*z^4 - 4*a^8*b^6*d*f^8*
z^4 + 48*a*b^4*c^9*d^9*z^4 - 24*b^10*c^4*d^7*f^2*z^4 - 16*b^12*c^2*d^6*f^3*
z^4 + 17920*a^7*c^7*d^5*f^4*z^4 - 14336*a^8*c^6*d^4*f^5*z^4 - 14336*a^6*c^8
*d^6*f^3*z^4 + 7168*a^9*c^5*d^3*f^6*z^4 + 7168*a^5*c^9*d^7*f^2*z^4 - 2048*a
^10*c^4*d^2*f^7*z^4 - 24*a^4*b^10*d^3*f^6*z^4 - 16*a^6*b^8*d^2*f^7*z^4 - 16
*a^2*b^12*d^4*f^5*z^4 - 192*a^2*b^2*c^10*d^9*z^4 - 4*b^14*d^5*f^4*z^4 - 4*b
^6*c^8*d^9*z^4 + 256*a^3*c^11*d^9*z^4 + 912*A*B*a^6*b*c^3*d*f^6*z^2 + 192*A
*B*a^4*b^5*c*d*f^6*z^2 + 920*A*B*a^4*b^3*c^3*d^2*f^5*z^2 - 480*A*B*a^2*b^5*
c^3*d^3*f^4*z^2 - 336*A*B*a^2*b^3*c^5*d^4*f^3*z^2 - 272*A*B*a^3*b^3*c^4*d^3
*f^4*z^2 + 240*A*B*a^3*b^5*c^2*d^2*f^5*z^2 + 192*A*B*a*b*c^8*d^6*f*z^2 - 24

96*A*B*a^5*b*c^4*d^2*f^5*z^2 + 1872*A*B*a^4*b*c^5*d^3*f^4*z^2 - 744*A*B*a^5
*b^3*c^2*d*f^6*z^2 - 720*A*B*a^2*b*c^7*d^5*f^2*z^2 + 504*A*B*a*b^3*c^6*d^5*
f^2*z^2 + 256*A*B*a^3*b*c^6*d^4*f^3*z^2 + 168*A*B*a*b^7*c^2*d^3*f^4*z^2 - 1
44*A*B*a^2*b^7*c*d^2*f^5*z^2 + 144*A*B*a*b^5*c^4*d^4*f^3*z^2 - 56*B^2*a*b^2
*c^7*d^6*f*z^2 - 36*B^2*a^5*b^4*c*d*f^6*z^2 - 16*B^2*a*b^8*c*d^3*f^4*z^2 -
164*A^2*a^3*b^6*c*d*f^6*z^2 - 16*A^2*a*b^8*c*d^2*f^5*z^2 - 96*A*B*b^5*c^5*d
^5*f^2*z^2 - 24*A*B*b^7*c^3*d^4*f^3*z^2 - 580*B^2*a^4*b^2*c^4*d^3*f^4*z^2 +
 536*B^2*a^3*b^4*c^3*d^3*f^4*z^2 - 348*B^2*a^4*b^4*c^2*d^2*f^5*z^2 + 316*B^
2*a^2*b^2*c^6*d^5*f^2*z^2 + 200*B^2*a^5*b^2*c^3*d^2*f^5*z^2 - 120*B^2*a^2*b
^4*c^4*d^4*f^3*z^2 - 66*B^2*a^2*b^6*c^2*d^3*f^4*z^2 - 16*B^2*a^3*b^2*c^5*d^
4*f^3*z^2 + 1952*A^2*a^4*b^2*c^4*d^2*f^5*z^2 - 1792*A^2*a^3*b^2*c^5*d^3*f^4
*z^2 - 1272*A^2*a^3*b^4*c^3*d^2*f^5*z^2 + 976*A^2*a^2*b^2*c^6*d^4*f^3*z^2 +
 960*A^2*a^2*b^4*c^4*d^3*f^4*z^2 + 282*A^2*a^2*b^6*c^2*d^2*f^5*z^2 - 72*A*B
*b^3*c^7*d^6*f*z^2 - 16*A*B*b^9*c*d^3*f^4*z^2 - 16*A*B*a^3*b^7*d*f^6*z^2 +
16*A*B*a*b^9*d^2*f^5*z^2 - 180*B^2*a*b^4*c^5*d^5*f^2*z^2 + 132*B^2*a^6*b^2*
c^2*d*f^6*z^2 + 108*B^2*a^3*b^6*c*d^2*f^5*z^2 + 20*B^2*a*b^6*c^3*d^4*f^3*z^
2 - 736*A^2*a^5*b^2*c^3*d*f^6*z^2 + 624*A^2*a^4*b^4*c^2*d*f^6*z^2 - 416*A^2
*a*b^2*c^7*d^5*f^2*z^2 - 276*A^2*a*b^4*c^5*d^4*f^3*z^2 - 196*A^2*a*b^6*c^3*
d^3*f^4*z^2 + 31*B^2*b^6*c^4*d^5*f^2*z^2 + 2*B^2*b^8*c^2*d^4*f^3*z^2 - 768*
B^2*a^5*c^5*d^3*f^4*z^2 + 512*B^2*a^6*c^4*d^2*f^5*z^2 + 512*B^2*a^4*c^6*d^4
*f^3*z^2 - 128*B^2*a^3*c^7*d^5*f^2*z^2 + 80*A^2*b^4*c^6*d^5*f^2*z^2 + 31*A^
2*b^6*c^4*d^4*f^3*z^2 + 14*A^2*b^8*c^2*d^3*f^4*z^2 - 1152*A^2*a^3*c^7*d^4*f
^3*z^2 + 1008*A^2*a^4*c^6*d^3*f^4*z^2 + 624*A^2*a^2*c^8*d^5*f^2*z^2 - 288*A
^2*a^5*c^5*d^2*f^5*z^2 - 10*B^2*a^2*b^8*d^2*f^5*z^2 - 48*A^2*a^6*b^2*c^2*f^
7*z^2 - 16*A*B*b*c^9*d^7*z^2 + 20*B^2*b^4*c^6*d^6*f*z^2 - 128*B^2*a^7*c^3*d
*f^6*z^2 + 64*A^2*b^2*c^8*d^6*f*z^2 - 112*A^2*a^6*c^4*d*f^6*z^2 + 3*B^2*a^4
*b^6*d*f^6*z^2 + 14*A^2*a^2*b^8*d*f^6*z^2 + 12*A^2*a^5*b^4*c*f^7*z^2 - 160*
A^2*a*c^9*d^6*f*z^2 + 3*B^2*b^10*d^3*f^4*z^2 - A^2*b^10*d^2*f^5*z^2 + 64*A^
2*a^7*c^3*f^7*z^2 + 4*B^2*b^2*c^8*d^7*z^2 - A^2*a^4*b^6*f^7*z^2 + 16*A^2*c^
10*d^7*z^2 - 160*A*B^2*a*b*c^6*d^4*f^2*z + 112*A*B^2*a^4*b*c^3*d*f^5*z - 24
*A*B^2*a^2*b^5*c*d*f^5*z + 480*A^2*B*a^2*b^2*c^4*d^2*f^4*z - 176*A*B^2*a^2*
b^3*c^3*d^2*f^4*z - 10*A^2*B*a*b^6*c*d*f^5*z + 384*A*B^2*a^2*b*c^5*d^3*f^3*
z - 352*A*B^2*a^3*b*c^4*d^2*f^4*z - 288*A^2*B*a*b^2*c^5*d^3*f^3*z - 160*A^2
*B*a^3*b^2*c^3*d*f^5*z - 148*A^2*B*a*b^4*c^3*d^2*f^4*z + 112*A*B^2*a*b^3*c^
4*d^3*f^3*z + 72*A^2*B*a^2*b^4*c^2*d*f^5*z + 72*A*B^2*a*b^5*c^2*d^2*f^4*z +
 48*A*B^2*a^3*b^3*c^2*d*f^5*z + 48*B^3*a*b^2*c^5*d^4*f^2*z - 36*B^3*a^4*b^2
*c^2*d*f^5*z - 4*B^3*a*b^4*c^3*d^3*f^3*z - 480*A^3*a^2*b*c^5*d^2*f^4*z - 16
0*A^3*a^2*b^3*c^3*d*f^5*z + 128*A^3*a*b^3*c^4*d^2*f^4*z + 112*A^2*B*b^4*c^4
*d^3*f^3*z - 64*A*B^2*b^5*c^3*d^3*f^3*z + 16*A^2*B*b^2*c^6*d^4*f^2*z + 16*A
*B^2*b^3*c^5*d^4*f^2*z - A^2*B*b^6*c^2*d^2*f^4*z + 448*A^2*B*a^3*c^5*d^2*f^
4*z - 352*A^2*B*a^2*c^6*d^3*f^3*z - 48*A^2*B*a^4*b^2*c^2*f^6*z + 12*B^3*a^3
*b^4*c*d*f^5*z - 10*B^3*a*b^6*c*d^2*f^4*z + 416*A^3*a^3*b*c^4*d*f^5*z + 224
*A^3*a*b*c^6*d^3*f^3*z + 24*A^3*a*b^5*c^2*d*f^5*z - 2*A*B^2*b^7*c*d^2*f^4*z
 - 272*A^2*B*a^4*c^4*d*f^5*z + 128*A^2*B*a*c^7*d^4*f^2*z + 12*A^2*B*a^3*b^4
*c*f^6*z - 120*B^3*a^2*b^2*c^4*d^3*f^3*z + 112*B^3*a^3*b^2*c^3*d^2*f^4*z +

```
16*A*B^2*b*c^7*d^5*f*z + 2*A*B^2*a*b^7*d*f^5*z - 2*A^3*b^7*c*d*f^5*z - 16*A
^2*B*c^8*d^5*f*z + 11*B^3*b^6*c^2*d^3*f^3*z - 8*B^3*b^4*c^4*d^4*f^2*z - 64*
A^3*b^3*c^5*d^3*f^3*z + 96*A^3*a^3*b^3*c^2*f^6*z - 4*B^3*b^2*c^6*d^5*f*z -
32*A^3*b*c^7*d^4*f^2*z - B^3*a^2*b^6*d*f^5*z - 128*A^3*a^4*b*c^3*f^6*z - 24
*A^3*a^2*b^5*c*f^6*z + 64*A^2*B*a^5*c^3*f^6*z - A^2*B*a^2*b^6*f^6*z + A^2*B
*b^8*d*f^5*z + 2*A^3*a*b^7*f^6*z + B^3*b^8*d^2*f^4*z + 32*A^3*B*a*b*c^4*d*f
^4 - 18*A^2*B^2*a*b^2*c^3*d*f^4 + 32*A*B^3*a*b*c^4*d^2*f^3 - 28*A*B^3*a^2*b
*c^3*d*f^4 + 6*A*B^3*a*b^3*c^2*d*f^4 - 10*A^3*B*b^3*c^3*d*f^4 - 4*A^3*B*b*c
^5*d^2*f^3 - 4*A*B^3*b*c^5*d^3*f^2 - 28*A^3*B*a^2*b*c^3*f^5 + 6*A^3*B*a*b^3
*c^2*f^5 + 9*A^2*B^2*b^2*c^4*d^2*f^3 - 3*A^2*B^2*a^2*b^2*c^2*f^5 - 10*B^4*a
*b^2*c^3*d^2*f^3 - 3*B^4*a^2*b^2*c^2*d*f^4 - 10*A*B^3*b^3*c^3*d^2*f^3 + 3*A
^2*B^2*b^4*c^2*d*f^4 + 36*A^2*B^2*a^2*c^4*d*f^4 - 24*A^2*B^2*a*c^5*d^2*f^3
+ 4*A^2*B^2*c^6*d^3*f^2 + 16*A^2*B^2*a^3*c^3*f^5 + 16*B^4*a^3*c^3*d*f^4 + 8
*A^4*b^2*c^4*d*f^4 - 8*A^4*a*b^2*c^3*f^5 - 24*A^4*a*c^5*d*f^4 + 3*B^4*b^4*c
^2*d^2*f^3 + 4*A^4*c^6*d^2*f^3 + 36*A^4*a^2*c^4*f^5 + B^4*b^2*c^4*d^3*f^2,
z, k), k, 1, 4)
```

3.6 $\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{d-fx^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 331

$$\begin{aligned}
\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{d-fx^2}\,dx &= -\frac{B\sqrt{a+bx+cx^2}}{f}-\frac{(bB+2Ac)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}f}\\
&\quad-\frac{\left(B\sqrt{d}-A\sqrt{f}\right)\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\operatorname{arctanh}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{d}f^{3/2}}\\
&\quad+\frac{\left(B\sqrt{d}+A\sqrt{f}\right)\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\operatorname{arctanh}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{d}f^{3/2}}
\end{aligned}$$

```
[Out] -1/2*(2*A*c+B*b)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/f/c^(1/
2)-B*(c*x^2+b*x+a)^(1/2)/f-1/2*arctanh(1/2*(b*d^(1/2)-2*a*f^(1/2)+x*(2*c*d^
(1/2)-b*f^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2))*(B
*d^(1/2)-A*f^(1/2))*(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2)/f^(3/2)/d^(1/2)+1/2*a
rctanh(1/2*(b*d^(1/2)+2*a*f^(1/2)+x*(2*c*d^(1/2)+b*f^(1/2)))/(c*x^2+b*x+a)^
(1/2)/(c*d+a*f+b*d^(1/2)*f^(1/2))^(1/2))*(B*d^(1/2)+A*f^(1/2))*(c*d+a*f+b*d
^(1/2)*f^(1/2))^(1/2)/f^(3/2)/d^(1/2)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1035, 1092, 635, 212, 1047, 738}

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{d-fx^2}\,dx$$
$$=-\frac{\left(B\sqrt{d}-A\sqrt{f}\right)\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\,\text{arctanh}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\sqrt{d}f^{3/2}}$$
$$+\frac{\left(A\sqrt{f}+B\sqrt{d}\right)\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\,\text{arctanh}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\sqrt{d}f^{3/2}}$$
$$-\frac{(2Ac+bB)\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}f}-\frac{B\sqrt{a+bx+cx^2}}{f}$$

```
[In] Int[((A + B*x)*Sqrt[a + b*x + c*x^2])/(d - f*x^2),x]
```

```
[Out] -((B*Sqrt[a + b*x + c*x^2])/f) - ((b*B + 2*A*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt
[c]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[c]*f) - ((B*Sqrt[d] - A*Sqrt[f])*Sqrt[
c*d - b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt
[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c
*x^2])])/(2*Sqrt[d]*f^(3/2)) + ((B*Sqrt[d] + A*Sqrt[f])*Sqrt[c*d + b*Sqrt[d
]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f
])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*Sq
rt[d]*f^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1035

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[h*(a + b*x + c*x^2)^p*((d + f*x^2)^(q +
 1)/(2*f*(p + q + 1))), x] - Dist[1/(2*f*(p + q + 1)), Int[(a + b*x + c*x^2
)^(p - 1)*(d + f*x^2)^q*Simp[h*p*(b*d) + a*(-2*g*f)*(p + q + 1) + (2*h*p*(c
*d - a*f) + b*(-2*g*f)*(p + q + 1))*x + (h*p*((-b)*f) + c*(-2*g*f)*(p + q +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*a
*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]
```

Rule 1047

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(-a)*c, 2]}, Dist[h/2 + c*(g/(2*q)
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - c*(g/(2*q)
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[(-a)*c]
```

Rule 1092

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
 (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + e*x + f
*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x
 + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f
, 0]
```

Rubi steps

$$\text{integral} = -\frac{B\sqrt{a+bx+cx^2}}{f}+\frac{\int \frac{\frac{1}{2}(bBd+2aAf)+(Bcd+Abf+aBf)x+\frac{1}{2}(bB+2Ac)fx^2}{\sqrt{a+bx+cx^2}(d-fx^2)}\,dx}{f}$$

$$= -\frac{B\sqrt{a+bx+cx^2}}{f}-\frac{\int \frac{-\frac{1}{2}(bB+2Ac)df-\frac{1}{2}f(bBd+2aAf)-f(Bcd+Abf+aBf)x}{\sqrt{a+bx+cx^2}(d-fx^2)}\,dx}{f^2} - \frac{(bB+2Ac)\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{2f}$$

$$= -\frac{B\sqrt{a+bx+cx^2}}{f} - \frac{(bB+2Ac)\mathrm{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{f} + \frac{\left(\left(B\sqrt{d}-A\sqrt{f}\right)\left(cd-b\sqrt{d}\sqrt{f}+af\right)\right)\int \frac{1}{\left(-\sqrt{d}\sqrt{f}-fx\right)\sqrt{a+bx+cx^2}}\,dx}{2\sqrt{d}f} + \frac{\left(\left(B\sqrt{d}+A\sqrt{f}\right)\left(cd+b\sqrt{d}\sqrt{f}+af\right)\right)\int \frac{1}{\left(\sqrt{d}\sqrt{f}-fx\right)\sqrt{a+bx+cx^2}}\,dx}{2\sqrt{d}f}$$

$$= -\frac{B\sqrt{a+bx+cx^2}}{f} - \frac{(bB+2Ac)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}f} - \frac{\left(\left(B\sqrt{d}-A\sqrt{f}\right)\left(cd-b\sqrt{d}\sqrt{f}+af\right)\right)\mathrm{Subst}\left(\int \frac{1}{4cdf-4b\sqrt{d}f^{3/2}+4af^2-x^2}\,dx, x, \frac{b\sqrt{d}\sqrt{f}-2af-\left(-2c\sqrt{d}\right.}{\sqrt{a+bx+cx^2}}\right.}{\sqrt{d}f} - \frac{\left(\left(B\sqrt{d}+A\sqrt{f}\right)\left(cd+b\sqrt{d}\sqrt{f}+af\right)\right)\mathrm{Subst}\left(\int \frac{1}{4cdf+4b\sqrt{d}f^{3/2}+4af^2-x^2}\,dx, x, \frac{-b\sqrt{d}\sqrt{f}-2af-\left(2c\sqrt{d}\right.}{\sqrt{a+bx+cx^2}}\right.}{\sqrt{d}f}$$

$$= -\frac{B\sqrt{a+bx+cx^2}}{f} - \frac{(bB+2Ac)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}f} - \frac{\left(B\sqrt{d}-A\sqrt{f}\right)\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\tanh^{-1}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{d}f^{3/2}} + \frac{\left(B\sqrt{d}+A\sqrt{f}\right)\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\tanh^{-1}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{d}f^{3/2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.95 (sec) , antiderivative size = 619, normalized size of antiderivative = 1.87

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{d-fx^2}\,dx = -\frac{2B\sqrt{a+x(b+cx)} - \frac{2(bB+2Ac)\mathrm{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a}-\sqrt{a+x(b+cx)}}\right)}{\sqrt{c}} + \mathrm{RootSum}\left[c^2d-b^2f+4\sqrt{a}bf\#1-2cd\#1^2-\right.}{}$$

```
[In] Integrate[((A + B*x)*Sqrt[a + b*x + c*x^2])/(d - f*x^2),x]
```

```
[Out] -1/2*(2*B*Sqrt[a + x*(b + c*x)] - (2*(b*B + 2*A*c)*ArcTanh[(Sqrt[c]*x)/(Sqr
t[a] - Sqrt[a + x*(b + c*x)])])/Sqrt[c] + RootSum[c^2*d - b^2*f + 4*Sqrt[a]
*b*f*#1 - 2*c*d*#1^2 - 4*a*f*#1^2 + d*#1^4 & , (-(A*c^2*d*Log[x]) + A*b^2*f
*Log[x] + a*b*B*f*Log[x] - a*A*c*f*Log[x] + A*c^2*d*Log[-Sqrt[a] + Sqrt[a +
 b*x + c*x^2] - x*#1] - A*b^2*f*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1
] - a*b*B*f*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] + a*A*c*f*Log[-Sqr
t[a] + Sqrt[a + b*x + c*x^2] - x*#1] - 2*Sqrt[a]*B*c*d*Log[x]*#1 - 2*Sqrt[a
]*A*b*f*Log[x]*#1 - 2*a^(3/2)*B*f*Log[x]*#1 + 2*Sqrt[a]*B*c*d*Log[-Sqrt[a]
+ Sqrt[a + b*x + c*x^2] - x*#1]*#1 + 2*Sqrt[a]*A*b*f*Log[-Sqrt[a] + Sqrt[a
+ b*x + c*x^2] - x*#1]*#1 + 2*a^(3/2)*B*f*Log[-Sqrt[a] + Sqrt[a + b*x + c*x
^2] - x*#1]*#1 + b*B*d*Log[x]*#1^2 + A*c*d*Log[x]*#1^2 + a*A*f*Log[x]*#1^2
- b*B*d*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1]*#1^2 - A*c*d*Log[-Sqrt
[a] + Sqrt[a + b*x + c*x^2] - x*#1]*#1^2 - a*A*f*Log[-Sqrt[a] + Sqrt[a + b*
x + c*x^2] - x*#1]*#1^2)/(-(Sqrt[a]*b*f) + c*d*#1 + 2*a*f*#1 - d*#1^3) & ])
/f
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 540 vs. $2(251) = 502$.

Time = 0.82 (sec) , antiderivative size = 541, normalized size of antiderivative = 1.63

method	result
risch	$-\frac{B\sqrt{c x^2+bx+a}}{f}-\frac{\frac{Bb\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c x^2+bx+a}\right)}{\sqrt{c}}+2A\sqrt{c}\,\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c x^2+bx+a}\right)-\frac{\left(A\sqrt{df}\,bf-Aa\,f^2-Acdf+B\sqrt{df}\,af+B\sqrt{df}\,cd-Bbdf\right)}{\cdots}}{\cdots}$
default	$\frac{\left(-Af-B\sqrt{df}\right)\left(\sqrt{\left(x-\frac{\sqrt{df}}{f}\right)^2c+\frac{\left(2c\sqrt{df}+bf\right)\left(x-\frac{\sqrt{df}}{f}\right)}{f}+\frac{b\sqrt{df}+fa+cd}{f}}+\frac{\left(2c\sqrt{df}+bf\right)\ln\left(\frac{\frac{2c\sqrt{df}+bf}{2f}+c\left(x-\frac{\sqrt{df}}{f}\right)}{\sqrt{c}}+\sqrt{\left(x-\frac{\sqrt{df}}{f}\right)^2c+\frac{(2c\sqrt{df}+\cdots}{\cdots}}\right)}{2f\sqrt{c}}\cdots\right.}{\cdots}$

```
[In] int((B*x+A)*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] -B*(c*x^2+b*x+a)^(1/2)/f-1/2/f*(B*b*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1
/2))/c^(1/2)+2*A*c^(1/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-(A*(d*
f)^(1/2)*b*f-A*a*f^2-A*c*d*f+B*(d*f)^(1/2)*a*f+B*(d*f)^(1/2)*c*d-B*b*d*f)/(
d*f)^(1/2)/f/(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+f
*a+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)
+f*a+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)
```

```
^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1/2)/f))-(A*(d*f)^
(1/2)*b*f+A*a*f^2+A*c*d*f+B*(d*f)^(1/2)*a*f+B*(d*f)^(1/2)*c*d+B*b*d*f)/(d*f
)^(1/2)/f/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+f*a+c*d)/f
+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f*a+c*d)/f)^(1
/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*
f)^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{d-fx^2}\,dx = \text{Timed out}$$

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{d-fx^2}\,dx = -\int \frac{A\sqrt{a+bx+cx^2}}{-d+fx^2}\,dx - \int \frac{Bx\sqrt{a+bx+cx^2}}{-d+fx^2}\,dx$$

```
[In] integrate((B*x+A)*(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
[Out] -Integral(A*sqrt(a + b*x + c*x**2)/(-d + f*x**2), x) - Integral(B*x*sqrt(a
+ b*x + c*x**2)/(-d + f*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{d-fx^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see `
assume?
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{d-fx^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{d-fx^2}\,dx = \int \frac{(A+B\,x)\,\sqrt{c\,x^2+b\,x+a}}{d-f\,x^2}\,dx$$

```
[In] int(((A + B*x)*(a + b*x + c*x^2)^(1/2))/(d - f*x^2),x)
```

```
[Out] int(((A + B*x)*(a + b*x + c*x^2)^(1/2))/(d - f*x^2), x)
```

3.7 $\int \frac{A+Bx}{\sqrt{a+bx+cx^2}(d-fx^2)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 249

$$\int \frac{A+Bx}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx=-\frac{\left(B-\frac{A\sqrt{f}}{\sqrt{d}}\right)\operatorname{arctanh}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{f}\sqrt{cd-b\sqrt{d}\sqrt{f}+af}}+\frac{\left(B+\frac{A\sqrt{f}}{\sqrt{d}}\right)\operatorname{arctanh}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{f}\sqrt{cd+b\sqrt{d}\sqrt{f}+af}}$$

```
[Out] -1/2*arctanh(1/2*(b*d^(1/2)-2*a*f^(1/2)+x*(2*c*d^(1/2)-b*f^(1/2)))/(c*x^2+b
*x+a)^(1/2)/(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2))*(B-A*f^(1/2)/d^(1/2))/f^(1/2
)/(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2)+1/2*arctanh(1/2*(b*d^(1/2)+2*a*f^(1/2)+
x*(2*c*d^(1/2)+b*f^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f+b*d^(1/2)*f^(1/2))^
(1/2))*(B+A*f^(1/2)/d^(1/2))/f^(1/2)/(c*d+a*f+b*d^(1/2)*f^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1047, 738, 212}

$$\int \frac{A+Bx}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = \frac{\left(\frac{A\sqrt{f}}{\sqrt{d}}+B\right)\operatorname{arctanh}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\sqrt{f}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}} - \frac{\left(B-\frac{A\sqrt{f}}{\sqrt{d}}\right)\operatorname{arctanh}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\sqrt{f}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}$$

```
[In] Int[(A + B*x)/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] -1/2*((B - (A*Sqrt[f])/Sqrt[d])*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqr
t[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x +
c*x^2])])/(Sqrt[f]*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]) + ((B + (A*Sqrt[f])
/Sqrt[d])*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(
2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[f]*S
qrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1047

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(-a)*c, 2]}, Dist[h/2 + c*(g/(2*q)
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - c*(g/(2*q)
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[(-a)*c]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\left(B-\frac{A\sqrt{f}}{\sqrt{d}}\right)\int \frac{1}{\left(-\sqrt{d}\sqrt{f}-fx\right)\sqrt{a+bx+cx^2}}\,dx + \frac{1}{2}\left(B+\frac{A\sqrt{f}}{\sqrt{d}}\right)\int \frac{1}{\left(\sqrt{d}\sqrt{f}-fx\right)\sqrt{a+bx+cx^2}}\,dx$$

$$= \left(-B - \frac{A\sqrt{f}}{\sqrt{d}}\right)\text{Subst}\left(\int \frac{1}{4cdf+4b\sqrt{d}f^{3/2}+4af^2-x^2}\,dx, x, \frac{-b\sqrt{d}\sqrt{f}-2af-\left(2c\sqrt{d}\sqrt{f}+bf\right)x}{\sqrt{a+bx+cx^2}}\right) + \left(-B + \frac{A\sqrt{f}}{\sqrt{d}}\right)\text{Subst}\left(\int \frac{1}{4cdf-4b\sqrt{d}f^{3/2}+4af^2-x^2}\,dx, x, \frac{b\sqrt{d}\sqrt{f}-2af-\left(-2c\sqrt{d}\sqrt{f}+bf\right)x}{\sqrt{a+bx+cx^2}}\right)$$

$$= -\frac{\left(B-\frac{A\sqrt{f}}{\sqrt{d}}\right)\tanh^{-1}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{f}\sqrt{cd-b\sqrt{d}\sqrt{f}+af}} + \frac{\left(B+\frac{A\sqrt{f}}{\sqrt{d}}\right)\tanh^{-1}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{f}\sqrt{cd+b\sqrt{d}\sqrt{f}+af}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.39 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = -\frac{1}{2}\text{RootSum}\Big[b^2d-a^2f-4b\sqrt{c}d\#1+4cd\#1^2+2af\#1^2 -f\#1^4\&, \frac{Ab\log\left(-\sqrt{c}x+\sqrt{a+bx+cx^2}-\#1\right)-aB\log\left(-\sqrt{c}x+\sqrt{a+bx+cx^2}-\#1\right)-2A\sqrt{c}\log \cdots}{b\sqrt{c}d-2cd\#1-af\#\cdots}$$

```
[In] Integrate[(A + B*x)/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] -1/2*RootSum[b^2*d - a^2*f - 4*b*Sqrt[c]*d*#1 + 4*c*d*#1^2 + 2*a*f*#1^2 - f
*#1^4 & , (A*b*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - a*B*Log[-(S
qrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*A*Sqrt[c]*Log[-(Sqrt[c]*x) + Sq
rt[a + b*x + c*x^2] - #1]*#1 + B*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] -
 #1]*#1^2)/(b*Sqrt[c]*d - 2*c*d*#1 - a*f*#1 + f*#1^3) & ]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 388 vs. $2(185) = 370$.

Time = 0.79 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.56

method	result
default	$-\frac{(-Af-B\sqrt{df})\ln\left(\frac{\frac{2b\sqrt{df}+2fa+2cd}{f}+\frac{(2c\sqrt{df}+bf)\left(x-\frac{\sqrt{df}}{f}\right)}{f}+2\sqrt{\frac{b\sqrt{df}+fa+cd}{f}}\sqrt{\left(x-\frac{\sqrt{df}}{f}\right)^2 c+\frac{(2c\sqrt{df}+bf)\left(x-\frac{\sqrt{df}}{f}\right)}{f}+\frac{b\sqrt{df}+fa+cd}{f}}}{x-\frac{\sqrt{df}}{f}}\right)}{2\sqrt{df}\,f\sqrt{\frac{b\sqrt{df}+fa+cd}{f}}} -$

```
[In] int((B*x+A)/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x,method=_RETURNVERBOSE)
[Out] -1/2*(-A*f-B*(d*f)^(1/2))/(d*f)^(1/2)/f/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*l
n((2*(b*(d*f)^(1/2)+f*a+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*
((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b
*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f)
)-1/2*(A*f-B*(d*f)^(1/2))/(d*f)^(1/2)/f/(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2
)*ln((2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2
)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*
c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/(
x+(d*f)^(1/2)/f))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6113 vs. $2(185) = 370$.

Time = 34.86 (sec) , antiderivative size = 6113, normalized size of antiderivative = 24.55

$$\int \frac{A+Bx}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = \text{Too large to display}$$

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F]

$$\int \frac{A+Bx}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = -\int \frac{A}{-d\sqrt{a+bx+cx^2}+fx^2\sqrt{a+bx+cx^2}}\,dx - \int \frac{Bx}{-d\sqrt{a+bx+cx^2}+fx^2\sqrt{a+bx+cx^2}}\,dx$$

```
[In] integrate((B*x+A)/(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(A/(-d*sqrt(a + b*x + c*x**2) + f*x**2*sqrt(a + b*x + c*x**2)), x)
 - Integral(B*x/(-d*sqrt(a + b*x + c*x**2) + f*x**2*sqrt(a + b*x + c*x**2))
, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see `
assume?
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+Bx}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument ValueBad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = \int \frac{A+B\,x}{(d-f\,x^2)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

```
[In] int((A + B*x)/((d - f*x^2)*(a + b*x + c*x^2)^(1/2)),x)
[Out] int((A + B*x)/((d - f*x^2)*(a + b*x + c*x^2)^(1/2)), x)
```

3.8 $\int \frac{A+Bx}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 381

$$\int \frac{A+Bx}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx = -\frac{2(aB(2c^2d-b^2f+2acf)+A(b^3f-bc(cd+3af))+c(Ab^2f+bB(cd-af)-2Ac(cd+af))x)}{(b^2-4ac)(b^2df-(cd+af)^2)\sqrt{a+bx+cx^2}} - \frac{\left(B\sqrt{d}-A\sqrt{f}\right)\sqrt{f}\operatorname{arctanh}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{d}\left(cd-b\sqrt{d}\sqrt{f}+af\right)^{3/2}} + \frac{\left(B\sqrt{d}+A\sqrt{f}\right)\sqrt{f}\operatorname{arctanh}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{d}\left(cd+b\sqrt{d}\sqrt{f}+af\right)^{3/2}}$$

```
[Out] -1/2*arctanh(1/2*(b*d^(1/2)-2*a*f^(1/2)+x*(2*c*d^(1/2)-b*f^(1/2)))/(c*x^2+b
*x+a)^(1/2)/(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2))*f^(1/2)*(B*d^(1/2)-A*f^(1/2)
)/d^(1/2)/(c*d+a*f-b*d^(1/2)*f^(1/2))^(3/2)+1/2*arctanh(1/2*(b*d^(1/2)+2*a*
f^(1/2)+x*(2*c*d^(1/2)+b*f^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f+b*d^(1/2)*f
^(1/2))^(1/2))*f^(1/2)*(B*d^(1/2)+A*f^(1/2))/d^(1/2)/(c*d+a*f+b*d^(1/2)*f^(
1/2))^(3/2)-2*(a*B*(2*a*c*f-b^2*f+2*c^2*d)+A*(b^3*f-b*c*(3*a*f+c*d))+c*(A*b
^2*f+b*B*(-a*f+c*d)-2*A*c*(a*f+c*d))*x)/(-4*a*c+b^2)/(b^2*d*f-(a*f+c*d)^2)/
(c*x^2+b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 380, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1032, 1047, 738, 212}

$$\int \frac{A+Bx}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx =$$
$$-\frac{\sqrt{f}\left(B\sqrt{d}-A\sqrt{f}\right)\operatorname{arctanh}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\sqrt{d}\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}}$$
$$+\frac{\sqrt{f}\left(A\sqrt{f}+B\sqrt{d}\right)\operatorname{arctanh}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\sqrt{d}\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}}$$
$$-\frac{2(cx(-2Ac(af+cd)+bB(cd-af)+Ab^2f)-Abc(3af+cd)+aB(2acf+b^2(-f)+2c^2d)+Ab^3f)}{(b^2-4ac)\sqrt{a+bx+cx^2}\,(b^2df-(af+cd)^2)}$$

```
[In] Int[(A + B*x)/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

```
[Out] (-2*(A*b^3*f - A*b*c*(c*d + 3*a*f) + a*B*(2*c^2*d - b^2*f + 2*a*c*f) + c*(A
*b^2*f + b*B*(c*d - a*f) - 2*A*c*(c*d + a*f))*x))/((b^2 - 4*a*c)*(b^2*d*f -
 (c*d + a*f)^2)*Sqrt[a + b*x + c*x^2]) - ((B*Sqrt[d] - A*Sqrt[f])*Sqrt[f]*A
rcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d
- b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[d]*(c*d - b*Sqr
t[d]*Sqrt[f] + a*f)^(3/2)) + ((B*Sqrt[d] + A*Sqrt[f])*Sqrt[f]*ArcTanh[(b*Sq
rt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*
Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[d]*(c*d + b*Sqrt[d]*Sqrt[f]
 + a*f)^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x + c*x^2)^(p + 1)*((d + f*x^2)^
(q + 1)/((b^2 - 4*a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)))*((g*c)*((-b)*(c*
d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f)) + c*(g*(2*c^2*d + b^2
*f - c*(2*a*f)) - h*(b*c*d + a*b*f))*x), x] + Dist[1/((b^2 - 4*a*c)*(b^2*d*
f + (c*d - a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Si
mp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d)*((-b)*f))*(p + 1) + (b^2*(g*f) - b*
(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(a*f*(p + 1) - c*d*(p + 2)) - (2*f*(
(g*c)*((-b)*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f)))*(p +
q + 2) - (b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(b*f*(p + 1)
))*x - c*f*(b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(2*p + 2*q
 + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*
a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] &&  !( !IntegerQ[p
] && ILtQ[q, -1])
```

Rule 1047

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(-a)*c, 2]}, Dist[h/2 + c*(g/(2*q)
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - c*(g/(2*q)
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[(-a)*c]
```

Rubi steps

$$\text{integral} = -\frac{2(Ab^3f - Abc(cd+3af) + aB(2c^2d - b^2f + 2acf) + c(Ab^2f + bB(cd-af) - 2Ac(cd+af))x)}{(b^2-4ac)(b^2df-(cd+af)^2)\sqrt{a+bx+cx^2}} + \frac{2\int \frac{\frac{1}{2}(b^2-4ac)f(bBd-A(cd+af))+\frac{1}{2}(b^2-4ac)f(Abf-B(cd+af))x}{\sqrt{a+bx+cx^2}(d-fx^2)}\,dx}{(b^2-4ac)(b^2df-(cd+af)^2)}$$

$$= -\frac{2(Ab^3f - Abc(cd+3af) + aB(2c^2d - b^2f + 2acf) + c(Ab^2f + bB(cd-af) - 2Ac(cd+af))}{(b^2-4ac)(b^2df-(cd+af)^2)\sqrt{a+bx+cx^2}} + \frac{\left(\left(B+\frac{A\sqrt{f}}{\sqrt{d}}\right)f\right)\int \frac{1}{\left(\sqrt{d}\sqrt{f}-fx\right)\sqrt{a+bx+cx^2}}\,dx}{2\left(cd+b\sqrt{d}\sqrt{f}+af\right)} - \frac{\left(\left(B\sqrt{d}-A\sqrt{f}\right)f\left(cd+b\sqrt{d}\sqrt{f}+af\right)\right)\int \frac{1}{\left(-\sqrt{d}\sqrt{f}-fx\right)\sqrt{a+bx+cx^2}}\,dx}{2\sqrt{d}\,(b^2df-(cd+af)^2)}$$

$$= -\frac{2(Ab^3f - Abc(cd+3af) + aB(2c^2d - b^2f + 2acf) + c(Ab^2f + bB(cd-af) - 2Ac(cd+af))\,x)}{(b^2-4ac)\left(b^2df-(cd+af)^2\right)\sqrt{a+bx+cx^2}}$$

$$- \frac{\left(\left(B+\frac{A\sqrt{f}}{\sqrt{d}}\right)f\right)\text{Subst}\left(\int \frac{1}{4cdf+4b\sqrt{d}f^{3/2}+4af^2-x^2}\,dx, x, \frac{-b\sqrt{d}\sqrt{f}-2af-\left(2c\sqrt{d}\sqrt{f}+bf\right)x}{\sqrt{a+bx+cx^2}}\right)}{cd+b\sqrt{d}\sqrt{f}+af}$$

$$+ \frac{\left(\left(B\sqrt{d}-A\sqrt{f}\right)f\left(cd+b\sqrt{d}\sqrt{f}+af\right)\right)\text{Subst}\left(\int \frac{1}{4cdf-4b\sqrt{d}f^{3/2}+4af^2-x^2}\,dx, x, \frac{b\sqrt{d}\sqrt{f}-2af-\left(-2c\sqrt{d}\right.}{\sqrt{a+bx+cx^2}}\right.}{\sqrt{d}\left(b^2df-(cd+af)^2\right)}$$

$$= -\frac{2(Ab^3f - Abc(cd+3af) + aB(2c^2d - b^2f + 2acf) + c(Ab^2f + bB(cd-af) - 2Ac(cd+af))\,x)}{(b^2-4ac)\left(b^2df-(cd+af)^2\right)\sqrt{a+bx+cx^2}}$$

$$- \frac{\left(B\sqrt{d}-A\sqrt{f}\right)\sqrt{f}\tanh^{-1}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{d}\left(cd-b\sqrt{d}\sqrt{f}+af\right)^{3/2}}$$

$$+ \frac{\left(B+\frac{A\sqrt{f}}{\sqrt{d}}\right)\sqrt{f}\tanh^{-1}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\left(cd+b\sqrt{d}\sqrt{f}+af\right)^{3/2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 1.15 (sec) , antiderivative size = 753, normalized size of antiderivative = 1.98

$$\int \frac{A+Bx}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx = \frac{4A(-b^3f+bc(cd+3af)-b^2cfx+2c^2(cd+af)x)+4B(-2a^2cf-bc^2}{}$$

```
[In] Integrate[(A + B*x)/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

```
[Out] (4*A*(-(b^3*f) + b*c*(c*d + 3*a*f) - b^2*c*f*x + 2*c^2*(c*d + a*f)*x) + 4*B
*(-2*a^2*c*f - b*c^2*d*x + a*(-2*c^2*d + b^2*f + b*c*f*x)) - (b^2 - 4*a*c)*
f*Sqrt[a + x*(b + c*x)]*RootSum[c^2*d - b^2*f + 4*Sqrt[a]*b*f*#1 - 2*c*d*#1
^2 - 4*a*f*#1^2 + d*#1^4 & , (2*b*B*c*d*Log[x] - A*c^2*d*Log[x] - A*b^2*f*L
og[x] + a*b*B*f*Log[x] - a*A*c*f*Log[x] - 2*b*B*c*d*Log[-Sqrt[a] + Sqrt[a +
 b*x + c*x^2] - x*#1] + A*c^2*d*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1
] + A*b^2*f*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] - a*b*B*f*Log[-Sqr
t[a] + Sqrt[a + b*x + c*x^2] - x*#1] + a*A*c*f*Log[-Sqrt[a] + Sqrt[a + b*x
+ c*x^2] - x*#1] - 2*Sqrt[a]*B*c*d*Log[x]*#1 + 2*Sqrt[a]*A*b*f*Log[x]*#1 -
2*a^(3/2)*B*f*Log[x]*#1 + 2*Sqrt[a]*B*c*d*Log[-Sqrt[a] + Sqrt[a + b*x + c*x
^2] - x*#1]*#1 - 2*Sqrt[a]*A*b*f*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#
```

```
1]*#1 + 2*a^(3/2)*B*f*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1]*#1 - b*B
*d*Log[x]*#1^2 + A*c*d*Log[x]*#1^2 + a*A*f*Log[x]*#1^2 + b*B*d*Log[-Sqrt[a]
 + Sqrt[a + b*x + c*x^2] - x*#1]*#1^2 - A*c*d*Log[-Sqrt[a] + Sqrt[a + b*x +
 c*x^2] - x*#1]*#1^2 - a*A*f*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1]*#
1^2)/(Sqrt[a]*b*f - c*d*#1 - 2*a*f*#1 + d*#1^3) & ])/(2*(b^2 - 4*a*c)*(-(c^
2*d^2) - 2*a*c*d*f + f*(b^2*d - a^2*f))*Sqrt[a + x*(b + c*x)])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 933 vs. $2(311) = 622$.

Time $= 0.79$ (sec) , antiderivative size $= 934$, normalized size of antiderivative $= 2.45$

method	result
default	$(Af-B\sqrt{df})\left(\frac{f}{(-b\sqrt{df}+fa+cd)\sqrt{\left(x+\frac{\sqrt{df}}{f}\right)^2 c+\frac{(-2c\sqrt{df}+bf)\left(x+\frac{\sqrt{df}}{f}\right)}{f}+\frac{-b\sqrt{df}+fa+cd}{f}}}-\frac{(-2c\sqrt{df}+bf \cdots}{(-b\sqrt{df}+fa+cd)\left(\frac{4c(-b\sqrt{df}+fa+cd)}{f}-\frac{(-2c\sqrt{df} \cdots}{f^2} \cdots\right.} \cdots\right.$

```
[In] int((B*x+A)/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(A*f-B*(d*f)^(1/2))/(d*f)^(1/2)/f*(f/(-b*(d*f)^(1/2)+f*a+c*d)/((x+(d*f)
^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1
/2)+f*a+c*d))^(1/2)-(-2*c*(d*f)^(1/2)+b*f)/(-b*(d*f)^(1/2)+f*a+c*d)*(2*c*(x
+(d*f)^(1/2)/f)+1/f*(-2*c*(d*f)^(1/2)+b*f))/(4*c/f*(-b*(d*f)^(1/2)+f*a+c*d)
-1/f^2*(-2*c*(d*f)^(1/2)+b*f)^2)/((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/
2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)-f/(-b*(d*f)^(
1/2)+f*a+c*d)/(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+
f*a+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2
)+f*a+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f
)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1/2)/f)))+1/2*(-A
*f-B*(d*f)^(1/2))/(d*f)^(1/2)/f*(1/(b*(d*f)^(1/2)+f*a+c*d)*f/((x-(d*f)^(1/2
)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/
f)^(1/2)-(2*c*(d*f)^(1/2)+b*f)/(b*(d*f)^(1/2)+f*a+c*d)*(2*c*(x-(d*f)^(1/2)/
f)+(2*c*(d*f)^(1/2)+b*f)/f)/(4*c*(b*(d*f)^(1/2)+f*a+c*d)/f-(2*c*(d*f)^(1/2)
+b*f)^2/f^2)/((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/
f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)-1/(b*(d*f)^(1/2)+f*a+c*d)*f/((b*(d*f)^(
1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+f*a+c*d)/f+(2*c*(d*f)^(1/2)+b*f
)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f
)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^
(1/2))/(x-(d*f)^(1/2)/f)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx = \text{Timed out}$$

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx = \text{Timed out}$$

```
[In] integrate((B*x+A)/(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see `
assume?
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+Bx}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument ValueDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx=\int \frac{A+B\,x}{(d-f\,x^2)\,\left(c\,x^2+b\,x+a\right)^{3/2}}\,dx$$

```
[In] int((A + B*x)/((d - f*x^2)*(a + b*x + c*x^2)^(3/2)),x)
[Out] int((A + B*x)/((d - f*x^2)*(a + b*x + c*x^2)^(3/2)), x)
```

3.9 $\int \frac{A+Bx}{(a+bx+cx^2)^{5/2}(d-fx^2)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 797

$$\int \frac{A+Bx}{(a+bx+cx^2)^{5/2}\,(d-fx^2)}\,dx =$$
$$-\frac{2(aB(2c^2d-b^2f+2acf)+A(b^3f-bc(cd+3af))+c(Ab^2f+bB(cd-af)-2Ac(cd+af))\,x)}{3\,(b^2-4ac)\,(b^2df-(cd+af)^2)\,(a+bx+cx^2)^{3/2}}$$
$$-\frac{2(3b^6Bdf^2+24a^2Bc^2f(cd+af)^2-Ab^5f^2(7cd+6af)-b^4Bf(7c^2d^2+14acdf-3a^2f^2)+Ab^3cf(15c^2d^2+}{}$$
$$-\frac{\left(B\sqrt{d}-A\sqrt{f}\right)f^{3/2}\operatorname{arctanh}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{d}\left(cd-b\sqrt{d}\sqrt{f}+af\right)^{5/2}}$$
$$+\frac{\left(B\sqrt{d}+A\sqrt{f}\right)f^{3/2}\operatorname{arctanh}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{d}\left(cd+b\sqrt{d}\sqrt{f}+af\right)^{5/2}}$$

```
[Out] -2/3*(a*B*(2*a*c*f-b^2*f+2*c^2*d)+A*(b^3*f-b*c*(3*a*f+c*d))+c*(A*b^2*f+b*B*
(-a*f+c*d)-2*A*c*(a*f+c*d))*x)/(-4*a*c+b^2)/(b^2*d*f-(a*f+c*d)^2)/(c*x^2+b*
x+a)^(3/2)-1/2*f^(3/2)*arctanh(1/2*(b*d^(1/2)-2*a*f^(1/2)+x*(2*c*d^(1/2)-b*
f^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2))*(B*d^(1/2)
-A*f^(1/2))/d^(1/2)/(c*d+a*f-b*d^(1/2)*f^(1/2))^(5/2)+1/2*f^(3/2)*arctanh(1
/2*(b*d^(1/2)+2*a*f^(1/2)+x*(2*c*d^(1/2)+b*f^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c
*d+a*f+b*d^(1/2)*f^(1/2))^(1/2))*(B*d^(1/2)+A*f^(1/2))/d^(1/2)/(c*d+a*f+b*d
^(1/2)*f^(1/2))^(5/2)-2/3*(3*b^6*B*d*f^2+24*a^2*B*c^2*f*(a*f+c*d)^2-A*b^5*f
^2*(6*a*f+7*c*d)-b^4*B*f*(-3*a^2*f^2+14*a*c*d*f+7*c^2*d^2)+A*b^3*c*f*(43*a^
2*f^2+46*a*c*d*f+15*c^2*d^2)+2*b^2*B*c*(-11*a^3*f^3+4*a^2*c*d*f^2+5*a*c^2*d
```

```
^2*f+2*c^3*d^3)-4*A*b*c^2*(17*a^3*f^3+24*a^2*c*d*f^2+9*a*c^2*d^2*f+2*c^3*d^
3)+c*(3*b^5*B*d*f^2-2*A*b^4*f^2*(3*a*f+4*c*d)-8*A*c^2*(a*f+c*d)^2*(5*a*f+2*
c*d)-b^3*B*f*(-3*a^2*f^2+10*a*c*d*f+17*c^2*d^2)+2*A*b^2*c*f*(19*a^2*f^2+22*
a*c*d*f+15*c^2*d^2)+4*b*B*c*(-5*a^3*f^3+4*a^2*c*d*f^2+11*a*c^2*d^2*f+2*c^3*
d^3))*x)/(-4*a*c+b^2)^2/(c^2*d^2+2*a*c*d*f-f*(-a^2*f+b^2*d))^2/(c*x^2+b*x+a
)^(1/2)
```

Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 796, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1032, 1078, 1047, 738, 212}

$$\int \frac{A+Bx}{(a+bx+cx^2)^{5/2}(d-fx^2)}dx =$$

$$-\frac{\left(B\sqrt{d}-A\sqrt{f}\right)\text{arctanh}\left(\frac{-2\sqrt{f}a+\left(2c\sqrt{d}-b\sqrt{f}\right)x+b\sqrt{d}}{2\sqrt{-\sqrt{d}\sqrt{f}b+cd+af}\sqrt{cx^2+bx+a}}\right)f^{3/2}}{2\sqrt{d}\left(-\sqrt{d}\sqrt{f}b+cd+af\right)^{5/2}}$$

$$+\frac{\left(\sqrt{f}A+B\sqrt{d}\right)\text{arctanh}\left(\frac{2\sqrt{f}a+\left(\sqrt{f}b+2c\sqrt{d}\right)x+b\sqrt{d}}{2\sqrt{\sqrt{d}\sqrt{f}b+cd+af}\sqrt{cx^2+bx+a}}\right)f^{3/2}}{2\sqrt{d}\left(\sqrt{d}\sqrt{f}b+cd+af\right)^{5/2}}$$

$$-\frac{2(3Bdf^2b^6-Af^2(7cd+6af)b^5-Bf(7c^2d^2+14acfd-3a^2f^2)b^4+Acf(15c^2d^2+46acfd+43a^2f^2)b^3-}{}$$

$$-\frac{2(Afb^3-Ac(cd+3af)b+aB(-fb^2+2c^2d+2acf)+c(Afb^2+B(cd-af)b-2Ac(cd+af))x)}{3(b^2-4ac)(b^2df-(cd+af)^2)(cx^2+bx+a)^{3/2}}$$

```
[In] Int[(A + B*x)/((a + b*x + c*x^2)^(5/2)*(d - f*x^2)),x]
```

```
[Out] (-2*(A*b^3*f - A*b*c*(c*d + 3*a*f) + a*B*(2*c^2*d - b^2*f + 2*a*c*f) + c*(A
*b^2*f + b*B*(c*d - a*f) - 2*A*c*(c*d + a*f))*x))/(3*(b^2 - 4*a*c)*(b^2*d*f
 - (c*d + a*f)^2)*(a + b*x + c*x^2)^(3/2)) - (2*(3*b^6*B*d*f^2 + 24*a^2*B*c
^2*f*(c*d + a*f)^2 - A*b^5*f^2*(7*c*d + 6*a*f) - b^4*B*f*(7*c^2*d^2 + 14*a*
c*d*f - 3*a^2*f^2) + A*b^3*c*f*(15*c^2*d^2 + 46*a*c*d*f + 43*a^2*f^2) + 2*b
^2*B*c*(2*c^3*d^3 + 5*a*c^2*d^2*f + 4*a^2*c*d*f^2 - 11*a^3*f^3) - 4*A*b*c^2
*(2*c^3*d^3 + 9*a*c^2*d^2*f + 24*a^2*c*d*f^2 + 17*a^3*f^3) + c*(3*b^5*B*d*f
^2 - 2*A*b^4*f^2*(4*c*d + 3*a*f) - 8*A*c^2*(c*d + a*f)^2*(2*c*d + 5*a*f) -
b^3*B*f*(17*c^2*d^2 + 10*a*c*d*f - 3*a^2*f^2) + 2*A*b^2*c*f*(15*c^2*d^2 + 2
2*a*c*d*f + 19*a^2*f^2) + 4*b*B*c*(2*c^3*d^3 + 11*a*c^2*d^2*f + 4*a^2*c*d*f
^2 - 5*a^3*f^3))*x))/(3*(b^2 - 4*a*c)^2*(c^2*d^2 + 2*a*c*d*f - f*(b^2*d - a
^2*f))^2*Sqrt[a + b*x + c*x^2]) - ((B*Sqrt[d] - A*Sqrt[f])*f^(3/2)*ArcTanh[
(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqr
t[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[d]*(c*d - b*Sqrt[d]*Sq
```

```
rt[f] + a*f)^(5/2)) + ((B*Sqrt[d] + A*Sqrt[f])*f^(3/2)*ArcTanh[(b*Sqrt[d] +
 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f]
 + a*f]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[d]*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)
^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x + c*x^2)^(p + 1)*((d + f*x^2)^
(q + 1)/((b^2 - 4*a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)))*((g*c)*((-b)*(c*
d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f)) + c*(g*(2*c^2*d + b^2
*f - c*(2*a*f)) - h*(b*c*d + a*b*f))*x), x] + Dist[1/((b^2 - 4*a*c)*(b^2*d*
f + (c*d - a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Si
mp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d)*((-b)*f))*(p + 1) + (b^2*(g*f) - b*
(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(a*f*(p + 1) - c*d*(p + 2)) - (2*f*(
(g*c)*((-b)*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f)))*(p +
q + 2) - (b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(b*f*(p + 1)
))*x - c*f*(b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(2*p + 2*q
 + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*
a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] &&  !( !IntegerQ[p
] && ILtQ[q, -1])
```

Rule 1047

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(-a)*c, 2]}, Dist[h/2 + c*(g/(2*q)
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - c*(g/(2*q)
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[(-a)*c]
```

Rule 1078

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x + c*x^2)^(p + 1
```

```
)*((d + f*x^2)^(q + 1)/((b^2 - 4*a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)))*(
(A*c - a*C)*((-b)*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(2*a*f))
+ c*(A*(2*c^2*d + b^2*f - c*(2*a*f)) - B*(b*c*d + a*b*f) + C*(b^2*d - 2*a*(
c*d - a*f)))*x), x] + Dist[1/((b^2 - 4*a*c)*(b^2*d*f + (c*d - a*f)^2)*(p +
1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Simp[(b*B - 2*A*c - 2*a*C)
*((c*d - a*f)^2 - (b*d)*((-b)*f))*(p + 1) + (b^2*(C*d + A*f) - b*(B*c*d + a
*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)
) - (2*f*((A*c - a*C)*((-b)*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c
*(2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d + a*B*f) + 2*(A*c*(c*d
 - a*f) - a*(c*C*d - a*C*f)))*(b*f*(p + 1)))*x - c*f*(b^2*(C*d + A*f) - b*(
B*c*d + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - a*C*f)))*(2*p + 2*q + 5)*x
^2, x], x], x] /; FreeQ[{a, b, c, d, f, A, B, C, q}, x] && NeQ[b^2 - 4*a*c,
 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] &&  !( !IntegerQ[p] &&
 ILtQ[q, -1]) &&  !IGtQ[q, 0]
```

Rubi steps

integral =
$$-\frac{2(Ab^3f-Abc(cd+3af)+aB(2c^2d-b^2f+2acf)+c(Ab^2f+bB(cd-af)-2Ac(cd+af))\,x)}{3\,(b^2-4ac)\,(b^2df-(cd+af)^2)\,(a+bx+cx^2)^{3/2}}$$
$$+\frac{2\int\frac{\frac{1}{2}(3b^3Bdf-4bBcd(cd+2af)-Ab^2f(7cd+3af)+4Ac(2c^2d^2+5acdf+3a^2f^2))+\frac{3}{2}(b^2-4ac)f(Abf-B(cd+af))x+2cf(Ab^2f+bB(cd-af)-}{(a+bx+cx^2)^{3/2}(d-fx^2)}}{3\,(b^2-4ac)\,(b^2df-(cd+af)^2)}$$
$$=-\frac{2(Ab^3f-Abc(cd+3af)+aB(2c^2d-b^2f+2acf)+c(Ab^2f+bB(cd-af)-2Ac(cd+af))}{3\,(b^2-4ac)\,(b^2df-(cd+af)^2)\,(a+bx+cx^2)^{3/2}}$$
$$-\frac{2(3b^6Bdf^2+24a^2Bc^2f(cd+af)^2-Ab^5f^2(7cd+6af)-b^4Bf(7c^2d^2+14acdf-3a^2f^2)+Ab^3}{}$$
$$+\frac{4\int\frac{\frac{3}{4}(b^2-4ac)^2f^2(Ab^2df-2bBd(cd+af)+A(cd+af)^2)-\frac{3}{4}(b^2-4ac)^2f^2(2Abf(cd+af)-B(c^2d^2+2acdf+f(b^2d+a^2f)))x}{\sqrt{a+bx+cx^2}(d-fx^2)}\,dx}{3\,(b^2-4ac)^2\,(b^2df-(cd+af)^2)^2}$$
$$=-\frac{2(Ab^3f-Abc(cd+3af)+aB(2c^2d-b^2f+2acf)+c(Ab^2f+bB(cd-af)-2Ac(cd+af))}{3\,(b^2-4ac)\,(b^2df-(cd+af)^2)\,(a+bx+cx^2)^{3/2}}$$
$$-\frac{2(3b^6Bdf^2+24a^2Bc^2f(cd+af)^2-Ab^5f^2(7cd+6af)-b^4Bf(7c^2d^2+14acdf-3a^2f^2)+Ab^3}{}$$
$$+\frac{\left(\left(B\sqrt{d}-A\sqrt{f}\right)f^2\right)\int\frac{1}{\left(-\sqrt{d}\sqrt{f}-fx\right)\sqrt{a+bx+cx^2}}\,dx}{2\sqrt{d}\left(cd-b\sqrt{d}\sqrt{f}+af\right)^2}$$
$$+\frac{\left(\left(B\sqrt{d}+A\sqrt{f}\right)f^2\right)\int\frac{1}{\left(\sqrt{d}\sqrt{f}-fx\right)\sqrt{a+bx+cx^2}}\,dx}{2\sqrt{d}\left(cd+b\sqrt{d}\sqrt{f}+af\right)^2}$$

$$= -\frac{2(Ab^3f - Abc(cd+3af) + aB(2c^2d - b^2f + 2acf) + c(Ab^2f + bB(cd-af) - 2Ac(cd+af))\,x)}{3\,(b^2-4ac)\,(b^2df - (cd+af)^2)\,(a+bx+cx^2)^{3/2}}$$

$$- \frac{2(3b^6Bdf^2 + 24a^2Bc^2f(cd+af)^2 - Ab^5f^2(7cd+6af) - b^4Bf(7c^2d^2 + 14acdf - 3a^2f^2) + Ab^3c}{}$$

$$- \frac{\left(\left(B\sqrt{d} - A\sqrt{f}\right)f^2\right)\text{Subst}\left(\int \frac{1}{4cdf - 4b\sqrt{d}f^{3/2} + 4af^2 - x^2}\,dx, x, \frac{b\sqrt{d}\sqrt{f} - 2af - \left(-2c\sqrt{d}\sqrt{f} + bf\right)x}{\sqrt{a+bx+cx^2}}\right)}{\sqrt{d}\left(cd - b\sqrt{d}\sqrt{f} + af\right)^2}$$

$$- \frac{\left(\left(B\sqrt{d} + A\sqrt{f}\right)f^2\right)\text{Subst}\left(\int \frac{1}{4cdf + 4b\sqrt{d}f^{3/2} + 4af^2 - x^2}\,dx, x, \frac{-b\sqrt{d}\sqrt{f} - 2af - \left(2c\sqrt{d}\sqrt{f} + bf\right)x}{\sqrt{a+bx+cx^2}}\right)}{\sqrt{d}\left(cd + b\sqrt{d}\sqrt{f} + af\right)^2}$$

$$= -\frac{2(Ab^3f - Abc(cd+3af) + aB(2c^2d - b^2f + 2acf) + c(Ab^2f + bB(cd-af) - 2Ac(cd+af))\,x)}{3\,(b^2-4ac)\,(b^2df - (cd+af)^2)\,(a+bx+cx^2)^{3/2}}$$

$$- \frac{2(3b^6Bdf^2 + 24a^2Bc^2f(cd+af)^2 - Ab^5f^2(7cd+6af) - b^4Bf(7c^2d^2 + 14acdf - 3a^2f^2) + Ab^3c}{}$$

$$- \frac{\left(B\sqrt{d} - A\sqrt{f}\right)f^{3/2}\tanh^{-1}\left(\frac{b\sqrt{d} - 2a\sqrt{f} + \left(2c\sqrt{d} - b\sqrt{f}\right)x}{2\sqrt{cd - b\sqrt{d}\sqrt{f} + af}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{d}\left(cd - b\sqrt{d}\sqrt{f} + af\right)^{5/2}}$$

$$+ \frac{\left(B\sqrt{d} + A\sqrt{f}\right)f^{3/2}\tanh^{-1}\left(\frac{b\sqrt{d} + 2a\sqrt{f} + \left(2c\sqrt{d} + b\sqrt{f}\right)x}{2\sqrt{cd + b\sqrt{d}\sqrt{f} + af}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{d}\left(cd + b\sqrt{d}\sqrt{f} + af\right)^{5/2}}$$

Mathematica [A] (verified)

Time = 12.52 (sec) , antiderivative size = 674, normalized size of antiderivative = 0.85

$$\int \frac{A+Bx}{(a+bx+cx^2)^{5/2}\,(d-fx^2)}\,dx = \frac{2\left(\frac{4c(-Ab^2f + bB(-cd+af) + 2Ac(cd+af))(b+2cx)}{(b^2-4ac)\sqrt{a+x(b+cx)}} - \frac{3f(b^4Bdf + 2c(cd+af)^2(-aB+Acx) + b^3f}{}\right.}{}$$

```
[In] Integrate[(A + B*x)/((a + b*x + c*x^2)^(5/2)*(d - f*x^2)),x]
```

```
[Out] (2*((4*c*(-(A*b^2*f) + b*B*(-(c*d) + a*f) + 2*A*c*(c*d + a*f))*(b + 2*c*x))
/((b^2 - 4*a*c)*Sqrt[a + x*(b + c*x)]) - (3*f*(b^4*B*d*f + 2*c*(c*d + a*f)^
```

```
2*(-(a*B) + A*c*x) + b^3*f*(-(A*(c*d + 2*a*f)) + B*c*d*x) + b*c*(c*d + a*f)
*(A*c*d + 5*a*A*f - 3*B*c*d*x + a*B*f*x) - b^2*(B*(c^2*d^2 + 2*a*c*d*f - a^
2*f^2) + 2*a*A*c*f^2*x)))/((c^2*d^2 + 2*a*c*d*f + f*(-(b^2*d) + a^2*f))*Sqr
t[a + x*(b + c*x)]) + (A*(b^3*f - b*c*(c*d + 3*a*f) + b^2*c*f*x - 2*c^2*(c*
d + a*f)*x) + B*(2*a^2*c*f + b*c^2*d*x + a*(2*c^2*d - b^2*f - b*c*f*x)))/(a
 + x*(b + c*x))^(3/2) + (3*(b^2 - 4*a*c)*f^(3/2)*(((-(B*Sqrt[d]) + A*Sqrt[f
])*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)^2*ArcTanh[(-2*a*Sqrt[f] + 2*c*Sqrt[d]*x
+ b*(Sqrt[d] - Sqrt[f]*x))/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a +
x*(b + c*x)])])/Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f] - ((B*Sqrt[d] + A*Sqrt[
f])*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^2*ArcTanh[(-2*(a*Sqrt[f] + c*Sqrt[d]*x)
 - b*(Sqrt[d] + Sqrt[f]*x))/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a +
 x*(b + c*x)])])/Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]))/(4*Sqrt[d]*(-(b^2*d*
f) + (c*d + a*f)^2))))/(3*(b^2 - 4*a*c)*(-(b^2*d*f) + (c*d + a*f)^2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1767 vs. 2(721) = 1442.

Time = 0.75 (sec) , antiderivative size = 1768, normalized size of antiderivative = 2.22

method	result	size
default	Expression too large to display	1768

```
[In] int((B*x+A)/(c*x^2+b*x+a)^(5/2)/(-f*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-A*f-B*(d*f)^(1/2))/(d*f)^(1/2)/f*(1/3/(b*(d*f)^(1/2)+f*a+c*d)*f/((x-(
d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+
f*a+c*d)/f)^(3/2)-1/2*(2*c*(d*f)^(1/2)+b*f)/(b*(d*f)^(1/2)+f*a+c*d)*(2/3*(2
*c*(x-(d*f)^(1/2)/f)+(2*c*(d*f)^(1/2)+b*f)/f)/(4*c*(b*(d*f)^(1/2)+f*a+c*d)/
f-(2*c*(d*f)^(1/2)+b*f)^2/f^2)/((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)
/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(3/2)+16/3*c/(4*c*(b*(d*f)^
(1/2)+f*a+c*d)/f-(2*c*(d*f)^(1/2)+b*f)^2/f^2)^2*(2*c*(x-(d*f)^(1/2)/f)+(2*c
*(d*f)^(1/2)+b*f)/f)/((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f
)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))+1/(b*(d*f)^(1/2)+f*a+c*d)*f*(1
/(b*(d*f)^(1/2)+f*a+c*d)*f/((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(
x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)-(2*c*(d*f)^(1/2)+b*f)/(b*
(d*f)^(1/2)+f*a+c*d)*(2*c*(x-(d*f)^(1/2)/f)+(2*c*(d*f)^(1/2)+b*f)/f)/(4*c*(
b*(d*f)^(1/2)+f*a+c*d)/f-(2*c*(d*f)^(1/2)+b*f)^2/f^2)/((x-(d*f)^(1/2)/f)^2*
c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2
)-1/(b*(d*f)^(1/2)+f*a+c*d)*f/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d
*f)^(1/2)+f*a+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^
(1/2)+f*a+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(
d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))))+1/2*(A
*f-B*(d*f)^(1/2))/(d*f)^(1/2)/f*(1/3*f/(-b*(d*f)^(1/2)+f*a+c*d)/((x+(d*f)^(
1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2
)+f*a+c*d))^(3/2)-1/2*(-2*c*(d*f)^(1/2)+b*f)/(-b*(d*f)^(1/2)+f*a+c*d)*(2/3*
```

```
(2*c*(x+(d*f)^(1/2)/f)+1/f*(-2*c*(d*f)^(1/2)+b*f))/(4*c/f*(-b*(d*f)^(1/2)+f
*a+c*d)-1/f^2*(-2*c*(d*f)^(1/2)+b*f)^2)/((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d
*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(3/2)+16/3*c
/(4*c/f*(-b*(d*f)^(1/2)+f*a+c*d)-1/f^2*(-2*c*(d*f)^(1/2)+b*f)^2)^2*(2*c*(x+
(d*f)^(1/2)/f)+1/f*(-2*c*(d*f)^(1/2)+b*f))/((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c
*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))+f/
(-b*(d*f)^(1/2)+f*a+c*d)*(f/(-b*(d*f)^(1/2)+f*a+c*d)/((x+(d*f)^(1/2)/f)^2*c
+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))
^(1/2)-(-2*c*(d*f)^(1/2)+b*f)/(-b*(d*f)^(1/2)+f*a+c*d)*(2*c*(x+(d*f)^(1/2)/
f)+1/f*(-2*c*(d*f)^(1/2)+b*f))/(4*c/f*(-b*(d*f)^(1/2)+f*a+c*d)-1/f^2*(-2*c*
(d*f)^(1/2)+b*f)^2)/((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d
*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)-f/(-b*(d*f)^(1/2)+f*a+c*d)
/(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1/f*
(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(
1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/
f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1/2)/f))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(a+bx+cx^2)^{5/2}(d-fx^2)}\,dx = \text{Timed out}$$

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)^(5/2)/(-f*x^2+d),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(a+bx+cx^2)^{5/2}(d-fx^2)}\,dx = \text{Timed out}$$

```
[In] integrate((B*x+A)/(c*x**2+b*x+a)**(5/2)/(-f*x**2+d),x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx}{\left(a+bx+cx^2\right)^{5/2}\left(d-fx^2\right)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)^(5/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see `
assume?
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+Bx}{\left(a+bx+cx^2\right)^{5/2}\left(d-fx^2\right)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)^(5/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument ValueDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{\left(a+bx+cx^2\right)^{5/2}\left(d-fx^2\right)}\,dx = \int \frac{A+B\,x}{\left(d-f\,x^2\right)\,\left(c\,x^2+b\,x+a\right)^{5/2}}\,dx$$

```
[In] int((A + B*x)/((d - f*x^2)*(a + b*x + c*x^2)^(5/2)),x)
```

```
[Out] int((A + B*x)/((d - f*x^2)*(a + b*x + c*x^2)^(5/2)), x)
```

3.10 $\int \frac{1+2x}{(-1+x^2)\sqrt{-1+x+x^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 47

$$\int \frac{1+2x}{(-1+x^2)\sqrt{-1+x+x^2}}\,dx = -\frac{1}{2}\arctan\left(\frac{3+x}{2\sqrt{-1+x+x^2}}\right)+\frac{3}{2}\text{arctanh}\left(\frac{1-3x}{2\sqrt{-1+x+x^2}}\right)$$

```
[Out] -1/2*arctan(1/2*(3+x)/(x^2+x-1)^(1/2))+3/2*arctanh(1/2*(1-3*x)/(x^2+x-1)^(1
/2))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {1047, 738, 212, 210}

$$\int \frac{1+2x}{(-1+x^2)\sqrt{-1+x+x^2}}\,dx = \frac{3}{2}\text{arctanh}\left(\frac{1-3x}{2\sqrt{x^2+x-1}}\right)-\frac{1}{2}\arctan\left(\frac{x+3}{2\sqrt{x^2+x-1}}\right)$$

```
[In] Int[(1 + 2*x)/((-1 + x^2)*Sqrt[-1 + x + x^2]),x]
```

```
[Out] -1/2*ArcTan[(3 + x)/(2*Sqrt[-1 + x + x^2])] + (3*ArcTanh[(1 - 3*x)/(2*Sqrt[
-1 + x + x^2])])/2
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1047

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(-a)*c, 2]}, Dist[h/2 + c*(g/(2*q)
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - c*(g/(2*q)
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[(-a)*c]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\int \frac{1}{(1+x)\sqrt{-1+x+x^2}}\,dx + \frac{3}{2}\int \frac{1}{(-1+x)\sqrt{-1+x+x^2}}\,dx \\
&= -\left(3\text{Subst}\left(\int \frac{1}{4-x^2}\,dx, x, \frac{-1+3x}{\sqrt{-1+x+x^2}}\right)\right) - \text{Subst}\left(\int \frac{1}{-4-x^2}\,dx, x, \frac{-3-x}{\sqrt{-1+x+x^2}}\right) \\
&= \frac{1}{2}\tan^{-1}\left(\frac{-3-x}{2\sqrt{-1+x+x^2}}\right) + \frac{3}{2}\tanh^{-1}\left(\frac{1-3x}{2\sqrt{-1+x+x^2}}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.79

$$
\begin{aligned}
\int \frac{1+2x}{(-1+x^2)\sqrt{-1+x+x^2}}\,dx = &-\arctan\left(1+x-\sqrt{-1+x+x^2}\right) \\
&- 3\text{arctanh}\left(1-x+\sqrt{-1+x+x^2}\right)
\end{aligned}
$$

```
[In] Integrate[(1 + 2*x)/((-1 + x^2)*Sqrt[-1 + x + x^2]),x]
[Out] -ArcTan[1 + x - Sqrt[-1 + x + x^2]] - 3*ArcTanh[1 - x + Sqrt[-1 + x + x^2]]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.98

method	result	size
default	$-\frac{3\,\mathrm{arctanh}\left(\frac{-1+3x}{2\sqrt{(-1+x)^{2}-2+3x}}\right)}{2}+\frac{\arctan\left(\frac{-3-x}{2\sqrt{(1+x)^{2}-2-x}}\right)}{2}$	46
trager	$-\frac{3\ln\left(-\frac{2\sqrt{x^{2}+x-1}-1+3x}{-1+x}\right)}{2}-\frac{\mathrm{RootOf}\left(\_Z^{2}+1\right)\ln\left(\frac{-\mathrm{RootOf}\left(\_Z^{2}+1\right)x+2\sqrt{x^{2}+x-1}-3\,\mathrm{RootOf}\left(\_Z^{2}+1\right)}{1+x}\right)}{2}$	70

```
[In] int(1/(x^2-1)/(x^2+x-1)^(1/2)*(1+2*x),x,method=_RETURNVERBOSE)
[Out] -3/2*arctanh(1/2*(-1+3*x)/((-1+x)^2-2+3*x)^(1/2))+1/2*arctan(1/2*(-3-x)/((1
+x)^2-2-x)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.98

$$\int \frac{1+2x}{(-1+x^2)\sqrt{-1+x+x^2}}\,dx = \arctan\left(-x+\sqrt{x^2+x-1}-1\right) - \frac{3}{2}\log\left(-x+\sqrt{x^2+x-1}+2\right) + \frac{3}{2}\log\left(-x+\sqrt{x^2+x-1}\right)$$

```
[In] integrate((1+2*x)/(x^2-1)/(x^2+x-1)^(1/2),x, algorithm="fricas")
[Out] arctan(-x + sqrt(x^2 + x - 1) - 1) - 3/2*log(-x + sqrt(x^2 + x - 1) + 2) +
3/2*log(-x + sqrt(x^2 + x - 1))
```

Sympy [F]

$$\int \frac{1+2x}{(-1+x^2)\sqrt{-1+x+x^2}}\,dx = \int \frac{2x+1}{(x-1)(x+1)\sqrt{x^2+x-1}}\,dx$$

```
[In] integrate((1+2*x)/(x**2-1)/(x**2+x-1)**(1/2),x)
[Out] Integral((2*x + 1)/((x - 1)*(x + 1)*sqrt(x**2 + x - 1)), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.38

$$\int \frac{1+2x}{(-1+x^2)\sqrt{-1+x+x^2}}\,dx = -\frac{1}{2}\arcsin\left(\frac{2\sqrt{5}x}{5\,|2\,x+2|}+\frac{6\sqrt{5}}{5\,|2\,x+2|}\right) - \frac{3}{2}\log\left(\frac{2\sqrt{x^2+x-1}}{|2\,x-2|}+\frac{2}{|2\,x-2|}+\frac{3}{2}\right)$$

```
[In] integrate((1+2*x)/(x^2-1)/(x^2+x-1)^(1/2),x, algorithm="maxima")
[Out] -1/2*arcsin(2/5*sqrt(5)*x/abs(2*x + 2) + 6/5*sqrt(5)/abs(2*x + 2)) - 3/2*lo
g(2*sqrt(x^2 + x - 1)/abs(2*x - 2) + 2/abs(2*x - 2) + 3/2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.02

$$\int \frac{1+2x}{(-1+x^2)\sqrt{-1+x+x^2}}\,dx = \arctan\left(-x+\sqrt{x^2+x-1}-1\right) - \frac{3}{2}\log\left(\left|-x+\sqrt{x^2+x-1}+2\right|\right) + \frac{3}{2}\log\left(\left|-x+\sqrt{x^2+x-1}\right|\right)$$

```
[In] integrate((1+2*x)/(x^2-1)/(x^2+x-1)^(1/2),x, algorithm="giac")
[Out] arctan(-x + sqrt(x^2 + x - 1) - 1) - 3/2*log(abs(-x + sqrt(x^2 + x - 1) + 2
)) + 3/2*log(abs(-x + sqrt(x^2 + x - 1)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+2x}{(-1+x^2)\sqrt{-1+x+x^2}}\,dx = \int \frac{2\,x+1}{(x^2-1)\,\sqrt{x^2+x-1}}\,dx$$

```
[In] int((2*x + 1)/((x^2 - 1)*(x + x^2 - 1)^(1/2)),x)
[Out] int((2*x + 1)/((x^2 - 1)*(x + x^2 - 1)^(1/2)), x)
```

3.11 $\int \frac{1+2x}{(1+x^2)\sqrt{-1+x+x^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 117

$$\int \frac{1+2x}{(1+x^2)\sqrt{-1+x+x^2}}\,dx = -\sqrt{\frac{1}{2}\left(2+\sqrt{5}\right)}\arctan\left(\frac{5+2\sqrt{5}-\sqrt{5}x}{\sqrt{10\left(2+\sqrt{5}\right)}\sqrt{-1+x+x^2}}\right) + \sqrt{\frac{1}{2}\left(-2+\sqrt{5}\right)}\text{arctanh}\left(\frac{5-2\sqrt{5}+\sqrt{5}x}{\sqrt{10\left(-2+\sqrt{5}\right)}\sqrt{-1+x+x^2}}\right)$$

```
[Out] 1/2*arctanh((5-2*5^(1/2)+x*5^(1/2))/(x^2+x-1)^(1/2)/(-20+10*5^(1/2))^(1/2))
*(-4+2*5^(1/2))^(1/2)-1/2*arctan((5+2*5^(1/2)-x*5^(1/2))/(x^2+x-1)^(1/2)/(2
0+10*5^(1/2))^(1/2))*(4+2*5^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {1050, 1044, 213, 209}

$$\int \frac{1+2x}{(1+x^2)\sqrt{-1+x+x^2}}\,dx = \sqrt{\frac{1}{2}\left(\sqrt{5}-2\right)}\text{arctanh}\left(\frac{\sqrt{5}x-2\sqrt{5}+5}{\sqrt{10\left(\sqrt{5}-2\right)}\sqrt{x^2+x-1}}\right) - \sqrt{\frac{1}{2}\left(2+\sqrt{5}\right)}\arctan\left(\frac{-\sqrt{5}x+2\sqrt{5}+5}{\sqrt{10\left(2+\sqrt{5}\right)}\sqrt{x^2+x-1}}\right)$$

```
[In] Int[(1 + 2*x)/((1 + x^2)*Sqrt[-1 + x + x^2]),x]
```

```
[Out] -(Sqrt[(2 + Sqrt[5])/2]*ArcTan[(5 + 2*Sqrt[5] - Sqrt[5]*x)/(Sqrt[10*(2 + Sq
rt[5])]*Sqrt[-1 + x + x^2])]) + Sqrt[(-2 + Sqrt[5])/2]*ArcTanh[(5 - 2*Sqrt[
5] + Sqrt[5]*x)/(Sqrt[10*(-2 + Sqrt[5])]*Sqrt[-1 + x + x^2])]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1044

```
Int[((g_) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f
_.)*(x_)^2]), x_Symbol] :> Dist[-2*a*g*h, Subst[Int[1/Simp[2*a^2*g*h*c + a*
e*x^2, x], x], x, Simp[a*h - g*c*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[
{a, c, d, e, f, g, h}, x] && EqQ[a*h^2*e + 2*g*h*(c*d - a*f) - g^2*c*e, 0]
```

Rule 1050

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 + a*c*e^2, 2]}, Dist
[1/(2*q), Int[Simp[(-a)*h*e - g*(c*d - a*f - q) + (h*(c*d - a*f + q) - g*c*
e)*x, x]/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Si
mp[(-a)*h*e - g*(c*d - a*f + q) + (h*(c*d - a*f - q) - g*c*e)*x, x]/((a + c
*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] &&
 NeQ[e^2 - 4*d*f, 0] && NegQ[(-a)*c]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\int \frac{-\sqrt{5}+\left(-5-2\sqrt{5}\right)x}{(1+x^2)\sqrt{-1+x+x^2}}\,dx}{2\sqrt{5}} + \frac{\int \frac{\sqrt{5}+\left(-5+2\sqrt{5}\right)x}{(1+x^2)\sqrt{-1+x+x^2}}\,dx}{2\sqrt{5}} \\
&= -\left(\left(-5+2\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{10\left(2-\sqrt{5}\right)+x^2}\,dx, x, \frac{-5+2\sqrt{5}-\sqrt{5}x}{\sqrt{-1+x+x^2}}\right)\right) \\
&\quad + \left(5+2\sqrt{5}\right)\text{Subst}\left(\int \frac{1}{10\left(2+\sqrt{5}\right)+x^2}\,dx, x, \frac{-5-2\sqrt{5}+\sqrt{5}x}{\sqrt{-1+x+x^2}}\right)
\end{aligned}
$$

$$= -\sqrt{\frac{1}{2}\left(2+\sqrt{5}\right)}\tan^{-1}\left(\frac{5+2\sqrt{5}-\sqrt{5}x}{\sqrt{10\left(2+\sqrt{5}\right)}\sqrt{-1+x+x^2}}\right) + \sqrt{\frac{1}{2}\left(-2+\sqrt{5}\right)}\tanh^{-1}\left(\frac{5-2\sqrt{5}+\sqrt{5}x}{\sqrt{10\left(-2+\sqrt{5}\right)}\sqrt{-1+x+x^2}}\right)$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.13 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.91

$$\int \frac{1+2x}{(1+x^2)\sqrt{-1+x+x^2}}\,dx = \frac{1}{2}\text{RootSum}\Big[2-4\#1+6\#1^2 +\#1^4\&, \frac{3\log\left(-x+\sqrt{-1+x+x^2}-\#1\right)-2\log\left(-x+\sqrt{-1+x+x^2}-\#1\right)\#1+2\log\left(-x+\sqrt{-1+\right.}{-1+3\#1+\#1^3}$$

```
[In] Integrate[(1 + 2*x)/((1 + x^2)*Sqrt[-1 + x + x^2]),x]
```

```
[Out] RootSum[2 - 4*#1 + 6*#1^2 + #1^4 & , (3*Log[-x + Sqrt[-1 + x + x^2] - #1] -
 2*Log[-x + Sqrt[-1 + x + x^2] - #1]*#1 + 2*Log[-x + Sqrt[-1 + x + x^2] - #
1]*#1^2)/(-1 + 3*#1 + #1^3) & ]/2
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.08 (sec) , antiderivative size = 246, normalized size of antiderivative = 2.10

method	result
trager	$\text{RootOf}\left(\text{RootOf}\left(16\_Z^4+16\_Z^2+5\right)^2+\_Z^2+1\right)\ln\left(-\frac{4\,\text{RootOf}\left(\text{RootOf}\left(16\_Z^4+16\_Z^2+5\right)^2+\_Z^2\right.}{}\right.$
default	$\frac{\sqrt{\frac{10\left(-\sqrt{5}-2+x\right)^2}{\left(-\sqrt{5}+2-x\right)^2}-\frac{5\sqrt{5}\left(-\sqrt{5}-2+x\right)^2}{\left(-\sqrt{5}+2-x\right)^2}+10+5\sqrt{5}}\,\sqrt{5}\left(\arctan\left(\frac{\sqrt{5}\sqrt{\left(-2+\sqrt{5}\right)\left(-\frac{\left(-\sqrt{5}-2+x\right)^2}{\left(-\sqrt{5}+2-x\right)^2}+4\sqrt{5}+9\right)}\sqrt{20+10\sqrt{5}}\left(\frac{\sqrt{5}\left(-\sqrt{5}-2+x\right)^2}{\left(-\sqrt{5}+2-x\right)^2}+\right.}{5\left(-\sqrt{5}+2-x\right)\left(\frac{\left(-\sqrt{5}-2+x\right)^4}{\left(-\sqrt{5}+2-x\right)^4}-\frac{18\left(-\sqrt{5}\right.}{\left(-\sqrt{5}+\right.}\right.}\right.\right.}{}$

```
[In] int((1+2*x)/(x^2+1)/(x^2+x-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] RootOf(RootOf(16*_Z^4+16*_Z^2+5)^2+_Z^2+1)*ln(-(4*RootOf(RootOf(16*_Z^4+16*
_Z^2+5)^2+_Z^2+1)*RootOf(16*_Z^4+16*_Z^2+5)^2*x+2*RootOf(RootOf(16*_Z^4+16*
_Z^2+5)^2+_Z^2+1)*x+(x^2+x-1)^(1/2)+RootOf(RootOf(16*_Z^4+16*_Z^2+5)^2+_Z^2
+1))/(4*x*RootOf(16*_Z^4+16*_Z^2+5)^2+2*x-1))-RootOf(16*_Z^4+16*_Z^2+5)*ln(
-(4*RootOf(16*_Z^4+16*_Z^2+5)^3*x+2*RootOf(16*_Z^4+16*_Z^2+5)*x+(x^2+x-1)^(
1/2)-RootOf(16*_Z^4+16*_Z^2+5))/(4*x*RootOf(16*_Z^4+16*_Z^2+5)^2+2*x+1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.41 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.79

$$
\begin{aligned}
\int \frac{1+2x}{(1+x^2)\sqrt{-1+x+x^2}}\,dx = & \frac{1}{2}\sqrt{i-2}\log\left(-x+\sqrt{i-2}+\sqrt{x^2+x-1}+i\right) \\
& -\frac{1}{2}\sqrt{i-2}\log\left(-x-\sqrt{i-2}+\sqrt{x^2+x-1}+i\right) \\
& +\frac{1}{2}\sqrt{-i-2}\log\left(-x+\sqrt{-i-2}+\sqrt{x^2+x-1}-i\right) \\
& -\frac{1}{2}\sqrt{-i-2}\log\left(-x-\sqrt{-i-2}+\sqrt{x^2+x-1}-i\right)
\end{aligned}
$$

```
[In] integrate((1+2*x)/(x^2+1)/(x^2+x-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(I - 2)*log(-x + sqrt(I - 2) + sqrt(x^2 + x - 1) + I) - 1/2*sqrt(I
- 2)*log(-x - sqrt(I - 2) + sqrt(x^2 + x - 1) + I) + 1/2*sqrt(-I - 2)*log(-
x + sqrt(-I - 2) + sqrt(x^2 + x - 1) - I) - 1/2*sqrt(-I - 2)*log(-x - sqrt(
-I - 2) + sqrt(x^2 + x - 1) - I)
```

Sympy [F]

$$
\int \frac{1+2x}{(1+x^2)\sqrt{-1+x+x^2}}\,dx = \int \frac{2x+1}{(x^2+1)\sqrt{x^2+x-1}}\,dx
$$

```
[In] integrate((1+2*x)/(x**2+1)/(x**2+x-1)**(1/2),x)
```

```
[Out] Integral((2*x + 1)/((x**2 + 1)*sqrt(x**2 + x - 1)), x)
```

Maxima [F]

$$\int \frac{1+2x}{(1+x^2)\sqrt{-1+x+x^2}}\,dx = \int \frac{2\,x+1}{\sqrt{x^2+x-1}(x^2+1)}\,dx$$

```
[In] integrate((1+2*x)/(x^2+1)/(x^2+x-1)^(1/2),x, algorithm="maxima")
[Out] integrate((2*x + 1)/(sqrt(x^2 + x - 1)*(x^2 + 1)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 457 vs. 2(86) = 172.

Time = 0.34 (sec) , antiderivative size = 457, normalized size of antiderivative = 3.91

$$\int \frac{1+2x}{(1+x^2)\sqrt{-1+x+x^2}}\,dx$$
$$= \frac{1}{4}\sqrt{2\sqrt{5}-4}\log\left(16\left(15\sqrt{5}\left(x-\sqrt{x^2+x-1}\right)+33\,x+5\sqrt{5}-33\sqrt{x^2+x-1}+2\sqrt{5\sqrt{5}+11}+11\right)\right.$$
$$+16\left(5\sqrt{5}\left(x-\sqrt{x^2+x-1}\right)+11\,x-5\sqrt{5}\sqrt{5\sqrt{5}+11}-15\sqrt{5}-11\sqrt{x^2+x-1}-11\sqrt{5\sqrt{5}+11}-\right.$$
$$-\frac{1}{4}\sqrt{2\sqrt{5}-4}\log\left(16\left(15\sqrt{5}\left(x-\sqrt{x^2+x-1}\right)+33\,x+5\sqrt{5}-33\sqrt{x^2+x-1}-2\sqrt{5\sqrt{5}+11}+11\right.\right.$$
$$+16\left(5\sqrt{5}\left(x-\sqrt{x^2+x-1}\right)+11\,x+5\sqrt{5}\sqrt{5\sqrt{5}+11}-15\sqrt{5}-11\sqrt{x^2+x-1}+11\sqrt{5\sqrt{5}+11}-\right.$$
$$+\frac{\sqrt{2\sqrt{5}-4}\left(\arctan(3)+\arctan\left(\frac{1}{10}\left(x-\sqrt{x^2+x-1}\right)\left(\sqrt{5}\sqrt{5\sqrt{5}+11}+4\sqrt{5}-5\sqrt{5\sqrt{5}+11}\right)-\frac{7}{10}\right.\right.}{2\left(\sqrt{5}-2\right)}$$
$$-\frac{\sqrt{2\sqrt{5}-4}\left(\arctan(3)+\arctan\left(-\frac{1}{10}\left(x-\sqrt{x^2+x-1}\right)\left(\sqrt{5}\sqrt{5\sqrt{5}+11}-4\sqrt{5}-5\sqrt{5\sqrt{5}+11}\right)+\right.\right.}{2\left(\sqrt{5}-2\right)}$$

```
[In] integrate((1+2*x)/(x^2+1)/(x^2+x-1)^(1/2),x, algorithm="giac")
[Out] 1/4*sqrt(2*sqrt(5) - 4)*log(16*(15*sqrt(5)*(x - sqrt(x^2 + x - 1)) + 33*x +
 5*sqrt(5) - 33*sqrt(x^2 + x - 1) + 2*sqrt(5*sqrt(5) + 11) + 11)^2 + 16*(5*
sqrt(5)*(x - sqrt(x^2 + x - 1)) + 11*x - 5*sqrt(5)*sqrt(5*sqrt(5) + 11) - 1
5*sqrt(5) - 11*sqrt(x^2 + x - 1) - 11*sqrt(5*sqrt(5) + 11) - 33)^2) - 1/4*s
qrt(2*sqrt(5) - 4)*log(16*(15*sqrt(5)*(x - sqrt(x^2 + x - 1)) + 33*x + 5*sq
rt(5) - 33*sqrt(x^2 + x - 1) - 2*sqrt(5*sqrt(5) + 11) + 11)^2 + 16*(5*sqrt(
5)*(x - sqrt(x^2 + x - 1)) + 11*x + 5*sqrt(5)*sqrt(5*sqrt(5) + 11) - 15*sqr
t(5) - 11*sqrt(x^2 + x - 1) + 11*sqrt(5*sqrt(5) + 11) - 33)^2) + 1/2*sqrt(2
```

```
*sqrt(5) - 4)*(arctan(3) + arctan(1/10*(x - sqrt(x^2 + x - 1))*(sqrt(5)*sqr
t(5*sqrt(5) + 11) + 4*sqrt(5) - 5*sqrt(5*sqrt(5) + 11)) - 7/10*sqrt(5)*sqrt
(5*sqrt(5) + 11) + 1/5*sqrt(5) + 3/2*sqrt(5*sqrt(5) + 11)))/(sqrt(5) - 2) -
 1/2*sqrt(2*sqrt(5) - 4)*(arctan(3) + arctan(-1/10*(x - sqrt(x^2 + x - 1))*
(sqrt(5)*sqrt(5*sqrt(5) + 11) - 4*sqrt(5) - 5*sqrt(5*sqrt(5) + 11)) + 7/10*
sqrt(5)*sqrt(5*sqrt(5) + 11) + 1/5*sqrt(5) - 3/2*sqrt(5*sqrt(5) + 11)))/(sq
rt(5) - 2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1+2x}{(1+x^2)\sqrt{-1+x+x^2}}\,dx = \int \frac{2\,x+1}{(x^2+1)\,\sqrt{x^2+x-1}}\,dx$$

```
[In] int((2*x + 1)/((x^2 + 1)*(x + x^2 - 1)^(1/2)),x)
[Out] int((2*x + 1)/((x^2 + 1)*(x + x^2 - 1)^(1/2)), x)
```

3.12 $\int \frac{a-c+bx}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 484

$$\int \frac{a-c+bx}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx =$$

$$-\frac{\sqrt{a^2+b^2+c\left(c-\sqrt{a^2+b^2-2ac+c^2}\right)-a\left(2c-\sqrt{a^2+b^2-2ac+c^2}\right)}\arctan\left(\frac{b\sqrt{a^2+\cdots}}{\sqrt{2}\sqrt[4]{a^2+b^2-2ac+c\cdots}}\right.}{\sqrt{2}\sqrt[4]{a^2+b^2-2ac+c^2}}$$

$$-\frac{\sqrt{a^2+b^2+c\left(c+\sqrt{a^2+b^2-2ac+c^2}\right)-a\left(2c+\sqrt{a^2+b^2-2ac+c^2}\right)}\operatorname{arctanh}\left(\frac{b\sqrt{a^2+\cdots}}{\sqrt{2}\sqrt[4]{a^2+b^2-2ac+c\cdots}}\right.}{\sqrt{2}\sqrt[4]{a^2+b^2-2ac+c^2}}$$

```
[Out] -1/2*arctan(1/2*(b*(a^2-2*a*c+b^2+c^2)^(1/2)-x*(b^2+(a-c)*(a-c+(a^2-2*a*c+b
^2+c^2)^(1/2))))/(a^2-2*a*c+b^2+c^2)^(1/4)*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(a^2
+b^2+c*(c-(a^2-2*a*c+b^2+c^2)^(1/2))-a*(2*c-(a^2-2*a*c+b^2+c^2)^(1/2)))^(1/
2))*(a^2+b^2+c*(c-(a^2-2*a*c+b^2+c^2)^(1/2))-a*(2*c-(a^2-2*a*c+b^2+c^2)^(1/
2)))^(1/2)/(a^2-2*a*c+b^2+c^2)^(1/4)*2^(1/2)-1/2*arctanh(1/2*(x*(b^2+(a-c)*
(a-c-(a^2-2*a*c+b^2+c^2)^(1/2)))+b*(a^2-2*a*c+b^2+c^2)^(1/2))/(a^2-2*a*c+b^
2+c^2)^(1/4)*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(a^2+b^2+c*(c+(a^2-2*a*c+b^2+c^2)^
(1/2))-a*(2*c+(a^2-2*a*c+b^2+c^2)^(1/2)))^(1/2))*(a^2+b^2+c*(c+(a^2-2*a*c+b
^2+c^2)^(1/2))-a*(2*c+(a^2-2*a*c+b^2+c^2)^(1/2)))^(1/2)/(a^2-2*a*c+b^2+c^2)
^(1/4)*2^(1/2)
```

Rubi [A] (verified)

Time = 22.98 (sec) , antiderivative size = 484, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1050, 1044, 214, 211}

$$\int \frac{a-c+bx}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx =$$

$$-\frac{\sqrt{-a\left(2c-\sqrt{a^2-2ac+b^2+c^2}\right)+c\left(c-\sqrt{a^2-2ac+b^2+c^2}\right)+a^2+b^2}\arctan\left(\frac{b\cdots}{\sqrt{2}\sqrt[4]{a^2-2ac+b^2\cdots}}\right)}{\sqrt{2}\sqrt[4]{a^2-2ac+b^2+c^2}}$$

$$-\frac{\sqrt{-a\left(\sqrt{a^2-2ac+b^2+c^2}+2c\right)+c\left(\sqrt{a^2-2ac+b^2+c^2}+c\right)+a^2+b^2}\,\text{arctanh}\left(\frac{x(\cdots}{\sqrt{2}\sqrt[4]{a^2-2ac+b^2\cdots}}\right)}{\sqrt{2}\sqrt[4]{a^2-2ac+b^2+c^2}}$$

```
[In] Int[(a - c + b*x)/((1 + x^2)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -((Sqrt[a^2 + b^2 + c*(c - Sqrt[a^2 + b^2 - 2*a*c + c^2]) - a*(2*c - Sqrt[a
^2 + b^2 - 2*a*c + c^2])]*ArcTan[(b*Sqrt[a^2 + b^2 - 2*a*c + c^2] - (b^2 +
(a - c)*(a - c + Sqrt[a^2 + b^2 - 2*a*c + c^2]))*x)/(Sqrt[2]*(a^2 + b^2 - 2
*a*c + c^2)^(1/4)*Sqrt[a^2 + b^2 + c*(c - Sqrt[a^2 + b^2 - 2*a*c + c^2]) -
a*(2*c - Sqrt[a^2 + b^2 - 2*a*c + c^2])]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*
(a^2 + b^2 - 2*a*c + c^2)^(1/4))) - (Sqrt[a^2 + b^2 + c*(c + Sqrt[a^2 + b^2
 - 2*a*c + c^2]) - a*(2*c + Sqrt[a^2 + b^2 - 2*a*c + c^2])]*ArcTanh[(b*Sqrt
[a^2 + b^2 - 2*a*c + c^2] + (b^2 + (a - c)*(a - c - Sqrt[a^2 + b^2 - 2*a*c
+ c^2]))*x)/(Sqrt[2]*(a^2 + b^2 - 2*a*c + c^2)^(1/4)*Sqrt[a^2 + b^2 + c*(c
+ Sqrt[a^2 + b^2 - 2*a*c + c^2]) - a*(2*c + Sqrt[a^2 + b^2 - 2*a*c + c^2])]
*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*(a^2 + b^2 - 2*a*c + c^2)^(1/4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1044

```
Int[((g_) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f
_.)*(x_)^2]), x_Symbol] :> Dist[-2*a*g*h, Subst[Int[1/Simp[2*a^2*g*h*c + a*
e*x^2, x], x], x, Simp[a*h - g*c*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[
```

```
{a, c, d, e, f, g, h}, x] && EqQ[a*h^2*e + 2*g*h*(c*d - a*f) - g^2*c*e, 0]
```

Rule 1050

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 + a*c*e^2, 2]}, Dist
[1/(2*q), Int[Simp[(-a)*h*e - g*(c*d - a*f - q) + (h*(c*d - a*f + q) - g*c*
e)*x, x]/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Si
mp[(-a)*h*e - g*(c*d - a*f + q) + (h*(c*d - a*f - q) - g*c*e)*x, x]/((a + c
*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] &&
 NeQ[e^2 - 4*d*f, 0] && NegQ[(-a)*c]
```

Rubi steps

$$\text{integral} = -\frac{\int \frac{-b^2-(a-c)\left(a-c+\sqrt{a^2+b^2-2ac+c^2}\right)-b\sqrt{a^2+b^2-2ac+c^2}x}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx}{2\sqrt{a^2+b^2-2ac+c^2}} + \frac{\int \frac{-b^2-(a-c)\left(a-c-\sqrt{a^2+b^2-2ac+c^2}\right)+b\sqrt{a^2+b^2-2ac+c^2}x}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx}{2\sqrt{a^2+b^2-2ac+c^2}}$$

$$= \left(b\left(b^2 +(a-c)\left(a-c-\sqrt{a^2+b^2-2ac+c^2}\right)\right)\right)\text{Subst}\left(\int \frac{1}{-2b\sqrt{a^2+b^2-2ac+c^2}\,(b^2+(a-c)\,(a-c-\right.$$

$$+ \left(b\left(b^2 +(a-c)\left(a-c+\sqrt{a^2+b^2-2ac+c^2}\right)\right)\right)\text{Subst}\left(\int \frac{1}{2b\sqrt{a^2+b^2-2ac+c^2}\,(b^2+(a-c)\,(a-c+\right.$$

$$= -\frac{\sqrt{a^2+b^2+c\left(c-\sqrt{a^2+b^2-2ac+c^2}\right)-a\left(2c-\sqrt{a^2+b^2-2ac+c^2}\right)}\tan^{-1}\left(\frac{}{\sqrt{2}\sqrt[4]{a^2+b^2-}\right.}{\sqrt{2}\sqrt[4]{a^2+b^2-2ac+c^2}}$$

$$-\frac{\sqrt{a^2+b^2+c\left(c+\sqrt{a^2+b^2-2ac+c^2}\right)-a\left(2c+\sqrt{a^2+b^2-2ac+c^2}\right)}\tanh^{-1}\left(\frac{}{\sqrt{2}\sqrt[4]{a^2+b^2-}\right.}{\sqrt{2}\sqrt[4]{a^2+b^2-2ac+c^2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.36 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.43

$$\int \frac{a-c+bx}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx=\frac{1}{2}\text{RootSum}\Big[a^2+b^2-4b\sqrt{c}\#1-2a\#1^2+4c\#1^2$$
$$+\#1^4\&,\frac{bc\log\left(-\sqrt{c}x+\sqrt{a+bx+cx^2}-\#1\right)+2a\sqrt{c}\log\left(-\sqrt{c}x+\sqrt{a+bx+cx^2}-\#1\right)\#1-2c^{3/2}\text{lo}}{b\sqrt{c}+a\#1-2c\#1}$$

```
[In] Integrate[(a - c + b*x)/((1 + x^2)*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] RootSum[a^2 + b^2 - 4*b*Sqrt[c]*#1 - 2*a*#1^2 + 4*c*#1^2 + #1^4 & , (b*c*Lo
g[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + 2*a*Sqrt[c]*Log[-(Sqrt[c]*x)
 + Sqrt[a + b*x + c*x^2] - #1]*#1 - 2*c^(3/2)*Log[-(Sqrt[c]*x) + Sqrt[a + b
*x + c*x^2] - #1]*#1 - b*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^
2)/(b*Sqrt[c] + a*#1 - 2*c*#1 - #1^3) & ]/2
```

Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 116.44 (sec) , antiderivative size = 6870946, normalized size of antiderivative = 14196.17

method	result	size
default	Expression too large to display	6870946

```
[In] int((b*x+a-c)/(x^2+1)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Fricas [A] (verification not implemented)

none

Time = 0.37 (sec) , antiderivative size = 307, normalized size of antiderivative = 0.63

$$\int \frac{a-c+bx}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx = -\frac{1}{4}\sqrt{-a+c+\sqrt{-b^2}}\log\left(\frac{2\,bcx+2\,\sqrt{cx^2+bx+a}\sqrt{-a+c+\sqrt{-b^2}}b+b^2+\sqrt{-b^2}(bx+2\,a)}{x}\right) +\frac{1}{4}\sqrt{-a+c+\sqrt{-b^2}}\log\left(\frac{2\,bcx-2\,\sqrt{cx^2+bx+a}\sqrt{-a+c+\sqrt{-b^2}}b+b^2+\sqrt{-b^2}(bx+2\,a)}{x}\right) -\frac{1}{4}\sqrt{-a+c-\sqrt{-b^2}}\log\left(\frac{2\,bcx+2\,\sqrt{cx^2+bx+a}\sqrt{-a+c-\sqrt{-b^2}}b+b^2-\sqrt{-b^2}(bx+2\,a)}{x}\right) +\frac{1}{4}\sqrt{-a+c-\sqrt{-b^2}}\log\left(\frac{2\,bcx-2\,\sqrt{cx^2+bx+a}\sqrt{-a+c-\sqrt{-b^2}}b+b^2-\sqrt{-b^2}(bx+2\,a)}{x}\right)$$

```
[In] integrate((b*x+a-c)/(x^2+1)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(-a + c + sqrt(-b^2))*log((2*b*c*x + 2*sqrt(c*x^2 + b*x + a)*sqrt(
-a + c + sqrt(-b^2))*b + b^2 + sqrt(-b^2)*(b*x + 2*a))/x) + 1/4*sqrt(-a + c
 + sqrt(-b^2))*log((2*b*c*x - 2*sqrt(c*x^2 + b*x + a)*sqrt(-a + c + sqrt(-b
^2))*b + b^2 + sqrt(-b^2)*(b*x + 2*a))/x) - 1/4*sqrt(-a + c - sqrt(-b^2))*l
og((2*b*c*x + 2*sqrt(c*x^2 + b*x + a)*sqrt(-a + c - sqrt(-b^2))*b + b^2 - s
qrt(-b^2)*(b*x + 2*a))/x) + 1/4*sqrt(-a + c - sqrt(-b^2))*log((2*b*c*x - 2*
sqrt(c*x^2 + b*x + a)*sqrt(-a + c - sqrt(-b^2))*b + b^2 - sqrt(-b^2)*(b*x +
 2*a))/x)
```

Sympy [F]

$$\int \frac{a-c+bx}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx = \int \frac{a+bx-c}{(x^2+1)\sqrt{a+bx+cx^2}}\,dx$$

```
[In] integrate((b*x+a-c)/(x**2+1)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((a + b*x - c)/((x**2 + 1)*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F]

$$\int \frac{a-c+bx}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx = \int \frac{bx+a-c}{\sqrt{cx^2+bx+a}(x^2+1)}\,dx$$

```
[In] integrate((b*x+a-c)/(x^2+1)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
[Out] integrate((b*x + a - c)/(sqrt(c*x^2 + b*x + a)*(x^2 + 1)), x)
```

Giac [F]

$$\int \frac{a-c+bx}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx = \int \frac{bx+a-c}{\sqrt{cx^2+bx+a}(x^2+1)}\,dx$$

```
[In] integrate((b*x+a-c)/(x^2+1)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate((b*x + a - c)/(sqrt(c*x^2 + b*x + a)*(x^2 + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a-c+bx}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx = \int \frac{a-c+b\,x}{(x^2+1)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

```
[In] int((a - c + b*x)/((x^2 + 1)*(a + b*x + c*x^2)^(1/2)),x)
[Out] int((a - c + b*x)/((x^2 + 1)*(a + b*x + c*x^2)^(1/2)), x)
```

3.13 $\int \frac{(A+Bx)(a+bx+cx^2)}{d+ex+fx^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 184

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{d+ex+fx^2}\,dx=-\frac{(Bce-bBf-Acf)x}{f^2}+\frac{Bcx^2}{2f}-\frac{\left(Af\left(ce^2-2cdf-bef+2af^2\right)+B\left(f\left(be^2-2bdf-aef\right)-c\left(e^3-3def\right)\right)\right)\operatorname{arctanh}\left(\frac{e+2fx}{\sqrt{e^2-4df}}\right)}{f^3\sqrt{e^2-4df}}-\frac{\left(Af(ce-bf)-B\left(ce^2-cdf-bef+af^2\right)\right)\log\left(d+ex+fx^2\right)}{2f^3}$$

```
[Out] -(-A*c*f-B*b*f+B*c*e)*x/f^2+1/2*B*c*x^2/f-1/2*(A*f*(-b*f+c*e)-B*(a*f^2-b*e*
f-c*d*f+c*e^2))*ln(f*x^2+e*x+d)/f^3-(A*f*(2*a*f^2-b*e*f-2*c*d*f+c*e^2)+B*(f
*(-a*e*f-2*b*d*f+b*e^2)-c*(-3*d*e*f+e^3)))*arctanh((2*f*x+e)/(-4*d*f+e^2)^(
1/2))/f^3/(-4*d*f+e^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {1642, 648, 632, 212, 642}

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{d+ex+fx^2}\,dx=-\frac{\operatorname{arctanh}\left(\frac{e+2fx}{\sqrt{e^2-4df}}\right)\left(Af\left(2af^2-bef-2cdf+ce^2\right)+Bf\left(-aef-2bdf+be^2\right)-Bc\left(e^3-3def\right)\right)}{f^3\sqrt{e^2-4df}}-\frac{\log\left(d+ex+fx^2\right)\left(Bf(be-af)+Af(ce-bf)-Bc\left(e^2-df\right)\right)}{2f^3}-\frac{x(-Acf-bBf+Bce)}{f^2}+\frac{Bcx^2}{2f}$$

```
[In] Int[((A + B*x)*(a + b*x + c*x^2))/(d + e*x + f*x^2),x]
```

```
[Out] -(((B*c*e - b*B*f - A*c*f)*x)/f^2) + (B*c*x^2)/(2*f) - ((B*f*(b*e^2 - 2*b*d
*f - a*e*f) - B*c*(e^3 - 3*d*e*f) + A*f*(c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2)
)*ArcTanh[(e + 2*f*x)/Sqrt[e^2 - 4*d*f]])/(f^3*Sqrt[e^2 - 4*d*f]) - ((B*f*(
b*e - a*f) + A*f*(c*e - b*f) - B*c*(e^2 - d*f))*Log[d + e*x + f*x^2])/(2*f^
3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

integral

$$= \int \left(-\frac{Bce - bBf - Acf}{f^2} + \frac{Bcx}{f} + \frac{-Af(cd - af) + Bd(ce - bf) - (Bf(be - af) + Af(ce - bf) - Bc(e^2 - df))\,x}{f^2\,(d + ex + fx^2)}\right) dx$$

$$=-\frac{(Bce-bBf-Acf)x}{f^2}+\frac{Bcx^2}{2f}+\frac{\int \frac{-Af(cd-af)+Bd(ce-bf)-(Bf(be-af)+Af(ce-bf)-Bc(e^2-df))x}{d+ex+fx^2}\,dx}{f^2}$$

$$=-\frac{(Bce-bBf-Acf)x}{f^2}+\frac{Bcx^2}{2f}+\frac{(-Bf(be-af)-Af(ce-bf)+Bc(e^2-df))\int \frac{e+2fx}{d+ex+fx^2}\,dx}{2f^3}+\frac{(Bf(be^2-2bdf-aef)-Bc(e^3-3def)+Af(ce^2-2cdf-bef+2af^2))\int \frac{1}{d+ex+fx^2}\,dx}{2f^3}$$

$$=-\frac{(Bce-bBf-Acf)x}{f^2}+\frac{Bcx^2}{2f}-\frac{(Bf(be-af)+Af(ce-bf)-Bc(e^2-df))\log(d+ex+fx^2)}{2f^3}-\frac{(Bf(be^2-2bdf-aef)-Bc(e^3-3def)+Af(ce^2-2cdf-bef+2af^2))\,\text{Subst}\left(\int \frac{1}{e^2-4df-x^2}\,dx,x,e+2\right.}{f^3}$$

$$=-\frac{(Bce-bBf-Acf)x}{f^2}+\frac{Bcx^2}{2f}-\frac{(Bf(be^2-2bdf-aef)-Bc(e^3-3def)+Af(ce^2-2cdf-bef+2af^2))\tanh^{-1}\left(\frac{e+2fx}{\sqrt{e^2-4df}}\right)}{f^3\sqrt{e^2-4df}}-\frac{(Bf(be-af)+Af(ce-bf)-Bc(e^2-df))\log(d+ex+fx^2)}{2f^3}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.95

$$\int \frac{(A+Bx)(a+bx+cx^2)}{d+ex+fx^2}\,dx$$

$$=\frac{2f(-Bce+bBf+Acf)x+Bcf^2x^2-\frac{2(Bf(-be^2+2bdf+aef)+Bc(e^3-3def)+Af(-ce^2+2cdf+bef-2af^2))\arctan\left(\frac{e+2fx}{\sqrt{-e^2+4df}}\right)}{\sqrt{-e^2+4df}}}{2f^3}$$

```
[In] Integrate[((A + B*x)*(a + b*x + c*x^2))/(d + e*x + f*x^2),x]
```

```
[Out] (2*f*(-(B*c*e) + b*B*f + A*c*f)*x + B*c*f^2*x^2 - (2*(B*f*(-(b*e^2) + 2*b*d
*f + a*e*f) + B*c*(e^3 - 3*d*e*f) + A*f*(-(c*e^2) + 2*c*d*f + b*e*f - 2*a*f
^2))*ArcTan[(e + 2*f*x)/Sqrt[-e^2 + 4*d*f]])/Sqrt[-e^2 + 4*d*f] + (B*f*(-(b
*e) + a*f) + A*f*(-(c*e) + b*f) + B*c*(e^2 - d*f))*Log[d + x*(e + f*x)])/(2
*f^3)
```

Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.03

method	result
default	$\frac{\frac{1}{2}Bc\,x^2f+Acfx+Bbfx-Bcex}{f^2}+\frac{\frac{\left(Ab\,f^2-Acef+Ba\,f^2-Bbef-Bcdf+Bc\,e^2\right)\ln\left(f\,x^2+ex+d\right)}{2f}+2\left(Aa\,f^2-Acdf-Bbdf+Bcde-\frac{\left(Ab\,f^2-Ace\right.}{}\right.}{f^2}$
risch	Expression too large to display

```
[In] int((B*x+A)*(c*x^2+b*x+a)/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f^2*(1/2*B*c*x^2*f+A*c*f*x+B*b*f*x-B*c*e*x)+1/f^2*(1/2*(A*b*f^2-A*c*e*f+B
*a*f^2-B*b*e*f-B*c*d*f+B*c*e^2)/f*ln(f*x^2+e*x+d)+2*(A*a*f^2-A*c*d*f-B*b*d*
f+B*c*d*e-1/2*(A*b*f^2-A*c*e*f+B*a*f^2-B*b*e*f-B*c*d*f+B*c*e^2)*e/f)/(4*d*f
-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 583, normalized size of antiderivative = 3.17

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{d+ex+fx^2}\,dx$$
$$=\left[\frac{(Bce^2f^2-4\,Bcdf^3)x^2-(Bce^3-2\,Aaf^3+(2\,(Bb+Ac)d+(Ba+Ab)e)f^2-(3\,Bcde+(Bb+Ac)e^2)}{}\right.$$

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] [1/2*((B*c*e^2*f^2 - 4*B*c*d*f^3)*x^2 - (B*c*e^3 - 2*A*a*f^3 + (2*(B*b + A*
c)*d + (B*a + A*b)*e)*f^2 - (3*B*c*d*e + (B*b + A*c)*e^2)*f)*sqrt(e^2 - 4*d
*f)*log((2*f^2*x^2 + 2*e*f*x + e^2 - 2*d*f - sqrt(e^2 - 4*d*f)*(2*f*x + e))
/(f*x^2 + e*x + d)) - 2*(B*c*e^3*f + 4*(B*b + A*c)*d*f^3 - (4*B*c*d*e + (B*
b + A*c)*e^2)*f^2)*x + (B*c*e^4 - 4*(B*a + A*b)*d*f^3 + (4*B*c*d^2 + 4*(B*b
 + A*c)*d*e + (B*a + A*b)*e^2)*f^2 - (5*B*c*d*e^2 + (B*b + A*c)*e^3)*f)*log
(f*x^2 + e*x + d))/(e^2*f^3 - 4*d*f^4), 1/2*((B*c*e^2*f^2 - 4*B*c*d*f^3)*x^
2 + 2*(B*c*e^3 - 2*A*a*f^3 + (2*(B*b + A*c)*d + (B*a + A*b)*e)*f^2 - (3*B*c
*d*e + (B*b + A*c)*e^2)*f)*sqrt(-e^2 + 4*d*f)*arctan(-sqrt(-e^2 + 4*d*f)*(2
*f*x + e)/(e^2 - 4*d*f)) - 2*(B*c*e^3*f + 4*(B*b + A*c)*d*f^3 - (4*B*c*d*e
+ (B*b + A*c)*e^2)*f^2)*x + (B*c*e^4 - 4*(B*a + A*b)*d*f^3 + (4*B*c*d^2 + 4
*(B*b + A*c)*d*e + (B*a + A*b)*e^2)*f^2 - (5*B*c*d*e^2 + (B*b + A*c)*e^3)*f
)*log(f*x^2 + e*x + d))/(e^2*f^3 - 4*d*f^4)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1260 vs. $2(175) = 350$.

Time = 6.33 (sec) , antiderivative size = 1260, normalized size of antiderivative = 6.85

$$\int \frac{(A+Bx)(a+bx+cx^2)}{d+ex+fx^2}\,dx = \frac{Bcx^2}{2f} + x\left(\frac{Ac}{f} + \frac{Bb}{f} - \frac{Bce}{f^2}\right)$$
$$+\left(-\frac{\sqrt{-4df+e^2}(-2Aaf^3 + Abef^2 + 2Acdf^2 - Ace^2f + Baef^2 + 2Bbdf^2 - Bbe^2f - 3Bcdef + Bce^3)}{2f^3\cdot(4df-e^2)}\right.$$
$$\left.+\frac{Abf^2 - Acef + Baf^2 - Bbef - Bcdf + Bce^2}{2f^3}\right)\log\left(x + \frac{-Aaef^2 + 2Abdf^2 - Acdef + 2Badf^2 - Bbd}{}\right.$$
$$+\left(\frac{\sqrt{-4df+e^2}(-2Aaf^3 + Abef^2 + 2Acdf^2 - Ace^2f + Baef^2 + 2Bbdf^2 - Bbe^2f - 3Bcdef + Bce^3)}{2f^3\cdot(4df-e^2)}\right.$$
$$\left.+\frac{Abf^2 - Acef + Baf^2 - Bbef - Bcdf + Bce^2}{2f^3}\right)\log\left(x + \frac{-Aaef^2 + 2Abdf^2 - Acdef + 2Badf^2 - Bbd}{}\right.$$

```
[In] integrate((B*x+A)*(c*x**2+b*x+a)/(f*x**2+e*x+d),x)
```

```
[Out] B*c*x**2/(2*f) + x*(A*c/f + B*b/f - B*c*e/f**2) + (-sqrt(-4*d*f + e**2)*(-2
*A*a*f**3 + A*b*e*f**2 + 2*A*c*d*f**2 - A*c*e**2*f + B*a*e*f**2 + 2*B*b*d*f
**2 - B*b*e**2*f - 3*B*c*d*e*f + B*c*e**3)/(2*f**3*(4*d*f - e**2)) + (A*b*f
**2 - A*c*e*f + B*a*f**2 - B*b*e*f - B*c*d*f + B*c*e**2)/(2*f**3))*log(x +
(-A*a*e*f**2 + 2*A*b*d*f**2 - A*c*d*e*f + 2*B*a*d*f**2 - B*b*d*e*f - 2*B*c*
d**2*f + B*c*d*e**2 - 4*d*f**3*(-sqrt(-4*d*f + e**2)*(-2*A*a*f**3 + A*b*e*f
**2 + 2*A*c*d*f**2 - A*c*e**2*f + B*a*e*f**2 + 2*B*b*d*f**2 - B*b*e**2*f -
3*B*c*d*e*f + B*c*e**3)/(2*f**3*(4*d*f - e**2)) + (A*b*f**2 - A*c*e*f + B*a
*f**2 - B*b*e*f - B*c*d*f + B*c*e**2)/(2*f**3)) + e**2*f**2*(-sqrt(-4*d*f +
 e**2)*(-2*A*a*f**3 + A*b*e*f**2 + 2*A*c*d*f**2 - A*c*e**2*f + B*a*e*f**2 +
 2*B*b*d*f**2 - B*b*e**2*f - 3*B*c*d*e*f + B*c*e**3)/(2*f**3*(4*d*f - e**2)
) + (A*b*f**2 - A*c*e*f + B*a*f**2 - B*b*e*f - B*c*d*f + B*c*e**2)/(2*f**3)
))/(-2*A*a*f**3 + A*b*e*f**2 + 2*A*c*d*f**2 - A*c*e**2*f + B*a*e*f**2 + 2*B
*b*d*f**2 - B*b*e**2*f - 3*B*c*d*e*f + B*c*e**3)) + (sqrt(-4*d*f + e**2)*(-
2*A*a*f**3 + A*b*e*f**2 + 2*A*c*d*f**2 - A*c*e**2*f + B*a*e*f**2 + 2*B*b*d*
f**2 - B*b*e**2*f - 3*B*c*d*e*f + B*c*e**3)/(2*f**3*(4*d*f - e**2)) + (A*b*
f**2 - A*c*e*f + B*a*f**2 - B*b*e*f - B*c*d*f + B*c*e**2)/(2*f**3))*log(x +
 (-A*a*e*f**2 + 2*A*b*d*f**2 - A*c*d*e*f + 2*B*a*d*f**2 - B*b*d*e*f - 2*B*c
*d**2*f + B*c*d*e**2 - 4*d*f**3*(sqrt(-4*d*f + e**2)*(-2*A*a*f**3 + A*b*e*f
**2 + 2*A*c*d*f**2 - A*c*e**2*f + B*a*e*f**2 + 2*B*b*d*f**2 - B*b*e**2*f -
3*B*c*d*e*f + B*c*e**3)/(2*f**3*(4*d*f - e**2)) + (A*b*f**2 - A*c*e*f + B*a
*f**2 - B*b*e*f - B*c*d*f + B*c*e**2)/(2*f**3)) + e**2*f**2*(sqrt(-4*d*f +
e**2)*(-2*A*a*f**3 + A*b*e*f**2 + 2*A*c*d*f**2 - A*c*e**2*f + B*a*e*f**2 +
2*B*b*d*f**2 - B*b*e**2*f - 3*B*c*d*e*f + B*c*e**3)/(2*f**3*(4*d*f - e**2))
```

```
 + (A*b*f**2 - A*c*e*f + B*a*f**2 - B*b*e*f - B*c*d*f + B*c*e**2)/(2*f**3))
)/(-2*A*a*f**3 + A*b*e*f**2 + 2*A*c*d*f**2 - A*c*e**2*f + B*a*e*f**2 + 2*B*
b*d*f**2 - B*b*e**2*f - 3*B*c*d*e*f + B*c*e**3))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(A+Bx)(a+bx+cx^2)}{d+ex+fx^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.03

$$\int \frac{(A+Bx)(a+bx+cx^2)}{d+ex+fx^2}\,dx = \frac{Bcfx^2-2\,Bcex+2\,Bbfx+2\,Acfx}{2\,f^2} + \frac{(Bce^2-Bcdf-Bbef-Acef+Baf^2+Abf^2)\log(fx^2+ex+d)}{2\,f^3} - \frac{(Bce^3-3\,Bcdef-Bbe^2f-Ace^2f+2\,Bbdf^2+2\,Acdf^2+Baef^2+Abef^2-2\,Aaf^3)\arctan\left(\frac{2\,fx+e}{\sqrt{-e^2+4\,df}}\right)}{\sqrt{-e^2+4\,df}\,f^3}$$

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] 1/2*(B*c*f*x^2 - 2*B*c*e*x + 2*B*b*f*x + 2*A*c*f*x)/f^2 + 1/2*(B*c*e^2 - B*
c*d*f - B*b*e*f - A*c*e*f + B*a*f^2 + A*b*f^2)*log(f*x^2 + e*x + d)/f^3 - (
B*c*e^3 - 3*B*c*d*e*f - B*b*e^2*f - A*c*e^2*f + 2*B*b*d*f^2 + 2*A*c*d*f^2 +
 B*a*e*f^2 + A*b*e*f^2 - 2*A*a*f^3)*arctan((2*f*x + e)/sqrt(-e^2 + 4*d*f))/
(sqrt(-e^2 + 4*d*f)*f^3)
```

Mupad [B] (verification not implemented)

Time = 13.66 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.48

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{d+ex+fx^2}\,dx = x\left(\frac{A\,c+B\,b}{f}-\frac{B\,c\,e}{f^2}\right)$$
$$-\frac{\ln\left(f\,x^2+e\,x+d\right)\,\left(B\,c\,e^4-4\,A\,b\,d\,f^3-4\,B\,a\,d\,f^3-A\,c\,e^3\,f-B\,b\,e^3\,f+A\,b\,e^2\,f^2+B\,a\,e^2\,f^2+4\,B\,c\,d^2}{2\,\left(4\,d\,f^4-e^2\,f^3\right)}$$
$$-\frac{\mathrm{atan}\left(\frac{e}{\sqrt{4\,d\,f-e^2}}+\frac{2\,f\,x}{\sqrt{4\,d\,f-e^2}}\right)\,\left(B\,c\,e^3-2\,A\,a\,f^3+A\,b\,e\,f^2+2\,A\,c\,d\,f^2+B\,a\,e\,f^2+2\,B\,b\,d\,f^2-A\,c\,e^2\,f-}{f^3\,\sqrt{4\,d\,f-e^2}}$$
$$+\frac{B\,c\,x^2}{2\,f}$$

```
[In] int(((A + B*x)*(a + b*x + c*x^2))/(d + e*x + f*x^2),x)
```

```
[Out] x*((A*c + B*b)/f - (B*c*e)/f^2) - (log(d + e*x + f*x^2)*(B*c*e^4 - 4*A*b*d*
f^3 - 4*B*a*d*f^3 - A*c*e^3*f - B*b*e^3*f + A*b*e^2*f^2 + B*a*e^2*f^2 + 4*B
*c*d^2*f^2 + 4*A*c*d*e*f^2 + 4*B*b*d*e*f^2 - 5*B*c*d*e^2*f))/(2*(4*d*f^4 -
e^2*f^3)) - (atan(e/(4*d*f - e^2)^(1/2) + (2*f*x)/(4*d*f - e^2)^(1/2))*(B*c
*e^3 - 2*A*a*f^3 + A*b*e*f^2 + 2*A*c*d*f^2 + B*a*e*f^2 + 2*B*b*d*f^2 - A*c*
e^2*f - B*b*e^2*f - 3*B*c*d*e*f))/(f^3*(4*d*f - e^2)^(1/2)) + (B*c*x^2)/(2*
f)
```

3.14 $\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{d+ex+fx^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 542

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{d+ex+fx^2}\,dx$$
$$= \frac{(B(ce-bf)\left(f(be-2af)-c(e^2-2df)\right)+Af(b^2f^2-2cf(be-af)+c^2(e^2-df)))\,x}{f^4}$$
$$-\frac{(Acf(ce-2bf)-B(b^2f^2-2cf(be-af)+c^2(e^2-df)))\,x^2}{2f^3}$$
$$-\frac{c(Bce-2bBf-Acf)x^3}{3f^2}+\frac{Bc^2x^4}{4f}$$
$$-\frac{(Af(c^2(e^4-4de^2f+2d^2f^2)-f^2(2abef-2a^2f^2-b^2(e^2-2df))+2cf(af(e^2-2df)-b(e^3-3def))}{}$$
$$+\frac{(Af(ce-bf)\left(f(be-2af)-c(e^2-2df)\right)+B(c^2(e^4-3de^2f+d^2f^2)-f^2(2abef-a^2f^2-b^2(e^2-df}{2f^5}$$

```
[Out] (B*(-b*f+c*e)*(f*(-2*a*f+b*e)-c*(-2*d*f+e^2))+A*f*(b^2*f^2-2*c*f*(-a*f+b*e)
+c^2*(-d*f+e^2)))*x/f^4-1/2*(A*c*f*(-2*b*f+c*e)-B*(b^2*f^2-2*c*f*(-a*f+b*e)
+c^2*(-d*f+e^2)))*x^2/f^3-1/3*c*(-A*c*f-2*B*b*f+B*c*e)*x^3/f^2+1/4*B*c^2*x^
4/f+1/2*(A*f*(-b*f+c*e)*(f*(-2*a*f+b*e)-c*(-2*d*f+e^2))+B*(c^2*(d^2*f^2-3*d
*e^2*f+e^4)-f^2*(2*a*b*e*f-a^2*f^2-b^2*(-d*f+e^2))+2*c*f*(a*f*(-d*f+e^2)-b*
(-2*d*e*f+e^3))))*ln(f*x^2+e*x+d)/f^5-(A*f*(c^2*(2*d^2*f^2-4*d*e^2*f+e^4)-f
^2*(2*a*b*e*f-2*a^2*f^2-b^2*(-2*d*f+e^2))+2*c*f*(a*f*(-2*d*f+e^2)-b*(-3*d*e
*f+e^3)))-B*(c^2*(5*d^2*e*f^2-5*d*e^3*f+e^5)+f^2*(a^2*e*f^2-2*a*b*f*(-2*d*f
+e^2)+b^2*(-3*d*e*f+e^3))+2*c*f*(a*e*f*(-3*d*f+e^2)-b*(2*d^2*f^2-4*d*e^2*f+
e^4))))*arctanh((2*f*x+e)/(-4*d*f+e^2)^(1/2))/f^5/(-4*d*f+e^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 542, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1025, 648, 632, 212, 642}

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{d+ex+fx^2}\,dx =$$

$$-\frac{\operatorname{arctanh}\left(\frac{e+2fx}{\sqrt{e^2-4df}}\right)\left(Af(-f^2(-2a^2f^2+2abef-(b^2(e^2-2df)))+2cf(af(e^2-2df)-b(e^3-3def))+c\right.}{}$$

$$+\frac{\log\left(d+ex+fx^2\right)\left(B(-f^2(-a^2f^2+2abef-(b^2(e^2-df)))+2cf(af(e^2-df)-b(e^3-2def))+c^2(d^2\right.}{2f^5}$$

$$+\frac{x(Af(-2cf(be-af)+b^2f^2+c^2(e^2-df))+B(ce-bf)\left(f(be-2af)-c(e^2-2df)\right))}{f^4}$$

$$-\frac{x^2(Acf(ce-2bf)-B(-2cf(be-af)+b^2f^2+c^2(e^2-df)))}{2f^3}$$

$$-\frac{cx^3(-Acf-2bBf+Bce)}{3f^2}+\frac{Bc^2x^4}{4f}$$

```
[In] Int[((A + B*x)*(a + b*x + c*x^2)^2)/(d + e*x + f*x^2),x]
```

```
[Out] ((B*(c*e - b*f)*(f*(b*e - 2*a*f) - c*(e^2 - 2*d*f)) + A*f*(b^2*f^2 - 2*c*f*
(b*e - a*f) + c^2*(e^2 - d*f)))*x)/f^4 - ((A*c*f*(c*e - 2*b*f) - B*(b^2*f^2
 - 2*c*f*(b*e - a*f) + c^2*(e^2 - d*f)))*x^2)/(2*f^3) - (c*(B*c*e - 2*b*B*f
 - A*c*f)*x^3)/(3*f^2) + (B*c^2*x^4)/(4*f) - ((A*f*(c^2*(e^4 - 4*d*e^2*f +
2*d^2*f^2) - f^2*(2*a*b*e*f - 2*a^2*f^2 - b^2*(e^2 - 2*d*f)) + 2*c*f*(a*f*(
e^2 - 2*d*f) - b*(e^3 - 3*d*e*f))) - B*(c^2*(e^5 - 5*d*e^3*f + 5*d^2*e*f^2)
 + f^2*(a^2*e*f^2 - 2*a*b*f*(e^2 - 2*d*f) + b^2*(e^3 - 3*d*e*f)) + 2*c*f*(a
*e*f*(e^2 - 3*d*f) - b*(e^4 - 4*d*e^2*f + 2*d^2*f^2))))*ArcTanh[(e + 2*f*x)
/Sqrt[e^2 - 4*d*f]])/(f^5*Sqrt[e^2 - 4*d*f]) + ((A*f*(c*e - b*f)*(f*(b*e -
2*a*f) - c*(e^2 - 2*d*f)) + B*(c^2*(e^4 - 3*d*e^2*f + d^2*f^2) - f^2*(2*a*b
*e*f - a^2*f^2 - b^2*(e^2 - d*f)) + 2*c*f*(a*f*(e^2 - d*f) - b*(e^3 - 2*d*e
*f))))*Log[d + e*x + f*x^2])/(2*f^5)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1025

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x +
c*x^2)^p*(d + e*x + f*x^2)^q*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && IGtQ[p, 0] && In
tegerQ[q]
```

Rubi steps

integral

$$= \int \left(\frac{B(ce-bf)\left(f(be-2af)-c(e^2-2df)\right)+Af\left(b^2f^2-2cf(be-af)+c^2(e^2-df)\right)}{f^4} - \frac{\left(Acf(ce-2bf)-B\left(b^2f^2-2cf(be-af)+c^2(e^2-df)\right)\right)x}{f^3} - \frac{c(Bce-2bBf-Acf)x^2}{f^2} + \frac{Bc^2x^3}{f} + \frac{-Bd(ce-bf)\left(f(be-2af)-c(e^2-2df)\right)+Af\left(2cdf(be-af)-f^2(b^2d-a^2f)-c^2d(e^2-df)\right)+(A \cdots}{\cdots} \right.$$

$$= \frac{\left(B(ce-bf)\left(f(be-2af)-c(e^2-2df)\right)+Af\left(b^2f^2-2cf(be-af)+c^2(e^2-df)\right)\right)x}{f^4} - \frac{\left(Acf(ce-2bf)-B\left(b^2f^2-2cf(be-af)+c^2(e^2-df)\right)\right)x^2}{2f^3} - \frac{c(Bce-2bBf-Acf)x^3}{3f^2} + \frac{Bc^2x^4}{4f} + \frac{\int \frac{-Bd(ce-bf)\left(f(be-2af)-c(e^2-2df)\right)+Af\left(2cdf(be-af)-f^2(b^2d-a^2f)-c^2d(e^2-df)\right)+\left(Af(ce-bf)\left(f(be-2af)-c(e^2-2df)\right) \cdots\right.}{d+ex+fx^2}}{f^4}$$

$$
\begin{aligned}
&= \frac{\left(B(ce-bf)\left(f(be-2af)-c(e^2-2df)\right)+Af(b^2f^2-2cf(be-af)+c^2(e^2-df))\right)x}{f^4} \\
&\quad - \frac{\left(Acf(ce-2bf)-B(b^2f^2-2cf(be-af)+c^2(e^2-df))\right)x^2}{2f^3} \\
&\quad - \frac{c(Bce-2bBf-Acf)x^3}{3f^2} + \frac{Bc^2x^4}{4f} \\
&\quad + \frac{(Af(ce-bf)\left(f(be-2af)-c(e^2-2df)\right)+B(c^2(e^4-3de^2f+d^2f^2)-f^2(2abef-a^2f^2-b^2(e}{2f^5} \\
&\quad + \frac{(Af(c^2(e^4-4de^2f+2d^2f^2)-f^2(2abef-2a^2f^2-b^2(e^2-2df))+2cf(af(e^2-2df)-b(e^3-3}{}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{\left(B(ce-bf)\left(f(be-2af)-c(e^2-2df)\right)+Af(b^2f^2-2cf(be-af)+c^2(e^2-df))\right)x}{f^4} \\
&\quad - \frac{\left(Acf(ce-2bf)-B(b^2f^2-2cf(be-af)+c^2(e^2-df))\right)x^2}{2f^3} \\
&\quad - \frac{c(Bce-2bBf-Acf)x^3}{3f^2} + \frac{Bc^2x^4}{4f} \\
&\quad + \frac{(Af(ce-bf)\left(f(be-2af)-c(e^2-2df)\right)+B(c^2(e^4-3de^2f+d^2f^2)-f^2(2abef-a^2f^2-b^2(e}{2f^5} \\
&\quad - \frac{(Af(c^2(e^4-4de^2f+2d^2f^2)-f^2(2abef-2a^2f^2-b^2(e^2-2df))+2cf(af(e^2-2df)-b(e^3-3}{}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{\left(B(ce-bf)\left(f(be-2af)-c(e^2-2df)\right)+Af(b^2f^2-2cf(be-af)+c^2(e^2-df))\right)x}{f^4} \\
&\quad - \frac{\left(Acf(ce-2bf)-B(b^2f^2-2cf(be-af)+c^2(e^2-df))\right)x^2}{2f^3} \\
&\quad - \frac{c(Bce-2bBf-Acf)x^3}{3f^2} + \frac{Bc^2x^4}{4f} \\
&\quad - \frac{(Af(c^2(e^4-4de^2f+2d^2f^2)-f^2(2abef-2a^2f^2-b^2(e^2-2df))+2cf(af(e^2-2df)-b(e^3-3}{} \\
&\quad + \frac{(Af(ce-bf)\left(f(be-2af)-c(e^2-2df)\right)+B(c^2(e^4-3de^2f+d^2f^2)-f^2(2abef-a^2f^2-b^2(e}{2f^5}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 535, normalized size of antiderivative = 0.99

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{d+ex+fx^2}\,dx$$

$$= \frac{12f(-B(ce-bf)\left(f(-be+2af)+c(e^2-2df)\right)+Af\left(b^2f^2+2cf(-be+af)+c^2(e^2-df)\right))x+6f^2(Ac}{}$$

```
[In] Integrate[((A + B*x)*(a + b*x + c*x^2)^2)/(d + e*x + f*x^2),x]
```

```
[Out] (12*f*(-(B*(c*e - b*f)*(f*(-(b*e) + 2*a*f) + c*(e^2 - 2*d*f))) + A*f*(b^2*f
^2 + 2*c*f*(-(b*e) + a*f) + c^2*(e^2 - d*f)))*x + 6*f^2*(A*c*f*(-(c*e) + 2*
b*f) + B*(b^2*f^2 + 2*c*f*(-(b*e) + a*f) + c^2*(e^2 - d*f)))*x^2 + 4*c*f^3*
(-(B*c*e) + 2*b*B*f + A*c*f)*x^3 + 3*B*c^2*f^4*x^4 - (12*(-(A*f*(c^2*(e^4 -
 4*d*e^2*f + 2*d^2*f^2) + f^2*(-2*a*b*e*f + 2*a^2*f^2 + b^2*(e^2 - 2*d*f))
+ 2*c*f*(a*f*(e^2 - 2*d*f) - b*(e^3 - 3*d*e*f)))) + B*(c^2*(e^5 - 5*d*e^3*f
 + 5*d^2*e*f^2) + f^2*(a^2*e*f^2 + 2*a*b*f*(-e^2 + 2*d*f) + b^2*(e^3 - 3*d*
e*f)) - 2*c*f*(-(a*e*f*(e^2 - 3*d*f)) + b*(e^4 - 4*d*e^2*f + 2*d^2*f^2))))*
ArcTan[(e + 2*f*x)/Sqrt[-e^2 + 4*d*f]])/Sqrt[-e^2 + 4*d*f] + 6*(A*f*(-(c*e)
 + b*f)*(f*(-(b*e) + 2*a*f) + c*(e^2 - 2*d*f)) + B*(c^2*(e^4 - 3*d*e^2*f +
d^2*f^2) + f^2*(-2*a*b*e*f + a^2*f^2 + b^2*(e^2 - d*f)) - 2*c*f*(a*f*(-e^2
+ d*f) + b*(e^3 - 2*d*e*f))))*Log[d + x*(e + f*x)])/(12*f^5)
```

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 800, normalized size of antiderivative = 1.48

method	result
default	$\frac{\frac{1}{4}Bc^2x^4f^3+\frac{1}{3}Ac^2f^3x^3+\frac{2}{3}Bbcf^3x^3-\frac{1}{3}Bc^2ef^2x^3+Abcf^3x^2-\frac{1}{2}Ac^2ef^2x^2+Bacf^3x^2+\frac{1}{2}Bb^2f^3x^2-Bbcef^2x^2-\frac{1}{2}Bc^2df^2x^2+\frac{1}{2}}{}$
risch	Expression too large to display

```
[In] int((B*x+A)*(c*x^2+b*x+a)^2/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f^4*(1/4*B*c^2*x^4*f^3+1/3*A*c^2*f^3*x^3+2/3*B*b*c*f^3*x^3-1/3*B*c^2*e*f^
2*x^3+A*b*c*f^3*x^2-1/2*A*c^2*e*f^2*x^2+B*a*c*f^3*x^2+1/2*B*b^2*f^3*x^2-B*b
*c*e*f^2*x^2-1/2*B*c^2*d*f^2*x^2+1/2*B*c^2*e^2*f*x^2+2*A*a*c*f^3*x+A*b^2*f^
3*x-2*A*b*c*e*f^2*x-A*c^2*d*f^2*x+A*c^2*e^2*f*x+2*B*a*b*f^3*x-2*B*a*c*e*f^2
*x-B*b^2*e*f^2*x-2*B*b*c*d*f^2*x+2*B*b*c*e^2*f*x+2*B*c^2*d*e*f*x-B*c^2*e^3*
x)+1/f^4*(1/2*(2*A*a*b*f^4-2*A*a*c*e*f^3-A*b^2*e*f^3-2*A*b*c*d*f^3+2*A*b*c*
e^2*f^2+2*A*c^2*d*e*f^2-A*c^2*e^3*f+B*a^2*f^4-2*B*a*b*e*f^3-2*B*a*c*d*f^3+2
```

```
*B*a*c*e^2*f^2-B*b^2*d*f^3+B*b^2*e^2*f^2+4*B*b*c*d*e*f^2-2*B*b*c*e^3*f+B*c^
2*d^2*f^2-3*B*c^2*d*e^2*f+B*c^2*e^4)/f*ln(f*x^2+e*x+d)+2*(A*a^2*f^4-2*A*a*c
*d*f^3-A*b^2*d*f^3+2*A*b*c*d*e*f^2+A*c^2*d^2*f^2-A*c^2*d*e^2*f-2*B*a*b*d*f^
3+2*B*a*c*d*e*f^2+B*b^2*d*e*f^2+2*B*b*c*d^2*f^2-2*B*b*c*d*e^2*f-2*B*c^2*d^2
*e*f+B*c^2*d*e^3-1/2*(2*A*a*b*f^4-2*A*a*c*e*f^3-A*b^2*e*f^3-2*A*b*c*d*f^3+2
*A*b*c*e^2*f^2+2*A*c^2*d*e*f^2-A*c^2*e^3*f+B*a^2*f^4-2*B*a*b*e*f^3-2*B*a*c*
d*f^3+2*B*a*c*e^2*f^2-B*b^2*d*f^3+B*b^2*e^2*f^2+4*B*b*c*d*e*f^2-2*B*b*c*e^3
*f+B*c^2*d^2*f^2-3*B*c^2*d*e^2*f+B*c^2*e^4)*e/f)/(4*d*f-e^2)^(1/2)*arctan((
2*f*x+e)/(4*d*f-e^2)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.42 (sec) , antiderivative size = 1837, normalized size of antiderivative = 3.39

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{d+ex+fx^2}\,dx = \text{Too large to display}$$

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)^2/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(B*c^2*e^2*f^4 - 4*B*c^2*d*f^5)*x^4 - 4*(B*c^2*e^3*f^3 + 4*(2*B*b*
c + A*c^2)*d*f^5 - (4*B*c^2*d*e + (2*B*b*c + A*c^2)*e^2)*f^4)*x^3 + 6*(B*c^
2*e^4*f^2 - 4*(B*b^2 + 2*(B*a + A*b)*c)*d*f^5 + (4*B*c^2*d^2 + 4*(2*B*b*c +
 A*c^2)*d*e + (B*b^2 + 2*(B*a + A*b)*c)*e^2)*f^4 - (5*B*c^2*d*e^2 + (2*B*b*
c + A*c^2)*e^3)*f^3)*x^2 - 6*(B*c^2*e^5 - 2*A*a^2*f^5 + (2*(2*B*a*b + A*b^2
 + 2*A*a*c)*d + (B*a^2 + 2*A*a*b)*e)*f^4 - (2*(2*B*b*c + A*c^2)*d^2 + 3*(B*
b^2 + 2*(B*a + A*b)*c)*d*e + (2*B*a*b + A*b^2 + 2*A*a*c)*e^2)*f^3 + (5*B*c^
2*d^2*e + 4*(2*B*b*c + A*c^2)*d*e^2 + (B*b^2 + 2*(B*a + A*b)*c)*e^3)*f^2 -
(5*B*c^2*d*e^3 + (2*B*b*c + A*c^2)*e^4)*f)*sqrt(e^2 - 4*d*f)*log((2*f^2*x^2
 + 2*e*f*x + e^2 - 2*d*f - sqrt(e^2 - 4*d*f)*(2*f*x + e))/(f*x^2 + e*x + d)
) - 12*(B*c^2*e^5*f + 4*(2*B*a*b + A*b^2 + 2*A*a*c)*d*f^5 - (4*(2*B*b*c + A
*c^2)*d^2 + 4*(B*b^2 + 2*(B*a + A*b)*c)*d*e + (2*B*a*b + A*b^2 + 2*A*a*c)*e
^2)*f^4 + (8*B*c^2*d^2*e + 5*(2*B*b*c + A*c^2)*d*e^2 + (B*b^2 + 2*(B*a + A*
b)*c)*e^3)*f^3 - (6*B*c^2*d*e^3 + (2*B*b*c + A*c^2)*e^4)*f^2)*x + 6*(B*c^2*
e^6 - 4*(B*a^2 + 2*A*a*b)*d*f^5 + (4*(B*b^2 + 2*(B*a + A*b)*c)*d^2 + 4*(2*B
*a*b + A*b^2 + 2*A*a*c)*d*e + (B*a^2 + 2*A*a*b)*e^2)*f^4 - (4*B*c^2*d^3 + 8
*(2*B*b*c + A*c^2)*d^2*e + 5*(B*b^2 + 2*(B*a + A*b)*c)*d*e^2 + (2*B*a*b + A
*b^2 + 2*A*a*c)*e^3)*f^3 + (13*B*c^2*d^2*e^2 + 6*(2*B*b*c + A*c^2)*d*e^3 +
(B*b^2 + 2*(B*a + A*b)*c)*e^4)*f^2 - (7*B*c^2*d*e^4 + (2*B*b*c + A*c^2)*e^5
)*f)*log(f*x^2 + e*x + d))/(e^2*f^5 - 4*d*f^6), 1/12*(3*(B*c^2*e^2*f^4 - 4*
B*c^2*d*f^5)*x^4 - 4*(B*c^2*e^3*f^3 + 4*(2*B*b*c + A*c^2)*d*f^5 - (4*B*c^2*
d*e + (2*B*b*c + A*c^2)*e^2)*f^4)*x^3 + 6*(B*c^2*e^4*f^2 - 4*(B*b^2 + 2*(B*
a + A*b)*c)*d*f^5 + (4*B*c^2*d^2 + 4*(2*B*b*c + A*c^2)*d*e + (B*b^2 + 2*(B*
a + A*b)*c)*e^2)*f^4 - (5*B*c^2*d*e^2 + (2*B*b*c + A*c^2)*e^3)*f^3)*x^2 + 1
```

```
2*(B*c^2*e^5 - 2*A*a^2*f^5 + (2*(2*B*a*b + A*b^2 + 2*A*a*c)*d + (B*a^2 + 2*
A*a*b)*e)*f^4 - (2*(2*B*b*c + A*c^2)*d^2 + 3*(B*b^2 + 2*(B*a + A*b)*c)*d*e
+ (2*B*a*b + A*b^2 + 2*A*a*c)*e^2)*f^3 + (5*B*c^2*d^2*e + 4*(2*B*b*c + A*c^
2)*d*e^2 + (B*b^2 + 2*(B*a + A*b)*c)*e^3)*f^2 - (5*B*c^2*d*e^3 + (2*B*b*c +
 A*c^2)*e^4)*f)*sqrt(-e^2 + 4*d*f)*arctan(-sqrt(-e^2 + 4*d*f)*(2*f*x + e)/(
e^2 - 4*d*f)) - 12*(B*c^2*e^5*f + 4*(2*B*a*b + A*b^2 + 2*A*a*c)*d*f^5 - (4*
(2*B*b*c + A*c^2)*d^2 + 4*(B*b^2 + 2*(B*a + A*b)*c)*d*e + (2*B*a*b + A*b^2
+ 2*A*a*c)*e^2)*f^4 + (8*B*c^2*d^2*e + 5*(2*B*b*c + A*c^2)*d*e^2 + (B*b^2 +
 2*(B*a + A*b)*c)*e^3)*f^3 - (6*B*c^2*d*e^3 + (2*B*b*c + A*c^2)*e^4)*f^2)*x
 + 6*(B*c^2*e^6 - 4*(B*a^2 + 2*A*a*b)*d*f^5 + (4*(B*b^2 + 2*(B*a + A*b)*c)*
d^2 + 4*(2*B*a*b + A*b^2 + 2*A*a*c)*d*e + (B*a^2 + 2*A*a*b)*e^2)*f^4 - (4*B
*c^2*d^3 + 8*(2*B*b*c + A*c^2)*d^2*e + 5*(B*b^2 + 2*(B*a + A*b)*c)*d*e^2 +
(2*B*a*b + A*b^2 + 2*A*a*c)*e^3)*f^3 + (13*B*c^2*d^2*e^2 + 6*(2*B*b*c + A*c
^2)*d*e^3 + (B*b^2 + 2*(B*a + A*b)*c)*e^4)*f^2 - (7*B*c^2*d*e^4 + (2*B*b*c
+ A*c^2)*e^5)*f)*log(f*x^2 + e*x + d))/(e^2*f^5 - 4*d*f^6)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4663 vs. $2(520) = 1040$.

Time = 82.39 (sec) , antiderivative size = 4663, normalized size of antiderivative = 8.60

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{d+ex+fx^2}\,dx = \text{Too large to display}$$

```
[In] integrate((B*x+A)*(c*x**2+b*x+a)**2/(f*x**2+e*x+d),x)
```

```
[Out] B*c**2*x**4/(4*f) + x**3*(A*c**2/(3*f) + 2*B*b*c/(3*f) - B*c**2*e/(3*f**2))
 + x**2*(A*b*c/f - A*c**2*e/(2*f**2) + B*a*c/f + B*b**2/(2*f) - B*b*c*e/f**
2 - B*c**2*d/(2*f**2) + B*c**2*e**2/(2*f**3)) + x*(2*A*a*c/f + A*b**2/f - 2
*A*b*c*e/f**2 - A*c**2*d/f**2 + A*c**2*e**2/f**3 + 2*B*a*b/f - 2*B*a*c*e/f*
*2 - B*b**2*e/f**2 - 2*B*b*c*d/f**2 + 2*B*b*c*e**2/f**3 + 2*B*c**2*d*e/f**3
 - B*c**2*e**3/f**4) + (-sqrt(-4*d*f + e**2)*(-2*A*a**2*f**5 + 2*A*a*b*e*f*
*4 + 4*A*a*c*d*f**4 - 2*A*a*c*e**2*f**3 + 2*A*b**2*d*f**4 - A*b**2*e**2*f**
3 - 6*A*b*c*d*e*f**3 + 2*A*b*c*e**3*f**2 - 2*A*c**2*d**2*f**3 + 4*A*c**2*d*
e**2*f**2 - A*c**2*e**4*f + B*a**2*e*f**4 + 4*B*a*b*d*f**4 - 2*B*a*b*e**2*f
**3 - 6*B*a*c*d*e*f**3 + 2*B*a*c*e**3*f**2 - 3*B*b**2*d*e*f**3 + B*b**2*e**
3*f**2 - 4*B*b*c*d**2*f**3 + 8*B*b*c*d*e**2*f**2 - 2*B*b*c*e**4*f + 5*B*c**
2*d**2*e*f**2 - 5*B*c**2*d*e**3*f + B*c**2*e**5)/(2*f**5*(4*d*f - e**2)) +
(2*A*a*b*f**4 - 2*A*a*c*e*f**3 - A*b**2*e*f**3 - 2*A*b*c*d*f**3 + 2*A*b*c*e
**2*f**2 + 2*A*c**2*d*e*f**2 - A*c**2*e**3*f + B*a**2*f**4 - 2*B*a*b*e*f**3
 - 2*B*a*c*d*f**3 + 2*B*a*c*e**2*f**2 - B*b**2*d*f**3 + B*b**2*e**2*f**2 +
4*B*b*c*d*e*f**2 - 2*B*b*c*e**3*f + B*c**2*d**2*f**2 - 3*B*c**2*d*e**2*f +
B*c**2*e**4)/(2*f**5))*log(x + (-A*a**2*e*f**4 + 4*A*a*b*d*f**4 - 2*A*a*c*d
*e*f**3 - A*b**2*d*e*f**3 - 4*A*b*c*d**2*f**3 + 2*A*b*c*d*e**2*f**2 + 3*A*c
```

```
**2*d**2*e*f**2 - A*c**2*d*e**3*f + 2*B*a**2*d*f**4 - 2*B*a*b*d*e*f**3 - 4*
B*a*c*d**2*f**3 + 2*B*a*c*d*e**2*f**2 - 2*B*b**2*d**2*f**3 + B*b**2*d*e**2*
f**2 + 6*B*b*c*d**2*e*f**2 - 2*B*b*c*d*e**3*f + 2*B*c**2*d**3*f**2 - 4*B*c*
*2*d**2*e**2*f + B*c**2*d*e**4 - 4*d*f**5*(-sqrt(-4*d*f + e**2)*(-2*A*a**2*
f**5 + 2*A*a*b*e*f**4 + 4*A*a*c*d*f**4 - 2*A*a*c*e**2*f**3 + 2*A*b**2*d*f**
4 - A*b**2*e**2*f**3 - 6*A*b*c*d*e*f**3 + 2*A*b*c*e**3*f**2 - 2*A*c**2*d**2
*f**3 + 4*A*c**2*d*e**2*f**2 - A*c**2*e**4*f + B*a**2*e*f**4 + 4*B*a*b*d*f*
*4 - 2*B*a*b*e**2*f**3 - 6*B*a*c*d*e*f**3 + 2*B*a*c*e**3*f**2 - 3*B*b**2*d*
e*f**3 + B*b**2*e**3*f**2 - 4*B*b*c*d**2*f**3 + 8*B*b*c*d*e**2*f**2 - 2*B*b
*c*e**4*f + 5*B*c**2*d**2*e*f**2 - 5*B*c**2*d*e**3*f + B*c**2*e**5)/(2*f**5
*(4*d*f - e**2)) + (2*A*a*b*f**4 - 2*A*a*c*e*f**3 - A*b**2*e*f**3 - 2*A*b*c
*d*f**3 + 2*A*b*c*e**2*f**2 + 2*A*c**2*d*e*f**2 - A*c**2*e**3*f + B*a**2*f*
*4 - 2*B*a*b*e*f**3 - 2*B*a*c*d*f**3 + 2*B*a*c*e**2*f**2 - B*b**2*d*f**3 +
B*b**2*e**2*f**2 + 4*B*b*c*d*e*f**2 - 2*B*b*c*e**3*f + B*c**2*d**2*f**2 - 3
*B*c**2*d*e**2*f + B*c**2*e**4)/(2*f**5)) + e**2*f**4*(-sqrt(-4*d*f + e**2)
*(-2*A*a**2*f**5 + 2*A*a*b*e*f**4 + 4*A*a*c*d*f**4 - 2*A*a*c*e**2*f**3 + 2*
A*b**2*d*f**4 - A*b**2*e**2*f**3 - 6*A*b*c*d*e*f**3 + 2*A*b*c*e**3*f**2 - 2
*A*c**2*d**2*f**3 + 4*A*c**2*d*e**2*f**2 - A*c**2*e**4*f + B*a**2*e*f**4 +
4*B*a*b*d*f**4 - 2*B*a*b*e**2*f**3 - 6*B*a*c*d*e*f**3 + 2*B*a*c*e**3*f**2 -
 3*B*b**2*d*e*f**3 + B*b**2*e**3*f**2 - 4*B*b*c*d**2*f**3 + 8*B*b*c*d*e**2*
f**2 - 2*B*b*c*e**4*f + 5*B*c**2*d**2*e*f**2 - 5*B*c**2*d*e**3*f + B*c**2*e
**5)/(2*f**5*(4*d*f - e**2)) + (2*A*a*b*f**4 - 2*A*a*c*e*f**3 - A*b**2*e*f*
*3 - 2*A*b*c*d*f**3 + 2*A*b*c*e**2*f**2 + 2*A*c**2*d*e*f**2 - A*c**2*e**3*f
 + B*a**2*f**4 - 2*B*a*b*e*f**3 - 2*B*a*c*d*f**3 + 2*B*a*c*e**2*f**2 - B*b*
*2*d*f**3 + B*b**2*e**2*f**2 + 4*B*b*c*d*e*f**2 - 2*B*b*c*e**3*f + B*c**2*d
**2*f**2 - 3*B*c**2*d*e**2*f + B*c**2*e**4)/(2*f**5)))/(-2*A*a**2*f**5 + 2*
A*a*b*e*f**4 + 4*A*a*c*d*f**4 - 2*A*a*c*e**2*f**3 + 2*A*b**2*d*f**4 - A*b**
2*e**2*f**3 - 6*A*b*c*d*e*f**3 + 2*A*b*c*e**3*f**2 - 2*A*c**2*d**2*f**3 + 4
*A*c**2*d*e**2*f**2 - A*c**2*e**4*f + B*a**2*e*f**4 + 4*B*a*b*d*f**4 - 2*B*
a*b*e**2*f**3 - 6*B*a*c*d*e*f**3 + 2*B*a*c*e**3*f**2 - 3*B*b**2*d*e*f**3 +
B*b**2*e**3*f**2 - 4*B*b*c*d**2*f**3 + 8*B*b*c*d*e**2*f**2 - 2*B*b*c*e**4*f
 + 5*B*c**2*d**2*e*f**2 - 5*B*c**2*d*e**3*f + B*c**2*e**5)) + (sqrt(-4*d*f
+ e**2)*(-2*A*a**2*f**5 + 2*A*a*b*e*f**4 + 4*A*a*c*d*f**4 - 2*A*a*c*e**2*f*
*3 + 2*A*b**2*d*f**4 - A*b**2*e**2*f**3 - 6*A*b*c*d*e*f**3 + 2*A*b*c*e**3*f
**2 - 2*A*c**2*d**2*f**3 + 4*A*c**2*d*e**2*f**2 - A*c**2*e**4*f + B*a**2*e*
f**4 + 4*B*a*b*d*f**4 - 2*B*a*b*e**2*f**3 - 6*B*a*c*d*e*f**3 + 2*B*a*c*e**3
*f**2 - 3*B*b**2*d*e*f**3 + B*b**2*e**3*f**2 - 4*B*b*c*d**2*f**3 + 8*B*b*c*
d*e**2*f**2 - 2*B*b*c*e**4*f + 5*B*c**2*d**2*e*f**2 - 5*B*c**2*d*e**3*f + B
*c**2*e**5)/(2*f**5*(4*d*f - e**2)) + (2*A*a*b*f**4 - 2*A*a*c*e*f**3 - A*b*
*2*e*f**3 - 2*A*b*c*d*f**3 + 2*A*b*c*e**2*f**2 + 2*A*c**2*d*e*f**2 - A*c**2
*e**3*f + B*a**2*f**4 - 2*B*a*b*e*f**3 - 2*B*a*c*d*f**3 + 2*B*a*c*e**2*f**2
 - B*b**2*d*f**3 + B*b**2*e**2*f**2 + 4*B*b*c*d*e*f**2 - 2*B*b*c*e**3*f + B
*c**2*d**2*f**2 - 3*B*c**2*d*e**2*f + B*c**2*e**4)/(2*f**5))*log(x + (-A*a*
*2*e*f**4 + 4*A*a*b*d*f**4 - 2*A*a*c*d*e*f**3 - A*b**2*d*e*f**3 - 4*A*b*c*d
**2*f**3 + 2*A*b*c*d*e**2*f**2 + 3*A*c**2*d**2*e*f**2 - A*c**2*d*e**3*f + 2
```

```
*B*a**2*d*f**4 - 2*B*a*b*d*e*f**3 - 4*B*a*c*d**2*f**3 + 2*B*a*c*d*e**2*f**2
 - 2*B*b**2*d**2*f**3 + B*b**2*d*e**2*f**2 + 6*B*b*c*d**2*e*f**2 - 2*B*b*c*
d*e**3*f + 2*B*c**2*d**3*f**2 - 4*B*c**2*d**2*e**2*f + B*c**2*d*e**4 - 4*d*
f**5*(sqrt(-4*d*f + e**2)*(-2*A*a**2*f**5 + 2*A*a*b*e*f**4 + 4*A*a*c*d*f**4
 - 2*A*a*c*e**2*f**3 + 2*A*b**2*d*f**4 - A*b**2*e**2*f**3 - 6*A*b*c*d*e*f**
3 + 2*A*b*c*e**3*f**2 - 2*A*c**2*d**2*f**3 + 4*A*c**2*d*e**2*f**2 - A*c**2*
e**4*f + B*a**2*e*f**4 + 4*B*a*b*d*f**4 - 2*B*a*b*e**2*f**3 - 6*B*a*c*d*e*f
**3 + 2*B*a*c*e**3*f**2 - 3*B*b**2*d*e*f**3 + B*b**2*e**3*f**2 - 4*B*b*c*d*
*2*f**3 + 8*B*b*c*d*e**2*f**2 - 2*B*b*c*e**4*f + 5*B*c**2*d**2*e*f**2 - 5*B
*c**2*d*e**3*f + B*c**2*e**5)/(2*f**5*(4*d*f - e**2)) + (2*A*a*b*f**4 - 2*A
*a*c*e*f**3 - A*b**2*e*f**3 - 2*A*b*c*d*f**3 + 2*A*b*c*e**2*f**2 + 2*A*c**2
*d*e*f**2 - A*c**2*e**3*f + B*a**2*f**4 - 2*B*a*b*e*f**3 - 2*B*a*c*d*f**3 +
 2*B*a*c*e**2*f**2 - B*b**2*d*f**3 + B*b**2*e**2*f**2 + 4*B*b*c*d*e*f**2 -
2*B*b*c*e**3*f + B*c**2*d**2*f**2 - 3*B*c**2*d*e**2*f + B*c**2*e**4)/(2*f**
5)) + e**2*f**4*(sqrt(-4*d*f + e**2)*(-2*A*a**2*f**5 + 2*A*a*b*e*f**4 + 4*A
*a*c*d*f**4 - 2*A*a*c*e**2*f**3 + 2*A*b**2*d*f**4 - A*b**2*e**2*f**3 - 6*A*
b*c*d*e*f**3 + 2*A*b*c*e**3*f**2 - 2*A*c**2*d**2*f**3 + 4*A*c**2*d*e**2*f**
2 - A*c**2*e**4*f + B*a**2*e*f**4 + 4*B*a*b*d*f**4 - 2*B*a*b*e**2*f**3 - 6*
B*a*c*d*e*f**3 + 2*B*a*c*e**3*f**2 - 3*B*b**2*d*e*f**3 + B*b**2*e**3*f**2 -
 4*B*b*c*d**2*f**3 + 8*B*b*c*d*e**2*f**2 - 2*B*b*c*e**4*f + 5*B*c**2*d**2*e
*f**2 - 5*B*c**2*d*e**3*f + B*c**2*e**5)/(2*f**5*(4*d*f - e**2)) + (2*A*a*b
*f**4 - 2*A*a*c*e*f**3 - A*b**2*e*f**3 - 2*A*b*c*d*f**3 + 2*A*b*c*e**2*f**2
 + 2*A*c**2*d*e*f**2 - A*c**2*e**3*f + B*a**2*f**4 - 2*B*a*b*e*f**3 - 2*B*a
*c*d*f**3 + 2*B*a*c*e**2*f**2 - B*b**2*d*f**3 + B*b**2*e**2*f**2 + 4*B*b*c*
d*e*f**2 - 2*B*b*c*e**3*f + B*c**2*d**2*f**2 - 3*B*c**2*d*e**2*f + B*c**2*e
**4)/(2*f**5)))/(-2*A*a**2*f**5 + 2*A*a*b*e*f**4 + 4*A*a*c*d*f**4 - 2*A*a*c
*e**2*f**3 + 2*A*b**2*d*f**4 - A*b**2*e**2*f**3 - 6*A*b*c*d*e*f**3 + 2*A*b*
c*e**3*f**2 - 2*A*c**2*d**2*f**3 + 4*A*c**2*d*e**2*f**2 - A*c**2*e**4*f + B
*a**2*e*f**4 + 4*B*a*b*d*f**4 - 2*B*a*b*e**2*f**3 - 6*B*a*c*d*e*f**3 + 2*B*
a*c*e**3*f**2 - 3*B*b**2*d*e*f**3 + B*b**2*e**3*f**2 - 4*B*b*c*d**2*f**3 +
8*B*b*c*d*e**2*f**2 - 2*B*b*c*e**4*f + 5*B*c**2*d**2*e*f**2 - 5*B*c**2*d*e*
*3*f + B*c**2*e**5))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{d+ex+fx^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)^2/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 743, normalized size of antiderivative = 1.37

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{d+ex+fx^2}\,dx$$
$$= \frac{3\,Bc^2f^3x^4 - 4\,Bc^2ef^2x^3 + 8\,Bbcf^3x^3 + 4\,Ac^2f^3x^3 + 6\,Bc^2e^2fx^2 - 6\,Bc^2df^2x^2 - 12\,Bbcef^2x^2 - 6\,Ac^2ef^2x}{}$$
$$+ \frac{(Bc^2e^4 - 3\,Bc^2de^2f - 2\,Bbce^3f - Ac^2e^3f + Bc^2d^2f^2 + 4\,Bbcdef^2 + 2\,Ac^2def^2 + Bb^2e^2f^2 + 2\,Bace^2f^2}{}$$
$$- \frac{(Bc^2e^5 - 5\,Bc^2de^3f - 2\,Bbce^4f - Ac^2e^4f + 5\,Bc^2d^2ef^2 + 8\,Bbcde^2f^2 + 4\,Ac^2de^2f^2 + Bb^2e^3f^2 + 2\,Bac}{}$$

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)^2/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] 1/12*(3*B*c^2*f^3*x^4 - 4*B*c^2*e*f^2*x^3 + 8*B*b*c*f^3*x^3 + 4*A*c^2*f^3*x
^3 + 6*B*c^2*e^2*f*x^2 - 6*B*c^2*d*f^2*x^2 - 12*B*b*c*e*f^2*x^2 - 6*A*c^2*e
*f^2*x^2 + 6*B*b^2*f^3*x^2 + 12*B*a*c*f^3*x^2 + 12*A*b*c*f^3*x^2 - 12*B*c^2
*e^3*x + 24*B*c^2*d*e*f*x + 24*B*b*c*e^2*f*x + 12*A*c^2*e^2*f*x - 24*B*b*c*
d*f^2*x - 12*A*c^2*d*f^2*x - 12*B*b^2*e*f^2*x - 24*B*a*c*e*f^2*x - 24*A*b*c
*e*f^2*x + 24*B*a*b*f^3*x + 12*A*b^2*f^3*x + 24*A*a*c*f^3*x)/f^4 + 1/2*(B*c
^2*e^4 - 3*B*c^2*d*e^2*f - 2*B*b*c*e^3*f - A*c^2*e^3*f + B*c^2*d^2*f^2 + 4*
B*b*c*d*e*f^2 + 2*A*c^2*d*e*f^2 + B*b^2*e^2*f^2 + 2*B*a*c*e^2*f^2 + 2*A*b*c
*e^2*f^2 - B*b^2*d*f^3 - 2*B*a*c*d*f^3 - 2*A*b*c*d*f^3 - 2*B*a*b*e*f^3 - A*
b^2*e*f^3 - 2*A*a*c*e*f^3 + B*a^2*f^4 + 2*A*a*b*f^4)*log(f*x^2 + e*x + d)/f
^5 - (B*c^2*e^5 - 5*B*c^2*d*e^3*f - 2*B*b*c*e^4*f - A*c^2*e^4*f + 5*B*c^2*d
^2*e*f^2 + 8*B*b*c*d*e^2*f^2 + 4*A*c^2*d*e^2*f^2 + B*b^2*e^3*f^2 + 2*B*a*c*
e^3*f^2 + 2*A*b*c*e^3*f^2 - 4*B*b*c*d^2*f^3 - 2*A*c^2*d^2*f^3 - 3*B*b^2*d*e
*f^3 - 6*B*a*c*d*e*f^3 - 6*A*b*c*d*e*f^3 - 2*B*a*b*e^2*f^3 - A*b^2*e^2*f^3
- 2*A*a*c*e^2*f^3 + 4*B*a*b*d*f^4 + 2*A*b^2*d*f^4 + 4*A*a*c*d*f^4 + B*a^2*e
*f^4 + 2*A*a*b*e*f^4 - 2*A*a^2*f^5)*arctan((2*f*x + e)/sqrt(-e^2 + 4*d*f))/
(sqrt(-e^2 + 4*d*f)*f^5)
```

Mupad [B] (verification not implemented)

Time = 15.37 (sec) , antiderivative size = 893, normalized size of antiderivative = 1.65

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{d+ex+fx^2}\,dx = x^3\left(\frac{A\,c^2+2\,B\,b\,c}{3\,f}-\frac{B\,c^2\,e}{3\,f^2}\right)+x\left(\frac{A\,b^2+2\,B\,a\,b+2\,A\,a\,c}{f}-\frac{d\left(\frac{A\,c^2+2\,B\,b\,c}{f}-\frac{B\,c^2\,e}{f^2}\right)}{f}+\frac{e\left(\frac{e\left(\frac{A\,c^2+2\,B\,b\,c}{f}-\frac{B\,c^2\,e}{f^2}\right)}{f}-\frac{B\,b^2+2\,A\,c\,b+2\,B\,a\,c}{f}+\frac{B\,c^2\,d}{f^2}\right)}{f}\right)-x^2\left(\frac{e\left(\frac{A\,c^2+2\,B\,b\,c}{f}-\frac{B\,c^2\,e}{f^2}\right)}{2\,f}-\frac{B\,b^2+2\,A\,c\,b+2\,B\,a\,c}{2\,f}+\frac{B\,c^2\,d}{2\,f^2}\right)-\frac{\ln\left(f\,x^2+e\,x+d\right)\left(-4\,B\,a^2\,d\,f^5+B\,a^2\,e^2\,f^4+8\,B\,a\,b\,d\,e\,f^4-8\,A\,a\,b\,d\,f^5-2\,B\,a\,b\,e^3\,f^3+2\,A\,a\,b\,e^2\,f^4\cdots\right.}{\cdots}+\frac{B\,c^2\,x^4}{4\,f}+\frac{\mathrm{atan}\left(\frac{e}{\sqrt{4\,d\,f-e^2}}+\frac{2\,f\,x}{\sqrt{4\,d\,f-e^2}}\right)\left(-B\,a^2\,e\,f^4+2\,A\,a^2\,f^5-4\,B\,a\,b\,d\,f^4+2\,B\,a\,b\,e^2\,f^3-2\,A\,a\,b\,e\,f^4+6\,B\,a\,c\cdots\right.}{\cdots}$$

```
[In] int(((A + B*x)*(a + b*x + c*x^2)^2)/(d + e*x + f*x^2),x)
```

```
[Out] x^3*((A*c^2 + 2*B*b*c)/(3*f) - (B*c^2*e)/(3*f^2)) + x*((A*b^2 + 2*A*a*c + 2
*B*a*b)/f - (d*((A*c^2 + 2*B*b*c)/f - (B*c^2*e)/f^2))/f + (e*((e*((A*c^2 +
2*B*b*c)/f - (B*c^2*e)/f^2))/f - (B*b^2 + 2*A*b*c + 2*B*a*c)/f + (B*c^2*d)/
f^2))/f) - x^2*((e*((A*c^2 + 2*B*b*c)/f - (B*c^2*e)/f^2))/(2*f) - (B*b^2 +
2*A*b*c + 2*B*a*c)/(2*f) + (B*c^2*d)/(2*f^2)) - (log(d + e*x + f*x^2)*(B*c^
2*e^6 - 4*B*a^2*d*f^5 - A*c^2*e^5*f - A*b^2*e^3*f^3 + B*a^2*e^2*f^4 + 4*B*b
^2*d^2*f^4 + B*b^2*e^4*f^2 - 4*B*c^2*d^3*f^3 + 6*A*c^2*d*e^3*f^2 - 8*A*c^2*
d^2*e*f^3 - 5*B*b^2*d*e^2*f^3 - 8*A*a*b*d*f^5 - 2*B*b*c*e^5*f + 13*B*c^2*d^
2*e^2*f^2 + 2*A*a*b*e^2*f^4 - 2*A*a*c*e^3*f^3 + 8*A*b*c*d^2*f^4 - 2*B*a*b*e
^3*f^3 + 8*B*a*c*d^2*f^4 + 2*A*b*c*e^4*f^2 + 2*B*a*c*e^4*f^2 + 4*A*b^2*d*e*
f^4 - 7*B*c^2*d*e^4*f - 10*A*b*c*d*e^2*f^3 - 10*B*a*c*d*e^2*f^3 + 12*B*b*c*
d*e^3*f^2 - 16*B*b*c*d^2*e*f^3 + 8*A*a*c*d*e*f^4 + 8*B*a*b*d*e*f^4))/(2*(4*
d*f^6 - e^2*f^5)) + (B*c^2*x^4)/(4*f) + (atan(e/(4*d*f - e^2)^(1/2) + (2*f*
x)/(4*d*f - e^2)^(1/2))*(2*A*a^2*f^5 - B*c^2*e^5 - 2*A*b^2*d*f^4 - B*a^2*e*
f^4 + A*c^2*e^4*f + A*b^2*e^2*f^3 + 2*A*c^2*d^2*f^3 - B*b^2*e^3*f^2 - 4*A*c
^2*d*e^2*f^2 - 5*B*c^2*d^2*e*f^2 - 2*A*a*b*e*f^4 - 4*A*a*c*d*f^4 - 4*B*a*b*
d*f^4 + 2*B*b*c*e^4*f + 2*A*a*c*e^2*f^3 + 2*B*a*b*e^2*f^3 - 2*A*b*c*e^3*f^2
 - 2*B*a*c*e^3*f^2 + 4*B*b*c*d^2*f^3 + 3*B*b^2*d*e*f^3 + 5*B*c^2*d*e^3*f -
8*B*b*c*d*e^2*f^2 + 6*A*b*c*d*e*f^3 + 6*B*a*c*d*e*f^3))/(f^5*(4*d*f - e^2)^
(1/2))
```

3.15 $\int \frac{A+Bx}{(a+bx+cx^2)(d+ex+fx^2)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 406

$$\begin{aligned}
&\int \frac{A+Bx}{(a+bx+cx^2)(d+ex+fx^2)}\,dx \\
&= -\frac{(Ab^2f+2c(Acd+aBe-aAf)-b(Bcd+Ace+aBf))\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\,(c^2d^2+f(b^2d-abe+a^2f)-c(bde-a(e^2-2df)))} \\
&\quad+\frac{(B(cde-2bdf+aef)-A(ce^2-2cdf-bef+2af^2))\operatorname{arctanh}\left(\frac{e+2fx}{\sqrt{e^2-4df}}\right)}{\sqrt{e^2-4df}\,(c^2d^2+f(b^2d-abe+a^2f)-c(bde-a(e^2-2df)))} \\
&\quad+\frac{(Bcd-Ace+Abf-aBf)\log(a+bx+cx^2)}{2(c^2d^2+f(b^2d-abe+a^2f)-c(bde-a(e^2-2df)))} \\
&\quad-\frac{(Bcd-Ace+Abf-aBf)\log(d+ex+fx^2)}{2(c^2d^2+f(b^2d-abe+a^2f)-c(bde-a(e^2-2df)))}
\end{aligned}$$

```
[Out] 1/2*(A*b*f-A*c*e-B*a*f+B*c*d)*ln(c*x^2+b*x+a)/(c^2*d^2+f*(a^2*f-a*b*e+b^2*d
)-c*(b*d*e-a*(-2*d*f+e^2)))-1/2*(A*b*f-A*c*e-B*a*f+B*c*d)*ln(f*x^2+e*x+d)/(
c^2*d^2+f*(a^2*f-a*b*e+b^2*d)-c*(b*d*e-a*(-2*d*f+e^2)))-(A*b^2*f+2*c*(-A*a*
f+A*c*d+B*a*e)-b*(A*c*e+B*a*f+B*c*d))*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))
/(c^2*d^2+f*(a^2*f-a*b*e+b^2*d)-c*(b*d*e-a*(-2*d*f+e^2)))/(-4*a*c+b^2)^(1/2
)+(B*(a*e*f-2*b*d*f+c*d*e)-A*(2*a*f^2-b*e*f-2*c*d*f+c*e^2))*arctanh((2*f*x+
e)/(-4*d*f+e^2)^(1/2))/(c^2*d^2+f*(a^2*f-a*b*e+b^2*d)-c*(b*d*e-a*(-2*d*f+e^
2)))/(-4*d*f+e^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 398, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1036, 648, 632, 212, 642}

$$\begin{aligned}
&\int \frac{A+Bx}{(a+bx+cx^2)(d+ex+fx^2)}\,dx \\
&= \frac{\operatorname{arctanh}\left(\frac{e+2fx}{\sqrt{e^2-4df}}\right)\left(B(aef-2bdf+cde)-A(2af^2-bef-2cdf+ce^2)\right)}{\sqrt{e^2-4df}\left(f\left(a^2f-abe+b^2d\right)+ac\left(e^2-2df\right)-bcde+c^2d^2\right)} \\
&\quad - \frac{\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(-b(aBf+Ace+Bcd)+2c(-aAf+aBe+Acd)+Ab^2f\right)}{\sqrt{b^2-4ac}\left(f\left(a^2f-abe+b^2d\right)+ac\left(e^2-2df\right)-bcde+c^2d^2\right)} \\
&\quad + \frac{\log\left(a+bx+cx^2\right)\left(-aBf+Abf-Ace+Bcd\right)}{2\left(f\left(a^2f-abe+b^2d\right)+ac\left(e^2-2df\right)-bcde+c^2d^2\right)} \\
&\quad - \frac{\log\left(d+ex+fx^2\right)\left(-aBf+Abf-Ace+Bcd\right)}{2\left(f\left(a^2f-abe+b^2d\right)+ac\left(e^2-2df\right)-bcde+c^2d^2\right)}
\end{aligned}$$

```
[In] Int[(A + B*x)/((a + b*x + c*x^2)*(d + e*x + f*x^2)),x]
```

```
[Out] -(((A*b^2*f + 2*c*(A*c*d + a*B*e - a*A*f) - b*(B*c*d + A*c*e + a*B*f))*ArcT
anh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*(c^2*d^2 - b*c*d*e +
 f*(b^2*d - a*b*e + a^2*f) + a*c*(e^2 - 2*d*f)))) + ((B*(c*d*e - 2*b*d*f +
a*e*f) - A*(c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2))*ArcTanh[(e + 2*f*x)/Sqrt[e^
2 - 4*d*f]])/(Sqrt[e^2 - 4*d*f]*(c^2*d^2 - b*c*d*e + f*(b^2*d - a*b*e + a^2
*f) + a*c*(e^2 - 2*d*f))) + ((B*c*d - A*c*e + A*b*f - a*B*f)*Log[a + b*x +
c*x^2])/(2*(c^2*d^2 - b*c*d*e + f*(b^2*d - a*b*e + a^2*f) + a*c*(e^2 - 2*d*
f))) - ((B*c*d - A*c*e + A*b*f - a*B*f)*Log[d + e*x + f*x^2])/(2*(c^2*d^2 -
 b*c*d*e + f*(b^2*d - a*b*e + a^2*f) + a*c*(e^2 - 2*d*f)))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1036

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_) + (e_.)*
(x_) + (f_.)*(x_)^2)), x_Symbol] :> With[{q = Simplify[c^2*d^2 - b*c*d*e +
a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2]}, Dist[1/q, Int[Simp[g*c
^2*d - g*b*c*e + a*h*c*e + g*b^2*f - a*b*h*f - a*g*c*f + c*(h*c*d - g*c*e +
 g*b*f - a*h*f)*x, x]/(a + b*x + c*x^2), x], x] + Dist[1/q, Int[Simp[(-h)*c
*d*e + g*c*e^2 + b*h*d*f - g*c*d*f - g*b*e*f + a*g*f^2 - f*(h*c*d - g*c*e +
 g*b*f - a*h*f)*x, x]/(d + e*x + f*x^2), x], x] /; NeQ[q, 0]] /; FreeQ[{a,
b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{aB(ce-bf)+A\left(c^2d+b^2f-c(be+af)\right)+c(Bcd-Ace+Abf-aBf)x}{a+bx+cx^2}\,dx}{c^2d^2-bcde+f\left(b^2d-abe+a^2f\right)+ac\left(e^2-2df\right)} + \frac{\int \frac{-Af(be-af)+Ac\left(e^2-df\right)-B(cde-bdf)-f(Bcd-Ace+Abf-aBf)x}{d+ex+fx^2}\,dx}{c^2d^2-bcde+f\left(b^2d-abe+a^2f\right)+ac\left(e^2-2df\right)}$$

$$= \frac{(Bcd-Ace+Abf-aBf)\int \frac{b+2cx}{a+bx+cx^2}\,dx}{2\left(c^2d^2-bcde+f\left(b^2d-abe+a^2f\right)+ac\left(e^2-2df\right)\right)} - \frac{(Bcd-Ace+Abf-aBf)\int \frac{e+2fx}{d+ex+fx^2}\,dx}{2\left(c^2d^2-bcde+f\left(b^2d-abe+a^2f\right)+ac\left(e^2-2df\right)\right)} + \frac{\left(Ab^2f+2c(Acd+aBe-aAf)-b(Bcd+Ace+aBf)\right)\int \frac{1}{a+bx+cx^2}\,dx}{2\left(c^2d^2-bcde+f\left(b^2d-abe+a^2f\right)+ac\left(e^2-2df\right)\right)} + \frac{\left(ef(Bcd-Ace+Abf-aBf)+2f\left(-Af(be-af)+Ac\left(e^2-df\right)-B(cde-bdf)\right)\right)\int \frac{1}{d+ex+fx^2}\,d}{2f\left(c^2d^2-bcde+f\left(b^2d-abe+a^2f\right)+ac\left(e^2-2df\right)\right)}$$

$$= \frac{(Bcd - Ace + Abf - aBf)\log(a + bx + cx^2)}{2(c^2d^2 - bcde + f(b^2d - abe + a^2f) + ac(e^2 - 2df))} - \frac{(Bcd - Ace + Abf - aBf)\log(d + ex + fx^2)}{2(c^2d^2 - bcde + f(b^2d - abe + a^2f) + ac(e^2 - 2df))} - \frac{(Ab^2f + 2c(Acd + aBe - aAf) - b(Bcd + Ace + aBf))\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b + 2cx\right)}{c^2d^2 - bcde + f(b^2d - abe + a^2f) + ac(e^2 - 2df)} - \frac{(ef(Bcd - Ace + Abf - aBf) + 2f(-Af(be - af) + Ac(e^2 - df) - B(cde - bdf)))\,\text{Subst}\left(\int \right.}{f(c^2d^2 - bcde + f(b^2d - abe + a^2f) + ac(e^2 - 2df))}$$

$$= -\frac{(Ab^2f + 2c(Acd + aBe - aAf) - b(Bcd + Ace + aBf))\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2 - 4ac}\,(c^2d^2 - bcde + f(b^2d - abe + a^2f) + ac(e^2 - 2df))} + \frac{(B(cde - 2bdf + aef) - A(ce^2 - 2cdf - bef + 2af^2))\tanh^{-1}\left(\frac{e+2fx}{\sqrt{e^2-4df}}\right)}{\sqrt{e^2 - 4df}\,(c^2d^2 - bcde + f(b^2d - abe + a^2f) + ac(e^2 - 2df))} + \frac{(Bcd - Ace + Abf - aBf)\log(a + bx + cx^2)}{2(c^2d^2 - bcde + f(b^2d - abe + a^2f) + ac(e^2 - 2df))} - \frac{(Bcd - Ace + Abf - aBf)\log(d + ex + fx^2)}{2(c^2d^2 - bcde + f(b^2d - abe + a^2f) + ac(e^2 - 2df))}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.66

$$\int \frac{A + Bx}{(a + bx + cx^2)(d + ex + fx^2)}\,dx$$
$$= \frac{\frac{2(Ab^2f+2c(Acd+aBe-aAf)-b(Bcd+Ace+aBf))\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}} - \frac{2(B(cde-2bdf+aef)+A(-ce^2+2cdf+bef-2af^2))\arctan\left(\frac{e+2f}{\sqrt{-e^2+}}\right.}{\sqrt{-e^2+4df}}}{2(c^2d^2 - bcde + f(b^2d - a}$$

```
[In] Integrate[(A + B*x)/((a + b*x + c*x^2)*(d + e*x + f*x^2)),x]
```

```
[Out] ((2*(A*b^2*f + 2*c*(A*c*d + a*B*e - a*A*f) - b*(B*c*d + A*c*e + a*B*f))*Arc
Tan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] - (2*(B*(c*d*e - 2*
b*d*f + a*e*f) + A*(-(c*e^2) + 2*c*d*f + b*e*f - 2*a*f^2))*ArcTan[(e + 2*f*
x)/Sqrt[-e^2 + 4*d*f]])/Sqrt[-e^2 + 4*d*f] + (B*c*d - A*c*e + A*b*f - a*B*f
)*Log[a + x*(b + c*x)] + (-(B*c*d) + A*c*e - A*b*f + a*B*f)*Log[d + x*(e +
f*x)])/(2*(c^2*d^2 - b*c*d*e + f*(b^2*d - a*b*e + a^2*f) + a*c*(e^2 - 2*d*f
)))
```

Maple [A] (verified)

Time = 1.57 (sec) , antiderivative size = 384, normalized size of antiderivative = 0.95

method	result
default	$\frac{\frac{\left(-Ab\,f^{2}+Acef+Ba\,f^{2}-Bcdf\right)\ln\left(f\,x^{2}+ex+d\right)}{2f}+\frac{2\left(Aa\,f^{2}-Abef-Acdf+Ac\,e^{2}+Bbdf-Bcde-\frac{\left(-Ab\,f^{2}+Acef+Ba\,f^{2}-Bcdf\right)e}{2f}\right)\arctan\left(\frac{2fx+e}{\sqrt{4df-e^{2}}}\right.}{\sqrt{4df-e^{2}}}}{a^{2}f^{2}-abef-2acdf+ac\,e^{2}+b^{2}df-bcde+c^{2}d^{2}}$
risch	Expression too large to display

[In] int((B*x+A)/(c*x^2+b*x+a)/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)

[Out] 1/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)*(1/2*(-A*b*f^2+A*c*e*f+B*a*f^2-B*c*d*f)/f*ln(f*x^2+e*x+d)+2*(A*a*f^2-A*b*e*f-A*c*d*f+A*c*e^2+B*b*d*f-B*c*d*e-1/2*(-A*b*f^2+A*c*e*f+B*a*f^2-B*c*d*f)*e/f)/(4*d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2)))+1/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)*(1/2*(A*b*c*f-A*c^2*e-B*a*c*f+B*c^2*d)/c*ln(c*x^2+b*x+a)+2*(-A*a*c*f+A*b^2*f-A*b*c*e+A*c^2*d-B*a*b*f+B*a*c*e-1/2*(A*b*c*f-A*c^2*e-B*a*c*f+B*c^2*d)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)))

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(a+bx+cx^2)(d+ex+fx^2)}\,dx = \text{Timed out}$$

[In] integrate((B*x+A)/(c*x^2+b*x+a)/(f*x^2+e*x+d),x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(a+bx+cx^2)(d+ex+fx^2)}\,dx = \text{Timed out}$$

[In] integrate((B*x+A)/(c*x**2+b*x+a)/(f*x**2+e*x+d),x)

[Out] Timed out

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx}{(a+bx+cx^2)(d+ex+fx^2)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.00

$$\begin{aligned}\int \frac{A+Bx}{(a+bx+cx^2)(d+ex+fx^2)}\,dx &= \frac{(Bcd-Ace-Baf+Abf)\log(cx^2+bx+a)}{2(c^2d^2-bcde+ace^2+b^2df-2\,acdf-abef+a^2f^2)} \\ &\quad - \frac{(Bcd-Ace-Baf+Abf)\log(fx^2+ex+d)}{2(c^2d^2-bcde+ace^2+b^2df-2\,acdf-abef+a^2f^2)} \\ &\quad - \frac{(Bbcd-2\,Ac^2d-2\,Bace+Abce+Babf-Ab^2f+2\,Aacf)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(c^2d^2-bcde+ace^2+b^2df-2\,acdf-abef+a^2f^2)\sqrt{-b^2+4\,ac}} \\ &\quad - \frac{(Bcde-Ace^2-2\,Bbdf+2\,Acdf+Baef+Abef-2\,Aaf^2)\arctan\left(\frac{2\,fx+e}{\sqrt{-e^2+4\,df}}\right)}{(c^2d^2-bcde+ace^2+b^2df-2\,acdf-abef+a^2f^2)\sqrt{-e^2+4\,df}}\end{aligned}$$

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] 1/2*(B*c*d - A*c*e - B*a*f + A*b*f)*log(c*x^2 + b*x + a)/(c^2*d^2 - b*c*d*e
 + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2) - 1/2*(B*c*d - A*c*e
- B*a*f + A*b*f)*log(f*x^2 + e*x + d)/(c^2*d^2 - b*c*d*e + a*c*e^2 + b^2*d*
f - 2*a*c*d*f - a*b*e*f + a^2*f^2) - (B*b*c*d - 2*A*c^2*d - 2*B*a*c*e + A*b
*c*e + B*a*b*f - A*b^2*f + 2*A*a*c*f)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c)
)/((c^2*d^2 - b*c*d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2)*
sqrt(-b^2 + 4*a*c)) - (B*c*d*e - A*c*e^2 - 2*B*b*d*f + 2*A*c*d*f + B*a*e*f
+ A*b*e*f - 2*A*a*f^2)*arctan((2*f*x + e)/sqrt(-e^2 + 4*d*f))/((c^2*d^2 - b
*c*d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2)*sqrt(-e^2 + 4*d
*f))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(a+bx+cx^2)\,(d+ex+fx^2)}\,dx=\text{Hanged}$$

```
[In] int((A + B*x)/((a + b*x + c*x^2)*(d + e*x + f*x^2)),x)
[Out] \text{Hanged}
```

3.16 $\int \frac{A+Bx}{(a+bx+cx^2)^2(d+ex+fx^2)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 1075

$$\int \frac{A+Bx}{(a+bx+cx^2)^2(d+ex+fx^2)}\,dx =$$

$$-\frac{Ac(2ace-b(cd+af))+(Ab-aB)(2c^2d+b^2f-c(be+2af))+c(Ab^2f+2c(Acd+aBe-aAf)-}{(b^2-4ac)((cd-af)^2-(bd-ae)(ce-bf))(a+bx+cx^2)}$$

$$-\frac{(b^5(Bd-Ae)f^2-2b^4f(Bcde-A(ce^2-cdf+af^2))-4c^2(A(c^3d^3-3a^3f^3-a^2cf(e^2-7df)+ac^2d(3}{}$$

$$+\frac{(B(c^2de(e^2-3df)-2cdf(be^2-2bdf-aef)+f^2(b^2de-4abdf+a^2ef))-A(c^2(e^4-4de^2f+2d^2f^2)}{\sqrt{e^2-4df}\,(c^2d^2+f(b^2d-abe+a^2}$$

$$+\frac{(A(ce-bf)(f(be-2af)-c(e^2-2df))-B(2cdf(be-af)-f^2(b^2d-a^2f)-c^2d(e^2-df)))\log(a+}{2(c^2d^2+f(b^2d-abe+a^2f)-c(bde-a(e^2-2df)))^2}$$

$$-\frac{(A(ce-bf)(f(be-2af)-c(e^2-2df))-B(2cdf(be-af)-f^2(b^2d-a^2f)-c^2d(e^2-df)))\log(d+}{2(c^2d^2+f(b^2d-abe+a^2f)-c(bde-a(e^2-2df)))^2}$$

```
[Out] (-A*c*(2*a*c*e-b*(a*f+c*d))-(A*b-B*a)*(2*c^2*d+b^2*f-c*(2*a*f+b*e))-c*(A*b^
2*f+2*c*(-A*a*f+A*c*d+B*a*e)-b*(A*c*e+B*a*f+B*c*d))*x)/(-4*a*c+b^2)/((-a*f+
c*d)^2-(-a*e+b*d)*(-b*f+c*e))/(c*x^2+b*x+a)-(b^5*(-A*e+B*d)*f^2-2*b^4*f*(B*
c*d*e-A*(a*f^2-c*d*f+c*e^2))-4*c^2*(A*(c^3*d^3-3*a^3*f^3-a^2*c*f*(-7*d*f+e^
2)+a*c^2*d*(-5*d*f+3*e^2))-a*B*e*(c^2*d^2-3*a^2*f^2-a*c*(-2*d*f+e^2)))-4*b^
2*c*(B*c^2*d^2*e+A*f*(2*c^2*d^2+3*a^2*f^2+3*a*c*(-d*f+e^2)))+2*b*c*(B*(c^3*
d^3+3*a^3*f^3+a*c^2*d*(-7*d*f+e^2)+3*a^2*c*f*(d*f+e^2))+A*c*e*(3*c^2*d^2+3*
a^2*f^2+a*c*(2*d*f+3*e^2)))-b^3*(A*c*e*(-4*a*f^2-2*c*d*f+c*e^2)+B*(4*a*c*d*
f^2+a^2*f^3-c^2*d*(5*d*f+e^2))))*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))/(-4*
a*c+b^2)^(3/2)/(c^2*d^2+f*(a^2*f-a*b*e+b^2*d)-c*(b*d*e-a*(-2*d*f+e^2)))^2+1
/2*(A*(-b*f+c*e)*(f*(-2*a*f+b*e)-c*(-2*d*f+e^2))-B*(2*c*d*f*(-a*f+b*e)-f^2*
```

```
(-a^2*f+b^2*d)-c^2*d*(-d*f+e^2)))*ln(c*x^2+b*x+a)/(c^2*d^2+f*(a^2*f-a*b*e+b
^2*d)-c*(b*d*e-a*(-2*d*f+e^2)))^2-1/2*(A*(-b*f+c*e)*(f*(-2*a*f+b*e)-c*(-2*d
*f+e^2))-B*(2*c*d*f*(-a*f+b*e)-f^2*(-a^2*f+b^2*d)-c^2*d*(-d*f+e^2)))*ln(f*x
^2+e*x+d)/(c^2*d^2+f*(a^2*f-a*b*e+b^2*d)-c*(b*d*e-a*(-2*d*f+e^2)))^2+(B*(c^
2*d*e*(-3*d*f+e^2)-2*c*d*f*(-a*e*f-2*b*d*f+b*e^2)+f^2*(a^2*e*f-4*a*b*d*f+b^
2*d*e))-A*(c^2*(2*d^2*f^2-4*d*e^2*f+e^4)-f^2*(2*a*b*e*f-2*a^2*f^2-b^2*(-2*d
*f+e^2))+2*c*f*(a*f*(-2*d*f+e^2)-b*(-3*d*e*f+e^3))))*arctanh((2*f*x+e)/(-4*
d*f+e^2)^(1/2))/(c^2*d^2+f*(a^2*f-a*b*e+b^2*d)-c*(b*d*e-a*(-2*d*f+e^2)))^2/
(-4*d*f+e^2)^(1/2)
```

Rubi [A] (verified)

Time = 2.48 (sec) , antiderivative size = 1067, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1030, 1086, 648, 632, 212, 642}

$$\int \frac{A+Bx}{(a+bx+cx^2)^2(d+ex+fx^2)}dx =$$

$$-\frac{Ac(2ace-b(cd+af))+(Ab-aB)(fb^2+2c^2d-c(be+2af))+c(Afb^2-(Bcd+Ace+aBf)b+2c(}{(b^2-4ac)((cd-af)^2-(bd-ae)(ce-bf))(cx^2+bx+a)}$$

$$-\frac{((Bd-Ae)f^2b^5-2f(-aAf^2+Bcde-Ac(e^2-df))b^4-(Ace(ce^2-4af^2-2cdf)+B(a^2f^3+4acdf^2}{}$$

$$+\frac{(B(de(e^2-3df)c^2-2df(be^2-afe-2bdf)c+f^2(efa^2-4bdfa+b^2de))-A((e^4-4dfe^2+2d^2f^2)c^2}{\sqrt{e^2-4df}(c^2d^2-bced+f(fa^2-}$$

$$+\frac{(A(ce-bf)(f(be-2af)-c(e^2-2df))-B(-d(e^2-df)c^2+2df(be-af)c-f^2(b^2d-a^2f)))\log(cx^2}{2(c^2d^2-bced+f(fa^2-bea+b^2d)+ac(e^2-2df))^2}$$

$$-\frac{(A(ce-bf)(f(be-2af)-c(e^2-2df))-B(-d(e^2-df)c^2+2df(be-af)c-f^2(b^2d-a^2f)))\log(fx}{2(c^2d^2-bced+f(fa^2-bea+b^2d)+ac(e^2-2df))^2}$$

```
[In] Int[(A + B*x)/((a + b*x + c*x^2)^2*(d + e*x + f*x^2)),x]
```

```
[Out] -((A*c*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e +
2*a*f)) + c*(A*b^2*f + 2*c*(A*c*d + a*B*e - a*A*f) - b*(B*c*d + A*c*e + a*B
*f))*x)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(a + b*x +
 c*x^2))) - ((b^5*(B*d - A*e)*f^2 - 2*b^4*f*(B*c*d*e - a*A*f^2 - A*c*(e^2 -
 d*f)) - 4*c^2*(A*(c^3*d^3 - 3*a^3*f^3 - a^2*c*f*(e^2 - 7*d*f) + a*c^2*d*(3
*e^2 - 5*d*f)) - a*B*e*(c^2*d^2 - 3*a^2*f^2 - a*c*(e^2 - 2*d*f))) - 4*b^2*(
B*c^3*d^2*e + A*c*f*(2*c^2*d^2 + 3*a^2*f^2 + 3*a*c*(e^2 - d*f))) + 2*b*c*(B
*(c^3*d^3 + 3*a^3*f^3 + a*c^2*d*(e^2 - 7*d*f) + 3*a^2*c*f*(e^2 + d*f)) + A*
c*e*(3*c^2*d^2 + 3*a^2*f^2 + a*c*(3*e^2 + 2*d*f))) - b^3*(A*c*e*(c*e^2 - 2*
c*d*f - 4*a*f^2) + B*(4*a*c*d*f^2 + a^2*f^3 - c^2*d*(e^2 + 5*d*f))))*ArcTan
h[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/((b^2 - 4*a*c)^(3/2)*(c^2*d^2 - b*c*d*e +
```

```
 f*(b^2*d - a*b*e + a^2*f) + a*c*(e^2 - 2*d*f))^2) + ((B*(c^2*d*e*(e^2 - 3*
d*f) - 2*c*d*f*(b*e^2 - 2*b*d*f - a*e*f) + f^2*(b^2*d*e - 4*a*b*d*f + a^2*e
*f)) - A*(c^2*(e^4 - 4*d*e^2*f + 2*d^2*f^2) - f^2*(2*a*b*e*f - 2*a^2*f^2 -
b^2*(e^2 - 2*d*f)) + 2*c*f*(a*f*(e^2 - 2*d*f) - b*(e^3 - 3*d*e*f))))*ArcTan
h[(e + 2*f*x)/Sqrt[e^2 - 4*d*f]])/(Sqrt[e^2 - 4*d*f]*(c^2*d^2 - b*c*d*e + f
*(b^2*d - a*b*e + a^2*f) + a*c*(e^2 - 2*d*f))^2) + ((A*(c*e - b*f)*(f*(b*e
- 2*a*f) - c*(e^2 - 2*d*f)) - B*(2*c*d*f*(b*e - a*f) - f^2*(b^2*d - a^2*f)
- c^2*d*(e^2 - d*f)))*Log[a + b*x + c*x^2])/(2*(c^2*d^2 - b*c*d*e + f*(b^2*
d - a*b*e + a^2*f) + a*c*(e^2 - 2*d*f))^2) - ((A*(c*e - b*f)*(f*(b*e - 2*a*
f) - c*(e^2 - 2*d*f)) - B*(2*c*d*f*(b*e - a*f) - f^2*(b^2*d - a^2*f) - c^2*
d*(e^2 - d*f)))*Log[d + e*x + f*x^2])/(2*(c^2*d^2 - b*c*d*e + f*(b^2*d - a*
b*e + a^2*f) + a*c*(e^2 - 2*d*f))^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1030

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x + c*x^2)^(p + 1)*
((d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
 - b*f))*(p + 1)))*(g*c*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d +
b^2*f - c*(b*e + 2*a*f)) + c*(g*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - h*(b*
c*d - 2*a*c*e + a*b*f))*x), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*
d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*
```

```
x^2)^q*Simp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)
 + (b^2*(g*f) - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*((-h)*c*
e)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((g*c)*(2*a*c*e - b*(c*d + a*f)) + (g*
b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((g*c)*(2*
a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(
p + q + 2) - (b^2*g*f - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*
((-h)*c*e)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(g*f) - b*(h*c
*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) + a*h*c*e))*(2*p + 2*q + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e
 - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
```

Rule 1086

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)), x_Symbol] :> With[{q = c^2*d^2 - b*c*
d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2}, Dist[1/q, Int[(A*
c^2*d - a*c*C*d - A*b*c*e + a*B*c*e + A*b^2*f - a*b*B*f - a*A*c*f + a^2*C*f
 + c*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B*f)*x)/(a + b*x + c*x^2),
x], x] + Dist[1/q, Int[(c*C*d^2 - B*c*d*e + A*c*e^2 + b*B*d*f - A*c*d*f - a
*C*d*f - A*b*e*f + a*A*f^2 - f*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B
*f)*x)/(d + e*x + f*x^2), x], x] /; NeQ[q, 0]] /; FreeQ[{a, b, c, d, e, f,
A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

integral =

$$-\frac{Ac(2ace-b(cd+af))+(Ab-aB)\left(2c^2d+b^2f-c(be+2af)\right)+c\left(Ab^2f+2c(Acd+aBe-aAf)-b(\right.}{\left(b^2-4ac\right)\left((cd-af)^2-(bd-ae)(ce-bf)\right)\left(a+bx+cx^2\right)}$$

$$-\frac{\int\frac{-b^3(Bdf-Aef)-bc(Bd(cd-3af)+Ae(cd+4af))+b^2\left(Bcde-A\left(ce^2-2cdf+af^2\right)\right)-2c\left(aBcde-A\left(c^2d^2+2ace^2-3acdf+2a^2f^2\right)\right)+\left(Ab^3f^2+b^2\right.}{(a+bx+c}}{\left(b^2-4ac\right)\left((cd-a\right.}$$

= Too large to display

Mathematica [A] (verified)

Time = 3.87 (sec) , antiderivative size = 952, normalized size of antiderivative = 0.89

$$\int\frac{A+Bx}{\left(a+bx+cx^2\right)^2\left(d+ex+fx^2\right)}\,dx$$

$$=\frac{-\frac{2\left(c^2d^2-bcde+f\left(b^2d-abe+a^2f\right)+ac\left(e^2-2df\right)\right)\left(A\left(b^3f+b^2c(-e+fx)+bc(-3af+c(d-ex))+2c^2(cdx+a(e-fx))\right)+B\left(2a^2cf-bc^2dx-a\left(b^2f+2c^2\right.\right.\right.}{\left(b^2-4ac\right)(a+x(b+cx))}}{}$$

```
[In] Integrate[(A + B*x)/((a + b*x + c*x^2)^2*(d + e*x + f*x^2)),x]
```

```
[Out] ((-2*(c^2*d^2 - b*c*d*e + f*(b^2*d - a*b*e + a^2*f) + a*c*(e^2 - 2*d*f))*(A
*(b^3*f + b^2*c*(-e + f*x) + b*c*(-3*a*f + c*(d - e*x)) + 2*c^2*(c*d*x + a*
(e - f*x))) + B*(2*a^2*c*f - b*c^2*d*x - a*(b^2*f + 2*c^2*(d - e*x) + b*c*(
-e + f*x)))))/((b^2 - 4*a*c)*(a + x*(b + c*x))) - (2*(b^5*(B*d - A*e)*f^2 +
 2*b^4*f*(-(B*c*d*e) + a*A*f^2 + A*c*(e^2 - d*f)) - 4*b^2*(B*c^3*d^2*e + A*
c*f*(2*c^2*d^2 + 3*a^2*f^2 + 3*a*c*(e^2 - d*f))) + 2*b*c*(B*(c^3*d^3 + 3*a^
3*f^3 + a*c^2*d*(e^2 - 7*d*f) + 3*a^2*c*f*(e^2 + d*f)) + A*c*e*(3*c^2*d^2 +
 3*a^2*f^2 + a*c*(3*e^2 + 2*d*f))) + 4*c^2*(a*B*e*(c^2*d^2 - 3*a^2*f^2 - a*
c*(e^2 - 2*d*f)) + A*(-(c^3*d^3) + 3*a^3*f^3 + a^2*c*f*(e^2 - 7*d*f) + a*c^
2*d*(-3*e^2 + 5*d*f))) + b^3*(A*c*e*(-(c*e^2) + 2*c*d*f + 4*a*f^2) + B*(-4*
a*c*d*f^2 - a^2*f^3 + c^2*d*(e^2 + 5*d*f))))*ArcTan[(b + 2*c*x)/Sqrt[-b^2 +
 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + (2*(B*(c^2*d*e*(-e^2 + 3*d*f) - 2*c*d*f*(-
(b*e^2) + 2*b*d*f + a*e*f) + f^2*(-(b^2*d*e) + 4*a*b*d*f - a^2*e*f)) + A*(c
^2*(e^4 - 4*d*e^2*f + 2*d^2*f^2) + f^2*(-2*a*b*e*f + 2*a^2*f^2 + b^2*(e^2 -
 2*d*f)) + 2*c*f*(a*f*(e^2 - 2*d*f) - b*(e^3 - 3*d*e*f))))*ArcTan[(e + 2*f*
x)/Sqrt[-e^2 + 4*d*f]])/Sqrt[-e^2 + 4*d*f] - (A*(c*e - b*f)*(f*(-(b*e) + 2*
a*f) + c*(e^2 - 2*d*f)) + B*(2*c*d*f*(b*e - a*f) + f^2*(-(b^2*d) + a^2*f) +
 c^2*d*(-e^2 + d*f)))*Log[a + x*(b + c*x)] + (A*(c*e - b*f)*(f*(-(b*e) + 2*
a*f) + c*(e^2 - 2*d*f)) + B*(2*c*d*f*(b*e - a*f) + f^2*(-(b^2*d) + a^2*f) +
 c^2*d*(-e^2 + d*f)))*Log[d + x*(e + f*x)])/(2*(c^2*d^2 - b*c*d*e + f*(b^2*
d - a*b*e + a^2*f) + a*c*(e^2 - 2*d*f))^2)
```

Maple [F(-1)]

Timed out.

hanged

```
[In] int((B*x+A)/(c*x^2+b*x+a)^2/(f*x^2+e*x+d),x)
```

```
[Out] int((B*x+A)/(c*x^2+b*x+a)^2/(f*x^2+e*x+d),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(a+bx+cx^2)^2\,(d+ex+fx^2)}\,dx = \text{Timed out}$$

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)^2/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(a+bx+cx^2)^2\,(d+ex+fx^2)}\,dx = \text{Timed out}$$

```
[In] integrate((B*x+A)/(c*x**2+b*x+a)**2/(f*x**2+e*x+d),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx}{(a+bx+cx^2)^2\,(d+ex+fx^2)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)^2/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re deta
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3236 vs. $2(1061) = 2122$.

Time = 0.33 (sec) , antiderivative size = 3236, normalized size of antiderivative = 3.01

$$\int \frac{A+Bx}{(a+bx+cx^2)^2\,(d+ex+fx^2)}\,dx = \text{Too large to display}$$

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)^2/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] 1/2*(B*c^2*d*e^2 - A*c^2*e^3 - B*c^2*d^2*f - 2*B*b*c*d*e*f + 2*A*c^2*d*e*f
+ 2*A*b*c*e^2*f + B*b^2*d*f^2 + 2*B*a*c*d*f^2 - 2*A*b*c*d*f^2 - A*b^2*e*f^2
 - 2*A*a*c*e*f^2 - B*a^2*f^3 + 2*A*a*b*f^3)*log(c*x^2 + b*x + a)/(c^4*d^4 -
 2*b*c^3*d^3*e + b^2*c^2*d^2*e^2 + 2*a*c^3*d^2*e^2 - 2*a*b*c^2*d*e^3 + a^2*
c^2*e^4 + 2*b^2*c^2*d^3*f - 4*a*c^3*d^3*f - 2*b^3*c*d^2*e*f + 2*a*b*c^2*d^2
*e*f + 4*a*b^2*c*d*e^2*f - 4*a^2*c^2*d*e^2*f - 2*a^2*b*c*e^3*f + b^4*d^2*f^
2 - 4*a*b^2*c*d^2*f^2 + 6*a^2*c^2*d^2*f^2 - 2*a*b^3*d*e*f^2 + 2*a^2*b*c*d*e
*f^2 + a^2*b^2*e^2*f^2 + 2*a^3*c*e^2*f^2 + 2*a^2*b^2*d*f^3 - 4*a^3*c*d*f^3
- 2*a^3*b*e*f^3 + a^4*f^4) - 1/2*(B*c^2*d*e^2 - A*c^2*e^3 - B*c^2*d^2*f - 2
*B*b*c*d*e*f + 2*A*c^2*d*e*f + 2*A*b*c*e^2*f + B*b^2*d*f^2 + 2*B*a*c*d*f^2
- 2*A*b*c*d*f^2 - A*b^2*e*f^2 - 2*A*a*c*e*f^2 - B*a^2*f^3 + 2*A*a*b*f^3)*lo
```

```
g(f*x^2 + e*x + d)/(c^4*d^4 - 2*b*c^3*d^3*e + b^2*c^2*d^2*e^2 + 2*a*c^3*d^2
*e^2 - 2*a*b*c^2*d*e^3 + a^2*c^2*e^4 + 2*b^2*c^2*d^3*f - 4*a*c^3*d^3*f - 2*
b^3*c*d^2*e*f + 2*a*b*c^2*d^2*e*f + 4*a*b^2*c*d*e^2*f - 4*a^2*c^2*d*e^2*f -
 2*a^2*b*c*e^3*f + b^4*d^2*f^2 - 4*a*b^2*c*d^2*f^2 + 6*a^2*c^2*d^2*f^2 - 2*
a*b^3*d*e*f^2 + 2*a^2*b*c*d*e*f^2 + a^2*b^2*e^2*f^2 + 2*a^3*c*e^2*f^2 + 2*a
^2*b^2*d*f^3 - 4*a^3*c*d*f^3 - 2*a^3*b*e*f^3 + a^4*f^4) + (2*B*b*c^4*d^3 -
4*A*c^5*d^3 - 4*B*b^2*c^3*d^2*e + 4*B*a*c^4*d^2*e + 6*A*b*c^4*d^2*e + B*b^3
*c^2*d*e^2 + 2*B*a*b*c^3*d*e^2 - 12*A*a*c^4*d*e^2 - A*b^3*c^2*e^3 - 4*B*a^2
*c^3*e^3 + 6*A*a*b*c^3*e^3 + 5*B*b^3*c^2*d^2*f - 14*B*a*b*c^3*d^2*f - 8*A*b
^2*c^3*d^2*f + 20*A*a*c^4*d^2*f - 2*B*b^4*c*d*e*f + 2*A*b^3*c^2*d*e*f + 8*B
*a^2*c^3*d*e*f + 4*A*a*b*c^3*d*e*f + 2*A*b^4*c*e^2*f + 6*B*a^2*b*c^2*e^2*f
- 12*A*a*b^2*c^2*e^2*f + 4*A*a^2*c^3*e^2*f + B*b^5*d*f^2 - 4*B*a*b^3*c*d*f^
2 - 2*A*b^4*c*d*f^2 + 6*B*a^2*b*c^2*d*f^2 + 12*A*a*b^2*c^2*d*f^2 - 28*A*a^2
*c^3*d*f^2 - A*b^5*e*f^2 + 4*A*a*b^3*c*e*f^2 - 12*B*a^3*c^2*e*f^2 + 6*A*a^2
*b*c^2*e*f^2 - B*a^2*b^3*f^3 + 2*A*a*b^4*f^3 + 6*B*a^3*b*c*f^3 - 12*A*a^2*b
^2*c*f^3 + 12*A*a^3*c^2*f^3)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^2*c
^4*d^4 - 4*a*c^5*d^4 - 2*b^3*c^3*d^3*e + 8*a*b*c^4*d^3*e + b^4*c^2*d^2*e^2
- 2*a*b^2*c^3*d^2*e^2 - 8*a^2*c^4*d^2*e^2 - 2*a*b^3*c^2*d*e^3 + 8*a^2*b*c^3
*d*e^3 + a^2*b^2*c^2*e^4 - 4*a^3*c^3*e^4 + 2*b^4*c^2*d^3*f - 12*a*b^2*c^3*d
^3*f + 16*a^2*c^4*d^3*f - 2*b^5*c*d^2*e*f + 10*a*b^3*c^2*d^2*e*f - 8*a^2*b*
c^3*d^2*e*f + 4*a*b^4*c*d*e^2*f - 20*a^2*b^2*c^2*d*e^2*f + 16*a^3*c^3*d*e^2
*f - 2*a^2*b^3*c*e^3*f + 8*a^3*b*c^2*e^3*f + b^6*d^2*f^2 - 8*a*b^4*c*d^2*f^
2 + 22*a^2*b^2*c^2*d^2*f^2 - 24*a^3*c^3*d^2*f^2 - 2*a*b^5*d*e*f^2 + 10*a^2*
b^3*c*d*e*f^2 - 8*a^3*b*c^2*d*e*f^2 + a^2*b^4*e^2*f^2 - 2*a^3*b^2*c*e^2*f^2
 - 8*a^4*c^2*e^2*f^2 + 2*a^2*b^4*d*f^3 - 12*a^3*b^2*c*d*f^3 + 16*a^4*c^2*d*
f^3 - 2*a^3*b^3*e*f^3 + 8*a^4*b*c*e*f^3 + a^4*b^2*f^4 - 4*a^5*c*f^4)*sqrt(-
b^2 + 4*a*c)) - (B*c^2*d*e^3 - A*c^2*e^4 - 3*B*c^2*d^2*e*f - 2*B*b*c*d*e^2*
f + 4*A*c^2*d*e^2*f + 2*A*b*c*e^3*f + 4*B*b*c*d^2*f^2 - 2*A*c^2*d^2*f^2 + B
*b^2*d*e*f^2 + 2*B*a*c*d*e*f^2 - 6*A*b*c*d*e*f^2 - A*b^2*e^2*f^2 - 2*A*a*c*
e^2*f^2 - 4*B*a*b*d*f^3 + 2*A*b^2*d*f^3 + 4*A*a*c*d*f^3 + B*a^2*e*f^3 + 2*A
*a*b*e*f^3 - 2*A*a^2*f^4)*arctan((2*f*x + e)/sqrt(-e^2 + 4*d*f))/((c^4*d^4
- 2*b*c^3*d^3*e + b^2*c^2*d^2*e^2 + 2*a*c^3*d^2*e^2 - 2*a*b*c^2*d*e^3 + a^2
*c^2*e^4 + 2*b^2*c^2*d^3*f - 4*a*c^3*d^3*f - 2*b^3*c*d^2*e*f + 2*a*b*c^2*d^
2*e*f + 4*a*b^2*c*d*e^2*f - 4*a^2*c^2*d*e^2*f - 2*a^2*b*c*e^3*f + b^4*d^2*f
^2 - 4*a*b^2*c*d^2*f^2 + 6*a^2*c^2*d^2*f^2 - 2*a*b^3*d*e*f^2 + 2*a^2*b*c*d*
e*f^2 + a^2*b^2*e^2*f^2 + 2*a^3*c*e^2*f^2 + 2*a^2*b^2*d*f^3 - 4*a^3*c*d*f^3
 - 2*a^3*b*e*f^3 + a^4*f^4)*sqrt(-e^2 + 4*d*f)) + (2*B*a*c^4*d^3 - A*b*c^4*
d^3 - 3*B*a*b*c^3*d^2*e + 2*A*b^2*c^3*d^2*e - 2*A*a*c^4*d^2*e + B*a*b^2*c^2
*d*e^2 - A*b^3*c^2*d*e^2 + 2*B*a^2*c^3*d*e^2 + A*a*b*c^3*d*e^2 - B*a^2*b*c^
2*e^3 + A*a*b^2*c^2*e^3 - 2*A*a^2*c^3*e^3 + 3*B*a*b^2*c^2*d^2*f - 2*A*b^3*c
^2*d^2*f - 6*B*a^2*c^3*d^2*f + 5*A*a*b*c^3*d^2*f - 2*B*a*b^3*c*d*e*f + 2*A*
b^4*c*d*e*f + 2*B*a^2*b*c^2*d*e*f - 6*A*a*b^2*c^2*d*e*f + 4*A*a^2*c^3*d*e*f
 + 2*B*a^2*b^2*c*e^2*f - 2*A*a*b^3*c*e^2*f - 2*B*a^3*c^2*e^2*f + 5*A*a^2*b*
c^2*e^2*f + B*a*b^4*d*f^2 - A*b^5*d*f^2 - 4*B*a^2*b^2*c*d*f^2 + 5*A*a*b^3*c
*d*f^2 + 6*B*a^3*c^2*d*f^2 - 7*A*a^2*b*c^2*d*f^2 - B*a^2*b^3*e*f^2 + A*a*b^
```

```
4*e*f^2 + B*a^3*b*c*e*f^2 - 2*A*a^2*b^2*c*e*f^2 - 2*A*a^3*c^2*e*f^2 + B*a^3
*b^2*f^3 - A*a^2*b^3*f^3 - 2*B*a^4*c*f^3 + 3*A*a^3*b*c*f^3 + (B*b*c^4*d^3 -
 2*A*c^5*d^3 - B*b^2*c^3*d^2*e - 2*B*a*c^4*d^2*e + 3*A*b*c^4*d^2*e + 3*B*a*
b*c^3*d*e^2 - A*b^2*c^3*d*e^2 - 2*A*a*c^4*d*e^2 - 2*B*a^2*c^3*e^3 + A*a*b*c
^3*e^3 + B*b^3*c^2*d^2*f - B*a*b*c^3*d^2*f - 3*A*b^2*c^3*d^2*f + 6*A*a*c^4*
d^2*f - 4*B*a*b^2*c^2*d*e*f + 2*A*b^3*c^2*d*e*f + 4*B*a^2*c^3*d*e*f - 2*A*a
*b*c^3*d*e*f + 3*B*a^2*b*c^2*e^2*f - 2*A*a*b^2*c^2*e^2*f + 2*A*a^2*c^3*e^2*
f + B*a*b^3*c*d*f^2 - A*b^4*c*d*f^2 - B*a^2*b*c^2*d*f^2 + 4*A*a*b^2*c^2*d*f
^2 - 6*A*a^2*c^3*d*f^2 - B*a^2*b^2*c*e*f^2 + A*a*b^3*c*e*f^2 - 2*B*a^3*c^2*
e*f^2 - A*a^2*b*c^2*e*f^2 + B*a^3*b*c*f^3 - A*a^2*b^2*c*f^3 + 2*A*a^3*c^2*f
^3)*x)/((c^2*d^2 - b*c*d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*
f^2)^2*(c*x^2 + b*x + a)*(b^2 - 4*a*c))
```

Mupad [B] (verification not implemented)

Time = 50.27 (sec) , antiderivative size = 118429, normalized size of antiderivative = 110.17

$$\int \frac{A+Bx}{(a+bx+cx^2)^2\,(d+ex+fx^2)}\,dx = \text{Too large to display}$$

```
[In] int((A + B*x)/((a + b*x + c*x^2)^2*(d + e*x + f*x^2)),x)
```

```
[Out] symsum(log((x*(4*A^3*b^3*c^4*f^6 + 16*B^3*a^3*c^4*f^6 - 3*B^3*a^2*b^2*c^3*f
^6 + 4*B^3*a^2*c^5*e^2*f^4 + B^3*b^2*c^5*d^2*f^4 - 16*A^3*a*b*c^5*f^6 + 16*
A^3*a*c^6*e*f^5 + 20*A^2*B*a^2*c^5*f^6 - 3*A^2*B*b^4*c^3*f^6 + 4*A^2*B*c^7*
d^2*f^4 - 16*B^3*a^2*c^5*d*f^5 - 4*A^3*b^2*c^5*e*f^5 + 6*B^3*a*b^2*c^4*d*f^
5 - 4*B^3*a^2*b*c^4*e*f^5 + A^2*B*b^2*c^5*e^2*f^4 - 24*A^2*B*a*c^6*d*f^5 +
6*A*B^2*a*b^3*c^3*f^6 - 28*A*B^2*a^2*b*c^4*f^6 + 8*A^2*B*a*b^2*c^4*f^6 - 4*
A*B^2*b*c^6*d^2*f^4 + 8*A*B^2*a^2*c^5*e*f^5 - 6*A*B^2*b^3*c^4*d*f^5 + 8*A^2
*B*b^2*c^5*d*f^5 + 2*A^2*B*b^3*c^4*e*f^5 - 4*B^3*a*b*c^5*d*e*f^4 - 4*A*B^2*
a*b*c^5*e^2*f^4 + 2*A*B^2*a*b^2*c^4*e*f^5 + 2*A*B^2*b^2*c^5*d*e*f^4 + 16*A*
B^2*a*b*c^5*d*f^5 - 12*A^2*B*a*b*c^5*e*f^5 + 8*A*B^2*a*c^6*d*e*f^4 - 4*A^2*
B*b*c^6*d*e*f^4))/(16*a^2*c^6*d^4 + a^4*b^4*f^4 + 16*a^4*c^4*e^4 + b^4*c^4*
d^4 + 16*a^6*c^2*f^4 + b^8*d^2*f^2 - 8*a*b^2*c^5*d^4 - 8*a^5*b^2*c*f^4 + 2*
a^2*b^6*d*f^3 - 2*a^3*b^5*e*f^3 - 64*a^3*c^5*d^3*f - 64*a^5*c^3*d*f^3 - 2*b
^5*c^3*d^3*e + 2*b^6*c^2*d^3*f + a^2*b^4*c^2*e^4 - 8*a^3*b^2*c^3*e^4 + 32*a
^3*c^5*d^2*e^2 + a^2*b^6*e^2*f^2 + 96*a^4*c^4*d^2*f^2 + b^6*c^2*d^2*e^2 + 3
2*a^5*c^3*e^2*f^2 - 2*a*b^7*d*e*f^2 - 2*b^7*c*d^2*e*f + 54*a^2*b^4*c^2*d^2*
f^2 - 112*a^3*b^2*c^3*d^2*f^2 + 16*a*b^3*c^4*d^3*e - 2*a*b^5*c^2*d*e^3 - 32
*a^2*b*c^5*d^3*e - 32*a^3*b*c^4*d*e^3 - 20*a*b^4*c^3*d^3*f - 12*a*b^6*c*d^2
*f^2 - 20*a^3*b^4*c*d*f^3 - 2*a^2*b^5*c*e^3*f - 32*a^4*b*c^3*e^3*f + 16*a^4
*b^3*c*e*f^3 - 32*a^5*b*c^2*e*f^3 - 64*a^4*c^4*d*e^2*f - 6*a*b^4*c^3*d^2*e^
2 + 16*a^2*b^3*c^3*d*e^3 + 64*a^2*b^2*c^4*d^3*f + 64*a^4*b^2*c^2*d*f^3 + 16
*a^3*b^3*c^2*e^3*f - 6*a^3*b^4*c*e^2*f^2 - 48*a^2*b^3*c^3*d^2*e*f - 36*a^2*
b^4*c^2*d*e^2*f + 96*a^3*b^2*c^3*d*e^2*f - 48*a^3*b^3*c^2*d*e*f^2 + 4*a*b^6
```

*c*d*e^2*f + 18*a*b^5*c^2*d^2*e*f + 18*a^2*b^5*c*d*e*f^2 + 32*a^3*b*c^4*d^2
*e*f + 32*a^4*b*c^3*d*e*f^2) - root(48416*a^6*b^2*c^6*d^4*e^2*f^4*z^4 - 415
44*a^5*b^4*c^5*d^4*e^2*f^4*z^4 - 31872*a^7*b^2*c^5*d^3*e^2*f^5*z^4 - 31872*
a^5*b^2*c^7*d^5*e^2*f^3*z^4 - 29184*a^6*b^2*c^6*d^3*e^4*f^3*z^4 + 28800*a^5
*b^4*c^5*d^3*e^4*f^3*z^4 + 21510*a^4*b^6*c^4*d^4*e^2*f^4*z^4 + 21408*a^6*b^
4*c^4*d^3*e^2*f^5*z^4 + 21408*a^4*b^4*c^6*d^5*e^2*f^3*z^4 - 18112*a^7*b^3*c
^4*d^2*e^3*f^5*z^4 - 18112*a^4*b^3*c^7*d^5*e^3*f^2*z^4 - 15600*a^5*b^5*c^4*
d^3*e^3*f^4*z^4 - 15600*a^4*b^5*c^5*d^4*e^3*f^3*z^4 + 15296*a^6*b^3*c^5*d^3
*e^3*f^4*z^4 + 15296*a^5*b^3*c^6*d^4*e^3*f^3*z^4 + 14016*a^7*b^2*c^5*d^2*e^
4*f^4*z^4 + 14016*a^5*b^2*c^7*d^4*e^4*f^2*z^4 - 13920*a^4*b^6*c^4*d^3*e^4*f
^3*z^4 - 11648*a^6*b^3*c^5*d^2*e^5*f^3*z^4 - 11648*a^5*b^3*c^6*d^3*e^5*f^2*
z^4 + 10432*a^6*b^2*c^6*d^2*e^6*f^2*z^4 + 9008*a^6*b^5*c^3*d^2*e^3*f^5*z^4
+ 9008*a^3*b^5*c^6*d^5*e^3*f^2*z^4 + 8544*a^5*b^5*c^4*d^2*e^5*f^3*z^4 + 854
4*a^4*b^5*c^5*d^3*e^5*f^2*z^4 - 8496*a^5*b^4*c^5*d^2*e^6*f^2*z^4 + 7488*a^8
*b^2*c^4*d^2*e^2*f^6*z^4 + 7488*a^4*b^2*c^8*d^6*e^2*f^2*z^4 + 7380*a^4*b^7*
c^3*d^3*e^3*f^4*z^4 + 7380*a^3*b^7*c^4*d^4*e^3*f^3*z^4 - 6720*a^3*b^8*c^3*d
^4*e^2*f^4*z^4 - 5784*a^5*b^6*c^3*d^3*e^2*f^5*z^4 - 5784*a^3*b^6*c^5*d^5*e^
2*f^3*z^4 - 3440*a^6*b^4*c^4*d^2*e^4*f^4*z^4 - 3440*a^4*b^4*c^6*d^4*e^4*f^2
*z^4 + 3360*a^3*b^8*c^3*d^3*e^4*f^3*z^4 + 3140*a^4*b^6*c^4*d^2*e^6*f^2*z^4
- 2760*a^4*b^7*c^3*d^2*e^5*f^3*z^4 - 2760*a^3*b^7*c^4*d^3*e^5*f^2*z^4 - 176
4*a^5*b^7*c^2*d^2*e^3*f^5*z^4 - 1764*a^2*b^7*c^5*d^5*e^3*f^2*z^4 - 1640*a^3
*b^9*c^2*d^3*e^3*f^4*z^4 - 1640*a^2*b^9*c^3*d^4*e^3*f^3*z^4 - 1604*a^6*b^6*
c^2*d^2*e^2*f^6*z^4 - 1604*a^2*b^6*c^6*d^6*e^2*f^2*z^4 - 1500*a^5*b^6*c^3*d
^2*e^4*f^4*z^4 - 1500*a^3*b^6*c^5*d^4*e^4*f^2*z^4 + 1140*a^2*b^10*c^2*d^4*e
^2*f^4*z^4 + 810*a^4*b^8*c^2*d^2*e^4*f^4*z^4 + 810*a^2*b^8*c^4*d^4*e^4*f^2*
z^4 - 544*a^3*b^8*c^3*d^2*e^6*f^2*z^4 + 416*a^3*b^9*c^2*d^2*e^5*f^3*z^4 + 4
16*a^2*b^9*c^3*d^3*e^5*f^2*z^4 - 384*a^2*b^10*c^2*d^3*e^4*f^3*z^4 + 180*a^4
*b^8*c^2*d^3*e^2*f^5*z^4 + 180*a^2*b^8*c^4*d^5*e^2*f^3*z^4 + 48*a^7*b^4*c^3
*d^2*e^2*f^6*z^4 + 48*a^3*b^4*c^7*d^6*e^2*f^2*z^4 + 36*a^2*b^10*c^2*d^2*e^6
*f^2*z^4 - 1024*a^10*b*c^3*d*e*f^8*z^4 - 1024*a^3*b*c^10*d^8*e*f*z^4 - 192*
a^8*b^5*c*d*e*f^8*z^4 - 192*a*b^5*c^8*d^8*e*f*z^4 + 16128*a^7*b^3*c^4*d^3*e
*f^6*z^4 + 16128*a^4*b^3*c^7*d^6*e*f^3*z^4 - 11712*a^6*b^5*c^3*d^3*e*f^6*z^
4 - 11712*a^3*b^5*c^6*d^6*e*f^3*z^4 + 11520*a^8*b*c^5*d^2*e^3*f^5*z^4 + 115
20*a^5*b*c^8*d^5*e^3*f^2*z^4 - 9984*a^6*b^3*c^5*d^4*e*f^5*z^4 - 9984*a^5*b^
3*c^6*d^5*e*f^4*z^4 + 8640*a^5*b^5*c^4*d^4*e*f^5*z^4 + 8640*a^4*b^5*c^5*d^5
*e*f^4*z^4 - 7424*a^7*b*c^6*d^3*e^3*f^4*z^4 - 7424*a^6*b*c^7*d^4*e^3*f^3*z^
4 - 6912*a^8*b^3*c^3*d^2*e*f^7*z^4 - 6912*a^3*b^3*c^8*d^7*e*f^2*z^4 + 4800*
a^7*b^3*c^4*d*e^5*f^4*z^4 + 4800*a^4*b^3*c^7*d^4*e^5*f*z^4 + 4608*a^7*b*c^6
*d^2*e^5*f^3*z^4 + 4608*a^6*b*c^7*d^3*e^5*f^2*z^4 - 4560*a^4*b^7*c^3*d^4*e*
f^5*z^4 - 4560*a^3*b^7*c^4*d^5*e*f^4*z^4 + 4176*a^5*b^7*c^2*d^3*e*f^6*z^4 +
4176*a^2*b^7*c^5*d^6*e*f^3*z^4 + 3264*a^7*b^5*c^2*d^2*e*f^7*z^4 + 3264*a^2
*b^5*c^7*d^7*e*f^2*z^4 + 3008*a^8*b^3*c^3*d*e^3*f^6*z^4 + 3008*a^3*b^3*c^8*
d^6*e^3*f*z^4 + 2880*a^6*b^3*c^5*d*e^7*f^2*z^4 + 2880*a^5*b^3*c^6*d^2*e^7*f
*z^4 - 2240*a^7*b^4*c^3*d*e^4*f^5*z^4 - 2240*a^3*b^4*c^7*d^5*e^4*f*z^4 - 14
88*a^5*b^5*c^4*d*e^7*f^2*z^4 - 1488*a^4*b^5*c^5*d^2*e^7*f*z^4 + 1440*a^3*b^

9*c^2*d^4*e*f^5*z^4 + 1440*a^2*b^9*c^3*d^5*e*f^4*z^4 - 1328*a^6*b^5*c^3*d*e
^5*f^4*z^4 - 1328*a^3*b^5*c^6*d^4*e^5*f*z^4 - 1152*a^7*b^2*c^5*d*e^6*f^3*z^
4 - 1152*a^5*b^2*c^7*d^3*e^6*f*z^4 - 1120*a^6*b^4*c^4*d*e^6*f^3*z^4 - 1120*
a^4*b^4*c^6*d^3*e^6*f*z^4 + 912*a^6*b^6*c^2*d*e^4*f^5*z^4 + 912*a^2*b^6*c^6
*d^5*e^4*f*z^4 + 872*a^5*b^6*c^3*d*e^6*f^3*z^4 + 872*a^3*b^6*c^5*d^3*e^6*f*
z^4 + 768*a^8*b^2*c^4*d*e^4*f^5*z^4 + 768*a^4*b^2*c^8*d^5*e^4*f*z^4 - 672*a
^8*b^4*c^2*d*e^2*f^7*z^4 - 672*a^2*b^4*c^8*d^7*e^2*f*z^4 - 624*a^7*b^5*c^2*
d*e^3*f^6*z^4 - 624*a^2*b^5*c^7*d^6*e^3*f*z^4 + 480*a^5*b^8*c*d^2*e^2*f^6*z
^4 + 480*a*b^8*c^5*d^6*e^2*f^2*z^4 + 316*a^4*b^7*c^3*d*e^7*f^2*z^4 + 316*a^
3*b^7*c^4*d^2*e^7*f*z^4 - 204*a^4*b^8*c^2*d*e^6*f^3*z^4 - 204*a^2*b^8*c^4*d
^3*e^6*f*z^4 + 168*a^3*b^10*c*d^3*e^2*f^5*z^4 + 168*a*b^10*c^3*d^5*e^2*f^3*
z^4 + 156*a^2*b^11*c*d^3*e^3*f^4*z^4 + 156*a*b^11*c^2*d^4*e^3*f^3*z^4 + 128
*a^9*b^2*c^3*d*e^2*f^7*z^4 + 128*a^3*b^2*c^9*d^7*e^2*f*z^4 - 124*a^3*b^10*c
*d^2*e^4*f^4*z^4 - 124*a*b^10*c^3*d^4*e^4*f^2*z^4 + 100*a^4*b^9*c*d^2*e^3*f
^5*z^4 + 100*a*b^9*c^4*d^5*e^3*f^2*z^4 + 36*a^5*b^7*c^2*d*e^5*f^4*z^4 + 36*
a^2*b^7*c^5*d^4*e^5*f*z^4 - 24*a^3*b^9*c^2*d*e^7*f^2*z^4 - 24*a^2*b^11*c*d^
2*e^5*f^3*z^4 - 24*a^2*b^9*c^3*d^2*e^7*f*z^4 - 24*a*b^11*c^2*d^3*e^5*f^2*z^
4 - 9216*a^8*b*c^5*d^3*e*f^6*z^4 - 9216*a^5*b*c^8*d^6*e*f^3*z^4 - 5376*a^8*
b*c^5*d*e^5*f^4*z^4 - 5376*a^5*b*c^8*d^4*e^5*f*z^4 + 5120*a^9*b*c^4*d^2*e*f
^7*z^4 + 5120*a^7*b*c^6*d^4*e*f^5*z^4 + 5120*a^6*b*c^7*d^5*e*f^4*z^4 + 5120
*a^4*b*c^9*d^7*e*f^2*z^4 - 4352*a^9*b*c^4*d*e^3*f^6*z^4 - 4352*a^4*b*c^9*d^
6*e^3*f*z^4 - 1792*a^7*b*c^6*d*e^7*f^2*z^4 - 1792*a^6*b*c^7*d^2*e^7*f*z^4 -
1600*a^6*b^2*c^6*d*e^8*f*z^4 + 912*a^5*b^4*c^5*d*e^8*f*z^4 + 768*a^9*b^3*c
^2*d*e*f^8*z^4 + 768*a^2*b^3*c^9*d^8*e*f*z^4 - 720*a^4*b^9*c*d^3*e*f^6*z^4
- 720*a*b^9*c^4*d^6*e*f^3*z^4 - 656*a^6*b^7*c*d^2*e*f^7*z^4 - 656*a*b^7*c^6
*d^7*e*f^2*z^4 - 240*a^2*b^11*c*d^4*e*f^5*z^4 - 240*a*b^11*c^2*d^5*e*f^4*z^
4 + 216*a^7*b^6*c*d*e^2*f^7*z^4 + 216*a*b^6*c^7*d^7*e^2*f*z^4 - 204*a^4*b^6
*c^4*d*e^8*f*z^4 - 144*a^5*b^8*c*d*e^4*f^5*z^4 - 144*a*b^8*c^5*d^5*e^4*f*z^
4 - 84*a*b^12*c*d^4*e^2*f^4*z^4 + 36*a^4*b^9*c*d*e^5*f^4*z^4 + 36*a*b^9*c^4
*d^4*e^5*f*z^4 + 20*a^6*b^7*c*d*e^3*f^6*z^4 + 20*a*b^7*c^6*d^6*e^3*f*z^4 +
16*a^3*b^10*c*d*e^6*f^3*z^4 + 16*a^3*b^8*c^3*d*e^8*f*z^4 + 16*a*b^12*c*d^3*
e^4*f^3*z^4 + 16*a*b^10*c^3*d^3*e^6*f*z^4 + 48*b^11*c^3*d^6*e*f^3*z^4 + 48*
b^9*c^5*d^7*e*f^2*z^4 - 20*b^8*c^6*d^7*e^2*f*z^4 + 8*b^10*c^4*d^5*e^4*f*z^4
- 4*b^13*c*d^4*e^3*f^3*z^4 - 4*b^11*c^3*d^4*e^5*f*z^4 + 4*b^9*c^5*d^6*e^3*
f*z^4 + 3072*a^9*c^5*d*e^4*f^5*z^4 + 3072*a^5*c^9*d^5*e^4*f*z^4 + 2560*a^8*
c^6*d*e^6*f^3*z^4 + 2560*a^6*c^8*d^3*e^6*f*z^4 + 1536*a^10*c^4*d*e^2*f^7*z^
4 + 1536*a^4*c^10*d^7*e^2*f*z^4 + 48*a^5*b^9*d^2*e*f^7*z^4 + 48*a^3*b^11*d^
3*e*f^6*z^4 - 20*a^6*b^8*d*e^2*f^7*z^4 + 8*a^4*b^10*d*e^4*f^5*z^4 + 4*a^5*b
^9*d*e^3*f^6*z^4 - 4*a^3*b^11*d*e^5*f^4*z^4 - 4*a*b^13*d^3*e^3*f^4*z^4 + 76
8*a^9*b*c^4*e^5*f^5*z^4 + 768*a^8*b*c^5*e^7*f^3*z^4 + 256*a^10*b*c^3*e^3*f^
7*z^4 - 192*a^6*b^3*c^5*e^9*f*z^4 - 68*a^7*b^6*c*e^4*f^6*z^4 + 48*a^8*b^5*c
*e^3*f^7*z^4 + 48*a^5*b^5*c^4*e^9*f*z^4 + 36*a^6*b^7*c*e^5*f^5*z^4 - 12*a^9
*b^4*c*e^2*f^8*z^4 - 4*a^4*b^9*c*e^7*f^3*z^4 - 4*a^4*b^7*c^3*e^9*f*z^4 + 38
4*a^5*b^8*c*d^3*f^7*z^4 + 384*a*b^8*c^5*d^7*f^3*z^4 + 288*a^3*b^10*c*d^4*f^
6*z^4 + 288*a*b^10*c^3*d^6*f^4*z^4 + 224*a^7*b^6*c*d^2*f^8*z^4 + 224*a*b^6*

c^7*d^8*f^2*z^4 - 192*a^10*b^2*c^2*d*f^9*z^4 - 192*a^2*b^2*c^10*d^9*f*z^4 +
768*a^5*b*c^8*d^3*e^7*z^4 + 768*a^4*b*c^9*d^5*e^5*z^4 + 256*a^3*b*c^10*d^7
*e^3*z^4 - 192*a^5*b^3*c^6*d*e^9*z^4 - 68*a*b^6*c^7*d^6*e^4*z^4 + 48*a^4*b^
5*c^5*d*e^9*z^4 + 48*a*b^5*c^8*d^7*e^3*z^4 + 36*a*b^7*c^6*d^5*e^5*z^4 - 12*
a*b^4*c^9*d^8*e^2*z^4 - 4*a^3*b^7*c^4*d*e^9*z^4 - 4*a*b^9*c^4*d^3*e^7*z^4 +
16*b^13*c*d^5*e*f^4*z^4 + 16*b^7*c^7*d^8*e*f*z^4 + 768*a^7*c^7*d*e^8*f*z^4
+ 16*a^7*b^7*d*e*f^8*z^4 + 16*a*b^13*d^4*e*f^5*z^4 + 256*a^7*b*c^6*e^9*f*z
^4 + 80*a*b^12*c*d^5*f^5*z^4 + 48*a^9*b^4*c*d*f^9*z^4 + 48*a*b^4*c^9*d^9*f*
z^4 + 256*a^6*b*c^7*d*e^9*z^4 - 42*b^10*c^4*d^6*e^2*f^2*z^4 - 20*b^12*c^2*d
^5*e^2*f^3*z^4 + 6*b^12*c^2*d^4*e^4*f^2*z^4 + 4*b^11*c^3*d^5*e^3*f^2*z^4 -
24960*a^7*c^7*d^4*e^2*f^4*z^4 + 18944*a^8*c^6*d^3*e^2*f^5*z^4 + 18944*a^6*c
^8*d^5*e^2*f^3*z^4 + 14336*a^7*c^7*d^3*e^4*f^3*z^4 - 9984*a^8*c^6*d^2*e^4*f
^4*z^4 - 9984*a^6*c^8*d^4*e^4*f^2*z^4 - 7936*a^9*c^5*d^2*e^2*f^6*z^4 - 7936
*a^5*c^9*d^6*e^2*f^2*z^4 - 4352*a^7*c^7*d^2*e^6*f^2*z^4 - 42*a^4*b^10*d^2*e
^2*f^6*z^4 - 20*a^2*b^12*d^3*e^2*f^5*z^4 + 6*a^2*b^12*d^2*e^4*f^4*z^4 + 4*a
^3*b^11*d^2*e^3*f^5*z^4 - 480*a^8*b^2*c^4*e^6*f^4*z^4 + 440*a^7*b^4*c^3*e^6
*f^4*z^4 - 320*a^8*b^3*c^3*e^5*f^5*z^4 - 320*a^7*b^3*c^4*e^7*f^3*z^4 + 240*
a^8*b^4*c^2*e^4*f^6*z^4 + 240*a^6*b^4*c^4*e^8*f^2*z^4 - 192*a^9*b^3*c^2*e^3
*f^7*z^4 - 192*a^9*b^2*c^3*e^4*f^6*z^4 - 192*a^7*b^2*c^5*e^8*f^2*z^4 - 90*a
^6*b^6*c^2*e^6*f^4*z^4 - 68*a^5*b^6*c^3*e^8*f^2*z^4 + 48*a^10*b^2*c^2*e^2*f
^8*z^4 - 48*a^7*b^5*c^2*e^5*f^5*z^4 - 48*a^6*b^5*c^3*e^7*f^3*z^4 + 36*a^5*b
^7*c^2*e^7*f^3*z^4 + 6*a^4*b^8*c^2*e^8*f^2*z^4 - 33920*a^6*b^2*c^6*d^5*f^5*
z^4 + 27936*a^5*b^4*c^5*d^5*f^5*z^4 + 26112*a^7*b^2*c^5*d^4*f^6*z^4 + 26112
*a^5*b^2*c^7*d^6*f^4*z^4 - 20352*a^6*b^4*c^4*d^4*f^6*z^4 - 20352*a^4*b^4*c^
6*d^6*f^4*z^4 - 13080*a^4*b^6*c^4*d^5*f^5*z^4 - 11520*a^8*b^2*c^4*d^3*f^7*z
^4 - 11520*a^4*b^2*c^8*d^7*f^3*z^4 + 8736*a^5*b^6*c^3*d^4*f^6*z^4 + 8736*a^
3*b^6*c^5*d^6*f^4*z^4 + 7488*a^7*b^4*c^3*d^3*f^7*z^4 + 7488*a^3*b^4*c^7*d^7
*f^3*z^4 + 3840*a^3*b^8*c^3*d^5*f^5*z^4 + 2560*a^9*b^2*c^3*d^2*f^8*z^4 + 25
60*a^3*b^2*c^9*d^8*f^2*z^4 - 2416*a^6*b^6*c^2*d^3*f^7*z^4 - 2416*a^2*b^6*c^
6*d^7*f^3*z^4 - 2160*a^4*b^8*c^2*d^4*f^6*z^4 - 2160*a^2*b^8*c^4*d^6*f^4*z^4
- 1152*a^8*b^4*c^2*d^2*f^8*z^4 - 1152*a^2*b^4*c^8*d^8*f^2*z^4 - 720*a^2*b^
10*c^2*d^5*f^5*z^4 - 480*a^4*b^2*c^8*d^4*e^6*z^4 + 440*a^3*b^4*c^7*d^4*e^6*
z^4 - 320*a^4*b^3*c^7*d^3*e^7*z^4 - 320*a^3*b^3*c^8*d^5*e^5*z^4 + 240*a^4*b
^4*c^6*d^2*e^8*z^4 + 240*a^2*b^4*c^8*d^6*e^4*z^4 - 192*a^5*b^2*c^7*d^2*e^8*
z^4 - 192*a^3*b^2*c^9*d^6*e^4*z^4 - 192*a^2*b^3*c^9*d^7*e^3*z^4 - 90*a^2*b^
6*c^6*d^4*e^6*z^4 - 68*a^3*b^6*c^5*d^2*e^8*z^4 - 48*a^3*b^5*c^6*d^3*e^7*z^4
- 48*a^2*b^5*c^7*d^5*e^5*z^4 + 48*a^2*b^2*c^10*d^8*e^2*z^4 + 36*a^2*b^7*c^
5*d^3*e^7*z^4 + 6*a^2*b^8*c^4*d^2*e^8*z^4 - 4*b^6*c^8*d^9*f*z^4 + 256*a^11*
c^3*d*f^9*z^4 + 256*a^3*c^11*d^9*f*z^4 - 4*a^8*b^6*d*f^9*z^4 - 384*a^9*c^5*
e^6*f^4*z^4 - 256*a^10*c^4*e^4*f^6*z^4 - 256*a^8*c^6*e^8*f^2*z^4 - 64*a^11*
c^3*e^2*f^8*z^4 - 24*b^10*c^4*d^7*f^3*z^4 - 16*b^12*c^2*d^6*f^4*z^4 - 16*b^
8*c^6*d^8*f^2*z^4 + 17920*a^7*c^7*d^5*f^5*z^4 - 14336*a^8*c^6*d^4*f^6*z^4 -
14336*a^6*c^8*d^6*f^4*z^4 + 7168*a^9*c^5*d^3*f^7*z^4 + 7168*a^5*c^9*d^7*f^
3*z^4 - 2048*a^10*c^4*d^2*f^8*z^4 - 2048*a^4*c^10*d^8*f^2*z^4 + 6*b^8*c^6*d
^6*e^4*z^4 + 6*a^6*b^8*e^4*f^6*z^4 - 4*b^9*c^5*d^5*e^5*z^4 - 4*b^7*c^7*d^7*

e^3*z^4 - 4*a^7*b^7*e^3*f^7*z^4 - 4*a^5*b^9*e^5*f^5*z^4 - 384*a^5*c^9*d^4*e
^6*z^4 - 256*a^6*c^8*d^2*e^8*z^4 - 256*a^4*c^10*d^6*e^4*z^4 - 64*a^3*c^11*d
^8*e^2*z^4 - 24*a^4*b^10*d^3*f^7*z^4 - 16*a^6*b^8*d^2*f^8*z^4 - 16*a^2*b^12
*d^4*f^6*z^4 + 48*a^6*b^2*c^6*e^10*z^4 - 12*a^5*b^4*c^5*e^10*z^4 - 4*b^14*d
^5*f^5*z^4 - 64*a^7*c^7*e^10*z^4 + b^14*d^4*e^2*f^4*z^4 + b^10*c^4*d^4*e^6*
z^4 + b^6*c^8*d^8*e^2*z^4 + a^8*b^6*e^2*f^8*z^4 + a^4*b^10*e^6*f^4*z^4 + a^
4*b^6*c^4*e^10*z^4 - 4820*A*B*a^4*b*c^5*d^2*e^2*f^4*z^2 + 2976*A*B*a^3*b*c^
6*d^3*e^2*f^3*z^2 - 2328*A*B*a^3*b*c^6*d^2*e^4*f^2*z^2 + 1848*A*B*a^2*b^4*c
^4*d^3*e*f^4*z^2 - 1768*A*B*a^3*b^4*c^3*d^2*e*f^5*z^2 + 1528*A*B*a^4*b^2*c^
4*d^2*e*f^5*z^2 - 1136*A*B*a^3*b^2*c^5*d^3*e*f^4*z^2 - 974*A*B*a^4*b^3*c^3*
d*e^2*f^5*z^2 + 692*A*B*a^2*b*c^7*d^4*e^2*f^2*z^2 + 588*A*B*a*b^6*c^3*d^2*e
^3*f^3*z^2 - 580*A*B*a^3*b^3*c^4*d*e^4*f^3*z^2 + 488*A*B*a^3*b^4*c^3*d*e^3*
f^4*z^2 - 444*A*B*a^2*b^2*c^6*d^2*e^5*f*z^2 - 412*A*B*a*b^5*c^4*d^2*e^4*f^2
*z^2 + 366*A*B*a^2*b^6*c^2*d^2*e*f^5*z^2 - 352*A*B*a^2*b^2*c^6*d^4*e*f^3*z^
2 + 326*A*B*a^2*b^4*c^4*d*e^5*f^2*z^2 + 324*A*B*a*b^5*c^4*d^3*e^2*f^3*z^2 -
302*A*B*a*b^3*c^6*d^4*e^2*f^2*z^2 - 296*A*B*a*b^7*c^2*d^2*e^2*f^4*z^2 + 12
2*A*B*a^4*b^2*c^4*d*e^3*f^4*z^2 - 122*A*B*a^2*b^6*c^2*d*e^3*f^4*z^2 - 84*A*
B*a^3*b^2*c^5*d*e^5*f^2*z^2 + 72*A*B*a*b^4*c^5*d^3*e^3*f^2*z^2 - 64*A*B*a^2
*b^5*c^3*d*e^4*f^3*z^2 + 60*A*B*a^3*b^5*c^2*d*e^2*f^5*z^2 + 1312*A*B*a^5*b*
c^4*d*e^2*f^5*z^2 + 1040*A*B*a^4*b*c^5*d*e^4*f^3*z^2 - 500*A*B*a*b^6*c^3*d^
3*e*f^4*z^2 - 376*A*B*a*b^2*c^7*d^5*e*f^2*z^2 + 276*A*B*a^4*b^4*c^2*d*e*f^6
*z^2 - 262*A*B*a^2*b^3*c^5*d*e^6*f*z^2 + 238*A*B*a*b^2*c^7*d^4*e^3*f*z^2 +
232*A*B*a^5*b^2*c^3*d*e*f^6*z^2 - 176*A*B*a^2*b*c^7*d^3*e^4*f*z^2 - 120*A*B
*a*b^6*c^3*d*e^5*f^2*z^2 - 108*A*B*a*b^4*c^5*d^4*e*f^3*z^2 + 68*A*B*a*b^7*c
^2*d*e^4*f^3*z^2 + 68*A*B*a*b^4*c^5*d^2*e^5*f*z^2 + 46*A*B*a^2*b^7*c*d*e^2*
f^5*z^2 - 36*A*B*a*b^3*c^6*d^3*e^4*f*z^2 - 1932*A*B*a^2*b^3*c^5*d^3*e^2*f^3
*z^2 - 1818*A*B*a^2*b^4*c^4*d^2*e^3*f^3*z^2 + 1620*A*B*a^3*b^3*c^4*d^2*e^2*
f^4*z^2 + 1560*A*B*a^2*b^3*c^5*d^2*e^4*f^2*z^2 + 1244*A*B*a^3*b^2*c^5*d^2*e
^3*f^3*z^2 + 820*A*B*a^2*b^2*c^6*d^3*e^3*f^2*z^2 + 480*A*B*a^2*b^5*c^3*d^2*
e^2*f^4*z^2 + 352*A*B*a^3*b*c^6*d*e^6*f*z^2 - 108*A*B*a^3*b^6*c*d*e*f^6*z^2
+ 82*A*B*a*b^5*c^4*d*e^6*f*z^2 - 64*A*B*a*b*c^8*d^5*e^2*f*z^2 + 16*A*B*a*b
^8*c*d^2*e*f^5*z^2 - 4*A*B*a*b^8*c*d*e^3*f^4*z^2 + 16*B^2*a*b*c^8*d^6*e*f*z
^2 + 56*A*B*b^2*c^8*d^6*e*f*z^2 - 8*A*B*b^9*c*d*e^4*f^3*z^2 - 8*A*B*b^7*c^3
*d*e^6*f*z^2 - 800*A*B*a^6*c^4*d*e*f^6*z^2 + 10*A*B*a^2*b^8*d*e*f^6*z^2 - 6
*A*B*a*b^9*d*e^2*f^5*z^2 - 12*A*B*a^5*b^4*c*e*f^7*z^2 + 912*A*B*a^6*b*c^3*d
*f^7*z^2 + 192*A*B*a^4*b^5*c*d*f^7*z^2 + 192*A*B*a*b*c^8*d^6*f^2*z^2 - 20*A
*B*a*b^4*c^5*d*e^7*z^2 + 4*A*B*a*b*c^8*d^4*e^4*z^2 + 2144*B^2*a^4*b*c^5*d^3
*e*f^4*z^2 - 1120*B^2*a^3*b*c^6*d^4*e*f^3*z^2 - 688*B^2*a^5*b*c^4*d^2*e*f^5
*z^2 - 256*B^2*a^3*b*c^6*d^2*e^5*f*z^2 + 152*B^2*a*b^3*c^6*d^5*e*f^2*z^2 +
120*B^2*a^5*b^3*c^2*d*e*f^6*z^2 - 116*B^2*a^5*b*c^4*d*e^3*f^4*z^2 + 110*B^2
*a*b^7*c^2*d^3*e*f^4*z^2 - 80*B^2*a^2*b*c^7*d^5*e*f^2*z^2 - 72*B^2*a*b^5*c^
4*d^4*e*f^3*z^2 - 48*B^2*a^4*b*c^5*d*e^5*f^2*z^2 - 46*B^2*a*b^3*c^6*d^4*e^3
*f*z^2 - 44*B^2*a*b^4*c^5*d^3*e^4*f*z^2 - 34*B^2*a*b^5*c^4*d^2*e^5*f*z^2 +
20*B^2*a^2*b*c^7*d^4*e^3*f*z^2 - 10*B^2*a^3*b^6*c*d*e^2*f^5*z^2 - 10*B^2*a^
2*b^7*c*d^2*e*f^5*z^2 - 10*B^2*a*b^2*c^7*d^5*e^2*f*z^2 - 7*B^2*a^2*b^4*c^4*

d*e^6*f*z^2 - 6*B^2*a^3*b^2*c^5*d*e^6*f*z^2 + 4*B^2*a*b^8*c*d^2*e^2*f^4*z^2
 - 2*B^2*a^2*b^7*c*d*e^3*f^4*z^2 + 3196*A^2*a^4*b*c^5*d*e^3*f^4*z^2 - 3184*
A^2*a^4*b*c^5*d^2*e*f^5*z^2 + 1568*A^2*a^3*b*c^6*d^3*e*f^4*z^2 + 1504*A^2*a
^3*b*c^6*d*e^5*f^2*z^2 - 656*A^2*a^4*b^3*c^3*d*e*f^6*z^2 - 400*A^2*a*b^6*c^
3*d*e^4*f^3*z^2 + 314*A^2*a*b^5*c^4*d*e^5*f^2*z^2 - 264*A^2*a^3*b^5*c^2*d*e
*f^6*z^2 + 240*A^2*a^2*b^2*c^6*d*e^6*f*z^2 - 224*A^2*a^2*b*c^7*d^4*e*f^3*z^
2 + 216*A^2*a*b^5*c^4*d^3*e*f^4*z^2 - 192*A^2*a^2*b*c^7*d^2*e^5*f*z^2 + 178
*A^2*a*b^7*c^2*d*e^3*f^4*z^2 - 154*A^2*a*b^7*c^2*d^2*e*f^5*z^2 + 128*A^2*a*
b^3*c^6*d^4*e*f^3*z^2 + 106*A^2*a*b^3*c^6*d^2*e^5*f*z^2 - 12*A^2*a*b^2*c^7*
d^3*e^4*f*z^2 - 58*A*B*b^8*c^2*d^2*e^3*f^3*z^2 + 40*A*B*b^7*c^3*d^2*e^4*f^2
*z^2 - 28*A*B*b^7*c^3*d^3*e^2*f^3*z^2 - 24*A*B*b^5*c^5*d^4*e^2*f^2*z^2 - 20
*A*B*b^6*c^4*d^3*e^3*f^2*z^2 + 2768*A*B*a^4*c^6*d^2*e^3*f^3*z^2 - 1712*A*B*
a^3*c^7*d^3*e^3*f^2*z^2 - 156*A*B*a^4*b^2*c^4*e^5*f^3*z^2 + 146*A*B*a^4*b^3
*c^3*e^4*f^4*z^2 - 106*A*B*a^5*b^2*c^3*e^3*f^5*z^2 + 90*A*B*a^5*b^3*c^2*e^2
*f^6*z^2 + 38*A*B*a^3*b^3*c^4*e^6*f^2*z^2 - 36*A*B*a^3*b^5*c^2*e^4*f^4*z^2
+ 16*A*B*a^3*b^4*c^3*e^5*f^3*z^2 - 9*A*B*a^4*b^4*c^2*e^3*f^5*z^2 - 8*A*B*a^
2*b^5*c^3*e^6*f^2*z^2 + 2*A*B*a^2*b^6*c^2*e^5*f^3*z^2 + 920*A*B*a^4*b^3*c^3
*d^2*f^6*z^2 - 480*A*B*a^2*b^5*c^3*d^3*f^5*z^2 - 336*A*B*a^2*b^3*c^5*d^4*f^
4*z^2 - 272*A*B*a^3*b^3*c^4*d^3*f^5*z^2 + 240*A*B*a^3*b^5*c^2*d^2*f^6*z^2 -
 32*A*B*a*c^9*d^6*e*f*z^2 - 792*B^2*a^2*b^3*c^5*d^3*e^3*f^2*z^2 + 714*B^2*a
^2*b^4*c^4*d^3*e^2*f^3*z^2 - 572*B^2*a^3*b^2*c^5*d^3*e^2*f^3*z^2 - 475*B^2*
a^2*b^2*c^6*d^4*e^2*f^2*z^2 + 265*B^2*a^4*b^2*c^4*d^2*e^2*f^4*z^2 + 260*B^2
*a^3*b^3*c^4*d^2*e^3*f^3*z^2 - 212*B^2*a^3*b^4*c^3*d^2*e^2*f^4*z^2 + 180*B^
2*a^3*b^2*c^5*d^2*e^4*f^2*z^2 - 158*B^2*a^2*b^4*c^4*d^2*e^4*f^2*z^2 + 47*B^
2*a^2*b^6*c^2*d^2*e^2*f^4*z^2 + 16*B^2*a^2*b^5*c^3*d^2*e^3*f^3*z^2 + 2752*A
^2*a^3*b^2*c^5*d^2*e^2*f^4*z^2 - 2148*A^2*a^2*b^4*c^4*d^2*e^2*f^4*z^2 + 206
4*A^2*a^2*b^3*c^5*d^2*e^3*f^3*z^2 - 424*A^2*a^2*b^2*c^6*d^3*e^2*f^3*z^2 - 1
98*A^2*a^2*b^2*c^6*d^2*e^4*f^2*z^2 - 272*B^2*a^6*b*c^3*d*e*f^6*z^2 - 24*B^2
*a^4*b^5*c*d*e*f^6*z^2 + 1808*A^2*a^5*b*c^4*d*e*f^6*z^2 - 244*A^2*a*b*c^8*d
^4*e^3*f*z^2 + 208*A^2*a*b*c^8*d^5*e*f^2*z^2 + 134*A^2*a^2*b^7*c*d*e*f^6*z^
2 - 76*A^2*a*b^4*c^5*d*e^6*f*z^2 + 4*A^2*a*b^8*c*d*e^2*f^5*z^2 + 148*A*B*b^
4*c^6*d^5*e*f^2*z^2 + 65*A*B*b^6*c^4*d^4*e*f^3*z^2 + 46*A*B*b^8*c^2*d^3*e*f
^4*z^2 - 38*A*B*b^3*c^7*d^5*e^2*f*z^2 + 34*A*B*b^9*c*d^2*e^2*f^4*z^2 - 29*A
*B*b^4*c^6*d^4*e^3*f*z^2 + 20*A*B*b^5*c^5*d^3*e^4*f*z^2 + 12*A*B*b^8*c^2*d*
e^5*f^2*z^2 - 7*A*B*b^6*c^4*d^2*e^5*f*z^2 - 2880*A*B*a^4*c^6*d^3*e*f^4*z^2
+ 2784*A*B*a^5*c^5*d^2*e*f^5*z^2 - 1112*A*B*a^5*c^5*d*e^3*f^4*z^2 + 896*A*B
*a^3*c^7*d^4*e*f^3*z^2 + 848*A*B*a^3*c^7*d^2*e^5*f*z^2 - 560*A*B*a^4*c^6*d*
e^5*f^2*z^2 + 96*A*B*a^2*c^8*d^5*e*f^2*z^2 - 88*A*B*a^2*c^8*d^4*e^3*f*z^2 -
 100*A*B*a^6*b*c^3*e^2*f^6*z^2 - 76*A*B*a^5*b*c^4*e^4*f^4*z^2 + 48*A*B*a^6*
b^2*c^2*e*f^7*z^2 - 42*A*B*a^3*b^2*c^5*e^7*f*z^2 + 36*A*B*a^4*b*c^5*e^6*f^2
*z^2 - 24*A*B*a^4*b^5*c*e^2*f^6*z^2 + 10*A*B*a^3*b^6*c*e^3*f^5*z^2 + 7*A*B*
a^2*b^4*c^4*e^7*f*z^2 + 2*A*B*a^2*b^7*c*e^4*f^4*z^2 - 2496*A*B*a^5*b*c^4*d^
2*f^6*z^2 + 1872*A*B*a^4*b*c^5*d^3*f^5*z^2 - 744*A*B*a^5*b^3*c^2*d*f^7*z^2
- 720*A*B*a^2*b*c^7*d^5*f^3*z^2 + 504*A*B*a*b^3*c^6*d^5*f^3*z^2 + 256*A*B*a
^3*b*c^6*d^4*f^4*z^2 + 168*A*B*a*b^7*c^2*d^3*f^5*z^2 - 144*A*B*a^2*b^7*c*d^

2*f^6*z^2 + 144*A*B*a*b^5*c^4*d^4*f^4*z^2 + 66*A*B*a^2*b^2*c^6*d*e^7*z^2 -
36*A*B*a*b^2*c^7*d^3*e^5*z^2 + 20*A*B*a*b^3*c^6*d^2*e^6*z^2 + 12*A*B*a^2*b*
c^7*d^2*e^6*z^2 + 1208*B^2*a^3*b*c^6*d^3*e^3*f^2*z^2 - 848*B^2*a^3*b^3*c^4*
d^3*e*f^4*z^2 + 672*B^2*a^2*b^3*c^5*d^4*e*f^3*z^2 - 632*B^2*a^4*b*c^5*d^2*e
^3*f^3*z^2 + 432*B^2*a^4*b^3*c^3*d^2*e*f^5*z^2 + 276*B^2*a^2*b^2*c^6*d^3*e^
4*f*z^2 - 196*B^2*a*b^6*c^3*d^3*e^2*f^3*z^2 - 168*B^2*a^2*b^5*c^3*d^3*e*f^4
*z^2 + 154*B^2*a^2*b^3*c^5*d^2*e^5*f*z^2 + 148*B^2*a*b^5*c^4*d^3*e^3*f^2*z^
2 + 96*B^2*a*b^4*c^5*d^4*e^2*f^2*z^2 - 72*B^2*a^3*b^5*c^2*d^2*e*f^5*z^2 + 7
0*B^2*a^5*b^2*c^3*d*e^2*f^5*z^2 - 60*B^2*a^4*b^3*c^3*d*e^3*f^4*z^2 + 52*B^2
*a*b^6*c^3*d^2*e^4*f^2*z^2 + 36*B^2*a^4*b^2*c^4*d*e^4*f^3*z^2 - 32*B^2*a*b^
7*c^2*d^2*e^3*f^3*z^2 + 24*B^2*a^3*b^5*c^2*d*e^3*f^4*z^2 + 15*B^2*a^4*b^4*c
^2*d*e^2*f^5*z^2 - 8*B^2*a^3*b^4*c^3*d*e^4*f^3*z^2 + 8*B^2*a^2*b^5*c^3*d*e^
5*f^2*z^2 - 2*B^2*a^3*b^3*c^4*d*e^5*f^2*z^2 - 2*B^2*a^2*b^6*c^2*d*e^4*f^3*z
^2 - 3176*A^2*a^3*b*c^6*d^2*e^3*f^3*z^2 - 2252*A^2*a^4*b^2*c^4*d*e^2*f^5*z^
2 + 1952*A^2*a^3*b^4*c^3*d*e^2*f^5*z^2 - 1496*A^2*a^3*b^3*c^4*d*e^3*f^4*z^2
+ 1378*A^2*a^2*b^4*c^4*d*e^4*f^3*z^2 + 1184*A^2*a^3*b^3*c^4*d^2*e*f^5*z^2
- 1166*A^2*a^2*b^3*c^5*d*e^5*f^2*z^2 - 1164*A^2*a^3*b^2*c^5*d*e^4*f^3*z^2 -
1152*A^2*a^2*b^3*c^5*d^3*e*f^4*z^2 + 578*A^2*a*b^6*c^3*d^2*e^2*f^4*z^2 - 5
48*A^2*a*b^5*c^4*d^2*e^3*f^3*z^2 + 440*A^2*a*b^2*c^7*d^4*e^2*f^2*z^2 - 412*
A^2*a^2*b^6*c^2*d*e^2*f^5*z^2 - 360*A^2*a*b^3*c^6*d^3*e^3*f^2*z^2 + 312*A^2
*a*b^4*c^5*d^3*e^2*f^3*z^2 + 248*A^2*a^2*b*c^7*d^3*e^3*f^2*z^2 - 224*A^2*a^
2*b^5*c^3*d*e^3*f^4*z^2 + 216*A^2*a^2*b^5*c^3*d^2*e*f^5*z^2 + 52*A^2*a*b^4*
c^5*d^2*e^4*f^2*z^2 - 16*B^2*b^3*c^7*d^6*e*f*z^2 - 14*B^2*b^9*c*d^3*e*f^4*z
^2 + 32*B^2*a^4*c^6*d*e^6*f*z^2 - 20*A^2*b^9*c*d*e^3*f^4*z^2 + 18*A^2*b^9*c
*d^2*e*f^5*z^2 + 8*A^2*b^6*c^4*d*e^6*f*z^2 - 360*A^2*a^3*c^7*d*e^6*f*z^2 +
136*A^2*a*c^9*d^5*e^2*f*z^2 + 2*B^2*a^3*b^7*d*e*f^6*z^2 + 2*B^2*a*b^9*d^2*e
*f^5*z^2 + 12*B^2*a^4*b*c^5*e^7*f*z^2 - 204*A^2*a^3*b*c^6*e^7*f*z^2 - 128*A
^2*a^6*b*c^3*e*f^7*z^2 - 48*A^2*a*b^5*c^4*e^7*f*z^2 - 36*B^2*a^5*b^4*c*d*f^
7*z^2 - 24*A^2*a^4*b^5*c*e*f^7*z^2 - 16*B^2*a*b^8*c*d^3*f^5*z^2 - 164*A^2*a
^3*b^6*c*d*f^7*z^2 - 16*A^2*a*b^8*c*d^2*f^6*z^2 + 4*B^2*a^3*b*c^6*d*e^7*z^2
- 4*B^2*a*b*c^8*d^5*e^3*z^2 + 48*A^2*a*b*c^8*d^3*e^5*z^2 + 36*A^2*a^2*b*c^
7*d*e^7*z^2 - 6*A^2*a*b^3*c^6*d*e^7*z^2 + 136*A*B*a^6*c^4*e^3*f^5*z^2 - 96*
A*B*b^5*c^5*d^5*f^3*z^2 + 80*A*B*a^5*c^5*e^5*f^3*z^2 - 72*A*B*b^3*c^7*d^6*f
^2*z^2 - 24*A*B*b^7*c^3*d^4*f^4*z^2 + 14*A*B*b^3*c^7*d^4*e^4*z^2 - 14*A*B*b
^2*c^8*d^5*e^3*z^2 - 2*A*B*b^5*c^5*d^2*e^6*z^2 - 2*A*B*b^4*c^6*d^3*e^5*z^2
+ 2*A*B*a^3*b^7*e^2*f^6*z^2 - A*B*a^2*b^8*e^3*f^5*z^2 + 16*A*B*a^2*c^8*d^3*
e^5*z^2 - 2*A*B*a^2*b^3*c^5*e^8*z^2 + 22*B^2*b^8*c^2*d^3*e^2*f^3*z^2 - 12*B
^2*b^7*c^3*d^3*e^3*f^2*z^2 + 12*B^2*b^6*c^4*d^4*e^2*f^2*z^2 - 6*B^2*b^8*c^2
*d^2*e^4*f^2*z^2 - 864*B^2*a^4*c^6*d^3*e^2*f^3*z^2 + 496*B^2*a^3*c^7*d^4*e^
2*f^2*z^2 + 224*B^2*a^5*c^5*d^2*e^2*f^4*z^2 + 136*B^2*a^4*c^6*d^2*e^4*f^2*z
^2 - 53*A^2*b^8*c^2*d^2*e^2*f^4*z^2 + 52*A^2*b^7*c^3*d^2*e^3*f^3*z^2 + 52*A
^2*b^5*c^5*d^3*e^3*f^2*z^2 - 36*A^2*b^6*c^4*d^3*e^2*f^3*z^2 - 12*A^2*b^4*c^
6*d^4*e^2*f^2*z^2 - 9*A^2*b^6*c^4*d^2*e^4*f^2*z^2 + 836*A^2*a^4*c^6*d^2*e^2
*f^4*z^2 - 668*A^2*a^2*c^8*d^4*e^2*f^2*z^2 + 656*A^2*a^3*c^7*d^2*e^4*f^2*z^
2 + 368*A^2*a^3*c^7*d^3*e^2*f^3*z^2 - 45*B^2*a^6*b^2*c^2*e^2*f^6*z^2 - 18*B

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^2*a^5*b^2*c^3*e^4*f^4*z^2 - 9*B^2*a^4*b^2*c^4*e^6*f^2*z^2 - 6*B^2*a^5*b^3*
c^2*e^3*f^5*z^2 + 3*B^2*a^4*b^4*c^2*e^4*f^4*z^2 - 2*B^2*a^4*b^3*c^3*e^5*f^3
*z^2 - 580*B^2*a^4*b^2*c^4*d^3*f^5*z^2 + 536*B^2*a^3*b^4*c^3*d^3*f^5*z^2 +
471*A^2*a^4*b^2*c^4*e^4*f^4*z^2 - 436*A^2*a^3*b^4*c^3*e^4*f^4*z^2 - 348*B^2
*a^4*b^4*c^2*d^2*f^6*z^2 + 316*B^2*a^2*b^2*c^6*d^5*f^3*z^2 + 310*A^2*a^3*b^
3*c^4*e^5*f^3*z^2 + 232*A^2*a^5*b^2*c^3*e^2*f^6*z^2 - 229*A^2*a^2*b^4*c^4*e
^6*f^2*z^2 - 216*A^2*a^4*b^4*c^2*e^2*f^6*z^2 + 204*A^2*a^4*b^3*c^3*e^3*f^5*
z^2 + 200*B^2*a^5*b^2*c^3*d^2*f^6*z^2 + 150*A^2*a^3*b^2*c^5*e^6*f^2*z^2 - 1
20*B^2*a^2*b^4*c^4*d^4*f^4*z^2 + 91*A^2*a^2*b^6*c^2*e^4*f^4*z^2 + 72*A^2*a^
3*b^5*c^2*e^3*f^5*z^2 - 66*B^2*a^2*b^6*c^2*d^3*f^5*z^2 + 44*A^2*a^2*b^5*c^3
*e^5*f^3*z^2 - 16*B^2*a^3*b^2*c^5*d^4*f^4*z^2 + 1952*A^2*a^4*b^2*c^4*d^2*f^
6*z^2 - 1792*A^2*a^3*b^2*c^5*d^3*f^5*z^2 - 1272*A^2*a^3*b^4*c^3*d^2*f^6*z^2
 + 976*A^2*a^2*b^2*c^6*d^4*f^4*z^2 + 960*A^2*a^2*b^4*c^4*d^3*f^5*z^2 + 282*
A^2*a^2*b^6*c^2*d^2*f^6*z^2 - 45*B^2*a^2*b^2*c^6*d^2*e^6*z^2 - 48*A^2*b*c^9
*d^6*e*f*z^2 - 14*A^2*a*b^9*d*e*f^6*z^2 - 7*A*B*b^10*d^2*e*f^5*z^2 + 2*A*B*
b^10*d*e^3*f^4*z^2 - 64*A*B*a^7*c^3*e*f^7*z^2 - 16*A*B*b^9*c*d^3*f^5*z^2 +
8*A*B*a^4*c^6*e^7*f*z^2 + 4*A*B*b*c^9*d^6*e^2*z^2 + 2*A*B*b^6*c^4*d*e^7*z^2
 - 120*A*B*a^3*c^7*d*e^7*z^2 - 16*A*B*a^3*b^7*d*f^7*z^2 + 16*A*B*a*b^9*d^2*
f^6*z^2 + 8*A*B*a*c^9*d^5*e^3*z^2 + 12*A*B*a^3*b*c^6*e^8*z^2 - 48*B^2*b^5*c
^5*d^5*e*f^2*z^2 + 15*B^2*b^4*c^6*d^5*e^2*f*z^2 - 14*B^2*b^7*c^3*d^4*e*f^3*
z^2 + 4*B^2*b^9*c*d^2*e^3*f^3*z^2 + 4*B^2*b^7*c^3*d^2*e^5*f*z^2 + 4*B^2*b^5
*c^5*d^4*e^3*f*z^2 - B^2*b^6*c^4*d^3*e^4*f*z^2 - 336*B^2*a^3*c^7*d^3*e^4*f*
z^2 + 112*B^2*a^5*c^5*d*e^4*f^3*z^2 - 112*A^2*b^3*c^7*d^5*e*f^2*z^2 + 80*B^
2*a^6*c^4*d*e^2*f^5*z^2 - 48*A^2*b^5*c^5*d^4*e*f^3*z^2 + 36*A^2*b^8*c^2*d*e
^4*f^3*z^2 + 36*A^2*b^3*c^7*d^4*e^3*f*z^2 - 28*A^2*b^7*c^3*d*e^5*f^2*z^2 +
20*A^2*b^2*c^8*d^5*e^2*f*z^2 + 16*B^2*a^2*c^8*d^5*e^2*f*z^2 - 14*A^2*b^7*c^
3*d^3*e*f^4*z^2 - 14*A^2*b^4*c^6*d^3*e^4*f*z^2 - 10*A^2*b^5*c^5*d^2*e^5*f*z
^2 - 1008*A^2*a^4*c^6*d*e^4*f^3*z^2 - 760*A^2*a^5*c^5*d*e^2*f^5*z^2 + 272*A
^2*a^2*c^8*d^3*e^4*f*z^2 + 48*B^2*a^5*b*c^4*e^5*f^3*z^2 + 36*B^2*a^6*b*c^3*
e^3*f^5*z^2 + 12*B^2*a^5*b^4*c*e^2*f^6*z^2 - 624*A^2*a^4*b*c^5*e^5*f^3*z^2
- 548*A^2*a^5*b*c^4*e^3*f^5*z^2 + 182*A^2*a^2*b^3*c^5*e^7*f*z^2 - 180*B^2*a
*b^4*c^5*d^5*f^3*z^2 + 132*B^2*a^6*b^2*c^2*d*f^7*z^2 + 108*B^2*a^3*b^6*c*d^
2*f^6*z^2 + 96*A^2*a^5*b^3*c^2*e*f^7*z^2 + 68*A^2*a*b^6*c^3*e^6*f^2*z^2 + 5
8*A^2*a^3*b^6*c*e^2*f^6*z^2 - 56*B^2*a*b^2*c^7*d^6*f^2*z^2 - 38*A^2*a^2*b^7
*c*e^3*f^5*z^2 - 36*A^2*a*b^7*c^2*e^5*f^3*z^2 + 20*B^2*a*b^6*c^3*d^4*f^4*z^
2 - 736*A^2*a^5*b^2*c^3*d*f^7*z^2 + 624*A^2*a^4*b^4*c^2*d*f^7*z^2 - 416*A^2
*a*b^2*c^7*d^5*f^3*z^2 - 276*A^2*a*b^4*c^5*d^4*f^4*z^2 - 196*A^2*a*b^6*c^3*
d^3*f^5*z^2 + 8*B^2*a*b^4*c^5*d^2*e^6*z^2 + 6*B^2*a*b^2*c^7*d^4*e^4*z^2 + 2
*B^2*a^2*b^3*c^5*d*e^7*z^2 + 2*B^2*a*b^3*c^6*d^3*e^5*z^2 - 18*A^2*a*b^2*c^7
*d^2*e^6*z^2 - 16*A*B*b*c^9*d^7*f*z^2 - B^2*b^10*d^2*e^2*f^4*z^2 + 48*B^2*a
^7*c^3*e^2*f^6*z^2 - 36*B^2*a^6*c^4*e^4*f^4*z^2 + 31*B^2*b^6*c^4*d^5*f^3*z^
2 - 24*B^2*a^5*c^5*e^6*f^2*z^2 + 20*B^2*b^4*c^6*d^6*f^2*z^2 - 6*A^2*b^8*c^2
*e^6*f^2*z^2 + 2*B^2*b^8*c^2*d^4*f^4*z^2 - 768*B^2*a^5*c^5*d^3*f^5*z^2 + 51
2*B^2*a^6*c^4*d^2*f^6*z^2 + 512*B^2*a^4*c^6*d^4*f^4*z^2 + 232*A^2*a^5*c^5*e
^4*f^4*z^2 + 188*A^2*a^4*c^6*e^6*f^2*z^2 - 128*B^2*a^3*c^7*d^5*f^3*z^2 + 92
```

*A^2*a^6*c^4*e^2*f^6*z^2 + 80*A^2*b^4*c^6*d^5*f^3*z^2 + 64*A^2*b^2*c^8*d^6*
f^2*z^2 + 31*A^2*b^6*c^4*d^4*f^4*z^2 + 14*A^2*b^8*c^2*d^3*f^5*z^2 - 5*B^2*b
^4*c^6*d^4*e^4*z^2 + 4*B^2*b^3*c^7*d^5*e^3*z^2 + 2*B^2*b^5*c^5*d^3*e^5*z^2
- B^2*b^6*c^4*d^2*e^6*z^2 - B^2*b^2*c^8*d^6*e^2*z^2 - B^2*a^4*b^6*e^2*f^6*z
^2 - 1152*A^2*a^3*c^7*d^4*f^4*z^2 + 1008*A^2*a^4*c^6*d^3*f^5*z^2 + 624*A^2*
a^2*c^8*d^5*f^3*z^2 - 288*A^2*a^5*c^5*d^2*f^6*z^2 + 56*B^2*a^3*c^7*d^2*e^6*
z^2 - 10*B^2*a^2*b^8*d^2*f^6*z^2 - 9*A^2*b^2*c^8*d^4*e^4*z^2 - 5*A^2*a^2*b^
8*e^2*f^6*z^2 - 4*B^2*a^2*c^8*d^4*e^4*z^2 + 3*A^2*b^4*c^6*d^2*e^6*z^2 - 2*A
^2*b^3*c^7*d^3*e^5*z^2 - 36*A^2*a^2*c^8*d^2*e^6*z^2 - 48*A^2*a^6*b^2*c^2*f^
8*z^2 - 45*A^2*a^2*b^2*c^6*e^8*z^2 + 4*A^2*b^10*d*e^2*f^5*z^2 + 4*B^2*b^2*c
^8*d^7*f*z^2 + 4*A^2*b^9*c*e^5*f^3*z^2 + 4*A^2*b^7*c^3*e^7*f*z^2 - 128*B^2*
a^7*c^3*d*f^7*z^2 - 160*A^2*a*c^9*d^6*f^2*z^2 - 112*A^2*a^6*c^4*d*f^7*z^2 +
12*A^2*b*c^9*d^5*e^3*z^2 + 4*A^2*a*b^9*e^3*f^5*z^2 + 3*B^2*a^4*b^6*d*f^7*z
^2 + 2*A^2*a^3*b^7*e*f^7*z^2 - 24*A^2*a*c^9*d^4*e^4*z^2 + 14*A^2*a^2*b^8*d*
f^7*z^2 + 12*A^2*a^5*b^4*c*f^8*z^2 + 12*A^2*a*b^4*c^5*e^8*z^2 + A*B*a^4*b^6
*e*f^7*z^2 + B^2*a^2*b^8*d*e^2*f^5*z^2 + 16*A^2*c^10*d^7*f*z^2 + 3*B^2*b^10
*d^3*f^5*z^2 - A^2*b^10*e^4*f^4*z^2 - 4*A^2*c^10*d^6*e^2*z^2 - A^2*b^10*d^2
*f^6*z^2 + 64*A^2*a^7*c^3*f^8*z^2 - 4*B^2*a^4*c^6*e^8*z^2 - A^2*b^6*c^4*e^8
*z^2 + 48*A^2*a^3*c^7*e^8*z^2 - A^2*a^4*b^6*f^8*z^2 + 720*A^2*B*a*b^2*c^5*d
^2*e^2*f^3*z - 600*A^2*B*a^2*b^2*c^4*d*e^2*f^4*z + 576*A*B^2*a^2*b^2*c^4*d^
2*e*f^4*z + 348*A*B^2*a*b^2*c^5*d^2*e^3*f^2*z - 336*A*B^2*a^2*b*c^5*d^2*e^2
*f^3*z - 260*A*B^2*a*b^3*c^4*d^2*e^2*f^3*z - 240*A*B^2*a^2*b^2*c^4*d*e^3*f^
3*z + 196*A*B^2*a^2*b^3*c^3*d*e^2*f^4*z + 172*A^2*B*a*b*c^6*d*e^5*f*z + 20*
A*B^2*a*b^6*c*d*e*f^5*z - 912*A^2*B*a^2*b*c^5*d^2*e*f^4*z - 644*A^2*B*a*b*c
^6*d^2*e^3*f^2*z - 432*A*B^2*a*b^2*c^5*d^3*e*f^3*z + 372*A^2*B*a^2*b*c^5*d*
e^3*f^3*z - 330*A^2*B*a*b^2*c^5*d*e^4*f^2*z + 312*A*B^2*a*b*c^6*d^3*e^2*f^2
*z - 208*A*B^2*a^3*b^2*c^3*d*e*f^5*z + 192*A^2*B*a^2*b^3*c^3*d*e*f^5*z + 17
2*A^2*B*a*b^3*c^4*d*e^3*f^3*z + 108*A*B^2*a^2*b*c^5*d*e^4*f^2*z + 104*A*B^2
*a^3*b*c^4*d*e^2*f^4*z - 80*A^2*B*a*b^3*c^4*d^2*e*f^4*z + 68*A^2*B*a*b^4*c^
3*d*e^2*f^4*z - 60*A*B^2*a*b^5*c^2*d*e^2*f^4*z + 58*A*B^2*a*b^3*c^4*d*e^4*f
^2*z - 36*A*B^2*a*b^4*c^3*d^2*e*f^4*z - 24*A*B^2*a^2*b^4*c^2*d*e*f^5*z + 24
*A*B^2*a*b^4*c^3*d*e^3*f^3*z + 592*A^2*B*a*b*c^6*d^3*e*f^3*z + 240*A^2*B*a^
3*b*c^4*d*e*f^5*z - 132*A*B^2*a*b*c^6*d^2*e^4*f*z - 60*A*B^2*a*b^2*c^5*d*e^
5*f*z - 48*A^2*B*a*b^5*c^2*d*e*f^5*z + 20*B^3*a*b*c^6*d^3*e^3*f*z + 16*B^3*
a^4*b*c^3*d*e*f^5*z - 16*B^3*a*b*c^6*d^4*e*f^2*z + 12*B^3*a^2*b*c^5*d*e^5*f
*z + 320*A^3*a*b*c^6*d*e^4*f^2*z + 40*A^3*a*b^4*c^3*d*e*f^5*z - 48*A^2*B*b*
c^7*d^4*e*f^2*z - 44*A^2*B*b^3*c^5*d*e^5*f*z - 20*A*B^2*b*c^7*d^4*e^2*f*z +
14*A*B^2*b^4*c^4*d*e^5*f*z + 12*A^2*B*b*c^7*d^3*e^3*f*z + 4*A*B^2*b^7*c*d*
e^2*f^4*z + 160*A*B^2*a^4*c^4*d*e*f^5*z + 152*A^2*B*a*c^7*d^2*e^4*f*z - 40*
A*B^2*a*c^7*d^3*e^3*f*z + 32*A*B^2*a*c^7*d^4*e*f^2*z - 16*A*B^2*a^2*c^6*d*e
^5*f*z + 128*A^2*B*a^4*b*c^3*e*f^6*z + 42*A^2*B*a*b^2*c^5*e^6*f*z + 24*A^2*
B*a^2*b^5*c*e*f^6*z - 12*A*B^2*a^3*b^4*c*e*f^6*z - 12*A*B^2*a^2*b*c^5*e^6*f
*z - 10*A^2*B*a*b^6*c*e^2*f^5*z - 160*A*B^2*a*b*c^6*d^4*f^3*z + 112*A*B^2*a
^4*b*c^3*d*f^6*z - 24*A*B^2*a^2*b^5*c*d*f^6*z - 84*B^3*a*b^2*c^5*d^3*e^2*f^
2*z - 80*B^3*a^2*b^3*c^3*d^2*e*f^4*z - 60*B^3*a^2*b*c^5*d^2*e^3*f^2*z - 20*

B^3*a^3*b^2*c^3*d*e^2*f^4*z - 20*B^3*a*b^3*c^4*d^2*e^3*f^2*z - 9*B^3*a^2*b^
2*c^4*d*e^4*f^2*z - 8*B^3*a*b^4*c^3*d^2*e^2*f^3*z + 6*B^3*a^2*b^4*c^2*d*e^2
*f^4*z - 4*B^3*a^2*b^3*c^3*d*e^3*f^3*z - 216*A^2*B*b^4*c^4*d^2*e^2*f^3*z +
196*A^2*B*b^3*c^5*d^2*e^3*f^2*z - 108*A*B^2*b^3*c^5*d^3*e^2*f^2*z - 94*A*B^
2*b^4*c^4*d^2*e^3*f^2*z + 88*A^2*B*b^2*c^6*d^3*e^2*f^2*z + 80*A*B^2*b^5*c^3
*d^2*e^2*f^3*z + 360*A^2*B*a^2*c^6*d^2*e^2*f^3*z + 8*A*B^2*a^2*c^6*d^2*e^3*
f^2*z + 153*A^2*B*a^2*b^2*c^4*e^4*f^3*z - 144*A^2*B*a^2*b^3*c^3*e^3*f^4*z +
80*A^2*B*a^3*b^2*c^3*e^2*f^5*z + 36*A*B^2*a^3*b^2*c^3*e^3*f^4*z + 12*A^2*B
*a^2*b^4*c^2*e^2*f^5*z + 12*A*B^2*a^3*b^3*c^2*e^2*f^5*z + 9*A*B^2*a^2*b^2*c
^4*e^5*f^2*z - 6*A*B^2*a^2*b^4*c^2*e^3*f^4*z + 4*A*B^2*a^2*b^3*c^3*e^4*f^3*
z + 480*A^2*B*a^2*b^2*c^4*d^2*f^5*z - 176*A*B^2*a^2*b^3*c^3*d^2*f^5*z - 10*
A^2*B*a*b^6*c*d*f^6*z + 16*A*B^2*a*b*c^6*d*e^6*z + 80*B^3*a*b^3*c^4*d^3*e*f
^3*z - 48*B^3*a^3*b*c^4*d^2*e*f^4*z + 48*B^3*a^2*b*c^5*d^3*e*f^3*z + 44*B^3
*a^3*b*c^4*d*e^3*f^3*z + 24*B^3*a*b^5*c^2*d^2*e*f^4*z + 18*B^3*a*b^2*c^5*d^
2*e^4*f*z + 696*A^3*a^2*b*c^5*d*e^2*f^4*z - 504*A^3*a*b*c^6*d^2*e^2*f^3*z -
192*A^3*a*b^2*c^5*d*e^3*f^3*z - 144*A^3*a^2*b^2*c^4*d*e*f^5*z + 96*A^3*a*b
^2*c^5*d^2*e*f^4*z - 72*A^3*a*b^3*c^4*d*e^2*f^4*z - 208*A^2*B*b^3*c^5*d^3*e
*f^3*z + 152*A*B^2*b^4*c^4*d^3*e*f^3*z + 80*A^2*B*b^5*c^3*d^2*e*f^4*z + 75*
A^2*B*b^4*c^4*d*e^4*f^2*z - 59*A^2*B*b^2*c^6*d^2*e^4*f*z - 52*A^2*B*b^5*c^3
*d*e^3*f^3*z + 42*A*B^2*b^3*c^5*d^2*e^4*f*z - 21*A*B^2*b^6*c^2*d^2*e*f^4*z
- 16*A*B^2*b^5*c^3*d*e^4*f^2*z + 16*A*B^2*b^2*c^6*d^4*e*f^2*z + 16*A*B^2*b^
2*c^6*d^3*e^3*f*z + 11*A^2*B*b^6*c^2*d*e^2*f^4*z + 4*A*B^2*b^6*c^2*d*e^3*f^
3*z - 256*A^2*B*a*c^7*d^3*e^2*f^2*z - 96*A*B^2*a^3*c^5*d^2*e*f^4*z - 36*A^2
*B*a^2*c^6*d*e^4*f^2*z - 32*A^2*B*a^3*c^5*d*e^2*f^4*z - 32*A*B^2*a^2*c^6*d^
3*e*f^3*z + 8*A*B^2*a^3*c^5*d*e^3*f^3*z - 96*A^2*B*a^3*b^3*c^2*e*f^6*z + 68
*A^2*B*a^3*b*c^4*e^3*f^4*z - 60*A*B^2*a^4*b*c^3*e^2*f^5*z - 60*A*B^2*a^3*b*
c^4*e^4*f^3*z + 48*A*B^2*a^4*b^2*c^2*e*f^6*z - 38*A^2*B*a*b^3*c^4*e^5*f^2*z
- 36*A^2*B*a^2*b*c^5*e^5*f^2*z + 36*A^2*B*a*b^5*c^2*e^3*f^4*z - 16*A^2*B*a
*b^4*c^3*e^4*f^3*z + 384*A*B^2*a^2*b*c^5*d^3*f^4*z - 352*A*B^2*a^3*b*c^4*d^
2*f^5*z - 288*A^2*B*a*b^2*c^5*d^3*f^4*z - 160*A^2*B*a^3*b^2*c^3*d*f^6*z - 1
48*A^2*B*a*b^4*c^3*d^2*f^5*z + 112*A*B^2*a*b^3*c^4*d^3*f^4*z + 72*A^2*B*a^2
*b^4*c^2*d*f^6*z + 72*A*B^2*a*b^5*c^2*d^2*f^5*z + 48*A*B^2*a^3*b^3*c^2*d*f^
6*z + 102*B^3*a^2*b^2*c^4*d^2*e^2*f^3*z - 32*B^3*b^5*c^3*d^3*e*f^3*z - 8*B^
3*b^3*c^5*d^3*e^3*f*z - 7*B^3*b^4*c^4*d^2*e^4*f*z + 5*B^3*b^2*c^6*d^4*e^2*f
*z + 80*A^3*b^2*c^6*d^3*e*f^3*z - 74*A^3*b^3*c^5*d*e^4*f^2*z - 64*A^3*b^4*c
^4*d^2*e*f^4*z + 60*A^3*b^4*c^4*d*e^3*f^3*z - 48*B^3*a^4*c^4*d*e^2*f^4*z -
24*B^3*a^3*c^5*d*e^4*f^2*z + 20*B^3*a^2*c^6*d^2*e^4*f*z - 16*A^3*b^5*c^3*d*
e^2*f^4*z + 8*A^3*b*c^7*d^3*e^2*f^2*z + 480*A^3*a^2*c^6*d^2*e*f^4*z - 392*A
^3*a^2*c^6*d*e^3*f^3*z + 280*A^3*a*c^7*d^2*e^3*f^2*z - 4*B^3*a^4*b*c^3*e^3*
f^4*z - 200*A^3*a^3*b*c^4*e^2*f^5*z - 144*A^3*a^2*b*c^5*e^4*f^3*z + 48*B^3*
a*b^2*c^5*d^4*f^3*z + 42*A^3*a*b^2*c^5*e^5*f^2*z - 36*B^3*a^4*b^2*c^2*d*f^6
*z - 32*A^3*a^3*b^2*c^3*e*f^6*z - 24*A^3*a^2*b^4*c^2*e*f^6*z - 24*A^3*a*b^5
*c^2*e^2*f^5*z + 10*A^3*a*b^3*c^4*e^4*f^3*z - 4*B^3*a*b^4*c^3*d^3*f^4*z - 4
*A^3*a*b^4*c^3*e^3*f^4*z - 480*A^3*a^2*b*c^5*d^2*f^5*z - 160*A^3*a^2*b^3*c^
3*d*f^6*z + 128*A^3*a*b^3*c^4*d^2*f^5*z + 8*A^2*B*b^5*c^3*e^5*f^2*z - 2*A^2

*B*b^6*c^2*e^4*f^3*z + 112*A^2*B*b^4*c^4*d^3*f^4*z - 92*A^2*B*a^4*c^4*e^2*f
^5*z - 64*A^2*B*a^3*c^5*e^4*f^3*z - 64*A*B^2*b^5*c^3*d^3*f^4*z + 24*A*B^2*a
^4*c^4*e^3*f^4*z + 24*A*B^2*a^3*c^5*e^5*f^2*z + 16*A^2*B*b^2*c^6*d^4*f^3*z
+ 16*A*B^2*b^3*c^5*d^4*f^3*z - A^2*B*b^6*c^2*d^2*f^5*z + 448*A^2*B*a^3*c^5*
d^2*f^5*z - 352*A^2*B*a^2*c^6*d^3*f^4*z - 5*A*B^2*b^2*c^6*d^2*e^5*z - 48*A^
2*B*a^4*b^2*c^2*f^7*z - 2*B^3*b^7*c*d^2*e*f^4*z + 34*A^3*b^2*c^6*d*e^5*f*z
+ 16*A^3*b*c^7*d^2*e^4*f*z + 2*A^3*b^6*c^2*d*e*f^5*z - 416*A^3*a^3*c^5*d*e*
f^5*z - 224*A^3*a*c^7*d^3*e*f^3*z + 12*B^3*a^3*b^4*c*d*f^6*z - 10*B^3*a*b^6
*c*d^2*f^5*z + 416*A^3*a^3*b*c^4*d*f^6*z + 224*A^3*a*b*c^6*d^3*f^4*z + 24*A
^3*a*b^5*c^2*d*f^6*z - 4*B^3*a*b*c^6*d^2*e^5*z + 20*A^2*B*c^8*d^4*e^2*f*z -
 7*A^2*B*b^4*c^4*e^6*f*z - 2*A^2*B*b^7*c*e^3*f^4*z - 64*A*B^2*a^5*c^3*e*f^6
*z + 16*A*B^2*b*c^7*d^5*f^2*z - 8*A^2*B*a^2*c^6*e^6*f*z - 2*A*B^2*b^7*c*d^2
*f^5*z - 272*A^2*B*a^4*c^4*d*f^6*z + 128*A^2*B*a*c^7*d^4*f^3*z + 9*A^2*B*b^
2*c^6*d*e^6*z - 4*A*B^2*b^3*c^5*d*e^6*z + 4*A*B^2*b*c^7*d^3*e^4*z + 8*A*B^2
*a*c^7*d^2*e^5*z + 12*A^2*B*a^3*b^4*c*f^7*z + 30*B^3*b^4*c^4*d^3*e^2*f^2*z
+ 8*B^3*b^5*c^3*d^2*e^3*f^2*z - 2*B^3*b^6*c^2*d^2*e^2*f^3*z + 152*A^3*b^3*c
^5*d^2*e^2*f^3*z - 108*A^3*b^2*c^6*d^2*e^3*f^2*z + 48*B^3*a^3*c^5*d^2*e^2*f
^3*z - 16*B^3*a^2*c^6*d^3*e^2*f^2*z - 3*B^3*a^4*b^2*c^2*e^2*f^5*z - 120*B^3
*a^2*b^2*c^4*d^3*f^4*z + 112*B^3*a^3*b^2*c^3*d^2*f^5*z + 112*A^3*a^2*b^3*c^
3*e^2*f^5*z + 12*A^3*a^2*b^2*c^4*e^3*f^4*z - 120*A^3*a*c^7*d*e^5*f*z - 52*A
^3*a*b*c^6*e^6*f*z + 10*A^3*a*b^6*c*e*f^6*z - 2*A*B^2*b^8*d*e*f^5*z - 2*A^2
*B*a*b^7*e*f^6*z - 24*A^2*B*a*c^7*d*e^6*z + 2*A*B^2*a*b^7*d*f^6*z - 12*A^2*
B*a*b*c^6*e^7*z - 2*A^3*b^7*c*d*f^6*z - 4*A^3*b*c^7*d*e^6*z + 16*B^3*a^5*c^
3*e^2*f^5*z + 11*B^3*b^6*c^2*d^3*f^4*z - 11*A^3*b^4*c^4*e^5*f^2*z - 8*B^3*b
^4*c^4*d^4*f^3*z - 4*B^3*b^2*c^6*d^5*f^2*z + 4*B^3*a^4*c^4*e^4*f^3*z + 4*A^
3*b^5*c^3*e^4*f^3*z - A^3*b^6*c^2*e^3*f^4*z + 136*A^3*a^3*c^5*e^3*f^4*z + 6
8*A^3*a^2*c^6*e^5*f^2*z - 64*A^3*b^3*c^5*d^3*f^4*z + 2*B^3*b^3*c^5*d^2*e^5*
z - B^3*b^2*c^6*d^3*e^4*z + 96*A^3*a^3*b^3*c^2*f^7*z + A*B^2*a^2*b^6*e*f^6*
z + 32*A^3*c^8*d^4*e*f^2*z - 24*A^3*c^8*d^3*e^3*f*z + 10*A^3*b^3*c^5*e^6*f*
z + 2*A^3*b^7*c*e^2*f^5*z + 128*A^3*a^4*c^4*e*f^6*z - 32*A^3*b*c^7*d^4*f^3*
z - 4*B^3*a^2*c^6*d*e^6*z - B^3*a^2*b^6*d*f^6*z - 128*A^3*a^4*b*c^3*f^7*z -
 24*A^3*a^2*b^5*c*f^7*z - 16*A^2*B*c^8*d^5*f^2*z - 4*A^2*B*c^8*d^3*e^4*z +
64*A^2*B*a^5*c^3*f^7*z + 2*A^2*B*b^3*c^5*e^7*z + 4*A*B^2*a^2*c^6*e^7*z - A^
2*B*a^2*b^6*f^7*z + 4*A^3*c^8*d^2*e^5*z - 3*A^3*b^2*c^6*e^7*z + A^2*B*b^8*d
*f^6*z - A^3*b^8*e*f^6*z + 16*A^3*a*c^7*e^7*z + 2*A^3*a*b^7*f^7*z + A^2*B*b
^8*e^2*f^5*z + B^3*b^8*d^2*f^5*z - 48*A^2*B^2*a*b*c^4*d*e*f^4 + 28*A*B^3*a*
b^2*c^3*d*e*f^4 - 16*A*B^3*a*b*c^4*d*e^2*f^3 + 16*A^3*B*a*c^5*d*e*f^4 + 32*
A^3*B*a*b*c^4*d*f^5 + 12*A^2*B^2*b^3*c^3*d*e*f^4 + 5*A*B^3*b^2*c^4*d^2*e*f^
3 + 4*A*B^3*b^3*c^3*d*e^2*f^3 + 24*A^2*B^2*a*c^5*d*e^2*f^3 + 24*A^2*B^2*a^2
*b*c^3*e*f^5 + 12*A^2*B^2*a*b*c^4*e^3*f^3 - 6*A^2*B^2*a*b^3*c^2*e*f^5 + 4*A
*B^3*a^2*b*c^3*e^2*f^4 + 3*A*B^3*a^2*b^2*c^2*e*f^5 - 18*A^2*B^2*a*b^2*c^3*d
*f^5 - 4*B^4*a^2*b*c^3*d*e*f^4 + 4*B^4*a*b*c^4*d^2*e*f^3 - 6*A*B^3*b^4*c^2*
d*e*f^4 + 4*A^3*B*b*c^5*d*e^2*f^3 - 2*A^3*B*b^2*c^4*d*e*f^4 - 8*A*B^3*a^2*c
^4*d*e*f^4 - 8*A*B^3*a*c^5*d^2*e*f^3 + 26*A^3*B*a*b^2*c^3*e*f^5 + 8*A^3*B*a
*b*c^4*e^2*f^4 + 32*A*B^3*a*b*c^4*d^2*f^4 - 28*A*B^3*a^2*b*c^3*d*f^5 + 6*A*

B^3*a*b^3*c^2*d*f^5 - 9*A^2*B^2*b^2*c^4*d*e^2*f^3 - 18*A^2*B^2*a*b^2*c^3*e^
2*f^4 - 4*A^3*B*c^6*d^2*e*f^3 - 3*A^3*B*b^4*c^2*e*f^5 - 44*A^3*B*a^2*c^4*e*
f^5 - 16*A^3*B*a*c^5*e^3*f^3 - 16*A*B^3*a^3*c^3*e*f^5 - 10*A^3*B*b^3*c^3*d*
f^5 - 4*A^3*B*b*c^5*d^2*f^4 - 4*A*B^3*b*c^5*d^3*f^3 - 28*A^3*B*a^2*b*c^3*f^
6 + 6*A^3*B*a*b^3*c^2*f^6 - 4*A^4*b*c^5*d*e*f^4 - 20*A^4*a*b*c^4*e*f^5 + 3*
A^2*B^2*b^4*c^2*e^2*f^4 - 2*A^2*B^2*b^3*c^3*e^3*f^3 + 12*A^2*B^2*a^2*c^4*e^
2*f^4 + 9*A^2*B^2*b^2*c^4*d^2*f^4 - 3*A^2*B^2*a^2*b^2*c^2*f^6 - 2*B^4*b^3*c
^3*d^2*e*f^3 + 4*B^4*a^2*c^4*d*e^2*f^3 - 10*B^4*a*b^2*c^3*d^2*f^4 - 3*B^4*a
^2*b^2*c^2*d*f^5 + 3*A^3*B*b^2*c^4*e^3*f^3 - 2*A^3*B*b^3*c^3*e^2*f^4 - 10*A
*B^3*b^3*c^3*d^2*f^4 - 4*A*B^3*a^2*c^4*e^3*f^3 + 3*A^2*B^2*b^4*c^2*d*f^5 +
36*A^2*B^2*a^2*c^4*d*f^5 - 24*A^2*B^2*a*c^5*d^2*f^4 + 4*A^2*B^2*c^6*d^3*f^3
 + 16*A^2*B^2*a^3*c^3*f^6 + 4*A^4*b^3*c^3*e*f^5 + 16*B^4*a^3*c^3*d*f^5 + 16
*A^4*a*c^5*e^2*f^4 + 8*A^4*b^2*c^4*d*f^5 - 8*A^4*a*b^2*c^3*f^6 - 24*A^4*a*c
^5*d*f^5 + 3*B^4*b^4*c^2*d^2*f^4 - 3*A^4*b^2*c^4*e^2*f^4 + 4*A^4*c^6*d^2*f^
4 + 36*A^4*a^2*c^4*f^6 + B^4*b^2*c^4*d^3*f^3, z, k)*(root(48416*a^6*b^2*c^6
*d^4*e^2*f^4*z^4 - 41544*a^5*b^4*c^5*d^4*e^2*f^4*z^4 - 31872*a^7*b^2*c^5*d^
3*e^2*f^5*z^4 - 31872*a^5*b^2*c^7*d^5*e^2*f^3*z^4 - 29184*a^6*b^2*c^6*d^3*e
^4*f^3*z^4 + 28800*a^5*b^4*c^5*d^3*e^4*f^3*z^4 + 21510*a^4*b^6*c^4*d^4*e^2*
f^4*z^4 + 21408*a^6*b^4*c^4*d^3*e^2*f^5*z^4 + 21408*a^4*b^4*c^6*d^5*e^2*f^3
*z^4 - 18112*a^7*b^3*c^4*d^2*e^3*f^5*z^4 - 18112*a^4*b^3*c^7*d^5*e^3*f^2*z^
4 - 15600*a^5*b^5*c^4*d^3*e^3*f^4*z^4 - 15600*a^4*b^5*c^5*d^4*e^3*f^3*z^4 +
 15296*a^6*b^3*c^5*d^3*e^3*f^4*z^4 + 15296*a^5*b^3*c^6*d^4*e^3*f^3*z^4 + 14
016*a^7*b^2*c^5*d^2*e^4*f^4*z^4 + 14016*a^5*b^2*c^7*d^4*e^4*f^2*z^4 - 13920
*a^4*b^6*c^4*d^3*e^4*f^3*z^4 - 11648*a^6*b^3*c^5*d^2*e^5*f^3*z^4 - 11648*a^
5*b^3*c^6*d^3*e^5*f^2*z^4 + 10432*a^6*b^2*c^6*d^2*e^6*f^2*z^4 + 9008*a^6*b^
5*c^3*d^2*e^3*f^5*z^4 + 9008*a^3*b^5*c^6*d^5*e^3*f^2*z^4 + 8544*a^5*b^5*c^4
*d^2*e^5*f^3*z^4 + 8544*a^4*b^5*c^5*d^3*e^5*f^2*z^4 - 8496*a^5*b^4*c^5*d^2*
e^6*f^2*z^4 + 7488*a^8*b^2*c^4*d^2*e^2*f^6*z^4 + 7488*a^4*b^2*c^8*d^6*e^2*f
^2*z^4 + 7380*a^4*b^7*c^3*d^3*e^3*f^4*z^4 + 7380*a^3*b^7*c^4*d^4*e^3*f^3*z^
4 - 6720*a^3*b^8*c^3*d^4*e^2*f^4*z^4 - 5784*a^5*b^6*c^3*d^3*e^2*f^5*z^4 - 5
784*a^3*b^6*c^5*d^5*e^2*f^3*z^4 - 3440*a^6*b^4*c^4*d^2*e^4*f^4*z^4 - 3440*a
^4*b^4*c^6*d^4*e^4*f^2*z^4 + 3360*a^3*b^8*c^3*d^3*e^4*f^3*z^4 + 3140*a^4*b^
6*c^4*d^2*e^6*f^2*z^4 - 2760*a^4*b^7*c^3*d^2*e^5*f^3*z^4 - 2760*a^3*b^7*c^4
*d^3*e^5*f^2*z^4 - 1764*a^5*b^7*c^2*d^2*e^3*f^5*z^4 - 1764*a^2*b^7*c^5*d^5*
e^3*f^2*z^4 - 1640*a^3*b^9*c^2*d^3*e^3*f^4*z^4 - 1640*a^2*b^9*c^3*d^4*e^3*f
^3*z^4 - 1604*a^6*b^6*c^2*d^2*e^2*f^6*z^4 - 1604*a^2*b^6*c^6*d^6*e^2*f^2*z^
4 - 1500*a^5*b^6*c^3*d^2*e^4*f^4*z^4 - 1500*a^3*b^6*c^5*d^4*e^4*f^2*z^4 + 1
140*a^2*b^10*c^2*d^4*e^2*f^4*z^4 + 810*a^4*b^8*c^2*d^2*e^4*f^4*z^4 + 810*a^
2*b^8*c^4*d^4*e^4*f^2*z^4 - 544*a^3*b^8*c^3*d^2*e^6*f^2*z^4 + 416*a^3*b^9*c
^2*d^2*e^5*f^3*z^4 + 416*a^2*b^9*c^3*d^3*e^5*f^2*z^4 - 384*a^2*b^10*c^2*d^3
*e^4*f^3*z^4 + 180*a^4*b^8*c^2*d^3*e^2*f^5*z^4 + 180*a^2*b^8*c^4*d^5*e^2*f^
3*z^4 + 48*a^7*b^4*c^3*d^2*e^2*f^6*z^4 + 48*a^3*b^4*c^7*d^6*e^2*f^2*z^4 + 3
6*a^2*b^10*c^2*d^2*e^6*f^2*z^4 - 1024*a^10*b*c^3*d*e*f^8*z^4 - 1024*a^3*b*c
^10*d^8*e*f*z^4 - 192*a^8*b^5*c*d*e*f^8*z^4 - 192*a*b^5*c^8*d^8*e*f*z^4 + 1
6128*a^7*b^3*c^4*d^3*e*f^6*z^4 + 16128*a^4*b^3*c^7*d^6*e*f^3*z^4 - 11712*a^

6*b^5*c^3*d^3*e*f^6*z^4 - 11712*a^3*b^5*c^6*d^6*e*f^3*z^4 + 11520*a^8*b*c^5
*d^2*e^3*f^5*z^4 + 11520*a^5*b*c^8*d^5*e^3*f^2*z^4 - 9984*a^6*b^3*c^5*d^4*e
*f^5*z^4 - 9984*a^5*b^3*c^6*d^5*e*f^4*z^4 + 8640*a^5*b^5*c^4*d^4*e*f^5*z^4
+ 8640*a^4*b^5*c^5*d^5*e*f^4*z^4 - 7424*a^7*b*c^6*d^3*e^3*f^4*z^4 - 7424*a^
6*b*c^7*d^4*e^3*f^3*z^4 - 6912*a^8*b^3*c^3*d^2*e*f^7*z^4 - 6912*a^3*b^3*c^8
*d^7*e*f^2*z^4 + 4800*a^7*b^3*c^4*d*e^5*f^4*z^4 + 4800*a^4*b^3*c^7*d^4*e^5*
f*z^4 + 4608*a^7*b*c^6*d^2*e^5*f^3*z^4 + 4608*a^6*b*c^7*d^3*e^5*f^2*z^4 - 4
560*a^4*b^7*c^3*d^4*e*f^5*z^4 - 4560*a^3*b^7*c^4*d^5*e*f^4*z^4 + 4176*a^5*b
^7*c^2*d^3*e*f^6*z^4 + 4176*a^2*b^7*c^5*d^6*e*f^3*z^4 + 3264*a^7*b^5*c^2*d^
2*e*f^7*z^4 + 3264*a^2*b^5*c^7*d^7*e*f^2*z^4 + 3008*a^8*b^3*c^3*d*e^3*f^6*z
^4 + 3008*a^3*b^3*c^8*d^6*e^3*f*z^4 + 2880*a^6*b^3*c^5*d*e^7*f^2*z^4 + 2880
*a^5*b^3*c^6*d^2*e^7*f*z^4 - 2240*a^7*b^4*c^3*d*e^4*f^5*z^4 - 2240*a^3*b^4*
c^7*d^5*e^4*f*z^4 - 1488*a^5*b^5*c^4*d*e^7*f^2*z^4 - 1488*a^4*b^5*c^5*d^2*e
^7*f*z^4 + 1440*a^3*b^9*c^2*d^4*e*f^5*z^4 + 1440*a^2*b^9*c^3*d^5*e*f^4*z^4
- 1328*a^6*b^5*c^3*d*e^5*f^4*z^4 - 1328*a^3*b^5*c^6*d^4*e^5*f*z^4 - 1152*a^
7*b^2*c^5*d*e^6*f^3*z^4 - 1152*a^5*b^2*c^7*d^3*e^6*f*z^4 - 1120*a^6*b^4*c^4
*d*e^6*f^3*z^4 - 1120*a^4*b^4*c^6*d^3*e^6*f*z^4 + 912*a^6*b^6*c^2*d*e^4*f^5
*z^4 + 912*a^2*b^6*c^6*d^5*e^4*f*z^4 + 872*a^5*b^6*c^3*d*e^6*f^3*z^4 + 872*
a^3*b^6*c^5*d^3*e^6*f*z^4 + 768*a^8*b^2*c^4*d*e^4*f^5*z^4 + 768*a^4*b^2*c^8
*d^5*e^4*f*z^4 - 672*a^8*b^4*c^2*d*e^2*f^7*z^4 - 672*a^2*b^4*c^8*d^7*e^2*f*
z^4 - 624*a^7*b^5*c^2*d*e^3*f^6*z^4 - 624*a^2*b^5*c^7*d^6*e^3*f*z^4 + 480*a
^5*b^8*c*d^2*e^2*f^6*z^4 + 480*a*b^8*c^5*d^6*e^2*f^2*z^4 + 316*a^4*b^7*c^3*
d*e^7*f^2*z^4 + 316*a^3*b^7*c^4*d^2*e^7*f*z^4 - 204*a^4*b^8*c^2*d*e^6*f^3*z
^4 - 204*a^2*b^8*c^4*d^3*e^6*f*z^4 + 168*a^3*b^10*c*d^3*e^2*f^5*z^4 + 168*a
*b^10*c^3*d^5*e^2*f^3*z^4 + 156*a^2*b^11*c*d^3*e^3*f^4*z^4 + 156*a*b^11*c^2
*d^4*e^3*f^3*z^4 + 128*a^9*b^2*c^3*d*e^2*f^7*z^4 + 128*a^3*b^2*c^9*d^7*e^2*
f*z^4 - 124*a^3*b^10*c*d^2*e^4*f^4*z^4 - 124*a*b^10*c^3*d^4*e^4*f^2*z^4 + 1
00*a^4*b^9*c*d^2*e^3*f^5*z^4 + 100*a*b^9*c^4*d^5*e^3*f^2*z^4 + 36*a^5*b^7*c
^2*d*e^5*f^4*z^4 + 36*a^2*b^7*c^5*d^4*e^5*f*z^4 - 24*a^3*b^9*c^2*d*e^7*f^2*
z^4 - 24*a^2*b^11*c*d^2*e^5*f^3*z^4 - 24*a^2*b^9*c^3*d^2*e^7*f*z^4 - 24*a*b
^11*c^2*d^3*e^5*f^2*z^4 - 9216*a^8*b*c^5*d^3*e*f^6*z^4 - 9216*a^5*b*c^8*d^6
*e*f^3*z^4 - 5376*a^8*b*c^5*d*e^5*f^4*z^4 - 5376*a^5*b*c^8*d^4*e^5*f*z^4 +
5120*a^9*b*c^4*d^2*e*f^7*z^4 + 5120*a^7*b*c^6*d^4*e*f^5*z^4 + 5120*a^6*b*c^
7*d^5*e*f^4*z^4 + 5120*a^4*b*c^9*d^7*e*f^2*z^4 - 4352*a^9*b*c^4*d*e^3*f^6*z
^4 - 4352*a^4*b*c^9*d^6*e^3*f*z^4 - 1792*a^7*b*c^6*d*e^7*f^2*z^4 - 1792*a^6
*b*c^7*d^2*e^7*f*z^4 - 1600*a^6*b^2*c^6*d*e^8*f*z^4 + 912*a^5*b^4*c^5*d*e^8
*f*z^4 + 768*a^9*b^3*c^2*d*e*f^8*z^4 + 768*a^2*b^3*c^9*d^8*e*f*z^4 - 720*a^
4*b^9*c*d^3*e*f^6*z^4 - 720*a*b^9*c^4*d^6*e*f^3*z^4 - 656*a^6*b^7*c*d^2*e*f
^7*z^4 - 656*a*b^7*c^6*d^7*e*f^2*z^4 - 240*a^2*b^11*c*d^4*e*f^5*z^4 - 240*a
*b^11*c^2*d^5*e*f^4*z^4 + 216*a^7*b^6*c*d*e^2*f^7*z^4 + 216*a*b^6*c^7*d^7*e
^2*f*z^4 - 204*a^4*b^6*c^4*d*e^8*f*z^4 - 144*a^5*b^8*c*d*e^4*f^5*z^4 - 144*
a*b^8*c^5*d^5*e^4*f*z^4 - 84*a*b^12*c*d^4*e^2*f^4*z^4 + 36*a^4*b^9*c*d*e^5*
f^4*z^4 + 36*a*b^9*c^4*d^4*e^5*f*z^4 + 20*a^6*b^7*c*d*e^3*f^6*z^4 + 20*a*b^
7*c^6*d^6*e^3*f*z^4 + 16*a^3*b^10*c*d*e^6*f^3*z^4 + 16*a^3*b^8*c^3*d*e^8*f*
z^4 + 16*a*b^12*c*d^3*e^4*f^3*z^4 + 16*a*b^10*c^3*d^3*e^6*f*z^4 + 48*b^11*c

^3*d^6*e*f^3*z^4 + 48*b^9*c^5*d^7*e*f^2*z^4 - 20*b^8*c^6*d^7*e^2*f*z^4 + 8*
b^10*c^4*d^5*e^4*f*z^4 - 4*b^13*c*d^4*e^3*f^3*z^4 - 4*b^11*c^3*d^4*e^5*f*z^
4 + 4*b^9*c^5*d^6*e^3*f*z^4 + 3072*a^9*c^5*d*e^4*f^5*z^4 + 3072*a^5*c^9*d^5
*e^4*f*z^4 + 2560*a^8*c^6*d*e^6*f^3*z^4 + 2560*a^6*c^8*d^3*e^6*f*z^4 + 1536
*a^10*c^4*d*e^2*f^7*z^4 + 1536*a^4*c^10*d^7*e^2*f*z^4 + 48*a^5*b^9*d^2*e*f^
7*z^4 + 48*a^3*b^11*d^3*e*f^6*z^4 - 20*a^6*b^8*d*e^2*f^7*z^4 + 8*a^4*b^10*d
*e^4*f^5*z^4 + 4*a^5*b^9*d*e^3*f^6*z^4 - 4*a^3*b^11*d*e^5*f^4*z^4 - 4*a*b^1
3*d^3*e^3*f^4*z^4 + 768*a^9*b*c^4*e^5*f^5*z^4 + 768*a^8*b*c^5*e^7*f^3*z^4 +
256*a^10*b*c^3*e^3*f^7*z^4 - 192*a^6*b^3*c^5*e^9*f*z^4 - 68*a^7*b^6*c*e^4*
f^6*z^4 + 48*a^8*b^5*c*e^3*f^7*z^4 + 48*a^5*b^5*c^4*e^9*f*z^4 + 36*a^6*b^7*
c*e^5*f^5*z^4 - 12*a^9*b^4*c*e^2*f^8*z^4 - 4*a^4*b^9*c*e^7*f^3*z^4 - 4*a^4*
b^7*c^3*e^9*f*z^4 + 384*a^5*b^8*c*d^3*f^7*z^4 + 384*a*b^8*c^5*d^7*f^3*z^4 +
288*a^3*b^10*c*d^4*f^6*z^4 + 288*a*b^10*c^3*d^6*f^4*z^4 + 224*a^7*b^6*c*d^
2*f^8*z^4 + 224*a*b^6*c^7*d^8*f^2*z^4 - 192*a^10*b^2*c^2*d*f^9*z^4 - 192*a^
2*b^2*c^10*d^9*f*z^4 + 768*a^5*b*c^8*d^3*e^7*z^4 + 768*a^4*b*c^9*d^5*e^5*z^
4 + 256*a^3*b*c^10*d^7*e^3*z^4 - 192*a^5*b^3*c^6*d*e^9*z^4 - 68*a*b^6*c^7*d
^6*e^4*z^4 + 48*a^4*b^5*c^5*d*e^9*z^4 + 48*a*b^5*c^8*d^7*e^3*z^4 + 36*a*b^7
*c^6*d^5*e^5*z^4 - 12*a*b^4*c^9*d^8*e^2*z^4 - 4*a^3*b^7*c^4*d*e^9*z^4 - 4*a
*b^9*c^4*d^3*e^7*z^4 + 16*b^13*c*d^5*e*f^4*z^4 + 16*b^7*c^7*d^8*e*f*z^4 + 7
68*a^7*c^7*d*e^8*f*z^4 + 16*a^7*b^7*d*e*f^8*z^4 + 16*a*b^13*d^4*e*f^5*z^4 +
256*a^7*b*c^6*e^9*f*z^4 + 80*a*b^12*c*d^5*f^5*z^4 + 48*a^9*b^4*c*d*f^9*z^4
+ 48*a*b^4*c^9*d^9*f*z^4 + 256*a^6*b*c^7*d*e^9*z^4 - 42*b^10*c^4*d^6*e^2*f
^2*z^4 - 20*b^12*c^2*d^5*e^2*f^3*z^4 + 6*b^12*c^2*d^4*e^4*f^2*z^4 + 4*b^11*
c^3*d^5*e^3*f^2*z^4 - 24960*a^7*c^7*d^4*e^2*f^4*z^4 + 18944*a^8*c^6*d^3*e^2
*f^5*z^4 + 18944*a^6*c^8*d^5*e^2*f^3*z^4 + 14336*a^7*c^7*d^3*e^4*f^3*z^4 -
9984*a^8*c^6*d^2*e^4*f^4*z^4 - 9984*a^6*c^8*d^4*e^4*f^2*z^4 - 7936*a^9*c^5*
d^2*e^2*f^6*z^4 - 7936*a^5*c^9*d^6*e^2*f^2*z^4 - 4352*a^7*c^7*d^2*e^6*f^2*z
^4 - 42*a^4*b^10*d^2*e^2*f^6*z^4 - 20*a^2*b^12*d^3*e^2*f^5*z^4 + 6*a^2*b^12
*d^2*e^4*f^4*z^4 + 4*a^3*b^11*d^2*e^3*f^5*z^4 - 480*a^8*b^2*c^4*e^6*f^4*z^4
+ 440*a^7*b^4*c^3*e^6*f^4*z^4 - 320*a^8*b^3*c^3*e^5*f^5*z^4 - 320*a^7*b^3*
c^4*e^7*f^3*z^4 + 240*a^8*b^4*c^2*e^4*f^6*z^4 + 240*a^6*b^4*c^4*e^8*f^2*z^4
- 192*a^9*b^3*c^2*e^3*f^7*z^4 - 192*a^9*b^2*c^3*e^4*f^6*z^4 - 192*a^7*b^2*
c^5*e^8*f^2*z^4 - 90*a^6*b^6*c^2*e^6*f^4*z^4 - 68*a^5*b^6*c^3*e^8*f^2*z^4 +
48*a^10*b^2*c^2*e^2*f^8*z^4 - 48*a^7*b^5*c^2*e^5*f^5*z^4 - 48*a^6*b^5*c^3*
e^7*f^3*z^4 + 36*a^5*b^7*c^2*e^7*f^3*z^4 + 6*a^4*b^8*c^2*e^8*f^2*z^4 - 3392
0*a^6*b^2*c^6*d^5*f^5*z^4 + 27936*a^5*b^4*c^5*d^5*f^5*z^4 + 26112*a^7*b^2*c
^5*d^4*f^6*z^4 + 26112*a^5*b^2*c^7*d^6*f^4*z^4 - 20352*a^6*b^4*c^4*d^4*f^6*
z^4 - 20352*a^4*b^4*c^6*d^6*f^4*z^4 - 13080*a^4*b^6*c^4*d^5*f^5*z^4 - 11520
*a^8*b^2*c^4*d^3*f^7*z^4 - 11520*a^4*b^2*c^8*d^7*f^3*z^4 + 8736*a^5*b^6*c^3
*d^4*f^6*z^4 + 8736*a^3*b^6*c^5*d^6*f^4*z^4 + 7488*a^7*b^4*c^3*d^3*f^7*z^4
+ 7488*a^3*b^4*c^7*d^7*f^3*z^4 + 3840*a^3*b^8*c^3*d^5*f^5*z^4 + 2560*a^9*b^
2*c^3*d^2*f^8*z^4 + 2560*a^3*b^2*c^9*d^8*f^2*z^4 - 2416*a^6*b^6*c^2*d^3*f^7
*z^4 - 2416*a^2*b^6*c^6*d^7*f^3*z^4 - 2160*a^4*b^8*c^2*d^4*f^6*z^4 - 2160*a
^2*b^8*c^4*d^6*f^4*z^4 - 1152*a^8*b^4*c^2*d^2*f^8*z^4 - 1152*a^2*b^4*c^8*d^
8*f^2*z^4 - 720*a^2*b^10*c^2*d^5*f^5*z^4 - 480*a^4*b^2*c^8*d^4*e^6*z^4 + 44

0*a^3*b^4*c^7*d^4*e^6*z^4 - 320*a^4*b^3*c^7*d^3*e^7*z^4 - 320*a^3*b^3*c^8*d
^5*e^5*z^4 + 240*a^4*b^4*c^6*d^2*e^8*z^4 + 240*a^2*b^4*c^8*d^6*e^4*z^4 - 19
2*a^5*b^2*c^7*d^2*e^8*z^4 - 192*a^3*b^2*c^9*d^6*e^4*z^4 - 192*a^2*b^3*c^9*d
^7*e^3*z^4 - 90*a^2*b^6*c^6*d^4*e^6*z^4 - 68*a^3*b^6*c^5*d^2*e^8*z^4 - 48*a
^3*b^5*c^6*d^3*e^7*z^4 - 48*a^2*b^5*c^7*d^5*e^5*z^4 + 48*a^2*b^2*c^10*d^8*e
^2*z^4 + 36*a^2*b^7*c^5*d^3*e^7*z^4 + 6*a^2*b^8*c^4*d^2*e^8*z^4 - 4*b^6*c^8
*d^9*f*z^4 + 256*a^11*c^3*d*f^9*z^4 + 256*a^3*c^11*d^9*f*z^4 - 4*a^8*b^6*d*
f^9*z^4 - 384*a^9*c^5*e^6*f^4*z^4 - 256*a^10*c^4*e^4*f^6*z^4 - 256*a^8*c^6*
e^8*f^2*z^4 - 64*a^11*c^3*e^2*f^8*z^4 - 24*b^10*c^4*d^7*f^3*z^4 - 16*b^12*c
^2*d^6*f^4*z^4 - 16*b^8*c^6*d^8*f^2*z^4 + 17920*a^7*c^7*d^5*f^5*z^4 - 14336
*a^8*c^6*d^4*f^6*z^4 - 14336*a^6*c^8*d^6*f^4*z^4 + 7168*a^9*c^5*d^3*f^7*z^4
 + 7168*a^5*c^9*d^7*f^3*z^4 - 2048*a^10*c^4*d^2*f^8*z^4 - 2048*a^4*c^10*d^8
*f^2*z^4 + 6*b^8*c^6*d^6*e^4*z^4 + 6*a^6*b^8*e^4*f^6*z^4 - 4*b^9*c^5*d^5*e^
5*z^4 - 4*b^7*c^7*d^7*e^3*z^4 - 4*a^7*b^7*e^3*f^7*z^4 - 4*a^5*b^9*e^5*f^5*z
^4 - 384*a^5*c^9*d^4*e^6*z^4 - 256*a^6*c^8*d^2*e^8*z^4 - 256*a^4*c^10*d^6*e
^4*z^4 - 64*a^3*c^11*d^8*e^2*z^4 - 24*a^4*b^10*d^3*f^7*z^4 - 16*a^6*b^8*d^2
*f^8*z^4 - 16*a^2*b^12*d^4*f^6*z^4 + 48*a^6*b^2*c^6*e^10*z^4 - 12*a^5*b^4*c
^5*e^10*z^4 - 4*b^14*d^5*f^5*z^4 - 64*a^7*c^7*e^10*z^4 + b^14*d^4*e^2*f^4*z
^4 + b^10*c^4*d^4*e^6*z^4 + b^6*c^8*d^8*e^2*z^4 + a^8*b^6*e^2*f^8*z^4 + a^4
*b^10*e^6*f^4*z^4 + a^4*b^6*c^4*e^10*z^4 - 4820*A*B*a^4*b*c^5*d^2*e^2*f^4*z
^2 + 2976*A*B*a^3*b*c^6*d^3*e^2*f^3*z^2 - 2328*A*B*a^3*b*c^6*d^2*e^4*f^2*z^
2 + 1848*A*B*a^2*b^4*c^4*d^3*e*f^4*z^2 - 1768*A*B*a^3*b^4*c^3*d^2*e*f^5*z^2
 + 1528*A*B*a^4*b^2*c^4*d^2*e*f^5*z^2 - 1136*A*B*a^3*b^2*c^5*d^3*e*f^4*z^2
- 974*A*B*a^4*b^3*c^3*d*e^2*f^5*z^2 + 692*A*B*a^2*b*c^7*d^4*e^2*f^2*z^2 + 5
88*A*B*a*b^6*c^3*d^2*e^3*f^3*z^2 - 580*A*B*a^3*b^3*c^4*d*e^4*f^3*z^2 + 488*
A*B*a^3*b^4*c^3*d*e^3*f^4*z^2 - 444*A*B*a^2*b^2*c^6*d^2*e^5*f*z^2 - 412*A*B
*a*b^5*c^4*d^2*e^4*f^2*z^2 + 366*A*B*a^2*b^6*c^2*d^2*e*f^5*z^2 - 352*A*B*a^
2*b^2*c^6*d^4*e*f^3*z^2 + 326*A*B*a^2*b^4*c^4*d*e^5*f^2*z^2 + 324*A*B*a*b^5
*c^4*d^3*e^2*f^3*z^2 - 302*A*B*a*b^3*c^6*d^4*e^2*f^2*z^2 - 296*A*B*a*b^7*c^
2*d^2*e^2*f^4*z^2 + 122*A*B*a^4*b^2*c^4*d*e^3*f^4*z^2 - 122*A*B*a^2*b^6*c^2
*d*e^3*f^4*z^2 - 84*A*B*a^3*b^2*c^5*d*e^5*f^2*z^2 + 72*A*B*a*b^4*c^5*d^3*e^
3*f^2*z^2 - 64*A*B*a^2*b^5*c^3*d*e^4*f^3*z^2 + 60*A*B*a^3*b^5*c^2*d*e^2*f^5
*z^2 + 1312*A*B*a^5*b*c^4*d*e^2*f^5*z^2 + 1040*A*B*a^4*b*c^5*d*e^4*f^3*z^2
- 500*A*B*a*b^6*c^3*d^3*e*f^4*z^2 - 376*A*B*a*b^2*c^7*d^5*e*f^2*z^2 + 276*A
*B*a^4*b^4*c^2*d*e*f^6*z^2 - 262*A*B*a^2*b^3*c^5*d*e^6*f*z^2 + 238*A*B*a*b^
2*c^7*d^4*e^3*f*z^2 + 232*A*B*a^5*b^2*c^3*d*e*f^6*z^2 - 176*A*B*a^2*b*c^7*d
^3*e^4*f*z^2 - 120*A*B*a*b^6*c^3*d*e^5*f^2*z^2 - 108*A*B*a*b^4*c^5*d^4*e*f^
3*z^2 + 68*A*B*a*b^7*c^2*d*e^4*f^3*z^2 + 68*A*B*a*b^4*c^5*d^2*e^5*f*z^2 + 4
6*A*B*a^2*b^7*c*d*e^2*f^5*z^2 - 36*A*B*a*b^3*c^6*d^3*e^4*f*z^2 - 1932*A*B*a
^2*b^3*c^5*d^3*e^2*f^3*z^2 - 1818*A*B*a^2*b^4*c^4*d^2*e^3*f^3*z^2 + 1620*A*
B*a^3*b^3*c^4*d^2*e^2*f^4*z^2 + 1560*A*B*a^2*b^3*c^5*d^2*e^4*f^2*z^2 + 1244
*A*B*a^3*b^2*c^5*d^2*e^3*f^3*z^2 + 820*A*B*a^2*b^2*c^6*d^3*e^3*f^2*z^2 + 48
0*A*B*a^2*b^5*c^3*d^2*e^2*f^4*z^2 + 352*A*B*a^3*b*c^6*d*e^6*f*z^2 - 108*A*B
*a^3*b^6*c*d*e*f^6*z^2 + 82*A*B*a*b^5*c^4*d*e^6*f*z^2 - 64*A*B*a*b*c^8*d^5*
e^2*f*z^2 + 16*A*B*a*b^8*c*d^2*e*f^5*z^2 - 4*A*B*a*b^8*c*d*e^3*f^4*z^2 + 16

*B^2*a*b*c^8*d^6*e*f*z^2 + 56*A*B*b^2*c^8*d^6*e*f*z^2 - 8*A*B*b^9*c*d*e^4*f
^3*z^2 - 8*A*B*b^7*c^3*d*e^6*f*z^2 - 800*A*B*a^6*c^4*d*e*f^6*z^2 + 10*A*B*a
^2*b^8*d*e*f^6*z^2 - 6*A*B*a*b^9*d*e^2*f^5*z^2 - 12*A*B*a^5*b^4*c*e*f^7*z^2
 + 912*A*B*a^6*b*c^3*d*f^7*z^2 + 192*A*B*a^4*b^5*c*d*f^7*z^2 + 192*A*B*a*b*
c^8*d^6*f^2*z^2 - 20*A*B*a*b^4*c^5*d*e^7*z^2 + 4*A*B*a*b*c^8*d^4*e^4*z^2 +
2144*B^2*a^4*b*c^5*d^3*e*f^4*z^2 - 1120*B^2*a^3*b*c^6*d^4*e*f^3*z^2 - 688*B
^2*a^5*b*c^4*d^2*e*f^5*z^2 - 256*B^2*a^3*b*c^6*d^2*e^5*f*z^2 + 152*B^2*a*b^
3*c^6*d^5*e*f^2*z^2 + 120*B^2*a^5*b^3*c^2*d*e*f^6*z^2 - 116*B^2*a^5*b*c^4*d
*e^3*f^4*z^2 + 110*B^2*a*b^7*c^2*d^3*e*f^4*z^2 - 80*B^2*a^2*b*c^7*d^5*e*f^2
*z^2 - 72*B^2*a*b^5*c^4*d^4*e*f^3*z^2 - 48*B^2*a^4*b*c^5*d*e^5*f^2*z^2 - 46
*B^2*a*b^3*c^6*d^4*e^3*f*z^2 - 44*B^2*a*b^4*c^5*d^3*e^4*f*z^2 - 34*B^2*a*b^
5*c^4*d^2*e^5*f*z^2 + 20*B^2*a^2*b*c^7*d^4*e^3*f*z^2 - 10*B^2*a^3*b^6*c*d*e
^2*f^5*z^2 - 10*B^2*a^2*b^7*c*d^2*e*f^5*z^2 - 10*B^2*a*b^2*c^7*d^5*e^2*f*z^
2 - 7*B^2*a^2*b^4*c^4*d*e^6*f*z^2 - 6*B^2*a^3*b^2*c^5*d*e^6*f*z^2 + 4*B^2*a
*b^8*c*d^2*e^2*f^4*z^2 - 2*B^2*a^2*b^7*c*d*e^3*f^4*z^2 + 3196*A^2*a^4*b*c^5
*d*e^3*f^4*z^2 - 3184*A^2*a^4*b*c^5*d^2*e*f^5*z^2 + 1568*A^2*a^3*b*c^6*d^3*
e*f^4*z^2 + 1504*A^2*a^3*b*c^6*d*e^5*f^2*z^2 - 656*A^2*a^4*b^3*c^3*d*e*f^6*
z^2 - 400*A^2*a*b^6*c^3*d*e^4*f^3*z^2 + 314*A^2*a*b^5*c^4*d*e^5*f^2*z^2 - 2
64*A^2*a^3*b^5*c^2*d*e*f^6*z^2 + 240*A^2*a^2*b^2*c^6*d*e^6*f*z^2 - 224*A^2*
a^2*b*c^7*d^4*e*f^3*z^2 + 216*A^2*a*b^5*c^4*d^3*e*f^4*z^2 - 192*A^2*a^2*b*c
^7*d^2*e^5*f*z^2 + 178*A^2*a*b^7*c^2*d*e^3*f^4*z^2 - 154*A^2*a*b^7*c^2*d^2*
e*f^5*z^2 + 128*A^2*a*b^3*c^6*d^4*e*f^3*z^2 + 106*A^2*a*b^3*c^6*d^2*e^5*f*z
^2 - 12*A^2*a*b^2*c^7*d^3*e^4*f*z^2 - 58*A*B*b^8*c^2*d^2*e^3*f^3*z^2 + 40*A
*B*b^7*c^3*d^2*e^4*f^2*z^2 - 28*A*B*b^7*c^3*d^3*e^2*f^3*z^2 - 24*A*B*b^5*c^
5*d^4*e^2*f^2*z^2 - 20*A*B*b^6*c^4*d^3*e^3*f^2*z^2 + 2768*A*B*a^4*c^6*d^2*e
^3*f^3*z^2 - 1712*A*B*a^3*c^7*d^3*e^3*f^2*z^2 - 156*A*B*a^4*b^2*c^4*e^5*f^3
*z^2 + 146*A*B*a^4*b^3*c^3*e^4*f^4*z^2 - 106*A*B*a^5*b^2*c^3*e^3*f^5*z^2 +
90*A*B*a^5*b^3*c^2*e^2*f^6*z^2 + 38*A*B*a^3*b^3*c^4*e^6*f^2*z^2 - 36*A*B*a^
3*b^5*c^2*e^4*f^4*z^2 + 16*A*B*a^3*b^4*c^3*e^5*f^3*z^2 - 9*A*B*a^4*b^4*c^2*
e^3*f^5*z^2 - 8*A*B*a^2*b^5*c^3*e^6*f^2*z^2 + 2*A*B*a^2*b^6*c^2*e^5*f^3*z^2
 + 920*A*B*a^4*b^3*c^3*d^2*f^6*z^2 - 480*A*B*a^2*b^5*c^3*d^3*f^5*z^2 - 336*
A*B*a^2*b^3*c^5*d^4*f^4*z^2 - 272*A*B*a^3*b^3*c^4*d^3*f^5*z^2 + 240*A*B*a^3
*b^5*c^2*d^2*f^6*z^2 - 32*A*B*a*c^9*d^6*e*f*z^2 - 792*B^2*a^2*b^3*c^5*d^3*e
^3*f^2*z^2 + 714*B^2*a^2*b^4*c^4*d^3*e^2*f^3*z^2 - 572*B^2*a^3*b^2*c^5*d^3*
e^2*f^3*z^2 - 475*B^2*a^2*b^2*c^6*d^4*e^2*f^2*z^2 + 265*B^2*a^4*b^2*c^4*d^2
*e^2*f^4*z^2 + 260*B^2*a^3*b^3*c^4*d^2*e^3*f^3*z^2 - 212*B^2*a^3*b^4*c^3*d^
2*e^2*f^4*z^2 + 180*B^2*a^3*b^2*c^5*d^2*e^4*f^2*z^2 - 158*B^2*a^2*b^4*c^4*d
^2*e^4*f^2*z^2 + 47*B^2*a^2*b^6*c^2*d^2*e^2*f^4*z^2 + 16*B^2*a^2*b^5*c^3*d^
2*e^3*f^3*z^2 + 2752*A^2*a^3*b^2*c^5*d^2*e^2*f^4*z^2 - 2148*A^2*a^2*b^4*c^4
*d^2*e^2*f^4*z^2 + 2064*A^2*a^2*b^3*c^5*d^2*e^3*f^3*z^2 - 424*A^2*a^2*b^2*c
^6*d^3*e^2*f^3*z^2 - 198*A^2*a^2*b^2*c^6*d^2*e^4*f^2*z^2 - 272*B^2*a^6*b*c^
3*d*e*f^6*z^2 - 24*B^2*a^4*b^5*c*d*e*f^6*z^2 + 1808*A^2*a^5*b*c^4*d*e*f^6*z
^2 - 244*A^2*a*b*c^8*d^4*e^3*f*z^2 + 208*A^2*a*b*c^8*d^5*e*f^2*z^2 + 134*A^
2*a^2*b^7*c*d*e*f^6*z^2 - 76*A^2*a*b^4*c^5*d*e^6*f*z^2 + 4*A^2*a*b^8*c*d*e^
2*f^5*z^2 + 148*A*B*b^4*c^6*d^5*e*f^2*z^2 + 65*A*B*b^6*c^4*d^4*e*f^3*z^2 +

46*A*B*b^8*c^2*d^3*e*f^4*z^2 - 38*A*B*b^3*c^7*d^5*e^2*f*z^2 + 34*A*B*b^9*c*d^2*e^2*f^4*z^2 - 29*A*B*b^4*c^6*d^4*e^3*f*z^2 + 20*A*B*b^5*c^5*d^3*e^4*f*z^2 + 12*A*B*b^8*c^2*d*e^5*f^2*z^2 - 7*A*B*b^6*c^4*d^2*e^5*f*z^2 - 2880*A*B*a^4*c^6*d^3*e*f^4*z^2 + 2784*A*B*a^5*c^5*d^2*e*f^5*z^2 - 1112*A*B*a^5*c^5*d*e^3*f^4*z^2 + 896*A*B*a^3*c^7*d^4*e*f^3*z^2 + 848*A*B*a^3*c^7*d^2*e^5*f*z^2 - 560*A*B*a^4*c^6*d*e^5*f^2*z^2 + 96*A*B*a^2*c^8*d^5*e*f^2*z^2 - 88*A*B*a^2*c^8*d^4*e^3*f*z^2 - 100*A*B*a^6*b*c^3*e^2*f^6*z^2 - 76*A*B*a^5*b*c^4*e^4*f^4*z^2 + 48*A*B*a^6*b^2*c^2*e*f^7*z^2 - 42*A*B*a^3*b^2*c^5*e^7*f*z^2 + 36*A*B*a^4*b*c^5*e^6*f^2*z^2 - 24*A*B*a^4*b^5*c*e^2*f^6*z^2 + 10*A*B*a^3*b^6*c*e^3*f^5*z^2 + 7*A*B*a^2*b^4*c^4*e^7*f*z^2 + 2*A*B*a^2*b^7*c*e^4*f^4*z^2 - 2496*A*B*a^5*b*c^4*d^2*f^6*z^2 + 1872*A*B*a^4*b*c^5*d^3*f^5*z^2 - 744*A*B*a^5*b^3*c^2*d*f^7*z^2 - 720*A*B*a^2*b*c^7*d^5*f^3*z^2 + 504*A*B*a*b^3*c^6*d^5*f^3*z^2 + 256*A*B*a^3*b*c^6*d^4*f^4*z^2 + 168*A*B*a*b^7*c^2*d^3*f^5*z^2 - 144*A*B*a^2*b^7*c*d^2*f^6*z^2 + 144*A*B*a*b^5*c^4*d^4*f^4*z^2 + 66*A*B*a^2*b^2*c^6*d*e^7*z^2 - 36*A*B*a*b^2*c^7*d^3*e^5*z^2 + 20*A*B*a*b^3*c^6*d^2*e^6*z^2 + 12*A*B*a^2*b*c^7*d^2*e^6*z^2 + 1208*B^2*a^3*b*c^6*d^3*e^3*f^2*z^2 - 848*B^2*a^3*b^3*c^4*d^3*e*f^4*z^2 + 672*B^2*a^2*b^3*c^5*d^4*e*f^3*z^2 - 632*B^2*a^4*b*c^5*d^2*e^3*f^3*z^2 + 432*B^2*a^4*b^3*c^3*d^2*e*f^5*z^2 + 276*B^2*a^2*b^2*c^6*d^3*e^4*f*z^2 - 196*B^2*a*b^6*c^3*d^3*e^2*f^3*z^2 - 168*B^2*a^2*b^5*c^3*d^3*e*f^4*z^2 + 154*B^2*a^2*b^3*c^5*d^2*e^5*f*z^2 + 148*B^2*a*b^5*c^4*d^3*e^3*f^2*z^2 + 96*B^2*a*b^4*c^5*d^4*e^2*f^2*z^2 - 72*B^2*a^3*b^5*c^2*d^2*e*f^5*z^2 + 70*B^2*a^5*b^2*c^3*d*e^2*f^5*z^2 - 60*B^2*a^4*b^3*c^3*d*e^3*f^4*z^2 + 52*B^2*a*b^6*c^3*d^2*e^4*f^2*z^2 + 36*B^2*a^4*b^2*c^4*d*e^4*f^3*z^2 - 32*B^2*a*b^7*c^2*d^2*e^3*f^3*z^2 + 24*B^2*a^3*b^5*c^2*d*e^3*f^4*z^2 + 15*B^2*a^4*b^4*c^2*d*e^2*f^5*z^2 - 8*B^2*a^3*b^4*c^3*d*e^4*f^3*z^2 + 8*B^2*a^2*b^5*c^3*d*e^5*f^2*z^2 - 2*B^2*a^3*b^3*c^4*d*e^5*f^2*z^2 - 2*B^2*a^2*b^6*c^2*d*e^4*f^3*z^2 - 3176*A^2*a^3*b*c^6*d^2*e^3*f^3*z^2 - 2252*A^2*a^4*b^2*c^4*d*e^2*f^5*z^2 + 1952*A^2*a^3*b^4*c^3*d*e^2*f^5*z^2 - 1496*A^2*a^3*b^3*c^4*d*e^3*f^4*z^2 + 1378*A^2*a^2*b^4*c^4*d*e^4*f^3*z^2 + 1184*A^2*a^3*b^3*c^4*d^2*e*f^5*z^2 - 1166*A^2*a^2*b^3*c^5*d*e^5*f^2*z^2 - 1164*A^2*a^3*b^2*c^5*d*e^4*f^3*z^2 - 1152*A^2*a^2*b^3*c^5*d^3*e*f^4*z^2 + 578*A^2*a*b^6*c^3*d^2*e^2*f^4*z^2 - 548*A^2*a*b^5*c^4*d^2*e^3*f^3*z^2 + 440*A^2*a*b^2*c^7*d^4*e^2*f^2*z^2 - 412*A^2*a^2*b^6*c^2*d*e^2*f^5*z^2 - 360*A^2*a*b^3*c^6*d^3*e^3*f^2*z^2 + 312*A^2*a*b^4*c^5*d^3*e^2*f^3*z^2 + 248*A^2*a^2*b*c^7*d^3*e^3*f^2*z^2 - 224*A^2*a^2*b^5*c^3*d*e^3*f^4*z^2 + 216*A^2*a^2*b^5*c^3*d^2*e*f^5*z^2 + 52*A^2*a*b^4*c^5*d^2*e^4*f^2*z^2 - 16*B^2*b^3*c^7*d^6*e*f*z^2 - 14*B^2*b^9*c*d^3*e*f^4*z^2 + 32*B^2*a^4*c^6*d*e^6*f*z^2 - 20*A^2*b^9*c*d*e^3*f^4*z^2 + 18*A^2*b^9*c*d^2*e*f^5*z^2 + 8*A^2*b^6*c^4*d*e^6*f*z^2 - 360*A^2*a^3*c^7*d*e^6*f*z^2 + 136*A^2*a*c^9*d^5*e^2*f*z^2 + 2*B^2*a^3*b^7*d*e*f^6*z^2 + 2*B^2*a*b^9*d^2*e*f^5*z^2 + 12*B^2*a^4*b*c^5*e^7*f*z^2 - 204*A^2*a^3*b*c^6*e^7*f*z^2 - 128*A^2*a^6*b*c^3*e*f^7*z^2 - 48*A^2*a*b^5*c^4*e^7*f*z^2 - 36*B^2*a^5*b^4*c*d*f^7*z^2 - 24*A^2*a^4*b^5*c*e*f^7*z^2 - 16*B^2*a*b^8*c*d^3*f^5*z^2 - 164*A^2*a^3*b^6*c*d*f^7*z^2 - 16*A^2*a*b^8*c*d^2*f^6*z^2 + 4*B^2*a^3*b*c^6*d*e^7*z^2 - 4*B^2*a*b*c^8*d^5*e^3*z^2 + 48*A^2*a*b*c^8*d^3*e^5*z^2 + 36*A^2*a^2*b*c^7*d*e^7*z^2 - 6*A^2*a*b^3*c^6*d*e^7*z^2 + 136*A*B*a^6

*c^4*e^3*f^5*z^2 - 96*A*B*b^5*c^5*d^5*f^3*z^2 + 80*A*B*a^5*c^5*e^5*f^3*z^2
- 72*A*B*b^3*c^7*d^6*f^2*z^2 - 24*A*B*b^7*c^3*d^4*f^4*z^2 + 14*A*B*b^3*c^7*
d^4*e^4*z^2 - 14*A*B*b^2*c^8*d^5*e^3*z^2 - 2*A*B*b^5*c^5*d^2*e^6*z^2 - 2*A*
B*b^4*c^6*d^3*e^5*z^2 + 2*A*B*a^3*b^7*e^2*f^6*z^2 - A*B*a^2*b^8*e^3*f^5*z^2
 + 16*A*B*a^2*c^8*d^3*e^5*z^2 - 2*A*B*a^2*b^3*c^5*e^8*z^2 + 22*B^2*b^8*c^2*
d^3*e^2*f^3*z^2 - 12*B^2*b^7*c^3*d^3*e^3*f^2*z^2 + 12*B^2*b^6*c^4*d^4*e^2*f
^2*z^2 - 6*B^2*b^8*c^2*d^2*e^4*f^2*z^2 - 864*B^2*a^4*c^6*d^3*e^2*f^3*z^2 +
496*B^2*a^3*c^7*d^4*e^2*f^2*z^2 + 224*B^2*a^5*c^5*d^2*e^2*f^4*z^2 + 136*B^2
*a^4*c^6*d^2*e^4*f^2*z^2 - 53*A^2*b^8*c^2*d^2*e^2*f^4*z^2 + 52*A^2*b^7*c^3*
d^2*e^3*f^3*z^2 + 52*A^2*b^5*c^5*d^3*e^3*f^2*z^2 - 36*A^2*b^6*c^4*d^3*e^2*f
^3*z^2 - 12*A^2*b^4*c^6*d^4*e^2*f^2*z^2 - 9*A^2*b^6*c^4*d^2*e^4*f^2*z^2 + 8
36*A^2*a^4*c^6*d^2*e^2*f^4*z^2 - 668*A^2*a^2*c^8*d^4*e^2*f^2*z^2 + 656*A^2*
a^3*c^7*d^2*e^4*f^2*z^2 + 368*A^2*a^3*c^7*d^3*e^2*f^3*z^2 - 45*B^2*a^6*b^2*
c^2*e^2*f^6*z^2 - 18*B^2*a^5*b^2*c^3*e^4*f^4*z^2 - 9*B^2*a^4*b^2*c^4*e^6*f^
2*z^2 - 6*B^2*a^5*b^3*c^2*e^3*f^5*z^2 + 3*B^2*a^4*b^4*c^2*e^4*f^4*z^2 - 2*B
^2*a^4*b^3*c^3*e^5*f^3*z^2 - 580*B^2*a^4*b^2*c^4*d^3*f^5*z^2 + 536*B^2*a^3*
b^4*c^3*d^3*f^5*z^2 + 471*A^2*a^4*b^2*c^4*e^4*f^4*z^2 - 436*A^2*a^3*b^4*c^3
*e^4*f^4*z^2 - 348*B^2*a^4*b^4*c^2*d^2*f^6*z^2 + 316*B^2*a^2*b^2*c^6*d^5*f^
3*z^2 + 310*A^2*a^3*b^3*c^4*e^5*f^3*z^2 + 232*A^2*a^5*b^2*c^3*e^2*f^6*z^2 -
 229*A^2*a^2*b^4*c^4*e^6*f^2*z^2 - 216*A^2*a^4*b^4*c^2*e^2*f^6*z^2 + 204*A^
2*a^4*b^3*c^3*e^3*f^5*z^2 + 200*B^2*a^5*b^2*c^3*d^2*f^6*z^2 + 150*A^2*a^3*b
^2*c^5*e^6*f^2*z^2 - 120*B^2*a^2*b^4*c^4*d^4*f^4*z^2 + 91*A^2*a^2*b^6*c^2*e
^4*f^4*z^2 + 72*A^2*a^3*b^5*c^2*e^3*f^5*z^2 - 66*B^2*a^2*b^6*c^2*d^3*f^5*z^
2 + 44*A^2*a^2*b^5*c^3*e^5*f^3*z^2 - 16*B^2*a^3*b^2*c^5*d^4*f^4*z^2 + 1952*
A^2*a^4*b^2*c^4*d^2*f^6*z^2 - 1792*A^2*a^3*b^2*c^5*d^3*f^5*z^2 - 1272*A^2*a
^3*b^4*c^3*d^2*f^6*z^2 + 976*A^2*a^2*b^2*c^6*d^4*f^4*z^2 + 960*A^2*a^2*b^4*
c^4*d^3*f^5*z^2 + 282*A^2*a^2*b^6*c^2*d^2*f^6*z^2 - 45*B^2*a^2*b^2*c^6*d^2*
e^6*z^2 - 48*A^2*b*c^9*d^6*e*f*z^2 - 14*A^2*a*b^9*d*e*f^6*z^2 - 7*A*B*b^10*
d^2*e*f^5*z^2 + 2*A*B*b^10*d*e^3*f^4*z^2 - 64*A*B*a^7*c^3*e*f^7*z^2 - 16*A*
B*b^9*c*d^3*f^5*z^2 + 8*A*B*a^4*c^6*e^7*f*z^2 + 4*A*B*b*c^9*d^6*e^2*z^2 + 2
*A*B*b^6*c^4*d*e^7*z^2 - 120*A*B*a^3*c^7*d*e^7*z^2 - 16*A*B*a^3*b^7*d*f^7*z
^2 + 16*A*B*a*b^9*d^2*f^6*z^2 + 8*A*B*a*c^9*d^5*e^3*z^2 + 12*A*B*a^3*b*c^6*
e^8*z^2 - 48*B^2*b^5*c^5*d^5*e*f^2*z^2 + 15*B^2*b^4*c^6*d^5*e^2*f*z^2 - 14*
B^2*b^7*c^3*d^4*e*f^3*z^2 + 4*B^2*b^9*c*d^2*e^3*f^3*z^2 + 4*B^2*b^7*c^3*d^2
*e^5*f*z^2 + 4*B^2*b^5*c^5*d^4*e^3*f*z^2 - B^2*b^6*c^4*d^3*e^4*f*z^2 - 336*
B^2*a^3*c^7*d^3*e^4*f*z^2 + 112*B^2*a^5*c^5*d*e^4*f^3*z^2 - 112*A^2*b^3*c^7
*d^5*e*f^2*z^2 + 80*B^2*a^6*c^4*d*e^2*f^5*z^2 - 48*A^2*b^5*c^5*d^4*e*f^3*z^
2 + 36*A^2*b^8*c^2*d*e^4*f^3*z^2 + 36*A^2*b^3*c^7*d^4*e^3*f*z^2 - 28*A^2*b^
7*c^3*d*e^5*f^2*z^2 + 20*A^2*b^2*c^8*d^5*e^2*f*z^2 + 16*B^2*a^2*c^8*d^5*e^2
*f*z^2 - 14*A^2*b^7*c^3*d^3*e*f^4*z^2 - 14*A^2*b^4*c^6*d^3*e^4*f*z^2 - 10*A
^2*b^5*c^5*d^2*e^5*f*z^2 - 1008*A^2*a^4*c^6*d*e^4*f^3*z^2 - 760*A^2*a^5*c^5
*d*e^2*f^5*z^2 + 272*A^2*a^2*c^8*d^3*e^4*f*z^2 + 48*B^2*a^5*b*c^4*e^5*f^3*z
^2 + 36*B^2*a^6*b*c^3*e^3*f^5*z^2 + 12*B^2*a^5*b^4*c*e^2*f^6*z^2 - 624*A^2*
a^4*b*c^5*e^5*f^3*z^2 - 548*A^2*a^5*b*c^4*e^3*f^5*z^2 + 182*A^2*a^2*b^3*c^5
*e^7*f*z^2 - 180*B^2*a*b^4*c^5*d^5*f^3*z^2 + 132*B^2*a^6*b^2*c^2*d*f^7*z^2

+ 108*B^2*a^3*b^6*c*d^2*f^6*z^2 + 96*A^2*a^5*b^3*c^2*e*f^7*z^2 + 68*A^2*a*b
^6*c^3*e^6*f^2*z^2 + 58*A^2*a^3*b^6*c*e^2*f^6*z^2 - 56*B^2*a*b^2*c^7*d^6*f^
2*z^2 - 38*A^2*a^2*b^7*c*e^3*f^5*z^2 - 36*A^2*a*b^7*c^2*e^5*f^3*z^2 + 20*B^
2*a*b^6*c^3*d^4*f^4*z^2 - 736*A^2*a^5*b^2*c^3*d*f^7*z^2 + 624*A^2*a^4*b^4*c
^2*d*f^7*z^2 - 416*A^2*a*b^2*c^7*d^5*f^3*z^2 - 276*A^2*a*b^4*c^5*d^4*f^4*z^
2 - 196*A^2*a*b^6*c^3*d^3*f^5*z^2 + 8*B^2*a*b^4*c^5*d^2*e^6*z^2 + 6*B^2*a*b
^2*c^7*d^4*e^4*z^2 + 2*B^2*a^2*b^3*c^5*d*e^7*z^2 + 2*B^2*a*b^3*c^6*d^3*e^5*
z^2 - 18*A^2*a*b^2*c^7*d^2*e^6*z^2 - 16*A*B*b*c^9*d^7*f*z^2 - B^2*b^10*d^2*
e^2*f^4*z^2 + 48*B^2*a^7*c^3*e^2*f^6*z^2 - 36*B^2*a^6*c^4*e^4*f^4*z^2 + 31*
B^2*b^6*c^4*d^5*f^3*z^2 - 24*B^2*a^5*c^5*e^6*f^2*z^2 + 20*B^2*b^4*c^6*d^6*f
^2*z^2 - 6*A^2*b^8*c^2*e^6*f^2*z^2 + 2*B^2*b^8*c^2*d^4*f^4*z^2 - 768*B^2*a^
5*c^5*d^3*f^5*z^2 + 512*B^2*a^6*c^4*d^2*f^6*z^2 + 512*B^2*a^4*c^6*d^4*f^4*z
^2 + 232*A^2*a^5*c^5*e^4*f^4*z^2 + 188*A^2*a^4*c^6*e^6*f^2*z^2 - 128*B^2*a^
3*c^7*d^5*f^3*z^2 + 92*A^2*a^6*c^4*e^2*f^6*z^2 + 80*A^2*b^4*c^6*d^5*f^3*z^2
+ 64*A^2*b^2*c^8*d^6*f^2*z^2 + 31*A^2*b^6*c^4*d^4*f^4*z^2 + 14*A^2*b^8*c^2
*d^3*f^5*z^2 - 5*B^2*b^4*c^6*d^4*e^4*z^2 + 4*B^2*b^3*c^7*d^5*e^3*z^2 + 2*B^
2*b^5*c^5*d^3*e^5*z^2 - B^2*b^6*c^4*d^2*e^6*z^2 - B^2*b^2*c^8*d^6*e^2*z^2 -
B^2*a^4*b^6*e^2*f^6*z^2 - 1152*A^2*a^3*c^7*d^4*f^4*z^2 + 1008*A^2*a^4*c^6*
d^3*f^5*z^2 + 624*A^2*a^2*c^8*d^5*f^3*z^2 - 288*A^2*a^5*c^5*d^2*f^6*z^2 + 5
6*B^2*a^3*c^7*d^2*e^6*z^2 - 10*B^2*a^2*b^8*d^2*f^6*z^2 - 9*A^2*b^2*c^8*d^4*
e^4*z^2 - 5*A^2*a^2*b^8*e^2*f^6*z^2 - 4*B^2*a^2*c^8*d^4*e^4*z^2 + 3*A^2*b^4
*c^6*d^2*e^6*z^2 - 2*A^2*b^3*c^7*d^3*e^5*z^2 - 36*A^2*a^2*c^8*d^2*e^6*z^2 -
48*A^2*a^6*b^2*c^2*f^8*z^2 - 45*A^2*a^2*b^2*c^6*e^8*z^2 + 4*A^2*b^10*d*e^2
*f^5*z^2 + 4*B^2*b^2*c^8*d^7*f*z^2 + 4*A^2*b^9*c*e^5*f^3*z^2 + 4*A^2*b^7*c^
3*e^7*f*z^2 - 128*B^2*a^7*c^3*d*f^7*z^2 - 160*A^2*a*c^9*d^6*f^2*z^2 - 112*A
^2*a^6*c^4*d*f^7*z^2 + 12*A^2*b*c^9*d^5*e^3*z^2 + 4*A^2*a*b^9*e^3*f^5*z^2 +
3*B^2*a^4*b^6*d*f^7*z^2 + 2*A^2*a^3*b^7*e*f^7*z^2 - 24*A^2*a*c^9*d^4*e^4*z
^2 + 14*A^2*a^2*b^8*d*f^7*z^2 + 12*A^2*a^5*b^4*c*f^8*z^2 + 12*A^2*a*b^4*c^5
*e^8*z^2 + A*B*a^4*b^6*e*f^7*z^2 + B^2*a^2*b^8*d*e^2*f^5*z^2 + 16*A^2*c^10*
d^7*f*z^2 + 3*B^2*b^10*d^3*f^5*z^2 - A^2*b^10*e^4*f^4*z^2 - 4*A^2*c^10*d^6*
e^2*z^2 - A^2*b^10*d^2*f^6*z^2 + 64*A^2*a^7*c^3*f^8*z^2 - 4*B^2*a^4*c^6*e^8
*z^2 - A^2*b^6*c^4*e^8*z^2 + 48*A^2*a^3*c^7*e^8*z^2 - A^2*a^4*b^6*f^8*z^2 +
720*A^2*B*a*b^2*c^5*d^2*e^2*f^3*z - 600*A^2*B*a^2*b^2*c^4*d*e^2*f^4*z + 57
6*A*B^2*a^2*b^2*c^4*d^2*e*f^4*z + 348*A*B^2*a*b^2*c^5*d^2*e^3*f^2*z - 336*A
*B^2*a^2*b*c^5*d^2*e^2*f^3*z - 260*A*B^2*a*b^3*c^4*d^2*e^2*f^3*z - 240*A*B^
2*a^2*b^2*c^4*d*e^3*f^3*z + 196*A*B^2*a^2*b^3*c^3*d*e^2*f^4*z + 172*A^2*B*a
*b*c^6*d*e^5*f*z + 20*A*B^2*a*b^6*c*d*e*f^5*z - 912*A^2*B*a^2*b*c^5*d^2*e*f
^4*z - 644*A^2*B*a*b*c^6*d^2*e^3*f^2*z - 432*A*B^2*a*b^2*c^5*d^3*e*f^3*z +
372*A^2*B*a^2*b*c^5*d*e^3*f^3*z - 330*A^2*B*a*b^2*c^5*d*e^4*f^2*z + 312*A*B
^2*a*b*c^6*d^3*e^2*f^2*z - 208*A*B^2*a^3*b^2*c^3*d*e*f^5*z + 192*A^2*B*a^2*
b^3*c^3*d*e*f^5*z + 172*A^2*B*a*b^3*c^4*d*e^3*f^3*z + 108*A*B^2*a^2*b*c^5*d
*e^4*f^2*z + 104*A*B^2*a^3*b*c^4*d*e^2*f^4*z - 80*A^2*B*a*b^3*c^4*d^2*e*f^4
*z + 68*A^2*B*a*b^4*c^3*d*e^2*f^4*z - 60*A*B^2*a*b^5*c^2*d*e^2*f^4*z + 58*A
*B^2*a*b^3*c^4*d*e^4*f^2*z - 36*A*B^2*a*b^4*c^3*d^2*e*f^4*z - 24*A*B^2*a^2*
b^4*c^2*d*e*f^5*z + 24*A*B^2*a*b^4*c^3*d*e^3*f^3*z + 592*A^2*B*a*b*c^6*d^3*

e*f^3*z + 240*A^2*B*a^3*b*c^4*d*e*f^5*z - 132*A*B^2*a*b*c^6*d^2*e^4*f*z - 6
0*A*B^2*a*b^2*c^5*d*e^5*f*z - 48*A^2*B*a*b^5*c^2*d*e*f^5*z + 20*B^3*a*b*c^6
*d^3*e^3*f*z + 16*B^3*a^4*b*c^3*d*e*f^5*z - 16*B^3*a*b*c^6*d^4*e*f^2*z + 12
*B^3*a^2*b*c^5*d*e^5*f*z + 320*A^3*a*b*c^6*d*e^4*f^2*z + 40*A^3*a*b^4*c^3*d
*e*f^5*z - 48*A^2*B*b*c^7*d^4*e*f^2*z - 44*A^2*B*b^3*c^5*d*e^5*f*z - 20*A*B
^2*b*c^7*d^4*e^2*f*z + 14*A*B^2*b^4*c^4*d*e^5*f*z + 12*A^2*B*b*c^7*d^3*e^3*
f*z + 4*A*B^2*b^7*c*d*e^2*f^4*z + 160*A*B^2*a^4*c^4*d*e*f^5*z + 152*A^2*B*a
*c^7*d^2*e^4*f*z - 40*A*B^2*a*c^7*d^3*e^3*f*z + 32*A*B^2*a*c^7*d^4*e*f^2*z
- 16*A*B^2*a^2*c^6*d*e^5*f*z + 128*A^2*B*a^4*b*c^3*e*f^6*z + 42*A^2*B*a*b^2
*c^5*e^6*f*z + 24*A^2*B*a^2*b^5*c*e*f^6*z - 12*A*B^2*a^3*b^4*c*e*f^6*z - 12
*A*B^2*a^2*b*c^5*e^6*f*z - 10*A^2*B*a*b^6*c*e^2*f^5*z - 160*A*B^2*a*b*c^6*d
^4*f^3*z + 112*A*B^2*a^4*b*c^3*d*f^6*z - 24*A*B^2*a^2*b^5*c*d*f^6*z - 84*B^
3*a*b^2*c^5*d^3*e^2*f^2*z - 80*B^3*a^2*b^3*c^3*d^2*e*f^4*z - 60*B^3*a^2*b*c
^5*d^2*e^3*f^2*z - 20*B^3*a^3*b^2*c^3*d*e^2*f^4*z - 20*B^3*a*b^3*c^4*d^2*e^
3*f^2*z - 9*B^3*a^2*b^2*c^4*d*e^4*f^2*z - 8*B^3*a*b^4*c^3*d^2*e^2*f^3*z + 6
*B^3*a^2*b^4*c^2*d*e^2*f^4*z - 4*B^3*a^2*b^3*c^3*d*e^3*f^3*z - 216*A^2*B*b^
4*c^4*d^2*e^2*f^3*z + 196*A^2*B*b^3*c^5*d^2*e^3*f^2*z - 108*A*B^2*b^3*c^5*d
^3*e^2*f^2*z - 94*A*B^2*b^4*c^4*d^2*e^3*f^2*z + 88*A^2*B*b^2*c^6*d^3*e^2*f^
2*z + 80*A*B^2*b^5*c^3*d^2*e^2*f^3*z + 360*A^2*B*a^2*c^6*d^2*e^2*f^3*z + 8*
A*B^2*a^2*c^6*d^2*e^3*f^2*z + 153*A^2*B*a^2*b^2*c^4*e^4*f^3*z - 144*A^2*B*a
^2*b^3*c^3*e^3*f^4*z + 80*A^2*B*a^3*b^2*c^3*e^2*f^5*z + 36*A*B^2*a^3*b^2*c^
3*e^3*f^4*z + 12*A^2*B*a^2*b^4*c^2*e^2*f^5*z + 12*A*B^2*a^3*b^3*c^2*e^2*f^5
*z + 9*A*B^2*a^2*b^2*c^4*e^5*f^2*z - 6*A*B^2*a^2*b^4*c^2*e^3*f^4*z + 4*A*B^
2*a^2*b^3*c^3*e^4*f^3*z + 480*A^2*B*a^2*b^2*c^4*d^2*f^5*z - 176*A*B^2*a^2*b
^3*c^3*d^2*f^5*z - 10*A^2*B*a*b^6*c*d*f^6*z + 16*A*B^2*a*b*c^6*d*e^6*z + 80
*B^3*a*b^3*c^4*d^3*e*f^3*z - 48*B^3*a^3*b*c^4*d^2*e*f^4*z + 48*B^3*a^2*b*c^
5*d^3*e*f^3*z + 44*B^3*a^3*b*c^4*d*e^3*f^3*z + 24*B^3*a*b^5*c^2*d^2*e*f^4*z
 + 18*B^3*a*b^2*c^5*d^2*e^4*f*z + 696*A^3*a^2*b*c^5*d*e^2*f^4*z - 504*A^3*a
*b*c^6*d^2*e^2*f^3*z - 192*A^3*a*b^2*c^5*d*e^3*f^3*z - 144*A^3*a^2*b^2*c^4*
d*e*f^5*z + 96*A^3*a*b^2*c^5*d^2*e*f^4*z - 72*A^3*a*b^3*c^4*d*e^2*f^4*z - 2
08*A^2*B*b^3*c^5*d^3*e*f^3*z + 152*A*B^2*b^4*c^4*d^3*e*f^3*z + 80*A^2*B*b^5
*c^3*d^2*e*f^4*z + 75*A^2*B*b^4*c^4*d*e^4*f^2*z - 59*A^2*B*b^2*c^6*d^2*e^4*
f*z - 52*A^2*B*b^5*c^3*d*e^3*f^3*z + 42*A*B^2*b^3*c^5*d^2*e^4*f*z - 21*A*B^
2*b^6*c^2*d^2*e*f^4*z - 16*A*B^2*b^5*c^3*d*e^4*f^2*z + 16*A*B^2*b^2*c^6*d^4
*e*f^2*z + 16*A*B^2*b^2*c^6*d^3*e^3*f*z + 11*A^2*B*b^6*c^2*d*e^2*f^4*z + 4*
A*B^2*b^6*c^2*d*e^3*f^3*z - 256*A^2*B*a*c^7*d^3*e^2*f^2*z - 96*A*B^2*a^3*c^
5*d^2*e*f^4*z - 36*A^2*B*a^2*c^6*d*e^4*f^2*z - 32*A^2*B*a^3*c^5*d*e^2*f^4*z
 - 32*A*B^2*a^2*c^6*d^3*e*f^3*z + 8*A*B^2*a^3*c^5*d*e^3*f^3*z - 96*A^2*B*a^
3*b^3*c^2*e*f^6*z + 68*A^2*B*a^3*b*c^4*e^3*f^4*z - 60*A*B^2*a^4*b*c^3*e^2*f
^5*z - 60*A*B^2*a^3*b*c^4*e^4*f^3*z + 48*A*B^2*a^4*b^2*c^2*e*f^6*z - 38*A^2
*B*a*b^3*c^4*e^5*f^2*z - 36*A^2*B*a^2*b*c^5*e^5*f^2*z + 36*A^2*B*a*b^5*c^2*
e^3*f^4*z - 16*A^2*B*a*b^4*c^3*e^4*f^3*z + 384*A*B^2*a^2*b*c^5*d^3*f^4*z -
352*A*B^2*a^3*b*c^4*d^2*f^5*z - 288*A^2*B*a*b^2*c^5*d^3*f^4*z - 160*A^2*B*a
^3*b^2*c^3*d*f^6*z - 148*A^2*B*a*b^4*c^3*d^2*f^5*z + 112*A*B^2*a*b^3*c^4*d^
3*f^4*z + 72*A^2*B*a^2*b^4*c^2*d*f^6*z + 72*A*B^2*a*b^5*c^2*d^2*f^5*z + 48*

A*B^2*a^3*b^3*c^2*d*f^6*z + 102*B^3*a^2*b^2*c^4*d^2*e^2*f^3*z - 32*B^3*b^5*
c^3*d^3*e*f^3*z - 8*B^3*b^3*c^5*d^3*e^3*f*z - 7*B^3*b^4*c^4*d^2*e^4*f*z + 5
*B^3*b^2*c^6*d^4*e^2*f*z + 80*A^3*b^2*c^6*d^3*e*f^3*z - 74*A^3*b^3*c^5*d*e^
4*f^2*z - 64*A^3*b^4*c^4*d^2*e*f^4*z + 60*A^3*b^4*c^4*d*e^3*f^3*z - 48*B^3*
a^4*c^4*d*e^2*f^4*z - 24*B^3*a^3*c^5*d*e^4*f^2*z + 20*B^3*a^2*c^6*d^2*e^4*f
*z - 16*A^3*b^5*c^3*d*e^2*f^4*z + 8*A^3*b*c^7*d^3*e^2*f^2*z + 480*A^3*a^2*c
^6*d^2*e*f^4*z - 392*A^3*a^2*c^6*d*e^3*f^3*z + 280*A^3*a*c^7*d^2*e^3*f^2*z
- 4*B^3*a^4*b*c^3*e^3*f^4*z - 200*A^3*a^3*b*c^4*e^2*f^5*z - 144*A^3*a^2*b*c
^5*e^4*f^3*z + 48*B^3*a*b^2*c^5*d^4*f^3*z + 42*A^3*a*b^2*c^5*e^5*f^2*z - 36
*B^3*a^4*b^2*c^2*d*f^6*z - 32*A^3*a^3*b^2*c^3*e*f^6*z - 24*A^3*a^2*b^4*c^2*
e*f^6*z - 24*A^3*a*b^5*c^2*e^2*f^5*z + 10*A^3*a*b^3*c^4*e^4*f^3*z - 4*B^3*a
*b^4*c^3*d^3*f^4*z - 4*A^3*a*b^4*c^3*e^3*f^4*z - 480*A^3*a^2*b*c^5*d^2*f^5*
z - 160*A^3*a^2*b^3*c^3*d*f^6*z + 128*A^3*a*b^3*c^4*d^2*f^5*z + 8*A^2*B*b^5
*c^3*e^5*f^2*z - 2*A^2*B*b^6*c^2*e^4*f^3*z + 112*A^2*B*b^4*c^4*d^3*f^4*z -
92*A^2*B*a^4*c^4*e^2*f^5*z - 64*A^2*B*a^3*c^5*e^4*f^3*z - 64*A*B^2*b^5*c^3*
d^3*f^4*z + 24*A*B^2*a^4*c^4*e^3*f^4*z + 24*A*B^2*a^3*c^5*e^5*f^2*z + 16*A^
2*B*b^2*c^6*d^4*f^3*z + 16*A*B^2*b^3*c^5*d^4*f^3*z - A^2*B*b^6*c^2*d^2*f^5*
z + 448*A^2*B*a^3*c^5*d^2*f^5*z - 352*A^2*B*a^2*c^6*d^3*f^4*z - 5*A*B^2*b^2
*c^6*d^2*e^5*z - 48*A^2*B*a^4*b^2*c^2*f^7*z - 2*B^3*b^7*c*d^2*e*f^4*z + 34*
A^3*b^2*c^6*d*e^5*f*z + 16*A^3*b*c^7*d^2*e^4*f*z + 2*A^3*b^6*c^2*d*e*f^5*z
- 416*A^3*a^3*c^5*d*e*f^5*z - 224*A^3*a*c^7*d^3*e*f^3*z + 12*B^3*a^3*b^4*c*
d*f^6*z - 10*B^3*a*b^6*c*d^2*f^5*z + 416*A^3*a^3*b*c^4*d*f^6*z + 224*A^3*a*
b*c^6*d^3*f^4*z + 24*A^3*a*b^5*c^2*d*f^6*z - 4*B^3*a*b*c^6*d^2*e^5*z + 20*A
^2*B*c^8*d^4*e^2*f*z - 7*A^2*B*b^4*c^4*e^6*f*z - 2*A^2*B*b^7*c*e^3*f^4*z -
64*A*B^2*a^5*c^3*e*f^6*z + 16*A*B^2*b*c^7*d^5*f^2*z - 8*A^2*B*a^2*c^6*e^6*f
*z - 2*A*B^2*b^7*c*d^2*f^5*z - 272*A^2*B*a^4*c^4*d*f^6*z + 128*A^2*B*a*c^7*
d^4*f^3*z + 9*A^2*B*b^2*c^6*d*e^6*z - 4*A*B^2*b^3*c^5*d*e^6*z + 4*A*B^2*b*c
^7*d^3*e^4*z + 8*A*B^2*a*c^7*d^2*e^5*z + 12*A^2*B*a^3*b^4*c*f^7*z + 30*B^3*
b^4*c^4*d^3*e^2*f^2*z + 8*B^3*b^5*c^3*d^2*e^3*f^2*z - 2*B^3*b^6*c^2*d^2*e^2
*f^3*z + 152*A^3*b^3*c^5*d^2*e^2*f^3*z - 108*A^3*b^2*c^6*d^2*e^3*f^2*z + 48
*B^3*a^3*c^5*d^2*e^2*f^3*z - 16*B^3*a^2*c^6*d^3*e^2*f^2*z - 3*B^3*a^4*b^2*c
^2*e^2*f^5*z - 120*B^3*a^2*b^2*c^4*d^3*f^4*z + 112*B^3*a^3*b^2*c^3*d^2*f^5*
z + 112*A^3*a^2*b^3*c^3*e^2*f^5*z + 12*A^3*a^2*b^2*c^4*e^3*f^4*z - 120*A^3*
a*c^7*d*e^5*f*z - 52*A^3*a*b*c^6*e^6*f*z + 10*A^3*a*b^6*c*e*f^6*z - 2*A*B^2
*b^8*d*e*f^5*z - 2*A^2*B*a*b^7*e*f^6*z - 24*A^2*B*a*c^7*d*e^6*z + 2*A*B^2*a
*b^7*d*f^6*z - 12*A^2*B*a*b*c^6*e^7*z - 2*A^3*b^7*c*d*f^6*z - 4*A^3*b*c^7*d
*e^6*z + 16*B^3*a^5*c^3*e^2*f^5*z + 11*B^3*b^6*c^2*d^3*f^4*z - 11*A^3*b^4*c
^4*e^5*f^2*z - 8*B^3*b^4*c^4*d^4*f^3*z - 4*B^3*b^2*c^6*d^5*f^2*z + 4*B^3*a^
4*c^4*e^4*f^3*z + 4*A^3*b^5*c^3*e^4*f^3*z - A^3*b^6*c^2*e^3*f^4*z + 136*A^3
*a^3*c^5*e^3*f^4*z + 68*A^3*a^2*c^6*e^5*f^2*z - 64*A^3*b^3*c^5*d^3*f^4*z +
2*B^3*b^3*c^5*d^2*e^5*z - B^3*b^2*c^6*d^3*e^4*z + 96*A^3*a^3*b^3*c^2*f^7*z
+ A*B^2*a^2*b^6*e*f^6*z + 32*A^3*c^8*d^4*e*f^2*z - 24*A^3*c^8*d^3*e^3*f*z +
10*A^3*b^3*c^5*e^6*f*z + 2*A^3*b^7*c*e^2*f^5*z + 128*A^3*a^4*c^4*e*f^6*z -
32*A^3*b*c^7*d^4*f^3*z - 4*B^3*a^2*c^6*d*e^6*z - B^3*a^2*b^6*d*f^6*z - 128
*A^3*a^4*b*c^3*f^7*z - 24*A^3*a^2*b^5*c*f^7*z - 16*A^2*B*c^8*d^5*f^2*z - 4*

A^2*B*c^8*d^3*e^4*z + 64*A^2*B*a^5*c^3*f^7*z + 2*A^2*B*b^3*c^5*e^7*z + 4*A*B^2*a^2*c^6*e^7*z - A^2*B*a^2*b^6*f^7*z + 4*A^3*c^8*d^2*e^5*z - 3*A^3*b^2*c^6*e^7*z + A^2*B*b^8*d*f^6*z - A^3*b^8*e*f^6*z + 16*A^3*a*c^7*e^7*z + 2*A^3*a*b^7*f^7*z + A^2*B*b^8*e^2*f^5*z + B^3*b^8*d^2*f^5*z - 48*A^2*B^2*a*b*c^4*d*e*f^4 + 28*A*B^3*a*b^2*c^3*d*e*f^4 - 16*A*B^3*a*b*c^4*d*e^2*f^3 + 16*A^3*B*a*c^5*d*e*f^4 + 32*A^3*B*a*b*c^4*d*f^5 + 12*A^2*B^2*b^3*c^3*d*e*f^4 + 5*A*B^3*b^2*c^4*d^2*e*f^3 + 4*A*B^3*b^3*c^3*d*e^2*f^3 + 24*A^2*B^2*a*c^5*d*e^2*f^3 + 24*A^2*B^2*a^2*b*c^3*e*f^5 + 12*A^2*B^2*a*b*c^4*e^3*f^3 - 6*A^2*B^2*a*b^3*c^2*e*f^5 + 4*A*B^3*a^2*b*c^3*e^2*f^4 + 3*A*B^3*a^2*b^2*c^2*e*f^5 - 18*A^2*B^2*a*b^2*c^3*d*f^5 - 4*B^4*a^2*b*c^3*d*e*f^4 + 4*B^4*a*b*c^4*d^2*e*f^3 - 6*A*B^3*b^4*c^2*d*e*f^4 + 4*A^3*B*b*c^5*d*e^2*f^3 - 2*A^3*B*b^2*c^4*d*e*f^4 - 8*A*B^3*a^2*c^4*d*e*f^4 - 8*A*B^3*a*c^5*d^2*e*f^3 + 26*A^3*B*a*b^2*c^3*e*f^5 + 8*A^3*B*a*b*c^4*e^2*f^4 + 32*A*B^3*a*b*c^4*d^2*f^4 - 28*A*B^3*a^2*b*c^3*d*f^5 + 6*A*B^3*a*b^3*c^2*d*f^5 - 9*A^2*B^2*b^2*c^4*d*e^2*f^3 - 18*A^2*B^2*a*b^2*c^3*e^2*f^4 - 4*A^3*B*c^6*d^2*e*f^3 - 3*A^3*B*b^4*c^2*e*f^5 - 44*A^3*B*a^2*c^4*e*f^5 - 16*A^3*B*a*c^5*e^3*f^3 - 16*A*B^3*a^3*c^3*e*f^5 - 10*A^3*B*b^3*c^3*d*f^5 - 4*A^3*B*b*c^5*d^2*f^4 - 4*A*B^3*b*c^5*d^3*f^3 - 28*A^3*B*a^2*b*c^3*f^6 + 6*A^3*B*a*b^3*c^2*f^6 - 4*A^4*b*c^5*d*e*f^4 - 20*A^4*a*b*c^4*e*f^5 + 3*A^2*B^2*b^4*c^2*e^2*f^4 - 2*A^2*B^2*b^3*c^3*e^3*f^3 + 12*A^2*B^2*a^2*c^4*e^2*f^4 + 9*A^2*B^2*b^2*c^4*d^2*f^4 - 3*A^2*B^2*a^2*b^2*c^2*f^6 - 2*B^4*b^3*c^3*d^2*e*f^3 + 4*B^4*a^2*c^4*d*e^2*f^3 - 10*B^4*a*b^2*c^3*d^2*f^4 - 3*B^4*a^2*b^2*c^2*d*f^5 + 3*A^3*B*b^2*c^4*e^3*f^3 - 2*A^3*B*b^3*c^3*e^2*f^4 - 10*A*B^3*b^3*c^3*d^2*f^4 - 4*A*B^3*a^2*c^4*e^3*f^3 + 3*A^2*B^2*b^4*c^2*d*f^5 + 36*A^2*B^2*a^2*c^4*d*f^5 - 24*A^2*B^2*a*c^5*d^2*f^4 + 4*A^2*B^2*c^6*d^3*f^3 + 16*A^2*B^2*a^3*c^3*f^6 + 4*A^4*b^3*c^3*e*f^5 + 16*B^4*a^3*c^3*d*f^5 + 16*A^4*a*c^5*e^2*f^4 + 8*A^4*b^2*c^4*d*f^5 - 8*A^4*a*b^2*c^3*f^6 - 24*A^4*a*c^5*d*f^5 + 3*B^4*b^4*c^2*d^2*f^4 - 3*A^4*b^2*c^4*e^2*f^4 + 4*A^4*c^6*d^2*f^4 + 36*A^4*a^2*c^4*f^6 + B^4*b^2*c^4*d^3*f^3, z, k)*(root(48416*a^6*b^2*c^6*d^4*e^2*f^4*z^4 - 41544*a^5*b^4*c^5*d^4*e^2*f^4*z^4 - 31872*a^7*b^2*c^5*d^3*e^2*f^5*z^4 - 31872*a^5*b^2*c^7*d^5*e^2*f^3*z^4 - 29184*a^6*b^2*c^6*d^3*e^4*f^3*z^4 + 28800*a^5*b^4*c^5*d^3*e^4*f^3*z^4 + 21510*a^4*b^6*c^4*d^4*e^2*f^4*z^4 + 21408*a^6*b^4*c^4*d^3*e^2*f^5*z^4 + 21408*a^4*b^4*c^6*d^5*e^2*f^3*z^4 - 18112*a^7*b^3*c^4*d^2*e^3*f^5*z^4 - 18112*a^4*b^3*c^7*d^5*e^3*f^2*z^4 - 15600*a^5*b^5*c^4*d^3*e^3*f^4*z^4 - 15600*a^4*b^5*c^5*d^4*e^3*f^3*z^4 + 15296*a^6*b^3*c^5*d^3*e^3*f^4*z^4 + 15296*a^5*b^3*c^6*d^4*e^3*f^3*z^4 + 14016*a^7*b^2*c^5*d^2*e^4*f^4*z^4 + 14016*a^5*b^2*c^7*d^4*e^4*f^2*z^4 - 13920*a^4*b^6*c^4*d^3*e^4*f^3*z^4 - 11648*a^6*b^3*c^5*d^2*e^5*f^3*z^4 - 11648*a^5*b^3*c^6*d^3*e^5*f^2*z^4 + 10432*a^6*b^2*c^6*d^2*e^6*f^2*z^4 + 9008*a^6*b^5*c^3*d^2*e^3*f^5*z^4 + 9008*a^3*b^5*c^6*d^5*e^3*f^2*z^4 + 8544*a^5*b^5*c^4*d^2*e^5*f^3*z^4 + 8544*a^4*b^5*c^5*d^3*e^5*f^2*z^4 - 8496*a^5*b^4*c^5*d^2*e^6*f^2*z^4 + 7488*a^8*b^2*c^4*d^2*e^2*f^6*z^4 + 7488*a^4*b^2*c^8*d^6*e^2*f^2*z^4 + 7380*a^4*b^7*c^3*d^3*e^3*f^4*z^4 + 7380*a^3*b^7*c^4*d^4*e^3*f^3*z^4 - 6720*a^3*b^8*c^3*d^4*e^2*f^4*z^4 - 5784*a^5*b^6*c^3*d^3*e^2*f^5*z^4 - 5784*a^3*b^6*c^5*d^5*e^2*f^3*z^4 - 3440*a^6*b^4*c^4*d^2*e^4*f^4*z^4 - 3440*a^4*b^4*c^6*d^4*e^4*f^2*z^4 + 3360*a^3*b^8*c^3*d^3*e^4

*f^3*z^4 + 3140*a^4*b^6*c^4*d^2*e^6*f^2*z^4 - 2760*a^4*b^7*c^3*d^2*e^5*f^3*
z^4 - 2760*a^3*b^7*c^4*d^3*e^5*f^2*z^4 - 1764*a^5*b^7*c^2*d^2*e^3*f^5*z^4 -
1764*a^2*b^7*c^5*d^5*e^3*f^2*z^4 - 1640*a^3*b^9*c^2*d^3*e^3*f^4*z^4 - 1640
*a^2*b^9*c^3*d^4*e^3*f^3*z^4 - 1604*a^6*b^6*c^2*d^2*e^2*f^6*z^4 - 1604*a^2*
b^6*c^6*d^6*e^2*f^2*z^4 - 1500*a^5*b^6*c^3*d^2*e^4*f^4*z^4 - 1500*a^3*b^6*c
^5*d^4*e^4*f^2*z^4 + 1140*a^2*b^10*c^2*d^4*e^2*f^4*z^4 + 810*a^4*b^8*c^2*d^
2*e^4*f^4*z^4 + 810*a^2*b^8*c^4*d^4*e^4*f^2*z^4 - 544*a^3*b^8*c^3*d^2*e^6*f
^2*z^4 + 416*a^3*b^9*c^2*d^2*e^5*f^3*z^4 + 416*a^2*b^9*c^3*d^3*e^5*f^2*z^4
- 384*a^2*b^10*c^2*d^3*e^4*f^3*z^4 + 180*a^4*b^8*c^2*d^3*e^2*f^5*z^4 + 180*
a^2*b^8*c^4*d^5*e^2*f^3*z^4 + 48*a^7*b^4*c^3*d^2*e^2*f^6*z^4 + 48*a^3*b^4*c
^7*d^6*e^2*f^2*z^4 + 36*a^2*b^10*c^2*d^2*e^6*f^2*z^4 - 1024*a^10*b*c^3*d*e*
f^8*z^4 - 1024*a^3*b*c^10*d^8*e*f*z^4 - 192*a^8*b^5*c*d*e*f^8*z^4 - 192*a*b
^5*c^8*d^8*e*f*z^4 + 16128*a^7*b^3*c^4*d^3*e*f^6*z^4 + 16128*a^4*b^3*c^7*d^
6*e*f^3*z^4 - 11712*a^6*b^5*c^3*d^3*e*f^6*z^4 - 11712*a^3*b^5*c^6*d^6*e*f^3
*z^4 + 11520*a^8*b*c^5*d^2*e^3*f^5*z^4 + 11520*a^5*b*c^8*d^5*e^3*f^2*z^4 -
9984*a^6*b^3*c^5*d^4*e*f^5*z^4 - 9984*a^5*b^3*c^6*d^5*e*f^4*z^4 + 8640*a^5*
b^5*c^4*d^4*e*f^5*z^4 + 8640*a^4*b^5*c^5*d^5*e*f^4*z^4 - 7424*a^7*b*c^6*d^3
*e^3*f^4*z^4 - 7424*a^6*b*c^7*d^4*e^3*f^3*z^4 - 6912*a^8*b^3*c^3*d^2*e*f^7*
z^4 - 6912*a^3*b^3*c^8*d^7*e*f^2*z^4 + 4800*a^7*b^3*c^4*d*e^5*f^4*z^4 + 480
0*a^4*b^3*c^7*d^4*e^5*f*z^4 + 4608*a^7*b*c^6*d^2*e^5*f^3*z^4 + 4608*a^6*b*c
^7*d^3*e^5*f^2*z^4 - 4560*a^4*b^7*c^3*d^4*e*f^5*z^4 - 4560*a^3*b^7*c^4*d^5*
e*f^4*z^4 + 4176*a^5*b^7*c^2*d^3*e*f^6*z^4 + 4176*a^2*b^7*c^5*d^6*e*f^3*z^4
+ 3264*a^7*b^5*c^2*d^2*e*f^7*z^4 + 3264*a^2*b^5*c^7*d^7*e*f^2*z^4 + 3008*a
^8*b^3*c^3*d*e^3*f^6*z^4 + 3008*a^3*b^3*c^8*d^6*e^3*f*z^4 + 2880*a^6*b^3*c^
5*d*e^7*f^2*z^4 + 2880*a^5*b^3*c^6*d^2*e^7*f*z^4 - 2240*a^7*b^4*c^3*d*e^4*f
^5*z^4 - 2240*a^3*b^4*c^7*d^5*e^4*f*z^4 - 1488*a^5*b^5*c^4*d*e^7*f^2*z^4 -
1488*a^4*b^5*c^5*d^2*e^7*f*z^4 + 1440*a^3*b^9*c^2*d^4*e*f^5*z^4 + 1440*a^2*
b^9*c^3*d^5*e*f^4*z^4 - 1328*a^6*b^5*c^3*d*e^5*f^4*z^4 - 1328*a^3*b^5*c^6*d
^4*e^5*f*z^4 - 1152*a^7*b^2*c^5*d*e^6*f^3*z^4 - 1152*a^5*b^2*c^7*d^3*e^6*f*
z^4 - 1120*a^6*b^4*c^4*d*e^6*f^3*z^4 - 1120*a^4*b^4*c^6*d^3*e^6*f*z^4 + 912
*a^6*b^6*c^2*d*e^4*f^5*z^4 + 912*a^2*b^6*c^6*d^5*e^4*f*z^4 + 872*a^5*b^6*c^
3*d*e^6*f^3*z^4 + 872*a^3*b^6*c^5*d^3*e^6*f*z^4 + 768*a^8*b^2*c^4*d*e^4*f^5
*z^4 + 768*a^4*b^2*c^8*d^5*e^4*f*z^4 - 672*a^8*b^4*c^2*d*e^2*f^7*z^4 - 672*
a^2*b^4*c^8*d^7*e^2*f*z^4 - 624*a^7*b^5*c^2*d*e^3*f^6*z^4 - 624*a^2*b^5*c^7
*d^6*e^3*f*z^4 + 480*a^5*b^8*c*d^2*e^2*f^6*z^4 + 480*a*b^8*c^5*d^6*e^2*f^2*
z^4 + 316*a^4*b^7*c^3*d*e^7*f^2*z^4 + 316*a^3*b^7*c^4*d^2*e^7*f*z^4 - 204*a
^4*b^8*c^2*d*e^6*f^3*z^4 - 204*a^2*b^8*c^4*d^3*e^6*f*z^4 + 168*a^3*b^10*c*d
^3*e^2*f^5*z^4 + 168*a*b^10*c^3*d^5*e^2*f^3*z^4 + 156*a^2*b^11*c*d^3*e^3*f^
4*z^4 + 156*a*b^11*c^2*d^4*e^3*f^3*z^4 + 128*a^9*b^2*c^3*d*e^2*f^7*z^4 + 12
8*a^3*b^2*c^9*d^7*e^2*f*z^4 - 124*a^3*b^10*c*d^2*e^4*f^4*z^4 - 124*a*b^10*c
^3*d^4*e^4*f^2*z^4 + 100*a^4*b^9*c*d^2*e^3*f^5*z^4 + 100*a*b^9*c^4*d^5*e^3*
f^2*z^4 + 36*a^5*b^7*c^2*d*e^5*f^4*z^4 + 36*a^2*b^7*c^5*d^4*e^5*f*z^4 - 24*
a^3*b^9*c^2*d*e^7*f^2*z^4 - 24*a^2*b^11*c*d^2*e^5*f^3*z^4 - 24*a^2*b^9*c^3*
d^2*e^7*f*z^4 - 24*a*b^11*c^2*d^3*e^5*f^2*z^4 - 9216*a^8*b*c^5*d^3*e*f^6*z^
4 - 9216*a^5*b*c^8*d^6*e*f^3*z^4 - 5376*a^8*b*c^5*d*e^5*f^4*z^4 - 5376*a^5*

b*c^8*d^4*e^5*f*z^4 + 5120*a^9*b*c^4*d^2*e*f^7*z^4 + 5120*a^7*b*c^6*d^4*e*f
^5*z^4 + 5120*a^6*b*c^7*d^5*e*f^4*z^4 + 5120*a^4*b*c^9*d^7*e*f^2*z^4 - 4352
*a^9*b*c^4*d*e^3*f^6*z^4 - 4352*a^4*b*c^9*d^6*e^3*f*z^4 - 1792*a^7*b*c^6*d*
e^7*f^2*z^4 - 1792*a^6*b*c^7*d^2*e^7*f*z^4 - 1600*a^6*b^2*c^6*d*e^8*f*z^4 +
912*a^5*b^4*c^5*d*e^8*f*z^4 + 768*a^9*b^3*c^2*d*e*f^8*z^4 + 768*a^2*b^3*c^
9*d^8*e*f*z^4 - 720*a^4*b^9*c*d^3*e*f^6*z^4 - 720*a*b^9*c^4*d^6*e*f^3*z^4 -
656*a^6*b^7*c*d^2*e*f^7*z^4 - 656*a*b^7*c^6*d^7*e*f^2*z^4 - 240*a^2*b^11*c
*d^4*e*f^5*z^4 - 240*a*b^11*c^2*d^5*e*f^4*z^4 + 216*a^7*b^6*c*d*e^2*f^7*z^4
+ 216*a*b^6*c^7*d^7*e^2*f*z^4 - 204*a^4*b^6*c^4*d*e^8*f*z^4 - 144*a^5*b^8*
c*d*e^4*f^5*z^4 - 144*a*b^8*c^5*d^5*e^4*f*z^4 - 84*a*b^12*c*d^4*e^2*f^4*z^4
+ 36*a^4*b^9*c*d*e^5*f^4*z^4 + 36*a*b^9*c^4*d^4*e^5*f*z^4 + 20*a^6*b^7*c*d
*e^3*f^6*z^4 + 20*a*b^7*c^6*d^6*e^3*f*z^4 + 16*a^3*b^10*c*d*e^6*f^3*z^4 + 1
6*a^3*b^8*c^3*d*e^8*f*z^4 + 16*a*b^12*c*d^3*e^4*f^3*z^4 + 16*a*b^10*c^3*d^3
*e^6*f*z^4 + 48*b^11*c^3*d^6*e*f^3*z^4 + 48*b^9*c^5*d^7*e*f^2*z^4 - 20*b^8*
c^6*d^7*e^2*f*z^4 + 8*b^10*c^4*d^5*e^4*f*z^4 - 4*b^13*c*d^4*e^3*f^3*z^4 - 4
*b^11*c^3*d^4*e^5*f*z^4 + 4*b^9*c^5*d^6*e^3*f*z^4 + 3072*a^9*c^5*d*e^4*f^5*
z^4 + 3072*a^5*c^9*d^5*e^4*f*z^4 + 2560*a^8*c^6*d*e^6*f^3*z^4 + 2560*a^6*c^
8*d^3*e^6*f*z^4 + 1536*a^10*c^4*d*e^2*f^7*z^4 + 1536*a^4*c^10*d^7*e^2*f*z^4
+ 48*a^5*b^9*d^2*e*f^7*z^4 + 48*a^3*b^11*d^3*e*f^6*z^4 - 20*a^6*b^8*d*e^2*
f^7*z^4 + 8*a^4*b^10*d*e^4*f^5*z^4 + 4*a^5*b^9*d*e^3*f^6*z^4 - 4*a^3*b^11*d
*e^5*f^4*z^4 - 4*a*b^13*d^3*e^3*f^4*z^4 + 768*a^9*b*c^4*e^5*f^5*z^4 + 768*a
^8*b*c^5*e^7*f^3*z^4 + 256*a^10*b*c^3*e^3*f^7*z^4 - 192*a^6*b^3*c^5*e^9*f*z
^4 - 68*a^7*b^6*c*e^4*f^6*z^4 + 48*a^8*b^5*c*e^3*f^7*z^4 + 48*a^5*b^5*c^4*e
^9*f*z^4 + 36*a^6*b^7*c*e^5*f^5*z^4 - 12*a^9*b^4*c*e^2*f^8*z^4 - 4*a^4*b^9*
c*e^7*f^3*z^4 - 4*a^4*b^7*c^3*e^9*f*z^4 + 384*a^5*b^8*c*d^3*f^7*z^4 + 384*a
*b^8*c^5*d^7*f^3*z^4 + 288*a^3*b^10*c*d^4*f^6*z^4 + 288*a*b^10*c^3*d^6*f^4*
z^4 + 224*a^7*b^6*c*d^2*f^8*z^4 + 224*a*b^6*c^7*d^8*f^2*z^4 - 192*a^10*b^2*
c^2*d*f^9*z^4 - 192*a^2*b^2*c^10*d^9*f*z^4 + 768*a^5*b*c^8*d^3*e^7*z^4 + 76
8*a^4*b*c^9*d^5*e^5*z^4 + 256*a^3*b*c^10*d^7*e^3*z^4 - 192*a^5*b^3*c^6*d*e^
9*z^4 - 68*a*b^6*c^7*d^6*e^4*z^4 + 48*a^4*b^5*c^5*d*e^9*z^4 + 48*a*b^5*c^8*
d^7*e^3*z^4 + 36*a*b^7*c^6*d^5*e^5*z^4 - 12*a*b^4*c^9*d^8*e^2*z^4 - 4*a^3*b
^7*c^4*d*e^9*z^4 - 4*a*b^9*c^4*d^3*e^7*z^4 + 16*b^13*c*d^5*e*f^4*z^4 + 16*b
^7*c^7*d^8*e*f*z^4 + 768*a^7*c^7*d*e^8*f*z^4 + 16*a^7*b^7*d*e*f^8*z^4 + 16*
a*b^13*d^4*e*f^5*z^4 + 256*a^7*b*c^6*e^9*f*z^4 + 80*a*b^12*c*d^5*f^5*z^4 +
48*a^9*b^4*c*d*f^9*z^4 + 48*a*b^4*c^9*d^9*f*z^4 + 256*a^6*b*c^7*d*e^9*z^4 -
42*b^10*c^4*d^6*e^2*f^2*z^4 - 20*b^12*c^2*d^5*e^2*f^3*z^4 + 6*b^12*c^2*d^4
*e^4*f^2*z^4 + 4*b^11*c^3*d^5*e^3*f^2*z^4 - 24960*a^7*c^7*d^4*e^2*f^4*z^4 +
18944*a^8*c^6*d^3*e^2*f^5*z^4 + 18944*a^6*c^8*d^5*e^2*f^3*z^4 + 14336*a^7*
c^7*d^3*e^4*f^3*z^4 - 9984*a^8*c^6*d^2*e^4*f^4*z^4 - 9984*a^6*c^8*d^4*e^4*f
^2*z^4 - 7936*a^9*c^5*d^2*e^2*f^6*z^4 - 7936*a^5*c^9*d^6*e^2*f^2*z^4 - 4352
*a^7*c^7*d^2*e^6*f^2*z^4 - 42*a^4*b^10*d^2*e^2*f^6*z^4 - 20*a^2*b^12*d^3*e^
2*f^5*z^4 + 6*a^2*b^12*d^2*e^4*f^4*z^4 + 4*a^3*b^11*d^2*e^3*f^5*z^4 - 480*a
^8*b^2*c^4*e^6*f^4*z^4 + 440*a^7*b^4*c^3*e^6*f^4*z^4 - 320*a^8*b^3*c^3*e^5*
f^5*z^4 - 320*a^7*b^3*c^4*e^7*f^3*z^4 + 240*a^8*b^4*c^2*e^4*f^6*z^4 + 240*a
^6*b^4*c^4*e^8*f^2*z^4 - 192*a^9*b^3*c^2*e^3*f^7*z^4 - 192*a^9*b^2*c^3*e^4*

f^6*z^4 - 192*a^7*b^2*c^5*e^8*f^2*z^4 - 90*a^6*b^6*c^2*e^6*f^4*z^4 - 68*a^5
*b^6*c^3*e^8*f^2*z^4 + 48*a^10*b^2*c^2*e^2*f^8*z^4 - 48*a^7*b^5*c^2*e^5*f^5
*z^4 - 48*a^6*b^5*c^3*e^7*f^3*z^4 + 36*a^5*b^7*c^2*e^7*f^3*z^4 + 6*a^4*b^8*
c^2*e^8*f^2*z^4 - 33920*a^6*b^2*c^6*d^5*f^5*z^4 + 27936*a^5*b^4*c^5*d^5*f^5
*z^4 + 26112*a^7*b^2*c^5*d^4*f^6*z^4 + 26112*a^5*b^2*c^7*d^6*f^4*z^4 - 2035
2*a^6*b^4*c^4*d^4*f^6*z^4 - 20352*a^4*b^4*c^6*d^6*f^4*z^4 - 13080*a^4*b^6*c
^4*d^5*f^5*z^4 - 11520*a^8*b^2*c^4*d^3*f^7*z^4 - 11520*a^4*b^2*c^8*d^7*f^3*
z^4 + 8736*a^5*b^6*c^3*d^4*f^6*z^4 + 8736*a^3*b^6*c^5*d^6*f^4*z^4 + 7488*a^
7*b^4*c^3*d^3*f^7*z^4 + 7488*a^3*b^4*c^7*d^7*f^3*z^4 + 3840*a^3*b^8*c^3*d^5
*f^5*z^4 + 2560*a^9*b^2*c^3*d^2*f^8*z^4 + 2560*a^3*b^2*c^9*d^8*f^2*z^4 - 24
16*a^6*b^6*c^2*d^3*f^7*z^4 - 2416*a^2*b^6*c^6*d^7*f^3*z^4 - 2160*a^4*b^8*c^
2*d^4*f^6*z^4 - 2160*a^2*b^8*c^4*d^6*f^4*z^4 - 1152*a^8*b^4*c^2*d^2*f^8*z^4
 - 1152*a^2*b^4*c^8*d^8*f^2*z^4 - 720*a^2*b^10*c^2*d^5*f^5*z^4 - 480*a^4*b^
2*c^8*d^4*e^6*z^4 + 440*a^3*b^4*c^7*d^4*e^6*z^4 - 320*a^4*b^3*c^7*d^3*e^7*z
^4 - 320*a^3*b^3*c^8*d^5*e^5*z^4 + 240*a^4*b^4*c^6*d^2*e^8*z^4 + 240*a^2*b^
4*c^8*d^6*e^4*z^4 - 192*a^5*b^2*c^7*d^2*e^8*z^4 - 192*a^3*b^2*c^9*d^6*e^4*z
^4 - 192*a^2*b^3*c^9*d^7*e^3*z^4 - 90*a^2*b^6*c^6*d^4*e^6*z^4 - 68*a^3*b^6*
c^5*d^2*e^8*z^4 - 48*a^3*b^5*c^6*d^3*e^7*z^4 - 48*a^2*b^5*c^7*d^5*e^5*z^4 +
 48*a^2*b^2*c^10*d^8*e^2*z^4 + 36*a^2*b^7*c^5*d^3*e^7*z^4 + 6*a^2*b^8*c^4*d
^2*e^8*z^4 - 4*b^6*c^8*d^9*f*z^4 + 256*a^11*c^3*d*f^9*z^4 + 256*a^3*c^11*d^
9*f*z^4 - 4*a^8*b^6*d*f^9*z^4 - 384*a^9*c^5*e^6*f^4*z^4 - 256*a^10*c^4*e^4*
f^6*z^4 - 256*a^8*c^6*e^8*f^2*z^4 - 64*a^11*c^3*e^2*f^8*z^4 - 24*b^10*c^4*d
^7*f^3*z^4 - 16*b^12*c^2*d^6*f^4*z^4 - 16*b^8*c^6*d^8*f^2*z^4 + 17920*a^7*c
^7*d^5*f^5*z^4 - 14336*a^8*c^6*d^4*f^6*z^4 - 14336*a^6*c^8*d^6*f^4*z^4 + 71
68*a^9*c^5*d^3*f^7*z^4 + 7168*a^5*c^9*d^7*f^3*z^4 - 2048*a^10*c^4*d^2*f^8*z
^4 - 2048*a^4*c^10*d^8*f^2*z^4 + 6*b^8*c^6*d^6*e^4*z^4 + 6*a^6*b^8*e^4*f^6*
z^4 - 4*b^9*c^5*d^5*e^5*z^4 - 4*b^7*c^7*d^7*e^3*z^4 - 4*a^7*b^7*e^3*f^7*z^4
 - 4*a^5*b^9*e^5*f^5*z^4 - 384*a^5*c^9*d^4*e^6*z^4 - 256*a^6*c^8*d^2*e^8*z^
4 - 256*a^4*c^10*d^6*e^4*z^4 - 64*a^3*c^11*d^8*e^2*z^4 - 24*a^4*b^10*d^3*f^
7*z^4 - 16*a^6*b^8*d^2*f^8*z^4 - 16*a^2*b^12*d^4*f^6*z^4 + 48*a^6*b^2*c^6*e
^10*z^4 - 12*a^5*b^4*c^5*e^10*z^4 - 4*b^14*d^5*f^5*z^4 - 64*a^7*c^7*e^10*z^
4 + b^14*d^4*e^2*f^4*z^4 + b^10*c^4*d^4*e^6*z^4 + b^6*c^8*d^8*e^2*z^4 + a^8
*b^6*e^2*f^8*z^4 + a^4*b^10*e^6*f^4*z^4 + a^4*b^6*c^4*e^10*z^4 - 4820*A*B*a
^4*b*c^5*d^2*e^2*f^4*z^2 + 2976*A*B*a^3*b*c^6*d^3*e^2*f^3*z^2 - 2328*A*B*a^
3*b*c^6*d^2*e^4*f^2*z^2 + 1848*A*B*a^2*b^4*c^4*d^3*e*f^4*z^2 - 1768*A*B*a^3
*b^4*c^3*d^2*e*f^5*z^2 + 1528*A*B*a^4*b^2*c^4*d^2*e*f^5*z^2 - 1136*A*B*a^3*
b^2*c^5*d^3*e*f^4*z^2 - 974*A*B*a^4*b^3*c^3*d*e^2*f^5*z^2 + 692*A*B*a^2*b*c
^7*d^4*e^2*f^2*z^2 + 588*A*B*a*b^6*c^3*d^2*e^3*f^3*z^2 - 580*A*B*a^3*b^3*c^
4*d*e^4*f^3*z^2 + 488*A*B*a^3*b^4*c^3*d*e^3*f^4*z^2 - 444*A*B*a^2*b^2*c^6*d
^2*e^5*f*z^2 - 412*A*B*a*b^5*c^4*d^2*e^4*f^2*z^2 + 366*A*B*a^2*b^6*c^2*d^2*
e*f^5*z^2 - 352*A*B*a^2*b^2*c^6*d^4*e*f^3*z^2 + 326*A*B*a^2*b^4*c^4*d*e^5*f
^2*z^2 + 324*A*B*a*b^5*c^4*d^3*e^2*f^3*z^2 - 302*A*B*a*b^3*c^6*d^4*e^2*f^2*
z^2 - 296*A*B*a*b^7*c^2*d^2*e^2*f^4*z^2 + 122*A*B*a^4*b^2*c^4*d*e^3*f^4*z^2
 - 122*A*B*a^2*b^6*c^2*d*e^3*f^4*z^2 - 84*A*B*a^3*b^2*c^5*d*e^5*f^2*z^2 + 7
2*A*B*a*b^4*c^5*d^3*e^3*f^2*z^2 - 64*A*B*a^2*b^5*c^3*d*e^4*f^3*z^2 + 60*A*B

*a^3*b^5*c^2*d*e^2*f^5*z^2 + 1312*A*B*a^5*b*c^4*d*e^2*f^5*z^2 + 1040*A*B*a^
4*b*c^5*d*e^4*f^3*z^2 - 500*A*B*a*b^6*c^3*d^3*e*f^4*z^2 - 376*A*B*a*b^2*c^7
*d^5*e*f^2*z^2 + 276*A*B*a^4*b^4*c^2*d*e*f^6*z^2 - 262*A*B*a^2*b^3*c^5*d*e^
6*f*z^2 + 238*A*B*a*b^2*c^7*d^4*e^3*f*z^2 + 232*A*B*a^5*b^2*c^3*d*e*f^6*z^2
 - 176*A*B*a^2*b*c^7*d^3*e^4*f*z^2 - 120*A*B*a*b^6*c^3*d*e^5*f^2*z^2 - 108*
A*B*a*b^4*c^5*d^4*e*f^3*z^2 + 68*A*B*a*b^7*c^2*d*e^4*f^3*z^2 + 68*A*B*a*b^4
*c^5*d^2*e^5*f*z^2 + 46*A*B*a^2*b^7*c*d*e^2*f^5*z^2 - 36*A*B*a*b^3*c^6*d^3*
e^4*f*z^2 - 1932*A*B*a^2*b^3*c^5*d^3*e^2*f^3*z^2 - 1818*A*B*a^2*b^4*c^4*d^2
*e^3*f^3*z^2 + 1620*A*B*a^3*b^3*c^4*d^2*e^2*f^4*z^2 + 1560*A*B*a^2*b^3*c^5*
d^2*e^4*f^2*z^2 + 1244*A*B*a^3*b^2*c^5*d^2*e^3*f^3*z^2 + 820*A*B*a^2*b^2*c^
6*d^3*e^3*f^2*z^2 + 480*A*B*a^2*b^5*c^3*d^2*e^2*f^4*z^2 + 352*A*B*a^3*b*c^6
*d*e^6*f*z^2 - 108*A*B*a^3*b^6*c*d*e*f^6*z^2 + 82*A*B*a*b^5*c^4*d*e^6*f*z^2
 - 64*A*B*a*b*c^8*d^5*e^2*f*z^2 + 16*A*B*a*b^8*c*d^2*e*f^5*z^2 - 4*A*B*a*b^
8*c*d*e^3*f^4*z^2 + 16*B^2*a*b*c^8*d^6*e*f*z^2 + 56*A*B*b^2*c^8*d^6*e*f*z^2
 - 8*A*B*b^9*c*d*e^4*f^3*z^2 - 8*A*B*b^7*c^3*d*e^6*f*z^2 - 800*A*B*a^6*c^4*
d*e*f^6*z^2 + 10*A*B*a^2*b^8*d*e*f^6*z^2 - 6*A*B*a*b^9*d*e^2*f^5*z^2 - 12*A
*B*a^5*b^4*c*e*f^7*z^2 + 912*A*B*a^6*b*c^3*d*f^7*z^2 + 192*A*B*a^4*b^5*c*d*
f^7*z^2 + 192*A*B*a*b*c^8*d^6*f^2*z^2 - 20*A*B*a*b^4*c^5*d*e^7*z^2 + 4*A*B*
a*b*c^8*d^4*e^4*z^2 + 2144*B^2*a^4*b*c^5*d^3*e*f^4*z^2 - 1120*B^2*a^3*b*c^6
*d^4*e*f^3*z^2 - 688*B^2*a^5*b*c^4*d^2*e*f^5*z^2 - 256*B^2*a^3*b*c^6*d^2*e^
5*f*z^2 + 152*B^2*a*b^3*c^6*d^5*e*f^2*z^2 + 120*B^2*a^5*b^3*c^2*d*e*f^6*z^2
 - 116*B^2*a^5*b*c^4*d*e^3*f^4*z^2 + 110*B^2*a*b^7*c^2*d^3*e*f^4*z^2 - 80*B
^2*a^2*b*c^7*d^5*e*f^2*z^2 - 72*B^2*a*b^5*c^4*d^4*e*f^3*z^2 - 48*B^2*a^4*b*
c^5*d*e^5*f^2*z^2 - 46*B^2*a*b^3*c^6*d^4*e^3*f*z^2 - 44*B^2*a*b^4*c^5*d^3*e
^4*f*z^2 - 34*B^2*a*b^5*c^4*d^2*e^5*f*z^2 + 20*B^2*a^2*b*c^7*d^4*e^3*f*z^2
 - 10*B^2*a^3*b^6*c*d*e^2*f^5*z^2 - 10*B^2*a^2*b^7*c*d^2*e*f^5*z^2 - 10*B^2*
a*b^2*c^7*d^5*e^2*f*z^2 - 7*B^2*a^2*b^4*c^4*d*e^6*f*z^2 - 6*B^2*a^3*b^2*c^5
*d*e^6*f*z^2 + 4*B^2*a*b^8*c*d^2*e^2*f^4*z^2 - 2*B^2*a^2*b^7*c*d*e^3*f^4*z^
2 + 3196*A^2*a^4*b*c^5*d*e^3*f^4*z^2 - 3184*A^2*a^4*b*c^5*d^2*e*f^5*z^2 + 1
568*A^2*a^3*b*c^6*d^3*e*f^4*z^2 + 1504*A^2*a^3*b*c^6*d*e^5*f^2*z^2 - 656*A^
2*a^4*b^3*c^3*d*e*f^6*z^2 - 400*A^2*a*b^6*c^3*d*e^4*f^3*z^2 + 314*A^2*a*b^5
*c^4*d*e^5*f^2*z^2 - 264*A^2*a^3*b^5*c^2*d*e*f^6*z^2 + 240*A^2*a^2*b^2*c^6*
d*e^6*f*z^2 - 224*A^2*a^2*b*c^7*d^4*e*f^3*z^2 + 216*A^2*a*b^5*c^4*d^3*e*f^4
*z^2 - 192*A^2*a^2*b*c^7*d^2*e^5*f*z^2 + 178*A^2*a*b^7*c^2*d*e^3*f^4*z^2 -
154*A^2*a*b^7*c^2*d^2*e*f^5*z^2 + 128*A^2*a*b^3*c^6*d^4*e*f^3*z^2 + 106*A^2
*a*b^3*c^6*d^2*e^5*f*z^2 - 12*A^2*a*b^2*c^7*d^3*e^4*f*z^2 - 58*A*B*b^8*c^2*
d^2*e^3*f^3*z^2 + 40*A*B*b^7*c^3*d^2*e^4*f^2*z^2 - 28*A*B*b^7*c^3*d^3*e^2*f
^3*z^2 - 24*A*B*b^5*c^5*d^4*e^2*f^2*z^2 - 20*A*B*b^6*c^4*d^3*e^3*f^2*z^2 +
2768*A*B*a^4*c^6*d^2*e^3*f^3*z^2 - 1712*A*B*a^3*c^7*d^3*e^3*f^2*z^2 - 156*A
*B*a^4*b^2*c^4*e^5*f^3*z^2 + 146*A*B*a^4*b^3*c^3*e^4*f^4*z^2 - 106*A*B*a^5*
b^2*c^3*e^3*f^5*z^2 + 90*A*B*a^5*b^3*c^2*e^2*f^6*z^2 + 38*A*B*a^3*b^3*c^4*e
^6*f^2*z^2 - 36*A*B*a^3*b^5*c^2*e^4*f^4*z^2 + 16*A*B*a^3*b^4*c^3*e^5*f^3*z^
2 - 9*A*B*a^4*b^4*c^2*e^3*f^5*z^2 - 8*A*B*a^2*b^5*c^3*e^6*f^2*z^2 + 2*A*B*a
^2*b^6*c^2*e^5*f^3*z^2 + 920*A*B*a^4*b^3*c^3*d^2*f^6*z^2 - 480*A*B*a^2*b^5*
c^3*d^3*f^5*z^2 - 336*A*B*a^2*b^3*c^5*d^4*f^4*z^2 - 272*A*B*a^3*b^3*c^4*d^3

*f^5*z^2 + 240*A*B*a^3*b^5*c^2*d^2*f^6*z^2 - 32*A*B*a*c^9*d^6*e*f*z^2 - 792
*B^2*a^2*b^3*c^5*d^3*e^3*f^2*z^2 + 714*B^2*a^2*b^4*c^4*d^3*e^2*f^3*z^2 - 57
2*B^2*a^3*b^2*c^5*d^3*e^2*f^3*z^2 - 475*B^2*a^2*b^2*c^6*d^4*e^2*f^2*z^2 + 2
65*B^2*a^4*b^2*c^4*d^2*e^2*f^4*z^2 + 260*B^2*a^3*b^3*c^4*d^2*e^3*f^3*z^2 -
212*B^2*a^3*b^4*c^3*d^2*e^2*f^4*z^2 + 180*B^2*a^3*b^2*c^5*d^2*e^4*f^2*z^2 -
158*B^2*a^2*b^4*c^4*d^2*e^4*f^2*z^2 + 47*B^2*a^2*b^6*c^2*d^2*e^2*f^4*z^2 +
16*B^2*a^2*b^5*c^3*d^2*e^3*f^3*z^2 + 2752*A^2*a^3*b^2*c^5*d^2*e^2*f^4*z^2
- 2148*A^2*a^2*b^4*c^4*d^2*e^2*f^4*z^2 + 2064*A^2*a^2*b^3*c^5*d^2*e^3*f^3*z
^2 - 424*A^2*a^2*b^2*c^6*d^3*e^2*f^3*z^2 - 198*A^2*a^2*b^2*c^6*d^2*e^4*f^2*
z^2 - 272*B^2*a^6*b*c^3*d*e*f^6*z^2 - 24*B^2*a^4*b^5*c*d*e*f^6*z^2 + 1808*A
^2*a^5*b*c^4*d*e*f^6*z^2 - 244*A^2*a*b*c^8*d^4*e^3*f*z^2 + 208*A^2*a*b*c^8*
d^5*e*f^2*z^2 + 134*A^2*a^2*b^7*c*d*e*f^6*z^2 - 76*A^2*a*b^4*c^5*d*e^6*f*z^
2 + 4*A^2*a*b^8*c*d*e^2*f^5*z^2 + 148*A*B*b^4*c^6*d^5*e*f^2*z^2 + 65*A*B*b^
6*c^4*d^4*e*f^3*z^2 + 46*A*B*b^8*c^2*d^3*e*f^4*z^2 - 38*A*B*b^3*c^7*d^5*e^2
*f*z^2 + 34*A*B*b^9*c*d^2*e^2*f^4*z^2 - 29*A*B*b^4*c^6*d^4*e^3*f*z^2 + 20*A
*B*b^5*c^5*d^3*e^4*f*z^2 + 12*A*B*b^8*c^2*d*e^5*f^2*z^2 - 7*A*B*b^6*c^4*d^2
*e^5*f*z^2 - 2880*A*B*a^4*c^6*d^3*e*f^4*z^2 + 2784*A*B*a^5*c^5*d^2*e*f^5*z^
2 - 1112*A*B*a^5*c^5*d*e^3*f^4*z^2 + 896*A*B*a^3*c^7*d^4*e*f^3*z^2 + 848*A*
B*a^3*c^7*d^2*e^5*f*z^2 - 560*A*B*a^4*c^6*d*e^5*f^2*z^2 + 96*A*B*a^2*c^8*d^
5*e*f^2*z^2 - 88*A*B*a^2*c^8*d^4*e^3*f*z^2 - 100*A*B*a^6*b*c^3*e^2*f^6*z^2
- 76*A*B*a^5*b*c^4*e^4*f^4*z^2 + 48*A*B*a^6*b^2*c^2*e*f^7*z^2 - 42*A*B*a^3*
b^2*c^5*e^7*f*z^2 + 36*A*B*a^4*b*c^5*e^6*f^2*z^2 - 24*A*B*a^4*b^5*c*e^2*f^6
*z^2 + 10*A*B*a^3*b^6*c*e^3*f^5*z^2 + 7*A*B*a^2*b^4*c^4*e^7*f*z^2 + 2*A*B*a
^2*b^7*c*e^4*f^4*z^2 - 2496*A*B*a^5*b*c^4*d^2*f^6*z^2 + 1872*A*B*a^4*b*c^5*
d^3*f^5*z^2 - 744*A*B*a^5*b^3*c^2*d*f^7*z^2 - 720*A*B*a^2*b*c^7*d^5*f^3*z^2
+ 504*A*B*a*b^3*c^6*d^5*f^3*z^2 + 256*A*B*a^3*b*c^6*d^4*f^4*z^2 + 168*A*B*
a*b^7*c^2*d^3*f^5*z^2 - 144*A*B*a^2*b^7*c*d^2*f^6*z^2 + 144*A*B*a*b^5*c^4*d
^4*f^4*z^2 + 66*A*B*a^2*b^2*c^6*d*e^7*z^2 - 36*A*B*a*b^2*c^7*d^3*e^5*z^2 +
20*A*B*a*b^3*c^6*d^2*e^6*z^2 + 12*A*B*a^2*b*c^7*d^2*e^6*z^2 + 1208*B^2*a^3*
b*c^6*d^3*e^3*f^2*z^2 - 848*B^2*a^3*b^3*c^4*d^3*e*f^4*z^2 + 672*B^2*a^2*b^3
*c^5*d^4*e*f^3*z^2 - 632*B^2*a^4*b*c^5*d^2*e^3*f^3*z^2 + 432*B^2*a^4*b^3*c^
3*d^2*e*f^5*z^2 + 276*B^2*a^2*b^2*c^6*d^3*e^4*f*z^2 - 196*B^2*a*b^6*c^3*d^3
*e^2*f^3*z^2 - 168*B^2*a^2*b^5*c^3*d^3*e*f^4*z^2 + 154*B^2*a^2*b^3*c^5*d^2*
e^5*f*z^2 + 148*B^2*a*b^5*c^4*d^3*e^3*f^2*z^2 + 96*B^2*a*b^4*c^5*d^4*e^2*f^
2*z^2 - 72*B^2*a^3*b^5*c^2*d^2*e*f^5*z^2 + 70*B^2*a^5*b^2*c^3*d*e^2*f^5*z^2
- 60*B^2*a^4*b^3*c^3*d*e^3*f^4*z^2 + 52*B^2*a*b^6*c^3*d^2*e^4*f^2*z^2 + 36
*B^2*a^4*b^2*c^4*d*e^4*f^3*z^2 - 32*B^2*a*b^7*c^2*d^2*e^3*f^3*z^2 + 24*B^2*
a^3*b^5*c^2*d*e^3*f^4*z^2 + 15*B^2*a^4*b^4*c^2*d*e^2*f^5*z^2 - 8*B^2*a^3*b^
4*c^3*d*e^4*f^3*z^2 + 8*B^2*a^2*b^5*c^3*d*e^5*f^2*z^2 - 2*B^2*a^3*b^3*c^4*d
*e^5*f^2*z^2 - 2*B^2*a^2*b^6*c^2*d*e^4*f^3*z^2 - 3176*A^2*a^3*b*c^6*d^2*e^3
*f^3*z^2 - 2252*A^2*a^4*b^2*c^4*d*e^2*f^5*z^2 + 1952*A^2*a^3*b^4*c^3*d*e^2*
f^5*z^2 - 1496*A^2*a^3*b^3*c^4*d*e^3*f^4*z^2 + 1378*A^2*a^2*b^4*c^4*d*e^4*f
^3*z^2 + 1184*A^2*a^3*b^3*c^4*d^2*e*f^5*z^2 - 1166*A^2*a^2*b^3*c^5*d*e^5*f^
2*z^2 - 1164*A^2*a^3*b^2*c^5*d*e^4*f^3*z^2 - 1152*A^2*a^2*b^3*c^5*d^3*e*f^4
*z^2 + 578*A^2*a*b^6*c^3*d^2*e^2*f^4*z^2 - 548*A^2*a*b^5*c^4*d^2*e^3*f^3*z^

2 + 440*A^2*a*b^2*c^7*d^4*e^2*f^2*z^2 - 412*A^2*a^2*b^6*c^2*d*e^2*f^5*z^2 -
 360*A^2*a*b^3*c^6*d^3*e^3*f^2*z^2 + 312*A^2*a*b^4*c^5*d^3*e^2*f^3*z^2 + 24
8*A^2*a^2*b*c^7*d^3*e^3*f^2*z^2 - 224*A^2*a^2*b^5*c^3*d*e^3*f^4*z^2 + 216*A
^2*a^2*b^5*c^3*d^2*e*f^5*z^2 + 52*A^2*a*b^4*c^5*d^2*e^4*f^2*z^2 - 16*B^2*b^
3*c^7*d^6*e*f*z^2 - 14*B^2*b^9*c*d^3*e*f^4*z^2 + 32*B^2*a^4*c^6*d*e^6*f*z^2
 - 20*A^2*b^9*c*d*e^3*f^4*z^2 + 18*A^2*b^9*c*d^2*e*f^5*z^2 + 8*A^2*b^6*c^4*
d*e^6*f*z^2 - 360*A^2*a^3*c^7*d*e^6*f*z^2 + 136*A^2*a*c^9*d^5*e^2*f*z^2 + 2
*B^2*a^3*b^7*d*e*f^6*z^2 + 2*B^2*a*b^9*d^2*e*f^5*z^2 + 12*B^2*a^4*b*c^5*e^7
*f*z^2 - 204*A^2*a^3*b*c^6*e^7*f*z^2 - 128*A^2*a^6*b*c^3*e*f^7*z^2 - 48*A^2
*a*b^5*c^4*e^7*f*z^2 - 36*B^2*a^5*b^4*c*d*f^7*z^2 - 24*A^2*a^4*b^5*c*e*f^7*
z^2 - 16*B^2*a*b^8*c*d^3*f^5*z^2 - 164*A^2*a^3*b^6*c*d*f^7*z^2 - 16*A^2*a*b
^8*c*d^2*f^6*z^2 + 4*B^2*a^3*b*c^6*d*e^7*z^2 - 4*B^2*a*b*c^8*d^5*e^3*z^2 +
48*A^2*a*b*c^8*d^3*e^5*z^2 + 36*A^2*a^2*b*c^7*d*e^7*z^2 - 6*A^2*a*b^3*c^6*d
*e^7*z^2 + 136*A*B*a^6*c^4*e^3*f^5*z^2 - 96*A*B*b^5*c^5*d^5*f^3*z^2 + 80*A*
B*a^5*c^5*e^5*f^3*z^2 - 72*A*B*b^3*c^7*d^6*f^2*z^2 - 24*A*B*b^7*c^3*d^4*f^4
*z^2 + 14*A*B*b^3*c^7*d^4*e^4*z^2 - 14*A*B*b^2*c^8*d^5*e^3*z^2 - 2*A*B*b^5*
c^5*d^2*e^6*z^2 - 2*A*B*b^4*c^6*d^3*e^5*z^2 + 2*A*B*a^3*b^7*e^2*f^6*z^2 - A
*B*a^2*b^8*e^3*f^5*z^2 + 16*A*B*a^2*c^8*d^3*e^5*z^2 - 2*A*B*a^2*b^3*c^5*e^8
*z^2 + 22*B^2*b^8*c^2*d^3*e^2*f^3*z^2 - 12*B^2*b^7*c^3*d^3*e^3*f^2*z^2 + 12
*B^2*b^6*c^4*d^4*e^2*f^2*z^2 - 6*B^2*b^8*c^2*d^2*e^4*f^2*z^2 - 864*B^2*a^4*
c^6*d^3*e^2*f^3*z^2 + 496*B^2*a^3*c^7*d^4*e^2*f^2*z^2 + 224*B^2*a^5*c^5*d^2
*e^2*f^4*z^2 + 136*B^2*a^4*c^6*d^2*e^4*f^2*z^2 - 53*A^2*b^8*c^2*d^2*e^2*f^4
*z^2 + 52*A^2*b^7*c^3*d^2*e^3*f^3*z^2 + 52*A^2*b^5*c^5*d^3*e^3*f^2*z^2 - 36
*A^2*b^6*c^4*d^3*e^2*f^3*z^2 - 12*A^2*b^4*c^6*d^4*e^2*f^2*z^2 - 9*A^2*b^6*c
^4*d^2*e^4*f^2*z^2 + 836*A^2*a^4*c^6*d^2*e^2*f^4*z^2 - 668*A^2*a^2*c^8*d^4*
e^2*f^2*z^2 + 656*A^2*a^3*c^7*d^2*e^4*f^2*z^2 + 368*A^2*a^3*c^7*d^3*e^2*f^3
*z^2 - 45*B^2*a^6*b^2*c^2*e^2*f^6*z^2 - 18*B^2*a^5*b^2*c^3*e^4*f^4*z^2 - 9*
B^2*a^4*b^2*c^4*e^6*f^2*z^2 - 6*B^2*a^5*b^3*c^2*e^3*f^5*z^2 + 3*B^2*a^4*b^4
*c^2*e^4*f^4*z^2 - 2*B^2*a^4*b^3*c^3*e^5*f^3*z^2 - 580*B^2*a^4*b^2*c^4*d^3*
f^5*z^2 + 536*B^2*a^3*b^4*c^3*d^3*f^5*z^2 + 471*A^2*a^4*b^2*c^4*e^4*f^4*z^2
 - 436*A^2*a^3*b^4*c^3*e^4*f^4*z^2 - 348*B^2*a^4*b^4*c^2*d^2*f^6*z^2 + 316*
B^2*a^2*b^2*c^6*d^5*f^3*z^2 + 310*A^2*a^3*b^3*c^4*e^5*f^3*z^2 + 232*A^2*a^5
*b^2*c^3*e^2*f^6*z^2 - 229*A^2*a^2*b^4*c^4*e^6*f^2*z^2 - 216*A^2*a^4*b^4*c^
2*e^2*f^6*z^2 + 204*A^2*a^4*b^3*c^3*e^3*f^5*z^2 + 200*B^2*a^5*b^2*c^3*d^2*f
^6*z^2 + 150*A^2*a^3*b^2*c^5*e^6*f^2*z^2 - 120*B^2*a^2*b^4*c^4*d^4*f^4*z^2
+ 91*A^2*a^2*b^6*c^2*e^4*f^4*z^2 + 72*A^2*a^3*b^5*c^2*e^3*f^5*z^2 - 66*B^2*
a^2*b^6*c^2*d^3*f^5*z^2 + 44*A^2*a^2*b^5*c^3*e^5*f^3*z^2 - 16*B^2*a^3*b^2*c
^5*d^4*f^4*z^2 + 1952*A^2*a^4*b^2*c^4*d^2*f^6*z^2 - 1792*A^2*a^3*b^2*c^5*d^
3*f^5*z^2 - 1272*A^2*a^3*b^4*c^3*d^2*f^6*z^2 + 976*A^2*a^2*b^2*c^6*d^4*f^4*
z^2 + 960*A^2*a^2*b^4*c^4*d^3*f^5*z^2 + 282*A^2*a^2*b^6*c^2*d^2*f^6*z^2 - 4
5*B^2*a^2*b^2*c^6*d^2*e^6*z^2 - 48*A^2*b*c^9*d^6*e*f*z^2 - 14*A^2*a*b^9*d*e
*f^6*z^2 - 7*A*B*b^10*d^2*e*f^5*z^2 + 2*A*B*b^10*d*e^3*f^4*z^2 - 64*A*B*a^7
*c^3*e*f^7*z^2 - 16*A*B*b^9*c*d^3*f^5*z^2 + 8*A*B*a^4*c^6*e^7*f*z^2 + 4*A*B
*b*c^9*d^6*e^2*z^2 + 2*A*B*b^6*c^4*d*e^7*z^2 - 120*A*B*a^3*c^7*d*e^7*z^2 -
16*A*B*a^3*b^7*d*f^7*z^2 + 16*A*B*a*b^9*d^2*f^6*z^2 + 8*A*B*a*c^9*d^5*e^3*z

^2 + 12*A*B*a^3*b*c^6*e^8*z^2 - 48*B^2*b^5*c^5*d^5*e*f^2*z^2 + 15*B^2*b^4*c
^6*d^5*e^2*f*z^2 - 14*B^2*b^7*c^3*d^4*e*f^3*z^2 + 4*B^2*b^9*c*d^2*e^3*f^3*z
^2 + 4*B^2*b^7*c^3*d^2*e^5*f*z^2 + 4*B^2*b^5*c^5*d^4*e^3*f*z^2 - B^2*b^6*c^
4*d^3*e^4*f*z^2 - 336*B^2*a^3*c^7*d^3*e^4*f*z^2 + 112*B^2*a^5*c^5*d*e^4*f^3
*z^2 - 112*A^2*b^3*c^7*d^5*e*f^2*z^2 + 80*B^2*a^6*c^4*d*e^2*f^5*z^2 - 48*A^
2*b^5*c^5*d^4*e*f^3*z^2 + 36*A^2*b^8*c^2*d*e^4*f^3*z^2 + 36*A^2*b^3*c^7*d^4
*e^3*f*z^2 - 28*A^2*b^7*c^3*d*e^5*f^2*z^2 + 20*A^2*b^2*c^8*d^5*e^2*f*z^2 +
16*B^2*a^2*c^8*d^5*e^2*f*z^2 - 14*A^2*b^7*c^3*d^3*e*f^4*z^2 - 14*A^2*b^4*c^
6*d^3*e^4*f*z^2 - 10*A^2*b^5*c^5*d^2*e^5*f*z^2 - 1008*A^2*a^4*c^6*d*e^4*f^3
*z^2 - 760*A^2*a^5*c^5*d*e^2*f^5*z^2 + 272*A^2*a^2*c^8*d^3*e^4*f*z^2 + 48*B
^2*a^5*b*c^4*e^5*f^3*z^2 + 36*B^2*a^6*b*c^3*e^3*f^5*z^2 + 12*B^2*a^5*b^4*c*
e^2*f^6*z^2 - 624*A^2*a^4*b*c^5*e^5*f^3*z^2 - 548*A^2*a^5*b*c^4*e^3*f^5*z^2
 + 182*A^2*a^2*b^3*c^5*e^7*f*z^2 - 180*B^2*a*b^4*c^5*d^5*f^3*z^2 + 132*B^2*
a^6*b^2*c^2*d*f^7*z^2 + 108*B^2*a^3*b^6*c*d^2*f^6*z^2 + 96*A^2*a^5*b^3*c^2*
e*f^7*z^2 + 68*A^2*a*b^6*c^3*e^6*f^2*z^2 + 58*A^2*a^3*b^6*c*e^2*f^6*z^2 - 5
6*B^2*a*b^2*c^7*d^6*f^2*z^2 - 38*A^2*a^2*b^7*c*e^3*f^5*z^2 - 36*A^2*a*b^7*c
^2*e^5*f^3*z^2 + 20*B^2*a*b^6*c^3*d^4*f^4*z^2 - 736*A^2*a^5*b^2*c^3*d*f^7*z
^2 + 624*A^2*a^4*b^4*c^2*d*f^7*z^2 - 416*A^2*a*b^2*c^7*d^5*f^3*z^2 - 276*A^
2*a*b^4*c^5*d^4*f^4*z^2 - 196*A^2*a*b^6*c^3*d^3*f^5*z^2 + 8*B^2*a*b^4*c^5*d
^2*e^6*z^2 + 6*B^2*a*b^2*c^7*d^4*e^4*z^2 + 2*B^2*a^2*b^3*c^5*d*e^7*z^2 + 2*
B^2*a*b^3*c^6*d^3*e^5*z^2 - 18*A^2*a*b^2*c^7*d^2*e^6*z^2 - 16*A*B*b*c^9*d^7
*f*z^2 - B^2*b^10*d^2*e^2*f^4*z^2 + 48*B^2*a^7*c^3*e^2*f^6*z^2 - 36*B^2*a^6
*c^4*e^4*f^4*z^2 + 31*B^2*b^6*c^4*d^5*f^3*z^2 - 24*B^2*a^5*c^5*e^6*f^2*z^2
 + 20*B^2*b^4*c^6*d^6*f^2*z^2 - 6*A^2*b^8*c^2*e^6*f^2*z^2 + 2*B^2*b^8*c^2*d^
4*f^4*z^2 - 768*B^2*a^5*c^5*d^3*f^5*z^2 + 512*B^2*a^6*c^4*d^2*f^6*z^2 + 512
*B^2*a^4*c^6*d^4*f^4*z^2 + 232*A^2*a^5*c^5*e^4*f^4*z^2 + 188*A^2*a^4*c^6*e^
6*f^2*z^2 - 128*B^2*a^3*c^7*d^5*f^3*z^2 + 92*A^2*a^6*c^4*e^2*f^6*z^2 + 80*A
^2*b^4*c^6*d^5*f^3*z^2 + 64*A^2*b^2*c^8*d^6*f^2*z^2 + 31*A^2*b^6*c^4*d^4*f^
4*z^2 + 14*A^2*b^8*c^2*d^3*f^5*z^2 - 5*B^2*b^4*c^6*d^4*e^4*z^2 + 4*B^2*b^3*
c^7*d^5*e^3*z^2 + 2*B^2*b^5*c^5*d^3*e^5*z^2 - B^2*b^6*c^4*d^2*e^6*z^2 - B^2
*b^2*c^8*d^6*e^2*z^2 - B^2*a^4*b^6*e^2*f^6*z^2 - 1152*A^2*a^3*c^7*d^4*f^4*z
^2 + 1008*A^2*a^4*c^6*d^3*f^5*z^2 + 624*A^2*a^2*c^8*d^5*f^3*z^2 - 288*A^2*a
^5*c^5*d^2*f^6*z^2 + 56*B^2*a^3*c^7*d^2*e^6*z^2 - 10*B^2*a^2*b^8*d^2*f^6*z^
2 - 9*A^2*b^2*c^8*d^4*e^4*z^2 - 5*A^2*a^2*b^8*e^2*f^6*z^2 - 4*B^2*a^2*c^8*d
^4*e^4*z^2 + 3*A^2*b^4*c^6*d^2*e^6*z^2 - 2*A^2*b^3*c^7*d^3*e^5*z^2 - 36*A^2
*a^2*c^8*d^2*e^6*z^2 - 48*A^2*a^6*b^2*c^2*f^8*z^2 - 45*A^2*a^2*b^2*c^6*e^8*
z^2 + 4*A^2*b^10*d*e^2*f^5*z^2 + 4*B^2*b^2*c^8*d^7*f*z^2 + 4*A^2*b^9*c*e^5*
f^3*z^2 + 4*A^2*b^7*c^3*e^7*f*z^2 - 128*B^2*a^7*c^3*d*f^7*z^2 - 160*A^2*a*c
^9*d^6*f^2*z^2 - 112*A^2*a^6*c^4*d*f^7*z^2 + 12*A^2*b*c^9*d^5*e^3*z^2 + 4*A
^2*a*b^9*e^3*f^5*z^2 + 3*B^2*a^4*b^6*d*f^7*z^2 + 2*A^2*a^3*b^7*e*f^7*z^2 -
24*A^2*a*c^9*d^4*e^4*z^2 + 14*A^2*a^2*b^8*d*f^7*z^2 + 12*A^2*a^5*b^4*c*f^8*
z^2 + 12*A^2*a*b^4*c^5*e^8*z^2 + A*B*a^4*b^6*e*f^7*z^2 + B^2*a^2*b^8*d*e^2*
f^5*z^2 + 16*A^2*c^10*d^7*f*z^2 + 3*B^2*b^10*d^3*f^5*z^2 - A^2*b^10*e^4*f^4
*z^2 - 4*A^2*c^10*d^6*e^2*z^2 - A^2*b^10*d^2*f^6*z^2 + 64*A^2*a^7*c^3*f^8*z
^2 - 4*B^2*a^4*c^6*e^8*z^2 - A^2*b^6*c^4*e^8*z^2 + 48*A^2*a^3*c^7*e^8*z^2 -

A^2*a^4*b^6*f^8*z^2 + 720*A^2*B*a*b^2*c^5*d^2*e^2*f^3*z - 600*A^2*B*a^2*b^
2*c^4*d*e^2*f^4*z + 576*A*B^2*a^2*b^2*c^4*d^2*e*f^4*z + 348*A*B^2*a*b^2*c^5
*d^2*e^3*f^2*z - 336*A*B^2*a^2*b*c^5*d^2*e^2*f^3*z - 260*A*B^2*a*b^3*c^4*d^
2*e^2*f^3*z - 240*A*B^2*a^2*b^2*c^4*d*e^3*f^3*z + 196*A*B^2*a^2*b^3*c^3*d*e
^2*f^4*z + 172*A^2*B*a*b*c^6*d*e^5*f*z + 20*A*B^2*a*b^6*c*d*e*f^5*z - 912*A
^2*B*a^2*b*c^5*d^2*e*f^4*z - 644*A^2*B*a*b*c^6*d^2*e^3*f^2*z - 432*A*B^2*a*
b^2*c^5*d^3*e*f^3*z + 372*A^2*B*a^2*b*c^5*d*e^3*f^3*z - 330*A^2*B*a*b^2*c^5
*d*e^4*f^2*z + 312*A*B^2*a*b*c^6*d^3*e^2*f^2*z - 208*A*B^2*a^3*b^2*c^3*d*e*
f^5*z + 192*A^2*B*a^2*b^3*c^3*d*e*f^5*z + 172*A^2*B*a*b^3*c^4*d*e^3*f^3*z +
108*A*B^2*a^2*b*c^5*d*e^4*f^2*z + 104*A*B^2*a^3*b*c^4*d*e^2*f^4*z - 80*A^2
*B*a*b^3*c^4*d^2*e*f^4*z + 68*A^2*B*a*b^4*c^3*d*e^2*f^4*z - 60*A*B^2*a*b^5*
c^2*d*e^2*f^4*z + 58*A*B^2*a*b^3*c^4*d*e^4*f^2*z - 36*A*B^2*a*b^4*c^3*d^2*e
*f^4*z - 24*A*B^2*a^2*b^4*c^2*d*e*f^5*z + 24*A*B^2*a*b^4*c^3*d*e^3*f^3*z +
592*A^2*B*a*b*c^6*d^3*e*f^3*z + 240*A^2*B*a^3*b*c^4*d*e*f^5*z - 132*A*B^2*a
*b*c^6*d^2*e^4*f*z - 60*A*B^2*a*b^2*c^5*d*e^5*f*z - 48*A^2*B*a*b^5*c^2*d*e*
f^5*z + 20*B^3*a*b*c^6*d^3*e^3*f*z + 16*B^3*a^4*b*c^3*d*e*f^5*z - 16*B^3*a*
b*c^6*d^4*e*f^2*z + 12*B^3*a^2*b*c^5*d*e^5*f*z + 320*A^3*a*b*c^6*d*e^4*f^2*
z + 40*A^3*a*b^4*c^3*d*e*f^5*z - 48*A^2*B*b*c^7*d^4*e*f^2*z - 44*A^2*B*b^3*
c^5*d*e^5*f*z - 20*A*B^2*b*c^7*d^4*e^2*f*z + 14*A*B^2*b^4*c^4*d*e^5*f*z + 1
2*A^2*B*b*c^7*d^3*e^3*f*z + 4*A*B^2*b^7*c*d*e^2*f^4*z + 160*A*B^2*a^4*c^4*d
*e*f^5*z + 152*A^2*B*a*c^7*d^2*e^4*f*z - 40*A*B^2*a*c^7*d^3*e^3*f*z + 32*A*
B^2*a*c^7*d^4*e*f^2*z - 16*A*B^2*a^2*c^6*d*e^5*f*z + 128*A^2*B*a^4*b*c^3*e*
f^6*z + 42*A^2*B*a*b^2*c^5*e^6*f*z + 24*A^2*B*a^2*b^5*c*e*f^6*z - 12*A*B^2*
a^3*b^4*c*e*f^6*z - 12*A*B^2*a^2*b*c^5*e^6*f*z - 10*A^2*B*a*b^6*c*e^2*f^5*z
- 160*A*B^2*a*b*c^6*d^4*f^3*z + 112*A*B^2*a^4*b*c^3*d*f^6*z - 24*A*B^2*a^2
*b^5*c*d*f^6*z - 84*B^3*a*b^2*c^5*d^3*e^2*f^2*z - 80*B^3*a^2*b^3*c^3*d^2*e*
f^4*z - 60*B^3*a^2*b*c^5*d^2*e^3*f^2*z - 20*B^3*a^3*b^2*c^3*d*e^2*f^4*z - 2
0*B^3*a*b^3*c^4*d^2*e^3*f^2*z - 9*B^3*a^2*b^2*c^4*d*e^4*f^2*z - 8*B^3*a*b^4
*c^3*d^2*e^2*f^3*z + 6*B^3*a^2*b^4*c^2*d*e^2*f^4*z - 4*B^3*a^2*b^3*c^3*d*e^
3*f^3*z - 216*A^2*B*b^4*c^4*d^2*e^2*f^3*z + 196*A^2*B*b^3*c^5*d^2*e^3*f^2*z
- 108*A*B^2*b^3*c^5*d^3*e^2*f^2*z - 94*A*B^2*b^4*c^4*d^2*e^3*f^2*z + 88*A^
2*B*b^2*c^6*d^3*e^2*f^2*z + 80*A*B^2*b^5*c^3*d^2*e^2*f^3*z + 360*A^2*B*a^2*
c^6*d^2*e^2*f^3*z + 8*A*B^2*a^2*c^6*d^2*e^3*f^2*z + 153*A^2*B*a^2*b^2*c^4*e
^4*f^3*z - 144*A^2*B*a^2*b^3*c^3*e^3*f^4*z + 80*A^2*B*a^3*b^2*c^3*e^2*f^5*z
+ 36*A*B^2*a^3*b^2*c^3*e^3*f^4*z + 12*A^2*B*a^2*b^4*c^2*e^2*f^5*z + 12*A*B
^2*a^3*b^3*c^2*e^2*f^5*z + 9*A*B^2*a^2*b^2*c^4*e^5*f^2*z - 6*A*B^2*a^2*b^4*
c^2*e^3*f^4*z + 4*A*B^2*a^2*b^3*c^3*e^4*f^3*z + 480*A^2*B*a^2*b^2*c^4*d^2*f
^5*z - 176*A*B^2*a^2*b^3*c^3*d^2*f^5*z - 10*A^2*B*a*b^6*c*d*f^6*z + 16*A*B^
2*a*b*c^6*d*e^6*z + 80*B^3*a*b^3*c^4*d^3*e*f^3*z - 48*B^3*a^3*b*c^4*d^2*e*f
^4*z + 48*B^3*a^2*b*c^5*d^3*e*f^3*z + 44*B^3*a^3*b*c^4*d*e^3*f^3*z + 24*B^3
*a*b^5*c^2*d^2*e*f^4*z + 18*B^3*a*b^2*c^5*d^2*e^4*f*z + 696*A^3*a^2*b*c^5*d
*e^2*f^4*z - 504*A^3*a*b*c^6*d^2*e^2*f^3*z - 192*A^3*a*b^2*c^5*d*e^3*f^3*z
- 144*A^3*a^2*b^2*c^4*d*e*f^5*z + 96*A^3*a*b^2*c^5*d^2*e*f^4*z - 72*A^3*a*b
^3*c^4*d*e^2*f^4*z - 208*A^2*B*b^3*c^5*d^3*e*f^3*z + 152*A*B^2*b^4*c^4*d^3*
e*f^3*z + 80*A^2*B*b^5*c^3*d^2*e*f^4*z + 75*A^2*B*b^4*c^4*d*e^4*f^2*z - 59*

A^2*B*b^2*c^6*d^2*e^4*f*z - 52*A^2*B*b^5*c^3*d*e^3*f^3*z + 42*A*B^2*b^3*c^5
*d^2*e^4*f*z - 21*A*B^2*b^6*c^2*d^2*e*f^4*z - 16*A*B^2*b^5*c^3*d*e^4*f^2*z
+ 16*A*B^2*b^2*c^6*d^4*e*f^2*z + 16*A*B^2*b^2*c^6*d^3*e^3*f*z + 11*A^2*B*b^
6*c^2*d*e^2*f^4*z + 4*A*B^2*b^6*c^2*d*e^3*f^3*z - 256*A^2*B*a*c^7*d^3*e^2*f
^2*z - 96*A*B^2*a^3*c^5*d^2*e*f^4*z - 36*A^2*B*a^2*c^6*d*e^4*f^2*z - 32*A^2
*B*a^3*c^5*d*e^2*f^4*z - 32*A*B^2*a^2*c^6*d^3*e*f^3*z + 8*A*B^2*a^3*c^5*d*e
^3*f^3*z - 96*A^2*B*a^3*b^3*c^2*e*f^6*z + 68*A^2*B*a^3*b*c^4*e^3*f^4*z - 60
*A*B^2*a^4*b*c^3*e^2*f^5*z - 60*A*B^2*a^3*b*c^4*e^4*f^3*z + 48*A*B^2*a^4*b^
2*c^2*e*f^6*z - 38*A^2*B*a*b^3*c^4*e^5*f^2*z - 36*A^2*B*a^2*b*c^5*e^5*f^2*z
+ 36*A^2*B*a*b^5*c^2*e^3*f^4*z - 16*A^2*B*a*b^4*c^3*e^4*f^3*z + 384*A*B^2*
a^2*b*c^5*d^3*f^4*z - 352*A*B^2*a^3*b*c^4*d^2*f^5*z - 288*A^2*B*a*b^2*c^5*d
^3*f^4*z - 160*A^2*B*a^3*b^2*c^3*d*f^6*z - 148*A^2*B*a*b^4*c^3*d^2*f^5*z +
112*A*B^2*a*b^3*c^4*d^3*f^4*z + 72*A^2*B*a^2*b^4*c^2*d*f^6*z + 72*A*B^2*a*b
^5*c^2*d^2*f^5*z + 48*A*B^2*a^3*b^3*c^2*d*f^6*z + 102*B^3*a^2*b^2*c^4*d^2*e
^2*f^3*z - 32*B^3*b^5*c^3*d^3*e*f^3*z - 8*B^3*b^3*c^5*d^3*e^3*f*z - 7*B^3*b
^4*c^4*d^2*e^4*f*z + 5*B^3*b^2*c^6*d^4*e^2*f*z + 80*A^3*b^2*c^6*d^3*e*f^3*z
- 74*A^3*b^3*c^5*d*e^4*f^2*z - 64*A^3*b^4*c^4*d^2*e*f^4*z + 60*A^3*b^4*c^4
*d*e^3*f^3*z - 48*B^3*a^4*c^4*d*e^2*f^4*z - 24*B^3*a^3*c^5*d*e^4*f^2*z + 20
*B^3*a^2*c^6*d^2*e^4*f*z - 16*A^3*b^5*c^3*d*e^2*f^4*z + 8*A^3*b*c^7*d^3*e^2
*f^2*z + 480*A^3*a^2*c^6*d^2*e*f^4*z - 392*A^3*a^2*c^6*d*e^3*f^3*z + 280*A^
3*a*c^7*d^2*e^3*f^2*z - 4*B^3*a^4*b*c^3*e^3*f^4*z - 200*A^3*a^3*b*c^4*e^2*f
^5*z - 144*A^3*a^2*b*c^5*e^4*f^3*z + 48*B^3*a*b^2*c^5*d^4*f^3*z + 42*A^3*a*
b^2*c^5*e^5*f^2*z - 36*B^3*a^4*b^2*c^2*d*f^6*z - 32*A^3*a^3*b^2*c^3*e*f^6*z
- 24*A^3*a^2*b^4*c^2*e*f^6*z - 24*A^3*a*b^5*c^2*e^2*f^5*z + 10*A^3*a*b^3*c
^4*e^4*f^3*z - 4*B^3*a*b^4*c^3*d^3*f^4*z - 4*A^3*a*b^4*c^3*e^3*f^4*z - 480*
A^3*a^2*b*c^5*d^2*f^5*z - 160*A^3*a^2*b^3*c^3*d*f^6*z + 128*A^3*a*b^3*c^4*d
^2*f^5*z + 8*A^2*B*b^5*c^3*e^5*f^2*z - 2*A^2*B*b^6*c^2*e^4*f^3*z + 112*A^2*
B*b^4*c^4*d^3*f^4*z - 92*A^2*B*a^4*c^4*e^2*f^5*z - 64*A^2*B*a^3*c^5*e^4*f^3
*z - 64*A*B^2*b^5*c^3*d^3*f^4*z + 24*A*B^2*a^4*c^4*e^3*f^4*z + 24*A*B^2*a^3
*c^5*e^5*f^2*z + 16*A^2*B*b^2*c^6*d^4*f^3*z + 16*A*B^2*b^3*c^5*d^4*f^3*z -
A^2*B*b^6*c^2*d^2*f^5*z + 448*A^2*B*a^3*c^5*d^2*f^5*z - 352*A^2*B*a^2*c^6*d
^3*f^4*z - 5*A*B^2*b^2*c^6*d^2*e^5*z - 48*A^2*B*a^4*b^2*c^2*f^7*z - 2*B^3*b
^7*c*d^2*e*f^4*z + 34*A^3*b^2*c^6*d*e^5*f*z + 16*A^3*b*c^7*d^2*e^4*f*z + 2*
A^3*b^6*c^2*d*e*f^5*z - 416*A^3*a^3*c^5*d*e*f^5*z - 224*A^3*a*c^7*d^3*e*f^3
*z + 12*B^3*a^3*b^4*c*d*f^6*z - 10*B^3*a*b^6*c*d^2*f^5*z + 416*A^3*a^3*b*c^
4*d*f^6*z + 224*A^3*a*b*c^6*d^3*f^4*z + 24*A^3*a*b^5*c^2*d*f^6*z - 4*B^3*a*
b*c^6*d^2*e^5*z + 20*A^2*B*c^8*d^4*e^2*f*z - 7*A^2*B*b^4*c^4*e^6*f*z - 2*A^
2*B*b^7*c*e^3*f^4*z - 64*A*B^2*a^5*c^3*e*f^6*z + 16*A*B^2*b*c^7*d^5*f^2*z -
8*A^2*B*a^2*c^6*e^6*f*z - 2*A*B^2*b^7*c*d^2*f^5*z - 272*A^2*B*a^4*c^4*d*f^
6*z + 128*A^2*B*a*c^7*d^4*f^3*z + 9*A^2*B*b^2*c^6*d*e^6*z - 4*A*B^2*b^3*c^5
*d*e^6*z + 4*A*B^2*b*c^7*d^3*e^4*z + 8*A*B^2*a*c^7*d^2*e^5*z + 12*A^2*B*a^3
*b^4*c*f^7*z + 30*B^3*b^4*c^4*d^3*e^2*f^2*z + 8*B^3*b^5*c^3*d^2*e^3*f^2*z -
2*B^3*b^6*c^2*d^2*e^2*f^3*z + 152*A^3*b^3*c^5*d^2*e^2*f^3*z - 108*A^3*b^2*
c^6*d^2*e^3*f^2*z + 48*B^3*a^3*c^5*d^2*e^2*f^3*z - 16*B^3*a^2*c^6*d^3*e^2*f
^2*z - 3*B^3*a^4*b^2*c^2*e^2*f^5*z - 120*B^3*a^2*b^2*c^4*d^3*f^4*z + 112*B^

3*a^3*b^2*c^3*d^2*f^5*z + 112*A^3*a^2*b^3*c^3*e^2*f^5*z + 12*A^3*a^2*b^2*c^
4*e^3*f^4*z - 120*A^3*a*c^7*d*e^5*f*z - 52*A^3*a*b*c^6*e^6*f*z + 10*A^3*a*b
^6*c*e*f^6*z - 2*A*B^2*b^8*d*e*f^5*z - 2*A^2*B*a*b^7*e*f^6*z - 24*A^2*B*a*c
^7*d*e^6*z + 2*A*B^2*a*b^7*d*f^6*z - 12*A^2*B*a*b*c^6*e^7*z - 2*A^3*b^7*c*d
*f^6*z - 4*A^3*b*c^7*d*e^6*z + 16*B^3*a^5*c^3*e^2*f^5*z + 11*B^3*b^6*c^2*d^
3*f^4*z - 11*A^3*b^4*c^4*e^5*f^2*z - 8*B^3*b^4*c^4*d^4*f^3*z - 4*B^3*b^2*c^
6*d^5*f^2*z + 4*B^3*a^4*c^4*e^4*f^3*z + 4*A^3*b^5*c^3*e^4*f^3*z - A^3*b^6*c
^2*e^3*f^4*z + 136*A^3*a^3*c^5*e^3*f^4*z + 68*A^3*a^2*c^6*e^5*f^2*z - 64*A^
3*b^3*c^5*d^3*f^4*z + 2*B^3*b^3*c^5*d^2*e^5*z - B^3*b^2*c^6*d^3*e^4*z + 96*
A^3*a^3*b^3*c^2*f^7*z + A*B^2*a^2*b^6*e*f^6*z + 32*A^3*c^8*d^4*e*f^2*z - 24
*A^3*c^8*d^3*e^3*f*z + 10*A^3*b^3*c^5*e^6*f*z + 2*A^3*b^7*c*e^2*f^5*z + 128
*A^3*a^4*c^4*e*f^6*z - 32*A^3*b*c^7*d^4*f^3*z - 4*B^3*a^2*c^6*d*e^6*z - B^3
*a^2*b^6*d*f^6*z - 128*A^3*a^4*b*c^3*f^7*z - 24*A^3*a^2*b^5*c*f^7*z - 16*A^
2*B*c^8*d^5*f^2*z - 4*A^2*B*c^8*d^3*e^4*z + 64*A^2*B*a^5*c^3*f^7*z + 2*A^2*
B*b^3*c^5*e^7*z + 4*A*B^2*a^2*c^6*e^7*z - A^2*B*a^2*b^6*f^7*z + 4*A^3*c^8*d
^2*e^5*z - 3*A^3*b^2*c^6*e^7*z + A^2*B*b^8*d*f^6*z - A^3*b^8*e*f^6*z + 16*A
^3*a*c^7*e^7*z + 2*A^3*a*b^7*f^7*z + A^2*B*b^8*e^2*f^5*z + B^3*b^8*d^2*f^5*
z - 48*A^2*B^2*a*b*c^4*d*e*f^4 + 28*A*B^3*a*b^2*c^3*d*e*f^4 - 16*A*B^3*a*b*
c^4*d*e^2*f^3 + 16*A^3*B*a*c^5*d*e*f^4 + 32*A^3*B*a*b*c^4*d*f^5 + 12*A^2*B^
2*b^3*c^3*d*e*f^4 + 5*A*B^3*b^2*c^4*d^2*e*f^3 + 4*A*B^3*b^3*c^3*d*e^2*f^3 +
24*A^2*B^2*a*c^5*d*e^2*f^3 + 24*A^2*B^2*a^2*b*c^3*e*f^5 + 12*A^2*B^2*a*b*c
^4*e^3*f^3 - 6*A^2*B^2*a*b^3*c^2*e*f^5 + 4*A*B^3*a^2*b*c^3*e^2*f^4 + 3*A*B^
3*a^2*b^2*c^2*e*f^5 - 18*A^2*B^2*a*b^2*c^3*d*f^5 - 4*B^4*a^2*b*c^3*d*e*f^4
+ 4*B^4*a*b*c^4*d^2*e*f^3 - 6*A*B^3*b^4*c^2*d*e*f^4 + 4*A^3*B*b*c^5*d*e^2*f
^3 - 2*A^3*B*b^2*c^4*d*e*f^4 - 8*A*B^3*a^2*c^4*d*e*f^4 - 8*A*B^3*a*c^5*d^2*
e*f^3 + 26*A^3*B*a*b^2*c^3*e*f^5 + 8*A^3*B*a*b*c^4*e^2*f^4 + 32*A*B^3*a*b*c
^4*d^2*f^4 - 28*A*B^3*a^2*b*c^3*d*f^5 + 6*A*B^3*a*b^3*c^2*d*f^5 - 9*A^2*B^2
*b^2*c^4*d*e^2*f^3 - 18*A^2*B^2*a*b^2*c^3*e^2*f^4 - 4*A^3*B*c^6*d^2*e*f^3 -
3*A^3*B*b^4*c^2*e*f^5 - 44*A^3*B*a^2*c^4*e*f^5 - 16*A^3*B*a*c^5*e^3*f^3 -
16*A*B^3*a^3*c^3*e*f^5 - 10*A^3*B*b^3*c^3*d*f^5 - 4*A^3*B*b*c^5*d^2*f^4 - 4
*A*B^3*b*c^5*d^3*f^3 - 28*A^3*B*a^2*b*c^3*f^6 + 6*A^3*B*a*b^3*c^2*f^6 - 4*A
^4*b*c^5*d*e*f^4 - 20*A^4*a*b*c^4*e*f^5 + 3*A^2*B^2*b^4*c^2*e^2*f^4 - 2*A^2
*B^2*b^3*c^3*e^3*f^3 + 12*A^2*B^2*a^2*c^4*e^2*f^4 + 9*A^2*B^2*b^2*c^4*d^2*f
^4 - 3*A^2*B^2*a^2*b^2*c^2*f^6 - 2*B^4*b^3*c^3*d^2*e*f^3 + 4*B^4*a^2*c^4*d*
e^2*f^3 - 10*B^4*a*b^2*c^3*d^2*f^4 - 3*B^4*a^2*b^2*c^2*d*f^5 + 3*A^3*B*b^2*
c^4*e^3*f^3 - 2*A^3*B*b^3*c^3*e^2*f^4 - 10*A*B^3*b^3*c^3*d^2*f^4 - 4*A*B^3*
a^2*c^4*e^3*f^3 + 3*A^2*B^2*b^4*c^2*d*f^5 + 36*A^2*B^2*a^2*c^4*d*f^5 - 24*A
^2*B^2*a*c^5*d^2*f^4 + 4*A^2*B^2*c^6*d^3*f^3 + 16*A^2*B^2*a^3*c^3*f^6 + 4*A
^4*b^3*c^3*e*f^5 + 16*B^4*a^3*c^3*d*f^5 + 16*A^4*a*c^5*e^2*f^4 + 8*A^4*b^2*
c^4*d*f^5 - 8*A^4*a*b^2*c^3*f^6 - 24*A^4*a*c^5*d*f^5 + 3*B^4*b^4*c^2*d^2*f^
4 - 3*A^4*b^2*c^4*e^2*f^4 + 4*A^4*c^6*d^2*f^4 + 36*A^4*a^2*c^4*f^6 + B^4*b^
2*c^4*d^3*f^3, z, k)*((64*a^9*c^4*e*f^8 - 64*a^6*c^7*e^7*f^2 - 64*a^7*c^6*e
^5*f^4 + 64*a^8*c^5*e^3*f^6 + 4*b^5*c^8*d^7*f^2 + 4*b^7*c^6*d^6*f^3 - 4*b^9
*c^4*d^5*f^4 - 4*b^11*c^2*d^4*f^5 - 612*a^2*b^5*c^6*d^5*f^4 - 712*a^2*b^7*c
^4*d^4*f^5 - 132*a^2*b^9*c^2*d^3*f^6 + 1696*a^3*b^3*c^7*d^5*f^4 + 2736*a^3*

b^5*c^5*d^4*f^5 + 896*a^3*b^7*c^3*d^3*f^6 - 5120*a^4*b^3*c^6*d^4*f^5 - 3140
*a^4*b^5*c^4*d^3*f^6 - 220*a^4*b^7*c^2*d^2*f^7 + 5664*a^5*b^3*c^5*d^3*f^6 +
 1128*a^5*b^5*c^3*d^2*f^7 - 2560*a^6*b^3*c^4*d^2*f^7 + 4*a^3*b^6*c^4*e^7*f^
2 - 6*a^3*b^7*c^3*e^6*f^3 + 4*a^3*b^8*c^2*e^5*f^4 - 36*a^4*b^4*c^5*e^7*f^2
+ 57*a^4*b^5*c^4*e^6*f^3 - 37*a^4*b^6*c^3*e^5*f^4 + 7*a^4*b^7*c^2*e^4*f^5 +
 96*a^5*b^2*c^6*e^7*f^2 - 168*a^5*b^3*c^5*e^6*f^3 + 100*a^5*b^4*c^4*e^5*f^4
 - 3*a^5*b^5*c^3*e^4*f^5 - 10*a^5*b^6*c^2*e^3*f^6 - 48*a^6*b^2*c^5*e^5*f^4
- 56*a^6*b^3*c^4*e^4*f^5 + 36*a^6*b^4*c^3*e^3*f^6 - 13*a^6*b^5*c^2*e^2*f^7
- 64*a^7*b^2*c^4*e^3*f^6 + 56*a^7*b^3*c^3*e^2*f^7 - 1472*a^4*c^9*d^4*e^3*f^
2 - 1088*a^5*c^8*d^2*e^5*f^2 + 3584*a^5*c^8*d^3*e^3*f^3 - 3200*a^6*c^7*d^2*
e^3*f^4 - 2*b^7*c^6*d^5*e^2*f^2 - b^8*c^5*d^4*e^3*f^2 + 4*b^9*c^4*d^3*e^4*f
^2 - 5*b^9*c^4*d^4*e^2*f^3 - 6*b^10*c^3*d^3*e^3*f^3 + 4*b^11*c^2*d^3*e^2*f^
4 + 8*a*b^11*c*d^3*f^6 + 8*a^5*b^7*c*d*f^8 - 448*a^8*b*c^4*d*f^8 - 16*a^5*b
*c^7*e^8*f - a^6*b^6*c*e*f^8 + 128*a^5*c^8*d*e^7*f + 128*a^8*c^5*d*e*f^7 -
b^12*c*d^3*e*f^5 - 32*a*b^3*c^9*d^7*f^2 - 24*a*b^5*c^7*d^6*f^3 + 88*a*b^7*c
^5*d^5*f^4 + 88*a*b^9*c^3*d^4*f^5 + 64*a^2*b*c^10*d^7*f^2 + 128*a^3*b*c^9*d
^6*f^3 + 16*a^3*b^9*c*d^2*f^7 - 1600*a^4*b*c^8*d^5*f^4 + 3840*a^5*b*c^7*d^4
*f^5 - 4160*a^6*b*c^6*d^3*f^6 - 92*a^6*b^5*c^2*d*f^8 + 2176*a^7*b*c^5*d^2*f
^7 + 352*a^7*b^3*c^3*d*f^8 - a^3*b^5*c^5*e^8*f - a^3*b^9*c*e^4*f^5 + 8*a^4*
b^3*c^6*e^8*f + a^4*b^8*c*e^3*f^6 + a^5*b^7*c*e^2*f^7 + 144*a^6*b*c^6*e^6*f
^3 + 80*a^7*b*c^5*e^4*f^5 + 12*a^7*b^4*c^2*e*f^8 - 80*a^8*b*c^4*e^2*f^7 - 4
8*a^8*b^2*c^3*e*f^8 + 128*a^3*c^10*d^5*e^3*f - 448*a^3*c^10*d^6*e*f^2 + 256
*a^4*c^9*d^3*e^5*f + 2176*a^4*c^9*d^5*e*f^3 - 4160*a^5*c^8*d^4*e*f^4 + 896*
a^6*c^7*d*e^5*f^3 + 3840*a^6*c^7*d^3*e*f^5 + 896*a^7*c^6*d*e^3*f^5 - 1600*a
^7*c^6*d^2*e*f^6 - b^5*c^8*d^6*e^2*f + b^6*c^7*d^5*e^3*f - 5*b^6*c^7*d^6*e*
f^2 + b^7*c^6*d^4*e^4*f - b^8*c^5*d^3*e^5*f + 5*b^8*c^5*d^5*e*f^3 + 9*b^10*
c^3*d^4*e*f^4 + 8*a*b^3*c^9*d^6*e^2*f + 12*a*b^4*c^8*d^6*e*f^2 - 27*a*b^5*c
^7*d^4*e^4*f + 18*a*b^6*c^6*d^3*e^5*f - 154*a*b^6*c^6*d^5*e*f^3 + a*b^7*c^5
*d^2*e^6*f - 200*a*b^8*c^4*d^4*e*f^4 - 2*a*b^10*c^2*d^3*e*f^5 + a*b^11*c*d^
2*e^2*f^5 - 16*a^2*b*c^10*d^6*e^2*f + a^2*b^6*c^5*d*e^7*f + a^2*b^10*c*d*e^
3*f^5 - 19*a^2*b^10*c*d^2*e*f^6 - 304*a^3*b*c^9*d^4*e^4*f + 10*a^3*b^9*c*d*
e^2*f^6 - 304*a^4*b*c^8*d^2*e^6*f - 48*a^4*b^2*c^7*d*e^7*f - 160*a^5*b*c^7*
d*e^6*f^2 + 214*a^5*b^6*c^2*d*e*f^7 - 2048*a^6*b*c^6*d*e^4*f^4 - 792*a^6*b^
4*c^3*d*e*f^7 - 1824*a^7*b*c^5*d*e^2*f^6 + 928*a^7*b^2*c^4*d*e*f^7 + 78*a*b
^5*c^7*d^5*e^2*f^2 + 10*a*b^6*c^6*d^4*e^3*f^2 - 68*a*b^7*c^5*d^3*e^4*f^2 +
129*a*b^7*c^5*d^4*e^2*f^3 - 4*a*b^8*c^4*d^2*e^5*f^2 + 96*a*b^8*c^4*d^3*e^3*
f^3 + 6*a*b^9*c^3*d^2*e^4*f^3 - 52*a*b^9*c^3*d^3*e^2*f^4 - 4*a*b^10*c^2*d^2
*e^3*f^4 - 48*a^2*b^2*c^9*d^5*e^3*f + 144*a^2*b^2*c^9*d^6*e*f^2 + 168*a^2*b
^3*c^8*d^4*e^4*f - 80*a^2*b^4*c^7*d^3*e^5*f + 1128*a^2*b^4*c^7*d^5*e*f^3 -
27*a^2*b^5*c^6*d^2*e^6*f + 1481*a^2*b^6*c^5*d^4*e*f^4 - 4*a^2*b^7*c^4*d*e^6
*f^2 + 6*a^2*b^8*c^3*d*e^5*f^3 + 126*a^2*b^8*c^3*d^3*e*f^5 - 4*a^2*b^9*c^2*
d*e^4*f^4 + 992*a^3*b*c^9*d^5*e^2*f^2 + 32*a^3*b^2*c^8*d^3*e^5*f - 2912*a^3
*b^2*c^8*d^5*e*f^3 + 168*a^3*b^3*c^7*d^2*e^6*f - 4812*a^3*b^4*c^6*d^4*e*f^4
 + 22*a^3*b^5*c^5*d*e^6*f^2 - 68*a^3*b^6*c^4*d*e^5*f^3 - 860*a^3*b^6*c^4*d^
3*e*f^5 + 80*a^3*b^7*c^3*d*e^4*f^4 - 44*a^3*b^8*c^2*d*e^3*f^5 + 232*a^3*b^8

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*c^2*d^2*e*f^6 + 1280*a^4*b*c^8*d^3*e^4*f^2 - 816*a^4*b*c^8*d^4*e^2*f^3 + 7
088*a^4*b^2*c^7*d^4*e*f^4 + 16*a^4*b^3*c^6*d*e^6*f^2 + 312*a^4*b^4*c^5*d*e^
5*f^3 + 2864*a^4*b^4*c^5*d^3*e*f^5 - 576*a^4*b^5*c^4*d*e^4*f^4 + 393*a^4*b^
6*c^3*d*e^3*f^5 - 995*a^4*b^6*c^3*d^2*e*f^6 - 78*a^4*b^7*c^2*d*e^2*f^6 + 99
2*a^5*b*c^7*d^2*e^4*f^3 - 3264*a^5*b*c^7*d^3*e^2*f^4 - 768*a^5*b^2*c^6*d*e^
5*f^3 - 5184*a^5*b^2*c^6*d^3*e*f^5 + 1792*a^5*b^3*c^5*d*e^4*f^4 - 1232*a^5*
b^4*c^4*d*e^3*f^5 + 1588*a^5*b^4*c^4*d^2*e*f^6 + 30*a^5*b^5*c^3*d*e^2*f^6 +
 5008*a^6*b*c^6*d^2*e^2*f^5 + 976*a^6*b^2*c^5*d*e^3*f^5 - 16*a^6*b^2*c^5*d^
2*e*f^6 + 944*a^6*b^3*c^4*d*e^2*f^6 - 19*a^4*b^8*c*d*e*f^7 - 528*a^2*b^3*c^
8*d^5*e^2*f^2 - 124*a^2*b^4*c^7*d^4*e^3*f^2 + 432*a^2*b^5*c^6*d^3*e^4*f^2 -
 843*a^2*b^5*c^6*d^4*e^2*f^3 + 83*a^2*b^6*c^5*d^2*e^5*f^2 - 722*a^2*b^6*c^5
*d^3*e^3*f^3 - 80*a^2*b^7*c^4*d^2*e^4*f^3 + 376*a^2*b^7*c^4*d^3*e^2*f^4 + 4
3*a^2*b^9*c^2*d^2*e^2*f^5 + 768*a^3*b^2*c^8*d^4*e^3*f^2 - 1216*a^3*b^3*c^7*
d^3*e^4*f^2 + 1832*a^3*b^3*c^7*d^4*e^2*f^3 - 540*a^3*b^4*c^6*d^2*e^5*f^2 +
2928*a^3*b^4*c^6*d^3*e^3*f^3 + 414*a^3*b^5*c^5*d^2*e^4*f^3 - 1740*a^3*b^5*c
^5*d^3*e^2*f^4 + 344*a^3*b^6*c^4*d^2*e^3*f^4 - 634*a^3*b^7*c^3*d^2*e^2*f^5
+ 1360*a^4*b^2*c^7*d^2*e^5*f^2 - 5664*a^4*b^2*c^7*d^3*e^3*f^3 - 1008*a^4*b^
3*c^6*d^2*e^4*f^3 + 4064*a^4*b^3*c^6*d^3*e^2*f^4 - 1928*a^4*b^4*c^5*d^2*e^3
*f^4 + 3065*a^4*b^5*c^4*d^2*e^2*f^5 + 4032*a^5*b^2*c^6*d^2*e^3*f^4 - 6376*a
^5*b^3*c^5*d^2*e^2*f^5)/(16*a^2*c^6*d^4 + a^4*b^4*f^4 + 16*a^4*c^4*e^4 + b^
4*c^4*d^4 + 16*a^6*c^2*f^4 + b^8*d^2*f^2 - 8*a*b^2*c^5*d^4 - 8*a^5*b^2*c*f^
4 + 2*a^2*b^6*d*f^3 - 2*a^3*b^5*e*f^3 - 64*a^3*c^5*d^3*f - 64*a^5*c^3*d*f^3
 - 2*b^5*c^3*d^3*e + 2*b^6*c^2*d^3*f + a^2*b^4*c^2*e^4 - 8*a^3*b^2*c^3*e^4
+ 32*a^3*c^5*d^2*e^2 + a^2*b^6*e^2*f^2 + 96*a^4*c^4*d^2*f^2 + b^6*c^2*d^2*e
^2 + 32*a^5*c^3*e^2*f^2 - 2*a*b^7*d*e*f^2 - 2*b^7*c*d^2*e*f + 54*a^2*b^4*c^
2*d^2*f^2 - 112*a^3*b^2*c^3*d^2*f^2 + 16*a*b^3*c^4*d^3*e - 2*a*b^5*c^2*d*e^
3 - 32*a^2*b*c^5*d^3*e - 32*a^3*b*c^4*d*e^3 - 20*a*b^4*c^3*d^3*f - 12*a*b^6
*c*d^2*f^2 - 20*a^3*b^4*c*d*f^3 - 2*a^2*b^5*c*e^3*f - 32*a^4*b*c^3*e^3*f +
16*a^4*b^3*c*e*f^3 - 32*a^5*b*c^2*e*f^3 - 64*a^4*c^4*d*e^2*f - 6*a*b^4*c^3*
d^2*e^2 + 16*a^2*b^3*c^3*d*e^3 + 64*a^2*b^2*c^4*d^3*f + 64*a^4*b^2*c^2*d*f^
3 + 16*a^3*b^3*c^2*e^3*f - 6*a^3*b^4*c*e^2*f^2 - 48*a^2*b^3*c^3*d^2*e*f - 3
6*a^2*b^4*c^2*d*e^2*f + 96*a^3*b^2*c^3*d*e^2*f - 48*a^3*b^3*c^2*d*e*f^2 + 4
*a*b^6*c*d*e^2*f + 18*a*b^5*c^2*d^2*e*f + 18*a^2*b^5*c*d*e*f^2 + 32*a^3*b*c
^4*d^2*e*f + 32*a^4*b*c^3*d*e*f^2) + (x*(128*a^9*c^4*f^9 - 2*a^6*b^6*c*f^9
- 640*a^8*c^5*d*f^8 + 96*a^5*c^8*e^8*f + 6*b^12*c*d^3*f^6 + 24*a^7*b^4*c^2*
f^9 - 96*a^8*b^2*c^3*f^9 + 128*a^2*c^11*d^7*f^2 - 640*a^3*c^10*d^6*f^3 + 11
52*a^4*c^9*d^5*f^4 - 640*a^5*c^8*d^4*f^5 - 640*a^6*c^7*d^3*f^6 + 1152*a^7*c
^6*d^2*f^7 + 288*a^6*c^7*e^6*f^3 + 416*a^7*c^6*e^4*f^5 + 352*a^8*c^5*e^2*f^
7 + 8*b^4*c^9*d^7*f^2 + 22*b^6*c^7*d^6*f^3 + 26*b^8*c^5*d^5*f^4 + 18*b^10*c
^3*d^4*f^5 + 672*a^2*b^2*c^9*d^6*f^3 + 1224*a^2*b^4*c^7*d^5*f^4 + 1202*a^2*
b^6*c^5*d^4*f^5 + 564*a^2*b^8*c^3*d^3*f^6 - 2048*a^3*b^2*c^8*d^5*f^4 - 2744
*a^3*b^4*c^6*d^4*f^5 - 1736*a^3*b^6*c^4*d^3*f^6 - 128*a^3*b^8*c^2*d^2*f^7 +
 2656*a^4*b^2*c^7*d^4*f^5 + 2648*a^4*b^4*c^5*d^3*f^6 + 570*a^4*b^6*c^3*d^2*
f^7 - 1344*a^5*b^2*c^6*d^3*f^6 - 904*a^5*b^4*c^4*d^2*f^7 - 160*a^6*b^2*c^5*
d^2*f^7 + 8*a^2*b^7*c^4*e^7*f^2 - 12*a^2*b^8*c^3*e^6*f^3 + 8*a^2*b^9*c^2*e^
```

5*f^4 - 90*a^3*b^5*c^5*e^7*f^2 + 132*a^3*b^6*c^4*e^6*f^3 - 76*a^3*b^7*c^3*e
^5*f^4 + 6*a^3*b^8*c^2*e^4*f^5 + 336*a^4*b^3*c^6*e^7*f^2 - 462*a^4*b^4*c^5*
e^6*f^3 + 164*a^4*b^5*c^4*e^5*f^4 + 106*a^4*b^6*c^3*e^4*f^5 - 56*a^4*b^7*c^
2*e^3*f^6 + 432*a^5*b^2*c^6*e^6*f^3 + 288*a^5*b^3*c^5*e^5*f^4 - 598*a^5*b^4
*c^4*e^4*f^5 + 102*a^5*b^5*c^3*e^3*f^6 + 90*a^5*b^6*c^2*e^2*f^7 + 720*a^6*b
^2*c^5*e^4*f^5 + 336*a^6*b^3*c^4*e^3*f^6 - 314*a^6*b^4*c^3*e^2*f^7 + 240*a^
7*b^2*c^4*e^2*f^7 + 64*a^3*c^10*d^5*e^2*f^2 - 768*a^4*c^9*d^3*e^4*f^2 + 416
*a^4*c^9*d^4*e^2*f^3 + 1856*a^5*c^8*d^2*e^4*f^3 - 1664*a^5*c^8*d^3*e^2*f^4
+ 2336*a^6*c^7*d^2*e^2*f^5 + 26*b^6*c^7*d^5*e^2*f^2 - 8*b^7*c^6*d^4*e^3*f^2
 - 10*b^8*c^5*d^3*e^4*f^2 + 58*b^8*c^5*d^4*e^2*f^3 + 8*b^9*c^4*d^2*e^5*f^2
- 12*b^9*c^4*d^3*e^3*f^3 - 12*b^10*c^3*d^2*e^4*f^3 + 36*b^10*c^3*d^3*e^2*f^
4 + 8*b^11*c^2*d^2*e^3*f^4 + 2*a^4*b^8*c*d*f^8 + 6*a^5*b^7*c*e*f^8 - 384*a^
8*b*c^4*e*f^8 - 64*a*b^2*c^10*d^7*f^2 - 216*a*b^4*c^8*d^6*f^3 - 300*a*b^6*c
^6*d^5*f^4 - 240*a*b^8*c^4*d^4*f^5 - 92*a*b^10*c^2*d^3*f^6 + 10*a^2*b^10*c*
d^2*f^7 - 12*a^5*b^6*c^2*d*f^8 - 40*a^6*b^4*c^3*d*f^8 + 384*a^7*b^2*c^4*d*f
^8 - 2*a^2*b^6*c^5*e^8*f - 2*a^2*b^10*c*e^4*f^5 + 22*a^3*b^4*c^6*e^8*f + 6*
a^3*b^9*c*e^3*f^6 - 80*a^4*b^2*c^7*e^8*f - 8*a^4*b^8*c*e^2*f^7 - 416*a^5*b*
c^7*e^7*f^2 - 960*a^6*b*c^6*e^5*f^4 - 72*a^6*b^5*c^2*e*f^8 - 928*a^7*b*c^5*
e^3*f^6 + 288*a^7*b^3*c^3*e*f^8 - 32*a^2*c^11*d^6*e^2*f + 32*a^3*c^10*d^4*e
^4*f + 160*a^4*c^9*d^2*e^6*f - 704*a^5*c^8*d*e^6*f^2 - 1536*a^6*c^7*d*e^4*f
^4 - 1472*a^7*c^6*d*e^2*f^6 - 2*b^4*c^9*d^6*e^2*f + 6*b^5*c^8*d^5*e^3*f - 2
4*b^5*c^8*d^6*e*f^2 - 8*b^6*c^7*d^4*e^4*f + 6*b^7*c^6*d^3*e^5*f - 58*b^7*c^
6*d^5*e*f^3 - 2*b^8*c^5*d^2*e^6*f - 60*b^9*c^4*d^4*e*f^4 - 26*b^11*c^2*d^3*
e*f^5 - 2*b^12*c*d^2*e^2*f^5 + 16*a*b^2*c^10*d^6*e^2*f - 48*a*b^3*c^9*d^5*e
^3*f + 192*a*b^3*c^9*d^6*e*f^2 + 66*a*b^4*c^8*d^4*e^4*f - 52*a*b^5*c^7*d^3*
e^5*f + 576*a*b^5*c^7*d^5*e*f^3 + 14*a*b^6*c^6*d^2*e^6*f + 718*a*b^7*c^5*d^
4*e*f^4 - 16*a*b^8*c^4*d*e^6*f^2 + 24*a*b^9*c^3*d*e^5*f^3 + 356*a*b^9*c^3*d
^3*e*f^5 - 16*a*b^10*c^2*d*e^4*f^4 + 96*a^2*b*c^10*d^5*e^3*f - 384*a^2*b*c^
10*d^6*e*f^2 - 42*a^2*b^5*c^6*d*e^7*f - 2*a^2*b^10*c*d*e^2*f^6 - 64*a^3*b*c
^9*d^3*e^5*f + 1792*a^3*b*c^9*d^5*e*f^3 + 144*a^3*b^3*c^7*d*e^7*f - 3712*a^
4*b*c^8*d^4*e*f^4 + 14*a^4*b^7*c^2*d*e*f^7 + 2368*a^5*b*c^7*d*e^5*f^3 + 460
8*a^5*b*c^7*d^3*e*f^5 + 192*a^5*b^5*c^3*d*e*f^7 + 3808*a^6*b*c^6*d*e^3*f^5
- 3712*a^6*b*c^6*d^2*e*f^6 - 1184*a^6*b^3*c^4*d*e*f^7 - 204*a*b^4*c^8*d^5*e
^2*f^2 + 46*a*b^5*c^7*d^4*e^3*f^2 + 132*a*b^6*c^6*d^3*e^4*f^2 - 590*a*b^6*c
^6*d^4*e^2*f^3 - 90*a*b^7*c^5*d^2*e^5*f^2 + 64*a*b^7*c^5*d^3*e^3*f^3 + 196*
a*b^8*c^4*d^2*e^4*f^3 - 408*a*b^8*c^4*d^3*e^2*f^4 - 188*a*b^9*c^3*d^2*e^3*f
^4 + 78*a*b^10*c^2*d^2*e^2*f^5 - 144*a^2*b^2*c^9*d^4*e^4*f + 128*a^2*b^3*c^
8*d^3*e^5*f - 1824*a^2*b^3*c^8*d^5*e*f^3 - 6*a^2*b^4*c^7*d^2*e^6*f - 3096*a
^2*b^5*c^6*d^4*e*f^4 + 190*a^2*b^6*c^5*d*e^6*f^2 - 316*a^2*b^7*c^4*d*e^5*f^
3 - 1908*a^2*b^7*c^4*d^3*e*f^5 + 228*a^2*b^8*c^3*d*e^4*f^4 - 58*a^2*b^9*c^2
*d*e^3*f^5 + 92*a^2*b^9*c^2*d^2*e*f^6 - 288*a^3*b*c^9*d^4*e^3*f^2 - 112*a^3
*b^2*c^8*d^2*e^6*f + 5664*a^3*b^3*c^7*d^4*e*f^4 - 796*a^3*b^4*c^6*d*e^6*f^2
 + 1524*a^3*b^5*c^5*d*e^5*f^3 + 5120*a^3*b^5*c^5*d^3*e*f^5 - 1176*a^3*b^6*c
^4*d*e^4*f^4 + 240*a^3*b^7*c^3*d*e^3*f^5 - 116*a^3*b^7*c^3*d^2*e*f^6 + 68*a
^3*b^8*c^2*d*e^2*f^6 + 192*a^4*b*c^8*d^2*e^5*f^2 + 2240*a^4*b*c^8*d^3*e^3*f

^3 + 1344*a^4*b^2*c^7*d*e^6*f^2 - 3168*a^4*b^3*c^6*d*e^5*f^3 - 7232*a^4*b^3
*c^6*d^3*e*f^5 + 2464*a^4*b^4*c^5*d*e^4*f^4 + 78*a^4*b^5*c^4*d*e^3*f^5 - 11
60*a^4*b^5*c^4*d^2*e*f^6 - 574*a^4*b^6*c^3*d*e^2*f^6 - 4928*a^5*b*c^7*d^2*e
^3*f^4 - 1152*a^5*b^2*c^6*d*e^4*f^4 - 2416*a^5*b^3*c^5*d*e^3*f^5 + 4096*a^5
*b^3*c^5*d^2*e*f^6 + 1748*a^5*b^4*c^4*d*e^2*f^6 - 1280*a^6*b^2*c^5*d*e^2*f^
6 + 4*a*b^7*c^5*d*e^7*f + 4*a*b^11*c*d*e^3*f^5 - 10*a*b^11*c*d^2*e*f^6 - 4*
a^3*b^9*c*d*e*f^7 - 160*a^4*b*c^8*d*e^7*f + 1792*a^7*b*c^5*d*e*f^7 + 384*a^
2*b^2*c^9*d^5*e^2*f^2 + 16*a^2*b^3*c^8*d^4*e^3*f^2 - 624*a^2*b^4*c^7*d^3*e^
4*f^2 + 1962*a^2*b^4*c^7*d^4*e^2*f^3 + 348*a^2*b^5*c^6*d^2*e^5*f^2 + 204*a^
2*b^5*c^6*d^3*e^3*f^3 - 1214*a^2*b^6*c^5*d^2*e^4*f^3 + 1636*a^2*b^6*c^5*d^3
*e^2*f^4 + 1520*a^2*b^7*c^4*d^2*e^3*f^4 - 750*a^2*b^8*c^3*d^2*e^2*f^5 + 121
6*a^3*b^2*c^8*d^3*e^4*f^2 - 2224*a^3*b^2*c^8*d^4*e^2*f^3 - 512*a^3*b^3*c^7*
d^2*e^5*f^2 - 1632*a^3*b^3*c^7*d^3*e^3*f^3 + 3492*a^3*b^4*c^6*d^2*e^4*f^3 -
 2824*a^3*b^4*c^6*d^3*e^2*f^4 - 5492*a^3*b^5*c^5*d^2*e^3*f^4 + 2868*a^3*b^6
*c^4*d^2*e^2*f^5 - 4480*a^4*b^2*c^7*d^2*e^4*f^3 + 2432*a^4*b^2*c^7*d^3*e^2*
f^4 + 8864*a^4*b^3*c^6*d^2*e^3*f^4 - 4206*a^4*b^4*c^5*d^2*e^2*f^5 + 432*a^5
*b^2*c^6*d^2*e^2*f^5))/(16*a^2*c^6*d^4 + a^4*b^4*f^4 + 16*a^4*c^4*e^4 + b^4
*c^4*d^4 + 16*a^6*c^2*f^4 + b^8*d^2*f^2 - 8*a*b^2*c^5*d^4 - 8*a^5*b^2*c*f^4
 + 2*a^2*b^6*d*f^3 - 2*a^3*b^5*e*f^3 - 64*a^3*c^5*d^3*f - 64*a^5*c^3*d*f^3
- 2*b^5*c^3*d^3*e + 2*b^6*c^2*d^3*f + a^2*b^4*c^2*e^4 - 8*a^3*b^2*c^3*e^4 +
 32*a^3*c^5*d^2*e^2 + a^2*b^6*e^2*f^2 + 96*a^4*c^4*d^2*f^2 + b^6*c^2*d^2*e^
2 + 32*a^5*c^3*e^2*f^2 - 2*a*b^7*d*e*f^2 - 2*b^7*c*d^2*e*f + 54*a^2*b^4*c^2
*d^2*f^2 - 112*a^3*b^2*c^3*d^2*f^2 + 16*a*b^3*c^4*d^3*e - 2*a*b^5*c^2*d*e^3
 - 32*a^2*b*c^5*d^3*e - 32*a^3*b*c^4*d*e^3 - 20*a*b^4*c^3*d^3*f - 12*a*b^6*
c*d^2*f^2 - 20*a^3*b^4*c*d*f^3 - 2*a^2*b^5*c*e^3*f - 32*a^4*b*c^3*e^3*f + 1
6*a^4*b^3*c*e*f^3 - 32*a^5*b*c^2*e*f^3 - 64*a^4*c^4*d*e^2*f - 6*a*b^4*c^3*d
^2*e^2 + 16*a^2*b^3*c^3*d*e^3 + 64*a^2*b^2*c^4*d^3*f + 64*a^4*b^2*c^2*d*f^3
 + 16*a^3*b^3*c^2*e^3*f - 6*a^3*b^4*c*e^2*f^2 - 48*a^2*b^3*c^3*d^2*e*f - 36
*a^2*b^4*c^2*d*e^2*f + 96*a^3*b^2*c^3*d*e^2*f - 48*a^3*b^3*c^2*d*e*f^2 + 4*
a*b^6*c*d*e^2*f + 18*a*b^5*c^2*d^2*e*f + 18*a^2*b^5*c*d*e*f^2 + 32*a^3*b*c^
4*d^2*e*f + 32*a^4*b*c^3*d*e*f^2)) - (64*A*a^7*c^4*f^8 - A*a^4*b^6*c*f^8 +
32*A*a*c^10*d^6*f^2 - 352*A*a^6*c^5*d*f^7 - A*b^10*c*d^2*f^6 + 8*B*a^4*c^7*
e^7*f - 64*B*a^7*c^4*e*f^7 + 12*A*a^5*b^4*c^2*f^8 - 48*A*a^6*b^2*c^3*f^8 -
224*A*a^2*c^9*d^5*f^3 + 640*A*a^3*c^8*d^4*f^4 - 960*A*a^4*c^7*d^3*f^5 + 800
*A*a^5*c^6*d^2*f^6 - 40*A*a^4*c^7*e^6*f^2 - 80*A*a^5*c^6*e^4*f^4 + 24*A*a^6
*c^5*e^2*f^6 - 8*A*b^2*c^9*d^6*f^2 - 16*A*b^4*c^7*d^5*f^3 - A*b^6*c^5*d^4*f
^4 + 6*A*b^8*c^3*d^3*f^5 + 16*B*a^5*c^6*e^5*f^3 - 56*B*a^6*c^5*e^3*f^5 + 4*
B*b^3*c^8*d^6*f^2 + 12*B*b^5*c^6*d^5*f^3 + 4*B*b^7*c^4*d^4*f^4 - 4*B*b^9*c^
2*d^3*f^5 + 120*A*a*b^2*c^8*d^5*f^3 + 60*A*a*b^4*c^6*d^4*f^4 - 36*A*a*b^6*c
^4*d^3*f^5 + 8*A*a*b^8*c^2*d^2*f^6 + 20*A*a^3*b^6*c^2*d*f^7 - 80*A*a^4*b^4*
c^3*d*f^7 + 216*A*a^5*b^2*c^4*d*f^7 + 4*A*a*b^6*c^4*e^6*f^2 - 6*A*a*b^7*c^3
*e^5*f^3 + 4*A*a*b^8*c^2*e^4*f^4 + 9*A*a^2*b^3*c^6*e^7*f + 2*A*a^2*b^8*c*e^
2*f^6 + 88*A*a^4*b*c^6*e^5*f^3 + 172*A*a^5*b*c^5*e^3*f^5 - 92*B*a*b^3*c^7*d
^5*f^3 - 72*B*a*b^5*c^5*d^4*f^4 + 20*B*a*b^7*c^3*d^3*f^5 + 176*B*a^2*b*c^8*
d^5*f^3 - 544*B*a^3*b*c^7*d^4*f^4 + 736*B*a^4*b*c^6*d^3*f^5 + 4*B*a^4*b^5*c

^2*d*f^7 - 464*B*a^5*b*c^5*d^2*f^6 - 44*B*a^5*b^3*c^3*d*f^7 - 2*B*a^3*b^2*c
^6*e^7*f - B*a^3*b^7*c*e^2*f^6 - 28*B*a^4*b*c^6*e^6*f^2 - 56*B*a^5*b*c^5*e^
4*f^4 - 12*B*a^5*b^4*c^2*e*f^7 + 36*B*a^6*b*c^4*e^2*f^6 + 48*B*a^6*b^2*c^3*
e*f^7 - 16*A*a^2*c^9*d^3*e^4*f + 48*A*a^4*c^7*d*e^4*f^3 - 168*A*a^5*c^6*d*e
^2*f^5 + 2*A*b^2*c^9*d^5*e^2*f - 3*A*b^3*c^8*d^4*e^3*f + 12*A*b^3*c^8*d^5*e
*f^2 - 4*A*b^7*c^4*d*e^5*f^2 - 12*A*b^7*c^4*d^3*e*f^4 + 6*A*b^8*c^3*d*e^4*f
^3 - 4*A*b^9*c^2*d*e^3*f^4 + 8*A*b^9*c^2*d^2*e*f^5 + 8*B*a^2*c^9*d^4*e^3*f
- 32*B*a^2*c^9*d^5*e*f^2 + 16*B*a^3*c^8*d^2*e^5*f + 64*B*a^3*c^8*d^4*e*f^3
+ 64*B*a^4*c^7*d^3*e*f^4 + 96*B*a^5*c^6*d*e^3*f^4 - 256*B*a^5*c^6*d^2*e*f^5
- B*b^3*c^8*d^5*e^2*f + 2*B*b^4*c^7*d^4*e^3*f - 8*B*b^4*c^7*d^5*e*f^2 - B*
b^6*c^5*d^2*e^5*f - 3*B*b^6*c^5*d^4*e*f^3 + 12*B*b^8*c^3*d^3*e*f^4 - 384*A*
a^2*b^2*c^7*d^4*f^4 - 32*A*a^2*b^4*c^5*d^3*f^5 - 14*A*a^2*b^6*c^3*d^2*f^6 +
560*A*a^3*b^2*c^6*d^3*f^5 + 56*A*a^3*b^4*c^4*d^2*f^6 - 456*A*a^4*b^2*c^5*d
^2*f^6 - 38*A*a^2*b^4*c^5*e^6*f^2 + 58*A*a^2*b^5*c^4*e^5*f^3 - 36*A*a^2*b^6
*c^3*e^4*f^4 + 5*A*a^2*b^7*c^2*e^3*f^5 + 98*A*a^3*b^2*c^6*e^6*f^2 - 158*A*a
^3*b^3*c^5*e^5*f^3 + 80*A*a^3*b^4*c^4*e^4*f^4 + 22*A*a^3*b^5*c^3*e^3*f^5 -
22*A*a^3*b^6*c^2*e^2*f^6 + 20*A*a^4*b^2*c^5*e^4*f^4 - 147*A*a^4*b^3*c^4*e^3
*f^5 + 80*A*a^4*b^4*c^3*e^2*f^6 - 102*A*a^5*b^2*c^4*e^2*f^6 + 360*B*a^2*b^3
*c^6*d^4*f^4 + 64*B*a^2*b^5*c^4*d^3*f^5 - 504*B*a^3*b^3*c^5*d^3*f^5 - 40*B*
a^3*b^5*c^3*d^2*f^6 + 276*B*a^4*b^3*c^4*d^2*f^6 + 7*B*a^3*b^3*c^5*e^6*f^2 -
8*B*a^3*b^4*c^4*e^5*f^3 + 2*B*a^3*b^5*c^3*e^4*f^4 + 2*B*a^3*b^6*c^2*e^3*f^
5 + 28*B*a^4*b^2*c^5*e^5*f^3 + 6*B*a^4*b^3*c^4*e^4*f^4 - 26*B*a^4*b^4*c^3*e
^3*f^5 + 11*B*a^4*b^5*c^2*e^2*f^6 + 86*B*a^5*b^2*c^4*e^3*f^5 - 37*B*a^5*b^3
*c^3*e^2*f^6 + 120*A*a^2*c^9*d^4*e^2*f^2 + 48*A*a^3*c^8*d^2*e^4*f^2 - 336*A
*a^3*c^8*d^3*e^2*f^3 + 368*A*a^4*c^7*d^2*e^2*f^4 + 4*A*b^4*c^7*d^4*e^2*f^2
- 5*A*b^6*c^5*d^2*e^4*f^2 + 6*A*b^6*c^5*d^3*e^2*f^3 + 16*A*b^7*c^4*d^2*e^3*
f^3 - 18*A*b^8*c^3*d^2*e^2*f^4 - 32*B*a^3*c^8*d^3*e^3*f^2 - 16*B*a^4*c^7*d^
2*e^3*f^3 - 3*B*b^5*c^6*d^4*e^2*f^2 + 4*B*b^6*c^5*d^3*e^3*f^2 + 4*B*b^7*c^4
*d^2*e^4*f^2 - 12*B*b^7*c^4*d^3*e^2*f^3 - 6*B*b^8*c^3*d^2*e^3*f^3 + 4*B*b^9
*c^2*d^2*e^2*f^4 - 2*A*a^2*b^8*c*d*f^7 - A*a*b^5*c^5*e^7*f - A*a*b^9*c*e^3*
f^5 - 20*A*a^3*b*c^7*e^7*f - 16*B*a*b*c^9*d^6*f^2 + 112*B*a^6*b*c^4*d*f^7 +
B*a^4*b^6*c*e*f^7 - 8*A*a*c^10*d^5*e^2*f - 8*A*a^3*c^8*d*e^6*f + A*b^6*c^5
*d*e^6*f + A*b^10*c*d*e^2*f^5 + 224*B*a^6*c^5*d*e*f^6 - B*b^10*c*d^2*e*f^5
+ 12*A*a*b*c^9*d^4*e^3*f - 48*A*a*b*c^9*d^5*e*f^2 - 10*A*a*b^4*c^6*d*e^6*f
+ 80*A*a^5*b*c^5*d*e*f^6 + 4*B*a*b*c^9*d^5*e^2*f + B*a*b^5*c^5*d*e^6*f + B*
a*b^9*c*d*e^2*f^5 + 4*B*a^3*b*c^7*d*e^6*f - 92*A*a^2*b^2*c^7*d^2*e^4*f^2 +
132*A*a^2*b^2*c^7*d^3*e^2*f^3 + 446*A*a^2*b^3*c^6*d^2*e^3*f^3 - 604*A*a^2*b
^4*c^5*d^2*e^2*f^4 + 340*A*a^3*b^2*c^6*d^2*e^2*f^4 + 72*B*a^2*b^2*c^7*d^3*e
^3*f^2 + 134*B*a^2*b^3*c^6*d^2*e^4*f^2 - 306*B*a^2*b^3*c^6*d^3*e^2*f^3 - 26
4*B*a^2*b^4*c^5*d^2*e^3*f^3 + 188*B*a^2*b^5*c^4*d^2*e^2*f^4 + 292*B*a^3*b^2
*c^6*d^2*e^3*f^3 + 6*B*a^3*b^3*c^5*d^2*e^2*f^4 + 4*A*a*b^2*c^8*d^3*e^4*f +
2*A*a*b^3*c^7*d^2*e^5*f - 16*A*a*b^3*c^7*d^4*e*f^3 + 48*A*a*b^5*c^5*d*e^5*f
^2 + 72*A*a*b^5*c^5*d^3*e*f^4 - 84*A*a*b^6*c^4*d*e^4*f^3 + 64*A*a*b^7*c^3*d
*e^3*f^4 - 88*A*a*b^7*c^3*d^2*e*f^5 - 18*A*a*b^8*c^2*d*e^2*f^5 - 8*A*a^2*b*
c^8*d^2*e^5*f + 64*A*a^2*b*c^8*d^4*e*f^3 + 26*A*a^2*b^2*c^7*d*e^6*f + 16*A*

a^2*b^7*c^2*d*e*f^6 + 192*A*a^3*b*c^7*d*e^5*f^2 + 96*A*a^3*b*c^7*d^3*e*f^4
- 136*A*a^3*b^5*c^3*d*e*f^6 + 80*A*a^4*b*c^6*d*e^3*f^4 - 192*A*a^4*b*c^6*d^
2*e*f^5 + 268*A*a^4*b^3*c^4*d*e*f^6 - 10*B*a*b^2*c^8*d^4*e^3*f + 40*B*a*b^2
*c^8*d^5*e*f^2 - 2*B*a*b^3*c^7*d^3*e^4*f + 8*B*a*b^4*c^6*d^2*e^5*f + 36*B*a
*b^4*c^6*d^4*e*f^3 - 4*B*a*b^6*c^4*d*e^5*f^2 - 88*B*a*b^6*c^4*d^3*e*f^4 + 6
*B*a*b^7*c^3*d*e^4*f^3 - 4*B*a*b^8*c^2*d*e^3*f^4 + 16*B*a*b^8*c^2*d^2*e*f^5
+ 8*B*a^2*b*c^8*d^3*e^4*f - 5*B*a^2*b^3*c^6*d*e^6*f - 8*B*a^3*b^6*c^2*d*e*
f^6 + 40*B*a^4*b*c^6*d*e^4*f^3 + 104*B*a^4*b^4*c^3*d*e*f^6 + 148*B*a^5*b*c^
5*d*e^2*f^5 - 344*B*a^5*b^2*c^4*d*e*f^6 - 46*A*a*b^2*c^8*d^4*e^2*f^2 - 4*A*
a*b^3*c^7*d^3*e^3*f^2 + 40*A*a*b^4*c^6*d^2*e^4*f^2 - 36*A*a*b^4*c^6*d^3*e^2
*f^3 - 158*A*a*b^5*c^5*d^2*e^3*f^3 + 196*A*a*b^6*c^4*d^2*e^2*f^4 + 16*A*a^2
*b*c^8*d^3*e^3*f^2 - 176*A*a^2*b^3*c^6*d*e^5*f^2 - 120*A*a^2*b^3*c^6*d^3*e*
f^4 + 380*A*a^2*b^4*c^5*d*e^4*f^3 - 324*A*a^2*b^5*c^4*d*e^3*f^4 + 272*A*a^2
*b^5*c^4*d^2*e*f^5 + 80*A*a^2*b^6*c^3*d*e^2*f^5 - 280*A*a^3*b*c^7*d^2*e^3*f
^3 - 572*A*a^3*b^2*c^6*d*e^4*f^3 + 508*A*a^3*b^3*c^5*d*e^3*f^4 - 144*A*a^3*
b^3*c^5*d^2*e*f^5 - 4*A*a^3*b^4*c^4*d*e^2*f^5 - 326*A*a^4*b^2*c^5*d*e^2*f^5
+ 31*B*a*b^3*c^7*d^4*e^2*f^2 - 32*B*a*b^4*c^6*d^3*e^3*f^2 - 40*B*a*b^5*c^5
*d^2*e^4*f^2 + 102*B*a*b^5*c^5*d^3*e^2*f^3 + 72*B*a*b^6*c^4*d^2*e^3*f^3 - 5
6*B*a*b^7*c^3*d^2*e^2*f^4 - 76*B*a^2*b*c^8*d^4*e^2*f^2 - 20*B*a^2*b^2*c^7*d
^2*e^5*f - 112*B*a^2*b^2*c^7*d^4*e*f^3 + 24*B*a^2*b^4*c^5*d*e^5*f^2 + 192*B
*a^2*b^4*c^5*d^3*e*f^4 - 42*B*a^2*b^5*c^4*d*e^4*f^3 + 32*B*a^2*b^6*c^3*d*e^
3*f^4 - 38*B*a^2*b^6*c^3*d^2*e*f^5 - 9*B*a^2*b^7*c^2*d*e^2*f^5 - 152*B*a^3*
b*c^7*d^2*e^4*f^2 + 360*B*a^3*b*c^7*d^3*e^2*f^3 - 32*B*a^3*b^2*c^6*d*e^5*f^
2 - 144*B*a^3*b^2*c^6*d^3*e*f^4 + 62*B*a^3*b^3*c^5*d*e^4*f^3 - 40*B*a^3*b^4
*c^4*d*e^3*f^4 - 152*B*a^3*b^4*c^4*d^2*e*f^5 + 14*B*a^3*b^5*c^3*d*e^2*f^5 -
472*B*a^4*b*c^6*d^2*e^2*f^4 - 120*B*a^4*b^2*c^5*d*e^3*f^4 + 512*B*a^4*b^2*
c^5*d^2*e*f^5 - 13*B*a^4*b^3*c^4*d*e^2*f^5)/(16*a^2*c^6*d^4 + a^4*b^4*f^4 +
16*a^4*c^4*e^4 + b^4*c^4*d^4 + 16*a^6*c^2*f^4 + b^8*d^2*f^2 - 8*a*b^2*c^5*
d^4 - 8*a^5*b^2*c*f^4 + 2*a^2*b^6*d*f^3 - 2*a^3*b^5*e*f^3 - 64*a^3*c^5*d^3*
f - 64*a^5*c^3*d*f^3 - 2*b^5*c^3*d^3*e + 2*b^6*c^2*d^3*f + a^2*b^4*c^2*e^4
- 8*a^3*b^2*c^3*e^4 + 32*a^3*c^5*d^2*e^2 + a^2*b^6*e^2*f^2 + 96*a^4*c^4*d^2
*f^2 + b^6*c^2*d^2*e^2 + 32*a^5*c^3*e^2*f^2 - 2*a*b^7*d*e*f^2 - 2*b^7*c*d^2
*e*f + 54*a^2*b^4*c^2*d^2*f^2 - 112*a^3*b^2*c^3*d^2*f^2 + 16*a*b^3*c^4*d^3*
e - 2*a*b^5*c^2*d*e^3 - 32*a^2*b*c^5*d^3*e - 32*a^3*b*c^4*d*e^3 - 20*a*b^4*
c^3*d^3*f - 12*a*b^6*c*d^2*f^2 - 20*a^3*b^4*c*d*f^3 - 2*a^2*b^5*c*e^3*f - 3
2*a^4*b*c^3*e^3*f + 16*a^4*b^3*c*e*f^3 - 32*a^5*b*c^2*e*f^3 - 64*a^4*c^4*d*
e^2*f - 6*a*b^4*c^3*d^2*e^2 + 16*a^2*b^3*c^3*d*e^3 + 64*a^2*b^2*c^4*d^3*f +
64*a^4*b^2*c^2*d*f^3 + 16*a^3*b^3*c^2*e^3*f - 6*a^3*b^4*c*e^2*f^2 - 48*a^2
*b^3*c^3*d^2*e*f - 36*a^2*b^4*c^2*d*e^2*f + 96*a^3*b^2*c^3*d*e^2*f - 48*a^3
*b^3*c^2*d*e*f^2 + 4*a*b^6*c*d*e^2*f + 18*a*b^5*c^2*d^2*e*f + 18*a^2*b^5*c*
d*e*f^2 + 32*a^3*b*c^4*d^2*e*f + 32*a^4*b*c^3*d*e*f^2) + (x*(64*B*a^7*c^4*f
^8 + 4*A*a^3*b^7*c*f^8 - 256*A*a^6*b*c^4*f^8 - B*a^4*b^6*c*f^8 + 48*A*a^3*c
^8*e^7*f + 256*A*a^6*c^5*e*f^7 - 320*B*a^6*c^5*d*f^7 + 3*B*b^10*c*d^2*f^6 -
48*A*a^4*b^5*c^2*f^8 + 192*A*a^5*b^3*c^3*f^8 + 12*B*a^5*b^4*c^2*f^8 - 48*B
*a^6*b^2*c^3*f^8 + 256*A*a^4*c^7*e^5*f^3 + 464*A*a^5*c^6*e^3*f^5 - 16*A*b^3

*c^8*d^5*f^3 - 48*A*b^5*c^6*d^4*f^4 - 36*A*b^7*c^4*d^3*f^5 - 4*A*b^9*c^2*d^
2*f^6 - 64*B*a^2*c^9*d^5*f^3 + 320*B*a^3*c^8*d^4*f^4 - 640*B*a^4*c^7*d^3*f^
5 + 640*B*a^5*c^6*d^2*f^6 - 16*B*a^4*c^7*e^6*f^2 - 64*B*a^5*c^6*e^4*f^4 + 1
6*B*a^6*c^5*e^2*f^6 + 4*B*b^4*c^7*d^5*f^3 + 23*B*b^6*c^5*d^4*f^4 + 22*B*b^8
*c^3*d^3*f^5 + 320*A*a*b^3*c^7*d^4*f^4 + 352*A*a*b^5*c^5*d^3*f^5 + 76*A*a*b
^7*c^3*d^2*f^6 - 512*A*a^2*b*c^8*d^4*f^4 - 60*A*a^2*b^7*c^2*d*f^7 + 1408*A*
a^3*b*c^7*d^3*f^5 + 352*A*a^3*b^5*c^3*d*f^7 - 1792*A*a^4*b*c^6*d^2*f^6 - 97
6*A*a^4*b^3*c^4*d*f^7 - 6*A*a*b^5*c^5*e^6*f^2 + 4*A*a*b^6*c^4*e^5*f^3 + 4*A
*a*b^7*c^3*e^4*f^4 - 6*A*a*b^8*c^2*e^3*f^5 - 20*A*a^2*b^2*c^7*e^7*f - 144*A
*a^3*b*c^7*e^6*f^2 + 68*A*a^3*b^6*c^2*e*f^7 - 640*A*a^4*b*c^6*e^4*f^4 - 240
*A*a^4*b^4*c^3*e*f^7 - 848*A*a^5*b*c^5*e^2*f^6 + 192*A*a^5*b^2*c^4*e*f^7 -
132*B*a*b^4*c^6*d^4*f^4 - 196*B*a*b^6*c^4*d^3*f^5 - 40*B*a*b^8*c^2*d^2*f^6
- 20*B*a^3*b^6*c^2*d*f^7 + 52*B*a^4*b^4*c^3*d*f^7 + 64*B*a^5*b^2*c^4*d*f^7
+ 2*B*a^2*b^3*c^6*e^7*f + B*a^2*b^8*c*e^2*f^6 + 16*B*a^4*b*c^6*e^5*f^3 + 12
0*B*a^5*b*c^5*e^3*f^5 + 64*A*a^2*c^9*d^2*e^5*f + 512*A*a^2*c^9*d^4*e*f^3 -
384*A*a^3*c^8*d*e^5*f^2 - 1408*A*a^3*c^8*d^3*e*f^4 - 1280*A*a^4*c^7*d*e^3*f
^4 + 1792*A*a^4*c^7*d^2*e*f^5 - 4*A*b^2*c^9*d^4*e^3*f + 16*A*b^2*c^9*d^5*e*
f^2 + 8*A*b^3*c^8*d^3*e^4*f - 2*A*b^4*c^7*d^2*e^5*f + 80*A*b^4*c^7*d^4*e*f^
3 + 6*A*b^6*c^5*d*e^5*f^2 + 76*A*b^6*c^5*d^3*e*f^4 - 4*A*b^7*c^4*d*e^4*f^3
- 4*A*b^8*c^3*d*e^3*f^4 - 2*A*b^8*c^3*d^2*e*f^5 + 6*A*b^9*c^2*d*e^2*f^5 - 3
2*B*a^2*c^9*d^3*e^4*f - 96*B*a^4*c^7*d*e^4*f^3 - 192*B*a^5*c^6*d*e^2*f^5 +
2*B*b^3*c^8*d^4*e^3*f - 8*B*b^3*c^8*d^5*e*f^2 - 6*B*b^4*c^7*d^3*e^4*f + 4*B
*b^5*c^6*d^2*e^5*f - 48*B*b^5*c^6*d^4*e*f^3 - 60*B*b^7*c^4*d^3*e*f^4 - 8*B*
b^9*c^2*d^2*e*f^5 + 4*A*a*b^9*c*d*f^7 - 2*A*b^10*c*d*e*f^6 - 1184*A*a^2*b^3
*c^6*d^3*f^5 - 544*A*a^2*b^5*c^4*d^2*f^6 + 1664*A*a^3*b^3*c^5*d^2*f^6 + 60*
A*a^2*b^3*c^6*e^6*f^2 - 30*A*a^2*b^4*c^5*e^5*f^3 - 64*A*a^2*b^5*c^4*e^4*f^4
 + 72*A*a^2*b^6*c^3*e^3*f^5 - 12*A*a^2*b^7*c^2*e^2*f^6 - 8*A*a^3*b^2*c^6*e^
5*f^3 + 352*A*a^3*b^3*c^5*e^4*f^4 - 268*A*a^3*b^4*c^4*e^3*f^5 - 52*A*a^3*b^
5*c^3*e^2*f^6 + 188*A*a^4*b^2*c^5*e^3*f^5 + 484*A*a^4*b^3*c^4*e^2*f^6 + 80*
B*a^2*b^2*c^7*d^4*f^4 + 520*B*a^2*b^4*c^5*d^3*f^5 + 210*B*a^2*b^6*c^3*d^2*f
^6 - 192*B*a^3*b^2*c^6*d^3*f^5 - 456*B*a^3*b^4*c^4*d^2*f^6 + 96*B*a^4*b^2*c
^5*d^2*f^6 - 7*B*a^2*b^4*c^5*e^6*f^2 + 8*B*a^2*b^5*c^4*e^5*f^3 - 2*B*a^2*b^
6*c^3*e^4*f^4 - 2*B*a^2*b^7*c^2*e^3*f^5 + 32*B*a^3*b^2*c^6*e^6*f^2 - 36*B*a
^3*b^3*c^5*e^5*f^3 - 4*B*a^3*b^4*c^4*e^4*f^4 + 28*B*a^3*b^5*c^3*e^3*f^5 - 1
2*B*a^3*b^6*c^2*e^2*f^6 + 64*B*a^4*b^2*c^5*e^4*f^4 - 110*B*a^4*b^3*c^4*e^3*
f^5 + 47*B*a^4*b^4*c^3*e^2*f^6 - 64*B*a^5*b^2*c^4*e^2*f^6 - 384*A*a^2*c^9*d
^3*e^3*f^2 + 1184*A*a^3*c^8*d^2*e^3*f^3 - 28*A*b^3*c^8*d^4*e^2*f^2 - 12*A*b
^4*c^7*d^3*e^3*f^2 + 20*A*b^5*c^6*d^2*e^4*f^2 - 36*A*b^5*c^6*d^3*e^2*f^3 -
44*A*b^6*c^5*d^2*e^3*f^3 + 32*A*b^7*c^4*d^2*e^2*f^4 + 144*B*a^2*c^9*d^4*e^2
*f^2 + 192*B*a^3*c^8*d^2*e^4*f^2 - 448*B*a^3*c^8*d^3*e^2*f^3 + 480*B*a^4*c^
7*d^2*e^2*f^4 + 23*B*b^4*c^7*d^4*e^2*f^2 - 4*B*b^5*c^6*d^3*e^3*f^2 - 13*B*b
^6*c^5*d^2*e^4*f^2 + 48*B*b^6*c^5*d^3*e^2*f^3 + 12*B*b^7*c^4*d^2*e^3*f^3 +
2*B*b^8*c^3*d^2*e^2*f^4 + 64*A*a*b*c^9*d^5*f^3 + 1088*A*a^5*b*c^5*d*f^7 + 2
*A*a*b^4*c^6*e^7*f + 2*A*a*b^9*c*e^2*f^6 - 6*A*a^2*b^8*c*e*f^7 + 2*B*a^2*b^
8*c*d*f^7 - 8*B*a^3*b*c^7*e^7*f + 16*A*a*c^10*d^4*e^3*f - 64*A*a*c^10*d^5*e

*f^2 - 1088*A*a^5*c^6*d*e*f^6 - 2*A*b^5*c^6*d*e^6*f - 32*B*a^3*c^8*d*e^6*f
- 32*A*a*b*c^9*d^3*e^4*f + 24*A*a*b^3*c^7*d*e^6*f + 24*A*a*b^8*c^2*d*e*f^6
- 64*A*a^2*b*c^8*d*e^6*f - 8*B*a*b*c^9*d^4*e^3*f + 32*B*a*b*c^9*d^5*e*f^2 -
6*B*a*b^4*c^6*d*e^6*f + 288*B*a^5*b*c^5*d*e*f^6 - 1496*A*a^2*b^2*c^7*d^2*e
^3*f^3 + 1496*A*a^2*b^3*c^6*d^2*e^2*f^4 - 272*B*a^2*b^2*c^7*d^2*e^4*f^2 + 6
40*B*a^2*b^2*c^7*d^3*e^2*f^3 + 636*B*a^2*b^3*c^6*d^2*e^3*f^3 - 286*B*a^2*b^
4*c^5*d^2*e^2*f^4 + 192*B*a^3*b^2*c^6*d^2*e^2*f^4 + 112*A*a*b*c^9*d^4*e^2*f
^2 - 8*A*a*b^2*c^8*d^2*e^5*f - 448*A*a*b^2*c^8*d^4*e*f^3 - 88*A*a*b^4*c^6*d
*e^5*f^2 - 576*A*a*b^4*c^6*d^3*e*f^4 + 84*A*a*b^5*c^5*d*e^4*f^3 + 32*A*a*b^
6*c^4*d*e^3*f^4 + 12*A*a*b^6*c^4*d^2*e*f^5 - 80*A*a*b^7*c^3*d*e^2*f^5 - 108
*A*a^2*b^6*c^3*d*e*f^6 + 736*A*a^3*b*c^7*d*e^4*f^3 + 192*A*a^3*b^4*c^4*d*e*
f^6 + 2048*A*a^4*b*c^6*d*e^2*f^5 + 208*A*a^4*b^2*c^5*d*e*f^6 + 32*B*a*b^2*c
^8*d^3*e^4*f - 20*B*a*b^3*c^7*d^2*e^5*f + 288*B*a*b^3*c^7*d^4*e*f^3 + 20*B*
a*b^5*c^5*d*e^5*f^2 + 480*B*a*b^5*c^5*d^3*e*f^4 - 20*B*a*b^6*c^4*d*e^4*f^3
+ 72*B*a*b^7*c^3*d^2*e*f^5 + 10*B*a*b^8*c^2*d*e^2*f^5 + 16*B*a^2*b*c^8*d^2*
e^5*f - 384*B*a^2*b*c^8*d^4*e*f^3 + 32*B*a^2*b^2*c^7*d*e^6*f + 44*B*a^2*b^7
*c^2*d*e*f^6 + 192*B*a^3*b*c^7*d*e^5*f^2 + 960*B*a^3*b*c^7*d^3*e*f^4 - 160*
B*a^3*b^5*c^3*d*e*f^6 + 544*B*a^4*b*c^6*d*e^3*f^4 - 896*B*a^4*b*c^6*d^2*e*f
^5 + 120*B*a^4*b^3*c^4*d*e*f^6 - 4*B*a*b^9*c*d*e*f^6 + 144*A*a*b^2*c^8*d^3*
e^3*f^2 - 144*A*a*b^3*c^7*d^2*e^4*f^2 + 112*A*a*b^3*c^7*d^3*e^2*f^3 + 476*A
*a*b^4*c^6*d^2*e^3*f^3 - 412*A*a*b^5*c^5*d^2*e^2*f^4 + 256*A*a^2*b*c^8*d^2*
e^4*f^2 + 128*A*a^2*b*c^8*d^3*e^2*f^3 + 352*A*a^2*b^2*c^7*d*e^5*f^2 + 1440*
A*a^2*b^2*c^7*d^3*e*f^4 - 456*A*a^2*b^3*c^6*d*e^4*f^3 - 116*A*a^2*b^4*c^5*d
*e^3*f^4 + 224*A*a^2*b^4*c^5*d^2*e*f^5 + 452*A*a^2*b^5*c^4*d*e^2*f^5 - 1440
*A*a^3*b*c^7*d^2*e^2*f^4 + 528*A*a^3*b^2*c^6*d*e^3*f^4 - 1408*A*a^3*b^2*c^6
*d^2*e*f^5 - 1424*A*a^3*b^3*c^5*d*e^2*f^5 - 128*B*a*b^2*c^8*d^4*e^2*f^2 + 8
*B*a*b^3*c^7*d^3*e^3*f^2 + 108*B*a*b^4*c^6*d^2*e^4*f^2 - 324*B*a*b^4*c^6*d^
3*e^2*f^3 - 164*B*a*b^5*c^5*d^2*e^3*f^3 + 44*B*a*b^6*c^4*d^2*e^2*f^4 + 32*B
*a^2*b*c^8*d^3*e^3*f^2 - 128*B*a^2*b^3*c^6*d*e^5*f^2 - 1200*B*a^2*b^3*c^6*d
^3*e*f^4 + 142*B*a^2*b^4*c^5*d*e^4*f^3 + 20*B*a^2*b^5*c^4*d*e^3*f^4 - 304*B
*a^2*b^5*c^4*d^2*e*f^5 - 112*B*a^2*b^6*c^3*d*e^2*f^5 - 688*B*a^3*b*c^7*d^2*
e^3*f^3 - 224*B*a^3*b^2*c^6*d*e^4*f^3 - 216*B*a^3*b^3*c^5*d*e^3*f^4 + 800*B
*a^3*b^3*c^5*d^2*e*f^5 + 460*B*a^3*b^4*c^4*d*e^2*f^5 - 640*B*a^4*b^2*c^5*d*
e^2*f^5))/(16*a^2*c^6*d^4 + a^4*b^4*f^4 + 16*a^4*c^4*e^4 + b^4*c^4*d^4 + 16
*a^6*c^2*f^4 + b^8*d^2*f^2 - 8*a*b^2*c^5*d^4 - 8*a^5*b^2*c*f^4 + 2*a^2*b^6*
d*f^3 - 2*a^3*b^5*e*f^3 - 64*a^3*c^5*d^3*f - 64*a^5*c^3*d*f^3 - 2*b^5*c^3*d
^3*e + 2*b^6*c^2*d^3*f + a^2*b^4*c^2*e^4 - 8*a^3*b^2*c^3*e^4 + 32*a^3*c^5*d
^2*e^2 + a^2*b^6*e^2*f^2 + 96*a^4*c^4*d^2*f^2 + b^6*c^2*d^2*e^2 + 32*a^5*c^
3*e^2*f^2 - 2*a*b^7*d*e*f^2 - 2*b^7*c*d^2*e*f + 54*a^2*b^4*c^2*d^2*f^2 - 11
2*a^3*b^2*c^3*d^2*f^2 + 16*a*b^3*c^4*d^3*e - 2*a*b^5*c^2*d*e^3 - 32*a^2*b*c
^5*d^3*e - 32*a^3*b*c^4*d*e^3 - 20*a*b^4*c^3*d^3*f - 12*a*b^6*c*d^2*f^2 - 2
0*a^3*b^4*c*d*f^3 - 2*a^2*b^5*c*e^3*f - 32*a^4*b*c^3*e^3*f + 16*a^4*b^3*c*e
*f^3 - 32*a^5*b*c^2*e*f^3 - 64*a^4*c^4*d*e^2*f - 6*a*b^4*c^3*d^2*e^2 + 16*a
^2*b^3*c^3*d*e^3 + 64*a^2*b^2*c^4*d^3*f + 64*a^4*b^2*c^2*d*f^3 + 16*a^3*b^3
*c^2*e^3*f - 6*a^3*b^4*c*e^2*f^2 - 48*a^2*b^3*c^3*d^2*e*f - 36*a^2*b^4*c^2*

d*e^2*f + 96*a^3*b^2*c^3*d*e^2*f - 48*a^3*b^3*c^2*d*e*f^2 + 4*a*b^6*c*d*e^2
*f + 18*a*b^5*c^2*d^2*e*f + 18*a^2*b^5*c*d*e*f^2 + 32*a^3*b*c^4*d^2*e*f + 3
2*a^4*b*c^3*d*e*f^2)) - (56*A^2*a^3*b^3*c^3*f^7 - 13*A^2*a^2*b^5*c^2*f^7 -
24*A^2*a^2*c^7*e^5*f^2 - 16*A^2*b^3*c^6*d^3*f^4 - A^2*b^5*c^4*d^2*f^5 + 2*A
^2*b^4*c^5*e^5*f^2 - A^2*b^5*c^4*e^4*f^3 - A^2*b^6*c^3*e^3*f^4 + 2*A^2*b^7*
c^2*e^2*f^5 - 12*B^2*a^4*c^5*e^3*f^4 - 8*B^2*b^3*c^6*d^4*f^3 - 9*B^2*b^5*c^
4*d^3*f^4 + 64*A*B*a^5*c^4*f^7 + A^2*a*b^7*c*f^7 - A^2*b^8*c*e*f^6 - 80*A^2
*a^4*b*c^4*f^7 - 16*A^2*b*c^8*d^4*f^3 + 28*A^2*a^4*c^5*e*f^6 - 2*A^2*b^7*c^
2*d*f^6 - A^2*b^3*c^6*e^6*f - 16*B^2*a^5*c^4*e*f^6 - 4*A^2*c^9*d^3*e^3*f +
12*A^2*c^9*d^4*e*f^2 + 48*A^2*a*b*c^7*d^3*f^4 + 22*A^2*a*b^5*c^3*d*f^6 + 48
*A^2*a^3*b*c^5*d*f^6 + 12*A^2*a*b^6*c^2*e*f^6 + 16*B^2*a*b*c^7*d^4*f^3 + 64
*B^2*a^4*b*c^4*d*f^6 - 16*A^2*a*c^8*d^3*e*f^3 + 80*A^2*a^3*c^6*d*e*f^5 + 4*
A^2*b*c^8*d^2*e^4*f + A^2*b^2*c^7*d*e^5*f + 2*A^2*b^6*c^3*d*e*f^5 + 4*B^2*a
^2*c^7*d*e^5*f - 64*B^2*a^4*c^5*d*e*f^5 + A*B*b^8*c*d*f^6 + 8*A^2*a*b^3*c^5
*d^2*f^5 - 64*A^2*a^2*b^3*c^4*d*f^6 - 2*A^2*a*b^2*c^6*e^5*f^2 - 10*A^2*a*b^
3*c^5*e^4*f^3 + 20*A^2*a*b^4*c^4*e^3*f^4 - 25*A^2*a*b^5*c^3*e^2*f^5 + 56*A^
2*a^2*b*c^6*e^4*f^3 - 44*A^2*a^2*b^4*c^3*e*f^6 - 76*A^2*a^3*b*c^5*e^2*f^5 +
40*A^2*a^3*b^2*c^4*e*f^6 + 56*B^2*a*b^3*c^5*d^3*f^4 - 2*B^2*a*b^5*c^3*d^2*
f^5 - 96*B^2*a^2*b*c^6*d^3*f^4 + 11*B^2*a^2*b^5*c^2*d*f^6 + 16*B^2*a^3*b*c^
5*d^2*f^5 - 40*B^2*a^3*b^3*c^3*d*f^6 + 16*B^2*a^4*b*c^4*e^2*f^5 + 3*B^2*a^4
*b^2*c^3*e*f^6 + 24*A^2*a*c^8*d^2*e^3*f^2 + 92*A^2*a^2*c^7*d*e^3*f^3 - 104*
A^2*a^2*c^7*d^2*e*f^4 - 4*A^2*b*c^8*d^3*e^2*f^2 + 20*A^2*b^2*c^7*d^3*e*f^3
- A^2*b^4*c^5*d*e^3*f^3 - 8*A^2*b^4*c^5*d^2*e*f^4 + 32*B^2*a^2*c^7*d^3*e*f^
3 - 8*B^2*a^3*c^6*d*e^3*f^3 + 48*B^2*a^3*c^6*d^2*e*f^4 - B^2*b^2*c^7*d^3*e^
3*f + 3*B^2*b^2*c^7*d^4*e*f^2 + B^2*b^3*c^6*d^2*e^4*f + 8*B^2*b^4*c^5*d^3*e
*f^3 - 3*B^2*b^6*c^3*d^2*e*f^4 - A*B*a^2*b^6*c*f^7 - 32*A*B*a*c^8*d^4*f^3 -
32*A*B*a^4*c^5*d*f^6 - 8*A*B*a^2*c^7*e^6*f + 4*A^2*a*b*c^7*e^6*f - B^2*a*b
^7*c*d*f^6 - 8*A^2*a*c^8*d*e^5*f - 65*A^2*a^2*b^2*c^5*e^3*f^4 + 88*A^2*a^2*
b^3*c^4*e^2*f^5 + 8*B^2*a^2*b^3*c^4*d^2*f^5 + 2*B^2*a^3*b^2*c^4*e^3*f^4 - 3
*B^2*a^3*b^3*c^3*e^2*f^5 - 13*A^2*b^2*c^7*d^2*e^3*f^2 + 16*A^2*b^3*c^6*d^2*
e^2*f^3 - 28*B^2*a^2*c^7*d^2*e^3*f^2 + B^2*b^3*c^6*d^3*e^2*f^2 - 5*B^2*b^4*
c^5*d^2*e^3*f^2 + 7*B^2*b^5*c^4*d^2*e^2*f^3 + 12*A*B*a^3*b^4*c^2*f^7 - 48*A
*B*a^4*b^2*c^3*f^7 + 160*A*B*a^2*c^7*d^3*f^4 - 160*A*B*a^3*c^6*d^2*f^5 - 16
*A*B*a^3*c^6*e^4*f^3 + 48*A*B*a^4*c^5*e^2*f^5 + 24*A*B*b^2*c^7*d^4*f^3 + 24
*A*B*b^4*c^5*d^3*f^4 - A*B*b^6*c^3*d^2*f^5 + 20*B^2*a*b^2*c^6*d^2*e^3*f^2 -
49*B^2*a*b^3*c^5*d^2*e^2*f^3 + 96*B^2*a^2*b*c^6*d^2*e^2*f^3 + 19*B^2*a^2*b
^2*c^5*d*e^3*f^3 - 102*B^2*a^2*b^2*c^5*d^2*e*f^4 + 3*B^2*a^2*b^3*c^4*d*e^2*
f^4 - 120*A*B*a*b^2*c^6*d^3*f^4 + 4*A*B*a*b^4*c^4*d^2*f^5 + 24*A*B*a^2*b^4*
c^3*d*f^6 - 8*A*B*a^3*b^2*c^4*d*f^6 - 7*A*B*a*b^3*c^5*e^5*f^2 + 8*A*B*a*b^4
*c^4*e^4*f^3 - 2*A*B*a*b^5*c^3*e^3*f^4 - 2*A*B*a*b^6*c^2*e^2*f^5 + 28*A*B*a
^2*b*c^6*e^5*f^2 - 11*A*B*a^2*b^5*c^2*e*f^6 + 72*A*B*a^3*b*c^5*e^3*f^4 + 34
*A*B*a^3*b^3*c^3*e*f^6 + 16*A*B*a*c^8*d^3*e^2*f^2 + 80*A*B*a^2*c^7*d*e^4*f^
2 + 16*A*B*a^3*c^6*d*e^2*f^4 - 4*A*B*b^2*c^7*d^2*e^4*f - 22*A*B*b^3*c^6*d^3
*e*f^3 + 3*A*B*b^4*c^5*d*e^4*f^2 - 6*A*B*b^5*c^4*d*e^3*f^3 + 15*A*B*b^5*c^4
*d^2*e*f^4 + 4*A*B*b^6*c^3*d*e^2*f^4 + 12*A^2*a*b*c^7*d*e^4*f^2 - 28*A^2*a*

b^4*c^4*d*e*f^5 - 2*B^2*a*b^2*c^6*d*e^5*f + 2*B^2*a*b^6*c^2*d*e*f^5 + A*B*a
*b^7*c*e*f^6 + 24*A*B*a^2*b^2*c^5*d^2*f^5 - 28*A*B*a^2*b^2*c^5*e^4*f^3 - 9*
A*B*a^2*b^3*c^4*e^3*f^4 + 29*A*B*a^2*b^4*c^3*e^2*f^5 - 100*A*B*a^3*b^2*c^4*
e^2*f^5 - 208*A*B*a^2*c^7*d^2*e^2*f^3 + 13*A*B*b^3*c^6*d^2*e^3*f^2 - 23*A*B
*b^4*c^5*d^2*e^2*f^3 - 52*A^2*a*b*c^7*d^2*e^2*f^3 - 34*A^2*a*b^2*c^6*d*e^3*
f^3 + 48*A^2*a*b^2*c^6*d^2*e*f^4 + 40*A^2*a*b^3*c^5*d*e^2*f^4 - 108*A^2*a^2
*b*c^6*d*e^2*f^4 + 36*A^2*a^2*b^2*c^5*d*e*f^5 - 8*B^2*a*b*c^7*d^3*e^2*f^2 -
24*B^2*a*b^2*c^6*d^3*e*f^3 + 7*B^2*a*b^3*c^5*d*e^4*f^2 - 8*B^2*a*b^4*c^4*d
*e^3*f^3 + 32*B^2*a*b^4*c^4*d^2*e*f^4 + 2*B^2*a*b^5*c^3*d*e^2*f^4 - 16*B^2*
a^2*b*c^6*d*e^4*f^2 - 20*B^2*a^2*b^4*c^3*d*e*f^5 + 8*B^2*a^3*b*c^5*d*e^2*f^
4 + 40*B^2*a^3*b^2*c^4*d*e*f^5 - 10*A*B*a*b^6*c^2*d*f^6 + 2*A*B*a*b^2*c^6*e
^6*f - 20*A*B*a^4*b*c^4*e*f^6 + 4*A*B*b*c^8*d^3*e^3*f - 12*A*B*b*c^8*d^4*e*
f^2 - 2*A*B*b^7*c^2*d*e*f^5 + 132*A*B*a*b^2*c^6*d^2*e^2*f^3 + 96*A*B*a^2*b^
2*c^5*d*e^2*f^4 + 24*A*B*a*b*c^7*d^3*e*f^3 + 20*A*B*a*b^5*c^3*d*e*f^5 - 24*
A*B*a^3*b*c^5*d*e*f^5 - 24*A*B*a*b*c^7*d^2*e^3*f^2 - 44*A*B*a*b^2*c^6*d*e^4
*f^2 + 84*A*B*a*b^3*c^5*d*e^3*f^3 - 122*A*B*a*b^3*c^5*d^2*e*f^4 - 54*A*B*a*
b^4*c^4*d*e^2*f^4 - 180*A*B*a^2*b*c^6*d*e^3*f^3 + 288*A*B*a^2*b*c^6*d^2*e*f
^4 - 18*A*B*a^2*b^3*c^4*d*e*f^5 + 4*A*B*a*b*c^7*d*e^5*f)/(16*a^2*c^6*d^4 +
a^4*b^4*f^4 + 16*a^4*c^4*e^4 + b^4*c^4*d^4 + 16*a^6*c^2*f^4 + b^8*d^2*f^2 -
8*a*b^2*c^5*d^4 - 8*a^5*b^2*c*f^4 + 2*a^2*b^6*d*f^3 - 2*a^3*b^5*e*f^3 - 64
*a^3*c^5*d^3*f - 64*a^5*c^3*d*f^3 - 2*b^5*c^3*d^3*e + 2*b^6*c^2*d^3*f + a^2
*b^4*c^2*e^4 - 8*a^3*b^2*c^3*e^4 + 32*a^3*c^5*d^2*e^2 + a^2*b^6*e^2*f^2 + 9
6*a^4*c^4*d^2*f^2 + b^6*c^2*d^2*e^2 + 32*a^5*c^3*e^2*f^2 - 2*a*b^7*d*e*f^2
- 2*b^7*c*d^2*e*f + 54*a^2*b^4*c^2*d^2*f^2 - 112*a^3*b^2*c^3*d^2*f^2 + 16*a
*b^3*c^4*d^3*e - 2*a*b^5*c^2*d*e^3 - 32*a^2*b*c^5*d^3*e - 32*a^3*b*c^4*d*e^
3 - 20*a*b^4*c^3*d^3*f - 12*a*b^6*c*d^2*f^2 - 20*a^3*b^4*c*d*f^3 - 2*a^2*b^
5*c*e^3*f - 32*a^4*b*c^3*e^3*f + 16*a^4*b^3*c*e*f^3 - 32*a^5*b*c^2*e*f^3 -
64*a^4*c^4*d*e^2*f - 6*a*b^4*c^3*d^2*e^2 + 16*a^2*b^3*c^3*d*e^3 + 64*a^2*b^
2*c^4*d^3*f + 64*a^4*b^2*c^2*d*f^3 + 16*a^3*b^3*c^2*e^3*f - 6*a^3*b^4*c*e^2
*f^2 - 48*a^2*b^3*c^3*d^2*e*f - 36*a^2*b^4*c^2*d*e^2*f + 96*a^3*b^2*c^3*d*e
^2*f - 48*a^3*b^3*c^2*d*e*f^2 + 4*a*b^6*c*d*e^2*f + 18*a*b^5*c^2*d^2*e*f +
18*a^2*b^5*c*d*e*f^2 + 32*a^3*b*c^4*d^2*e*f + 32*a^4*b*c^3*d*e*f^2) + (x*(1
04*A^2*a^4*c^5*f^7 - 32*B^2*a^5*c^4*f^7 + 8*A^2*c^9*d^4*f^3 + A^2*b^8*c*f^7
+ 50*A^2*a^2*b^4*c^3*f^7 - 96*A^2*a^3*b^2*c^4*f^7 - 12*B^2*a^3*b^4*c^2*f^7
+ 42*B^2*a^4*b^2*c^3*f^7 + 208*A^2*a^2*c^7*d^2*f^5 + 36*A^2*a^2*c^7*e^4*f^
3 + 72*A^2*a^3*c^6*e^2*f^5 + 8*A^2*b^2*c^7*d^3*f^4 + 18*A^2*b^4*c^5*d^2*f^5
- 32*B^2*a^2*c^7*d^3*f^4 + 32*B^2*a^3*c^6*d^2*f^5 - 2*A^2*b^3*c^6*e^5*f^2
+ A^2*b^4*c^5*e^4*f^3 + A^2*b^6*c^3*e^2*f^5 + 24*B^2*a^3*c^6*e^4*f^3 + 56*B
^2*a^4*c^5*e^2*f^5 + 2*B^2*b^2*c^7*d^4*f^3 - 6*B^2*b^4*c^5*d^3*f^4 + 9*B^2*
b^6*c^3*d^2*f^5 - 16*A^2*c^9*d^3*e^2*f^2 - 12*A^2*a*b^6*c^2*f^7 + B^2*a^2*b
^6*c*f^7 - 64*A^2*a*c^8*d^3*f^4 - 256*A^2*a^3*c^6*d*f^6 + 2*A^2*b^6*c^3*d*f
^6 + 32*B^2*a^4*c^5*d*f^6 + A^2*b^2*c^7*e^6*f - 2*A^2*b^7*c^2*e*f^6 + 4*B^2
*a^2*c^7*e^6*f + 4*A^2*c^9*d^2*e^4*f - 36*A^2*a*b^4*c^4*d*f^6 - 4*A^2*a*b*c
^7*e^5*f^2 + 22*A^2*a*b^5*c^3*e*f^6 - 16*A^2*a^3*b*c^5*e*f^6 - 2*B^2*a*b^6*
c^2*d*f^6 - 40*B^2*a^4*b*c^4*e*f^6 + 8*A^2*a*c^8*d*e^4*f^2 + 16*A^2*b*c^8*d

^3*e*f^3 + 6*A^2*b^5*c^4*d*e*f^5 - 2*A*B*a*b^7*c*f^7 - 144*A^2*a*b^2*c^6*d^2*f^5 + 168*A^2*a^2*b^2*c^5*d*f^6 + 2*A^2*a*b^2*c^6*e^4*f^3 + 10*A^2*a*b^3*c^5*e^3*f^4 - 18*A^2*a*b^4*c^4*e^2*f^5 - 80*A^2*a^2*b*c^6*e^3*f^4 - 56*A^2*a^2*b^3*c^4*e*f^6 + 24*B^2*a*b^2*c^6*d^3*f^4 - 64*B^2*a*b^4*c^4*d^2*f^5 + 26*B^2*a^2*b^4*c^3*d*f^6 - 88*B^2*a^3*b^2*c^4*d*f^6 - 12*B^2*a^2*b*c^6*e^5*f^2 - 40*B^2*a^3*b*c^5*e^3*f^4 + 6*B^2*a^3*b^3*c^3*e*f^6 + 8*A^2*a*c^8*d^2*e^2*f^3 - 128*A^2*a^2*c^7*d*e^2*f^4 + 8*A^2*b*c^8*d^2*e^3*f^2 + 4*A^2*b^2*c^7*d*e^4*f^2 + 10*A^2*b^3*c^6*d*e^3*f^3 - 18*A^2*b^4*c^5*d*e^2*f^4 - 32*B^2*a^2*c^7*d*e^4*f^2 - 80*B^2*a^3*c^6*d*e^2*f^4 + B^2*b^2*c^7*d^2*e^4*f + 10*B^2*b^3*c^6*d^3*e*f^3 - 12*B^2*b^5*c^4*d^2*e*f^4 + 72*A*B*a^4*b*c^4*f^7 - 8*A*B*b*c^8*d^4*f^3 - 176*A*B*a^4*c^5*e*f^6 + 2*A*B*b^7*c^2*d*f^6 - 4*A^2*b*c^8*d*e^5*f + 54*A^2*a^2*b^2*c^5*e^2*f^5 + 84*B^2*a^2*b^2*c^5*d^2*f^5 + 9*B^2*a^2*b^2*c^5*e^4*f^3 + 2*B^2*a^3*b^2*c^4*e^2*f^5 - 26*A^2*b^2*c^7*d^2*e^2*f^3 + 88*B^2*a^2*c^7*d^2*e^2*f^3 - 4*B^2*b^2*c^7*d^3*e^2*f^2 - 4*B^2*b^3*c^6*d^2*e^3*f^2 + 8*B^2*b^4*c^5*d^2*e^2*f^3 + 24*A*B*a^2*b^5*c^2*f^7 - 84*A*B*a^3*b^3*c^3*f^7 + 8*A*B*a^2*c^7*e^5*f^2 - 32*A*B*a^3*c^6*e^3*f^4 + 4*A*B*b^3*c^6*d^3*f^4 - 20*A*B*b^5*c^4*d^2*f^5 - 4*B^2*a*b*c^7*d*e^5*f - 78*B^2*a*b^2*c^6*d^2*e^2*f^3 - 48*B^2*a^2*b^2*c^5*d*e^2*f^4 + 148*A*B*a*b^3*c^5*d^2*f^5 - 192*A*B*a^2*b*c^6*d^2*f^5 - 4*A*B*a^2*b^3*c^4*d*f^6 + 10*A*B*a*b^2*c^6*e^5*f^2 - 2*A*B*a*b^3*c^5*e^4*f^3 - 12*A*B*a*b^4*c^4*e^3*f^4 + 8*A*B*a*b^5*c^3*e^2*f^5 - 52*A*B*a^2*b*c^6*e^4*f^3 - 44*A*B*a^2*b^4*c^3*e*f^6 - 48*A*B*a^3*b*c^5*e^2*f^5 + 204*A*B*a^3*b^2*c^4*e*f^6 - 48*A*B*a*c^8*d^2*e^3*f^2 - 48*A*B*a^2*c^7*d*e^3*f^3 - 16*A*B*a^2*c^7*d^2*e*f^4 + 16*A*B*b*c^8*d^3*e^2*f^2 - 28*A*B*b^2*c^7*d^3*e*f^3 - 6*A*B*b^3*c^6*d*e^4*f^2 + 8*A*B*b^4*c^5*d^2*e*f^4 + 12*A*B*b^5*c^4*d*e^2*f^4 - 40*A^2*a*b*c^7*d*e^3*f^3 + 80*A^2*a*b*c^7*d^2*e*f^4 - 24*A^2*a*b^3*c^5*d*e*f^5 + 48*A^2*a^2*b*c^6*d*e*f^5 - 24*B^2*a*b*c^7*d^3*e*f^3 - 8*B^2*a*b^5*c^3*d*e*f^5 + 56*B^2*a^3*b*c^5*d*e*f^5 - 4*A*B*a*b*c^7*e^6*f + 8*A*B*a*c^8*d*e^5*f + 96*A*B*a^2*b^2*c^5*e^3*f^4 - 36*A*B*a^2*b^3*c^4*e^2*f^5 + 4*A*B*b^2*c^7*d^2*e^3*f^2 + 8*A*B*b^3*c^6*d^2*e^2*f^3 + 84*A^2*a*b^2*c^6*d*e^2*f^4 + 24*B^2*a*b*c^7*d^2*e^3*f^2 + 14*B^2*a*b^2*c^6*d*e^4*f^2 - 16*B^2*a*b^3*c^5*d*e^3*f^3 + 98*B^2*a*b^3*c^5*d^2*e*f^4 + 12*B^2*a*b^4*c^4*d*e^2*f^4 + 64*B^2*a^2*b*c^6*d*e^3*f^3 - 120*B^2*a^2*b*c^6*d^2*e*f^4 + 30*B^2*a^2*b^3*c^4*d*e*f^5 + 16*A*B*a*b*c^7*d^3*f^4 - 12*A*B*a*b^5*c^3*d*f^6 + 112*A*B*a^3*b*c^5*d*f^6 + 2*A*B*a*b^6*c^2*e*f^6 + 48*A*B*a*c^8*d^3*e*f^3 + 144*A*B*a^3*c^6*d*e*f^5 - 4*A*B*b*c^8*d^2*e^4*f + 2*A*B*b^2*c^7*d*e^5*f - 10*A*B*b^6*c^3*d*e*f^5 + 100*A*B*a*b^4*c^4*d*e*f^5 + 64*A*B*a*b*c^7*d^2*e^2*f^3 + 12*A*B*a*b^2*c^6*d*e^3*f^3 - 108*A*B*a*b^2*c^6*d^2*e*f^4 - 100*A*B*a*b^3*c^5*d*e^2*f^4 + 288*A*B*a^2*b*c^6*d*e^2*f^4 - 324*A*B*a^2*b^2*c^5*d*e*f^5))/(16*a^2*c^6*d^4 + a^4*b^4*f^4 + 16*a^4*c^4*e^4 + b^4*c^4*d^4 + 16*a^6*c^2*f^4 + b^8*d^2*f^2 - 8*a*b^2*c^5*d^4 - 8*a^5*b^2*c*f^4 + 2*a^2*b^6*d*f^3 - 2*a^3*b^5*e*f^3 - 64*a^3*c^5*d^3*f - 64*a^5*c^3*d*f^3 - 2*b^5*c^3*d^3*e + 2*b^6*c^2*d^3*f + a^2*b^4*c^2*e^4 - 8*a^3*b^2*c^3*e^4 + 32*a^3*c^5*d^2*e^2 + a^2*b^6*e^2*f^2 + 96*a^4*c^4*d^2*f^2 + b^6*c^2*d^2*e^2 + 32*a^5*c^3*e^2*f^2 - 2*a*b^7*d*e*f^2 - 2*b^7*c*d^2*e*f + 54*a^2*b^4*c^2*d^2*f^2 - 112*a^3*b^2*c^3*d^2*f^2 + 16*a*b^3*c^4*d^3*e - 2*a*b^5*c^2*d

*e^3 - 32*a^2*b*c^5*d^3*e - 32*a^3*b*c^4*d*e^3 - 20*a*b^4*c^3*d^3*f - 12*a*
b^6*c*d^2*f^2 - 20*a^3*b^4*c*d*f^3 - 2*a^2*b^5*c*e^3*f - 32*a^4*b*c^3*e^3*f
 + 16*a^4*b^3*c*e*f^3 - 32*a^5*b*c^2*e*f^3 - 64*a^4*c^4*d*e^2*f - 6*a*b^4*c
^3*d^2*e^2 + 16*a^2*b^3*c^3*d*e^3 + 64*a^2*b^2*c^4*d^3*f + 64*a^4*b^2*c^2*d
*f^3 + 16*a^3*b^3*c^2*e^3*f - 6*a^3*b^4*c*e^2*f^2 - 48*a^2*b^3*c^3*d^2*e*f
- 36*a^2*b^4*c^2*d*e^2*f + 96*a^3*b^2*c^3*d*e^2*f - 48*a^3*b^3*c^2*d*e*f^2
+ 4*a*b^6*c*d*e^2*f + 18*a*b^5*c^2*d^2*e*f + 18*a^2*b^5*c*d*e*f^2 + 32*a^3*
b*c^4*d^2*e*f + 32*a^4*b*c^3*d*e*f^2)) - (3*A^3*b^2*c^5*e^2*f^4 - 4*A^3*c^7
*d^2*f^4 - 12*A^3*a^2*c^5*f^6 - B^3*b^3*c^4*d^2*f^4 + 16*A^3*a*c^6*d*f^5 -
16*A*B^2*a^3*c^4*f^6 + 2*A^3*a*b^2*c^4*f^6 - 16*A^3*a*c^6*e^2*f^4 - 6*A^3*b
^2*c^5*d*f^5 - 3*A^3*b^3*c^4*e*f^5 + 4*B^3*a*b*c^5*d^2*f^4 + 3*B^3*a*b^3*c^
3*d*f^5 - 12*B^3*a^2*b*c^4*d*f^5 + 8*B^3*a^2*c^5*d*e*f^4 + 3*A*B^2*a^2*b^2*
c^3*f^6 - 12*A*B^2*a^2*c^5*e^2*f^4 + A*B^2*b^2*c^5*d^2*f^4 - 3*A^2*B*b^3*c^
4*e^2*f^4 + 16*A^3*a*b*c^5*e*f^5 + 4*A^3*b*c^6*d*e*f^4 - 3*A^2*B*a*b^3*c^3*
f^6 + 16*A^2*B*a^2*b*c^4*f^6 - 8*A*B^2*a*c^6*d^2*f^4 + 24*A*B^2*a^2*c^5*d*f
^5 - 3*A*B^2*b^4*c^3*d*f^5 + 4*A^2*B*b*c^6*d^2*f^4 - 8*A^2*B*a^2*c^5*e*f^5
+ 9*A^2*B*b^3*c^4*d*f^5 + 3*A^2*B*b^4*c^3*e*f^5 + 4*A*B^2*a*b^2*c^4*d*f^5 -
 3*A*B^2*a*b^3*c^3*e*f^5 + 16*A*B^2*a^2*b*c^4*e*f^5 + 16*A^2*B*a*b*c^5*e^2*
f^4 - 14*A^2*B*a*b^2*c^4*e*f^5 + 4*A*B^2*b^3*c^4*d*e*f^4 - 8*A^2*B*b^2*c^5*
d*e*f^4 - 2*B^3*a*b^2*c^4*d*e*f^4 + 2*A*B^2*a*b^2*c^4*e^2*f^4 - 28*A^2*B*a*
b*c^5*d*f^5 + 16*A^2*B*a*c^6*d*e*f^4 - 12*A*B^2*a*b*c^5*d*e*f^4)/(16*a^2*c^
6*d^4 + a^4*b^4*f^4 + 16*a^4*c^4*e^4 + b^4*c^4*d^4 + 16*a^6*c^2*f^4 + b^8*d
^2*f^2 - 8*a*b^2*c^5*d^4 - 8*a^5*b^2*c*f^4 + 2*a^2*b^6*d*f^3 - 2*a^3*b^5*e*
f^3 - 64*a^3*c^5*d^3*f - 64*a^5*c^3*d*f^3 - 2*b^5*c^3*d^3*e + 2*b^6*c^2*d^3
*f + a^2*b^4*c^2*e^4 - 8*a^3*b^2*c^3*e^4 + 32*a^3*c^5*d^2*e^2 + a^2*b^6*e^2
*f^2 + 96*a^4*c^4*d^2*f^2 + b^6*c^2*d^2*e^2 + 32*a^5*c^3*e^2*f^2 - 2*a*b^7*
d*e*f^2 - 2*b^7*c*d^2*e*f + 54*a^2*b^4*c^2*d^2*f^2 - 112*a^3*b^2*c^3*d^2*f^
2 + 16*a*b^3*c^4*d^3*e - 2*a*b^5*c^2*d*e^3 - 32*a^2*b*c^5*d^3*e - 32*a^3*b*
c^4*d*e^3 - 20*a*b^4*c^3*d^3*f - 12*a*b^6*c*d^2*f^2 - 20*a^3*b^4*c*d*f^3 -
2*a^2*b^5*c*e^3*f - 32*a^4*b*c^3*e^3*f + 16*a^4*b^3*c*e*f^3 - 32*a^5*b*c^2*
e*f^3 - 64*a^4*c^4*d*e^2*f - 6*a*b^4*c^3*d^2*e^2 + 16*a^2*b^3*c^3*d*e^3 + 6
4*a^2*b^2*c^4*d^3*f + 64*a^4*b^2*c^2*d*f^3 + 16*a^3*b^3*c^2*e^3*f - 6*a^3*b
^4*c*e^2*f^2 - 48*a^2*b^3*c^3*d^2*e*f - 36*a^2*b^4*c^2*d*e^2*f + 96*a^3*b^2
*c^3*d*e^2*f - 48*a^3*b^3*c^2*d*e*f^2 + 4*a*b^6*c*d*e^2*f + 18*a*b^5*c^2*d^
2*e*f + 18*a^2*b^5*c*d*e*f^2 + 32*a^3*b*c^4*d^2*e*f + 32*a^4*b*c^3*d*e*f^2)
)*root(48416*a^6*b^2*c^6*d^4*e^2*f^4*z^4 - 41544*a^5*b^4*c^5*d^4*e^2*f^4*z^
4 - 31872*a^7*b^2*c^5*d^3*e^2*f^5*z^4 - 31872*a^5*b^2*c^7*d^5*e^2*f^3*z^4 -
 29184*a^6*b^2*c^6*d^3*e^4*f^3*z^4 + 28800*a^5*b^4*c^5*d^3*e^4*f^3*z^4 + 21
510*a^4*b^6*c^4*d^4*e^2*f^4*z^4 + 21408*a^6*b^4*c^4*d^3*e^2*f^5*z^4 + 21408
*a^4*b^4*c^6*d^5*e^2*f^3*z^4 - 18112*a^7*b^3*c^4*d^2*e^3*f^5*z^4 - 18112*a^
4*b^3*c^7*d^5*e^3*f^2*z^4 - 15600*a^5*b^5*c^4*d^3*e^3*f^4*z^4 - 15600*a^4*b
^5*c^5*d^4*e^3*f^3*z^4 + 15296*a^6*b^3*c^5*d^3*e^3*f^4*z^4 + 15296*a^5*b^3*
c^6*d^4*e^3*f^3*z^4 + 14016*a^7*b^2*c^5*d^2*e^4*f^4*z^4 + 14016*a^5*b^2*c^7
*d^4*e^4*f^2*z^4 - 13920*a^4*b^6*c^4*d^3*e^4*f^3*z^4 - 11648*a^6*b^3*c^5*d^
2*e^5*f^3*z^4 - 11648*a^5*b^3*c^6*d^3*e^5*f^2*z^4 + 10432*a^6*b^2*c^6*d^2*e

^6*f^2*z^4 + 9008*a^6*b^5*c^3*d^2*e^3*f^5*z^4 + 9008*a^3*b^5*c^6*d^5*e^3*f^2*z^4 + 8544*a^5*b^5*c^4*d^2*e^5*f^3*z^4 + 8544*a^4*b^5*c^5*d^3*e^5*f^2*z^4 - 8496*a^5*b^4*c^5*d^2*e^6*f^2*z^4 + 7488*a^8*b^2*c^4*d^2*e^2*f^6*z^4 + 7488*a^4*b^2*c^8*d^6*e^2*f^2*z^4 + 7380*a^4*b^7*c^3*d^3*e^3*f^4*z^4 + 7380*a^3*b^7*c^4*d^4*e^3*f^3*z^4 - 6720*a^3*b^8*c^3*d^4*e^2*f^4*z^4 - 5784*a^5*b^6*c^3*d^3*e^2*f^5*z^4 - 5784*a^3*b^6*c^5*d^5*e^2*f^3*z^4 - 3440*a^6*b^4*c^4*d^2*e^4*f^4*z^4 - 3440*a^4*b^4*c^6*d^4*e^4*f^2*z^4 + 3360*a^3*b^8*c^3*d^3*e^4*f^3*z^4 + 3140*a^4*b^6*c^4*d^2*e^6*f^2*z^4 - 2760*a^4*b^7*c^3*d^2*e^5*f^3*z^4 - 2760*a^3*b^7*c^4*d^3*e^5*f^2*z^4 - 1764*a^5*b^7*c^2*d^2*e^3*f^5*z^4 - 1764*a^2*b^7*c^5*d^5*e^3*f^2*z^4 - 1640*a^3*b^9*c^2*d^3*e^3*f^4*z^4 - 1640*a^2*b^9*c^3*d^4*e^3*f^3*z^4 - 1604*a^6*b^6*c^2*d^2*e^2*f^6*z^4 - 1604*a^2*b^6*c^6*d^6*e^2*f^2*z^4 - 1500*a^5*b^6*c^3*d^2*e^4*f^4*z^4 - 1500*a^3*b^6*c^5*d^4*e^4*f^2*z^4 + 1140*a^2*b^10*c^2*d^4*e^2*f^4*z^4 + 810*a^4*b^8*c^2*d^2*e^4*f^4*z^4 + 810*a^2*b^8*c^4*d^4*e^4*f^2*z^4 - 544*a^3*b^8*c^3*d^2*e^6*f^2*z^4 + 416*a^3*b^9*c^2*d^2*e^5*f^3*z^4 + 416*a^2*b^9*c^3*d^3*e^5*f^2*z^4 - 384*a^2*b^10*c^2*d^3*e^4*f^3*z^4 + 180*a^4*b^8*c^2*d^3*e^2*f^5*z^4 + 180*a^2*b^8*c^4*d^5*e^2*f^3*z^4 + 48*a^7*b^4*c^3*d^2*e^2*f^6*z^4 + 48*a^3*b^4*c^7*d^6*e^2*f^2*z^4 + 36*a^2*b^10*c^2*d^2*e^6*f^2*z^4 - 1024*a^10*b*c^3*d*e*f^8*z^4 - 1024*a^3*b*c^10*d^8*e*f*z^4 - 192*a^8*b^5*c*d*e*f^8*z^4 - 192*a*b^5*c^8*d^8*e*f*z^4 + 16128*a^7*b^3*c^4*d^3*e*f^6*z^4 + 16128*a^4*b^3*c^7*d^6*e*f^3*z^4 - 11712*a^6*b^5*c^3*d^3*e*f^6*z^4 - 11712*a^3*b^5*c^6*d^6*e*f^3*z^4 + 11520*a^8*b*c^5*d^2*e^3*f^5*z^4 + 11520*a^5*b*c^8*d^5*e^3*f^2*z^4 - 9984*a^6*b^3*c^5*d^4*e*f^5*z^4 - 9984*a^5*b^3*c^6*d^5*e*f^4*z^4 + 8640*a^5*b^5*c^4*d^4*e*f^5*z^4 + 8640*a^4*b^5*c^5*d^5*e*f^4*z^4 - 7424*a^7*b*c^6*d^3*e^3*f^4*z^4 - 7424*a^6*b*c^7*d^4*e^3*f^3*z^4 - 6912*a^8*b^3*c^3*d^2*e*f^7*z^4 - 6912*a^3*b^3*c^8*d^7*e*f^2*z^4 + 4800*a^7*b^3*c^4*d*e^5*f^4*z^4 + 4800*a^4*b^3*c^7*d^4*e^5*f*z^4 + 4608*a^7*b*c^6*d^2*e^5*f^3*z^4 + 4608*a^6*b*c^7*d^3*e^5*f^2*z^4 - 4560*a^4*b^7*c^3*d^4*e*f^5*z^4 - 4560*a^3*b^7*c^4*d^5*e*f^4*z^4 + 4176*a^5*b^7*c^2*d^3*e*f^6*z^4 + 4176*a^2*b^7*c^5*d^6*e*f^3*z^4 + 3264*a^7*b^5*c^2*d^2*e*f^7*z^4 + 3264*a^2*b^5*c^7*d^7*e*f^2*z^4 + 3008*a^8*b^3*c^3*d*e^3*f^6*z^4 + 3008*a^3*b^3*c^8*d^6*e^3*f*z^4 + 2880*a^6*b^3*c^5*d*e^7*f^2*z^4 + 2880*a^5*b^3*c^6*d^2*e^7*f*z^4 - 2240*a^7*b^4*c^3*d*e^4*f^5*z^4 - 2240*a^3*b^4*c^7*d^5*e^4*f*z^4 - 1488*a^5*b^5*c^4*d*e^7*f^2*z^4 - 1488*a^4*b^5*c^5*d^2*e^7*f*z^4 + 1440*a^3*b^9*c^2*d^4*e*f^5*z^4 + 1440*a^2*b^9*c^3*d^5*e*f^4*z^4 - 1328*a^6*b^5*c^3*d*e^5*f^4*z^4 - 1328*a^3*b^5*c^6*d^4*e^5*f*z^4 - 1152*a^7*b^2*c^5*d*e^6*f^3*z^4 - 1152*a^5*b^2*c^7*d^3*e^6*f*z^4 - 1120*a^6*b^4*c^4*d*e^6*f^3*z^4 - 1120*a^4*b^4*c^6*d^3*e^6*f*z^4 + 912*a^6*b^6*c^2*d*e^4*f^5*z^4 + 912*a^2*b^6*c^6*d^5*e^4*f*z^4 + 872*a^5*b^6*c^3*d*e^6*f^3*z^4 + 872*a^3*b^6*c^5*d^3*e^6*f*z^4 + 768*a^8*b^2*c^4*d*e^4*f^5*z^4 + 768*a^4*b^2*c^8*d^5*e^4*f*z^4 - 672*a^8*b^4*c^2*d*e^2*f^7*z^4 - 672*a^2*b^4*c^8*d^7*e^2*f*z^4 - 624*a^7*b^5*c^2*d*e^3*f^6*z^4 - 624*a^2*b^5*c^7*d^6*e^3*f*z^4 + 480*a^5*b^8*c*d^2*e^2*f^6*z^4 + 480*a*b^8*c^5*d^6*e^2*f^2*z^4 + 316*a^4*b^7*c^3*d*e^7*f^2*z^4 + 316*a^3*b^7*c^4*d^2*e^7*f*z^4 - 204*a^4*b^8*c^2*d*e^6*f^3*z^4 - 204*a^2*b^8*c^4*d^3*e^6*f*z^4 + 168*a^3*b^10*c*d^3*e^2*f^5*z^4 + 168*a*b^10*c^3*d^5*e^2*f^3*z^4 + 156*a^2*b^11*c*d^3*e^3*

f^4*z^4 + 156*a*b^11*c^2*d^4*e^3*f^3*z^4 + 128*a^9*b^2*c^3*d*e^2*f^7*z^4 +
128*a^3*b^2*c^9*d^7*e^2*f*z^4 - 124*a^3*b^10*c*d^2*e^4*f^4*z^4 - 124*a*b^10
*c^3*d^4*e^4*f^2*z^4 + 100*a^4*b^9*c*d^2*e^3*f^5*z^4 + 100*a*b^9*c^4*d^5*e^
3*f^2*z^4 + 36*a^5*b^7*c^2*d*e^5*f^4*z^4 + 36*a^2*b^7*c^5*d^4*e^5*f*z^4 - 2
4*a^3*b^9*c^2*d*e^7*f^2*z^4 - 24*a^2*b^11*c*d^2*e^5*f^3*z^4 - 24*a^2*b^9*c^
3*d^2*e^7*f*z^4 - 24*a*b^11*c^2*d^3*e^5*f^2*z^4 - 9216*a^8*b*c^5*d^3*e*f^6*
z^4 - 9216*a^5*b*c^8*d^6*e*f^3*z^4 - 5376*a^8*b*c^5*d*e^5*f^4*z^4 - 5376*a^
5*b*c^8*d^4*e^5*f*z^4 + 5120*a^9*b*c^4*d^2*e*f^7*z^4 + 5120*a^7*b*c^6*d^4*e
*f^5*z^4 + 5120*a^6*b*c^7*d^5*e*f^4*z^4 + 5120*a^4*b*c^9*d^7*e*f^2*z^4 - 43
52*a^9*b*c^4*d*e^3*f^6*z^4 - 4352*a^4*b*c^9*d^6*e^3*f*z^4 - 1792*a^7*b*c^6*
d*e^7*f^2*z^4 - 1792*a^6*b*c^7*d^2*e^7*f*z^4 - 1600*a^6*b^2*c^6*d*e^8*f*z^4
 + 912*a^5*b^4*c^5*d*e^8*f*z^4 + 768*a^9*b^3*c^2*d*e*f^8*z^4 + 768*a^2*b^3*
c^9*d^8*e*f*z^4 - 720*a^4*b^9*c*d^3*e*f^6*z^4 - 720*a*b^9*c^4*d^6*e*f^3*z^4
 - 656*a^6*b^7*c*d^2*e*f^7*z^4 - 656*a*b^7*c^6*d^7*e*f^2*z^4 - 240*a^2*b^11
*c*d^4*e*f^5*z^4 - 240*a*b^11*c^2*d^5*e*f^4*z^4 + 216*a^7*b^6*c*d*e^2*f^7*z
^4 + 216*a*b^6*c^7*d^7*e^2*f*z^4 - 204*a^4*b^6*c^4*d*e^8*f*z^4 - 144*a^5*b^
8*c*d*e^4*f^5*z^4 - 144*a*b^8*c^5*d^5*e^4*f*z^4 - 84*a*b^12*c*d^4*e^2*f^4*z
^4 + 36*a^4*b^9*c*d*e^5*f^4*z^4 + 36*a*b^9*c^4*d^4*e^5*f*z^4 + 20*a^6*b^7*c
*d*e^3*f^6*z^4 + 20*a*b^7*c^6*d^6*e^3*f*z^4 + 16*a^3*b^10*c*d*e^6*f^3*z^4 +
 16*a^3*b^8*c^3*d*e^8*f*z^4 + 16*a*b^12*c*d^3*e^4*f^3*z^4 + 16*a*b^10*c^3*d
^3*e^6*f*z^4 + 48*b^11*c^3*d^6*e*f^3*z^4 + 48*b^9*c^5*d^7*e*f^2*z^4 - 20*b^
8*c^6*d^7*e^2*f*z^4 + 8*b^10*c^4*d^5*e^4*f*z^4 - 4*b^13*c*d^4*e^3*f^3*z^4 -
 4*b^11*c^3*d^4*e^5*f*z^4 + 4*b^9*c^5*d^6*e^3*f*z^4 + 3072*a^9*c^5*d*e^4*f^
5*z^4 + 3072*a^5*c^9*d^5*e^4*f*z^4 + 2560*a^8*c^6*d*e^6*f^3*z^4 + 2560*a^6*
c^8*d^3*e^6*f*z^4 + 1536*a^10*c^4*d*e^2*f^7*z^4 + 1536*a^4*c^10*d^7*e^2*f*z
^4 + 48*a^5*b^9*d^2*e*f^7*z^4 + 48*a^3*b^11*d^3*e*f^6*z^4 - 20*a^6*b^8*d*e^
2*f^7*z^4 + 8*a^4*b^10*d*e^4*f^5*z^4 + 4*a^5*b^9*d*e^3*f^6*z^4 - 4*a^3*b^11
*d*e^5*f^4*z^4 - 4*a*b^13*d^3*e^3*f^4*z^4 + 768*a^9*b*c^4*e^5*f^5*z^4 + 768
*a^8*b*c^5*e^7*f^3*z^4 + 256*a^10*b*c^3*e^3*f^7*z^4 - 192*a^6*b^3*c^5*e^9*f
*z^4 - 68*a^7*b^6*c*e^4*f^6*z^4 + 48*a^8*b^5*c*e^3*f^7*z^4 + 48*a^5*b^5*c^4
*e^9*f*z^4 + 36*a^6*b^7*c*e^5*f^5*z^4 - 12*a^9*b^4*c*e^2*f^8*z^4 - 4*a^4*b^
9*c*e^7*f^3*z^4 - 4*a^4*b^7*c^3*e^9*f*z^4 + 384*a^5*b^8*c*d^3*f^7*z^4 + 384
*a*b^8*c^5*d^7*f^3*z^4 + 288*a^3*b^10*c*d^4*f^6*z^4 + 288*a*b^10*c^3*d^6*f^
4*z^4 + 224*a^7*b^6*c*d^2*f^8*z^4 + 224*a*b^6*c^7*d^8*f^2*z^4 - 192*a^10*b^
2*c^2*d*f^9*z^4 - 192*a^2*b^2*c^10*d^9*f*z^4 + 768*a^5*b*c^8*d^3*e^7*z^4 +
768*a^4*b*c^9*d^5*e^5*z^4 + 256*a^3*b*c^10*d^7*e^3*z^4 - 192*a^5*b^3*c^6*d*
e^9*z^4 - 68*a*b^6*c^7*d^6*e^4*z^4 + 48*a^4*b^5*c^5*d*e^9*z^4 + 48*a*b^5*c^
8*d^7*e^3*z^4 + 36*a*b^7*c^6*d^5*e^5*z^4 - 12*a*b^4*c^9*d^8*e^2*z^4 - 4*a^3
*b^7*c^4*d*e^9*z^4 - 4*a*b^9*c^4*d^3*e^7*z^4 + 16*b^13*c*d^5*e*f^4*z^4 + 16
*b^7*c^7*d^8*e*f*z^4 + 768*a^7*c^7*d*e^8*f*z^4 + 16*a^7*b^7*d*e*f^8*z^4 + 1
6*a*b^13*d^4*e*f^5*z^4 + 256*a^7*b*c^6*e^9*f*z^4 + 80*a*b^12*c*d^5*f^5*z^4
+ 48*a^9*b^4*c*d*f^9*z^4 + 48*a*b^4*c^9*d^9*f*z^4 + 256*a^6*b*c^7*d*e^9*z^4
 - 42*b^10*c^4*d^6*e^2*f^2*z^4 - 20*b^12*c^2*d^5*e^2*f^3*z^4 + 6*b^12*c^2*d
^4*e^4*f^2*z^4 + 4*b^11*c^3*d^5*e^3*f^2*z^4 - 24960*a^7*c^7*d^4*e^2*f^4*z^4
 + 18944*a^8*c^6*d^3*e^2*f^5*z^4 + 18944*a^6*c^8*d^5*e^2*f^3*z^4 + 14336*a^

7*c^7*d^3*e^4*f^3*z^4 - 9984*a^8*c^6*d^2*e^4*f^4*z^4 - 9984*a^6*c^8*d^4*e^4
*f^2*z^4 - 7936*a^9*c^5*d^2*e^2*f^6*z^4 - 7936*a^5*c^9*d^6*e^2*f^2*z^4 - 43
52*a^7*c^7*d^2*e^6*f^2*z^4 - 42*a^4*b^10*d^2*e^2*f^6*z^4 - 20*a^2*b^12*d^3*
e^2*f^5*z^4 + 6*a^2*b^12*d^2*e^4*f^4*z^4 + 4*a^3*b^11*d^2*e^3*f^5*z^4 - 480
*a^8*b^2*c^4*e^6*f^4*z^4 + 440*a^7*b^4*c^3*e^6*f^4*z^4 - 320*a^8*b^3*c^3*e^
5*f^5*z^4 - 320*a^7*b^3*c^4*e^7*f^3*z^4 + 240*a^8*b^4*c^2*e^4*f^6*z^4 + 240
*a^6*b^4*c^4*e^8*f^2*z^4 - 192*a^9*b^3*c^2*e^3*f^7*z^4 - 192*a^9*b^2*c^3*e^
4*f^6*z^4 - 192*a^7*b^2*c^5*e^8*f^2*z^4 - 90*a^6*b^6*c^2*e^6*f^4*z^4 - 68*a
^5*b^6*c^3*e^8*f^2*z^4 + 48*a^10*b^2*c^2*e^2*f^8*z^4 - 48*a^7*b^5*c^2*e^5*f
^5*z^4 - 48*a^6*b^5*c^3*e^7*f^3*z^4 + 36*a^5*b^7*c^2*e^7*f^3*z^4 + 6*a^4*b^
8*c^2*e^8*f^2*z^4 - 33920*a^6*b^2*c^6*d^5*f^5*z^4 + 27936*a^5*b^4*c^5*d^5*f
^5*z^4 + 26112*a^7*b^2*c^5*d^4*f^6*z^4 + 26112*a^5*b^2*c^7*d^6*f^4*z^4 - 20
352*a^6*b^4*c^4*d^4*f^6*z^4 - 20352*a^4*b^4*c^6*d^6*f^4*z^4 - 13080*a^4*b^6
*c^4*d^5*f^5*z^4 - 11520*a^8*b^2*c^4*d^3*f^7*z^4 - 11520*a^4*b^2*c^8*d^7*f^
3*z^4 + 8736*a^5*b^6*c^3*d^4*f^6*z^4 + 8736*a^3*b^6*c^5*d^6*f^4*z^4 + 7488*
a^7*b^4*c^3*d^3*f^7*z^4 + 7488*a^3*b^4*c^7*d^7*f^3*z^4 + 3840*a^3*b^8*c^3*d
^5*f^5*z^4 + 2560*a^9*b^2*c^3*d^2*f^8*z^4 + 2560*a^3*b^2*c^9*d^8*f^2*z^4 -
2416*a^6*b^6*c^2*d^3*f^7*z^4 - 2416*a^2*b^6*c^6*d^7*f^3*z^4 - 2160*a^4*b^8*
c^2*d^4*f^6*z^4 - 2160*a^2*b^8*c^4*d^6*f^4*z^4 - 1152*a^8*b^4*c^2*d^2*f^8*z
^4 - 1152*a^2*b^4*c^8*d^8*f^2*z^4 - 720*a^2*b^10*c^2*d^5*f^5*z^4 - 480*a^4*
b^2*c^8*d^4*e^6*z^4 + 440*a^3*b^4*c^7*d^4*e^6*z^4 - 320*a^4*b^3*c^7*d^3*e^7
*z^4 - 320*a^3*b^3*c^8*d^5*e^5*z^4 + 240*a^4*b^4*c^6*d^2*e^8*z^4 + 240*a^2*
b^4*c^8*d^6*e^4*z^4 - 192*a^5*b^2*c^7*d^2*e^8*z^4 - 192*a^3*b^2*c^9*d^6*e^4
*z^4 - 192*a^2*b^3*c^9*d^7*e^3*z^4 - 90*a^2*b^6*c^6*d^4*e^6*z^4 - 68*a^3*b^
6*c^5*d^2*e^8*z^4 - 48*a^3*b^5*c^6*d^3*e^7*z^4 - 48*a^2*b^5*c^7*d^5*e^5*z^4
 + 48*a^2*b^2*c^10*d^8*e^2*z^4 + 36*a^2*b^7*c^5*d^3*e^7*z^4 + 6*a^2*b^8*c^4
*d^2*e^8*z^4 - 4*b^6*c^8*d^9*f*z^4 + 256*a^11*c^3*d*f^9*z^4 + 256*a^3*c^11*
d^9*f*z^4 - 4*a^8*b^6*d*f^9*z^4 - 384*a^9*c^5*e^6*f^4*z^4 - 256*a^10*c^4*e^
4*f^6*z^4 - 256*a^8*c^6*e^8*f^2*z^4 - 64*a^11*c^3*e^2*f^8*z^4 - 24*b^10*c^4
*d^7*f^3*z^4 - 16*b^12*c^2*d^6*f^4*z^4 - 16*b^8*c^6*d^8*f^2*z^4 + 17920*a^7
*c^7*d^5*f^5*z^4 - 14336*a^8*c^6*d^4*f^6*z^4 - 14336*a^6*c^8*d^6*f^4*z^4 +
7168*a^9*c^5*d^3*f^7*z^4 + 7168*a^5*c^9*d^7*f^3*z^4 - 2048*a^10*c^4*d^2*f^8
*z^4 - 2048*a^4*c^10*d^8*f^2*z^4 + 6*b^8*c^6*d^6*e^4*z^4 + 6*a^6*b^8*e^4*f^
6*z^4 - 4*b^9*c^5*d^5*e^5*z^4 - 4*b^7*c^7*d^7*e^3*z^4 - 4*a^7*b^7*e^3*f^7*z
^4 - 4*a^5*b^9*e^5*f^5*z^4 - 384*a^5*c^9*d^4*e^6*z^4 - 256*a^6*c^8*d^2*e^8*
z^4 - 256*a^4*c^10*d^6*e^4*z^4 - 64*a^3*c^11*d^8*e^2*z^4 - 24*a^4*b^10*d^3*
f^7*z^4 - 16*a^6*b^8*d^2*f^8*z^4 - 16*a^2*b^12*d^4*f^6*z^4 + 48*a^6*b^2*c^6
*e^10*z^4 - 12*a^5*b^4*c^5*e^10*z^4 - 4*b^14*d^5*f^5*z^4 - 64*a^7*c^7*e^10*
z^4 + b^14*d^4*e^2*f^4*z^4 + b^10*c^4*d^4*e^6*z^4 + b^6*c^8*d^8*e^2*z^4 + a
^8*b^6*e^2*f^8*z^4 + a^4*b^10*e^6*f^4*z^4 + a^4*b^6*c^4*e^10*z^4 - 4820*A*B
*a^4*b*c^5*d^2*e^2*f^4*z^2 + 2976*A*B*a^3*b*c^6*d^3*e^2*f^3*z^2 - 2328*A*B*
a^3*b*c^6*d^2*e^4*f^2*z^2 + 1848*A*B*a^2*b^4*c^4*d^3*e*f^4*z^2 - 1768*A*B*a
^3*b^4*c^3*d^2*e*f^5*z^2 + 1528*A*B*a^4*b^2*c^4*d^2*e*f^5*z^2 - 1136*A*B*a^
3*b^2*c^5*d^3*e*f^4*z^2 - 974*A*B*a^4*b^3*c^3*d*e^2*f^5*z^2 + 692*A*B*a^2*b
*c^7*d^4*e^2*f^2*z^2 + 588*A*B*a*b^6*c^3*d^2*e^3*f^3*z^2 - 580*A*B*a^3*b^3*

c^4*d*e^4*f^3*z^2 + 488*A*B*a^3*b^4*c^3*d*e^3*f^4*z^2 - 444*A*B*a^2*b^2*c^6
*d^2*e^5*f*z^2 - 412*A*B*a*b^5*c^4*d^2*e^4*f^2*z^2 + 366*A*B*a^2*b^6*c^2*d^
2*e*f^5*z^2 - 352*A*B*a^2*b^2*c^6*d^4*e*f^3*z^2 + 326*A*B*a^2*b^4*c^4*d*e^5
*f^2*z^2 + 324*A*B*a*b^5*c^4*d^3*e^2*f^3*z^2 - 302*A*B*a*b^3*c^6*d^4*e^2*f^
2*z^2 - 296*A*B*a*b^7*c^2*d^2*e^2*f^4*z^2 + 122*A*B*a^4*b^2*c^4*d*e^3*f^4*z
^2 - 122*A*B*a^2*b^6*c^2*d*e^3*f^4*z^2 - 84*A*B*a^3*b^2*c^5*d*e^5*f^2*z^2 +
 72*A*B*a*b^4*c^5*d^3*e^3*f^2*z^2 - 64*A*B*a^2*b^5*c^3*d*e^4*f^3*z^2 + 60*A
*B*a^3*b^5*c^2*d*e^2*f^5*z^2 + 1312*A*B*a^5*b*c^4*d*e^2*f^5*z^2 + 1040*A*B*
a^4*b*c^5*d*e^4*f^3*z^2 - 500*A*B*a*b^6*c^3*d^3*e*f^4*z^2 - 376*A*B*a*b^2*c
^7*d^5*e*f^2*z^2 + 276*A*B*a^4*b^4*c^2*d*e*f^6*z^2 - 262*A*B*a^2*b^3*c^5*d*
e^6*f*z^2 + 238*A*B*a*b^2*c^7*d^4*e^3*f*z^2 + 232*A*B*a^5*b^2*c^3*d*e*f^6*z
^2 - 176*A*B*a^2*b*c^7*d^3*e^4*f*z^2 - 120*A*B*a*b^6*c^3*d*e^5*f^2*z^2 - 10
8*A*B*a*b^4*c^5*d^4*e*f^3*z^2 + 68*A*B*a*b^7*c^2*d*e^4*f^3*z^2 + 68*A*B*a*b
^4*c^5*d^2*e^5*f*z^2 + 46*A*B*a^2*b^7*c*d*e^2*f^5*z^2 - 36*A*B*a*b^3*c^6*d^
3*e^4*f*z^2 - 1932*A*B*a^2*b^3*c^5*d^3*e^2*f^3*z^2 - 1818*A*B*a^2*b^4*c^4*d
^2*e^3*f^3*z^2 + 1620*A*B*a^3*b^3*c^4*d^2*e^2*f^4*z^2 + 1560*A*B*a^2*b^3*c^
5*d^2*e^4*f^2*z^2 + 1244*A*B*a^3*b^2*c^5*d^2*e^3*f^3*z^2 + 820*A*B*a^2*b^2*
c^6*d^3*e^3*f^2*z^2 + 480*A*B*a^2*b^5*c^3*d^2*e^2*f^4*z^2 + 352*A*B*a^3*b*c
^6*d*e^6*f*z^2 - 108*A*B*a^3*b^6*c*d*e*f^6*z^2 + 82*A*B*a*b^5*c^4*d*e^6*f*z
^2 - 64*A*B*a*b*c^8*d^5*e^2*f*z^2 + 16*A*B*a*b^8*c*d^2*e*f^5*z^2 - 4*A*B*a*
b^8*c*d*e^3*f^4*z^2 + 16*B^2*a*b*c^8*d^6*e*f*z^2 + 56*A*B*b^2*c^8*d^6*e*f*z
^2 - 8*A*B*b^9*c*d*e^4*f^3*z^2 - 8*A*B*b^7*c^3*d*e^6*f*z^2 - 800*A*B*a^6*c^
4*d*e*f^6*z^2 + 10*A*B*a^2*b^8*d*e*f^6*z^2 - 6*A*B*a*b^9*d*e^2*f^5*z^2 - 12
*A*B*a^5*b^4*c*e*f^7*z^2 + 912*A*B*a^6*b*c^3*d*f^7*z^2 + 192*A*B*a^4*b^5*c*
d*f^7*z^2 + 192*A*B*a*b*c^8*d^6*f^2*z^2 - 20*A*B*a*b^4*c^5*d*e^7*z^2 + 4*A*
B*a*b*c^8*d^4*e^4*z^2 + 2144*B^2*a^4*b*c^5*d^3*e*f^4*z^2 - 1120*B^2*a^3*b*c
^6*d^4*e*f^3*z^2 - 688*B^2*a^5*b*c^4*d^2*e*f^5*z^2 - 256*B^2*a^3*b*c^6*d^2*
e^5*f*z^2 + 152*B^2*a*b^3*c^6*d^5*e*f^2*z^2 + 120*B^2*a^5*b^3*c^2*d*e*f^6*z
^2 - 116*B^2*a^5*b*c^4*d*e^3*f^4*z^2 + 110*B^2*a*b^7*c^2*d^3*e*f^4*z^2 - 80
*B^2*a^2*b*c^7*d^5*e*f^2*z^2 - 72*B^2*a*b^5*c^4*d^4*e*f^3*z^2 - 48*B^2*a^4*
b*c^5*d*e^5*f^2*z^2 - 46*B^2*a*b^3*c^6*d^4*e^3*f*z^2 - 44*B^2*a*b^4*c^5*d^3
*e^4*f*z^2 - 34*B^2*a*b^5*c^4*d^2*e^5*f*z^2 + 20*B^2*a^2*b*c^7*d^4*e^3*f*z^
2 - 10*B^2*a^3*b^6*c*d*e^2*f^5*z^2 - 10*B^2*a^2*b^7*c*d^2*e*f^5*z^2 - 10*B^
2*a*b^2*c^7*d^5*e^2*f*z^2 - 7*B^2*a^2*b^4*c^4*d*e^6*f*z^2 - 6*B^2*a^3*b^2*c
^5*d*e^6*f*z^2 + 4*B^2*a*b^8*c*d^2*e^2*f^4*z^2 - 2*B^2*a^2*b^7*c*d*e^3*f^4*
z^2 + 3196*A^2*a^4*b*c^5*d*e^3*f^4*z^2 - 3184*A^2*a^4*b*c^5*d^2*e*f^5*z^2 +
 1568*A^2*a^3*b*c^6*d^3*e*f^4*z^2 + 1504*A^2*a^3*b*c^6*d*e^5*f^2*z^2 - 656*
A^2*a^4*b^3*c^3*d*e*f^6*z^2 - 400*A^2*a*b^6*c^3*d*e^4*f^3*z^2 + 314*A^2*a*b
^5*c^4*d*e^5*f^2*z^2 - 264*A^2*a^3*b^5*c^2*d*e*f^6*z^2 + 240*A^2*a^2*b^2*c^
6*d*e^6*f*z^2 - 224*A^2*a^2*b*c^7*d^4*e*f^3*z^2 + 216*A^2*a*b^5*c^4*d^3*e*f
^4*z^2 - 192*A^2*a^2*b*c^7*d^2*e^5*f*z^2 + 178*A^2*a*b^7*c^2*d*e^3*f^4*z^2
- 154*A^2*a*b^7*c^2*d^2*e*f^5*z^2 + 128*A^2*a*b^3*c^6*d^4*e*f^3*z^2 + 106*A
^2*a*b^3*c^6*d^2*e^5*f*z^2 - 12*A^2*a*b^2*c^7*d^3*e^4*f*z^2 - 58*A*B*b^8*c^
2*d^2*e^3*f^3*z^2 + 40*A*B*b^7*c^3*d^2*e^4*f^2*z^2 - 28*A*B*b^7*c^3*d^3*e^2
*f^3*z^2 - 24*A*B*b^5*c^5*d^4*e^2*f^2*z^2 - 20*A*B*b^6*c^4*d^3*e^3*f^2*z^2

+ 2768*A*B*a^4*c^6*d^2*e^3*f^3*z^2 - 1712*A*B*a^3*c^7*d^3*e^3*f^2*z^2 - 156
*A*B*a^4*b^2*c^4*e^5*f^3*z^2 + 146*A*B*a^4*b^3*c^3*e^4*f^4*z^2 - 106*A*B*a^
5*b^2*c^3*e^3*f^5*z^2 + 90*A*B*a^5*b^3*c^2*e^2*f^6*z^2 + 38*A*B*a^3*b^3*c^4
*e^6*f^2*z^2 - 36*A*B*a^3*b^5*c^2*e^4*f^4*z^2 + 16*A*B*a^3*b^4*c^3*e^5*f^3*
z^2 - 9*A*B*a^4*b^4*c^2*e^3*f^5*z^2 - 8*A*B*a^2*b^5*c^3*e^6*f^2*z^2 + 2*A*B
*a^2*b^6*c^2*e^5*f^3*z^2 + 920*A*B*a^4*b^3*c^3*d^2*f^6*z^2 - 480*A*B*a^2*b^
5*c^3*d^3*f^5*z^2 - 336*A*B*a^2*b^3*c^5*d^4*f^4*z^2 - 272*A*B*a^3*b^3*c^4*d
^3*f^5*z^2 + 240*A*B*a^3*b^5*c^2*d^2*f^6*z^2 - 32*A*B*a*c^9*d^6*e*f*z^2 - 7
92*B^2*a^2*b^3*c^5*d^3*e^3*f^2*z^2 + 714*B^2*a^2*b^4*c^4*d^3*e^2*f^3*z^2 -
572*B^2*a^3*b^2*c^5*d^3*e^2*f^3*z^2 - 475*B^2*a^2*b^2*c^6*d^4*e^2*f^2*z^2 +
265*B^2*a^4*b^2*c^4*d^2*e^2*f^4*z^2 + 260*B^2*a^3*b^3*c^4*d^2*e^3*f^3*z^2
- 212*B^2*a^3*b^4*c^3*d^2*e^2*f^4*z^2 + 180*B^2*a^3*b^2*c^5*d^2*e^4*f^2*z^2
- 158*B^2*a^2*b^4*c^4*d^2*e^4*f^2*z^2 + 47*B^2*a^2*b^6*c^2*d^2*e^2*f^4*z^2
+ 16*B^2*a^2*b^5*c^3*d^2*e^3*f^3*z^2 + 2752*A^2*a^3*b^2*c^5*d^2*e^2*f^4*z^
2 - 2148*A^2*a^2*b^4*c^4*d^2*e^2*f^4*z^2 + 2064*A^2*a^2*b^3*c^5*d^2*e^3*f^3
*z^2 - 424*A^2*a^2*b^2*c^6*d^3*e^2*f^3*z^2 - 198*A^2*a^2*b^2*c^6*d^2*e^4*f^
2*z^2 - 272*B^2*a^6*b*c^3*d*e*f^6*z^2 - 24*B^2*a^4*b^5*c*d*e*f^6*z^2 + 1808
*A^2*a^5*b*c^4*d*e*f^6*z^2 - 244*A^2*a*b*c^8*d^4*e^3*f*z^2 + 208*A^2*a*b*c^
8*d^5*e*f^2*z^2 + 134*A^2*a^2*b^7*c*d*e*f^6*z^2 - 76*A^2*a*b^4*c^5*d*e^6*f*
z^2 + 4*A^2*a*b^8*c*d*e^2*f^5*z^2 + 148*A*B*b^4*c^6*d^5*e*f^2*z^2 + 65*A*B*
b^6*c^4*d^4*e*f^3*z^2 + 46*A*B*b^8*c^2*d^3*e*f^4*z^2 - 38*A*B*b^3*c^7*d^5*e
^2*f*z^2 + 34*A*B*b^9*c*d^2*e^2*f^4*z^2 - 29*A*B*b^4*c^6*d^4*e^3*f*z^2 + 20
*A*B*b^5*c^5*d^3*e^4*f*z^2 + 12*A*B*b^8*c^2*d*e^5*f^2*z^2 - 7*A*B*b^6*c^4*d
^2*e^5*f*z^2 - 2880*A*B*a^4*c^6*d^3*e*f^4*z^2 + 2784*A*B*a^5*c^5*d^2*e*f^5*
z^2 - 1112*A*B*a^5*c^5*d*e^3*f^4*z^2 + 896*A*B*a^3*c^7*d^4*e*f^3*z^2 + 848*
A*B*a^3*c^7*d^2*e^5*f*z^2 - 560*A*B*a^4*c^6*d*e^5*f^2*z^2 + 96*A*B*a^2*c^8*
d^5*e*f^2*z^2 - 88*A*B*a^2*c^8*d^4*e^3*f*z^2 - 100*A*B*a^6*b*c^3*e^2*f^6*z^
2 - 76*A*B*a^5*b*c^4*e^4*f^4*z^2 + 48*A*B*a^6*b^2*c^2*e*f^7*z^2 - 42*A*B*a^
3*b^2*c^5*e^7*f*z^2 + 36*A*B*a^4*b*c^5*e^6*f^2*z^2 - 24*A*B*a^4*b^5*c*e^2*f
^6*z^2 + 10*A*B*a^3*b^6*c*e^3*f^5*z^2 + 7*A*B*a^2*b^4*c^4*e^7*f*z^2 + 2*A*B
*a^2*b^7*c*e^4*f^4*z^2 - 2496*A*B*a^5*b*c^4*d^2*f^6*z^2 + 1872*A*B*a^4*b*c^
5*d^3*f^5*z^2 - 744*A*B*a^5*b^3*c^2*d*f^7*z^2 - 720*A*B*a^2*b*c^7*d^5*f^3*z
^2 + 504*A*B*a*b^3*c^6*d^5*f^3*z^2 + 256*A*B*a^3*b*c^6*d^4*f^4*z^2 + 168*A*
B*a*b^7*c^2*d^3*f^5*z^2 - 144*A*B*a^2*b^7*c*d^2*f^6*z^2 + 144*A*B*a*b^5*c^4
*d^4*f^4*z^2 + 66*A*B*a^2*b^2*c^6*d*e^7*z^2 - 36*A*B*a*b^2*c^7*d^3*e^5*z^2
+ 20*A*B*a*b^3*c^6*d^2*e^6*z^2 + 12*A*B*a^2*b*c^7*d^2*e^6*z^2 + 1208*B^2*a^
3*b*c^6*d^3*e^3*f^2*z^2 - 848*B^2*a^3*b^3*c^4*d^3*e*f^4*z^2 + 672*B^2*a^2*b
^3*c^5*d^4*e*f^3*z^2 - 632*B^2*a^4*b*c^5*d^2*e^3*f^3*z^2 + 432*B^2*a^4*b^3*
c^3*d^2*e*f^5*z^2 + 276*B^2*a^2*b^2*c^6*d^3*e^4*f*z^2 - 196*B^2*a*b^6*c^3*d
^3*e^2*f^3*z^2 - 168*B^2*a^2*b^5*c^3*d^3*e*f^4*z^2 + 154*B^2*a^2*b^3*c^5*d^
2*e^5*f*z^2 + 148*B^2*a*b^5*c^4*d^3*e^3*f^2*z^2 + 96*B^2*a*b^4*c^5*d^4*e^2*
f^2*z^2 - 72*B^2*a^3*b^5*c^2*d^2*e*f^5*z^2 + 70*B^2*a^5*b^2*c^3*d*e^2*f^5*z
^2 - 60*B^2*a^4*b^3*c^3*d*e^3*f^4*z^2 + 52*B^2*a*b^6*c^3*d^2*e^4*f^2*z^2 +
36*B^2*a^4*b^2*c^4*d*e^4*f^3*z^2 - 32*B^2*a*b^7*c^2*d^2*e^3*f^3*z^2 + 24*B^
2*a^3*b^5*c^2*d*e^3*f^4*z^2 + 15*B^2*a^4*b^4*c^2*d*e^2*f^5*z^2 - 8*B^2*a^3*

b^4*c^3*d*e^4*f^3*z^2 + 8*B^2*a^2*b^5*c^3*d*e^5*f^2*z^2 - 2*B^2*a^3*b^3*c^4
*d*e^5*f^2*z^2 - 2*B^2*a^2*b^6*c^2*d*e^4*f^3*z^2 - 3176*A^2*a^3*b*c^6*d^2*e
^3*f^3*z^2 - 2252*A^2*a^4*b^2*c^4*d*e^2*f^5*z^2 + 1952*A^2*a^3*b^4*c^3*d*e^
2*f^5*z^2 - 1496*A^2*a^3*b^3*c^4*d*e^3*f^4*z^2 + 1378*A^2*a^2*b^4*c^4*d*e^4
*f^3*z^2 + 1184*A^2*a^3*b^3*c^4*d^2*e*f^5*z^2 - 1166*A^2*a^2*b^3*c^5*d*e^5*
f^2*z^2 - 1164*A^2*a^3*b^2*c^5*d*e^4*f^3*z^2 - 1152*A^2*a^2*b^3*c^5*d^3*e*f
^4*z^2 + 578*A^2*a*b^6*c^3*d^2*e^2*f^4*z^2 - 548*A^2*a*b^5*c^4*d^2*e^3*f^3*
z^2 + 440*A^2*a*b^2*c^7*d^4*e^2*f^2*z^2 - 412*A^2*a^2*b^6*c^2*d*e^2*f^5*z^2
 - 360*A^2*a*b^3*c^6*d^3*e^3*f^2*z^2 + 312*A^2*a*b^4*c^5*d^3*e^2*f^3*z^2 +
248*A^2*a^2*b*c^7*d^3*e^3*f^2*z^2 - 224*A^2*a^2*b^5*c^3*d*e^3*f^4*z^2 + 216
*A^2*a^2*b^5*c^3*d^2*e*f^5*z^2 + 52*A^2*a*b^4*c^5*d^2*e^4*f^2*z^2 - 16*B^2*
b^3*c^7*d^6*e*f*z^2 - 14*B^2*b^9*c*d^3*e*f^4*z^2 + 32*B^2*a^4*c^6*d*e^6*f*z
^2 - 20*A^2*b^9*c*d*e^3*f^4*z^2 + 18*A^2*b^9*c*d^2*e*f^5*z^2 + 8*A^2*b^6*c^
4*d*e^6*f*z^2 - 360*A^2*a^3*c^7*d*e^6*f*z^2 + 136*A^2*a*c^9*d^5*e^2*f*z^2 +
 2*B^2*a^3*b^7*d*e*f^6*z^2 + 2*B^2*a*b^9*d^2*e*f^5*z^2 + 12*B^2*a^4*b*c^5*e
^7*f*z^2 - 204*A^2*a^3*b*c^6*e^7*f*z^2 - 128*A^2*a^6*b*c^3*e*f^7*z^2 - 48*A
^2*a*b^5*c^4*e^7*f*z^2 - 36*B^2*a^5*b^4*c*d*f^7*z^2 - 24*A^2*a^4*b^5*c*e*f^
7*z^2 - 16*B^2*a*b^8*c*d^3*f^5*z^2 - 164*A^2*a^3*b^6*c*d*f^7*z^2 - 16*A^2*a
*b^8*c*d^2*f^6*z^2 + 4*B^2*a^3*b*c^6*d*e^7*z^2 - 4*B^2*a*b*c^8*d^5*e^3*z^2
+ 48*A^2*a*b*c^8*d^3*e^5*z^2 + 36*A^2*a^2*b*c^7*d*e^7*z^2 - 6*A^2*a*b^3*c^6
*d*e^7*z^2 + 136*A*B*a^6*c^4*e^3*f^5*z^2 - 96*A*B*b^5*c^5*d^5*f^3*z^2 + 80*
A*B*a^5*c^5*e^5*f^3*z^2 - 72*A*B*b^3*c^7*d^6*f^2*z^2 - 24*A*B*b^7*c^3*d^4*f
^4*z^2 + 14*A*B*b^3*c^7*d^4*e^4*z^2 - 14*A*B*b^2*c^8*d^5*e^3*z^2 - 2*A*B*b^
5*c^5*d^2*e^6*z^2 - 2*A*B*b^4*c^6*d^3*e^5*z^2 + 2*A*B*a^3*b^7*e^2*f^6*z^2 -
 A*B*a^2*b^8*e^3*f^5*z^2 + 16*A*B*a^2*c^8*d^3*e^5*z^2 - 2*A*B*a^2*b^3*c^5*e
^8*z^2 + 22*B^2*b^8*c^2*d^3*e^2*f^3*z^2 - 12*B^2*b^7*c^3*d^3*e^3*f^2*z^2 +
12*B^2*b^6*c^4*d^4*e^2*f^2*z^2 - 6*B^2*b^8*c^2*d^2*e^4*f^2*z^2 - 864*B^2*a^
4*c^6*d^3*e^2*f^3*z^2 + 496*B^2*a^3*c^7*d^4*e^2*f^2*z^2 + 224*B^2*a^5*c^5*d
^2*e^2*f^4*z^2 + 136*B^2*a^4*c^6*d^2*e^4*f^2*z^2 - 53*A^2*b^8*c^2*d^2*e^2*f
^4*z^2 + 52*A^2*b^7*c^3*d^2*e^3*f^3*z^2 + 52*A^2*b^5*c^5*d^3*e^3*f^2*z^2 -
36*A^2*b^6*c^4*d^3*e^2*f^3*z^2 - 12*A^2*b^4*c^6*d^4*e^2*f^2*z^2 - 9*A^2*b^6
*c^4*d^2*e^4*f^2*z^2 + 836*A^2*a^4*c^6*d^2*e^2*f^4*z^2 - 668*A^2*a^2*c^8*d^
4*e^2*f^2*z^2 + 656*A^2*a^3*c^7*d^2*e^4*f^2*z^2 + 368*A^2*a^3*c^7*d^3*e^2*f
^3*z^2 - 45*B^2*a^6*b^2*c^2*e^2*f^6*z^2 - 18*B^2*a^5*b^2*c^3*e^4*f^4*z^2 -
9*B^2*a^4*b^2*c^4*e^6*f^2*z^2 - 6*B^2*a^5*b^3*c^2*e^3*f^5*z^2 + 3*B^2*a^4*b
^4*c^2*e^4*f^4*z^2 - 2*B^2*a^4*b^3*c^3*e^5*f^3*z^2 - 580*B^2*a^4*b^2*c^4*d^
3*f^5*z^2 + 536*B^2*a^3*b^4*c^3*d^3*f^5*z^2 + 471*A^2*a^4*b^2*c^4*e^4*f^4*z
^2 - 436*A^2*a^3*b^4*c^3*e^4*f^4*z^2 - 348*B^2*a^4*b^4*c^2*d^2*f^6*z^2 + 31
6*B^2*a^2*b^2*c^6*d^5*f^3*z^2 + 310*A^2*a^3*b^3*c^4*e^5*f^3*z^2 + 232*A^2*a
^5*b^2*c^3*e^2*f^6*z^2 - 229*A^2*a^2*b^4*c^4*e^6*f^2*z^2 - 216*A^2*a^4*b^4*
c^2*e^2*f^6*z^2 + 204*A^2*a^4*b^3*c^3*e^3*f^5*z^2 + 200*B^2*a^5*b^2*c^3*d^2
*f^6*z^2 + 150*A^2*a^3*b^2*c^5*e^6*f^2*z^2 - 120*B^2*a^2*b^4*c^4*d^4*f^4*z^
2 + 91*A^2*a^2*b^6*c^2*e^4*f^4*z^2 + 72*A^2*a^3*b^5*c^2*e^3*f^5*z^2 - 66*B^
2*a^2*b^6*c^2*d^3*f^5*z^2 + 44*A^2*a^2*b^5*c^3*e^5*f^3*z^2 - 16*B^2*a^3*b^2
*c^5*d^4*f^4*z^2 + 1952*A^2*a^4*b^2*c^4*d^2*f^6*z^2 - 1792*A^2*a^3*b^2*c^5*

d^3*f^5*z^2 - 1272*A^2*a^3*b^4*c^3*d^2*f^6*z^2 + 976*A^2*a^2*b^2*c^6*d^4*f^
4*z^2 + 960*A^2*a^2*b^4*c^4*d^3*f^5*z^2 + 282*A^2*a^2*b^6*c^2*d^2*f^6*z^2 -
 45*B^2*a^2*b^2*c^6*d^2*e^6*z^2 - 48*A^2*b*c^9*d^6*e*f*z^2 - 14*A^2*a*b^9*d
*e*f^6*z^2 - 7*A*B*b^10*d^2*e*f^5*z^2 + 2*A*B*b^10*d*e^3*f^4*z^2 - 64*A*B*a
^7*c^3*e*f^7*z^2 - 16*A*B*b^9*c*d^3*f^5*z^2 + 8*A*B*a^4*c^6*e^7*f*z^2 + 4*A
*B*b*c^9*d^6*e^2*z^2 + 2*A*B*b^6*c^4*d*e^7*z^2 - 120*A*B*a^3*c^7*d*e^7*z^2
- 16*A*B*a^3*b^7*d*f^7*z^2 + 16*A*B*a*b^9*d^2*f^6*z^2 + 8*A*B*a*c^9*d^5*e^3
*z^2 + 12*A*B*a^3*b*c^6*e^8*z^2 - 48*B^2*b^5*c^5*d^5*e*f^2*z^2 + 15*B^2*b^4
*c^6*d^5*e^2*f*z^2 - 14*B^2*b^7*c^3*d^4*e*f^3*z^2 + 4*B^2*b^9*c*d^2*e^3*f^3
*z^2 + 4*B^2*b^7*c^3*d^2*e^5*f*z^2 + 4*B^2*b^5*c^5*d^4*e^3*f*z^2 - B^2*b^6*
c^4*d^3*e^4*f*z^2 - 336*B^2*a^3*c^7*d^3*e^4*f*z^2 + 112*B^2*a^5*c^5*d*e^4*f
^3*z^2 - 112*A^2*b^3*c^7*d^5*e*f^2*z^2 + 80*B^2*a^6*c^4*d*e^2*f^5*z^2 - 48*
A^2*b^5*c^5*d^4*e*f^3*z^2 + 36*A^2*b^8*c^2*d*e^4*f^3*z^2 + 36*A^2*b^3*c^7*d
^4*e^3*f*z^2 - 28*A^2*b^7*c^3*d*e^5*f^2*z^2 + 20*A^2*b^2*c^8*d^5*e^2*f*z^2
+ 16*B^2*a^2*c^8*d^5*e^2*f*z^2 - 14*A^2*b^7*c^3*d^3*e*f^4*z^2 - 14*A^2*b^4*
c^6*d^3*e^4*f*z^2 - 10*A^2*b^5*c^5*d^2*e^5*f*z^2 - 1008*A^2*a^4*c^6*d*e^4*f
^3*z^2 - 760*A^2*a^5*c^5*d*e^2*f^5*z^2 + 272*A^2*a^2*c^8*d^3*e^4*f*z^2 + 48
*B^2*a^5*b*c^4*e^5*f^3*z^2 + 36*B^2*a^6*b*c^3*e^3*f^5*z^2 + 12*B^2*a^5*b^4*
c*e^2*f^6*z^2 - 624*A^2*a^4*b*c^5*e^5*f^3*z^2 - 548*A^2*a^5*b*c^4*e^3*f^5*z
^2 + 182*A^2*a^2*b^3*c^5*e^7*f*z^2 - 180*B^2*a*b^4*c^5*d^5*f^3*z^2 + 132*B^
2*a^6*b^2*c^2*d*f^7*z^2 + 108*B^2*a^3*b^6*c*d^2*f^6*z^2 + 96*A^2*a^5*b^3*c^
2*e*f^7*z^2 + 68*A^2*a*b^6*c^3*e^6*f^2*z^2 + 58*A^2*a^3*b^6*c*e^2*f^6*z^2 -
 56*B^2*a*b^2*c^7*d^6*f^2*z^2 - 38*A^2*a^2*b^7*c*e^3*f^5*z^2 - 36*A^2*a*b^7
*c^2*e^5*f^3*z^2 + 20*B^2*a*b^6*c^3*d^4*f^4*z^2 - 736*A^2*a^5*b^2*c^3*d*f^7
*z^2 + 624*A^2*a^4*b^4*c^2*d*f^7*z^2 - 416*A^2*a*b^2*c^7*d^5*f^3*z^2 - 276*
A^2*a*b^4*c^5*d^4*f^4*z^2 - 196*A^2*a*b^6*c^3*d^3*f^5*z^2 + 8*B^2*a*b^4*c^5
*d^2*e^6*z^2 + 6*B^2*a*b^2*c^7*d^4*e^4*z^2 + 2*B^2*a^2*b^3*c^5*d*e^7*z^2 +
2*B^2*a*b^3*c^6*d^3*e^5*z^2 - 18*A^2*a*b^2*c^7*d^2*e^6*z^2 - 16*A*B*b*c^9*d
^7*f*z^2 - B^2*b^10*d^2*e^2*f^4*z^2 + 48*B^2*a^7*c^3*e^2*f^6*z^2 - 36*B^2*a
^6*c^4*e^4*f^4*z^2 + 31*B^2*b^6*c^4*d^5*f^3*z^2 - 24*B^2*a^5*c^5*e^6*f^2*z^
2 + 20*B^2*b^4*c^6*d^6*f^2*z^2 - 6*A^2*b^8*c^2*e^6*f^2*z^2 + 2*B^2*b^8*c^2*
d^4*f^4*z^2 - 768*B^2*a^5*c^5*d^3*f^5*z^2 + 512*B^2*a^6*c^4*d^2*f^6*z^2 + 5
12*B^2*a^4*c^6*d^4*f^4*z^2 + 232*A^2*a^5*c^5*e^4*f^4*z^2 + 188*A^2*a^4*c^6*
e^6*f^2*z^2 - 128*B^2*a^3*c^7*d^5*f^3*z^2 + 92*A^2*a^6*c^4*e^2*f^6*z^2 + 80
*A^2*b^4*c^6*d^5*f^3*z^2 + 64*A^2*b^2*c^8*d^6*f^2*z^2 + 31*A^2*b^6*c^4*d^4*
f^4*z^2 + 14*A^2*b^8*c^2*d^3*f^5*z^2 - 5*B^2*b^4*c^6*d^4*e^4*z^2 + 4*B^2*b^
3*c^7*d^5*e^3*z^2 + 2*B^2*b^5*c^5*d^3*e^5*z^2 - B^2*b^6*c^4*d^2*e^6*z^2 - B
^2*b^2*c^8*d^6*e^2*z^2 - B^2*a^4*b^6*e^2*f^6*z^2 - 1152*A^2*a^3*c^7*d^4*f^4
*z^2 + 1008*A^2*a^4*c^6*d^3*f^5*z^2 + 624*A^2*a^2*c^8*d^5*f^3*z^2 - 288*A^2
*a^5*c^5*d^2*f^6*z^2 + 56*B^2*a^3*c^7*d^2*e^6*z^2 - 10*B^2*a^2*b^8*d^2*f^6*
z^2 - 9*A^2*b^2*c^8*d^4*e^4*z^2 - 5*A^2*a^2*b^8*e^2*f^6*z^2 - 4*B^2*a^2*c^8
*d^4*e^4*z^2 + 3*A^2*b^4*c^6*d^2*e^6*z^2 - 2*A^2*b^3*c^7*d^3*e^5*z^2 - 36*A
^2*a^2*c^8*d^2*e^6*z^2 - 48*A^2*a^6*b^2*c^2*f^8*z^2 - 45*A^2*a^2*b^2*c^6*e^
8*z^2 + 4*A^2*b^10*d*e^2*f^5*z^2 + 4*B^2*b^2*c^8*d^7*f*z^2 + 4*A^2*b^9*c*e^
5*f^3*z^2 + 4*A^2*b^7*c^3*e^7*f*z^2 - 128*B^2*a^7*c^3*d*f^7*z^2 - 160*A^2*a

*c^9*d^6*f^2*z^2 - 112*A^2*a^6*c^4*d*f^7*z^2 + 12*A^2*b*c^9*d^5*e^3*z^2 + 4
*A^2*a*b^9*e^3*f^5*z^2 + 3*B^2*a^4*b^6*d*f^7*z^2 + 2*A^2*a^3*b^7*e*f^7*z^2
- 24*A^2*a*c^9*d^4*e^4*z^2 + 14*A^2*a^2*b^8*d*f^7*z^2 + 12*A^2*a^5*b^4*c*f^
8*z^2 + 12*A^2*a*b^4*c^5*e^8*z^2 + A*B*a^4*b^6*e*f^7*z^2 + B^2*a^2*b^8*d*e^
2*f^5*z^2 + 16*A^2*c^10*d^7*f*z^2 + 3*B^2*b^10*d^3*f^5*z^2 - A^2*b^10*e^4*f
^4*z^2 - 4*A^2*c^10*d^6*e^2*z^2 - A^2*b^10*d^2*f^6*z^2 + 64*A^2*a^7*c^3*f^8
*z^2 - 4*B^2*a^4*c^6*e^8*z^2 - A^2*b^6*c^4*e^8*z^2 + 48*A^2*a^3*c^7*e^8*z^2
- A^2*a^4*b^6*f^8*z^2 + 720*A^2*B*a*b^2*c^5*d^2*e^2*f^3*z - 600*A^2*B*a^2*
b^2*c^4*d*e^2*f^4*z + 576*A*B^2*a^2*b^2*c^4*d^2*e*f^4*z + 348*A*B^2*a*b^2*c
^5*d^2*e^3*f^2*z - 336*A*B^2*a^2*b*c^5*d^2*e^2*f^3*z - 260*A*B^2*a*b^3*c^4*
d^2*e^2*f^3*z - 240*A*B^2*a^2*b^2*c^4*d*e^3*f^3*z + 196*A*B^2*a^2*b^3*c^3*d
*e^2*f^4*z + 172*A^2*B*a*b*c^6*d*e^5*f*z + 20*A*B^2*a*b^6*c*d*e*f^5*z - 912
*A^2*B*a^2*b*c^5*d^2*e*f^4*z - 644*A^2*B*a*b*c^6*d^2*e^3*f^2*z - 432*A*B^2*
a*b^2*c^5*d^3*e*f^3*z + 372*A^2*B*a^2*b*c^5*d*e^3*f^3*z - 330*A^2*B*a*b^2*c
^5*d*e^4*f^2*z + 312*A*B^2*a*b*c^6*d^3*e^2*f^2*z - 208*A*B^2*a^3*b^2*c^3*d*
e*f^5*z + 192*A^2*B*a^2*b^3*c^3*d*e*f^5*z + 172*A^2*B*a*b^3*c^4*d*e^3*f^3*z
+ 108*A*B^2*a^2*b*c^5*d*e^4*f^2*z + 104*A*B^2*a^3*b*c^4*d*e^2*f^4*z - 80*A
^2*B*a*b^3*c^4*d^2*e*f^4*z + 68*A^2*B*a*b^4*c^3*d*e^2*f^4*z - 60*A*B^2*a*b^
5*c^2*d*e^2*f^4*z + 58*A*B^2*a*b^3*c^4*d*e^4*f^2*z - 36*A*B^2*a*b^4*c^3*d^2
*e*f^4*z - 24*A*B^2*a^2*b^4*c^2*d*e*f^5*z + 24*A*B^2*a*b^4*c^3*d*e^3*f^3*z
+ 592*A^2*B*a*b*c^6*d^3*e*f^3*z + 240*A^2*B*a^3*b*c^4*d*e*f^5*z - 132*A*B^2
*a*b*c^6*d^2*e^4*f*z - 60*A*B^2*a*b^2*c^5*d*e^5*f*z - 48*A^2*B*a*b^5*c^2*d*
e*f^5*z + 20*B^3*a*b*c^6*d^3*e^3*f*z + 16*B^3*a^4*b*c^3*d*e*f^5*z - 16*B^3*
a*b*c^6*d^4*e*f^2*z + 12*B^3*a^2*b*c^5*d*e^5*f*z + 320*A^3*a*b*c^6*d*e^4*f^
2*z + 40*A^3*a*b^4*c^3*d*e*f^5*z - 48*A^2*B*b*c^7*d^4*e*f^2*z - 44*A^2*B*b^
3*c^5*d*e^5*f*z - 20*A*B^2*b*c^7*d^4*e^2*f*z + 14*A*B^2*b^4*c^4*d*e^5*f*z +
12*A^2*B*b*c^7*d^3*e^3*f*z + 4*A*B^2*b^7*c*d*e^2*f^4*z + 160*A*B^2*a^4*c^4
*d*e*f^5*z + 152*A^2*B*a*c^7*d^2*e^4*f*z - 40*A*B^2*a*c^7*d^3*e^3*f*z + 32*
A*B^2*a*c^7*d^4*e*f^2*z - 16*A*B^2*a^2*c^6*d*e^5*f*z + 128*A^2*B*a^4*b*c^3*
e*f^6*z + 42*A^2*B*a*b^2*c^5*e^6*f*z + 24*A^2*B*a^2*b^5*c*e*f^6*z - 12*A*B^
2*a^3*b^4*c*e*f^6*z - 12*A*B^2*a^2*b*c^5*e^6*f*z - 10*A^2*B*a*b^6*c*e^2*f^5
*z - 160*A*B^2*a*b*c^6*d^4*f^3*z + 112*A*B^2*a^4*b*c^3*d*f^6*z - 24*A*B^2*a
^2*b^5*c*d*f^6*z - 84*B^3*a*b^2*c^5*d^3*e^2*f^2*z - 80*B^3*a^2*b^3*c^3*d^2*
e*f^4*z - 60*B^3*a^2*b*c^5*d^2*e^3*f^2*z - 20*B^3*a^3*b^2*c^3*d*e^2*f^4*z -
20*B^3*a*b^3*c^4*d^2*e^3*f^2*z - 9*B^3*a^2*b^2*c^4*d*e^4*f^2*z - 8*B^3*a*b
^4*c^3*d^2*e^2*f^3*z + 6*B^3*a^2*b^4*c^2*d*e^2*f^4*z - 4*B^3*a^2*b^3*c^3*d*
e^3*f^3*z - 216*A^2*B*b^4*c^4*d^2*e^2*f^3*z + 196*A^2*B*b^3*c^5*d^2*e^3*f^2
*z - 108*A*B^2*b^3*c^5*d^3*e^2*f^2*z - 94*A*B^2*b^4*c^4*d^2*e^3*f^2*z + 88*
A^2*B*b^2*c^6*d^3*e^2*f^2*z + 80*A*B^2*b^5*c^3*d^2*e^2*f^3*z + 360*A^2*B*a^
2*c^6*d^2*e^2*f^3*z + 8*A*B^2*a^2*c^6*d^2*e^3*f^2*z + 153*A^2*B*a^2*b^2*c^4
*e^4*f^3*z - 144*A^2*B*a^2*b^3*c^3*e^3*f^4*z + 80*A^2*B*a^3*b^2*c^3*e^2*f^5
*z + 36*A*B^2*a^3*b^2*c^3*e^3*f^4*z + 12*A^2*B*a^2*b^4*c^2*e^2*f^5*z + 12*A
*B^2*a^3*b^3*c^2*e^2*f^5*z + 9*A*B^2*a^2*b^2*c^4*e^5*f^2*z - 6*A*B^2*a^2*b^
4*c^2*e^3*f^4*z + 4*A*B^2*a^2*b^3*c^3*e^4*f^3*z + 480*A^2*B*a^2*b^2*c^4*d^2
*f^5*z - 176*A*B^2*a^2*b^3*c^3*d^2*f^5*z - 10*A^2*B*a*b^6*c*d*f^6*z + 16*A*

B^2*a*b*c^6*d*e^6*z + 80*B^3*a*b^3*c^4*d^3*e*f^3*z - 48*B^3*a^3*b*c^4*d^2*e
*f^4*z + 48*B^3*a^2*b*c^5*d^3*e*f^3*z + 44*B^3*a^3*b*c^4*d*e^3*f^3*z + 24*B
^3*a*b^5*c^2*d^2*e*f^4*z + 18*B^3*a*b^2*c^5*d^2*e^4*f*z + 696*A^3*a^2*b*c^5
*d*e^2*f^4*z - 504*A^3*a*b*c^6*d^2*e^2*f^3*z - 192*A^3*a*b^2*c^5*d*e^3*f^3*
z - 144*A^3*a^2*b^2*c^4*d*e*f^5*z + 96*A^3*a*b^2*c^5*d^2*e*f^4*z - 72*A^3*a
*b^3*c^4*d*e^2*f^4*z - 208*A^2*B*b^3*c^5*d^3*e*f^3*z + 152*A*B^2*b^4*c^4*d^
3*e*f^3*z + 80*A^2*B*b^5*c^3*d^2*e*f^4*z + 75*A^2*B*b^4*c^4*d*e^4*f^2*z - 5
9*A^2*B*b^2*c^6*d^2*e^4*f*z - 52*A^2*B*b^5*c^3*d*e^3*f^3*z + 42*A*B^2*b^3*c
^5*d^2*e^4*f*z - 21*A*B^2*b^6*c^2*d^2*e*f^4*z - 16*A*B^2*b^5*c^3*d*e^4*f^2*
z + 16*A*B^2*b^2*c^6*d^4*e*f^2*z + 16*A*B^2*b^2*c^6*d^3*e^3*f*z + 11*A^2*B*
b^6*c^2*d*e^2*f^4*z + 4*A*B^2*b^6*c^2*d*e^3*f^3*z - 256*A^2*B*a*c^7*d^3*e^2
*f^2*z - 96*A*B^2*a^3*c^5*d^2*e*f^4*z - 36*A^2*B*a^2*c^6*d*e^4*f^2*z - 32*A
^2*B*a^3*c^5*d*e^2*f^4*z - 32*A*B^2*a^2*c^6*d^3*e*f^3*z + 8*A*B^2*a^3*c^5*d
*e^3*f^3*z - 96*A^2*B*a^3*b^3*c^2*e*f^6*z + 68*A^2*B*a^3*b*c^4*e^3*f^4*z -
60*A*B^2*a^4*b*c^3*e^2*f^5*z - 60*A*B^2*a^3*b*c^4*e^4*f^3*z + 48*A*B^2*a^4*
b^2*c^2*e*f^6*z - 38*A^2*B*a*b^3*c^4*e^5*f^2*z - 36*A^2*B*a^2*b*c^5*e^5*f^2
*z + 36*A^2*B*a*b^5*c^2*e^3*f^4*z - 16*A^2*B*a*b^4*c^3*e^4*f^3*z + 384*A*B^
2*a^2*b*c^5*d^3*f^4*z - 352*A*B^2*a^3*b*c^4*d^2*f^5*z - 288*A^2*B*a*b^2*c^5
*d^3*f^4*z - 160*A^2*B*a^3*b^2*c^3*d*f^6*z - 148*A^2*B*a*b^4*c^3*d^2*f^5*z
+ 112*A*B^2*a*b^3*c^4*d^3*f^4*z + 72*A^2*B*a^2*b^4*c^2*d*f^6*z + 72*A*B^2*a
*b^5*c^2*d^2*f^5*z + 48*A*B^2*a^3*b^3*c^2*d*f^6*z + 102*B^3*a^2*b^2*c^4*d^2
*e^2*f^3*z - 32*B^3*b^5*c^3*d^3*e*f^3*z - 8*B^3*b^3*c^5*d^3*e^3*f*z - 7*B^3
*b^4*c^4*d^2*e^4*f*z + 5*B^3*b^2*c^6*d^4*e^2*f*z + 80*A^3*b^2*c^6*d^3*e*f^3
*z - 74*A^3*b^3*c^5*d*e^4*f^2*z - 64*A^3*b^4*c^4*d^2*e*f^4*z + 60*A^3*b^4*c
^4*d*e^3*f^3*z - 48*B^3*a^4*c^4*d*e^2*f^4*z - 24*B^3*a^3*c^5*d*e^4*f^2*z +
20*B^3*a^2*c^6*d^2*e^4*f*z - 16*A^3*b^5*c^3*d*e^2*f^4*z + 8*A^3*b*c^7*d^3*e
^2*f^2*z + 480*A^3*a^2*c^6*d^2*e*f^4*z - 392*A^3*a^2*c^6*d*e^3*f^3*z + 280*
A^3*a*c^7*d^2*e^3*f^2*z - 4*B^3*a^4*b*c^3*e^3*f^4*z - 200*A^3*a^3*b*c^4*e^2
*f^5*z - 144*A^3*a^2*b*c^5*e^4*f^3*z + 48*B^3*a*b^2*c^5*d^4*f^3*z + 42*A^3*
a*b^2*c^5*e^5*f^2*z - 36*B^3*a^4*b^2*c^2*d*f^6*z - 32*A^3*a^3*b^2*c^3*e*f^6
*z - 24*A^3*a^2*b^4*c^2*e*f^6*z - 24*A^3*a*b^5*c^2*e^2*f^5*z + 10*A^3*a*b^3
*c^4*e^4*f^3*z - 4*B^3*a*b^4*c^3*d^3*f^4*z - 4*A^3*a*b^4*c^3*e^3*f^4*z - 48
0*A^3*a^2*b*c^5*d^2*f^5*z - 160*A^3*a^2*b^3*c^3*d*f^6*z + 128*A^3*a*b^3*c^4
*d^2*f^5*z + 8*A^2*B*b^5*c^3*e^5*f^2*z - 2*A^2*B*b^6*c^2*e^4*f^3*z + 112*A^
2*B*b^4*c^4*d^3*f^4*z - 92*A^2*B*a^4*c^4*e^2*f^5*z - 64*A^2*B*a^3*c^5*e^4*f
^3*z - 64*A*B^2*b^5*c^3*d^3*f^4*z + 24*A*B^2*a^4*c^4*e^3*f^4*z + 24*A*B^2*a
^3*c^5*e^5*f^2*z + 16*A^2*B*b^2*c^6*d^4*f^3*z + 16*A*B^2*b^3*c^5*d^4*f^3*z
- A^2*B*b^6*c^2*d^2*f^5*z + 448*A^2*B*a^3*c^5*d^2*f^5*z - 352*A^2*B*a^2*c^6
*d^3*f^4*z - 5*A*B^2*b^2*c^6*d^2*e^5*z - 48*A^2*B*a^4*b^2*c^2*f^7*z - 2*B^3
*b^7*c*d^2*e*f^4*z + 34*A^3*b^2*c^6*d*e^5*f*z + 16*A^3*b*c^7*d^2*e^4*f*z +
2*A^3*b^6*c^2*d*e*f^5*z - 416*A^3*a^3*c^5*d*e*f^5*z - 224*A^3*a*c^7*d^3*e*f
^3*z + 12*B^3*a^3*b^4*c*d*f^6*z - 10*B^3*a*b^6*c*d^2*f^5*z + 416*A^3*a^3*b*
c^4*d*f^6*z + 224*A^3*a*b*c^6*d^3*f^4*z + 24*A^3*a*b^5*c^2*d*f^6*z - 4*B^3*
a*b*c^6*d^2*e^5*z + 20*A^2*B*c^8*d^4*e^2*f*z - 7*A^2*B*b^4*c^4*e^6*f*z - 2*
A^2*B*b^7*c*e^3*f^4*z - 64*A*B^2*a^5*c^3*e*f^6*z + 16*A*B^2*b*c^7*d^5*f^2*z

 - 8*A^2*B*a^2*c^6*e^6*f*z - 2*A*B^2*b^7*c*d^2*f^5*z - 272*A^2*B*a^4*c^4*d*
f^6*z + 128*A^2*B*a*c^7*d^4*f^3*z + 9*A^2*B*b^2*c^6*d*e^6*z - 4*A*B^2*b^3*c
^5*d*e^6*z + 4*A*B^2*b*c^7*d^3*e^4*z + 8*A*B^2*a*c^7*d^2*e^5*z + 12*A^2*B*a
^3*b^4*c*f^7*z + 30*B^3*b^4*c^4*d^3*e^2*f^2*z + 8*B^3*b^5*c^3*d^2*e^3*f^2*z
 - 2*B^3*b^6*c^2*d^2*e^2*f^3*z + 152*A^3*b^3*c^5*d^2*e^2*f^3*z - 108*A^3*b^
2*c^6*d^2*e^3*f^2*z + 48*B^3*a^3*c^5*d^2*e^2*f^3*z - 16*B^3*a^2*c^6*d^3*e^2
*f^2*z - 3*B^3*a^4*b^2*c^2*e^2*f^5*z - 120*B^3*a^2*b^2*c^4*d^3*f^4*z + 112*
B^3*a^3*b^2*c^3*d^2*f^5*z + 112*A^3*a^2*b^3*c^3*e^2*f^5*z + 12*A^3*a^2*b^2*
c^4*e^3*f^4*z - 120*A^3*a*c^7*d*e^5*f*z - 52*A^3*a*b*c^6*e^6*f*z + 10*A^3*a
*b^6*c*e*f^6*z - 2*A*B^2*b^8*d*e*f^5*z - 2*A^2*B*a*b^7*e*f^6*z - 24*A^2*B*a
*c^7*d*e^6*z + 2*A*B^2*a*b^7*d*f^6*z - 12*A^2*B*a*b*c^6*e^7*z - 2*A^3*b^7*c
*d*f^6*z - 4*A^3*b*c^7*d*e^6*z + 16*B^3*a^5*c^3*e^2*f^5*z + 11*B^3*b^6*c^2*
d^3*f^4*z - 11*A^3*b^4*c^4*e^5*f^2*z - 8*B^3*b^4*c^4*d^4*f^3*z - 4*B^3*b^2*
c^6*d^5*f^2*z + 4*B^3*a^4*c^4*e^4*f^3*z + 4*A^3*b^5*c^3*e^4*f^3*z - A^3*b^6
*c^2*e^3*f^4*z + 136*A^3*a^3*c^5*e^3*f^4*z + 68*A^3*a^2*c^6*e^5*f^2*z - 64*
A^3*b^3*c^5*d^3*f^4*z + 2*B^3*b^3*c^5*d^2*e^5*z - B^3*b^2*c^6*d^3*e^4*z + 9
6*A^3*a^3*b^3*c^2*f^7*z + A*B^2*a^2*b^6*e*f^6*z + 32*A^3*c^8*d^4*e*f^2*z -
24*A^3*c^8*d^3*e^3*f*z + 10*A^3*b^3*c^5*e^6*f*z + 2*A^3*b^7*c*e^2*f^5*z + 1
28*A^3*a^4*c^4*e*f^6*z - 32*A^3*b*c^7*d^4*f^3*z - 4*B^3*a^2*c^6*d*e^6*z - B
^3*a^2*b^6*d*f^6*z - 128*A^3*a^4*b*c^3*f^7*z - 24*A^3*a^2*b^5*c*f^7*z - 16*
A^2*B*c^8*d^5*f^2*z - 4*A^2*B*c^8*d^3*e^4*z + 64*A^2*B*a^5*c^3*f^7*z + 2*A^
2*B*b^3*c^5*e^7*z + 4*A*B^2*a^2*c^6*e^7*z - A^2*B*a^2*b^6*f^7*z + 4*A^3*c^8
*d^2*e^5*z - 3*A^3*b^2*c^6*e^7*z + A^2*B*b^8*d*f^6*z - A^3*b^8*e*f^6*z + 16
*A^3*a*c^7*e^7*z + 2*A^3*a*b^7*f^7*z + A^2*B*b^8*e^2*f^5*z + B^3*b^8*d^2*f^
5*z - 48*A^2*B^2*a*b*c^4*d*e*f^4 + 28*A*B^3*a*b^2*c^3*d*e*f^4 - 16*A*B^3*a*
b*c^4*d*e^2*f^3 + 16*A^3*B*a*c^5*d*e*f^4 + 32*A^3*B*a*b*c^4*d*f^5 + 12*A^2*
B^2*b^3*c^3*d*e*f^4 + 5*A*B^3*b^2*c^4*d^2*e*f^3 + 4*A*B^3*b^3*c^3*d*e^2*f^3
 + 24*A^2*B^2*a*c^5*d*e^2*f^3 + 24*A^2*B^2*a^2*b*c^3*e*f^5 + 12*A^2*B^2*a*b
*c^4*e^3*f^3 - 6*A^2*B^2*a*b^3*c^2*e*f^5 + 4*A*B^3*a^2*b*c^3*e^2*f^4 + 3*A*
B^3*a^2*b^2*c^2*e*f^5 - 18*A^2*B^2*a*b^2*c^3*d*f^5 - 4*B^4*a^2*b*c^3*d*e*f^
4 + 4*B^4*a*b*c^4*d^2*e*f^3 - 6*A*B^3*b^4*c^2*d*e*f^4 + 4*A^3*B*b*c^5*d*e^2
*f^3 - 2*A^3*B*b^2*c^4*d*e*f^4 - 8*A*B^3*a^2*c^4*d*e*f^4 - 8*A*B^3*a*c^5*d^
2*e*f^3 + 26*A^3*B*a*b^2*c^3*e*f^5 + 8*A^3*B*a*b*c^4*e^2*f^4 + 32*A*B^3*a*b
*c^4*d^2*f^4 - 28*A*B^3*a^2*b*c^3*d*f^5 + 6*A*B^3*a*b^3*c^2*d*f^5 - 9*A^2*B
^2*b^2*c^4*d*e^2*f^3 - 18*A^2*B^2*a*b^2*c^3*e^2*f^4 - 4*A^3*B*c^6*d^2*e*f^3
 - 3*A^3*B*b^4*c^2*e*f^5 - 44*A^3*B*a^2*c^4*e*f^5 - 16*A^3*B*a*c^5*e^3*f^3
- 16*A*B^3*a^3*c^3*e*f^5 - 10*A^3*B*b^3*c^3*d*f^5 - 4*A^3*B*b*c^5*d^2*f^4 -
 4*A*B^3*b*c^5*d^3*f^3 - 28*A^3*B*a^2*b*c^3*f^6 + 6*A^3*B*a*b^3*c^2*f^6 - 4
*A^4*b*c^5*d*e*f^4 - 20*A^4*a*b*c^4*e*f^5 + 3*A^2*B^2*b^4*c^2*e^2*f^4 - 2*A
^2*B^2*b^3*c^3*e^3*f^3 + 12*A^2*B^2*a^2*c^4*e^2*f^4 + 9*A^2*B^2*b^2*c^4*d^2
*f^4 - 3*A^2*B^2*a^2*b^2*c^2*f^6 - 2*B^4*b^3*c^3*d^2*e*f^3 + 4*B^4*a^2*c^4*
d*e^2*f^3 - 10*B^4*a*b^2*c^3*d^2*f^4 - 3*B^4*a^2*b^2*c^2*d*f^5 + 3*A^3*B*b^
2*c^4*e^3*f^3 - 2*A^3*B*b^3*c^3*e^2*f^4 - 10*A*B^3*b^3*c^3*d^2*f^4 - 4*A*B^
3*a^2*c^4*e^3*f^3 + 3*A^2*B^2*b^4*c^2*d*f^5 + 36*A^2*B^2*a^2*c^4*d*f^5 - 24
*A^2*B^2*a*c^5*d^2*f^4 + 4*A^2*B^2*c^6*d^3*f^3 + 16*A^2*B^2*a^3*c^3*f^6 + 4

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*A^4*b^3*c^3*e*f^5 + 16*B^4*a^3*c^3*d*f^5 + 16*A^4*a*c^5*e^2*f^4 + 8*A^4*b^
2*c^4*d*f^5 - 8*A^4*a*b^2*c^3*f^6 - 24*A^4*a*c^5*d*f^5 + 3*B^4*b^4*c^2*d^2*
f^4 - 3*A^4*b^2*c^4*e^2*f^4 + 4*A^4*c^6*d^2*f^4 + 36*A^4*a^2*c^4*f^6 + B^4*
b^2*c^4*d^3*f^3, z, k), k, 1, 4) - ((A*b^3*f + 2*A*a*c^2*e + A*b*c^2*d - 2*
B*a*c^2*d - A*b^2*c*e - B*a*b^2*f + 2*B*a^2*c*f - 3*A*a*b*c*f + B*a*b*c*e)/
(a^2*b^2*f^2 - 4*a^3*c*f^2 - 4*a*c^3*d^2 - 4*a^2*c^2*e^2 + b^2*c^2*d^2 + b^
4*d*f - a*b^3*e*f - b^3*c*d*e + a*b^2*c*e^2 + 8*a^2*c^2*d*f + 4*a*b*c^2*d*e
 - 6*a*b^2*c*d*f + 4*a^2*b*c*e*f) - (x*(2*A*a*c^2*f - 2*A*c^3*d + A*b*c^2*e
 - 2*B*a*c^2*e + B*b*c^2*d - A*b^2*c*f + B*a*b*c*f))/(a^2*b^2*f^2 - 4*a^3*c
*f^2 - 4*a*c^3*d^2 - 4*a^2*c^2*e^2 + b^2*c^2*d^2 + b^4*d*f - a*b^3*e*f - b^
3*c*d*e + a*b^2*c*e^2 + 8*a^2*c^2*d*f + 4*a*b*c^2*d*e - 6*a*b^2*c*d*f + 4*a
^2*b*c*e*f))/(a + b*x + c*x^2)
```

3.17 $\int \frac{g+hx}{(a+bx+cx^2)(ad+bdx+cdx^2)^2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 140

$$\int \frac{g+hx}{(a+bx+cx^2)(ad+bdx+cdx^2)^2}\,dx = -\frac{bg-2ah+(2cg-bh)x}{2(b^2-4ac)d^2(a+bx+cx^2)^2} + \frac{3(2cg-bh)(b+2cx)}{2(b^2-4ac)^2d^2(a+bx+cx^2)} - \frac{6c(2cg-bh)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}d^2}$$

```
[Out] 1/2*(-b*g+2*a*h-(-b*h+2*c*g)*x)/(-4*a*c+b^2)/d^2/(c*x^2+b*x+a)^2+3/2*(-b*h+
2*c*g)*(2*c*x+b)/(-4*a*c+b^2)^2/d^2/(c*x^2+b*x+a)-6*c*(-b*h+2*c*g)*arctanh(
(2*c*x+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(5/2)/d^2
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {1012, 652, 628, 632, 212}

$$\int \frac{g+hx}{(a+bx+cx^2)(ad+bdx+cdx^2)^2}\,dx = -\frac{6c(2cg-bh)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{d^2(b^2-4ac)^{5/2}} + \frac{3(b+2cx)(2cg-bh)}{2d^2(b^2-4ac)^2(a+bx+cx^2)} - \frac{-2ah+x(2cg-bh)+bg}{2d^2(b^2-4ac)(a+bx+cx^2)^2}$$

```
[In] Int[(g + h*x)/((a + b*x + c*x^2)*(a*d + b*d*x + c*d*x^2)^2),x]
```

```
[Out] -1/2*(b*g - 2*a*h + (2*c*g - b*h)*x)/((b^2 - 4*a*c)*d^2*(a + b*x + c*x^2)^2
) + (3*(2*c*g - b*h)*(b + 2*c*x))/(2*(b^2 - 4*a*c)^2*d^2*(a + b*x + c*x^2))
 - (6*c*(2*c*g - b*h)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/((b^2 - 4*a*c
)^(5/2)*d^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1012

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_
) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Dist[(c/f)^p, Int[(g + h*
x)^m*(d + e*x + f*x^2)^(p + q), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, p,
 q}, x] && EqQ[c*d - a*f, 0] && EqQ[b*d - a*e, 0] && (IntegerQ[p] || GtQ[c/
f, 0]) && ( !IntegerQ[q] || LeafCount[d + e*x + f*x^2] <= LeafCount[a + b*x
 + c*x^2])
```

Rubi steps

$$\text{integral} = \frac{\int \frac{g+hx}{(a+bx+cx^2)^3}\,dx}{d^2}$$

$$= -\frac{bg-2ah+(2cg-bh)x}{2\left(b^2-4ac\right)d^2\left(a+bx+cx^2\right)^2}-\frac{(3(2cg-bh))\int\frac{1}{(a+bx+cx^2)^2}\,dx}{2\left(b^2-4ac\right)d^2}$$

$$= -\frac{bg-2ah+(2cg-bh)x}{2\left(b^2-4ac\right)d^2\left(a+bx+cx^2\right)^2}+\frac{3(2cg-bh)(b+2cx)}{2\left(b^2-4ac\right)^2d^2\left(a+bx+cx^2\right)}+\frac{(3c(2cg-bh))\int\frac{1}{a+bx+cx^2}\,dx}{\left(b^2-4ac\right)^2d^2}$$

$$= -\frac{bg-2ah+(2cg-bh)x}{2\left(b^2-4ac\right)d^2\left(a+bx+cx^2\right)^2}+\frac{3(2cg-bh)(b+2cx)}{2\left(b^2-4ac\right)^2d^2\left(a+bx+cx^2\right)}-\frac{(6c(2cg-bh))\text{Subst}\left(\int\frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx\right)}{\left(b^2-4ac\right)^2d^2}$$

$$= -\frac{bg-2ah+(2cg-bh)x}{2\left(b^2-4ac\right)d^2\left(a+bx+cx^2\right)^2}+\frac{3(2cg-bh)(b+2cx)}{2\left(b^2-4ac\right)^2d^2\left(a+bx+cx^2\right)}-\frac{6c(2cg-bh)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{5/2}d^2}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.94

$$\int\frac{g+hx}{\left(a+bx+cx^2\right)\left(ad+bdx+cdx^2\right)^2}\,dx$$
$$=\frac{\frac{(b^2-4ac)(-bg+2ah-2cgx+bhx)}{(a+x(b+cx))^2}+\frac{3(2cg-bh)(b+2cx)}{a+x(b+cx)}-\frac{12c(-2cg+bh)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}}{2\left(b^2-4ac\right)^2d^2}$$

```
[In] Integrate[(g + h*x)/((a + b*x + c*x^2)*(a*d + b*d*x + c*d*x^2)^2),x]
```

```
[Out] (((b^2 - 4*a*c)*(-(b*g) + 2*a*h - 2*c*g*x + b*h*x))/(a + x*(b + c*x))^2 + (
3*(2*c*g - b*h)*(b + 2*c*x))/(a + x*(b + c*x)) - (12*c*(-2*c*g + b*h)*ArcTa
n[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c])/(2*(b^2 - 4*a*c)^2*d
^2)
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.01

method	result
default	$\frac{\frac{bg-2ah+(-bh+2cg)x}{2\left(4ac-b^{2}\right)\left(c\,x^{2}+bx+a\right)^{2}}+\frac{3(-bh+2cg)\left(\frac{2cx+b}{\left(4ac-b^{2}\right)\left(c\,x^{2}+bx+a\right)}+\frac{4c\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^{2}}}\right)}{\left(4ac-b^{2}\right)^{\frac{3}{2}}}\right)}{2\left(4ac-b^{2}\right)}}{d^{2}}$
risch	$\frac{-\frac{3c^{2}(bh-2cg)x^{3}}{16a^{2}c^{2}-8a\,b^{2}c+b^{4}}-\frac{9bc(bh-2cg)x^{2}}{2\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}-\frac{\left(5abch-10a\,c^{2}g+b^{3}h-2b^{2}cg\right)x}{16a^{2}c^{2}-8a\,b^{2}c+b^{4}}-\frac{8a^{2}ch+a\,b^{2}h-10abcg+b^{3}g}{2\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}}{\left(c\,x^{2}+bx+a\right)^{2}d^{2}}-\frac{3c\ln\left(\left(32a^{2}c^{3}-16a\,b^{2}c^{2}+2b^{4}\right.\right.}{}$

```
[In] int((h*x+g)/(c*x^2+b*x+a)/(c*d*x^2+b*d*x+a*d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^2*(1/2*(b*g-2*a*h+(-b*h+2*c*g)*x)/(4*a*c-b^2)/(c*x^2+b*x+a)^2+3/2*(-b*h
+2*c*g)/(4*a*c-b^2)*((2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)+4*c/(4*a*c-b^2)^(3
/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 565 vs. 2(132) = 264.

Time = 0.35 (sec) , antiderivative size = 1150, normalized size of antiderivative = 8.21

$$\int \frac{g+hx}{(a+bx+cx^2)(ad+bdx+cdx^2)^2}\,dx = \text{Too large to display}$$

```
[In] integrate((h*x+g)/(c*x^2+b*x+a)/(c*d*x^2+b*d*x+a*d)^2,x, algorithm="fricas"
)
```

```
[Out] [1/2*(6*(2*(b^2*c^3 - 4*a*c^4)*g - (b^3*c^2 - 4*a*b*c^3)*h)*x^3 + 9*(2*(b^3
*c^2 - 4*a*b*c^3)*g - (b^4*c - 4*a*b^2*c^2)*h)*x^2 - 6*(2*a^2*c^2*g - a^2*b
*c*h + (2*c^4*g - b*c^3*h)*x^4 + 2*(2*b*c^3*g - b^2*c^2*h)*x^3 + (2*(b^2*c^
2 + 2*a*c^3)*g - (b^3*c + 2*a*b*c^2)*h)*x^2 + 2*(2*a*b*c^2*g - a*b^2*c*h)*x
)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a
*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - (b^5 - 14*a*b^3*c + 40*a^2*b*c^2)*g -
 (a*b^4 + 4*a^2*b^2*c - 32*a^3*c^2)*h + 2*(2*(b^4*c + a*b^2*c^2 - 20*a^2*c^
3)*g - (b^5 + a*b^3*c - 20*a^2*b*c^2)*h)*x)/((b^6*c^2 - 12*a*b^4*c^3 + 48*a
^2*b^2*c^4 - 64*a^3*c^5)*d^2*x^4 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3
 - 64*a^3*b*c^4)*d^2*x^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*
c^3 - 128*a^4*c^4)*d^2*x^2 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*
a^4*b*c^3)*d^2*x + (a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*d
^2), 1/2*(6*(2*(b^2*c^3 - 4*a*c^4)*g - (b^3*c^2 - 4*a*b*c^3)*h)*x^3 + 9*(2*
(b^3*c^2 - 4*a*b*c^3)*g - (b^4*c - 4*a*b^2*c^2)*h)*x^2 - 12*(2*a^2*c^2*g -
a^2*b*c*h + (2*c^4*g - b*c^3*h)*x^4 + 2*(2*b*c^3*g - b^2*c^2*h)*x^3 + (2*(b
```

```
^2*c^2 + 2*a*c^3)*g - (b^3*c + 2*a*b*c^2)*h)*x^2 + 2*(2*a*b*c^2*g - a*b^2*c
*h)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a
*c)) - (b^5 - 14*a*b^3*c + 40*a^2*b*c^2)*g - (a*b^4 + 4*a^2*b^2*c - 32*a^3*
c^2)*h + 2*(2*(b^4*c + a*b^2*c^2 - 20*a^2*c^3)*g - (b^5 + a*b^3*c - 20*a^2*
b*c^2)*h)*x)/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^2*x^
4 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d^2*x^3 + (b^8
 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d^2*x^2 + 2*
(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d^2*x + (a^2*b^6 - 1
2*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*d^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 709 vs. $2(133) = 266$.

Time = 1.09 (sec) , antiderivative size = 709, normalized size of antiderivative = 5.06

$$\int \frac{g + hx}{(a + bx + cx^2)(ad + bdx + cdx^2)^2}\,dx$$

$$= \frac{3c\sqrt{-\frac{1}{(4ac-b^2)^5}}(bh-2cg)\log\left(x + \frac{-192a^3c^4\sqrt{-\frac{1}{(4ac-b^2)^5}}(bh-2cg)+144a^2b^2c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}(bh-2cg)-36ab^4c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}}{6bc^2h-12c^3g}\right.}{d^2}$$

$$- \frac{3c\sqrt{-\frac{1}{(4ac-b^2)^5}}(bh-2cg)\log\left(x + \frac{192a^3c^4\sqrt{-\frac{1}{(4ac-b^2)^5}}(bh-2cg)-144a^2b^2c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}(bh-2cg)+36ab^4c^2\sqrt{-\frac{1}{(4ac-b^2)}}}{6bc^2h-12c^3g}\right.}{d^2}$$

$$+ \frac{-8a^2ch - ab^2h + 10abcg - b^3g + x^3(-6bc^2h + 12c^3g) + x^2(-9b^2ch}{32a^4c^2d^2 - 16a^3b^2cd^2 + 2a^2b^4d^2 + x^4\cdot(32a^2c^4d^2 - 16ab^2c^3d^2 + 2b^4c^2d^2) + x^3\cdot(64a^2bc^3d^2 - 32ab^3c^2d^2}$$

```
[In] integrate((h*x+g)/(c*x**2+b*x+a)/(c*d*x**2+b*d*x+a*d)**2,x)
```

```
[Out] 3*c*sqrt(-1/(4*a*c - b**2)**5)*(b*h - 2*c*g)*log(x + (-192*a**3*c**4*sqrt(-
1/(4*a*c - b**2)**5)*(b*h - 2*c*g) + 144*a**2*b**2*c**3*sqrt(-1/(4*a*c - b*
*2)**5)*(b*h - 2*c*g) - 36*a*b**4*c**2*sqrt(-1/(4*a*c - b**2)**5)*(b*h - 2*
c*g) + 3*b**6*c*sqrt(-1/(4*a*c - b**2)**5)*(b*h - 2*c*g) + 3*b**2*c*h - 6*b
*c**2*g)/(6*b*c**2*h - 12*c**3*g))/d**2 - 3*c*sqrt(-1/(4*a*c - b**2)**5)*(b
*h - 2*c*g)*log(x + (192*a**3*c**4*sqrt(-1/(4*a*c - b**2)**5)*(b*h - 2*c*g)
 - 144*a**2*b**2*c**3*sqrt(-1/(4*a*c - b**2)**5)*(b*h - 2*c*g) + 36*a*b**4*
c**2*sqrt(-1/(4*a*c - b**2)**5)*(b*h - 2*c*g) - 3*b**6*c*sqrt(-1/(4*a*c - b
**2)**5)*(b*h - 2*c*g) + 3*b**2*c*h - 6*b*c**2*g)/(6*b*c**2*h - 12*c**3*g))
/d**2 + (-8*a**2*c*h - a*b**2*h + 10*a*b*c*g - b**3*g + x**3*(-6*b*c**2*h +
 12*c**3*g) + x**2*(-9*b**2*c*h + 18*b*c**2*g) + x*(-10*a*b*c*h + 20*a*c**2
*g - 2*b**3*h + 4*b**2*c*g))/(32*a**4*c**2*d**2 - 16*a**3*b**2*c*d**2 + 2*a
**2*b**4*d**2 + x**4*(32*a**2*c**4*d**2 - 16*a*b**2*c**3*d**2 + 2*b**4*c**2
*d**2) + x**3*(64*a**2*b*c**3*d**2 - 32*a*b**3*c**2*d**2 + 4*b**5*c*d**2) +
```

```
 x**2*(64*a**3*c**3*d**2 - 12*a*b**4*c*d**2 + 2*b**6*d**2) + x*(64*a**3*b*c
**2*d**2 - 32*a**2*b**3*c*d**2 + 4*a*b**5*d**2))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{g+hx}{(a+bx+cx^2)(ad+bdx+cdx^2)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((h*x+g)/(c*x^2+b*x+a)/(c*d*x^2+b*d*x+a*d)^2,x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.56

$$\int \frac{g+hx}{(a+bx+cx^2)(ad+bdx+cdx^2)^2}\,dx = \frac{6\,(2\,c^2g-bch)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^4d^2-8\,ab^2cd^2+16\,a^2c^2d^2)\sqrt{-b^2+4\,ac}} + \frac{12\,c^3gx^3-6\,bc^2hx^3+18\,bc^2gx^2-9\,b^2chx^2+4\,b^2cgx+20\,ac^2gx-2\,b^3hx-10\,abchx-b^3g+10\,abcg-}{2\,(b^4d^2-8\,ab^2cd^2+16\,a^2c^2d^2)(cx^2+bx+a)^2}$$

```
[In] integrate((h*x+g)/(c*x^2+b*x+a)/(c*d*x^2+b*d*x+a*d)^2,x, algorithm="giac")
```

```
[Out] 6*(2*c^2*g - b*c*h)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^4*d^2 - 8*a*
b^2*c*d^2 + 16*a^2*c^2*d^2)*sqrt(-b^2 + 4*a*c)) + 1/2*(12*c^3*g*x^3 - 6*b*c
^2*h*x^3 + 18*b*c^2*g*x^2 - 9*b^2*c*h*x^2 + 4*b^2*c*g*x + 20*a*c^2*g*x - 2*
b^3*h*x - 10*a*b*c*h*x - b^3*g + 10*a*b*c*g - a*b^2*h - 8*a^2*c*h)/((b^4*d^
2 - 8*a*b^2*c*d^2 + 16*a^2*c^2*d^2)*(c*x^2 + b*x + a)^2)
```

Mupad [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 395, normalized size of antiderivative = 2.82

$$\int \frac{g+hx}{(a+bx+cx^2)(ad+bdx+cdx^2)^2}\,dx = \frac{6\,c\,\mathrm{atan}\left(\frac{d^2\left(\frac{6\,c^2\,x\,(b\,h-2\,c\,g)}{d^2\,\left(4\,a\,c-b^2\right)^{5/2}}+\frac{3\,c\,(b\,h-2\,c\,g)\,\left(16\,a^2\,b\,c^2\,d^2-8\,a\,b^3\,c\,d^2+b^5\,d^2\right)}{d^4\,\left(4\,a\,c-b^2\right)^{5/2}\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}\right)\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}{6\,c^2\,g-3\,b\,c\,h}\right)(b\,h-2\,c\,g)}{d^2\,\left(4\,a\,c-b^2\right)^{5/2}} - \frac{\frac{8\,c\,h\,a^2+h\,a\,b^2-10\,c\,g\,a\,b+g\,b^3}{2\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}+\frac{x\,\left(b^2+5\,a\,c\right)\,(b\,h-2\,c\,g)}{16\,a^2\,c^2-8\,a\,b^2\,c+b^4}+\frac{3\,c^2\,x^3\,(b\,h-2\,c\,g)}{16\,a^2\,c^2-8\,a\,b^2\,c+b^4}+\frac{9\,b\,c\,x^2\,(b\,h-2\,c\,g)}{2\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}}{x^2\,\left(b^2\,d^2+2\,a\,c\,d^2\right)+a^2\,d^2+c^2\,d^2\,x^4+2\,a\,b\,d^2\,x+2\,b\,c\,d^2\,x^3}$$

```
[In] int((g + h*x)/((a*d + b*d*x + c*d*x^2)^2*(a + b*x + c*x^2)),x)
```

```
[Out] (6*c*atan((d^2*((6*c^2*x*(b*h - 2*c*g))/(d^2*(4*a*c - b^2)^(5/2)) + (3*c*(b
*h - 2*c*g)*(b^5*d^2 + 16*a^2*b*c^2*d^2 - 8*a*b^3*c*d^2))/(d^4*(4*a*c - b^2
)^(5/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(6
*c^2*g - 3*b*c*h))*(b*h - 2*c*g))/(d^2*(4*a*c - b^2)^(5/2)) - ((b^3*g + a*b
^2*h + 8*a^2*c*h - 10*a*b*c*g)/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x*(5*a
*c + b^2)*(b*h - 2*c*g))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) + (3*c^2*x^3*(b*h -
 2*c*g))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) + (9*b*c*x^2*(b*h - 2*c*g))/(2*(b^4
 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^2*(b^2*d^2 + 2*a*c*d^2) + a^2*d^2 + c^2*d^2
*x^4 + 2*a*b*d^2*x + 2*b*c*d^2*x^3)
```

3.18 $\int \frac{g+hx}{(a+bx+cx^2)^2(ad+bdx+cdx^2)}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 140

$$\int \frac{g+hx}{(a+bx+cx^2)^2(ad+bdx+cdx^2)}\,dx = -\frac{bg-2ah+(2cg-bh)x}{2(b^2-4ac)d(a+bx+cx^2)^2} + \frac{3(2cg-bh)(b+2cx)}{2(b^2-4ac)^2d(a+bx+cx^2)} - \frac{6c(2cg-bh)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}d}$$

```
[Out] 1/2*(-b*g+2*a*h-(-b*h+2*c*g)*x)/(-4*a*c+b^2)/d/(c*x^2+b*x+a)^2+3/2*(-b*h+2*
c*g)*(2*c*x+b)/(-4*a*c+b^2)^2/d/(c*x^2+b*x+a)-6*c*(-b*h+2*c*g)*arctanh((2*c
*x+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(5/2)/d
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {1012, 652, 628, 632, 212}

$$\int \frac{g+hx}{(a+bx+cx^2)^2(ad+bdx+cdx^2)}\,dx = -\frac{6c(2cg-bh)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{d(b^2-4ac)^{5/2}} + \frac{3(b+2cx)(2cg-bh)}{2d(b^2-4ac)^2(a+bx+cx^2)} - \frac{-2ah+x(2cg-bh)+bg}{2d(b^2-4ac)(a+bx+cx^2)^2}$$

```
[In] Int[(g + h*x)/((a + b*x + c*x^2)^2*(a*d + b*d*x + c*d*x^2)),x]
```

```
[Out] -1/2*(b*g - 2*a*h + (2*c*g - b*h)*x)/((b^2 - 4*a*c)*d*(a + b*x + c*x^2)^2)
+ (3*(2*c*g - b*h)*(b + 2*c*x))/(2*(b^2 - 4*a*c)^2*d*(a + b*x + c*x^2)) - (
6*c*(2*c*g - b*h)*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/((b^2 - 4*a*c)^(5
/2)*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1012

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_
) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Dist[(c/f)^p, Int[(g + h*
x)^m*(d + e*x + f*x^2)^(p + q), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, p,
 q}, x] && EqQ[c*d - a*f, 0] && EqQ[b*d - a*e, 0] && (IntegerQ[p] || GtQ[c/
f, 0]) && ( !IntegerQ[q] || LeafCount[d + e*x + f*x^2] <= LeafCount[a + b*x
 + c*x^2])
```

Rubi steps

$$\text{integral} = \frac{\int \frac{g+hx}{(a+bx+cx^2)^3}\,dx}{d}$$

$$= -\frac{bg - 2ah + (2cg - bh)x}{2(b^2 - 4ac)d(a + bx + cx^2)^2} - \frac{(3(2cg - bh))\int \frac{1}{(a+bx+cx^2)^2}dx}{2(b^2 - 4ac)d}$$

$$= -\frac{bg - 2ah + (2cg - bh)x}{2(b^2 - 4ac)d(a + bx + cx^2)^2} + \frac{3(2cg - bh)(b + 2cx)}{2(b^2 - 4ac)^2 d(a + bx + cx^2)} + \frac{(3c(2cg - bh))\int \frac{1}{a+bx+cx^2}dx}{(b^2 - 4ac)^2 d}$$

$$= -\frac{bg - 2ah + (2cg - bh)x}{2(b^2 - 4ac)d(a + bx + cx^2)^2} + \frac{3(2cg - bh)(b + 2cx)}{2(b^2 - 4ac)^2 d(a + bx + cx^2)} - \frac{(6c(2cg - bh))\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}dx, x, b + 2cx\right)}{(b^2 - 4ac)^2 d}$$

$$= -\frac{bg - 2ah + (2cg - bh)x}{2(b^2 - 4ac)d(a + bx + cx^2)^2} + \frac{3(2cg - bh)(b + 2cx)}{2(b^2 - 4ac)^2 d(a + bx + cx^2)} - \frac{6c(2cg - bh)\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2 - 4ac)^{5/2} d}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.94

$$\int \frac{g + hx}{(a + bx + cx^2)^2 (ad + bdx + cdx^2)} dx$$

$$= \frac{\frac{(b^2-4ac)(-bg+2ah-2cgx+bhx)}{(a+x(b+cx))^2} + \frac{3(2cg-bh)(b+2cx)}{a+x(b+cx)} - \frac{12c(-2cg+bh)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}}{2(b^2 - 4ac)^2 d}$$

```
[In] Integrate[(g + h*x)/((a + b*x + c*x^2)^2*(a*d + b*d*x + c*d*x^2)),x]
```

```
[Out] (((b^2 - 4*a*c)*(-(b*g) + 2*a*h - 2*c*g*x + b*h*x))/(a + x*(b + c*x))^2 + (
3*(2*c*g - b*h)*(b + 2*c*x))/(a + x*(b + c*x)) - (12*c*(-2*c*g + b*h)*ArcTa
n[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c])/(2*(b^2 - 4*a*c)^2*d
)
```

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.01

method	result
default	$\frac{\frac{bg-2ah+(-bh+2cg)x}{2(4ac-b^2)(c\,x^2+bx+a)^2}+\frac{3(-bh+2cg)\left(\frac{2cx+b}{(4ac-b^2)(c\,x^2+bx+a)}+\frac{4c\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{\frac{3}{2}}}\right)}{2(4ac-b^2)}}{d}$
risch	$\frac{-\frac{3c^2(bh-2cg)x^3}{16a^2c^2-8a\,b^2c+b^4}-\frac{9bc(bh-2cg)x^2}{2(16a^2c^2-8a\,b^2c+b^4)}-\frac{(5abch-10a\,c^2g+b^3h-2b^2cg)x}{16a^2c^2-8a\,b^2c+b^4}-\frac{8a^2ch+a\,b^2h-10abcg+b^3g}{2(16a^2c^2-8a\,b^2c+b^4)}}{(c\,x^2+bx+a)^2d}-\frac{3c\ln\left((32a^2c^3-16a\,b^2c^2+2\right.}{}$

```
[In] int((h*x+g)/(c*x^2+b*x+a)^2/(c*d*x^2+b*d*x+a*d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2*(b*g-2*a*h+(-b*h+2*c*g)*x)/(4*a*c-b^2)/(c*x^2+b*x+a)^2+3/2*(-b*h+2
*c*g)/(4*a*c-b^2)*((2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)+4*c/(4*a*c-b^2)^(3/2
)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 555 vs. 2(132) = 264.

Time = 0.34 (sec) , antiderivative size = 1130, normalized size of antiderivative = 8.07

$$\int \frac{g+hx}{(a+bx+cx^2)^2\,(ad+bdx+cdx^2)}\,dx = \text{Too large to display}$$

```
[In] integrate((h*x+g)/(c*x^2+b*x+a)^2/(c*d*x^2+b*d*x+a*d),x, algorithm="fricas"
)
```

```
[Out] [1/2*(6*(2*(b^2*c^3 - 4*a*c^4)*g - (b^3*c^2 - 4*a*b*c^3)*h)*x^3 + 9*(2*(b^3
*c^2 - 4*a*b*c^3)*g - (b^4*c - 4*a*b^2*c^2)*h)*x^2 - 6*(2*a^2*c^2*g - a^2*b
*c*h + (2*c^4*g - b*c^3*h)*x^4 + 2*(2*b*c^3*g - b^2*c^2*h)*x^3 + (2*(b^2*c^
2 + 2*a*c^3)*g - (b^3*c + 2*a*b*c^2)*h)*x^2 + 2*(2*a*b*c^2*g - a*b^2*c*h)*x
)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2 - 4*a
*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - (b^5 - 14*a*b^3*c + 40*a^2*b*c^2)*g -
 (a*b^4 + 4*a^2*b^2*c - 32*a^3*c^2)*h + 2*(2*(b^4*c + a*b^2*c^2 - 20*a^2*c^
3)*g - (b^5 + a*b^3*c - 20*a^2*b*c^2)*h)*x)/((b^6*c^2 - 12*a*b^4*c^3 + 48*a
^2*b^2*c^4 - 64*a^3*c^5)*d*x^4 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 -
 64*a^3*b*c^4)*d*x^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3
- 128*a^4*c^4)*d*x^2 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*
c^3)*d*x + (a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3)*d), 1/2*(
6*(2*(b^2*c^3 - 4*a*c^4)*g - (b^3*c^2 - 4*a*b*c^3)*h)*x^3 + 9*(2*(b^3*c^2 -
 4*a*b*c^3)*g - (b^4*c - 4*a*b^2*c^2)*h)*x^2 - 12*(2*a^2*c^2*g - a^2*b*c*h
+ (2*c^4*g - b*c^3*h)*x^4 + 2*(2*b*c^3*g - b^2*c^2*h)*x^3 + (2*(b^2*c^2 + 2
```

```
*a*c^3)*g - (b^3*c + 2*a*b*c^2)*h)*x^2 + 2*(2*a*b*c^2*g - a*b^2*c*h)*x)*sqr
t(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^2 - 4*a*c)) - (b^
5 - 14*a*b^3*c + 40*a^2*b*c^2)*g - (a*b^4 + 4*a^2*b^2*c - 32*a^3*c^2)*h + 2
*(2*(b^4*c + a*b^2*c^2 - 20*a^2*c^3)*g - (b^5 + a*b^3*c - 20*a^2*b*c^2)*h)*
x)/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d*x^4 + 2*(b^7*c
 - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*d*x^3 + (b^8 - 10*a*b^6*c
+ 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*d*x^2 + 2*(a*b^7 - 12*a^2*
b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*d*x + (a^2*b^6 - 12*a^3*b^4*c + 48*a
^4*b^2*c^2 - 64*a^5*c^3)*d)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 680 vs. $2(128) = 256$.

Time = 1.08 (sec) , antiderivative size = 680, normalized size of antiderivative = 4.86

$$\int \frac{g+hx}{(a+bx+cx^2)^2\,(ad+bdx+cdx^2)}\,dx$$

$$= \frac{3c\sqrt{-\frac{1}{(4ac-b^2)^5}}(bh-2cg)\log\left(x+\frac{-192a^3c^4\sqrt{-\frac{1}{(4ac-b^2)^5}}(bh-2cg)+144a^2b^2c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}(bh-2cg)-36ab^4c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}(b}{6bc^2h-12c^3g}\right.}{d}$$

$$-\frac{3c\sqrt{-\frac{1}{(4ac-b^2)^5}}(bh-2cg)\log\left(x+\frac{192a^3c^4\sqrt{-\frac{1}{(4ac-b^2)^5}}(bh-2cg)-144a^2b^2c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}(bh-2cg)+36ab^4c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}(}{6bc^2h-12c^3g}\right.}{d}$$

$$+\frac{-8a^2ch-ab^2h+10abcg-b^3g+x^3(-6bc^2h+12c^3g)+x^2(-9b^2ch+18bc^2g}{32a^4c^2d-16a^3b^2cd+2a^2b^4d+x^4\cdot(32a^2c^4d-16ab^2c^3d+2b^4c^2d)+x^3\cdot(64a^2bc^3d-32ab^3c^2d+4b^5cd)}$$

```
[In] integrate((h*x+g)/(c*x**2+b*x+a)**2/(c*d*x**2+b*d*x+a*d),x)
```

```
[Out] 3*c*sqrt(-1/(4*a*c - b**2)**5)*(b*h - 2*c*g)*log(x + (-192*a**3*c**4*sqrt(-
1/(4*a*c - b**2)**5)*(b*h - 2*c*g) + 144*a**2*b**2*c**3*sqrt(-1/(4*a*c - b*
*2)**5)*(b*h - 2*c*g) - 36*a*b**4*c**2*sqrt(-1/(4*a*c - b**2)**5)*(b*h - 2*
c*g) + 3*b**6*c*sqrt(-1/(4*a*c - b**2)**5)*(b*h - 2*c*g) + 3*b**2*c*h - 6*b
*c**2*g)/(6*b*c**2*h - 12*c**3*g))/d - 3*c*sqrt(-1/(4*a*c - b**2)**5)*(b*h
- 2*c*g)*log(x + (192*a**3*c**4*sqrt(-1/(4*a*c - b**2)**5)*(b*h - 2*c*g) -
144*a**2*b**2*c**3*sqrt(-1/(4*a*c - b**2)**5)*(b*h - 2*c*g) + 36*a*b**4*c**
2*sqrt(-1/(4*a*c - b**2)**5)*(b*h - 2*c*g) - 3*b**6*c*sqrt(-1/(4*a*c - b**2
)**5)*(b*h - 2*c*g) + 3*b**2*c*h - 6*b*c**2*g)/(6*b*c**2*h - 12*c**3*g))/d
+ (-8*a**2*c*h - a*b**2*h + 10*a*b*c*g - b**3*g + x**3*(-6*b*c**2*h + 12*c*
*3*g) + x**2*(-9*b**2*c*h + 18*b*c**2*g) + x*(-10*a*b*c*h + 20*a*c**2*g - 2
*b**3*h + 4*b**2*c*g))/(32*a**4*c**2*d - 16*a**3*b**2*c*d + 2*a**2*b**4*d +
 x**4*(32*a**2*c**4*d - 16*a*b**2*c**3*d + 2*b**4*c**2*d) + x**3*(64*a**2*b
*c**3*d - 32*a*b**3*c**2*d + 4*b**5*c*d) + x**2*(64*a**3*c**3*d - 12*a*b**4
*c*d + 2*b**6*d) + x*(64*a**3*b*c**2*d - 32*a**2*b**3*c*d + 4*a*b**5*d))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{g+hx}{(a+bx+cx^2)^2(ad+bdx+cdx^2)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((h*x+g)/(c*x^2+b*x+a)^2/(c*d*x^2+b*d*x+a*d),x, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.48

$$\int \frac{g+hx}{(a+bx+cx^2)^2(ad+bdx+cdx^2)}\,dx = \frac{6\,(2\,c^2g-bch)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^4d-8\,ab^2cd+16\,a^2c^2d)\sqrt{-b^2+4\,ac}} + \frac{12\,c^3gx^3-6\,bc^2hx^3+18\,bc^2gx^2-9\,b^2chx^2+4\,b^2cgx+20\,ac^2gx-2\,b^3hx-10\,abchx-b^3g+10\,abcg-}{2\,(b^4d-8\,ab^2cd+16\,a^2c^2d)(cx^2+bx+a)^2}$$

```
[In] integrate((h*x+g)/(c*x^2+b*x+a)^2/(c*d*x^2+b*d*x+a*d),x, algorithm="giac")
```

```
[Out] 6*(2*c^2*g - b*c*h)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^4*d - 8*a*b^
2*c*d + 16*a^2*c^2*d)*sqrt(-b^2 + 4*a*c)) + 1/2*(12*c^3*g*x^3 - 6*b*c^2*h*x
^3 + 18*b*c^2*g*x^2 - 9*b^2*c*h*x^2 + 4*b^2*c*g*x + 20*a*c^2*g*x - 2*b^3*h*
x - 10*a*b*c*h*x - b^3*g + 10*a*b*c*g - a*b^2*h - 8*a^2*c*h)/((b^4*d - 8*a*
b^2*c*d + 16*a^2*c^2*d)*(c*x^2 + b*x + a)^2)
```

Mupad [B] (verification not implemented)

Time = 12.76 (sec) , antiderivative size = 375, normalized size of antiderivative = 2.68

$$\int \frac{g+hx}{(a+bx+cx^2)^2(ad+bdx+cdx^2)}\,dx = \frac{6\,c\,\mathrm{atan}\left(\frac{d\,\left(\frac{6\,c^2\,x\,(b\,h-2\,c\,g)}{d\,{\left(4\,a\,c-b^2\right)}^{5/2}}+\frac{3\,c\,(b\,h-2\,c\,g)\,\left(16\,d\,a^2\,b\,c^2-8\,d\,a\,b^3\,c+d\,b^5\right)}{d^2\,{\left(4\,a\,c-b^2\right)}^{5/2}\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}\right)\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}{6\,c^2\,g-3\,b\,c\,h}\right)\,(b\,h-2\,c\,g)}{d\,{\left(4\,a\,c-b^2\right)}^{5/2}} - \frac{\frac{8\,c\,h\,a^2+h\,a\,b^2-10\,c\,g\,a\,b+g\,b^3}{2\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{x\,\left(b^2+5\,a\,c\right)\,(b\,h-2\,c\,g)}{16\,a^2\,c^2-8\,a\,b^2\,c+b^4}+\frac{3\,c^2\,x^3\,(b\,h-2\,c\,g)}{16\,a^2\,c^2-8\,a\,b^2\,c+b^4}+\frac{9\,b\,c\,x^2\,(b\,h-2\,c\,g)}{2\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}}{a^2\,d+x^2\,\left(d\,b^2+2\,a\,c\,d\right)+c^2\,d\,x^4+2\,b\,c\,d\,x^3+2\,a\,b\,d\,x}$$

```
[In] int((g + h*x)/((a*d + b*d*x + c*d*x^2)*(a + b*x + c*x^2)^2),x)
```

```
[Out] (6*c*atan((d*((6*c^2*x*(b*h - 2*c*g))/(d*(4*a*c - b^2)^(5/2)) + (3*c*(b*h -
 2*c*g)*(b^5*d - 8*a*b^3*c*d + 16*a^2*b*c^2*d))/(d^2*(4*a*c - b^2)^(5/2)*(b
^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(6*c^2*g - 3
*b*c*h))*(b*h - 2*c*g))/(d*(4*a*c - b^2)^(5/2)) - ((b^3*g + a*b^2*h + 8*a^2
*c*h - 10*a*b*c*g)/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x*(5*a*c + b^2)*(b
*h - 2*c*g))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) + (3*c^2*x^3*(b*h - 2*c*g))/(b^
4 + 16*a^2*c^2 - 8*a*b^2*c) + (9*b*c*x^2*(b*h - 2*c*g))/(2*(b^4 + 16*a^2*c^
2 - 8*a*b^2*c)))/(a^2*d + x^2*(b^2*d + 2*a*c*d) + c^2*d*x^4 + 2*b*c*d*x^3 +
 2*a*b*d*x)
```

3.19 $\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 617

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx$$
$$=\frac{B\sqrt{a+bx+cx^2}}{f}-\frac{(2Bce-bBf-2Acf)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}f^2}$$
$$+\frac{\left(2f(Af(cd-af)-Bd(ce-bf))-\left(e-\sqrt{e^2-4df}\right)\left(Af(ce-bf)+B(f(be-af)-c(e^2-df))\right)\right)\text{ar}}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{}}}$$
$$-\frac{\left(2f(Af(cd-af)-Bd(ce-bf))-\left(e+\sqrt{e^2-4df}\right)\left(Af(ce-bf)+B(f(be-af)-c(e^2-df))\right)\right)\text{ar}}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{}}}$$

```
[Out] -1/2*(-2*A*c*f-B*b*f+2*B*c*e)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(
1/2))/f^2/c^(1/2)+B*(c*x^2+b*x+a)^(1/2)/f+1/2*arctanh(1/4*(4*a*f+2*x*(b*f-c
*(e-(-4*d*f+e^2)^(1/2)))-b*(e-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+b*x+a)^(1
/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2))*(2*f
*(A*f*(-a*f+c*d)-B*d*(-b*f+c*e))-(A*f*(-b*f+c*e)+B*(f*(-a*f+b*e)-c*(-d*f+e^
2)))*(e-(-4*d*f+e^2)^(1/2)))/f^2*2^(1/2)/(-4*d*f+e^2)^(1/2)/(c*e^2-2*c*d*f-
b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)-1/2*arctanh(1/4*(4*a*f-b
*(e+(-4*d*f+e^2)^(1/2))+2*x*(b*f-c*(e+(-4*d*f+e^2)^(1/2))))*2^(1/2)/(c*x^2+
b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1
/2))*(2*f*(A*f*(-a*f+c*d)-B*d*(-b*f+c*e))-(A*f*(-b*f+c*e)+B*(f*(-a*f+b*e)-c
*(-d*f+e^2)))*(e+(-4*d*f+e^2)^(1/2)))/f^2*2^(1/2)/(-4*d*f+e^2)^(1/2)/(c*e^2
-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)
```

Rubi [A] (warning: unable to verify)

Time = 5.61 (sec) , antiderivative size = 615, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1033, 1090, 635, 212, 1046, 738}

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx$$

$$= \frac{\left(2f(Af(cd-af)-Bd(ce-bf))-\left(e-\sqrt{e^2-4df}\right)\left(Bf(be-af)+Af(ce-bf)-Bc(e^2-df)\right)\right)\text{arctan}}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf}}$$

$$-\frac{\left(2f(Af(cd-af)-Bd(ce-bf))-\left(\sqrt{e^2-4df}+e\right)\left(Bf(be-af)+Af(ce-bf)-Bc(e^2-df)\right)\right)\text{arct}}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cd}}$$

$$-\frac{\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(-2Acf-bBf+2Bce)}{2\sqrt{c}f^2}+\frac{B\sqrt{a+bx+cx^2}}{f}$$

```
[In] Int[((A + B*x)*Sqrt[a + b*x + c*x^2])/(d + e*x + f*x^2),x]
```

```
[Out] (B*Sqrt[a + b*x + c*x^2])/f - ((2*B*c*e - b*B*f - 2*A*c*f)*ArcTanh[(b + 2*c
*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[c]*f^2) + ((2*f*(A*f*(c*d -
 a*f) - B*d*(c*e - b*f)) - (e - Sqrt[e^2 - 4*d*f])*(B*f*(b*e - a*f) + A*f*(
c*e - b*f) - B*c*(e^2 - d*f)))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) +
 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b
*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(S
qrt[2]*f^2*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e
- b*f)*Sqrt[e^2 - 4*d*f]]) - ((2*f*(A*f*(c*d - a*f) - B*d*(c*e - b*f)) - (e
 + Sqrt[e^2 - 4*d*f])*(B*f*(b*e - a*f) + A*f*(c*e - b*f) - B*c*(e^2 - d*f))
)*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4
*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*
Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*f^2*Sqrt[e^2 - 4*d*f]*
Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[h*(a + b*x + c*x^2)^p*((d
+ e*x + f*x^2)^(q + 1)/(2*f*(p + q + 1))), x] - Dist[1/(2*f*(p + q + 1)), I
nt[(a + b*x + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[h*p*(b*d - a*e) + a*(
h*e - 2*g*f)*(p + q + 1) + (2*h*p*(c*d - a*f) + b*(h*e - 2*g*f)*(p + q + 1)
)*x + (h*p*(c*e - b*f) + c*(h*e - 2*g*f)*(p + q + 1))*x^2, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d
*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]
```

Rule 1046

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1090

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqr
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{B\sqrt{a+bx+cx^2}}{f} - \frac{\int \frac{\frac{1}{2}(bBd-2aAf)-\frac{1}{2}(2Abf-B(2cd+be-2af))x+\frac{1}{2}(2Bce-bBf-2Acf)x^2}{\sqrt{a+bx+cx^2}(d+ex+fx^2)}\,dx}{f} \\ &= \frac{B\sqrt{a+bx+cx^2}}{f} \\ &\quad - \frac{\int \frac{\frac{1}{2}f(bBd-2aAf)-\frac{1}{2}d(2Bce-bBf-2Acf)+\left(-\frac{1}{2}e(2Bce-bBf-2Acf)+\frac{1}{2}f(-2Abf+B(2cd+be-2af))\right)x}{\sqrt{a+bx+cx^2}(d+ex+fx^2)}\,dx}{f^2} \\ &\quad - \frac{(2Bce-bBf-2Acf)\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{2f^2} \end{aligned}$$

$$= \frac{B\sqrt{a+bx+cx^2}}{f} - \frac{(2Bce-bBf-2Acf)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{f^2}$$
$$+ \frac{\left(2f(Af(cd-af)-Bd(ce-bf)) - \left(e+\sqrt{e^2-4df}\right)\left(Bf(be-af)+Af(ce-bf)-Bc(e^2-d\right.\right.}{f^2\sqrt{e^2-4df}}$$
$$- \frac{\left(2f\left(\frac{1}{2}f(bBd-2aAf)-\frac{1}{2}d(2Bce-bBf-2Acf)\right)-\left(e-\sqrt{e^2-4df}\right)\left(-\frac{1}{2}e(2Bce-bBf-2A\right.\right.}{f^2\sqrt{e^2-4df}}$$

$$= \frac{B\sqrt{a+bx+cx^2}}{f} - \frac{(2Bce-bBf-2Acf)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}f^2}$$
$$- \frac{\left(2\left(2f(Af(cd-af)-Bd(ce-bf)) - \left(e+\sqrt{e^2-4df}\right)\left(Bf(be-af)+Af(ce-bf)-Bc(e^2-\right.\right.\right.}{f^2\sqrt{}}$$
$$+ \frac{\left(2\left(2f\left(\frac{1}{2}f(bBd-2aAf)-\frac{1}{2}d(2Bce-bBf-2Acf)\right)-\left(e-\sqrt{e^2-4df}\right)\left(-\frac{1}{2}e(2Bce-bBf-2\right.\right.\right.}{}$$

$$= \frac{B\sqrt{a+bx+cx^2}}{f} - \frac{(2Bce-bBf-2Acf)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}f^2}$$
$$+ \frac{\left(2f(Af(cd-af)-Bd(ce-bf)) - \left(e-\sqrt{e^2-4df}\right)\left(Bf(be-af)+Af(ce-bf)-Bc(e^2-d\right.\right.}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{ce^2-2cdf-bef+2af^2-(ce-}}$$
$$- \frac{\left(2f(Af(cd-af)-Bd(ce-bf)) - \left(e+\sqrt{e^2-4df}\right)\left(Bf(be-af)+Af(ce-bf)-Bc(e^2-d\right.\right.}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{ce^2-2cdf-bef+2af^2+(ce-}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 1.28 (sec) , antiderivative size = 1115, normalized size of antiderivative = 1.81

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx$$
$$= \frac{Bf\sqrt{a+x(b+cx)} + \frac{(-2Bce+bBf+2Acf)\text{arctanh}\left(\frac{\sqrt{c}x}{-\sqrt{a}+\sqrt{a+x(b+cx)}}\right)}{\sqrt{c}} - \text{RootSum}\left[c^2d-bce+b^2f+2\sqrt{a}ce\#1-\right.}{}$$

```
[In] Integrate[((A + B*x)*Sqrt[a + b*x + c*x^2])/(d + e*x + f*x^2),x]
```

```
[Out] (B*f*Sqrt[a + x*(b + c*x)] + ((-2*B*c*e + b*B*f + 2*A*c*f)*ArcTanh[(Sqrt[c]
*x)/(-Sqrt[a] + Sqrt[a + x*(b + c*x)])])/Sqrt[c] - RootSum[c^2*d - b*c*e +
b^2*f + 2*Sqrt[a]*c*e*#1 - 4*Sqrt[a]*b*f*#1 - 2*c*d*#1^2 + b*e*#1^2 + 4*a*f
*#1^2 - 2*Sqrt[a]*e*#1^3 + d*#1^4 & , (-(B*c^2*d*e*Log[x]) + b*B*c*e^2*Log[
x] + A*c^2*d*f*Log[x] - b^2*B*e*f*Log[x] - A*b*c*e*f*Log[x] + A*b^2*f^2*Log
[x] + a*b*B*f^2*Log[x] - a*A*c*f^2*Log[x] + B*c^2*d*e*Log[-Sqrt[a] + Sqrt[a
 + b*x + c*x^2] - x*#1] - b*B*c*e^2*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] -
x*#1] - A*c^2*d*f*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] + b^2*B*e*f*
Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] + A*b*c*e*f*Log[-Sqrt[a] + Sqr
t[a + b*x + c*x^2] - x*#1] - A*b^2*f^2*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2]
 - x*#1] - a*b*B*f^2*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] + a*A*c*f
^2*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] - 2*Sqrt[a]*B*c*e^2*Log[x]*
#1 + 2*Sqrt[a]*B*c*d*f*Log[x]*#1 + 2*Sqrt[a]*b*B*e*f*Log[x]*#1 + 2*Sqrt[a]*
A*c*e*f*Log[x]*#1 - 2*Sqrt[a]*A*b*f^2*Log[x]*#1 - 2*a^(3/2)*B*f^2*Log[x]*#1
 + 2*Sqrt[a]*B*c*e^2*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1]*#1 - 2*Sq
rt[a]*B*c*d*f*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1]*#1 - 2*Sqrt[a]*b
*B*e*f*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1]*#1 - 2*Sqrt[a]*A*c*e*f*
Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1]*#1 + 2*Sqrt[a]*A*b*f^2*Log[-Sq
rt[a] + Sqrt[a + b*x + c*x^2] - x*#1]*#1 + 2*a^(3/2)*B*f^2*Log[-Sqrt[a] + S
qrt[a + b*x + c*x^2] - x*#1]*#1 + B*c*d*e*Log[x]*#1^2 - b*B*d*f*Log[x]*#1^2
 - A*c*d*f*Log[x]*#1^2 + a*A*f^2*Log[x]*#1^2 - B*c*d*e*Log[-Sqrt[a] + Sqrt[
a + b*x + c*x^2] - x*#1]*#1^2 + b*B*d*f*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2
] - x*#1]*#1^2 + A*c*d*f*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1]*#1^2
- a*A*f^2*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1]*#1^2)/(-(Sqrt[a]*c*e
) + 2*Sqrt[a]*b*f + 2*c*d*#1 - b*e*#1 - 4*a*f*#1 + 3*Sqrt[a]*e*#1^2 - 2*d*#
1^3) & ])/f^2
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1175 vs. 2(558) = 1116.

Time = 0.99 (sec) , antiderivative size = 1176, normalized size of antiderivative = 1.91

method	result	size
risch	Expression too large to display	1176
default	Expression too large to display	1595

```
[In] int((B*x+A)*(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] B*(c*x^2+b*x+a)^(1/2)/f+1/2/f*(1/f*(2*A*c*f+B*b*f-2*B*c*e)*ln((1/2*b+c*x)/c
^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-1/2*(2*A*b*f^2*(-4*d*f+e^2)^(1/2)-2*A*c
*e*f*(-4*d*f+e^2)^(1/2)+4*A*a*f^3-2*A*b*e*f^2-4*A*c*d*f^2+2*A*c*e^2*f+2*B*a
*f^2*(-4*d*f+e^2)^(1/2)-2*B*b*e*f*(-4*d*f+e^2)^(1/2)-2*B*c*d*f*(-4*d*f+e^2)
^(1/2)+2*B*c*e^2*(-4*d*f+e^2)^(1/2)-2*B*a*e*f^2-4*B*b*d*f^2+2*B*b*e^2*f+6*B
*c*d*e*f-2*B*c*e^3)/f^2/(-4*d*f+e^2)^(1/2)*2^(1/2)/((b*f*(-4*d*f+e^2)^(1/2)
```

```
-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((b*f*(-
4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+(c
*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+1/2*2^(1/2
)*((b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e
^2)/f^2)^(1/2)*(4*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2*c+4*(c*(-4*d*f+e^2)^(
1/2)+b*f-c*e)/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+2*(b*f*(-4*d*f+e^2)^(1/2)
-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2/f*(
-e+(-4*d*f+e^2)^(1/2))))-1/2*(2*A*b*f^2*(-4*d*f+e^2)^(1/2)-2*A*c*e*f*(-4*d*
f+e^2)^(1/2)-4*A*a*f^3+2*A*b*e*f^2+4*A*c*d*f^2-2*A*c*e^2*f+2*B*a*f^2*(-4*d*
f+e^2)^(1/2)-2*B*b*e*f*(-4*d*f+e^2)^(1/2)-2*B*c*d*f*(-4*d*f+e^2)^(1/2)+2*B*
c*e^2*(-4*d*f+e^2)^(1/2)+2*B*a*e*f^2+4*B*b*d*f^2-2*B*b*e^2*f-6*B*c*d*e*f+2*
B*c*e^3)/f^2/(-4*d*f+e^2)^(1/2)*2^(1/2)/((-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e
^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-b*f*(-4*d*f+e^2
)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+1/f*(-c*(-4
*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-b*
f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^
2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4/f*(-c*(-4*d*f+e^2)^(1/2)
+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*(-b*f*(-4*d*f+e^2)^(1/2)+(-4*d
*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*
f+e^2)^(1/2))/f)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx = \text{Timed out}$$

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx = \int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx$$

```
[In] integrate((B*x+A)*(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)
[Out] Integral((A + B*x)*sqrt(a + b*x + c*x**2)/(d + e*x + f*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re deta
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx = \int \frac{(A+B\,x)\,\sqrt{c\,x^2+b\,x+a}}{f\,x^2+e\,x+d}\,dx$$

```
[In] int(((A + B*x)*(a + b*x + c*x^2)^(1/2))/(d + e*x + f*x^2),x)
```

```
[Out] int(((A + B*x)*(a + b*x + c*x^2)^(1/2))/(d + e*x + f*x^2), x)
```

3.20 $\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{d+ex+fx^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 1092

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{d+ex+fx^2}\,dx =$$

$$-\frac{(2Acf(4ce-5bf)-B(b^2f^2-2cf(5be-4af)+8c^2(e^2-df))+2cf(2Bce-bBf-2Acf)x)\sqrt{a+bx+c}}{8cf^3}$$

$$+\frac{B(a+bx+cx^2)^{3/2}}{3f}$$

$$+\frac{(2Acf(3b^2f^2-12cf(be-af)+8c^2(e^2-df))-B(b^3f^3+6bcf^2(be-2af)-24c^2f(be^2-bdf-aef)+16}{16c^{3/2}f^4}$$

$$-\frac{(2cf(Bd(ce-bf)(ce^2-2cdf-bef+2af^2)+Af(2cdf(be-af)-f^2(b^2d-a^2f)-c^2d(e^2-df)))-c(e}{}$$

$$+\frac{(2f(Bd(ce-bf)(ce^2-2cdf-bef+2af^2)+Af(2cdf(be-af)-f^2(b^2d-a^2f)-c^2d(e^2-df)))-(e+}{}$$

```
[Out] 1/3*B*(c*x^2+b*x+a)^(3/2)/f+1/16*(2*A*c*f*(3*b^2*f^2-12*c*f*(-a*f+b*e)+8*c^
2*(-d*f+e^2))-B*(b^3*f^3+6*b*c*f^2*(-2*a*f+b*e)-24*c^2*f*(-a*e*f-b*d*f+b*e^
2)+16*c^3*(-2*d*e*f+e^3)))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2
))/c^(3/2)/f^4-1/8*(2*A*c*f*(-5*b*f+4*c*e)-B*(b^2*f^2-2*c*f*(-4*a*f+5*b*e)+
8*c^2*(-d*f+e^2))+2*c*f*(-2*A*c*f-B*b*f+2*B*c*e)*x)*(c*x^2+b*x+a)^(1/2)/c/f
^3-1/2*arctanh(1/4*(4*a*f+2*x*(b*f-c*(e-(-4*d*f+e^2)^(1/2)))-b*(e-(-4*d*f+e
^2)^(1/2)))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+
c*e)*(-4*d*f+e^2)^(1/2))^(1/2))*(2*c*f*(B*d*(-b*f+c*e)*(2*a*f^2-b*e*f-2*c*d
*f+c*e^2)+A*f*(2*c*d*f*(-a*f+b*e)-f^2*(-a^2*f+b^2*d)-c^2*d*(-d*f+e^2)))-c*(
```

```
A*f*(-b*f+c*e)*(f*(-2*a*f+b*e)-c*(-2*d*f+e^2))+B*(c^2*(d^2*f^2-3*d*e^2*f+e^
4)-f^2*(2*a*b*e*f-a^2*f^2-b^2*(-d*f+e^2))+2*c*f*(a*f*(-d*f+e^2)-b*(-2*d*e*f
+e^3))))*(e-(-4*d*f+e^2)^(1/2)))/c/f^4*2^(1/2)/(-4*d*f+e^2)^(1/2)/(c*e^2-2*
c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)+1/2*arctanh(1/4*(4
*a*f-b*(e+(-4*d*f+e^2)^(1/2))+2*x*(b*f-c*(e+(-4*d*f+e^2)^(1/2))))*2^(1/2)/(
c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/
2))^(1/2))*(2*f*(B*d*(-b*f+c*e)*(2*a*f^2-b*e*f-2*c*d*f+c*e^2)+A*f*(2*c*d*f*
(-a*f+b*e)-f^2*(-a^2*f+b^2*d)-c^2*d*(-d*f+e^2)))-(A*f*(-b*f+c*e)*(f*(-2*a*f
+b*e)-c*(-2*d*f+e^2))+B*(c^2*(d^2*f^2-3*d*e^2*f+e^4)-f^2*(2*a*b*e*f-a^2*f^2
-b^2*(-d*f+e^2))+2*c*f*(a*f*(-d*f+e^2)-b*(-2*d*e*f+e^3))))*(e+(-4*d*f+e^2)^
(1/2)))/f^4*2^(1/2)/(-4*d*f+e^2)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c
*e)*(-4*d*f+e^2)^(1/2))^(1/2)
```

Rubi [A] (warning: unable to verify)

Time = 16.38 (sec) , antiderivative size = 1092, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {1033, 1080, 1090, 635, 212, 1046, 738}

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{d+ex+fx^2}\,dx = \frac{B\left(cx^2+bx+a\right)^{3/2}}{3f}$$

$$-\frac{\left(2Acf(4ce-5bf)-B\left(8\left(e^2-df\right)c^2-2f(5be-4af)c+b^2f^2\right)+2cf(2Bce-bBf-2Acf)x\right)\sqrt{cx^2+bx}}{8cf^3}$$

$$+\frac{\left(2Acf\left(8\left(e^2-df\right)c^2-12f(be-af)c+3b^2f^2\right)-B\left(16\left(e^3-2def\right)c^3-24f\left(be^2-afe-bdf\right)c^2+6bf^2(be\right.\right.}{16c^{3/2}f^4}$$

$$-\frac{\left(2cf\left(Bd(ce-bf)\left(ce^2-bfe+2af^2-2cdf\right)+Af\left(-d\left(e^2-df\right)c^2+2df(be-af)c-f^2\left(b^2d-a^2f\right)\right)\right)-\right.}{}$$

$$+\frac{\left(2f\left(Bd(ce-bf)\left(ce^2-bfe+2af^2-2cdf\right)+Af\left(-d\left(e^2-df\right)c^2+2df(be-af)c-f^2\left(b^2d-a^2f\right)\right)\right)-\left(e\right.\right.}{}$$

```
[In] Int[((A + B*x)*(a + b*x + c*x^2)^(3/2))/(d + e*x + f*x^2),x]
```

```
[Out] -1/8*((2*A*c*f*(4*c*e - 5*b*f) - B*(b^2*f^2 - 2*c*f*(5*b*e - 4*a*f) + 8*c^2
*(e^2 - d*f)) + 2*c*f*(2*B*c*e - b*B*f - 2*A*c*f)*x)*Sqrt[a + b*x + c*x^2])
/(c*f^3) + (B*(a + b*x + c*x^2)^(3/2))/(3*f) + ((2*A*c*f*(3*b^2*f^2 - 12*c*
f*(b*e - a*f) + 8*c^2*(e^2 - d*f)) - B*(b^3*f^3 + 6*b*c*f^2*(b*e - 2*a*f) -
 24*c^2*f*(b*e^2 - b*d*f - a*e*f) + 16*c^3*(e^3 - 2*d*e*f)))*ArcTanh[(b + 2
*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(3/2)*f^4) - ((2*c*f*(B*d*(
c*e - b*f)*(c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2) + A*f*(2*c*d*f*(b*e - a*f) -
 f^2*(b^2*d - a^2*f) - c^2*d*(e^2 - d*f))) - c*(e - Sqrt[e^2 - 4*d*f])*(A*f
*(c*e - b*f)*(f*(b*e - 2*a*f) - c*(e^2 - 2*d*f)) + B*(c^2*(e^4 - 3*d*e^2*f
```

```
+ d^2*f^2) - f^2*(2*a*b*e*f - a^2*f^2 - b^2*(e^2 - d*f)) + 2*c*f*(a*f*(e^2
- d*f) - b*(e^3 - 2*d*e*f)))))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) +
 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b
*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(S
qrt[2]*c*f^4*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*
e - b*f)*Sqrt[e^2 - 4*d*f]]) + ((2*f*(B*d*(c*e - b*f)*(c*e^2 - 2*c*d*f - b*
e*f + 2*a*f^2) + A*f*(2*c*d*f*(b*e - a*f) - f^2*(b^2*d - a^2*f) - c^2*d*(e^
2 - d*f))) - (e + Sqrt[e^2 - 4*d*f])*(A*f*(c*e - b*f)*(f*(b*e - 2*a*f) - c*
(e^2 - 2*d*f)) + B*(c^2*(e^4 - 3*d*e^2*f + d^2*f^2) - f^2*(2*a*b*e*f - a^2*
f^2 - b^2*(e^2 - d*f)) + 2*c*f*(a*f*(e^2 - d*f) - b*(e^3 - 2*d*e*f)))))*Arc
Tanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]
))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[
e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*f^4*Sqrt[e^2 - 4*d*f]*Sqrt[
c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

<u>Rule 212</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

<u>Rule 635</u>

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

<u>Rule 738</u>

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

<u>Rule 1033</u>

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[h*(a + b*x + c*x^2)^p*((d
+ e*x + f*x^2)^(q + 1)/(2*f*(p + q + 1))), x] - Dist[1/(2*f*(p + q + 1)), I
nt[(a + b*x + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[h*p*(b*d - a*e) + a*(
h*e - 2*g*f)*(p + q + 1) + (2*h*p*(c*d - a*f) + b*(h*e - 2*g*f)*(p + q + 1)
)*x + (h*p*(c*e - b*f) + c*(h*e - 2*g*f)*(p + q + 1))*x^2, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d
*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]
```

<u>Rule 1046</u>

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1080

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(B*c*f*(2*p
 + 2*q + 3) + C*(b*f*p - c*e*(2*p + q + 2)) + 2*c*C*f*(p + q + 1)*x)*(a + b
*x + c*x^2)^p*((d + e*x + f*x^2)^(q + 1)/(2*c*f^2*(p + q + 1)*(2*p + 2*q +
3))), x] - Dist[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + b*x + c*x
^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[p*(b*d - a*e)*(C*(c*e - b*f)*(q + 1) -
 c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(b^2*C*d*f*p + a*c*(C*(2*d*f
- e^2*(2*p + q + 2)) + f*(B*e - 2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)
*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*e
*f*p*(b^2 - 4*a*c) - b*c*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e +
2*A*f)*(2*p + 2*q + 3))))*x + (p*(c*e - b*f)*(C*(c*e - b*f)*(q + 1) - c*(C*
e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(e^
2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3))))*x^2,
x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*
q + 3, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1090

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqr
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\text{integral} = \frac{B(a+bx+cx^2)^{3/2}}{3f} - \frac{\int \frac{\sqrt{a+bx+cx^2}\left(\frac{3}{2}(bBd-2aAf)-\frac{3}{2}(2Abf-B(2cd+be-2af))x+\frac{3}{2}(2Bce-bBf-2Acf)x^2\right)}{d+ex+fx^2}\,dx}{3f}$$

$$= -\frac{(2Acf(4ce-5bf)-B(b^2f^2-2cf(5be-4af)+8c^2(e^2-df))+2cf(2Bce-bBf-2Acf)x)\sqrt{}}{8cf^3}$$

$$+\frac{B(a+bx+cx^2)^{3/2}}{3f}$$

$$+\frac{\int \frac{-\frac{3}{8}\left(b^3Bdf^2-10b^2cdf(Be-Af)-8acf(Bcde-Af(cd-2af))-4bcd\left(2Acef-B\left(2ce^2-2cdf+5af^2\right)\right)\right)+\frac{3}{8}\left(2Acf\left(8c^2de-4acef-bf(5\right.\right.}{}}{}$$

$$= -\frac{(2Acf(4ce-5bf)-B(b^2f^2-2cf(5be-4af)+8c^2(e^2-df))+2cf(2Bce-bBf-2Acf)x)\sqrt{}}{8cf^3}$$

$$+\frac{B(a+bx+cx^2)^{3/2}}{3f}$$

$$+\frac{\int \frac{-\frac{3}{8}d\left(2Acf\left(3b^2f^2-12cf(be-af)+8c^2\left(e^2-df\right)\right)-B\left(b^3f^3+6bcf^2(be-2af)-24c^2f\left(be^2-bdf-aef\right)+16c^3\left(e^3-2def\right)\right)\right)-\frac{3}{8}f\left(b^3Bdf\right.}{}}{}$$

$$+\frac{(2Acf(3b^2f^2-12cf(be-af)+8c^2(e^2-df))-B(b^3f^3+6bcf^2(be-2af)-24c^2f(be^2-bdf-}{16cf^4}$$

$$= -\frac{(2Acf(4ce-5bf)-B(b^2f^2-2cf(5be-4af)+8c^2(e^2-df))+2cf(2Bce-bBf-2Acf)x)\sqrt{}}{8cf^3}$$

$$+\frac{B(a+bx+cx^2)^{3/2}}{3f}$$

$$+\frac{(2Acf(3b^2f^2-12cf(be-af)+8c^2(e^2-df))-B(b^3f^3+6bcf^2(be-2af)-24c^2f(be^2-bdf-}{8cf^4}$$

$$-\frac{\left(Af\left(c^2\left(e^4-4de^2f+2d^2f^2+e^3\sqrt{e^2-4df}-2def\sqrt{e^2-4df}\right)+f^2\left(2a^2f^2-2abf\left(e+\sqrt{e^2-4d}\right.\right.\right.\right.}{}$$

$$-$$

$$-\frac{\left(2f\left(-\frac{3}{8}d(2Acf(3b^2f^2-12cf(be-af)+8c^2(e^2-df))-B(b^3f^3+6bcf^2(be-2af)-24c^2f(be\right.\right.}{}$$

$$= -\frac{(2Acf(4ce-5bf)-B(b^2f^2-2cf(5be-4af)+8c^2(e^2-df))+2cf(2Bce-bBf-2Acf)x)}{8cf^3}$$
$$+\frac{B(a+bx+cx^2)^{3/2}}{3f}$$
$$+\frac{(2Acf(3b^2f^2-12cf(be-af)+8c^2(e^2-df))-B(b^3f^3+6bcf^2(be-2af)-24c^2f(be^2-bdf}{16c^{3/2}f^4}$$
$$+\frac{\left(2\left(Af\left(c^2\left(e^4-4de^2f+2d^2f^2+e^3\sqrt{e^2-4df}-2def\sqrt{e^2-4df}\right)+f^2\left(2a^2f^2-2abf\left(e+\sqrt{e^2}\right.\right.\right.\right.\right.}{}$$
$$+$$
$$-\frac{\left(2f\left(-\frac{3}{8}d(2Acf(3b^2f^2-12cf(be-af)+8c^2(e^2-df))-B(b^3f^3+6bcf^2(be-2af)-24c^2f(b\right.\right.}{}$$

$$= -\frac{(2Acf(4ce-5bf)-B(b^2f^2-2cf(5be-4af)+8c^2(e^2-df))+2cf(2Bce-bBf-2Acf)x)}{8cf^3}$$
$$+\frac{B(a+bx+cx^2)^{3/2}}{3f}$$
$$+\frac{(2Acf(3b^2f^2-12cf(be-af)+8c^2(e^2-df))-B(b^3f^3+6bcf^2(be-2af)-24c^2f(be^2-bdf}{16c^{3/2}f^4}$$
$$+\frac{\left(2cf(Bd(ce-bf)\left(f(be-2af)-c(e^2-2df)\right)-Af(2cdf(be-af)-f^2(b^2d-a^2f)-c^2d(e^2\right.}{}$$
$$+\frac{\left(Af\left(c^2\left(e^4-4de^2f+2d^2f^2+e^3\sqrt{e^2-4df}-2def\sqrt{e^2-4df}\right)+f^2\left(2a^2f^2-2abf\left(e+\sqrt{e^2-}\right.\right.\right.\right.}{}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 6.53 (sec) , antiderivative size = 3516, normalized size of antiderivative = 3.22

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{d+ex+fx^2}\,dx = \text{Result too large to show}$$

```
[In] Integrate[((A + B*x)*(a + b*x + c*x^2)^(3/2))/(d + e*x + f*x^2),x]
[Out] (Sqrt[c]*f*Sqrt[a + x*(b + c*x)]*(6*A*c*f*(-4*c*e + 5*b*f + 2*c*f*x) + B*(3
*b^2*f^2 + 2*c*f*(-15*b*e + 16*a*f + 7*b*f*x) + 4*c^2*(6*e^2 - 6*d*f - 3*e*
```

```
f*x + 2*f^2*x^2))) + 3*(8*A*c^2*(2*c*d + 3*b*e)*f^2 + B*(16*c^3*e^3 + 6*b^2
*c*e*f^2 + 24*c^2*(b*d + a*e)*f^2 + b^3*f^3))*ArcTanh[(Sqrt[c]*x)/(Sqrt[a]
- Sqrt[a + x*(b + c*x)])] + 6*c*f*(2*B*(8*c^2*d*e + 6*b*c*e^2 + 3*a*b*f^2)
+ A*(8*c^2*e^2 + 3*b^2*f^2 + 12*a*c*f^2))*ArcTanh[(Sqrt[c]*x)/(-Sqrt[a] + S
qrt[a + x*(b + c*x)])] + 24*c^(3/2)*RootSum[c^2*d - b*c*e + b^2*f + 2*Sqrt[
a]*c*e*#1 - 4*Sqrt[a]*b*f*#1 - 2*c*d*#1^2 + b*e*#1^2 + 4*a*f*#1^2 - 2*Sqrt[
a]*e*#1^3 + d*#1^4 & , (B*c^3*d*e^3*Log[x] - b*B*c^2*e^4*Log[x] - 2*B*c^3*d
^2*e*f*Log[x] + b*B*c^2*d*e^2*f*Log[x] - A*c^3*d*e^2*f*Log[x] + 2*b^2*B*c*e
^3*f*Log[x] + A*b*c^2*e^3*f*Log[x] + b*B*c^2*d^2*f^2*Log[x] + A*c^3*d^2*f^2
*Log[x] - 3*b^2*B*c*d*e*f^2*Log[x] + 2*a*B*c^2*d*e*f^2*Log[x] - b^3*B*e^2*f
^2*Log[x] - 2*A*b^2*c*e^2*f^2*Log[x] - 2*a*b*B*c*e^2*f^2*Log[x] + b^3*B*d*f
^3*Log[x] + A*b^2*c*d*f^3*Log[x] - 2*a*A*c^2*d*f^3*Log[x] + A*b^3*e*f^3*Log
[x] + 2*a*b^2*B*e*f^3*Log[x] + 2*a*A*b*c*e*f^3*Log[x] - 2*a*A*b^2*f^4*Log[x
] - a^2*b*B*f^4*Log[x] + a^2*A*c*f^4*Log[x] - B*c^3*d*e^3*Log[-Sqrt[a] + Sq
rt[a + b*x + c*x^2] - x*#1] + b*B*c^2*e^4*Log[-Sqrt[a] + Sqrt[a + b*x + c*x
^2] - x*#1] + 2*B*c^3*d^2*e*f*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1]
- b*B*c^2*d*e^2*f*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] + A*c^3*d*e^
2*f*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] - 2*b^2*B*c*e^3*f*Log[-Sqr
t[a] + Sqrt[a + b*x + c*x^2] - x*#1] - A*b*c^2*e^3*f*Log[-Sqrt[a] + Sqrt[a
+ b*x + c*x^2] - x*#1] - b*B*c^2*d^2*f^2*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^
2] - x*#1] - A*c^3*d^2*f^2*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] + 3
*b^2*B*c*d*e*f^2*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] - 2*a*B*c^2*d
*e*f^2*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] + b^3*B*e^2*f^2*Log[-Sq
rt[a] + Sqrt[a + b*x + c*x^2] - x*#1] + 2*A*b^2*c*e^2*f^2*Log[-Sqrt[a] + Sq
rt[a + b*x + c*x^2] - x*#1] + 2*a*b*B*c*e^2*f^2*Log[-Sqrt[a] + Sqrt[a + b*x
 + c*x^2] - x*#1] - b^3*B*d*f^3*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1
] - A*b^2*c*d*f^3*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] + 2*a*A*c^2*
d*f^3*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] - A*b^3*e*f^3*Log[-Sqrt[
a] + Sqrt[a + b*x + c*x^2] - x*#1] - 2*a*b^2*B*e*f^3*Log[-Sqrt[a] + Sqrt[a
+ b*x + c*x^2] - x*#1] - 2*a*A*b*c*e*f^3*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^
2] - x*#1] + 2*a*A*b^2*f^4*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] + a
^2*b*B*f^4*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] - a^2*A*c*f^4*Log[-
Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] + 2*Sqrt[a]*B*c^2*e^4*Log[x]*#1 - 6
*Sqrt[a]*B*c^2*d*e^2*f*Log[x]*#1 - 4*Sqrt[a]*b*B*c*e^3*f*Log[x]*#1 - 2*Sqrt
[a]*A*c^2*e^3*f*Log[x]*#1 + 2*Sqrt[a]*B*c^2*d^2*f^2*Log[x]*#1 + 8*Sqrt[a]*b
*B*c*d*e*f^2*Log[x]*#1 + 4*Sqrt[a]*A*c^2*d*e*f^2*Log[x]*#1 + 2*Sqrt[a]*b^2*
B*e^2*f^2*Log[x]*#1 + 4*Sqrt[a]*A*b*c*e^2*f^2*Log[x]*#1 + 4*a^(3/2)*B*c*e^2
*f^2*Log[x]*#1 - 2*Sqrt[a]*b^2*B*d*f^3*Log[x]*#1 - 4*Sqrt[a]*A*b*c*d*f^3*Lo
g[x]*#1 - 4*a^(3/2)*B*c*d*f^3*Log[x]*#1 - 2*Sqrt[a]*A*b^2*e*f^3*Log[x]*#1 -
 4*a^(3/2)*b*B*e*f^3*Log[x]*#1 - 4*a^(3/2)*A*c*e*f^3*Log[x]*#1 + 4*a^(3/2)*
A*b*f^4*Log[x]*#1 + 2*a^(5/2)*B*f^4*Log[x]*#1 - 2*Sqrt[a]*B*c^2*e^4*Log[-Sq
rt[a] + Sqrt[a + b*x + c*x^2] - x*#1]*#1 + 6*Sqrt[a]*B*c^2*d*e^2*f*Log[-Sqr
t[a] + Sqrt[a + b*x + c*x^2] - x*#1]*#1 + 4*Sqrt[a]*b*B*c*e^3*f*Log[-Sqrt[a
] + Sqrt[a + b*x + c*x^2] - x*#1]*#1 + 2*Sqrt[a]*A*c^2*e^3*f*Log[-Sqrt[a] +
 Sqrt[a + b*x + c*x^2] - x*#1]*#1 - 2*Sqrt[a]*B*c^2*d^2*f^2*Log[-Sqrt[a] +
```

```
Sqrt[a + b*x + c*x^2] - x*#1]*#1 - 8*Sqrt[a]*b*B*c*d*e*f^2*Log[-Sqrt[a] + S
qrt[a + b*x + c*x^2] - x*#1]*#1 - 4*Sqrt[a]*A*c^2*d*e*f^2*Log[-Sqrt[a] + Sq
rt[a + b*x + c*x^2] - x*#1]*#1 - 2*Sqrt[a]*b^2*B*e^2*f^2*Log[-Sqrt[a] + Sqr
t[a + b*x + c*x^2] - x*#1]*#1 - 4*Sqrt[a]*A*b*c*e^2*f^2*Log[-Sqrt[a] + Sqrt
[a + b*x + c*x^2] - x*#1]*#1 - 4*a^(3/2)*B*c*e^2*f^2*Log[-Sqrt[a] + Sqrt[a
+ b*x + c*x^2] - x*#1]*#1 + 2*Sqrt[a]*b^2*B*d*f^3*Log[-Sqrt[a] + Sqrt[a + b
*x + c*x^2] - x*#1]*#1 + 4*Sqrt[a]*A*b*c*d*f^3*Log[-Sqrt[a] + Sqrt[a + b*x
+ c*x^2] - x*#1]*#1 + 4*a^(3/2)*B*c*d*f^3*Log[-Sqrt[a] + Sqrt[a + b*x + c*x
^2] - x*#1]*#1 + 2*Sqrt[a]*A*b^2*e*f^3*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2]
 - x*#1]*#1 + 4*a^(3/2)*b*B*e*f^3*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*
#1]*#1 + 4*a^(3/2)*A*c*e*f^3*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1]*#
1 - 4*a^(3/2)*A*b*f^4*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1]*#1 - 2*a
^(5/2)*B*f^4*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1]*#1 - B*c^2*d*e^3*
Log[x]*#1^2 + 2*B*c^2*d^2*e*f*Log[x]*#1^2 + 2*b*B*c*d*e^2*f*Log[x]*#1^2 + A
*c^2*d*e^2*f*Log[x]*#1^2 - 2*b*B*c*d^2*f^2*Log[x]*#1^2 - A*c^2*d^2*f^2*Log[
x]*#1^2 - b^2*B*d*e*f^2*Log[x]*#1^2 - 2*A*b*c*d*e*f^2*Log[x]*#1^2 - 2*a*B*c
*d*e*f^2*Log[x]*#1^2 + A*b^2*d*f^3*Log[x]*#1^2 + 2*a*b*B*d*f^3*Log[x]*#1^2
+ 2*a*A*c*d*f^3*Log[x]*#1^2 - a^2*A*f^4*Log[x]*#1^2 + B*c^2*d*e^3*Log[-Sqrt
[a] + Sqrt[a + b*x + c*x^2] - x*#1]*#1^2 - 2*B*c^2*d^2*e*f*Log[-Sqrt[a] + S
qrt[a + b*x + c*x^2] - x*#1]*#1^2 - 2*b*B*c*d*e^2*f*Log[-Sqrt[a] + Sqrt[a +
 b*x + c*x^2] - x*#1]*#1^2 - A*c^2*d*e^2*f*Log[-Sqrt[a] + Sqrt[a + b*x + c*
x^2] - x*#1]*#1^2 + 2*b*B*c*d^2*f^2*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] -
x*#1]*#1^2 + A*c^2*d^2*f^2*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1]*#1^
2 + b^2*B*d*e*f^2*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1]*#1^2 + 2*A*b
*c*d*e*f^2*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1]*#1^2 + 2*a*B*c*d*e*
f^2*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1]*#1^2 - A*b^2*d*f^3*Log[-Sq
rt[a] + Sqrt[a + b*x + c*x^2] - x*#1]*#1^2 - 2*a*b*B*d*f^3*Log[-Sqrt[a] + S
qrt[a + b*x + c*x^2] - x*#1]*#1^2 - 2*a*A*c*d*f^3*Log[-Sqrt[a] + Sqrt[a + b
*x + c*x^2] - x*#1]*#1^2 + a^2*A*f^4*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] -
 x*#1]*#1^2)/(-(Sqrt[a]*c*e) + 2*Sqrt[a]*b*f + 2*c*d*#1 - b*e*#1 - 4*a*f*#1
 + 3*Sqrt[a]*e*#1^2 - 2*d*#1^3) & ])/(24*c^(3/2)*f^4)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2277 vs. 2(1027) = 2054.

Time = 1.08 (sec) , antiderivative size = 2278, normalized size of antiderivative = 2.09

method	result	size
risch	Expression too large to display	2278
default	Expression too large to display	2908

```
[In] int((B*x+A)*(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24/c*(8*B*c^2*f^2*x^2+12*A*c^2*f^2*x+14*B*b*c*f^2*x-12*B*c^2*e*f*x+30*A*b
*c*f^2-24*A*c^2*e*f+32*B*a*c*f^2+3*B*b^2*f^2-30*B*b*c*e*f-24*B*c^2*d*f+24*B
```

*c^2*e^2)*(c*x^2+b*x+a)^(1/2)/f^3+1/16/f^3/c*(1/f*(24*A*a*c^2*f^3+6*A*b^2*c
*f^3-24*A*b*c^2*e*f^2-16*A*c^3*d*f^2+16*A*c^3*e^2*f+12*B*a*b*c*f^3-24*B*a*c
^2*e*f^2-B*b^3*f^3-6*B*b^2*c*e*f^2-24*B*b*c^2*d*f^2+24*B*b*c^2*e^2*f+32*B*c
^3*d*e*f-16*B*c^3*e^3)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-
8/f^2*c*(-2*A*a*c*e*f^3*(-4*d*f+e^2)^(1/2)-2*A*f^3*b*c*d*(-4*d*f+e^2)^(1/2)
+2*A*b*c*e^2*f^2*(-4*d*f+e^2)^(1/2)+2*A*c^2*d*e*f^2*(-4*d*f+e^2)^(1/2)-2*B*
a*b*e*f^3*(-4*d*f+e^2)^(1/2)-2*B*a*c*d*f^3*(-4*d*f+e^2)^(1/2)+2*B*a*c*e^2*f
^2*(-4*d*f+e^2)^(1/2)-2*B*b*c*e^3*f*(-4*d*f+e^2)^(1/2)-3*B*c^2*d*e^2*f*(-4*
d*f+e^2)^(1/2)+6*A*b*c*d*e*f^3+6*B*a*c*d*e*f^3-8*B*b*c*d*e^2*f^2-B*b^2*e^3*
f^2+A*c^2*e^4*f-B*a^2*e*f^4-2*A*b^2*d*f^4+A*b^2*e^2*f^3+2*A*c^2*d^2*f^3+4*B
*b*c*d*e*f^2*(-4*d*f+e^2)^(1/2)+2*A*a^2*f^5-B*c^2*e^5-4*A*a*c*d*f^4+2*A*a*c
*e^2*f^3-2*A*b*c*e^3*f^2-4*A*c^2*d*e^2*f^2-4*B*a*b*d*f^4+2*B*a*b*e^2*f^3-2*
B*a*c*e^3*f^2+3*B*b^2*d*e*f^3+4*B*b*c*d^2*f^3+2*B*b*c*e^4*f-5*B*c^2*d^2*e*f
^2+5*B*c^2*d*e^3*f-2*A*a*b*e*f^4+B*a^2*f^4*(-4*d*f+e^2)^(1/2)+B*c^2*e^4*(-4
*d*f+e^2)^(1/2)+2*A*a*b*f^4*(-4*d*f+e^2)^(1/2)-A*f^3*b^2*e*(-4*d*f+e^2)^(1/
2)-A*f*c^2*e^3*(-4*d*f+e^2)^(1/2)-B*b^2*d*f^3*(-4*d*f+e^2)^(1/2)+B*f^2*b^2*
e^2*(-4*d*f+e^2)^(1/2)+B*c^2*d^2*f^2*(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)
*2^(1/2)/((b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*
d*f+c*e^2)/f^2)^(1/2)*ln(((b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*
a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2/f*(-
e+(-4*d*f+e^2)^(1/2)))+1/2*2^(1/2)*((b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1
/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2/f*(-e+(-4*d*f+e^2
)^(1/2)))^2*c+4*(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2/f*(-e+(-4*d*f+e^2)^
(1/2)))+2*(b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*
d*f+c*e^2)/f^2)^(1/2))/(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2))))-8/f^2*c*(-2*A*a*c
*e*f^3*(-4*d*f+e^2)^(1/2)-2*A*f^3*b*c*d*(-4*d*f+e^2)^(1/2)+2*A*b*c*e^2*f^2*
(-4*d*f+e^2)^(1/2)+2*A*c^2*d*e*f^2*(-4*d*f+e^2)^(1/2)-2*B*a*b*e*f^3*(-4*d*f
+e^2)^(1/2)-2*B*a*c*d*f^3*(-4*d*f+e^2)^(1/2)+2*B*a*c*e^2*f^2*(-4*d*f+e^2)^(
1/2)-2*B*b*c*e^3*f*(-4*d*f+e^2)^(1/2)-3*B*c^2*d*e^2*f*(-4*d*f+e^2)^(1/2)-6*
A*b*c*d*e*f^3-6*B*a*c*d*e*f^3+8*B*b*c*d*e^2*f^2+B*b^2*e^3*f^2-A*c^2*e^4*f+B
*a^2*e*f^4+2*A*b^2*d*f^4-A*b^2*e^2*f^3-2*A*c^2*d^2*f^3+4*B*b*c*d*e*f^2*(-4*
d*f+e^2)^(1/2)-2*A*a^2*f^5+B*c^2*e^5+4*A*a*c*d*f^4-2*A*a*c*e^2*f^3+2*A*b*c*
e^3*f^2+4*A*c^2*d*e^2*f^2+4*B*a*b*d*f^4-2*B*a*b*e^2*f^3+2*B*a*c*e^3*f^2-3*B
*b^2*d*e*f^3-4*B*b*c*d^2*f^3-2*B*b*c*e^4*f+5*B*c^2*d^2*e*f^2-5*B*c^2*d*e^3*
f+2*A*a*b*e*f^4+B*a^2*f^4*(-4*d*f+e^2)^(1/2)+B*c^2*e^4*(-4*d*f+e^2)^(1/2)+2
*A*a*b*f^4*(-4*d*f+e^2)^(1/2)-A*f^3*b^2*e*(-4*d*f+e^2)^(1/2)-A*f*c^2*e^3*(-
4*d*f+e^2)^(1/2)-B*b^2*d*f^3*(-4*d*f+e^2)^(1/2)+B*f^2*b^2*e^2*(-4*d*f+e^2)^
(1/2)+B*c^2*d^2*f^2*(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)*2^(1/2)/((-b*f*(
-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^
(1/2)*ln(((-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c
*d*f+c*e^2)/f^2+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^
(1/2))/f)+1/2*2^(1/2)*((-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*
f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c
+4/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*(-b
f(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f

```
^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{d+ex+fx^2}\,dx=\text{Timed out}$$

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{d+ex+fx^2}\,dx=\text{Timed out}$$

```
[In] integrate((B*x+A)*(c*x**2+b*x+a)**(3/2)/(f*x**2+e*x+d),x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{d+ex+fx^2}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re deta
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{d+ex+fx^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((B*x+A)*(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{d+ex+fx^2}\,dx = \int \frac{(A+B\,x)\,\left(c\,x^2+b\,x+a\right)^{3/2}}{f\,x^2+e\,x+d}\,dx$$

```
[In] int(((A + B*x)*(a + b*x + c*x^2)^(3/2))/(d + e*x + f*x^2),x)
```

```
[Out] int(((A + B*x)*(a + b*x + c*x^2)^(3/2))/(d + e*x + f*x^2), x)
```

3.21 $\int \frac{A+Bx}{(a+bx+cx^2)\sqrt{d+ex+fx^2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 416

$$\int \frac{A+Bx}{(a+bx+cx^2)\sqrt{d+ex+fx^2}}\,dx = \frac{\left(bB-2Ac-B\sqrt{b^2-4ac}\right)\operatorname{arctanh}\left(\frac{4cd-\left(b-\sqrt{b^2-4ac}\right)e+2\left(ce-\left(b-\sqrt{b^2-4ac}\right)f\right)x}{2\sqrt{2}\sqrt{2c^2d-bce+b^2f-2acf+\sqrt{b^2-4ac}(ce-bf)}\sqrt{d+ex+fx^2}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{2c^2d-bce+b^2f-2acf+\sqrt{b^2-4ac}(ce-bf)}} + \frac{\left(2Ac-B\left(b+\sqrt{b^2-4ac}\right)\right)\operatorname{arctanh}\left(\frac{4cd-\left(b+\sqrt{b^2-4ac}\right)e+2\left(ce-\left(b+\sqrt{b^2-4ac}\right)f\right)x}{2\sqrt{2}\sqrt{2c^2d-bce+b^2f-2acf-\sqrt{b^2-4ac}(ce-bf)}\sqrt{d+ex+fx^2}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{2c^2d-bce+b^2f-2acf-\sqrt{b^2-4ac}(ce-bf)}}$$

```
[Out] 1/2*arctanh(1/4*(4*c*d-e*(b+(-4*a*c+b^2)^(1/2))+2*x*(c*e-f*(b+(-4*a*c+b^2)^
(1/2))))*2^(1/2)/(f*x^2+e*x+d)^(1/2)/(2*c^2*d-b*c*e+b^2*f-2*a*c*f-(-b*f+c*e
)*(-4*a*c+b^2)^(1/2))^(1/2))*(2*A*c-B*(b+(-4*a*c+b^2)^(1/2)))*2^(1/2)/(-4*a
*c+b^2)^(1/2)/(2*c^2*d-b*c*e+b^2*f-2*a*c*f-(-b*f+c*e)*(-4*a*c+b^2)^(1/2))^(
1/2)+1/2*arctanh(1/4*(4*c*d+2*x*(c*e-f*(b-(-4*a*c+b^2)^(1/2)))-e*(b-(-4*a*c
+b^2)^(1/2)))*2^(1/2)/(f*x^2+e*x+d)^(1/2)/(2*c^2*d-b*c*e+b^2*f-2*a*c*f+(-b*
f+c*e)*(-4*a*c+b^2)^(1/2))^(1/2))*(B*b-2*A*c-B*(-4*a*c+b^2)^(1/2))*2^(1/2)/
(-4*a*c+b^2)^(1/2)/(2*c^2*d-b*c*e+b^2*f-2*a*c*f+(-b*f+c*e)*(-4*a*c+b^2)^(1/
2))^(1/2)
```

Rubi [A] (verified)

Time = 1.74 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {1046, 738, 212}

$$\int \frac{A+Bx}{(a+bx+cx^2)\sqrt{d+ex+fx^2}}\,dx$$
$$= \frac{\left(-B\sqrt{b^2-4ac}-2Ac+bB\right)\operatorname{arctanh}\left(\frac{2x\left(ce-f\left(b-\sqrt{b^2-4ac}\right)\right)-e\left(b-\sqrt{b^2-4ac}\right)+4cd}{2\sqrt{2}\sqrt{d+ex+fx^2}\sqrt{\sqrt{b^2-4ac}(ce-bf)-2acf+b^2f-bce+2c^2d}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}(ce-bf)-2acf+b^2f-bce+2c^2d}}$$
$$+\frac{\left(2Ac-B\left(\sqrt{b^2-4ac}+b\right)\right)\operatorname{arctanh}\left(\frac{2x\left(ce-f\left(\sqrt{b^2-4ac}+b\right)\right)-e\left(\sqrt{b^2-4ac}+b\right)+4cd}{2\sqrt{2}\sqrt{d+ex+fx^2}\sqrt{-\sqrt{b^2-4ac}(ce-bf)-2acf+b^2f-bce+2c^2d}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\sqrt{b^2-4ac}(ce-bf)-2acf+b^2f-bce+2c^2d}}$$

```
[In] Int[(A + B*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]),x]
```

```
[Out] ((b*B - 2*A*c - B*Sqrt[b^2 - 4*a*c])*ArcTanh[(4*c*d - (b - Sqrt[b^2 - 4*a*c
])*e + 2*(c*e - (b - Sqrt[b^2 - 4*a*c])*f)*x)/(2*Sqrt[2]*Sqrt[2*c^2*d - b*c
*e + b^2*f - 2*a*c*f + Sqrt[b^2 - 4*a*c]*(c*e - b*f)]*Sqrt[d + e*x + f*x^2]
)])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c^2*d - b*c*e + b^2*f - 2*a*c*f + Sqr
t[b^2 - 4*a*c]*(c*e - b*f)]) + ((2*A*c - B*(b + Sqrt[b^2 - 4*a*c]))*ArcTanh
[(4*c*d - (b + Sqrt[b^2 - 4*a*c])*e + 2*(c*e - (b + Sqrt[b^2 - 4*a*c])*f)*x
)/(2*Sqrt[2]*Sqrt[2*c^2*d - b*c*e + b^2*f - 2*a*c*f - Sqrt[b^2 - 4*a*c]*(c*
e - b*f)]*Sqrt[d + e*x + f*x^2]))])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c^2*d
- b*c*e + b^2*f - 2*a*c*f - Sqrt[b^2 - 4*a*c]*(c*e - b*f)])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1046

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
```

```
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\text{integral} = \frac{\left(2Ac - B\left(b - \sqrt{b^2-4ac}\right)\right)\int \frac{1}{\left(b-\sqrt{b^2-4ac}+2cx\right)\sqrt{d+ex+fx^2}}\,dx}{\sqrt{b^2-4ac}} - \frac{\left(2Ac - B\left(b + \sqrt{b^2-4ac}\right)\right)\int \frac{1}{\left(b+\sqrt{b^2-4ac}+2cx\right)\sqrt{d+ex+fx^2}}\,dx}{\sqrt{b^2-4ac}}$$

$$= \frac{\left(2\left(bB - 2Ac - B\sqrt{b^2-4ac}\right)\right)\text{Subst}\left(\int \frac{1}{16c^2d-8c\left(b-\sqrt{b^2-4ac}\right)e+4\left(b-\sqrt{b^2-4ac}\right)^2f-x^2}\,dx, x, \frac{4cd-\left(b-\sqrt{b^2-4ac}\right.}{}\right.}{\sqrt{b^2-4ac}} + \frac{\left(2\left(2Ac - B\left(b+\sqrt{b^2-4ac}\right)\right)\right)\text{Subst}\left(\int \frac{1}{16c^2d-8c\left(b+\sqrt{b^2-4ac}\right)e+4\left(b+\sqrt{b^2-4ac}\right)^2f-x^2}\,dx, x, \frac{4cd-\left(b+\sqrt{b^2}\right.}{}\right.}{\sqrt{b^2-4ac}}$$

$$= \frac{\left(bB - 2Ac - B\sqrt{b^2-4ac}\right)\tanh^{-1}\left(\frac{4cd-\left(b-\sqrt{b^2-4ac}\right)e+2\left(ce-\left(b-\sqrt{b^2-4ac}\right)f\right)x}{2\sqrt{2}\sqrt{2c^2d-bce+b^2f-2acf+\sqrt{b^2-4ac}(ce-bf)}\sqrt{d+ex+fx^2}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{2c^2d-bce+b^2f-2acf+\sqrt{b^2-4ac}(ce-bf)}} + \frac{\left(2Ac - B\left(b+\sqrt{b^2-4ac}\right)\right)\tanh^{-1}\left(\frac{4cd-\left(b+\sqrt{b^2-4ac}\right)e+2\left(ce-\left(b+\sqrt{b^2-4ac}\right)f\right)x}{2\sqrt{2}\sqrt{2c^2d-bce+b^2f-2acf-\sqrt{b^2-4ac}(ce-bf)}\sqrt{d+ex+fx^2}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{2c^2d-bce+b^2f-2acf-\sqrt{b^2-4ac}(ce-bf)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.44 (sec) , antiderivative size = 278, normalized size of antiderivative = 0.67

$$\int \frac{A+Bx}{(a+bx+cx^2)\sqrt{d+ex+fx^2}}\,dx = -\text{RootSum}\Big[cd^2 - bde + ae^2 + 2bd\sqrt{f}\#1 - 4ae\sqrt{f}\#1 - 2cd\#1^2 + be\#1^2 + 4af\#1^2 - 2b\sqrt{f}\#1^3 + c\#1^4\&, \frac{Bd\log\left(-\sqrt{f}x+\sqrt{d+ex+fx^2}-\#1\right) - Ae\log\left(-\sqrt{f}x+\sqrt{d+ex+fx^2}-\#1\right) + 2A\sqrt{f}\,\text{lo}}{bd\sqrt{f} - 2ae\sqrt{f} - 2cd\#1 + be\#1 + 4}$$

```
[In] Integrate[(A + B*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]),x]
```

```
[Out] -RootSum[c*d^2 - b*d*e + a*e^2 + 2*b*d*Sqrt[f]*#1 - 4*a*e*Sqrt[f]*#1 - 2*c*
d*#1^2 + b*e*#1^2 + 4*a*f*#1^2 - 2*b*Sqrt[f]*#1^3 + c*#1^4 & , (B*d*Log[-(S
qrt[f]*x) + Sqrt[d + e*x + f*x^2] - #1] - A*e*Log[-(Sqrt[f]*x) + Sqrt[d + e
*x + f*x^2] - #1] + 2*A*Sqrt[f]*Log[-(Sqrt[f]*x) + Sqrt[d + e*x + f*x^2] -
#1]*#1 - B*Log[-(Sqrt[f]*x) + Sqrt[d + e*x + f*x^2] - #1]*#1^2)/(b*d*Sqrt[f
] - 2*a*e*Sqrt[f] - 2*c*d*#1 + b*e*#1 + 4*a*f*#1 - 3*b*Sqrt[f]*#1^2 + 2*c*#
1^3) & ]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 804 vs. $2(370) = 740$.

Time = 1.01 (sec) , antiderivative size = 805, normalized size of antiderivative = 1.94

method	result
default	$-\frac{\left(-2Ac+B\sqrt{-4ac+b^2}+Bb\right)\ln\left(-\frac{-\sqrt{-4ac+b^2}\,bf+ce\sqrt{-4ac+b^2}+2acf-b^2f+bce-2c^2d}{c^2}-\frac{\left(f\sqrt{-4ac+b^2}+bf-ce\right)\left(x+\frac{b+\sqrt{-4ac+b^2}}{2c}\right)}{c}+\frac{\sqrt{-}}{}\right.}{\sqrt{-4ac+b^2}\,}$

```
[In] int((B*x+A)/(c*x^2+b*x+a)/(f*x^2+e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(-2*A*c+B*(-4*a*c+b^2)^(1/2)+B*b)/(-4*a*c+b^2)^(1/2)/c/(-2*(-(-4*a*c+b^2)^
(1/2)*b*f+c*e*(-4*a*c+b^2)^(1/2)+2*a*c*f-b^2*f+b*c*e-2*c^2*d)/c^2)^(1/2)*ln
((-(-(-4*a*c+b^2)^(1/2)*b*f+c*e*(-4*a*c+b^2)^(1/2)+2*a*c*f-b^2*f+b*c*e-2*c^
2*d)/c^2-(f*(-4*a*c+b^2)^(1/2)+b*f-c*e)/c*(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)+
1/2*(-2*(-(-4*a*c+b^2)^(1/2)*b*f+c*e*(-4*a*c+b^2)^(1/2)+2*a*c*f-b^2*f+b*c*e
-2*c^2*d)/c^2)^(1/2)*(4*f*(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)^2-4*(f*(-4*a*c+b
^2)^(1/2)+b*f-c*e)/c*(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)-2*(-(-4*a*c+b^2)^(1/2
)*b*f+c*e*(-4*a*c+b^2)^(1/2)+2*a*c*f-b^2*f+b*c*e-2*c^2*d)/c^2)^(1/2))/(x+1/
2*(b+(-4*a*c+b^2)^(1/2))/c))-(2*A*c+B*(-4*a*c+b^2)^(1/2)-B*b)/(-4*a*c+b^2)^
(1/2)/c/(-2*((-4*a*c+b^2)^(1/2)*b*f-c*e*(-4*a*c+b^2)^(1/2)+2*a*c*f-b^2*f+b*
c*e-2*c^2*d)/c^2)^(1/2)*ln((-((-4*a*c+b^2)^(1/2)*b*f-c*e*(-4*a*c+b^2)^(1/2)
+2*a*c*f-b^2*f+b*c*e-2*c^2*d)/c^2-(-f*(-4*a*c+b^2)^(1/2)+b*f-c*e)/c*(x-1/2/
c*(-b+(-4*a*c+b^2)^(1/2)))+1/2*(-2*((-4*a*c+b^2)^(1/2)*b*f-c*e*(-4*a*c+b^2)
^(1/2)+2*a*c*f-b^2*f+b*c*e-2*c^2*d)/c^2)^(1/2)*(4*f*(x-1/2/c*(-b+(-4*a*c+b^
2)^(1/2)))^2-4*(-f*(-4*a*c+b^2)^(1/2)+b*f-c*e)/c*(x-1/2/c*(-b+(-4*a*c+b^2)^
(1/2)))-2*((-4*a*c+b^2)^(1/2)*b*f-c*e*(-4*a*c+b^2)^(1/2)+2*a*c*f-b^2*f+b*c*
e-2*c^2*d)/c^2)^(1/2))/(x-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 22103 vs. 2(369) = 738.

Time = 277.46 (sec) , antiderivative size = 22103, normalized size of antiderivative = 53.13

$$\int \frac{A+Bx}{(a+bx+cx^2)\sqrt{d+ex+fx^2}}\,dx = \text{Too large to display}$$

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)/(f*x^2+e*x+d)^(1/2),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F]

$$\int \frac{A+Bx}{(a+bx+cx^2)\sqrt{d+ex+fx^2}}\,dx = \int \frac{A+Bx}{(a+bx+cx^2)\sqrt{d+ex+fx^2}}\,dx$$

```
[In] integrate((B*x+A)/(c*x**2+b*x+a)/(f*x**2+e*x+d)**(1/2),x)
[Out] Integral((A + B*x)/((a + b*x + c*x**2)*sqrt(d + e*x + f*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx}{(a+bx+cx^2)\sqrt{d+ex+fx^2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)/(f*x^2+e*x+d)^(1/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+Bx}{(a+bx+cx^2)\sqrt{d+ex+fx^2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)/(f*x^2+e*x+d)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{%%{poly1[%%%{-4,[3,2,0]%%%}+%%%{16,[1,3,1]%%%},%%%{4,[4,2,0]%
%%}+%%
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(a+bx+cx^2)\sqrt{d+ex+fx^2}}\,dx=\int \frac{A+B\,x}{\left(c\,x^2+b\,x+a\right)\,\sqrt{f\,x^2+e\,x+d}}\,dx$$

```
[In] int((A + B*x)/((a + b*x + c*x^2)*(d + e*x + f*x^2)^(1/2)),x)
[Out] int((A + B*x)/((a + b*x + c*x^2)*(d + e*x + f*x^2)^(1/2)), x)
```

3.22 $\int \frac{A+Bx}{(a+cx^2)\sqrt{d+ex+fx^2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 780

$$\int \frac{A+Bx}{(a+cx^2)\sqrt{d+ex+fx^2}}\,dx$$

$$= \frac{\sqrt{aBe+A\left(cd-af-\sqrt{c^2d^2+a^2f^2+ac\left(e^2-2df\right)}\right)}\sqrt{-Ace+B\left(cd-af+\sqrt{c^2d^2+a^2f^2+ac\left(e^2-\right.}\right.}}{\sqrt{2}\sqrt{a}\sqrt{c}\sqrt{e}\sqrt{c^2d^2}}$$

$$-\frac{\sqrt{-Ace+B\left(cd-af-\sqrt{c^2d^2+a^2f^2+ac\left(e^2-2df\right)}\right)}\sqrt{aBe+A\left(cd-af+\sqrt{c^2d^2+a^2f^2+ac\left(e\right.}\right.}}{\sqrt{2}\sqrt{a}\sqrt{c}\sqrt{e}\sqrt{c^2}}$$

```
[Out] -1/2*arctanh(1/2*e^(1/2)*(a*(A*c*e-B*(c*d-a*f-(c^2*d^2+a^2*f^2+a*c*(-2*d*f+
e^2))^(1/2)))-c*x*(B*a*e+A*(c*d-a*f+(c^2*d^2+a^2*f^2+a*c*(-2*d*f+e^2))^(1/2
))))*2^(1/2)/a^(1/2)/c^(1/2)/(f*x^2+e*x+d)^(1/2)/(-A*c*e+B*(c*d-a*f-(c^2*d^
2+a^2*f^2+a*c*(-2*d*f+e^2))^(1/2)))^(1/2)/(B*a*e+A*(c*d-a*f+(c^2*d^2+a^2*f^
2+a*c*(-2*d*f+e^2))^(1/2)))^(1/2))*(-A*c*e+B*(c*d-a*f-(c^2*d^2+a^2*f^2+a*c*
(-2*d*f+e^2))^(1/2)))^(1/2)*(B*a*e+A*(c*d-a*f+(c^2*d^2+a^2*f^2+a*c*(-2*d*f+
e^2))^(1/2)))^(1/2)*2^(1/2)/a^(1/2)/c^(1/2)/e^(1/2)/(c^2*d^2+a^2*f^2+a*c*(-
2*d*f+e^2))^(1/2)+1/2*arctanh(1/2*e^(1/2)*(-c*x*(B*a*e+A*(c*d-a*f-(c^2*d^2+
a^2*f^2+a*c*(-2*d*f+e^2))^(1/2)))+a*(A*c*e-B*(c*d-a*f+(c^2*d^2+a^2*f^2+a*c*
(-2*d*f+e^2))^(1/2))))*2^(1/2)/a^(1/2)/c^(1/2)/(f*x^2+e*x+d)^(1/2)/(B*a*e+A
*(c*d-a*f-(c^2*d^2+a^2*f^2+a*c*(-2*d*f+e^2))^(1/2)))^(1/2)/(-A*c*e+B*(c*d-a
*f+(c^2*d^2+a^2*f^2+a*c*(-2*d*f+e^2))^(1/2)))^(1/2))*(B*a*e+A*(c*d-a*f-(c^2
*d^2+a^2*f^2+a*c*(-2*d*f+e^2))^(1/2)))^(1/2)*(-A*c*e+B*(c*d-a*f+(c^2*d^2+a^
2*f^2+a*c*(-2*d*f+e^2))^(1/2)))^(1/2)*2^(1/2)/a^(1/2)/c^(1/2)/e^(1/2)/(c^2*
d^2+a^2*f^2+a*c*(-2*d*f+e^2))^(1/2)
```

Rubi [A] (verified)

Time = 2.38 (sec) , antiderivative size = 780, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1050, 1044, 214}

$$\int \frac{A+Bx}{(a+cx^2)\sqrt{d+ex+fx^2}}\,dx$$

$$= \frac{\sqrt{A\left(-\sqrt{a^2f^2+ac\left(e^2-2df\right)+c^2d^2}-af+cd\right)+aBe}\sqrt{B\left(\sqrt{a^2f^2+ac\left(e^2-2df\right)+c^2d^2}-af+cd\right)}}{\sqrt{2}\sqrt{a}\sqrt{c}\sqrt{e}\sqrt{a^2f^2+}}$$

$$-\frac{\sqrt{B\left(-\sqrt{a^2f^2+ac\left(e^2-2df\right)+c^2d^2}-af+cd\right)-Ace}\sqrt{A\left(\sqrt{a^2f^2+ac\left(e^2-2df\right)+c^2d^2}-af+cd\right)}}{\sqrt{2}\sqrt{a}\sqrt{c}\sqrt{e}\sqrt{a^2f^2}}$$

```
[In] Int[(A + B*x)/((a + c*x^2)*Sqrt[d + e*x + f*x^2]),x]
```

```
[Out] (Sqrt[a*B*e + A*(c*d - a*f - Sqrt[c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f)])]*
Sqrt[-(A*c*e) + B*(c*d - a*f + Sqrt[c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f)])
]*ArcTanh[(Sqrt[e]*(a*(A*c*e - B*(c*d - a*f + Sqrt[c^2*d^2 + a^2*f^2 + a*c*
(e^2 - 2*d*f)])) - c*(a*B*e + A*(c*d - a*f - Sqrt[c^2*d^2 + a^2*f^2 + a*c*(
e^2 - 2*d*f)]))*x))/(Sqrt[2]*Sqrt[a]*Sqrt[c]*Sqrt[a*B*e + A*(c*d - a*f - Sq
rt[c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f)])]*Sqrt[-(A*c*e) + B*(c*d - a*f +
Sqrt[c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f)])]*Sqrt[d + e*x + f*x^2])])/(Sqr
t[2]*Sqrt[a]*Sqrt[c]*Sqrt[e]*Sqrt[c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f)]) -
 (Sqrt[-(A*c*e) + B*(c*d - a*f - Sqrt[c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f)
])]*Sqrt[a*B*e + A*(c*d - a*f + Sqrt[c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f)]
)]*ArcTanh[(Sqrt[e]*(a*(A*c*e - B*(c*d - a*f - Sqrt[c^2*d^2 + a^2*f^2 + a*c
*(e^2 - 2*d*f)])) - c*(a*B*e + A*(c*d - a*f + Sqrt[c^2*d^2 + a^2*f^2 + a*c*
(e^2 - 2*d*f)]))*x))/(Sqrt[2]*Sqrt[a]*Sqrt[c]*Sqrt[-(A*c*e) + B*(c*d - a*f
- Sqrt[c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f)])]*Sqrt[a*B*e + A*(c*d - a*f +
 Sqrt[c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f)])]*Sqrt[d + e*x + f*x^2])])/(Sq
rt[2]*Sqrt[a]*Sqrt[c]*Sqrt[e]*Sqrt[c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f)])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1044

```
Int[((g_) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f
_.)*(x_)^2]), x_Symbol] :> Dist[-2*a*g*h, Subst[Int[1/Simp[2*a^2*g*h*c + a*
e*x^2, x], x], x, Simp[a*h - g*c*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[
```

```
{a, c, d, e, f, g, h}, x] && EqQ[a*h^2*e + 2*g*h*(c*d - a*f) - g^2*c*e, 0]
```

Rule 1050

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 + a*c*e^2, 2]}, Dist
[1/(2*q), Int[Simp[(-a)*h*e - g*(c*d - a*f - q) + (h*(c*d - a*f + q) - g*c*
e)*x, x]/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Si
mp[(-a)*h*e - g*(c*d - a*f + q) + (h*(c*d - a*f - q) - g*c*e)*x, x]/((a + c
*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] &&
 NeQ[e^2 - 4*d*f, 0] && NegQ[(-a)*c]
```

Rubi steps

$$\text{integral} = -\frac{\int \frac{-aBe-A\left(cd-af+\sqrt{c^2d^2+a^2f^2+ac(e^2-2df)}\right)+\left(-Ace+B\left(cd-af-\sqrt{c^2d^2+a^2f^2+ac(e^2-2df)}\right)\right)x}{(a+cx^2)\sqrt{d+ex+fx^2}}\,dx}{2\sqrt{c^2d^2+a^2f^2+ac\left(e^2-2df\right)}} + \frac{\int \frac{-aBe-A\left(cd-af-\sqrt{c^2d^2+a^2f^2+ac(e^2-2df)}\right)+\left(-Ace+B\left(cd-af+\sqrt{c^2d^2+a^2f^2+ac(e^2-2df)}\right)\right)x}{(a+cx^2)\sqrt{d+ex+fx^2}}\,dx}{2\sqrt{c^2d^2+a^2f^2+ac\left(e^2-2df\right)}}$$

$$= \frac{\left(a\left(Ace-B\left(cd-af-\sqrt{c^2d^2+a^2f^2+ac\left(e^2-2df\right)}\right)\right)\right)\left(aBe+A\left(cd-af+\sqrt{c^2d^2+a^2f^2+\cdots\right.\right.}{\cdots} - \frac{\left(a\left(aBe+A\left(cd-af-\sqrt{c^2d^2+a^2f^2+ac\left(e^2-2df\right)}\right)\right)\right)\left(Ace-B\left(cd-af+\sqrt{c^2d^2+a^2f^2\cdots\right.\right.}{\cdots}$$

$$= \frac{\sqrt{aBe+A\left(cd-af-\sqrt{c^2d^2+a^2f^2+ac\left(e^2-2df\right)}\right)}\sqrt{-Ace+B\left(cd-af+\sqrt{c^2d^2+a^2f^2+\cdots\right.}}{\sqrt{2}\sqrt{a}\sqrt{c}\sqrt{e\cdots}} - \frac{\sqrt{-Ace+B\left(cd-af-\sqrt{c^2d^2+a^2f^2+ac\left(e^2-2df\right)}\right)}\sqrt{aBe+A\left(cd-af+\sqrt{c^2d^2+a^2f^2\cdots\right.}}{\sqrt{2}\sqrt{a}\sqrt{c}\cdots}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.37 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.28

$$\int \frac{A+Bx}{(a+cx^2)\sqrt{d+ex+fx^2}}\,dx = \frac{1}{2}\text{RootSum}\left[cd^2+ae^2-4ae\sqrt{f}\#1-2cd\#1^2+4af\#1^2\right.$$
$$\left.+c\#1^4\&,\frac{Bd\log\left(-\sqrt{f}x+\sqrt{d+ex+fx^2}-\#1\right)-Ae\log\left(-\sqrt{f}x+\sqrt{d+ex+fx^2}-\#1\right)+2A\sqrt{f}\log}{ae\sqrt{f}+cd\#1-2af\#}\right.$$

```
[In] Integrate[(A + B*x)/((a + c*x^2)*Sqrt[d + e*x + f*x^2]),x]
```

```
[Out] RootSum[c*d^2 + a*e^2 - 4*a*e*Sqrt[f]*#1 - 2*c*d*#1^2 + 4*a*f*#1^2 + c*#1^4
 & , (B*d*Log[-(Sqrt[f]*x) + Sqrt[d + e*x + f*x^2] - #1] - A*e*Log[-(Sqrt[f
]*x) + Sqrt[d + e*x + f*x^2] - #1] + 2*A*Sqrt[f]*Log[-(Sqrt[f]*x) + Sqrt[d
+ e*x + f*x^2] - #1]*#1 - B*Log[-(Sqrt[f]*x) + Sqrt[d + e*x + f*x^2] - #1]*
#1^2)/(a*e*Sqrt[f] + c*d*#1 - 2*a*f*#1 - c*#1^3) & ]/2
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 425, normalized size of antiderivative = 0.54

method	result
default	$-\frac{(-Ac+B\sqrt{-ac})\ln\left(\frac{-\frac{2(e\sqrt{-ac}+fa-cd)}{c}+\frac{(-2\sqrt{-ac}\,f+ce)\left(x+\frac{\sqrt{-ac}}{c}\right)}{c}+2\sqrt{-\frac{e\sqrt{-ac}+fa-cd}{c}}\sqrt{f\left(x+\frac{\sqrt{-ac}}{c}\right)^2+\frac{(-2\sqrt{-ac}\,f+ce)\left(x+\frac{\sqrt{-ac}}{c}\right.}{c}}}{x+\frac{\sqrt{-ac}}{c}}\right.}{2\sqrt{-ac}\,c\sqrt{-\frac{e\sqrt{-ac}+fa-cd}{c}}}$

```
[In] int((B*x+A)/(c*x^2+a)/(f*x^2+e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(-A*c+B*(-a*c)^(1/2))/(-a*c)^(1/2)/c/(-(e*(-a*c)^(1/2)+f*a-c*d)/c)^(1/
2)*ln((-2*(e*(-a*c)^(1/2)+f*a-c*d)/c+1/c*(-2*(-a*c)^(1/2)*f+c*e)*(x+(-a*c)^
(1/2)/c)+2*(-(e*(-a*c)^(1/2)+f*a-c*d)/c)^(1/2)*(f*(x+(-a*c)^(1/2)/c)^2+1/c*
(-2*(-a*c)^(1/2)*f+c*e)*(x+(-a*c)^(1/2)/c)-(e*(-a*c)^(1/2)+f*a-c*d)/c)^(1/2
))/(x+(-a*c)^(1/2)/c))-1/2*(A*c+B*(-a*c)^(1/2))/(-a*c)^(1/2)/c/(-(-e*(-a*c)
^(1/2)+f*a-c*d)/c)^(1/2)*ln((-2*(-e*(-a*c)^(1/2)+f*a-c*d)/c+(2*(-a*c)^(1/2)
*f+c*e)/c*(x-(-a*c)^(1/2)/c)+2*(-(-e*(-a*c)^(1/2)+f*a-c*d)/c)^(1/2)*(f*(x-(
-a*c)^(1/2)/c)^2+(2*(-a*c)^(1/2)*f+c*e)/c*(x-(-a*c)^(1/2)/c)-(-e*(-a*c)^(1/
2)+f*a-c*d)/c)^(1/2))/(x-(-a*c)^(1/2)/c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6861 vs. 2(703) = 1406.

Time = 28.21 (sec) , antiderivative size = 6861, normalized size of antiderivative = 8.80

$$\int \frac{A+Bx}{(a+cx^2)\sqrt{d+ex+fx^2}}\,dx = \text{Too large to display}$$

```
[In] integrate((B*x+A)/(c*x^2+a)/(f*x^2+e*x+d)^(1/2),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F]

$$\int \frac{A+Bx}{(a+cx^2)\sqrt{d+ex+fx^2}}\,dx = \int \frac{A+Bx}{(a+cx^2)\sqrt{d+ex+fx^2}}\,dx$$

```
[In] integrate((B*x+A)/(c*x**2+a)/(f*x**2+e*x+d)**(1/2),x)
[Out] Integral((A + B*x)/((a + c*x**2)*sqrt(d + e*x + f*x**2)), x)
```

Maxima [F]

$$\int \frac{A+Bx}{(a+cx^2)\sqrt{d+ex+fx^2}}\,dx = \int \frac{Bx+A}{(cx^2+a)\sqrt{fx^2+ex+d}}\,dx$$

```
[In] integrate((B*x+A)/(c*x^2+a)/(f*x^2+e*x+d)^(1/2),x, algorithm="maxima")
[Out] integrate((B*x + A)/((c*x^2 + a)*sqrt(f*x^2 + e*x + d)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+Bx}{(a+cx^2)\sqrt{d+ex+fx^2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((B*x+A)/(c*x^2+a)/(f*x^2+e*x+d)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const
gen &
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(a+cx^2)\sqrt{d+ex+fx^2}}\,dx = \int \frac{A+B\,x}{(c\,x^2+a)\,\sqrt{f\,x^2+e\,x+d}}\,dx$$

```
[In] int((A + B*x)/((a + c*x^2)*(d + e*x + f*x^2)^(1/2)),x)
[Out] int((A + B*x)/((a + c*x^2)*(d + e*x + f*x^2)^(1/2)), x)
```

3.23 $\int \frac{A+Bx}{(a+bx+cx^2)\sqrt{d+fx^2}}\,dx$

Optimal result

Integrand size = 29, antiderivative size = 302

$$\int \frac{A+Bx}{(a+bx+cx^2)\sqrt{d+fx^2}}\,dx = \frac{\left(bB-2Ac-B\sqrt{b^2-4ac}\right)\operatorname{arctanh}\left(\frac{2cd-\left(b-\sqrt{b^2-4ac}\right)fx}{\sqrt{2}\sqrt{2c^2d-2acf+b\left(b-\sqrt{b^2-4ac}\right)f}\sqrt{d+fx^2}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{2c^2d-2acf+b\left(b-\sqrt{b^2-4ac}\right)f}} + \frac{\left(2Ac-B\left(b+\sqrt{b^2-4ac}\right)\right)\operatorname{arctanh}\left(\frac{2cd-\left(b+\sqrt{b^2-4ac}\right)fx}{\sqrt{2}\sqrt{2c^2d-2acf+b\left(b+\sqrt{b^2-4ac}\right)f}\sqrt{d+fx^2}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{2c^2d-2acf+b\left(b+\sqrt{b^2-4ac}\right)f}}$$

```
[Out] 1/2*arctanh(1/2*(2*c*d-f*x*(b-(-4*a*c+b^2)^(1/2)))*2^(1/2)/(f*x^2+d)^(1/2)/
(2*c^2*d-2*a*c*f+b*f*(b-(-4*a*c+b^2)^(1/2)))^(1/2))*(B*b-2*A*c-B*(-4*a*c+b^
2)^(1/2))*2^(1/2)/(-4*a*c+b^2)^(1/2)/(2*c^2*d-2*a*c*f+b*f*(b-(-4*a*c+b^2)^(
1/2)))^(1/2)+1/2*arctanh(1/2*(2*c*d-f*x*(b+(-4*a*c+b^2)^(1/2)))*2^(1/2)/(f*
x^2+d)^(1/2)/(2*c^2*d-2*a*c*f+b*f*(b+(-4*a*c+b^2)^(1/2)))^(1/2))*(2*A*c-B*(
b+(-4*a*c+b^2)^(1/2)))*2^(1/2)/(-4*a*c+b^2)^(1/2)/(2*c^2*d-2*a*c*f+b*f*(b+(
-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {1048, 739, 212}

$$\int \frac{A+Bx}{(a+bx+cx^2)\sqrt{d+fx^2}}\,dx$$
$$=\frac{\left(-B\sqrt{b^2-4ac}-2Ac+bB\right)\operatorname{arctanh}\left(\frac{2cd-fx\left(b-\sqrt{b^2-4ac}\right)}{\sqrt{2}\sqrt{d+fx^2}\sqrt{bf\left(b-\sqrt{b^2-4ac}\right)-2acf+2c^2d}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{bf\left(b-\sqrt{b^2-4ac}\right)-2acf+2c^2d}}$$
$$+\frac{\left(2Ac-B\left(\sqrt{b^2-4ac}+b\right)\right)\operatorname{arctanh}\left(\frac{2cd-fx\left(\sqrt{b^2-4ac}+b\right)}{\sqrt{2}\sqrt{d+fx^2}\sqrt{bf\left(\sqrt{b^2-4ac}+b\right)-2acf+2c^2d}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{bf\left(\sqrt{b^2-4ac}+b\right)-2acf+2c^2d}}$$

```
[In] Int[(A + B*x)/((a + b*x + c*x^2)*Sqrt[d + f*x^2]),x]
```

```
[Out] ((b*B - 2*A*c - B*Sqrt[b^2 - 4*a*c])*ArcTanh[(2*c*d - (b - Sqrt[b^2 - 4*a*c
])*f*x)/(Sqrt[2]*Sqrt[2*c^2*d - 2*a*c*f + b*(b - Sqrt[b^2 - 4*a*c])*f]*Sqrt
[d + f*x^2])])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c^2*d - 2*a*c*f + b*(b - S
qrt[b^2 - 4*a*c])*f]) + ((2*A*c - B*(b + Sqrt[b^2 - 4*a*c]))*ArcTanh[(2*c*d
 - (b + Sqrt[b^2 - 4*a*c])*f*x)/(Sqrt[2]*Sqrt[2*c^2*d - 2*a*c*f + b*(b + Sq
rt[b^2 - 4*a*c])*f]*Sqrt[d + f*x^2])])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c^
2*d - 2*a*c*f + b*(b + Sqrt[b^2 - 4*a*c])*f])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1048

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
```

```
b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\text{integral} = \frac{\left(2Ac - B\left(b - \sqrt{b^2-4ac}\right)\right)\int \frac{1}{\left(b-\sqrt{b^2-4ac}+2cx\right)\sqrt{d+fx^2}}\,dx}{\sqrt{b^2-4ac}} - \frac{\left(2Ac - B\left(b + \sqrt{b^2-4ac}\right)\right)\int \frac{1}{\left(b+\sqrt{b^2-4ac}+2cx\right)\sqrt{d+fx^2}}\,dx}{\sqrt{b^2-4ac}}$$

$$= -\frac{\left(2Ac - B\left(b - \sqrt{b^2-4ac}\right)\right)\text{Subst}\left(\int \frac{1}{4c^2d+\left(b-\sqrt{b^2-4ac}\right)^2f-x^2}\,dx, x, \frac{2cd-\left(b-\sqrt{b^2-4ac}\right)fx}{\sqrt{d+fx^2}}\right)}{\sqrt{b^2-4ac}} + \frac{\left(2Ac - B\left(b + \sqrt{b^2-4ac}\right)\right)\text{Subst}\left(\int \frac{1}{4c^2d+\left(b+\sqrt{b^2-4ac}\right)^2f-x^2}\,dx, x, \frac{2cd-\left(b+\sqrt{b^2-4ac}\right)fx}{\sqrt{d+fx^2}}\right)}{\sqrt{b^2-4ac}}$$

$$= \frac{\left(bB - 2Ac - B\sqrt{b^2-4ac}\right)\tanh^{-1}\left(\frac{2cd-\left(b-\sqrt{b^2-4ac}\right)fx}{\sqrt{2}\sqrt{2c^2d-2acf+b\left(b-\sqrt{b^2-4ac}\right)f}\sqrt{d+fx^2}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{2c^2d-2acf+b\left(b-\sqrt{b^2-4ac}\right)f}} + \frac{\left(2Ac - B\left(b + \sqrt{b^2-4ac}\right)\right)\tanh^{-1}\left(\frac{2cd-\left(b+\sqrt{b^2-4ac}\right)fx}{\sqrt{2}\sqrt{2c^2d-2acf+b\left(b+\sqrt{b^2-4ac}\right)f}\sqrt{d+fx^2}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{2c^2d-2acf+b\left(b+\sqrt{b^2-4ac}\right)f}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.38 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.65

$$\int \frac{A+Bx}{(a+bx+cx^2)\sqrt{d+fx^2}}\,dx$$

$$= -\text{RootSum}\Big[cd^2 + 2bd\sqrt{f}\#1 - 2cd\#1^2 + 4af\#1^2 - 2b\sqrt{f}\#1^3 + c\#1^4\&, \frac{Bd\log\left(-\sqrt{f}x+\sqrt{d+fx^2}-\#1\right) + 2A\sqrt{f}\log\left(-\sqrt{f}x+\sqrt{d+fx^2}-\#1\right)\#1 - B\log\left(-\sqrt{f}\right.}{bd\sqrt{f} - 2cd\#1 + 4af\#1 - 3b\sqrt{f}\#1^2 + 2c\#1^3}$$

```
[In] Integrate[(A + B*x)/((a + b*x + c*x^2)*Sqrt[d + f*x^2]),x]
```

```
[Out] -RootSum[c*d^2 + 2*b*d*Sqrt[f]*#1 - 2*c*d*#1^2 + 4*a*f*#1^2 - 2*b*Sqrt[f]*#
1^3 + c*#1^4 & , (B*d*Log[-(Sqrt[f]*x) + Sqrt[d + f*x^2] - #1] + 2*A*Sqrt[f
]*Log[-(Sqrt[f]*x) + Sqrt[d + f*x^2] - #1]*#1 - B*Log[-(Sqrt[f]*x) + Sqrt[d
 + f*x^2] - #1]*#1^2)/(b*d*Sqrt[f] - 2*c*d*#1 + 4*a*f*#1 - 3*b*Sqrt[f]*#1^2
 + 2*c*#1^3) & ]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 638 vs. $2(266) = 532$.

Time = 0.77 (sec) , antiderivative size = 639, normalized size of antiderivative = 2.12

method	result
default	$-\frac{\left(-2Ac+B\sqrt{-4ac+b^2}+Bb\right)\ln\left(\frac{-\frac{-\sqrt{-4ac+b^2}\,bf+2acf-b^2f-2c^2d}{c^2}-\frac{f\left(b+\sqrt{-4ac+b^2}\right)\left(x+\frac{b+\sqrt{-4ac+b^2}}{2c}\right)}{c}+\frac{\sqrt{-\frac{2\left(-\sqrt{-4ac+b^2}\,bf+2acf\right.}{c^2}}}{}}{x}\right.}{\sqrt{-4ac+b^2}\,c\sqrt{-\frac{2\left(-\sqrt{-4ac+b^2}\,bf+\right.}{c^2}}}$

```
[In] int((B*x+A)/(c*x^2+b*x+a)/(f*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(-2*A*c+B*(-4*a*c+b^2)^(1/2)+B*b)/(-4*a*c+b^2)^(1/2)/c/(-2*(-(-4*a*c+b^2)^
(1/2)*b*f+2*a*c*f-b^2*f-2*c^2*d)/c^2)^(1/2)*ln((-(-(-4*a*c+b^2)^(1/2)*b*f+2
*a*c*f-b^2*f-2*c^2*d)/c^2-f*(b+(-4*a*c+b^2)^(1/2))/c*(x+1/2*(b+(-4*a*c+b^2)
^(1/2))/c)+1/2*(-2*(-(-4*a*c+b^2)^(1/2)*b*f+2*a*c*f-b^2*f-2*c^2*d)/c^2)^(1/
2)*(4*f*(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)^2-4*f*(b+(-4*a*c+b^2)^(1/2))/c*(x+
1/2*(b+(-4*a*c+b^2)^(1/2))/c)-2*(-(-4*a*c+b^2)^(1/2)*b*f+2*a*c*f-b^2*f-2*c^
2*d)/c^2)^(1/2))/(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c))-(2*A*c+B*(-4*a*c+b^2)^(1
/2)-B*b)/(-4*a*c+b^2)^(1/2)/c/(-2*((-4*a*c+b^2)^(1/2)*b*f+2*a*c*f-b^2*f-2*c
^2*d)/c^2)^(1/2)*ln((-((-4*a*c+b^2)^(1/2)*b*f+2*a*c*f-b^2*f-2*c^2*d)/c^2-f*
(b-(-4*a*c+b^2)^(1/2))/c*(x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))+1/2*(-2*((-4*a*c
+b^2)^(1/2)*b*f+2*a*c*f-b^2*f-2*c^2*d)/c^2)^(1/2)*(4*f*(x-1/2/c*(-b+(-4*a*c
+b^2)^(1/2)))^2-4*f*(b-(-4*a*c+b^2)^(1/2))/c*(x-1/2/c*(-b+(-4*a*c+b^2)^(1/2
)))-2*((-4*a*c+b^2)^(1/2)*b*f+2*a*c*f-b^2*f-2*c^2*d)/c^2)^(1/2))/(x-1/2/c*(
-b+(-4*a*c+b^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8977 vs. $2(263) = 526$.

Time = 43.72 (sec) , antiderivative size = 8977, normalized size of antiderivative = 29.73

$$\int \frac{A+Bx}{(a+bx+cx^2)\sqrt{d+fx^2}}\,dx = \text{Too large to display}$$

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)/(f*x^2+d)^(1/2),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F]

$$\int \frac{A+Bx}{(a+bx+cx^2)\sqrt{d+fx^2}}\,dx = \int \frac{A+Bx}{\sqrt{d+fx^2}\,(a+bx+cx^2)}\,dx$$

```
[In] integrate((B*x+A)/(c*x**2+b*x+a)/(f*x**2+d)**(1/2),x)
[Out] Integral((A + B*x)/(sqrt(d + f*x**2)*(a + b*x + c*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx}{(a+bx+cx^2)\sqrt{d+fx^2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)/(f*x^2+d)^(1/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [F(-2)]

Exception generated.

$$\int \frac{A+Bx}{(a+bx+cx^2)\sqrt{d+fx^2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((B*x+A)/(c*x^2+b*x+a)/(f*x^2+d)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{%%{poly1[%%%{-4,[3,2,0]%%%}+%%%{16,[1,3,1]%%%},%%%{4,[4,2,0]%
%%}+%%
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(a+bx+cx^2)\sqrt{d+fx^2}}\,dx = \int \frac{A+B\,x}{\sqrt{f\,x^2+d}\,\left(c\,x^2+b\,x+a\right)}\,dx$$

```
[In] int((A + B*x)/((d + f*x^2)^(1/2)*(a + b*x + c*x^2)),x)
[Out] int((A + B*x)/((d + f*x^2)^(1/2)*(a + b*x + c*x^2)), x)
```

3.24 $\int \frac{A+Bx}{(a+cx^2)\sqrt{d+fx^2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 101

$$\int \frac{A+Bx}{(a+cx^2)\sqrt{d+fx^2}}\,dx=\frac{A\arctan\left(\frac{\sqrt{cd-af}\,x}{\sqrt{a}\sqrt{d+fx^2}}\right)}{\sqrt{a}\sqrt{cd-af}}-\frac{B\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d+fx^2}}{\sqrt{cd-af}}\right)}{\sqrt{c}\sqrt{cd-af}}$$

```
[Out] A*arctan(x*(-a*f+c*d)^(1/2)/a^(1/2)/(f*x^2+d)^(1/2))/a^(1/2)/(-a*f+c*d)^(1/
2)-B*arctanh(c^(1/2)*(f*x^2+d)^(1/2)/(-a*f+c*d)^(1/2))/c^(1/2)/(-a*f+c*d)^(
1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1024, 385, 211, 455, 65, 214}

$$\int \frac{A+Bx}{(a+cx^2)\sqrt{d+fx^2}}\,dx=\frac{A\arctan\left(\frac{x\sqrt{cd-af}}{\sqrt{a}\sqrt{d+fx^2}}\right)}{\sqrt{a}\sqrt{cd-af}}-\frac{B\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d+fx^2}}{\sqrt{cd-af}}\right)}{\sqrt{c}\sqrt{cd-af}}$$

```
[In] Int[(A + B*x)/((a + c*x^2)*Sqrt[d + f*x^2]),x]
```

```
[Out] (A*ArcTan[(Sqrt[c*d - a*f]*x)/(Sqrt[a]*Sqrt[d + f*x^2])])/(Sqrt[a]*Sqrt[c*d
 - a*f]) - (B*ArcTanh[(Sqrt[c]*Sqrt[d + f*x^2])/Sqrt[c*d - a*f]])/(Sqrt[c]*
Sqrt[c*d - a*f])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 1024

```
Int[((g_) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q
_), x_Symbol] :> Dist[g, Int[(a + c*x^2)^p*(d + f*x^2)^q, x], x] + Dist[h,
Int[x*(a + c*x^2)^p*(d + f*x^2)^q, x], x] /; FreeQ[{a, c, d, f, g, h, p, q}
, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= A\int \frac{1}{(a+cx^2)\sqrt{d+fx^2}}\,dx + B\int \frac{x}{(a+cx^2)\sqrt{d+fx^2}}\,dx \\
&= A\text{Subst}\left(\int \frac{1}{a-(-cd+af)x^2}\,dx, x, \frac{x}{\sqrt{d+fx^2}}\right) + \frac{1}{2}B\text{Subst}\left(\int \frac{1}{(a+cx)\sqrt{d+fx}}\,dx, x, x^2\right) \\
&= \frac{A\tan^{-1}\left(\frac{\sqrt{cd-af}x}{\sqrt{a}\sqrt{d+fx^2}}\right)}{\sqrt{a}\sqrt{cd-af}} + \frac{B\text{Subst}\left(\int \frac{1}{a-\frac{cd}{f}+\frac{cx^2}{f}}\,dx, x, \sqrt{d+fx^2}\right)}{f} \\
&= \frac{A\tan^{-1}\left(\frac{\sqrt{cd-af}x}{\sqrt{a}\sqrt{d+fx^2}}\right)}{\sqrt{a}\sqrt{cd-af}} - \frac{B\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{d+fx^2}}{\sqrt{cd-af}}\right)}{\sqrt{c}\sqrt{cd-af}}
\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 349 vs. 2(101) = 202.

Time = 1.45 (sec) , antiderivative size = 349, normalized size of antiderivative = 3.46

$$\int \frac{A+Bx}{(a+cx^2)\sqrt{d+fx^2}}\,dx$$
$$= \frac{\sqrt{a}B\left(\left(\sqrt{a}\sqrt{f}+\sqrt{-cd+af}\right)\sqrt{-cd+2af-2\sqrt{a}\sqrt{f}\sqrt{-cd+af}}\arctan\left(\frac{\sqrt{c}\left(\sqrt{f}x-\sqrt{d+fx^2}\right)}{\sqrt{-cd+2af-2\sqrt{a}\sqrt{f}\sqrt{-cd+af}}}\right)+(\ldots}{}$$

```
[In] Integrate[(A + B*x)/((a + c*x^2)*Sqrt[d + f*x^2]),x]
```

```
[Out] (Sqrt[a]*B*((Sqrt[a]*Sqrt[f] + Sqrt[-(c*d) + a*f])*Sqrt[-(c*d) + 2*a*f - 2*
Sqrt[a]*Sqrt[f]*Sqrt[-(c*d) + a*f]]*ArcTan[(Sqrt[c]*(Sqrt[f]*x - Sqrt[d + f
*x^2]))/Sqrt[-(c*d) + 2*a*f - 2*Sqrt[a]*Sqrt[f]*Sqrt[-(c*d) + a*f]]] + (-(S
qrt[a]*Sqrt[f]) + Sqrt[-(c*d) + a*f])*Sqrt[-(c*d) + 2*a*f + 2*Sqrt[a]*Sqrt[
f]*Sqrt[-(c*d) + a*f]]*ArcTan[(Sqrt[c]*(Sqrt[f]*x - Sqrt[d + f*x^2]))/Sqrt[
-(c*d) + 2*a*f + 2*Sqrt[a]*Sqrt[f]*Sqrt[-(c*d) + a*f]]]) + A*c^(3/2)*d*ArcT
anh[(a*Sqrt[f] + c*x*(Sqrt[f]*x - Sqrt[d + f*x^2]))/(Sqrt[a]*Sqrt[-(c*d) +
a*f])])/(Sqrt[a]*c^(3/2)*d*Sqrt[-(c*d) + a*f])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 336 vs. 2(81) = 162.

Time = 0.58 (sec) , antiderivative size = 337, normalized size of antiderivative = 3.34

method	result
default	$-\frac{\left(-Ac+B\sqrt{-ac}\right)\ln\left(\frac{-\frac{2(fa-cd)}{c}-\frac{2f\sqrt{-ac}\left(x+\frac{\sqrt{-ac}}{c}\right)}{c}+2\sqrt{-\frac{fa-cd}{c}}\sqrt{f\left(x+\frac{\sqrt{-ac}}{c}\right)^2-\frac{2f\sqrt{-ac}\left(x+\frac{\sqrt{-ac}}{c}\right)}{c}-\frac{fa-cd}{c}}}{x+\frac{\sqrt{-ac}}{c}}\right)}{2\sqrt{-ac}\,c\sqrt{-\frac{fa-cd}{c}}}-\frac{(Ac+B\sqrt{\ldots}}{}$

```
[In] int((B*x+A)/(c*x^2+a)/(f*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(-A*c+B*(-a*c)^(1/2))/(-a*c)^(1/2)/c/(-(a*f-c*d)/c)^(1/2)*ln((-2*(a*f-
c*d)/c-2*f*(-a*c)^(1/2)/c*(x+(-a*c)^(1/2)/c)+2*(-(a*f-c*d)/c)^(1/2)*(f*(x+(
-a*c)^(1/2)/c)^2-2*f*(-a*c)^(1/2)/c*(x+(-a*c)^(1/2)/c)-(a*f-c*d)/c)^(1/2))/
(x+(-a*c)^(1/2)/c))-1/2*(A*c+B*(-a*c)^(1/2))/(-a*c)^(1/2)/c/(-(a*f-c*d)/c)^
(1/2)*ln((-2*(a*f-c*d)/c+2*f*(-a*c)^(1/2)/c*(x-(-a*c)^(1/2)/c)+2*(-(a*f-c*d
)/c)^(1/2)*(f*(x-(-a*c)^(1/2)/c)^2+2*f*(-a*c)^(1/2)/c*(x-(-a*c)^(1/2)/c)-(a
*f-c*d)/c)^(1/2))/(x-(-a*c)^(1/2)/c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1515 vs. 2(81) = 162.

Time = 0.37 (sec) , antiderivative size = 1515, normalized size of antiderivative = 15.00

$$\int \frac{A+Bx}{(a+cx^2)\sqrt{d+fx^2}}\,dx = \text{Too large to display}$$

```
[In] integrate((B*x+A)/(c*x^2+a)/(f*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt((B^2*a - A^2*c + 2*(a*c^2*d - a^2*c*f)*sqrt(-A^2*B^2/(a*c^3*d^2 -
 2*a^2*c^2*d*f + a^3*c*f^2)))/(a*c^2*d - a^2*c*f))*log(((A*B^3*a + A^3*B*c)
*f*x + (A^2*B*c^2*d - A^2*B*a*c*f + (B*a*c^3*d^2 - 2*B*a^2*c^2*d*f + B*a^3*
c*f^2)*sqrt(-A^2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d*f + a^3*c*f^2)))*sqrt(f*x^2 +
 d)*sqrt((B^2*a - A^2*c + 2*(a*c^2*d - a^2*c*f)*sqrt(-A^2*B^2/(a*c^3*d^2 -
2*a^2*c^2*d*f + a^3*c*f^2)))/(a*c^2*d - a^2*c*f)) + sqrt(-A^2*B^2/(a*c^3*d^
2 - 2*a^2*c^2*d*f + a^3*c*f^2))*((B^2*a*c^2 + A^2*c^3)*d^2 - (B^2*a^2*c + A
^2*a*c^2)*d*f))/x) + 1/4*sqrt((B^2*a - A^2*c + 2*(a*c^2*d - a^2*c*f)*sqrt(-
A^2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d*f + a^3*c*f^2)))/(a*c^2*d - a^2*c*f))*log(
((A*B^3*a + A^3*B*c)*f*x - (A^2*B*c^2*d - A^2*B*a*c*f + (B*a*c^3*d^2 - 2*B*
a^2*c^2*d*f + B*a^3*c*f^2)*sqrt(-A^2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d*f + a^3*c
*f^2)))*sqrt(f*x^2 + d)*sqrt((B^2*a - A^2*c + 2*(a*c^2*d - a^2*c*f)*sqrt(-A
^2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d*f + a^3*c*f^2)))/(a*c^2*d - a^2*c*f)) + sqr
t(-A^2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d*f + a^3*c*f^2))*((B^2*a*c^2 + A^2*c^3)*
d^2 - (B^2*a^2*c + A^2*a*c^2)*d*f))/x) - 1/4*sqrt((B^2*a - A^2*c - 2*(a*c^2
*d - a^2*c*f)*sqrt(-A^2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d*f + a^3*c*f^2)))/(a*c^
2*d - a^2*c*f))*log(((A*B^3*a + A^3*B*c)*f*x + (A^2*B*c^2*d - A^2*B*a*c*f -
 (B*a*c^3*d^2 - 2*B*a^2*c^2*d*f + B*a^3*c*f^2)*sqrt(-A^2*B^2/(a*c^3*d^2 - 2
*a^2*c^2*d*f + a^3*c*f^2)))*sqrt(f*x^2 + d)*sqrt((B^2*a - A^2*c - 2*(a*c^2*
d - a^2*c*f)*sqrt(-A^2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d*f + a^3*c*f^2)))/(a*c^2
*d - a^2*c*f)) - sqrt(-A^2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d*f + a^3*c*f^2))*((B
^2*a*c^2 + A^2*c^3)*d^2 - (B^2*a^2*c + A^2*a*c^2)*d*f))/x) + 1/4*sqrt((B^2*
a - A^2*c - 2*(a*c^2*d - a^2*c*f)*sqrt(-A^2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d*f
+ a^3*c*f^2)))/(a*c^2*d - a^2*c*f))*log(((A*B^3*a + A^3*B*c)*f*x - (A^2*B*c
^2*d - A^2*B*a*c*f - (B*a*c^3*d^2 - 2*B*a^2*c^2*d*f + B*a^3*c*f^2)*sqrt(-A^
2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d*f + a^3*c*f^2)))*sqrt(f*x^2 + d)*sqrt((B^2*a
 - A^2*c - 2*(a*c^2*d - a^2*c*f)*sqrt(-A^2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d*f +
 a^3*c*f^2)))/(a*c^2*d - a^2*c*f)) - sqrt(-A^2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d
*f + a^3*c*f^2))*((B^2*a*c^2 + A^2*c^3)*d^2 - (B^2*a^2*c + A^2*a*c^2)*d*f))
/x)
```

Sympy [F]

$$\int \frac{A+Bx}{(a+cx^2)\sqrt{d+fx^2}}\,dx = \int \frac{A+Bx}{(a+cx^2)\sqrt{d+fx^2}}\,dx$$

```
[In] integrate((B*x+A)/(c*x**2+a)/(f*x**2+d)**(1/2),x)
[Out] Integral((A + B*x)/((a + c*x**2)*sqrt(d + f*x**2)), x)
```

Maxima [F]

$$\int \frac{A+Bx}{(a+cx^2)\sqrt{d+fx^2}}\,dx = \int \frac{Bx+A}{(cx^2+a)\sqrt{fx^2+d}}\,dx$$

```
[In] integrate((B*x+A)/(c*x^2+a)/(f*x^2+d)^(1/2),x, algorithm="maxima")
[Out] integrate((B*x + A)/((c*x^2 + a)*sqrt(f*x^2 + d)), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(a+cx^2)\sqrt{d+fx^2}}\,dx = \text{Timed out}$$

```
[In] integrate((B*x+A)/(c*x^2+a)/(f*x^2+d)^(1/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(a+cx^2)\sqrt{d+fx^2}}\,dx = \begin{cases} \frac{B\,\mathrm{atan}\left(\frac{c\,\sqrt{f\,x^2+d}}{\sqrt{a\,c\,f-c^2\,d}}\right)}{\sqrt{a\,c\,f-c^2\,d}}+\frac{A\,\mathrm{atan}\left(\frac{x\,\sqrt{c\,d-a\,f}}{\sqrt{a}\,\sqrt{f\,x^2+d}}\right)}{\sqrt{-a\,(a\,f-c\,d)}} & \text{if } 0<c\,d-a\,f \\ \frac{A\,\ln\left(\frac{\sqrt{a\,(f\,x^2+d)}+x\,\sqrt{a\,f-c\,d}}{\sqrt{a\,(f\,x^2+d)}-x\,\sqrt{a\,f-c\,d}}\right)}{2\,\sqrt{a\,(a\,f-c\,d)}}+\frac{B\,\mathrm{atan}\left(\frac{c\,\sqrt{f\,x^2+d}}{\sqrt{a\,c\,f-c^2\,d}}\right)}{\sqrt{a\,c\,f-c^2\,d}} & \text{if } c\,d-a\,f<0 \\ \int \frac{A+B\,x}{(c\,x^2+a)\,\sqrt{f\,x^2+d}}\,dx & \text{if } c\,d-a\,f\notin\mathbb{R}\vee a\,f=c\,d \end{cases}$$

```
[In] int((A + B*x)/((a + c*x^2)*(d + f*x^2)^(1/2)),x)
```

```
[Out] piecewise(0 < - a*f + c*d, (B*atan((c*(d + f*x^2)^(1/2))/(- c^2*d + a*c*f)^
(1/2)))/(- c^2*d + a*c*f)^(1/2) + (A*atan((x*(- a*f + c*d)^(1/2))/(a^(1/2)*
(d + f*x^2)^(1/2))))/(-a*(a*f - c*d))^(1/2), - a*f + c*d < 0, (A*log(((a*(d
 + f*x^2))^(1/2) + x*(a*f - c*d)^(1/2))/((a*(d + f*x^2))^(1/2) - x*(a*f - c
*d)^(1/2))))/(2*(a*(a*f - c*d))^(1/2)) + (B*atan((c*(d + f*x^2)^(1/2))/(- c
^2*d + a*c*f)^(1/2)))/(- c^2*d + a*c*f)^(1/2), ~in(- a*f + c*d, 'real') | a
*f == c*d, int((A + B*x)/((a + c*x^2)*(d + f*x^2)^(1/2)), x))
```

3.25 $\int \frac{2+x}{(2+4x-3x^2)\sqrt{1+3x-2x^2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 139

$$\int \frac{2+x}{(2+4x-3x^2)\sqrt{1+3x-2x^2}}\,dx = \frac{1}{2}\sqrt{-\frac{13}{5}+\sqrt{10}}\arctan\left(\frac{3(4-\sqrt{10})+(1+4\sqrt{10})\,x}{2\sqrt{1+\sqrt{10}}\sqrt{1+3x-2x^2}}\right) + \frac{1}{2}\sqrt{\frac{13}{5}+\sqrt{10}}\,\text{arctanh}\left(\frac{3(4+\sqrt{10})+(1-4\sqrt{10})\,x}{2\sqrt{-1+\sqrt{10}}\sqrt{1+3x-2x^2}}\right)$$

```
[Out] 1/10*arctan(1/2*(12-3*10^(1/2)+x*(1+4*10^(1/2)))/(-2*x^2+3*x+1)^(1/2)/(1+10
^(1/2))^(1/2))*(-65+25*10^(1/2))^(1/2)+1/10*arctanh(1/2*(x*(1-4*10^(1/2))+1
2+3*10^(1/2))/(-2*x^2+3*x+1)^(1/2)/(-1+10^(1/2))^(1/2))*(65+25*10^(1/2))^(1
/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1046, 738, 210, 212}

$$\int \frac{2+x}{(2+4x-3x^2)\sqrt{1+3x-2x^2}}\,dx = \frac{1}{2}\sqrt{\sqrt{10}-\frac{13}{5}}\arctan\left(\frac{(1+4\sqrt{10})\,x+3(4-\sqrt{10})}{2\sqrt{1+\sqrt{10}}\sqrt{-2x^2+3x+1}}\right) + \frac{1}{2}\sqrt{\frac{13}{5}+\sqrt{10}}\,\text{arctanh}\left(\frac{(1-4\sqrt{10})\,x+3(4+\sqrt{10})}{2\sqrt{\sqrt{10}-1}\sqrt{-2x^2+3x+1}}\right)$$

```
[In] Int[(2 + x)/((2 + 4*x - 3*x^2)*Sqrt[1 + 3*x - 2*x^2]),x]
```

```
[Out] (Sqrt[-13/5 + Sqrt[10]]*ArcTan[(3*(4 - Sqrt[10]) + (1 + 4*Sqrt[10])*x)/(2*S
qrt[1 + Sqrt[10]]*Sqrt[1 + 3*x - 2*x^2])])/2 + (Sqrt[13/5 + Sqrt[10]]*ArcTa
nh[(3*(4 + Sqrt[10]) + (1 - 4*Sqrt[10])*x)/(2*Sqrt[-1 + Sqrt[10]]*Sqrt[1 +
3*x - 2*x^2])])/2
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1046

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{5}\left(5-4\sqrt{10}\right)\int\frac{1}{\left(4-2\sqrt{10}-6x\right)\sqrt{1+3x-2x^2}}\,dx\\ &\quad+\frac{1}{5}\left(5+4\sqrt{10}\right)\int\frac{1}{\left(4+2\sqrt{10}-6x\right)\sqrt{1+3x-2x^2}}\,dx\end{aligned}$$

$$= -\left(\frac{1}{5}\left(2\left(5-4\sqrt{10}\right)\right)\text{Subst}\left(\int \frac{1}{144+72\left(4-2\sqrt{10}\right)-8\left(4-2\sqrt{10}\right)^2-x^2}\,dx,x,\frac{-12-3(4-\right.\right.$$

$$-\frac{1}{5}\left(2\left(5\right.\right.$$

$$\left.\left.+4\sqrt{10}\right)\right)\text{Subst}\left(\int \frac{1}{144+72\left(4+2\sqrt{10}\right)-8\left(4+2\sqrt{10}\right)^2-x^2}\,dx,x,\frac{-12-3\left(4+2\sqrt{10}\right)-(}{\sqrt{1+3x}}\right.$$

$$= \frac{1}{10}\sqrt{-65+25\sqrt{10}}\tan^{-1}\left(\frac{3\left(4-\sqrt{10}\right)+\left(1+4\sqrt{10}\right)x}{2\sqrt{1+\sqrt{10}}\sqrt{1+3x-2x^2}}\right)$$

$$+\frac{1}{10}\sqrt{65+25\sqrt{10}}\tanh^{-1}\left(\frac{3\left(4+\sqrt{10}\right)+\left(1-4\sqrt{10}\right)x}{2\sqrt{-1+\sqrt{10}}\sqrt{1+3x-2x^2}}\right)$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.19 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.07

$$\int \frac{2+x}{(2+4x-3x^2)\sqrt{1+3x-2x^2}}\,dx = -\frac{1}{2}\text{RootSum}\left[5+20\#1+8\#1^2-8\#1^3\right.$$

$$+2\#1^4\&,\frac{-7\log(x)+7\log\left(-1+\sqrt{1+3x-2x^2}-x\#1\right)+2\log(x)\#1-2\log\left(-1+\sqrt{1+3x-2x^2}-\right.}{5+4\#1-6\#1^2+2\#1^3}$$

```
[In] Integrate[(2 + x)/((2 + 4*x - 3*x^2)*Sqrt[1 + 3*x - 2*x^2]),x]
```

```
[Out] -1/2*RootSum[5 + 20*#1 + 8*#1^2 - 8*#1^3 + 2*#1^4 & , (-7*Log[x] + 7*Log[-1
 + Sqrt[1 + 3*x - 2*x^2] - x*#1] + 2*Log[x]*#1 - 2*Log[-1 + Sqrt[1 + 3*x -
2*x^2] - x*#1]*#1 - 2*Log[x]*#1^2 + 2*Log[-1 + Sqrt[1 + 3*x - 2*x^2] - x*#1
]*#1^2)/(5 + 4*#1 - 6*#1^2 + 2*#1^3) & ]
```

Maple [A] (verified)

Time = 2.37 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.27

method	result
default	$-\frac{\left(-8+\sqrt{10}\right)\sqrt{10}\,\arctan\left(\frac{-1-\sqrt{10}+\frac{9\left(\frac{1}{3}+\frac{4\sqrt{10}}{3}\right)\left(x-\frac{2}{3}+\frac{\sqrt{10}}{3}\right)}{2}}{\sqrt{1+\sqrt{10}}\sqrt{-18\left(x-\frac{2}{3}+\frac{\sqrt{10}}{3}\right)^{2}+9\left(\frac{1}{3}+\frac{4\sqrt{10}}{3}\right)\left(x-\frac{2}{3}+\frac{\sqrt{10}}{3}\right)-1-\sqrt{10}}}\right)}{20\sqrt{1+\sqrt{10}}}+\frac{\left(8+\sqrt{10}\right)\sqrt{10}\,\operatorname{arctanh}\left(\frac{}{\sqrt{-1+\sqrt{10}}}\right.}{}$
trager	$-\frac{\operatorname{RootOf}\left(\_Z^{2}+100\operatorname{RootOf}\left(400\_Z^{4}-520\_Z^{2}-81\right)^{2}-130\right)\ln\left(\frac{129200x\operatorname{RootOf}\left(400\_Z^{4}-520\_Z^{2}-81\right)^{4}\operatorname{RootOf}\left(\_Z^{2}+100\operatorname{Ro}\right.}{}\right.}{}$

[In] int((2+x)/(-3*x^2+4*x+2)/(-2*x^2+3*x+1)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/20*(-8+10^(1/2))*10^(1/2)/(1+10^(1/2))^(1/2)*arctan(9/2*(-2/9-2/9*10^(1/2)+(1/3+4/3*10^(1/2))*(x-2/3+1/3*10^(1/2)))/(1+10^(1/2))^(1/2)/(-18*(x-2/3+1/3*10^(1/2))^2+9*(1/3+4/3*10^(1/2))*(x-2/3+1/3*10^(1/2))-1-10^(1/2))^(1/2))+1/20*(8+10^(1/2))*10^(1/2)/(-1+10^(1/2))^(1/2)*arctanh(9/2*(-2/9+2/9*10^(1/2)+(1/3-4/3*10^(1/2))*(x-2/3-1/3*10^(1/2)))/(-1+10^(1/2))^(1/2)/(-18*(x-2/3-1/3*10^(1/2))^2+9*(1/3-4/3*10^(1/2))*(x-2/3-1/3*10^(1/2))-1+10^(1/2))^(1/2))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 309 vs. $2(99) = 198$.

Time = 0.29 (sec) , antiderivative size = 309, normalized size of antiderivative = 2.22

$$\int \frac{2+x}{(2+4x-3x^2)\sqrt{1+3x-2x^2}}\,dx =$$

$$-\frac{1}{10}\sqrt{5}\sqrt{5\sqrt{5}\sqrt{2}+13}\log\left(\frac{9\sqrt{5}\sqrt{2}x+\left(4\sqrt{5}x-7\sqrt{2}x\right)\sqrt{5\sqrt{5}\sqrt{2}+13}-18x+18\sqrt{-2x^2+3x+1}}{x}\right.$$
$$+\frac{1}{10}\sqrt{5}\sqrt{5\sqrt{5}\sqrt{2}+13}\log\left(\frac{9\sqrt{5}\sqrt{2}x-\left(4\sqrt{5}x-7\sqrt{2}x\right)\sqrt{5\sqrt{5}\sqrt{2}+13}-18x+18\sqrt{-2x^2+3x+1}}{x}\right.$$
$$+\frac{1}{10}\sqrt{5}\sqrt{-5\sqrt{5}\sqrt{2}+13}\log\left(-\frac{9\sqrt{5}\sqrt{2}x+\left(4\sqrt{5}x+7\sqrt{2}x\right)\sqrt{-5\sqrt{5}\sqrt{2}+13}+18x-18\sqrt{-2x^2+3x}}{x}\right.$$
$$-\frac{1}{10}\sqrt{5}\sqrt{-5\sqrt{5}\sqrt{2}+13}\log\left(-\frac{9\sqrt{5}\sqrt{2}x-\left(4\sqrt{5}x+7\sqrt{2}x\right)\sqrt{-5\sqrt{5}\sqrt{2}+13}+18x-18\sqrt{-2x^2+3x}}{x}\right.$$

[In] integrate((2+x)/(-3*x^2+4*x+2)/(-2*x^2+3*x+1)^(1/2),x, algorithm="fricas")

[Out] -1/10*sqrt(5)*sqrt(5*sqrt(5)*sqrt(2) + 13)*log((9*sqrt(5)*sqrt(2)*x + (4*sqrt(5)*x - 7*sqrt(2)*x)*sqrt(5*sqrt(5)*sqrt(2) + 13) - 18*x + 18*sqrt(-2*x^2 + 3*x + 1) - 18)/x) + 1/10*sqrt(5)*sqrt(5*sqrt(5)*sqrt(2) + 13)*log((9*sqr

```
t(5)*sqrt(2)*x - (4*sqrt(5)*x - 7*sqrt(2)*x)*sqrt(5*sqrt(5)*sqrt(2) + 13) -
 18*x + 18*sqrt(-2*x^2 + 3*x + 1) - 18)/x) + 1/10*sqrt(5)*sqrt(-5*sqrt(5)*s
qrt(2) + 13)*log(-(9*sqrt(5)*sqrt(2)*x + (4*sqrt(5)*x + 7*sqrt(2)*x)*sqrt(-
5*sqrt(5)*sqrt(2) + 13) + 18*x - 18*sqrt(-2*x^2 + 3*x + 1) + 18)/x) - 1/10*
sqrt(5)*sqrt(-5*sqrt(5)*sqrt(2) + 13)*log(-(9*sqrt(5)*sqrt(2)*x - (4*sqrt(5
)*x + 7*sqrt(2)*x)*sqrt(-5*sqrt(5)*sqrt(2) + 13) + 18*x - 18*sqrt(-2*x^2 +
3*x + 1) + 18)/x)
```

Sympy [F]

$$\int \frac{2+x}{(2+4x-3x^2)\sqrt{1+3x-2x^2}}\,dx = -\int \frac{x}{3x^2\sqrt{-2x^2+3x+1}-4x\sqrt{-2x^2+3x+1}-2\sqrt{-2x^2+3x+1}}\,dx - \int \frac{2}{3x^2\sqrt{-2x^2+3x+1}-4x\sqrt{-2x^2+3x+1}-2\sqrt{-2x^2+3x+1}}\,dx$$

```
[In] integrate((2+x)/(-3*x**2+4*x+2)/(-2*x**2+3*x+1)**(1/2),x)
```

```
[Out] -Integral(x/(3*x**2*sqrt(-2*x**2 + 3*x + 1) - 4*x*sqrt(-2*x**2 + 3*x + 1) -
 2*sqrt(-2*x**2 + 3*x + 1)), x) - Integral(2/(3*x**2*sqrt(-2*x**2 + 3*x + 1
) - 4*x*sqrt(-2*x**2 + 3*x + 1) - 2*sqrt(-2*x**2 + 3*x + 1)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 361 vs. $2(99) = 198$.

Time = 0.31 (sec) , antiderivative size = 361, normalized size of antiderivative = 2.60

$$\int \frac{2+x}{(2+4x-3x^2)\sqrt{1+3x-2x^2}}\,dx = -\frac{1}{20}\sqrt{10}\left(\frac{\sqrt{10}\arcsin\left(\frac{8\sqrt{17}\sqrt{10}x}{17|6x+2\sqrt{10}-4|}+\frac{2\sqrt{17}x}{17|6x+2\sqrt{10}-4|}-\frac{6\sqrt{17}\sqrt{10}}{17|6x+2\sqrt{10}-4|}+\frac{24\sqrt{17}}{17|6x+2\sqrt{10}-4|}\right)}{\sqrt{\sqrt{10}+1}}-\frac{\sqrt{10}\log\left(-\right.}{}\right.$$

```
[In] integrate((2+x)/(-3*x^2+4*x+2)/(-2*x^2+3*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/20*sqrt(10)*(sqrt(10)*arcsin(8/17*sqrt(17)*sqrt(10)*x/abs(6*x + 2*sqrt(1
0) - 4) + 2/17*sqrt(17)*x/abs(6*x + 2*sqrt(10) - 4) - 6/17*sqrt(17)*sqrt(10
)/abs(6*x + 2*sqrt(10) - 4) + 24/17*sqrt(17)/abs(6*x + 2*sqrt(10) - 4))/sqr
t(sqrt(10) + 1) - sqrt(10)*log(-2/9*sqrt(10) + 2/3*sqrt(-2*x^2 + 3*x + 1)*s
qrt(sqrt(10) - 1)/abs(6*x - 2*sqrt(10) - 4) + 2/9*sqrt(10)/abs(6*x - 2*sqrt
(10) - 4) - 2/9/abs(6*x - 2*sqrt(10) - 4) + 1/18)/sqrt(sqrt(10) - 1) - 8*ar
```

```
csin(8/17*sqrt(17)*sqrt(10)*x/abs(6*x + 2*sqrt(10) - 4) + 2/17*sqrt(17)*x/a
bs(6*x + 2*sqrt(10) - 4) - 6/17*sqrt(17)*sqrt(10)/abs(6*x + 2*sqrt(10) - 4)
 + 24/17*sqrt(17)/abs(6*x + 2*sqrt(10) - 4))/sqrt(sqrt(10) + 1) - 8*log(-2/
9*sqrt(10) + 2/3*sqrt(-2*x^2 + 3*x + 1)*sqrt(sqrt(10) - 1)/abs(6*x - 2*sqrt
(10) - 4) + 2/9*sqrt(10)/abs(6*x - 2*sqrt(10) - 4) - 2/9/abs(6*x - 2*sqrt(1
0) - 4) + 1/18)/sqrt(sqrt(10) - 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 16965 vs. $2(99) = 198$.

Time = 255.51 (sec) , antiderivative size = 16965, normalized size of antiderivative = 122.05

$$\int \frac{2+x}{(2+4x-3x^2)\sqrt{1+3x-2x^2}}\,dx = \text{Too large to display}$$

```
[In] integrate((2+x)/(-3*x^2+4*x+2)/(-2*x^2+3*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/5*sqrt(25*sqrt(10) - 65)*(arctan(254400194096332582542150133/3623768829973
47899477595900837238918963320*(sqrt(34) + 2*sqrt(10) + sqrt(2) + sqrt(25*sqr
t(10) - 65))^15 - 6111149415804811055946029/1288451139546125864809229869643
5161563136*(sqrt(34) + 2*sqrt(10) + sqrt(2) + sqrt(25*sqrt(10) - 65))^14 -
958578619223566161086771177/14495075319893915979103836033489556758528*(sqrt
(34) + 2*sqrt(10) + sqrt(2) + sqrt(25*sqrt(10) - 65))^13 - 3257049408428409
227173436832461/579803012795756639164153441339582270341120*(sqrt(34) + 2*sq
rt(10) + sqrt(2) + sqrt(25*sqrt(10) - 65))^12 + 973696292113400209458145664
321/36237688299734789947759590083723891896320*(sqrt(34) + 2*sqrt(10) + sqrt
(2) + sqrt(25*sqrt(10) - 65))^11 - 6554584888977842514948162319687/15670351
697182611869301444360529250549760*(sqrt(34) + 2*sqrt(10) + sqrt(2) + sqrt(2
5*sqrt(10) - 65))^10 + 1404976614114289514312381195621513/72475376599469579
895519180167447783792640*(sqrt(34) + 2*sqrt(10) + sqrt(2) + sqrt(25*sqrt(10
) - 65))^9 - 46373592974030119798255326246458659/57980301279575663916415344
1339582270341120*(sqrt(34) + 2*sqrt(10) + sqrt(2) + sqrt(25*sqrt(10) - 65))
^8 + 29889765665562355905164717800682989/1207922943324492998258653002790796
3965440*(sqrt(34) + 2*sqrt(10) + sqrt(2) + sqrt(25*sqrt(10) - 65))^7 - 2587
1159491632638141490098189566285219/5798030127957566391641534413395822703411
20*(sqrt(34) + 2*sqrt(10) + sqrt(2) + sqrt(25*sqrt(10) - 65))^6 + 654417817
6466447186063829124503215171/24158458866489859965173060055815927930880*(sqr
t(34) + 2*sqrt(10) + sqrt(2) + sqrt(25*sqrt(10) - 65))^5 - 2009747540984165
333420183109756091353911/579803012795756639164153441339582270341120*(sqrt(3
4) + 2*sqrt(10) + sqrt(2) + sqrt(25*sqrt(10) - 65))^4 + 8217576380122268984
65002503929523567243/36237688299734789947759590083723891896320*(sqrt(34) +
2*sqrt(10) + sqrt(2) + sqrt(25*sqrt(10) - 65))^3 + 190391794603083901259595
42196557505686397/193267670931918879721384480446527423447040*(sqrt(34) + 2*
sqrt(10) + sqrt(2) + sqrt(25*sqrt(10) - 65))^2 - 1/350746251863327397899814
```

```
1175803844364533760*(5106074524584970848245397072*(sqrt(34) + 2*sqrt(10) +
sqrt(2) + sqrt(25*sqrt(10) - 65))^15 + 23168875587412489272401512799*(sqrt(
34) + 2*sqrt(10) + sqrt(2) + sqrt(25*sqrt(10) - 65))^14 - 50446925126786006
0784974492040*(sqrt(34) + 2*sqrt(10) + sqrt(2) + sqrt(25*sqrt(10) - 65))^13
 - 43836659106325893987507329228413*(sqrt(34) + 2*sqrt(10) + sqrt(2) + sqrt
(25*sqrt(10) - 65))^12 - 19995148767884444687437321870384*(sqrt(34) + 2*sqr
t(10) + sqrt(2) + sqrt(25*sqrt(10) - 65))^11 - 1961293287684629368051748556
354875*(sqrt(34) + 2*sqrt(10) + sqrt(2) + sqrt(25*sqrt(10) - 65))^10 + 1288
24075374110892795964678784373544*(sqrt(34) + 2*sqrt(10) + sqrt(2) + sqrt(25
*sqrt(10) - 65))^9 + 171744045117561837297108203102026205*(sqrt(34) + 2*sqr
t(10) + sqrt(2) + sqrt(25*sqrt(10) - 65))^8 + 15422222207892153975247464058
255245808*(sqrt(34) + 2*sqrt(10) + sqrt(2) + sqrt(25*sqrt(10) - 65))^7 - 23
8378160201730861411295472519662563627*(sqrt(34) + 2*sqrt(10) + sqrt(2) + sq
rt(25*sqrt(10) - 65))^6 + 206613174416554389673766686041326876776*(sqrt(34)
 + 2*sqrt(10) + sqrt(2) + sqrt(25*sqrt(10) - 65))^5 - 176838429667120321620
50098866604612487463*(sqrt(34) + 2*sqrt(10) + sqrt(2) + sqrt(25*sqrt(10) -
65))^4 + 30534240562049276130565631848366412993648*(sqrt(34) + 2*sqrt(10) +
 sqrt(2) + sqrt(25*sqrt(10) - 65))^3 + 148750199616689964060433922338235186
0839855*(sqrt(34) + 2*sqrt(10) + sqrt(2) + sqrt(25*sqrt(10) - 65))^2 - 1448
666703478082146222118160600037828672584*sqrt(34) - 289733340695616429244423
6321200075657345168*sqrt(10) - 1448666703478082146222118160600037828672584*
sqrt(2) - 1448666703478082146222118160600037828672584*sqrt(25*sqrt(10) - 65
) - 15599184271177254163382904923119849666422561)*(2*sqrt(2)*sqrt(-2*x^2 +
3*x + 1) - sqrt(17))/(4*x - 3) - 1811042235549317357139119769672328200929/1
610563924432657331011537337054395195392*sqrt(34) - 181104223554931735713911
9769672328200929/805281962216328665505768668527197597696*sqrt(10) - 1811042
235549317357139119769672328200929/1610563924432657331011537337054395195392*
sqrt(2) - 1811042235549317357139119769672328200929/161056392443265733101153
7337054395195392*sqrt(25*sqrt(10) - 65) - 154665465664344716772123904320326
2850067/21474185659102097746820497827391935938560) - arctan(-9/150272*(2387
9312122748629007113688927675929025*(sqrt(34) + 2*sqrt(10) + sqrt(2) + sqrt(
25*sqrt(10) - 65))^15 + 113230069499892206161818848907359417242*(sqrt(34) +
 2*sqrt(10) + sqrt(2) + sqrt(25*sqrt(10) - 65))^14 - 1358394428828297284262
972577458756060500*(sqrt(34) + 2*sqrt(10) + sqrt(2) + sqrt(25*sqrt(10) - 65
))^13 - 196383881389829711586972774164645208099234*(sqrt(34) + 2*sqrt(10) +
 sqrt(2) + sqrt(25*sqrt(10) - 65))^12 - 14350821823182215623282772135987142
6370075*(sqrt(34) + 2*sqrt(10) + sqrt(2) + sqrt(25*sqrt(10) - 65))^11 - 172
31641334142821702395417825566187691727930*(sqrt(34) + 2*sqrt(10) + sqrt(2)
+ sqrt(25*sqrt(10) - 65))^10 + 55997501682000406940835742993358534385032245
0*(sqrt(34) + 2*sqrt(10) + sqrt(2) + sqrt(25*sqrt(10) - 65))^9 + 4090536056
643979708105051615058182028180810*(sqrt(34) + 2*sqrt(10) + sqrt(2) + sqrt(2
5*sqrt(10) - 65))^8 + 90178271684816652713969406894205843692259645775*(sqrt
(34) + 2*sqrt(10) + sqrt(2) + sqrt(25*sqrt(10) - 65))^7 - 10421556996935915
06311767149099777117935909012946*(sqrt(34) + 2*sqrt(10) + sqrt(2) + sqrt(25
*sqrt(10) - 65))^6 + 4808899344936606299190903351525275961959980182800*(sqr
```

t(34) + 2*sqrt(10) + sqrt(2) + sqrt(25*sqrt(10) - 65))^5 - 1046428789568204
1228806937902715126682099476745 4134*(sqrt(34) + 2*sqrt(10) + sqrt(2) + sqrt
(25*sqrt(10) - 65))^4 + 2767408784159098836594901472954527749779842803092 75
*(sqrt(34) + 2*sqrt(10) + sqrt(2) + sqrt(25*sqrt(10) - 65))^3 + 34322427449
0906385041397295686476019543631365 6232130*(sqrt(34) + 2*sqrt(10) + sqrt(2)
+ sqrt(25*sqrt(10) - 65))^2 - 762120314390729356805469900754861499286441595
6426950*sqrt(34) - 15242406287814587136109398015097229985728831912853900*sq
rt(10) - 7621203143907293568054699007548614992864415956426950*sqrt(2) - 762
1203143907293568054699007548614992864415956426950*sqrt(25*sqrt(10) - 65) -
1426718691548554608018046700325808022849249673029 1618)/(1360691944599296895
579524560521201*(sqrt(34) + 2*sqrt(10) + sqrt(2) + sqrt(25*sqrt(10) - 65))^
15 + 8825348319343288362396585946189478*(sqrt(34) + 2*sqrt(10) + sqrt(2) +
sqrt(25*sqrt(10) - 65))^14 - 70182260746945612674928548803252031*(sqrt(34)
+ 2*sqrt(10) + sqrt(2) + sqrt(25*sqrt(10) - 65))^13 - 113316914743503137766
29173915699682596*(sqrt(34) + 2*sqrt(10) + sqrt(2) + sqrt(25*sqrt(10) - 65)
)^12 - 27552869451681202807696939322040938311*(sqrt(34) + 2*sqrt(10) + sqrt
(2) + sqrt(25*sqrt(10) - 65))^11 - 966380143141288377066814615598992497578*
(sqrt(34) + 2*sqrt(10) + sqrt(2) + sqrt(25*sqrt(10) - 65))^10 + 30086401731
19432185709577855576301584 8129*(sqrt(34) + 2*sqrt(10) + sqrt(2) + sqrt(25*s
qrt(10) - 65))^9 + 596300866097558451819403840043731963 50776*(sqrt(34) + 2*
sqrt(10) + sqrt(2) + sqrt(25*sqrt(10) - 65))^8 + 504723186225331497504267 91
93134843236780191*(sqrt(34) + 2*sqrt(10) + sqrt(2) + sqrt(25*sqrt(10) - 65)
)^7 - 4991764551352161936467307573169512678 7116726*(sqrt(34) + 2*sqrt(10) +
 sqrt(2) + sqrt(25*sqrt(10) - 65))^6 + 154121479125644930629896778190392664
903113999*(sqrt(34) + 2*sqrt(10) + sqrt(2) + sqrt(25*sqrt(10) - 65))^5 - 52
588940920191859028386474126689924387120 36900*(sqrt(34) + 2*sqrt(10) + sqrt(
2) + sqrt(25*sqrt(10) - 65))^4 + 363879705436335962775515936606165374825569
4919*(sqrt(34) + 2*sqrt(10) + sqrt(2) + sqrt(25*sqrt(10) - 65))^3 + 2449028
532811045070697868675630395881958071 48218*(sqrt(34) + 2*sqrt(10) + sqrt(2)
+ sqrt(25*sqrt(10) - 65))^2 - 208505227462429628893046473220180843272227623
265*sqrt(34) - 417010454924859257786092946440361686544455246530*sqrt(10) -
208505227462429628893046473220180843272227623265*sqrt(2) - 2085052274624296
28893046473220180843272227623265*sqrt(25*sqrt(10) - 65) - 17202405517090301
4962584848164405900614118399 3568))) - 1/10*sqrt(25*sqrt(10) + 65)*log(abs(1
988241701145789790384428579341345199690995312821229387863919767653806746296
555597778262375515688819414211443040291648146796967328145162376262709903090
94199201595097127260639833824360090108494014421401600000000000000000000*sqrt
(34)*sqrt(10)*sqrt(2)*(5*sqrt(10) + 13)^6 + 336695989368432521479624483972 7
823471117157412178753308092709209206324817758415638186821829924153907965275
383517523512473215682697357824151052949321704014721871811070519504382838151
202468189043687424000000000000000000000*sqrt(34)*sqrt(10)*sqrt(2)*(2*sqrt(2)*
sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*(5*sqrt(10) + 13)^6/(4*x - 3) + 15294166
931890690695264835225702655382238425483240226060491690520413898048435043059
832787503966837072417011100309935754975361287139578172125097768485314938400
122699779020049217986489237700653385724723200000000000000000000*sqrt(34)*sqr

```
t(10)*sqrt(25*sqrt(10) + 65)*(5*sqrt(10) + 13)^6 + 152941669318906906952648
352257026553822384254832402260604916905204138980484350430598327875039668370
724170111003099357549753612871395781721250977684853149384001226997790200492
1798648923770065338572472320000000000000000000*sqrt(34)*sqrt(2)*sqrt(25*sqr
t(10) + 65)*(5*sqrt(10) + 13)^6 + 15294166931890690695264835225702655382238
425483240226060491690520413898048435043059832787503966837072417011100309935
754975361287139578172125097768485314938400122699779020049217986489237700653
38572472320000000000000000000*sqrt(10)*sqrt(2)*sqrt(25*sqrt(10) + 65)*(5*sq
rt(10) + 13)^6 + 2589969148987942472920188338252171900859351855522117929302
084007081788321352627413989862946095503006127134910398095009594781294382582
941654656114862849242093747546977322695679106270155744760802836480000000000
000000000*sqrt(34)*sqrt(10)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*s
qrt(25*sqrt(10) + 65)*(5*sqrt(10) + 13)^6/(4*x - 3) + 258996914898794247292
018833825217190085935185552211792930208400708178832135262741398986294609550
300612713491039809500959478129438258294165465611486284924209374754697732269
5679106270155744760802836480000000000000000000*sqrt(34)*sqrt(2)*(2*sqrt(2)*
sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*sqrt(25*sqrt(10) + 65)*(5*sqrt(10) + 13)
^6/(4*x - 3) + 258996914898794247292018833825217190085935185552211792930208
400708178832135262741398986294609550300612713491039809500959478129438258294
165465611486284924209374754697732269567910627015574476080283648000000000000
0000000*sqrt(10)*sqrt(2)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*sqrt
(25*sqrt(10) + 65)*(5*sqrt(10) + 13)^6/(4*x - 3) + 546220247567524667688029
829489380549365658052972865216446131804300496358872680109279742410855958466
872036110725354848391977188826413506147324920303046962085718667849250716043
499517472775023335204454400000000000000000000*sqrt(34)*(5*sqrt(10) + 13)^7
+ 54622024756752466768802982948938054936565805297286521644613180430049635888
726801092797424108559584668720361107253548483919771888264135061473249203030
4696208571866784925071604349951747277502333520445440000000000000000000*sqr
t(10)*(5*sqrt(10) + 13)^7 + 54622024756752466768802982948938054936565805297
286521644613180430049635887268010927974241085595846687203611072535484839197
718882641350614732492030304696208571866784925071604349951747277502333520445
4400000000000000000000*sqrt(2)*(5*sqrt(10) + 13)^7 + 9249889817814080260429
244065186328217354828055436135461793157168149244004830812192820939093198225
021882624679993196462838504622794939077338057553081604436049098382061866770
28252239341337414572441600000000000000000000*sqrt(34)*(2*sqrt(2)*sqrt(-2*x^
2 + 3*x + 1) - sqrt(17))*(5*sqrt(10) + 13)^7/(4*x - 3) + 924988981781408026
042924406518632821735482805543613546179315716814924400483081219282093909319
822502188262467999319646283850462279493907733805755308160443604909838206186
677028252239341337414572441600000000000000000000*sqrt(10)*(2*sqrt(2)*sqrt(-
2*x^2 + 3*x + 1) - sqrt(17))*(5*sqrt(10) + 13)^7/(4*x - 3) + 92498898178140
802604292440651863282173548280554361354617931571681492440048308121928209390
931982250218826246799931964628385046227949390773380575530816044360490983820
6186677028252239341337414572441600000000000000000000*sqrt(2)*(2*sqrt(2)*sqr
t(-2*x^2 + 3*x + 1) - sqrt(17))*(5*sqrt(10) + 13)^7/(4*x - 3) + 61665932118
760535069528293767908854782365520369574236411954381120994960032205414618806
```

```
260621321500145884164533287976418923364151966260515587050353877362906993989
2137457784685501492894224943048294400000000000000000000*(2*sqrt(2)*sqrt(-2*x
^2 + 3*x + 1) - sqrt(17))*sqrt(25*sqrt(10) + 65)*(5*sqrt(10) + 13)^7/(4*x -
 3) + 364146831711683111792019886326253699577105368648576810964087869533664
239248453406186494940570638977914690740483569898927984792550942337431549946
868697974723812445232833810695666344981850015556802969600000000000000000000*
sqrt(25*sqrt(10) + 65)*(5*sqrt(10) + 13)^7 - 797053589106426667125823136608
027440567024434925937834912064526930315010317548227185394217418663442762208
153550492456003177828045592129645683447345132703141614349068289647973195354
689965252939992415600640000000000000000000*sqrt(34)*sqrt(10)*sqrt(2)*sqrt(25*
sqrt(10) + 65)*(5*sqrt(10) + 13)^5 + 25404826848046380624689450671475768480
756994407667211482125651118446316921804761536471226719853953113351180108062
378343108304545962582269365692072696636476659499161091369771667496096175857
850842989199360000000000000000000000*sqrt(34)*sqrt(10)*sqrt(2)*(2*sqrt(2)*sqrt(
-2*x^2 + 3*x + 1) - sqrt(17))*sqrt(25*sqrt(10) + 65)*(5*sqrt(10) + 13)^5/(4
*x - 3) - 3321056621276777796909297358667810023626018121914076454669355288
763125429897842799391425725777643448425339731270519000132409501899672068570
143639380529297567264544512068665549806445415218872499683983360000000000000
00000*sqrt(34)*sqrt(10)*(5*sqrt(10) + 13)^6 - 3321056621276777796909297358
667810023626018121914076454669355288763125429897842799391425725777643448425
339731270519000132409501899672068570143639380529297567264544512068665549806
445415218872499683983360000000000000000000000*sqrt(34)*sqrt(2)*(5*sqrt(10) + 13
)^6 - 33210566212767777969092973586678100236260181219140764546693552887631
254298978427993914257257776434484253397312705190001324095018996720685701436
393805292975672645445120686655498064454152188724996839833600000000000000000
0*sqrt(10)*sqrt(2)*(5*sqrt(10) + 13)^6 + 1058534452001932526028727111311490
353364874766986133811755235463268596538408531730686301113327248046389632504
502599097629512689415107594556903836362359853194145798378807073819479004007
3274104517912166400000000000000000000000*sqrt(34)*sqrt(10)*(2*sqrt(2)*sqrt(-2*x
^2 + 3*x + 1) - sqrt(17))*(5*sqrt(10) + 13)^6/(4*x - 3) + 10585344520019325
260287271113114903533648747669861338117552354632685965384085317306863011133
272480463896325045025990976295126894151075945569038363623598531941457983788
0707381947900400732741045179121664000000000000000000000*sqrt(34)*sqrt(2)*(2*s
qrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*(5*sqrt(10) + 13)^6/(4*x - 3) + 1
058534452001932526028727111311490353364874766986133811755235463268596538408
531730686301113327248046389632504502599097629512689415107594556903836362359
8531941457983788070738194790040073274104517912166400000000000000000000000*sqrt(
10)*sqrt(2)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*(5*sqrt(10) + 13)
^6/(4*x - 3) - 255465893944367521514686902758983154027892447091646741958995
04068279327253767571384147250558290494960327184408669630001018534629997477
450539566456773302288978958034708220504229280349347837480766799872000000000
000000000*sqrt(34)*sqrt(25*sqrt(10) + 65)*(5*sqrt(10) + 13)^6 - 25546589394
436752151468690275898315402789244709164674195899504068279327253767571384147
25055829049496032718440866963000101853462999747745053956645677330228897895
80347082205042292803493478374807667998720000000000000000000*sqrt(10)*sqrt(25
```

```
*sqrt(10) + 65)*(5*sqrt(10) + 13)^6 - 2554658939443675215146869027589831540
278924470916467419589950406827932725376757138414725055829049496032718440866
963000010185346299974774505395664567733022889789580347082205042292803493478
374807667998720000000000000000000*sqrt(2)*sqrt(25*sqrt(10) + 65)*(5*sqrt(10)
 + 13)^6 + 8142572707707173277144054702396079641268267438354875475040272794
373819526219474851433085487132677279920250034635377674073174533962366111976
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) - sqrt(17))*sqrt(25*sqrt(10) + 65)*(5*sqrt(10) + 13)^6/(4*x - 3) + 814257
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00000000*sqrt(34)*sqrt(2)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*(5*
sqrt(10) + 13)^4/(4*x - 3) + 16710165413463155520330940708162360857286327016
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29597347840000000000000000000*sqrt(10)*sqrt(2)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x +
 1) - sqrt(17))*(5*sqrt(10) + 13)^4/(4*x - 3) - 131681502548507541017198661
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)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*sqrt(25*sqrt(10) + 65)*(5*sqrt(10) + 1
3)^4/(4*x - 3) + 709311277685013953881693416072460231923240659213444202243
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25*sqrt(10) + 65)*(5*sqrt(10) + 13)^4/(4*x - 3) + 270452126948768145761719
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*x^2 + 3*x + 1) - sqrt(17))*sqrt(25*sqrt(10) + 65)*(5*sqrt(10) + 13)^4/(4*x
 - 3) + 16761827091401102166079578488744810476579163133379209818699498139 49
1214169898591250497624392907137567571902302213218951796747597552104141119 77
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000000*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*(5*sqrt(10) + 13)^5/(4
*x - 3) - 1975461741083530204465239695417236562262770518431254638465930423 3
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))*(5*sqrt(10) + 13)^3/(4*x - 3) - 83394064180899590785952015052300498184 95
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*(5*sqrt(10) + 13)^3 - 2261086610891060483856524971679485119194642235797886
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 + 13)^3 - 4225202809434381706834944071469586565932243867686295073434205634
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2*x^2 + 3*x + 1) - sqrt(17))*sqrt(25*sqrt(10) + 65)*(5*sqrt(10) + 13)^3/(4*
x - 3) - 1746336911549781810327907459779124274454448577771156158179677525 03
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t(25*sqrt(10) + 65)*(5*sqrt(10) + 13)^3/(4*x - 3) - 974647869052434014735 77
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- 3) - 1318774441239243093482785428979667792332837296592715599042984198285902
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*sqrt(10) + 13)^4/(4*x - 3) - 272621025987788689601038044650885025352397961
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sqrt(25*sqrt(10) + 65)*(5*sqrt(10) + 13)^4/(4*x - 3) + 2105092353155836516963
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(5*sqrt(10) + 13)^2 - 1067959701549473972666879822348520629728709827958955476
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- 1746986824494206794686738833944638027733482209140504053839776398053835667
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rt(-2*x^2 + 3*x + 1) - sqrt(17))*(5*sqrt(10) + 13)^3/(4*x - 3) - 9523425876
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qrt(2)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*(5*sqrt(10) + 13)^3/(4
*x - 3) - 16586275824704628356259802202714916761823945858547444331199366127
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 + 13)^3 - 4700293946026298637104599449278768660027934035569707532683307896
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*sqrt(25*sqrt(10) + 65)*(5*sqrt(10) + 13)^3 - 11909196551595961152476667381
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^2 + 3*x + 1) - sqrt(17))*sqrt(25*sqrt(10) + 65)*(5*sqrt(10) + 13)^3/(4*x -
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0) + 65)*(5*sqrt(10) + 13)^3/(4*x - 3) - 3845257812629980148449066645669399
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*x + 1) - sqrt(17))*sqrt(25*sqrt(10) + 65)*(5*sqrt(10) + 13)^3/(4*x - 3) -
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2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*(5*sqrt(10) + 13)^4/(4*x - 3)
 - 9991365635841946469737875764748065901689058668549343639020795148552053730
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*sqrt(34)*sqrt(10)*sqrt(2)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*(5
*sqrt(10) + 13)^2/(4*x - 3) + 8699661822886238077327565134536520202309228 34
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) + 65)*(5*sqrt(10) + 13)^2/(4*x - 3) + 137957130558882290668237287962406 34
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*x^2 + 3*x + 1) - sqrt(17))*sqrt(25*sqrt(10) + 65)*(5*sqrt(10) + 13)^2/(4*x
 - 3) + 244516378461602593585714502685213186474719648504762608345999809687 1
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(25*sqrt(10) + 65)*(5*sqrt(10) + 13)^2/(4*x - 3) - 678190988078077576172073
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) + 13)^3/(4*x - 3) - 30198915933962350836023940043842647346583013627181748187623622485555023223689584657520213250930967046223007129005821293529505648640805771394748054992664731069210783423508256822341179788440391649244620386415411200000000000000000*sqrt(2)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17)
)*(5*sqrt(10) + 13)^3/(4*x - 3) - 6645253607135804295510704433994742685574873388998494289943960121375981074968535369742630779703267729667956089538738494719244036362503010825670807955512547987482859757011488396075280185477333425927699397659852800000000000000000*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(
17))*sqrt(25*sqrt(10) + 65)*(5*sqrt(10) + 13)^3/(4*x - 3) - 15071418855295863871501903845011692289690429536373591716212364299872774844774093690130292509770093718811057329138476969648649938682650740203690553160562029210970630433483103118916762781892345625630295057990942720000000000000000*sqrt(25*sqrt(
10) + 65)*(5*sqrt(10) + 13)^3 + 7764908123959061423189628790844628771775998245320244755394392717046187290987541556471775216675912510841536560941531859088740305302998107499338652167225083494412949962296599647007897278106825330538673280455572193280000000000000000*sqrt(34)*sqrt(10)*sqrt(2)*sqrt(25*sqrt(10
) + 65)*(5*sqrt(10) + 13) + 37880903767967642771571576799786788032929416885138782964941551466666362743849138152771645885779236615474432329326187253340675929767731236137516063114382891274466221183675955701987979856586861123889180438345809920000000000000000*sqrt(34)*sqrt(10)*sqrt(2)*(2*sqrt(2)*sqrt(-2*x
^2 + 3*x + 1) - sqrt(17))*sqrt(25*sqrt(10) + 65)*(5*sqrt(10) + 13)/(4*x - 3
) + 11244997418964559389880836470540269963682986186165196928720167902435060918873119137123957415001200968068392434287934942496213233543864510920954207476015817552042255377241327618900740409106320310593220611047161856000000000000000*sqrt(34)*sqrt(10)*(5*sqrt(10) + 13)^2 + 20626968941444999507203266513183922789595664071434636513986934352417366736287242340203001692317929620158581760596503154565361021747926082701525948659770018339403014196663097373359163226822703743868351453784021401600000000000000000*sqrt(34)*sqrt(2)*(5*sqrt(
10) + 13)^2 + 39183171142335996418889680285975206934572878941354584681785824055649093194090166686159806893616143433301748027667720209940249431131880718241454950507604256942667466920541269371272047483196329258580488793649840128000000000000000*sqrt(10)*sqrt(2)*(5*sqrt(10) + 13)^2 + 4494365204691095268240928128170316262612719008013894066046592910193575979599613524415761718467259345722512389997434235403074262725856369254920953547370510154191222422861702580404637348903466824707227484547973120000000000000000*sqrt(34)*sqrt(10)*
(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*(5*sqrt(10) + 13)^2/(4*x - 3)
 + 6782535905228467100932753029286083781137513177835574205804243951019823641
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2592512814969662920411023164601611550150073735198548637855845311204399008941342720000000000000000*sqrt(34)*sqrt(10)*sqrt(2)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*(5*sqrt(10) + 13)/(4*x - 3) + 104565451796009860667761927791432715395981217460064583733861247898638920334998734059797117311222888793132702563146865777527079006808973547964119493268525303524037607121189391670052852081605275742226220399341233766400000000000000*sqrt(34)*sqrt(10)*sqrt(25*sqrt(10) + 65)*(5*sqrt(10) + 13) + 235156132928913200115692213346344456456035484401886002214419970795205407154802345604032862846012363103235164718617888571060823198052160222076051681234443485511470780380902205323010338213926337863360923606224522444800000000000000*sqrt(34)*sqrt(2)*sqrt(25*sqrt(10) + 65)*(5*sqrt(10) + 13) + 441946690536863590129393863801962136355394377335211225496279760579222937967026012513956451280577221849495864670058781392776244801103691564353788062421472851567102959892817756676889795419476215584702395103047162265600000000000000*sqrt(10)*sqrt(2)*sqrt(25*sqrt(10) + 65)*(5*sqrt(10) + 13) + 559716322518312285236507546561818827154396181254847485945808330638496248627446550144167866731499616933380809785744130178972716475532089188487973958729322959023864058300756107707646446555912567149326435722849157120000000000000*sqrt(34)*sqrt(10)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*sqrt(25*sqrt(10) + 65)*(5*sqrt(10) + 13)/(4*x - 3) + 115599081796257517221756961388178672386726146835200659851674941736763041586401047232966236787653508566084454698712018159534383550001191262340791233765734481642978354975162904424420860616459391162081925316319641600000000000000*sqrt(34)*sqrt(2)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*sqrt(25*sqrt(10) + 65)*(5*sqrt(10) + 13)/(4*x - 3) + 228513859436978503718640554393866405036062703472622067265044757285634919378403998673097940432582120711109471335587064634749004641212980624892401916191723322419098831072805734862896568734720200274692662144394067968000000000000000*sqrt(10)*sqrt(2)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*sqrt(25*sqrt(10) + 65)*(5*sqrt(10) + 13)/(4*x - 3) + 32064478570866809299761751115112174312530911369764180273489559135834750380723777355834471198759282815638256131691199927780062236847619568835580313794941894106309342842985671543622495584445006763916561713988422860800000000000000*sqrt(34)*(5*sqrt(10) + 13)^2 + 60466522785660170815906656571673791991110398713315366394847249633276744389892411899061229419018526564207011144173739546027997150235247360481463074290661594573199921773207888808402184102875102349916347500563791872000000000000000*sqrt(10)*(5*sqrt(10) + 13)^2 + 11030365211027487607875317475796232278483549927468603701932498600185519267157247265696969243460666147217886647496931249866448782793817872566691948404744119475754333488425568717240630001025414975068697478952197816320000000000000*sqrt(2)*(5*sqrt(10) + 13)^2 + 152168664039545906579348621997087127501529979779943067814424845650507755105412659455965708188691400905255002476911058398831769280001879803285658692790649833377585649789616636479503159445672584385932734078008688640000000000000000*sqrt(34)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*(5*sqrt(10) + 13)^2/(4*x - 3) + 29992437626373290989822654871906713666030283411280675616314797754072959994587626495402541328980716334943252051790054011200852777062877377204617119218573788166190342316633851935315686501838838

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 1) - sqrt(17))*(5*sqrt(10) + 13)^2/(4*x - 3) + 440743038586116042702165343
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2*x^2 + 3*x + 1) - sqrt(17))*(5*sqrt(10) + 13)^2/(4*x - 3) + 66577405998801
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rt(-2*x^2 + 3*x + 1) - sqrt(17))*sqrt(25*sqrt(10) + 65)*(5*sqrt(10) + 13)^2
/(4*x - 3) + 15378020585412345197279118141970356582383781892276415296195146
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t(10)*sqrt(2)*sqrt(25*sqrt(10) + 65) + 415702711442587835441940328386919596
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)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*sqrt(25*sqrt(10) + 65)/(4*x - 3) + 242
570285234764487197602645201278486382781422650936065163726636030680500922414
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 + 120473733949038273358122374877491185364483973823641087245394884048722983
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sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*(5*sqrt(10) + 13)/(4*x - 3) + 4223891539
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*sqrt(2)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*(5*sqrt(10) + 13)/(4
*x - 3) + 80647600784930331248903485864677645181787547779214581956105378565
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990286907985961931011281109172503278101663038291262120976193296237854720000
0000000000*sqrt(10)*sqrt(2)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*
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(5*sqrt(10) + 13)/(4*x - 3) + 335324624410721589928446991810847370375285412
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) + 13) + 61615025021385512967970531557515840593040234012916276953397710533
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x^2 + 3*x + 1) - sqrt(17))*sqrt(25*sqrt(10) + 65)*(5*sqrt(10) + 13)/(4*x -
3) + 3411766075478957389448011693836200053107188320886909097007474400223840
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10) + 65)*(5*sqrt(10) + 13)/(4*x - 3) + 71559656464536076529641180550715709
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3*x + 1) - sqrt(17))*sqrt(25*sqrt(10) + 65)*(5*sqrt(10) + 13)/(4*x - 3) + 8
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*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*(5*sqrt(10) + 13)^2/(4*x - 3
) + 20386008031550319239480673505248621628918434479632764525451716299401230
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0000000*(5*sqrt(10) + 13)^2 - 143199811900423823203216986398217265436490388
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10)*sqrt(2)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))/(4*x - 3) + 36840
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t(34)*sqrt(10)*sqrt(25*sqrt(10) + 65) + 13889738778295040455225321892570667
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) + 65) + 2371676480369317592833707388333390425552167670547615036285971061
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2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*sqrt(25*sqrt(10) + 65)/(4*x -
3) + 8567464115006848426016623828299023547820825224052474338985762172984641
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5*sqrt(10) + 65)/(4*x - 3) + 1270877955563433562544142108403636636931478940
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x + 1) - sqrt(17))*sqrt(25*sqrt(10) + 65)/(4*x - 3) + 807742674559437526307
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0) + 13) + 1457776248648864039787941273092915202142934370146033202252071102
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00*sqrt(34)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*(5*sqrt(10) + 13)
/(4*x - 3) + 76934493382915034304814565880048711394400328765827694467819654
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rt(10) + 13)/(4*x - 3) + 22818497351339696579385755558537486753774559720254
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887074288435200000000000000*sqrt(2)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqr
t(17))*(5*sqrt(10) + 13)/(4*x - 3) + 10065955588049281425813291295068905951
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sqrt(17))*sqrt(25*sqrt(10) + 65)*(5*sqrt(10) + 13)/(4*x - 3) + 19172287451
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sqrt(10) + 65)(5*sqrt(10) + 13) - 520536908355638289077479947968694726770
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) - 11410796897784691555347841868048205338551343800474908522212891412976009
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597669711052800000000000000*sqrt(34)*sqrt(10)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x +
1) - sqrt(17))/(4*x - 3) - 17457711021519057749557008965712274823215423151
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818525158013519691225180521800820291434271486348861487096146943818343592843
797659123712000000000000000*sqrt(34)*sqrt(2)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x
+ 1) - sqrt(17))/(4*x - 3) - 2472112556077812821544030572128214962446956647
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774315179212800000000000000*sqrt(10)*sqrt(2)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x
+ 1) - sqrt(17))/(4*x - 3) + 8574403650090272268350255504357018873001589384
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)*sqrt(25*sqrt(10) + 65) + 684495633451291163077312214116620435786205721452
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*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*sqrt(25*sqrt(10) + 65)/(4*x - 3
) + 20052507206085411073985028996812350703346708460422869935252269542587303
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00000*sqrt(10)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*sqrt(25*sqrt(1

```
0) + 65)/(4*x - 3) + 678212111410655267980318000290358979373634910023249245
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183075840000000000000000*sqrt(2)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17)
)*sqrt(25*sqrt(10) + 65)/(4*x - 3) + 31487933333905433948984535294475166907
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 sqrt(17))*(5*sqrt(10) + 13)/(4*x - 3) + 4100779890204823239114498330943942
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t(2) - 34170775753270895615035728241295105152952336033977067580641055881751
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00000000*sqrt(34)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))/(4*x - 3) -
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00*sqrt(10)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))/(4*x - 3) - 54654
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t(2)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))/(4*x - 3) + 218636225652
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sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*sqrt(25*sqrt(10) + 65)/(4*x - 3) + 85011
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rt(25*sqrt(10) + 65) + 9638444812622591952531284885578386507663329105824932
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583678976000000000000000000*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))/(4*x
 - 3) + 5779624748888358248079088656544958164016354733130769752830692689118
960768365167768460203123970113680631868400805568458604697488638472210755381
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000000000000)) + 1/10*sqrt(25*sqrt(10) + 65)*log(abs(-430021700303653941656
```

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)*(5*sqrt(10) + 13)^6 - 913093997474970813187945219370491950125291109107979
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000000000000000000*sqrt(34)*sqrt(10)*sqrt(2)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x
+ 1) - sqrt(17))*(5*sqrt(10) + 13)^6/(4*x - 3) - 33078592331050303204366491
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(10) + 65)*(5*sqrt(10) + 13)^6 - 330785923310503032043664919762836830312334
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t(10) + 13)^6 - 330785923310503032043664919762836830312334698347931048446360
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4)*sqrt(10)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*sqrt(25*sqrt(10)
+ 65)*(5*sqrt(10) + 13)^6/(4*x - 3) - 70237999805766985629841939951576303855
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3*x + 1) - sqrt(17))*sqrt(25*sqrt(10) + 65)*(5*sqrt(10) + 13)^6/(4*x - 3) -
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(10)*sqrt(2)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*sqrt(25*sqrt(10)
+ 65)*(5*sqrt(10) + 13)^6/(4*x - 3) - 118137829753751082872737471343870296
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qrt(10) + 13)^7 - 118137829753751082872737471343870296540119535124261088730
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x + 1) - sqrt(17))*(5*sqrt(10) + 13)^7/(4*x - 3) - 250849999306310662963721
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^2 + 3*x + 1) - sqrt(17))*(5*sqrt(10) + 13)^7/(4*x - 3) - 25084999930631066
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t(-2*x^2 + 3*x + 1) - sqrt(17))*(5*sqrt(10) + 13)^7/(4*x - 3) - 16723333287
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2*x^2 + 3*x + 1) - sqrt(17))*sqrt(25*sqrt(10) + 65)*(5*sqrt(10) + 13)^7/(4*
x - 3) - 787585531691673885818249808959135310267463567495073924872287372613
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(25*sqrt(10) + 65)*(5*sqrt(10) + 13)^5 - 1058878051797988055005406933927920
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sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*sqrt(25*sqrt(10) + 65)*(5*sqrt(10) + 13)
^5/(4*x - 3) + 775521370528249819726185365088416501218603746527558094281134
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 + 13)^6 + 7755213705282498197261853650884165012186037465275580942811347705
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(-2*x^2 + 3*x + 1) - sqrt(17))*(5*sqrt(10) + 13)^6/(4*x - 3) - 441199188249
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*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*(5*sqrt(10) + 13)^6/(4*x - 3
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) - 441199188249161689585586222469967081125204968670517833246023525031974151447123033682845440731837198472329684791192458842109216028825366272101903288774993913556516920836654229602682018046350030826110976000000000000000000*sqrt(10)*sqrt(2)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*(5*sqrt(10) + 13)^6/(4*x - 3) + 59655490040634601517398874237570500093738749732889084175471905426899011825473880736729269253078286867200671658235730386503224294063506120470177680009849562970005161902124942006045144266930186994319360000000000000000000*sqrt(34)*sqrt(25*sqrt(10) + 65)*(5*sqrt(10) + 13)^6 + 596554900406346015173988742375705000937387497328890841754719054268990118254738807367292692530782868672006716582357303865032242940635061204701776800098495629700051619021249420060451442669301869943193600000000000000000000*sqrt(10)*sqrt(25*sqrt(10) + 65)*(5*sqrt(10) + 13)^6 + 596554900406346015173988742375705000937387497328890841754719054268990118254738807367292692530782868672006716582357303865032242940635061204701776800098495629700051619021249420060451442669301869943193600000000000000000000*sqrt(2)*sqrt(25*sqrt(10) + 65)*(5*sqrt(10) + 13)^6 - 339383990960893607373527863438436216250157668208090640958479634639980116497786948986804185178336306517176680608609583724699396945250281747770694837519226087351166862182041715078986167727961562173931520000000000000000000*sqrt(34)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*sqrt(25*sqrt(10) + 65)*(5*sqrt(10) + 13)^6/(4*x - 3) - 339383990960893607373527863438436216250157668208090640958479634639980116497786948986804185178336306517176680608609583724699396945250281747770694837519226087351166862182041715078986167727961562173931520000000000000000000*sqrt(10)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*sqrt(25*sqrt(10) + 65)*(5*sqrt(10) + 13)^6/(4*x - 3) - 339383990960893607373527863438436216250157668208090640958479634639980116497786948986804185178336306517176680608609583724699396945250281747770694837519226087351166862182041715078986167727961562173931520000000000000000000*sqrt(2)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*sqrt(25*sqrt(10) + 65)*(5*sqrt(10) + 13)^6/(4*x - 3) - 121208568200319145490545665513727220089342024360032371770885583799992898749209624638144351849405823756134528788789137044535498909017957767060962441971152174053988165065014898242495059902843415062118400000000000000000000*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*(5*sqrt(10) + 13)^7/(4*x - 3) + 2130553215736950054192816937056089289062098204746038720552568050960678993766924312026045330467081673828595416365561799517972296216553790016792060000351770106071612925075890785930183723818935249797120000000000000000000000*(5*sqrt(10) + 13)^7 - 39570435207992611849181059491425410485939741306891469398218389238467932462364479488162273815597307987492148139086591587477897286644357575572030951732457301956869048644610167025055624877164313187334043467776000000000000000000*sqrt(34)*sqrt(10)*sqrt(2)*(5*sqrt(10) + 13)^5 - 778643583067129358312388631866242463567031734106463253670324289053850881578276677741958499128938139000344014656745384434572213277944419624317443369574886346796312094159917410177981216539226710672756703232000000000000000000*sqrt(34)*sqrt(10)*sqrt(2)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*(5*sqrt(10) + 13)^5/(4*x - 3) - 4285347012121538292940010259599919742135087200462921071515556543121626890864006486793248841632061014157935

```
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) + 65)*(5*sqrt(10) + 13)^5 - 703748589406492351954330239202672217033371189
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(10) + 13)^5 - 152939025398950791993531787893071294549295859614820663638104
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rt(34)*sqrt(10)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*sqrt(25*sqrt(
10) + 65)*(5*sqrt(10) + 13)^5/(4*x - 3) - 143833300076827606719798220446570
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*x^2 + 3*x + 1) - sqrt(17))*sqrt(25*sqrt(10) + 65)*(5*sqrt(10) + 13)^5/(4*x
 - 3) - 3191473475920220028518837025657049434856275211415943709309740069126
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5*sqrt(10) + 65)*(5*sqrt(10) + 13)^5/(4*x - 3) - 32189669227294874275808855
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 - 665914052515871896083500075933909585915801755585240122690786282136845729
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*sqrt(10)*(5*sqrt(10) + 13)^6 - 7805865059301796138586372481183596870907444
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)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*(5*sqrt(10) + 13)^6/(4*x - 3) - 139065
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*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*(5*sqrt(10) + 13)^6/(4*x - 3
) - 16341452807917486162788305004806012478985450411259697303240661369785725
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097809552521442994781305342739633182817063100054052536320000000000000000000
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0*sqrt(2)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*(5*sqrt(10) + 13)^6
/(4*x - 3) - 130386016458211282345489244751689798300530598594859105175364l8
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000000000*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*sqrt(25*sqrt(10) +
65)*(5*sqrt(10) + 13)^6/(4*x - 3) - 622503553295197341001862723972670932603
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523428906926080000000000000000000*sqrt(25*sqrt(10) + 65)*(5*sqrt(10) + 13)^6
 + 447497414323453307486715808415563617617934070901279669387882109742235967
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4)*sqrt(10)*sqrt(2)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*sqrt(25*s
qrt(10) + 65)*(5*sqrt(10) + 13)^4/(4*x - 3) + 237397883283023850570538766663
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+ 13)^5 + 29199458776821263787926221633556246091620416514506494699068091467
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00000000*sqrt(34)*sqrt(2)*(5*sqrt(10) + 13)^5 + 455784701223778999805432565
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770939607766660557747047628800000000000000000000*sqrt(10)*sqrt(2)*(5*sqrt(10)
 + 13)^5 - 7364040088838401816522880466837310292914636400035545968195213912
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720777151621073000356341800283785584530098422671219386806320496640000000000
0000000*sqrt(34)*sqrt(10)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*(5*
sqrt(10) + 13)^5/(4*x - 3) - 3842446294157938476334815052872299280412906619
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4533207040000000000000000000*sqrt(34)*sqrt(2)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x +
1) - sqrt(17))*(5*sqrt(10) + 13)^5/(4*x - 3) - 1316057314998023336038239607
143800403377723555776533678102754385611599358099287459567235597352520585667
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qrt(-2*x^2 + 3*x + 1) - sqrt(17))*(5*sqrt(10) + 13)^5/(4*x - 3) + 277857967
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rt(25*sqrt(10) + 65)*(5*sqrt(10) + 13)^5 + 42675807031370563330237584351617
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*sqrt(10) + 13)^5 + 476391438027513621764851706881833343709827641914645753
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)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*sqrt(25*sqrt(10) + 65)*(5*s
qrt(10) + 13)^5/(4*x - 3) - 12670076110271598121866388929986496451415996638
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(17))*sqrt(25*sqrt(10) + 65)*(5*sqrt(10) + 13)^5/(4*x - 3) - 15493750915066
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2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*sqrt(25*sqrt(10) + 65)*(5*sqrt(10) +
13)^5/(4*x - 3) - 674986234011037151606294946239426379184068182838106253791
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13)^6/(4*x - 3) + 203666574981652341200196113634686669887218875774619451234
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 + 13)^4 + 141220190584234751974437160617025542508021516309815045163711877013
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0000000*sqrt(34)*sqrt(10)*sqrt(2)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sq
rt(17))*(5*sqrt(10) + 13)^4/(4*x - 3) - 165903952685017175966556826200143450
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qrt(34)*sqrt(10)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*sqrt(25*sqrt
(10) + 65)*(5*sqrt(10) + 13)^4/(4*x - 3) - 255048036802984383489276670374666
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-2*x^2 + 3*x + 1) - sqrt(17))*sqrt(25*sqrt(10) + 65)*(5*sqrt(10) + 13)^4/(4
*x - 3) - 369517783079003520416706836751835802156195393221479403425589599951
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t(25*sqrt(10) + 65)*(5*sqrt(10) + 13)^4/(4*x - 3) - 14019664832755956496226
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)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*(5*sqrt(10) + 13)^5/(4*x -
3) - 41665755252624329745812042214264543852459719564520218170800288342696551
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000*sqrt(10)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*(5*sqrt(10) + 13
)^5/(4*x - 3) - 13013976515250508162915675415804338327847148623955842612720
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rt(10) + 13)^5/(4*x - 3) - 14426040851790824011348733317123305133077861574422
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887662080000000000000000000*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*sqr
t(25*sqrt(10) + 65)*(5*sqrt(10) + 13)^5/(4*x - 3) - 73788778916814074131703
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5*sqrt(10) + 13)^5 + 1579555784164651649985287206063801100037614715241730426
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qrt(10) + 13)^3 + 469413338012819356375748842595876944943903692926268823768
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4000000000000000*sqrt(34)*sqrt(10)*sqrt(2)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x +
1) - sqrt(17))*sqrt(25*sqrt(10) + 65)*(5*sqrt(10) + 13)^3/(4*x - 3) + 12524
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(34)*sqrt(10)*(5*sqrt(10) + 13)^4 + 224781955134548402273786675448322975784
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-2*x^2 + 3*x + 1) - sqrt(17))*(5*sqrt(10) + 13)^4/(4*x - 3) + 3423803642832
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(2)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*(5*sqrt(10) + 13)^4/(4*x
- 3) + 84015280726121635800453532528765667616613303010462885763805808736234
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rt(10) + 13)^4/(4*x - 3) + 288326361599803095661606445233752359960769385560
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)^4 + 572072774958708990911115200371177904562442645174084315632207746861881
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sqrt(10) + 65)(5*sqrt(10) + 13)^4 + 3312607045827487032633036550847798066

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+ 1) - sqrt(17))*sqrt(25*sqrt(10) + 65)*(5*sqrt(10) + 13)^4/(4*x - 3) + 760
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(10)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*sqrt(25*sqrt(10) + 65)*(
5*sqrt(10) + 13)^4/(4*x - 3) + 10789894726834870089092162899356106434035615
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qrt(17))*sqrt(25*sqrt(10) + 65)*(5*sqrt(10) + 13)^4/(4*x - 3) - 58636018981
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(10)*sqrt(2)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*(5*sqrt(10) + 13
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2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*sqrt(25*sqrt(10) + 65)*(5*sqrt
(10) + 13)^3/(4*x - 3) + 15543778285433189264141664816244457782347533664141

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) - sqrt(17))*sqrt(25*sqrt(10) + 65)*(5*sqrt(10) + 13)^3/(4*x - 3) + 299637
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+ 65)*(5*sqrt(10) + 13)^3/(4*x - 3) + 2058614649151037287535948118819822941
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(10)*(5*sqrt(10) + 13)^4 + 353280273803162708176870447942372592972778159781
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2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*(5*sqrt(10) + 13)^4/(4*x - 3) + 17064
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 - 3) + 207660092546597843412212652268736800703078037218654083612970970383138
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+ 13)^4/(4*x - 3) + 648017101060658313848756889594054193672919669748652166227
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rt(10) + 65)*(5*sqrt(10) + 13)^4/(4*x - 3) - 4716185423529149601070339709768
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^2 + 3*x + 1) - sqrt(17))*(5*sqrt(10) + 13)^3/(4*x - 3) + 93630134054999056
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*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*(5*sqrt(10) + 13)^3/(4*x - 3
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+ 3*x + 1) - sqrt(17))*sqrt(25*sqrt(10) + 65)*(5*sqrt(10) + 13)^3/(4*x - 3)
+ 327565838664439769181148203845137052062924679032521096361653150328302956
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 + 65)*(5*sqrt(10) + 13)^3/(4*x - 3) + 7598468688410081850102439933390392418
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x + 1) - sqrt(17))*sqrt(25*sqrt(10) + 65)*(5*sqrt(10) + 13)^3/(4*x - 3) + 4
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(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*(5*sqrt(10) + 13)^4/(4*x - 3)
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)*(5*sqrt(10) + 13)^2/(4*x - 3) - 186554281391705577838183763400339408658329
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)*(5*sqrt(10) + 13)^2 - 1171161752075319362282992254207885440120479095842947
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0) + 13)^2 - 23604978831104209778234139935427683197491821522217286057792950
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) + 13)^3/(4*x - 3) + 904436623599804766592941062787172791550915733285371 71
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6566656000000000000000000*sqrt(10)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17
))*(5*sqrt(10) + 13)^3/(4*x - 3) + 590909355609342279199778333295193859 2771
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 1) - sqrt(17))*(5*sqrt(10) + 13)^3/(4*x - 3) + 109689595110851992187103488
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3*x + 1) - sqrt(17))*sqrt(25*sqrt(10) + 65)*(5*sqrt(10) + 13)^3/(4*x - 3) +
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)*sqrt(25*sqrt(10) + 65)*(5*sqrt(10) + 13) - 554813250958058796781903093426
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10) + 13)/(4*x - 3) - 239725860908962188005790447469642017880660779505341 61
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5*sqrt(10) + 65)*(5*sqrt(10) + 13)^2/(4*x - 3) - 37200007809299254334634206
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(-2*x^2 + 3*x + 1) - sqrt(17))*sqrt(25*sqrt(10) + 65)*(5*sqrt(10) + 13)^2/(
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/(4*x - 3) - 17448665376025286585985819205723159408847448093241231715861957
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)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*sqrt(25*sqrt(10) + 65)*(5*s
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) + 13) - 1913442119796232686917894822637662064220821359175859799821091688 9
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000000000000*sqrt(34)*sqrt(10)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17)
)*(5*sqrt(10) + 13)/(4*x - 3) - 51030182667731409919402733068757915755154 23
2237777577302831139602762613479005860077991682697841955158988995143833364 66
8963117961176196431191598761982514983968079611088201303178361569268696870 71
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+ 3*x + 1) - sqrt(17))*(5*sqrt(10) + 13)/(4*x - 3) - 952561750596980857784 5
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*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*(5*sqrt(10) + 13)/(4*x - 3) - 7
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0*sqrt(34)*sqrt(25*sqrt(10) + 65)*(5*sqrt(10) + 13) - 1306960374911109113 42
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qrt(-2*x^2 + 3*x + 1) - sqrt(17))*sqrt(25*sqrt(10) + 65)*(5*sqrt(10) + 13)/
(4*x - 3) - 1029689523995620495466992696467308453710045953976153169373946 71
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25*sqrt(10) + 65)*(5*sqrt(10) + 13)/(4*x - 3) - 136469696032340855400612406
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+ 3*x + 1) - sqrt(17))*(5*sqrt(10) + 13)^2/(4*x - 3) - 4349065716082543255 0
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sqrt(-2*x^2 + 3*x + 1) - sqrt(17))/(4*x - 3) - 7705537919673617018422304359
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+ 3*x + 1) - sqrt(17))*sqrt(25*sqrt(10) + 65)/(4*x - 3) - 10837503079896122
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2)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*sqrt(25*sqrt(10) + 65)/(4*
x - 3) - 175241864253867535815024771977091498742613903841835831894313669131
354503384707378950767724751382591875663986333205324078748951934944667940365
324953843501586124397318593456565001703716423692744158489605371307071897600
00000000000*sqrt(10)*sqrt(2)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*
sqrt(25*sqrt(10) + 65)/(4*x - 3) - 1696839481769345178370342505918637961182
536064400048020999831050572070565427276024036282099751821875816842875031814
676229707688469483665807299779577142479311970756739421216466328382903499905
392900433151439652519936000000000000000*sqrt(34)*(5*sqrt(10) + 13) - 3062666
260307891027462784130827408104496110052366731670087165853651002094355962448
286669871906200107199804407395394384159252499374003324854138181504174684767
846141499214629814970078324152857036049365599689092628480000000000000000*sqr
t(10)*(5*sqrt(10) + 13) - 8156173780064043160166590269760920460985507812510
234301605825384508383141469054516428806586459331213389069116899842270420472
092843084009119048034872429100581842068701047726786499786795634234262535029
264218633273344000000000000000*sqrt(2)*(5*sqrt(10) + 13) - 61411047125061336
217266656591146795483940526703801134944718849742438410189193315631236091628
648917611365542636437959244708686340745116072347052792439239702096137624995
834398471638395925140652071754498293447314636800000000000000000*sqrt(34)*(2*sq
```

```
rt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*(5*sqrt(10) + 13)/(4*x - 3) - 1088
1497020847195609499263632638197967594063035531112911368613942122521948392347
5765667542081833941066414231320200574537173222419049420387554171176019181252
828273522093367330168861292434552071750655436318564674814607360000000000000000*
sqrt(10)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*(5*sqrt(10) + 13)/(4
*x - 3) - 3032562662108122518819383867914003008118176372756046411982484682828
6530804673677369402454476631158680838688345058138337040831927980437020769707
2379994638487085402804993697534914674702524155362226194276018593050342195207
0000000000000*sqrt(2)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*(5*sqrt
(10) + 13)/(4*x - 3) - 14391224238494047738024924111201627736687143894208244
6779495242932109253584477182185681486343028269425302797364208225591275751144
1501084572860576872079429066931107526599485079542237434975279061962282713191
4493915430912000000000000000*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*sq
rt(25*sqrt(10) + 65)*(5*sqrt(10) + 13)/(4*x - 3) - 405086009916944233015649
3858804895375789421063912249459526923995012751609882966945018417758476755411
8944689625002661183068885020696103825064354896694626622281779840708847770433
66911451453808203107566293535913567846400000000000000000*sqrt(25*sqrt(10) + 6
5)*(5*sqrt(10) + 13) + 10743465436434116301134120796185824983284123465936511
4705620598815302043153555713200574215029438096221771902531825746082128039277
7075170223024150648093604413512107902808353218128593487566271600889311028055
3159182336000000000000000000*sqrt(34)*sqrt(10) + 1530721755475057937908183179999
5425640862363817766593207787403797290011192755933101864227590813029141423255
0220708010706837925872254379579678474496924508559531530055066131381302261066
49286976781912983700400902603014144000000000000000*sqrt(34)*sqrt(2) + 2412182
8459368467946504198161771522547760599469574120934570810803008719638559030088
4482934388126755552727031847663045591641655573765907047194089151492222718255
92603525930692032428756794360701563131494486819784255078400000000000000*sqrt
(10)*sqrt(2) + 4424770105202430920746273910888315203604154965814156505072066
3758068190391267436252894427465692016579126994414743364283678787268165911399
8746032318065090526661742460152921319564330419978657582283992988411098793244
672000000000000*sqrt(34)*sqrt(10)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt
(17))/(4*x - 3) + 48254128821803814747073071619901517971813421496191918218000
8899120583270108689200126808354473960621709906162002480443001035356536239033
2305626073271710523564756560900614563523139781062909665985010172895649563933
47200000000000000000*sqrt(34)*sqrt(2)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sq
rt(17))/(4*x - 3) + 961509578692555089744639805959543640722860060266201848999
5259267616106222751485017009985924747177834855052116581891666856953029683433
0390317076537929279625291311185222547598112950799637276166231738472485795088
30592000000000000000000*sqrt(10)*sqrt(2)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) -
sqrt(17))/(4*x - 3) - 182201452989772213228973042616045158178610606611890844
7852438222323978949631016682666206940168649789653704266449434965899827637622
5251842543336952665677265864990164706414149324651413449599152042674807752111
1700312064000000000000000000*sqrt(34)*sqrt(25*sqrt(10) + 65) - 3629030807166434
8751297240264356733692624397366512255981963965096117195195362433518489726066
6072665009731883749542476232864414287759395928321297428377725157989716758888
```

2321596527910891636226961835623250772709259345920000000000000*sqrt(10)*sqrt
(25*sqrt(10) + 65) - 14337908385572800467067688752098400568433506801874932
099787806584069975469268374982615862302423123280309853616849814846500156800
934848425931197390025741847738574935273050565192842601644440001967346842571
99727837184000000000000*sqrt(2)*sqrt(25*sqrt(10) + 65) - 75762766785723440
057381613733747136841997295501269154575160134841239342017068104818248126204
378703048058365397070517744471487256267602592259977313079633144215554209407
305973169388091630580912119194152892099670835200000000000000*sqrt(34)*(2*sqr
t(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*sqrt(25*sqrt(10) + 65)/(4*x - 3) -
135430090299845438675930247658152721323790451802658991605281222269524653348
146485584591042745983935330726924080217607464184825120879116453241393281961
1747682370548611921233590652122076908660693062514354979115892736000000000000
00*sqrt(10)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*sqrt(25*sqrt(10)
+ 65)/(4*x - 3) - 592473398908132091674400808002937580378562040157336087432
817740641672914206631544237674585425599440327213644239977539523045097241798
023296063669728652338364061085804041162317703315123491844008576154682654023
745536000000000000000*sqrt(2)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*
sqrt(25*sqrt(10) + 65)/(4*x - 3) - 3511873937628599549413799858905490343196
240983316764197586331096915851555866575492296543498549026132857311665268611
955558125686116714955930850762201741610483863771573032518089576947736052623
941776194616278302851072000000000000000*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) -
sqrt(17))*(5*sqrt(10) + 13)/(4*x - 3) - 80630452705894674411605139603242312
9943394333076555035489054631615566125320804771778832379760017624492406723784
098965834334885104641415364807978827365128468838669327141173646911982926270
38231533120813072732127232000000000000000*sqrt(34) - 4734025366235559181270
529251880229266499044117525719135570744388218784611933373547867391648155960
900511651319004801255352949292026260063532595693119612825447457644941862809
38790816089424539201657745469094651494400000000000000000000*sqrt(10) + 187120
204560840330549977778488408799619962745303300723530639751073634980009333022
450944864462151351427298192307286509110725575384458424045812370989692433185
424249007614789885766976313508329857550270091586647254630400000000000000*sqr
t(2) + 12877930446168420364011730136648950154738627684465244541467662503326
394323984434212331342561951572031913449543140786664097311958374972581319359
577465571542975313174288786415177034488662721188922842059249359477800960000
000000000*sqrt(34)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))/(4*x - 3)
+ 1769529440412918255352455049097539769994147508052348719989007250920398756
797173151607083126243910337596800547122660637817362748719843504434383716912
302806627645020281740744977735463780133845743048313596957759111168000000000
0000*sqrt(10)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))/(4*x - 3) + 275
489577554052621387343350628960795376126348546258471762202827455081382825656
567744540113459710857644507040531059125933401037631847040316463631568552715
72672063968744191361726282828359864317925980776345397886976000000000000000*
sqrt(2)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))/(4*x - 3) - 463126073
980293664465956016559182546894536703323849198560335491341212396990578954883
355645302358346416201768556180602078557650494763056655901999340776107939734

```
46000854244826453370553386871911980101789501433577472000000000000000*(2*sqr
t(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*sqrt(25*sqrt(10) + 65)/(4*x - 3) -
176970845715038517457198405400821463370110658859455989220963086595400812414
658223244438102584054454839465750001262282770571555365430449159271187568155
192854825795801485666779437776546816152762863515898367047001702400000000000
000*sqrt(25*sqrt(10) + 65) - 6346351378391825833348061748496871591916857394
415238979271455933360879790289907072145148562670589215958755891779474469319
351952746156887522755285446949664008154572169788027293732146300742799867941
478672214523904000000000000000*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17)
)/(4*x - 3) - 1220916004172746191189909966420174155917556907993082266155587
233998093086894831710153870376131328332349012713700661046094831714732983671
194929435021365479720369781702976000328711396937511205268245301395356258108
70272000000000000000))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2+x}{(2+4x-3x^2)\sqrt{1+3x-2x^2}}\,dx=\int \frac{x+2}{\sqrt{-2\,x^2+3\,x+1}\,\left(-3\,x^2+4\,x+2\right)}\,dx$$

```
[In] int((x + 2)/((3*x - 2*x^2 + 1)^(1/2)*(4*x - 3*x^2 + 2)),x)
[Out] int((x + 2)/((3*x - 2*x^2 + 1)^(1/2)*(4*x - 3*x^2 + 2)), x)
```

3.26 $\int \frac{2+x}{(2+4x-3x^2)(1+3x-2x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 166

$$\int \frac{2+x}{(2+4x-3x^2)(1+3x-2x^2)^{3/2}}\,dx = -\frac{2(15+14x)}{17\sqrt{1+3x-2x^2}} - \frac{9}{2}\sqrt{\frac{1}{5}\left(-3+\sqrt{10}\right)}\arctan\left(\frac{3(4-\sqrt{10})+(1+4\sqrt{10})\,x}{2\sqrt{1+\sqrt{10}}\sqrt{1+3x-2x^2}}\right) + \frac{9}{2}\sqrt{\frac{1}{5}\left(3+\sqrt{10}\right)}\operatorname{arctanh}\left(\frac{3(4+\sqrt{10})+(1-4\sqrt{10})\,x}{2\sqrt{-1+\sqrt{10}}\sqrt{1+3x-2x^2}}\right)$$

```
[Out] -2/17*(15+14*x)/(-2*x^2+3*x+1)^(1/2)-9/10*arctan(1/2*(12-3*10^(1/2)+x*(1+4*
10^(1/2)))/(-2*x^2+3*x+1)^(1/2)/(1+10^(1/2))^(1/2))*(-15+5*10^(1/2))^(1/2)+
9/10*arctanh(1/2*(x*(1-4*10^(1/2))+12+3*10^(1/2))/(-2*x^2+3*x+1)^(1/2)/(-1+
10^(1/2))^(1/2))*(15+5*10^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1030, 12, 1046, 738, 210, 212}

$$\int \frac{2+x}{(2+4x-3x^2)(1+3x-2x^2)^{3/2}}\,dx = -\frac{9}{2}\sqrt{\frac{1}{5}\left(\sqrt{10}-3\right)}\arctan\left(\frac{(1+4\sqrt{10})\,x+3(4-\sqrt{10})}{2\sqrt{1+\sqrt{10}}\sqrt{-2x^2+3x+1}}\right) + \frac{9}{2}\sqrt{\frac{1}{5}\left(3+\sqrt{10}\right)}\operatorname{arctanh}\left(\frac{(1-4\sqrt{10})\,x+3(4+\sqrt{10})}{2\sqrt{\sqrt{10}-1}\sqrt{-2x^2+3x+1}}\right) - \frac{2(14x+15)}{17\sqrt{-2x^2+3x+1}}$$

```
[In] Int[(2 + x)/((2 + 4*x - 3*x^2)*(1 + 3*x - 2*x^2)^(3/2)),x]
```

```
[Out] (-2*(15 + 14*x))/(17*Sqrt[1 + 3*x - 2*x^2]) - (9*Sqrt[(-3 + Sqrt[10])/5]*Ar
cTan[(3*(4 - Sqrt[10]) + (1 + 4*Sqrt[10])*x)/(2*Sqrt[1 + Sqrt[10]]*Sqrt[1 +
 3*x - 2*x^2])])/2 + (9*Sqrt[(3 + Sqrt[10])/5]*ArcTanh[(3*(4 + Sqrt[10]) +
(1 - 4*Sqrt[10])*x)/(2*Sqrt[-1 + Sqrt[10]]*Sqrt[1 + 3*x - 2*x^2])])/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1030

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x + c*x^2)^(p + 1)*
((d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
 - b*f))*(p + 1)))*(g*c*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d +
b^2*f - c*(b*e + 2*a*f)) + c*(g*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - h*(b*
c*d - 2*a*c*e + a*b*f))*x), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*
d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*
x^2)^q*Simp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)
 + (b^2*(g*f) - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*((-h)*c*
e)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((g*c)*(2*a*c*e - b*(c*d + a*f)) + (g*
b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((g*c)*(2*
a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(
p + q + 2) - (b^2*g*f - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*
((-h)*c*e)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(g*f) - b*(h*c
```

```
*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) + a*h*c*e))*(2*p + 2*q + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e
 - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
```

Rule 1046

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\text{integral} = -\frac{2(15+14x)}{17\sqrt{1+3x-2x^2}} + \frac{2}{17}\int \frac{153x}{2\left(2+4x-3x^2\right)\sqrt{1+3x-2x^2}}\,dx$$

$$= -\frac{2(15+14x)}{17\sqrt{1+3x-2x^2}} + 9\int \frac{x}{\left(2+4x-3x^2\right)\sqrt{1+3x-2x^2}}\,dx$$

$$= -\frac{2(15+14x)}{17\sqrt{1+3x-2x^2}} + \frac{1}{5}\left(9\left(5-\sqrt{10}\right)\right)\int \frac{1}{\left(4-2\sqrt{10}-6x\right)\sqrt{1+3x-2x^2}}\,dx + \frac{1}{5}\left(9\left(5+\sqrt{10}\right)\right)\int \frac{1}{\left(4+2\sqrt{10}-6x\right)\sqrt{1+3x-2x^2}}\,dx$$

$$= -\frac{2(15+14x)}{17\sqrt{1+3x-2x^2}} - \frac{1}{5}\left(18\left(5-\sqrt{10}\right)\right)\text{Subst}\left(\int \frac{1}{144+72\left(4-2\sqrt{10}\right)-8\left(4-2\sqrt{10}\right)^2-x^2}\,dx, x, \frac{-12-3\left(4-2\sqrt{10}\right)-(18}{\sqrt{1+3x-}}\right.$$
$$- \frac{1}{5}\left(18\left(5+\sqrt{10}\right)\right)\text{Subst}\left(\int \frac{1}{144+72\left(4+2\sqrt{10}\right)-8\left(4+2\sqrt{10}\right)^2-x^2}\,dx, x, \frac{-12-3\left(4+2\sqrt{10}\right)-(18}{\sqrt{1+3x-}}\right.$$

$$= -\frac{2(15+14x)}{17\sqrt{1+3x-2x^2}} - \frac{9}{2}\sqrt{\frac{1}{5}\left(-3+\sqrt{10}\right)}\tan^{-1}\left(\frac{3\left(4-\sqrt{10}\right)+\left(1+4\sqrt{10}\right)x}{2\sqrt{1+\sqrt{10}}\sqrt{1+3x-2x^2}}\right) + \frac{9}{2}\sqrt{\frac{1}{5}\left(3+\sqrt{10}\right)}\tanh^{-1}\left(\frac{3\left(4+\sqrt{10}\right)+\left(1-4\sqrt{10}\right)x}{2\sqrt{-1+\sqrt{10}}\sqrt{1+3x-2x^2}}\right)$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.33 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.83

$$\int \frac{2+x}{(2+4x-3x^2)(1+3x-2x^2)^{3/2}}\,dx =$$

$$-\frac{2(15+14x)}{17\sqrt{1+3x-2x^2}}+\frac{9}{2}\text{RootSum}\Bigg[5+20\#1+8\#1^2-8\#1^3$$

$$+2\#1^4\&,\frac{3\log(x)-3\log\left(-1+\sqrt{1+3x-2x^2}-x\#1\right)-2\log(x)\#1+2\log\left(-1+\sqrt{1+3x-2x^2}-x\#1\right.}{5+4\#1-6\#1^2+2\#1^3}$$

```
[In] Integrate[(2 + x)/((2 + 4*x - 3*x^2)*(1 + 3*x - 2*x^2)^(3/2)),x]
```

```
[Out] (-2*(15 + 14*x))/(17*Sqrt[1 + 3*x - 2*x^2]) + (9*RootSum[5 + 20*#1 + 8*#1^2
 - 8*#1^3 + 2*#1^4 & , (3*Log[x] - 3*Log[-1 + Sqrt[1 + 3*x - 2*x^2] - x*#1]
 - 2*Log[x]*#1 + 2*Log[-1 + Sqrt[1 + 3*x - 2*x^2] - x*#1]*#1)/(5 + 4*#1 - 6
*#1^2 + 2*#1^3) & ])/2
```

Maple [A] (verified)

Time = 2.40 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.17

method	result
risch	$-\frac{2(15+14x)}{17\sqrt{-2x^2+3x+1}}-\frac{9\left(-2+\sqrt{10}\right)\sqrt{10}\,\arctan\left(\frac{-1-\sqrt{10}+\frac{9\left(\frac{1}{3}+\frac{4\sqrt{10}}{3}\right)\left(x-\frac{2}{3}+\frac{\sqrt{10}}{3}\right)}{2}}{\sqrt{1+\sqrt{10}}\sqrt{-18\left(x-\frac{2}{3}+\frac{\sqrt{10}}{3}\right)^2+9\left(\frac{1}{3}+\frac{4\sqrt{10}}{3}\right)\left(x-\frac{2}{3}+\frac{\sqrt{10}}{3}\right)-1-\sqrt{10}}}\right)}{20\sqrt{1+\sqrt{10}}}+\frac{9\left(2+\sqrt{10}\right.}{}$
default	$-\frac{\left(8+\sqrt{10}\right)\sqrt{10}\left(\frac{1}{3\left(-\frac{1}{9}+\frac{\sqrt{10}}{9}\right)\sqrt{-2\left(x-\frac{2}{3}-\frac{\sqrt{10}}{3}\right)^2+\left(\frac{1}{3}-\frac{4\sqrt{10}}{3}\right)\left(x-\frac{2}{3}-\frac{\sqrt{10}}{3}\right)-\frac{1}{9}+\frac{\sqrt{10}}{9}}}-\frac{\left(\frac{1}{3}-\right.}{3\left(-\frac{1}{9}+\frac{\sqrt{10}}{9}\right)\left(\frac{8}{9}-\frac{8\sqrt{10}}{9}-\left(\frac{1}{3}-\frac{4\sqrt{10}}{3}\right)^2\right)\sqrt{-}}\right.}{}$
trager	$\frac{2(15+14x)\sqrt{-2x^2+3x+1}}{17(2x^2-3x-1)}-18\,\text{RootOf}\left(6400\_Z^4-480\_Z^2-1\right)\ln\left(-\frac{8595200x\,\text{RootOf}\left(6400\_Z^4-480\_Z^2-\right.}{}\right.$

```
[In] int((2+x)/(-3*x^2+4*x+2)/(-2*x^2+3*x+1)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/17*(15+14*x)/(-2*x^2+3*x+1)^(1/2)-9/20*(-2+10^(1/2))*10^(1/2)/(1+10^(1/2
))^(1/2)*arctan(9/2*(-2/9-2/9*10^(1/2)+(1/3+4/3*10^(1/2))*(x-2/3+1/3*10^(1/
```

```
2)))/(1+10^(1/2))^(1/2)/(-18*(x-2/3+1/3*10^(1/2))^2+9*(1/3+4/3*10^(1/2))*(x
-2/3+1/3*10^(1/2))-1-10^(1/2))^(1/2))+9/20*(2+10^(1/2))*10^(1/2)/(-1+10^(1/
2))^(1/2)*arctanh(9/2*(-2/9+2/9*10^(1/2)+(1/3-4/3*10^(1/2))*(x-2/3-1/3*10^(
1/2)))/(-1+10^(1/2))^(1/2)/(-18*(x-2/3-1/3*10^(1/2))^2+9*(1/3-4/3*10^(1/2))
*(x-2/3-1/3*10^(1/2))-1+10^(1/2))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 361 vs. $2(118) = 236$.

Time = 0.31 (sec) , antiderivative size = 361, normalized size of antiderivative = 2.17

$$\int \frac{2+x}{(2+4x-3x^2)(1+3x-2x^2)^{3/2}}\,dx =$$
$$-\frac{17\sqrt{5}(2x^2-3x-1)\sqrt{-81\sqrt{10}+243}\log\left(-\frac{45\sqrt{10}x+\left(3\sqrt{10}\sqrt{5}x+10\sqrt{5}x\right)\sqrt{-81\sqrt{10}+243}+90x-90\sqrt{-2x^2+3x+1}+90}{x}\right)}{}$$

```
[In] integrate((2+x)/(-3*x^2+4*x+2)/(-2*x^2+3*x+1)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/170*(17*sqrt(5)*(2*x^2 - 3*x - 1)*sqrt(-81*sqrt(10) + 243)*log(-(45*sqrt
(10)*x + (3*sqrt(10)*sqrt(5)*x + 10*sqrt(5)*x)*sqrt(-81*sqrt(10) + 243) + 9
0*x - 90*sqrt(-2*x^2 + 3*x + 1) + 90)/x) - 17*sqrt(5)*(2*x^2 - 3*x - 1)*sqr
t(-81*sqrt(10) + 243)*log(-(45*sqrt(10)*x - (3*sqrt(10)*sqrt(5)*x + 10*sqrt
(5)*x)*sqrt(-81*sqrt(10) + 243) + 90*x - 90*sqrt(-2*x^2 + 3*x + 1) + 90)/x)
 + 153*sqrt(5)*(2*x^2 - 3*x - 1)*sqrt(sqrt(10) + 3)*log(9*(5*sqrt(10)*x + (
3*sqrt(10)*sqrt(5)*x - 10*sqrt(5)*x)*sqrt(sqrt(10) + 3) - 10*x + 10*sqrt(-2
*x^2 + 3*x + 1) - 10)/x) - 153*sqrt(5)*(2*x^2 - 3*x - 1)*sqrt(sqrt(10) + 3)
*log(9*(5*sqrt(10)*x - (3*sqrt(10)*sqrt(5)*x - 10*sqrt(5)*x)*sqrt(sqrt(10)
+ 3) - 10*x + 10*sqrt(-2*x^2 + 3*x + 1) - 10)/x) + 600*x^2 - 20*sqrt(-2*x^2
 + 3*x + 1)*(14*x + 15) - 900*x - 300)/(2*x^2 - 3*x - 1)
```

Sympy [F]

$$\int \frac{2+x}{(2+4x-3x^2)(1+3x-2x^2)^{3/2}}\,dx =$$
$$-\int \frac{x}{-6x^4\sqrt{-2x^2+3x+1}+17x^3\sqrt{-2x^2+3x+1}-5x^2\sqrt{-2x^2+3x+1}-10x\sqrt{-2x^2+3x+1}-2\sqrt{-2x}}$$
$$-\int \frac{2}{-6x^4\sqrt{-2x^2+3x+1}+17x^3\sqrt{-2x^2+3x+1}-5x^2\sqrt{-2x^2+3x+1}-10x\sqrt{-2x^2+3x+1}-2\sqrt{-2x}}$$

```
[In] integrate((2+x)/(-3*x**2+4*x+2)/(-2*x**2+3*x+1)**(3/2),x)
```

```
[Out] -Integral(x/(-6*x**4*sqrt(-2*x**2 + 3*x + 1) + 17*x**3*sqrt(-2*x**2 + 3*x +
 1) - 5*x**2*sqrt(-2*x**2 + 3*x + 1) - 10*x*sqrt(-2*x**2 + 3*x + 1) - 2*sqr
t(-2*x**2 + 3*x + 1)), x) - Integral(2/(-6*x**4*sqrt(-2*x**2 + 3*x + 1) + 1
7*x**3*sqrt(-2*x**2 + 3*x + 1) - 5*x**2*sqrt(-2*x**2 + 3*x + 1) - 10*x*sqrt
(-2*x**2 + 3*x + 1) - 2*sqrt(-2*x**2 + 3*x + 1)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 678 vs. $2(118) = 236$.

Time = 0.29 (sec) , antiderivative size = 678, normalized size of antiderivative = 4.08

$$\int \frac{2+x}{(2+4x-3x^2)(1+3x-2x^2)^{3/2}}\,dx = \frac{1}{340}\sqrt{10}\left(\frac{124\sqrt{10}x}{\sqrt{10}\sqrt{-2\,x^2+3\,x+1}+\sqrt{-2\,x^2+3\,x+1}} - \frac{}{\sqrt{10}\sqrt{-2}}\right.$$

```
[In] integrate((2+x)/(-3*x^2+4*x+2)/(-2*x^2+3*x+1)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/340*sqrt(10)*(124*sqrt(10)*x/(sqrt(10)*sqrt(-2*x^2 + 3*x + 1) + sqrt(-2*x
^2 + 3*x + 1)) - 124*sqrt(10)*x/(sqrt(10)*sqrt(-2*x^2 + 3*x + 1) - sqrt(-2*
x^2 + 3*x + 1)) + 153*sqrt(10)*arcsin(8/17*sqrt(17)*sqrt(10)*x/abs(6*x + 2*
sqrt(10) - 4) + 2/17*sqrt(17)*x/abs(6*x + 2*sqrt(10) - 4) - 6/17*sqrt(17)*s
qrt(10)/abs(6*x + 2*sqrt(10) - 4) + 24/17*sqrt(17)/abs(6*x + 2*sqrt(10) - 4
))/(sqrt(10)*sqrt(sqrt(10) + 1) + sqrt(sqrt(10) + 1)) - 128*x/(sqrt(10)*sqr
t(-2*x^2 + 3*x + 1) + sqrt(-2*x^2 + 3*x + 1)) - 128*x/(sqrt(10)*sqrt(-2*x^2
 + 3*x + 1) - sqrt(-2*x^2 + 3*x + 1)) - 1224*arcsin(8/17*sqrt(17)*sqrt(10)*
x/abs(6*x + 2*sqrt(10) - 4) + 2/17*sqrt(17)*x/abs(6*x + 2*sqrt(10) - 4) - 6
/17*sqrt(17)*sqrt(10)/abs(6*x + 2*sqrt(10) - 4) + 24/17*sqrt(17)/abs(6*x +
2*sqrt(10) - 4))/(sqrt(10)*sqrt(sqrt(10) + 1) + sqrt(sqrt(10) + 1)) + 153*s
qrt(10)*log(-2/9*sqrt(10) + 2/3*sqrt(-2*x^2 + 3*x + 1)*sqrt(sqrt(10) - 1)/a
bs(6*x - 2*sqrt(10) - 4) + 2/9*sqrt(10)/abs(6*x - 2*sqrt(10) - 4) - 2/9/abs
(6*x - 2*sqrt(10) - 4) + 1/18)/(sqrt(10) - 1)^(3/2) - 42*sqrt(10)/(sqrt(10)
*sqrt(-2*x^2 + 3*x + 1) + sqrt(-2*x^2 + 3*x + 1)) + 42*sqrt(10)/(sqrt(10)*s
qrt(-2*x^2 + 3*x + 1) - sqrt(-2*x^2 + 3*x + 1)) + 1224*log(-2/9*sqrt(10) +
2/3*sqrt(-2*x^2 + 3*x + 1)*sqrt(sqrt(10) - 1)/abs(6*x - 2*sqrt(10) - 4) + 2
/9*sqrt(10)/abs(6*x - 2*sqrt(10) - 4) - 2/9/abs(6*x - 2*sqrt(10) - 4) + 1/1
8)/(sqrt(10) - 1)^(3/2) - 312/(sqrt(10)*sqrt(-2*x^2 + 3*x + 1) + sqrt(-2*x^
2 + 3*x + 1)) - 312/(sqrt(10)*sqrt(-2*x^2 + 3*x + 1) - sqrt(-2*x^2 + 3*x +
1)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 16200 vs. $2(118) = 236$.

Time = 199.98 (sec) , antiderivative size = 16200, normalized size of antiderivative = 97.59

$$\int \frac{2+x}{(2+4x-3x^2)(1+3x-2x^2)^{3/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((2+x)/(-3*x^2+4*x+2)/(-2*x^2+3*x+1)^(3/2),x, algorithm="giac")
```

```
[Out] 9/5*sqrt(5*sqrt(10) - 15)*(arctan(80715006817815942 74179/61050658134324300 7
7384327396413440*(sqrt(34) + 2*sqrt(10) + sqrt(2) + sqrt(5*sqrt(10) - 15))^
15 + 1048472895661306156 2911/195362106029837762476 29847668523008*(sqrt(34)
+ 2*sqrt(10) + sqrt(2) + sqrt(5*sqrt(10) - 15))^14 - 392953500112821193 3447
79/61050658134324300773843273964 1344*(sqrt(34) + 2*sqrt(10) + sqrt(2) + sqr
t(5*sqrt(10) - 15))^13 - 2327787389694333388 70776481/97681053014918881 23814
9238342615040*(sqrt(34) + 2*sqrt(10) + sqrt(2) + sqrt(5*sqrt(10) - 15))^12
+ 387456520966533311136353209/3052532906716215038692163698206720*(sqrt(34)
+ 2*sqrt(10) + sqrt(2) + sqrt(5*sqrt(10) - 15))^11 + 5089330466552602451278
1737621/9768105301491888123814923834261 5040*(sqrt(34) + 2*sqrt(10) + sqrt(2
) + sqrt(5*sqrt(10) - 15))^10 - 5390728443663071579857 8564023/6105065813432
4300773843273964 13440*(sqrt(34) + 2*sqrt(10) + sqrt(2) + sqrt(5*sqrt(10) -
15))^9 - 39757056444471217190059991 76019/976810530149188812381492383426 1504
0*(sqrt(34) + 2*sqrt(10) + sqrt(2) + sqrt(5*sqrt(10) - 15))^8 + 40923441491
788207049248439479/678340645936936675264925266268160*(sqrt(34) + 2*sqrt(10)
 + sqrt(2) + sqrt(5*sqrt(10) - 15))^7 - 1647091282014574981574395493 36639/9
768105301491888123814923834261 5040*(sqrt(34) + 2*sqrt(10) + sqrt(2) + sqrt(
5*sqrt(10) - 15))^6 + 386839319957929368106171089 53/1177674732529403950112 7
17476160*(sqrt(34) + 2*sqrt(10) + sqrt(2) + sqrt(5*sqrt(10) - 15))^5 - 4655
039540820226689943605414024731/97681053014918881238149238342615040*(sqrt(34
) + 2*sqrt(10) + sqrt(2) + sqrt(5*sqrt(10) - 15))^4 + 662689461333257856102
491005356043/763133226679053759673040924551680*(sqrt(34) + 2*sqrt(10) + sqr
t(2) + sqrt(5*sqrt(10) - 15))^3 - 131055812500493196480903641310169049/9768
1053014918881238149238342615040*(sqrt(34) + 2*sqrt(10) + sqrt(2) + sqrt(5*s
qrt(10) - 15))^2 - 1/179582859004350866276289753568346 1120*(366927382479901
05192656*(sqrt(34) + 2*sqrt(10) + sqrt(2) + sqrt(5*sqrt(10) - 15))^15 - 198
6244356566939292154419*(sqrt(34) + 2*sqrt(10) + sqrt(2) + sqrt(5*sqrt(10) -
 15))^14 - 19111253078186401600075520*(sqrt(34) + 2*sqrt(10) + sqrt(2) + sq
rt(5*sqrt(10) - 15))^13 + 905963034535140752179553873*(sqrt(34) + 2*sqrt(10
) + sqrt(2) + sqrt(5*sqrt(10) - 15))^12 + 7398048389824609184699915168*(sqr
t(34) + 2*sqrt(10) + sqrt(2) + sqrt(5*sqrt(10) - 15))^11 - 175200720418394 2
2537285028205*(sqrt(34) + 2*sqrt(10) + sqrt(2) + sqrt(5*sqrt(10) - 15))^10
 - 109425052155794305371612 5835888*(sqrt(34) + 2*sqrt(10) + sqrt(2) + sqrt(
5*sqrt(10) - 15))^9 + 1145314928529711942244002 0000915*(sqrt(34) + 2*sqrt(1
0) + sqrt(2) + sqrt(5*sqrt(10) - 15))^8 + 673496478981994550096459660 45584*
```

```
(sqrt(34) + 2*sqrt(10) + sqrt(2) + sqrt(5*sqrt(10) - 15))^7 - 4171721815901
779400419757304279 3*(sqrt(34) + 2*sqrt(10) + sqrt(2) + sqrt(5*sqrt(10) - 15
))^6 + 369049422191788814007186699515004 8*(sqrt(34) + 2*sqrt(10) + sqrt(2)
+ sqrt(5*sqrt(10) - 15))^5 - 5386200460101891757817039941271947 7*(sqrt(34)
+ 2*sqrt(10) + sqrt(2) + sqrt(5*sqrt(10) - 15))^4 + 72170801951172731726787
094549559104*(sqrt(34) + 2*sqrt(10) + sqrt(2) + sqrt(5*sqrt(10) - 15))^3 -
14575762236771983907480606559386560 15*(sqrt(34) + 2*sqrt(10) + sqrt(2) + sq
rt(5*sqrt(10) - 15))^2 + 14173378430857593111034350714968347 68*sqrt(34) + 2
83467568617151862220687014299366953 6*sqrt(10) + 14173378430857593111034350 7
1496834768*sqrt(2) + 14173378430857593111034350714968347 68*sqrt(5*sqrt(10)
- 15) + 42764408426567906812279579993126715 61)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x
 + 1) - sqrt(17))/(4*x - 3) - 15049877799596395830870760142185 13/4070043875
62162005158955159760896*sqrt(34) - 15049877799596395830870760142185 13/20350
21937810810025794775798804 48*sqrt(10) - 15049877799596395830870760142185 13/
40700438756216200515895515976089 6*sqrt(2) - 1504987779959639583087076014218
513/40700438756216200515895515976089 6*sqrt(5*sqrt(10) - 15) + 2247459686114
6777604409740107185439/10853450334990986804238804260290560) - arctan(1/256*
(11430313685741577637656157115277 0*(sqrt(34) + 2*sqrt(10) + sqrt(2) + sqrt(
5*sqrt(10) - 15))^15 + 58476302956114681110288270679222*(sqrt(34) + 2*sqrt(
10) + sqrt(2) + sqrt(5*sqrt(10) - 15))^14 - 558208677232735405518244949980 8
1975*(sqrt(34) + 2*sqrt(10) + sqrt(2) + sqrt(5*sqrt(10) - 15))^13 - 2111115
0758868014315824807080403349 4*(sqrt(34) + 2*sqrt(10) + sqrt(2) + sqrt(5*sqr
t(10) - 15))^12 + 11052754313849367315502078961981215060*(sqrt(34) + 2*sqrt
(10) + sqrt(2) + sqrt(5*sqrt(10) - 15))^11 + 462802164557291881462248748267
96136370*(sqrt(34) + 2*sqrt(10) + sqrt(2) + sqrt(5*sqrt(10) - 15))^10 - 781
7397584655762979988225613131705028 35*(sqrt(34) + 2*sqrt(10) + sqrt(2) + sqr
t(5*sqrt(10) - 15))^9 - 36676796684397597542993680848908160016 90*(sqrt(34)
+ 2*sqrt(10) + sqrt(2) + sqrt(5*sqrt(10) - 15))^8 + 665905014701232157719 61
9325002593623773 0*(sqrt(34) + 2*sqrt(10) + sqrt(2) + sqrt(5*sqrt(10) - 15))
^7 - 136144132837200942691703032514785604247086*(sqrt(34) + 2*sqrt(10) + sq
rt(2) + sqrt(5*sqrt(10) - 15))^6 + 282024386593886504952426174062383324634 7
335*(sqrt(34) + 2*sqrt(10) + sqrt(2) + sqrt(5*sqrt(10) - 15))^5 - 364300843
19002830177097651359209193900801 94*(sqrt(34) + 2*sqrt(10) + sqrt(2) + sqrt(
5*sqrt(10) - 15))^4 + 678314347075029286783503843822748963692772 80*(sqrt(34
) + 2*sqrt(10) + sqrt(2) + sqrt(5*sqrt(10) - 15))^3 - 10896078448218051341 4
59063473427275067086417 0*(sqrt(34) + 2*sqrt(10) + sqrt(2) + sqrt(5*sqrt(10)
 - 15))^2 - 48490858500176162409069804119229055937365396 5*sqrt(34) - 969817
17000352324818139608238458111874730793 0*sqrt(10) - 484908585001761624090698
04119229055937365396 5*sqrt(2) - 484908585001761624090698041192290559373653 9
65*sqrt(5*sqrt(10) - 15) + 260470101189894166600286144617437493108298562)/(
5698873346795173420026762309*(sqrt(34) + 2*sqrt(10) + sqrt(2) + sqrt(5*sqrt
(10) - 15))^15 - 6553358686688014207240921636 66*(sqrt(34) + 2*sqrt(10) + sq
rt(2) + sqrt(5*sqrt(10) - 15))^14 - 3627131409667939848420802969739*(sqrt(3
4) + 2*sqrt(10) + sqrt(2) + sqrt(5*sqrt(10) - 15))^13 + 306072128210808386 9
1060259267681 2*(sqrt(34) + 2*sqrt(10) + sqrt(2) + sqrt(5*sqrt(10) - 15))^12
```

+ 1989759942645029950433729166442061*(sqrt(34) + 2*sqrt(10) + sqrt(2) + sq
rt(5*sqrt(10) - 15))^11 - 58350710786461173166030708406696034*(sqrt(34) + 2
*sqrt(10) + sqrt(2) + sqrt(5*sqrt(10) - 15))^10 - 3366473918242251682244333
04935481799*(sqrt(34) + 2*sqrt(10) + sqrt(2) + sqrt(5*sqrt(10) - 15))^9 + 3
788240291717116022622743286571658128*(sqrt(34) + 2*sqrt(10) + sqrt(2) + sqr
t(5*sqrt(10) - 15))^8 + 21902424357130024617206358111721823579*(sqrt(34) +
2*sqrt(10) + sqrt(2) + sqrt(5*sqrt(10) - 15))^7 - 1608521439494971283989696
5015516588478*(sqrt(34) + 2*sqrt(10) + sqrt(2) + sqrt(5*sqrt(10) - 15))^6 +
1034034206299622977195958121191377271011*(sqrt(34) + 2*sqrt(10) + sqrt(2)
+ sqrt(5*sqrt(10) - 15))^5 - 15502261507370095606858566273324029461700*(sqr
t(34) + 2*sqrt(10) + sqrt(2) + sqrt(5*sqrt(10) - 15))^4 + 20864835524597361
622030489851611876126211*(sqrt(34) + 2*sqrt(10) + sqrt(2) + sqrt(5*sqrt(10)
- 15))^3 - 433397870190449097071646281383200946482446*(sqrt(34) + 2*sqrt(1
0) + sqrt(2) + sqrt(5*sqrt(10) - 15))^2 + 421575977955046072750707213401588
325665295*sqrt(34) + 843151955910092145501414426803176651330590*sqrt(10) +
421575977955046072750707213401588325665295*sqrt(2) + 4215759779550460727507
07213401588325665295*sqrt(5*sqrt(10) - 15) + 126654657725763940817373879583
4376742692696))) + 9/10*sqrt(5*sqrt(10) + 15)*log(abs(168713070829668086264
046067355656578177114075764587270602965403560739421916775237151970742653370
4329336834375736081735560550606175512433786880000000000000000000*sqrt(34)*sq
rt(10)*sqrt(2)*(sqrt(10) + 3)^6 - 17180464995907963778504641885137870980502
555236707852328038682270887767457078067864303585523267254920804044433429901
232660989379960306306908160000000000000000000*sqrt(34)*sqrt(10)*sqrt(2)*(2*s
qrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*(sqrt(10) + 3)^6/(4*x - 3) + 1297
792852535908355877277441197358293670108275112209773868964642774918630129040
2857843903281028494841052572121046782581235004662888557182976000000000000000
0000*sqrt(34)*sqrt(10)*sqrt(5*sqrt(10) + 15)*(sqrt(10) + 3)^6 + 12977928525
359083558772774411973582936701082751122097738689646427749186301290402857843
903281028494841052572121046782581235004662888557182976000000000000000000*sq
rt(34)*sqrt(2)*sqrt(5*sqrt(10) + 15)*(sqrt(10) + 3)^6 + 1297792852535908355
877277441197358293670108275112209773868964642774918630129040285784390328102
849484105257212104678258123500466288855718297600000000000000000000*sqrt(10)*s
qrt(2)*sqrt(5*sqrt(10) + 15)*(sqrt(10) + 3)^6 - 132157423045445875219266476
039522084465404271051598864061836017468367441977523598956181427102055807083
108034103306932558930687538156202360832000000000000000000000*sqrt(34)*sqrt(10)
*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*sqrt(5*sqrt(10) + 15)*(sqrt(
10) + 3)^6/(4*x - 3) - 1321574230454458752192664760395220844654042710515988
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3*x + 1) - sqrt(17))*sqrt(5*sqrt(10) + 15)*(sqrt(10) + 3)^6/(4*x - 3) - 132
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5*sqrt(10) + 15)*(sqrt(10) + 3)^6/(4*x - 3) + 46349744733425298424188480042
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) + 3)^7/(4*x - 3) - 471990796590878125783094557284007444519300968041424514
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sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*sqrt(5*sqrt(10) + 15)*(sqrt(10) + 3)^7/(
4*x - 3) + 308998298222835322827923200285085308016692446455288041397372533
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sqrt(10) + 15)*(sqrt(10) + 3)^5/(4*x - 3) + 59973871877298658933229630717 87
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2*x^2 + 3*x + 1) - sqrt(17))*(sqrt(10) + 3)^6/(4*x - 3) - 26203319159108992
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)*sqrt(2)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*(sqrt(10) + 3)^6/(4
*x - 3) - 262033191591089924505829936211947895119925834219800625377709133 90
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rt(17))*(sqrt(10) + 3)^6/(4*x - 3) + 461337475979220453332535620906717019 01
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 + 461337475979220453332535620906717019016192961251913301153781645394877685
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t(10) + 3)^6/(4*x - 3) - 2015639935316076342352537970861137654768660263229 2
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) - sqrt(17))*sqrt(5*sqrt(10) + 15)*(sqrt(10) + 3)^6/(4*x - 3) - 2015639935
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qrt(2)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*sqrt(5*sqrt(10) + 15)*
(sqrt(10) + 3)^6/(4*x - 3) - 719871405470027265125906418164692019560235808 2
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rt(17))*(sqrt(10) + 3)^7/(4*x - 3) + 1647633842782930190473341503238275067 9
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*sqrt(34)*sqrt(10)*sqrt(2)*(sqrt(10) + 3)^5 - 2437328305898101792218139820 1
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)*sqrt(2)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*(sqrt(10) + 3)^5/(4
*x - 3) + 247329446101026881882619046194040753514950054058924712974061329 51
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^5 + 35530581143201445709160852930166096354830306254826056615995918779860 62
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3*x + 1) - sqrt(17))*sqrt(5*sqrt(10) + 15)*(sqrt(10) + 3)^5/(4*x - 3) + 159
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-2*x^2 + 3*x + 1) - sqrt(17))*(sqrt(10) + 3)^6/(4*x - 3) - 2984886218409751
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 + 1) - sqrt(17))*(sqrt(10) + 3)^5/(4*x - 3) - 446233288828877010309634935
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+ 2720639867747134839914954915209530612051807883837843015863896151035877153
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(sqrt(10) + 3)^5/(4*x - 3) - 9506956647175370523445216632712707701327336579
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23520000000000000000*sqrt(34)*sqrt(10)*sqrt(2)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x
+ 1) - sqrt(17))*sqrt(5*sqrt(10) + 15)*(sqrt(10) + 3)^3/(4*x - 3) + 9558820
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rt(10) + 3)^4 + 653388684605717744445468547310118152019402915438206432215558
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) + 3)^4/(4*x - 3) - 271003951306423192247696388304459264148393862982137045
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 3*x + 1) - sqrt(17))*(sqrt(10) + 3)^4/(4*x - 3) - 454252255218744076831230
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rt(2)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*(sqrt(10) + 3)^4/(4*x -
 3) + 553219473429251995768560192785867240625680681964183154984539609431764
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8800000000000000*sqrt(34)*sqrt(5*sqrt(10) + 15)*(sqrt(10) + 3)^4 + 1051830
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)*sqrt(5*sqrt(10) + 15)*(sqrt(10) + 3)^4 - 883795128764742184675702282726O2
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127752198762817295600309665103043624960000000000000000*sqrt(34)*(2*sqrt(2)*
sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*sqrt(5*sqrt(10) + 15)*(sqrt(10) + 3)^4/(
4*x - 3) - 182528982285449683939915760832149878700828073690697740170951389O
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7))*sqrt(5*sqrt(10) + 15)*(sqrt(10) + 3)^4/(4*x - 3) - 686158837968234077797
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*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*sqrt(5*sqrt(10) + 15)*(sqrt(
10) + 3)^4/(4*x - 3) - 4244802876324778189252738006911066118853807291605234
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463171827106840576000000000000000000*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqr
t(17))*(sqrt(10) + 3)^5/(4*x - 3) + 134755983961043729961323904462561042113
686401240276424809230485262050423644585563788327093220090871505392085408395
666025679874158642550761796403200000000000000000*(sqrt(10) + 3)^5 + 1194026

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*sqrt(10)*sqrt(2)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*(sqrt(10) +
 3)^3/(4*x - 3) + 752369073691486708873274319205338255725200606317640113137
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+ 3)^3 - 769727644622633840166232001366056897531984558900105222860831471223
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- sqrt(17))*sqrt(5*sqrt(10) + 15)*(sqrt(10) + 3)^3/(4*x - 3) - 207669113480
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2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*sqrt(5*sqrt(10) + 15)*(sqrt(10
) + 3)^3/(4*x - 3) + 250259287437818576779281072530163966736808296181115570
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) + 3)^4/(4*x - 3) - 455727683791196802528398799551215245668706369216184801
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 1) - sqrt(17))*(sqrt(10) + 3)^4/(4*x - 3) - 110665333767505731586726962747
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t(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*(sqrt(10) + 3)^4/(4*x - 3) - 179383
838411804040787429931458035330427025367810527790917234223040629086391181963

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)*(sqrt(10) + 3)^4/(4*x - 3) + 180450482161511112894353377003565046512142300
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) + 3)^4 - 3561345066066752352506251560859104470525836285247863304123713582
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qrt(10) + 3)^2 + 340657368472336988643198146385253191740787265397758622694500
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2*x^2 + 3*x + 1) - sqrt(17))*sqrt(5*sqrt(10) + 15)*(sqrt(10) + 3)^2/(4*x -
3) - 2950680951105857811130637846974113646129746495411834318969670659535236
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34)*sqrt(2)*(sqrt(10) + 3)^3 - 8899738025586877447684280484971207674010462400
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)^3 + 3432221521160513039940969987334740034319113162199983375254805502752280
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632448000000000000000*sqrt(34)*sqrt(10)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) -
sqrt(17))*(sqrt(10) + 3)^3/(4*x - 3) + 629538235406984823852571734719943360
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(sqrt(10) + 3)^3/(4*x - 3) - 311900075495725223622429393233813312323120375800
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qrt(10) + 3)^3 - 636773468865181163646240132047855497302515605027453120617200
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^3 + 732297411783602706359979663944980076996476143256800994959223209751295900
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) + 15)*(sqrt(10) + 3)^3/(4*x - 3) + 1673542111088603226837772913507001194900
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2093063960437662148979966830698299392000000000000000*sqrt(10)*(2*sqrt(2)*sqr
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t(-2*x^2 + 3*x + 1) - sqrt(17))*sqrt(5*sqrt(10) + 15)*(sqrt(10) + 3)^3/(4*x
 - 3) + 20520836068310701466896775223442686210599959618587141131278342822111
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02004480000000000000000*sqrt(2)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17)
)*sqrt(5*sqrt(10) + 15)*(sqrt(10) + 3)^3/(4*x - 3) + 890329912638297509863
72113812861220413356814298180344113552343761412912118484301332322196166178100
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2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*(sqrt(10) + 3)^4/(4*x - 3) + 17896295
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 + 3)^2 + 1209202157768421429126548506639757134284529087115371496049464292550
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+ 3*x + 1) - sqrt(17))*(sqrt(10) + 3)^2/(4*x - 3) - 287216620441230219018833
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sqrt(10)*sqrt(5*sqrt(10) + 15)*(sqrt(10) + 3)^2 - 62661283232649259688479415
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rt(2)*sqrt(5*sqrt(10) + 15)*(sqrt(10) + 3)^2 - 116723914879149147123426051143
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(2)*sqrt(5*sqrt(10) + 15)*(sqrt(10) + 3)^2 + 19255373016187636302220324604909
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0)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*sqrt(5*sqrt(10) + 15)*(sqr
t(10) + 3)^2/(4*x - 3) + 4446968330855820374311294314736462947726307910651945
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*x^2 + 3*x + 1) - sqrt(17))*sqrt(5*sqrt(10) + 15)*(sqrt(10) + 3)^2/(4*x - 3
) + 822570121229768285302146953273000683980007930498078612423454307313363359
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t(17))*sqrt(5*sqrt(10) + 15)*(sqrt(10) + 3)^2/(4*x - 3) + 2835772688821587012
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t(34)*(sqrt(10) + 3)^3 + 487102597741042801185385405433746096773747004696992
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000000*sqrt(2)*(sqrt(10) + 3)^3 - 425146095888891784469774219942917314367 70
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7928700017418423620244505664708335309026012685164376281001570764022014267179030155053473694434656924840493056000000000000000*sqrt(34)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*(sqrt(10) + 3)^3/(4*x - 3) - 76085928746757361467373913290842728657077503291818946134666241365275713923337210747584360514666849314369764027915445935313058284165610329403484117401600000000000000*sqrt(10)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*(sqrt(10) + 3)^3/(4*x - 3) - 2553111750627157364743205454210777018367868726743341013507722349520167279352324123494516619615995099051182949378889906373860847947230964115063378542592000000000000000*sqrt(2)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*(sqrt(10) + 3)^3/(4*x - 3) - 1438144487389573292786402627242403513961108827359169857124967428095259630661426696377676585131077318635993331566043744288995564151046914015740567748608000000000000000*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*sqrt(5*sqrt(10) + 15)*(sqrt(10) + 3)^3/(4*x - 3) + 14092946633540434692138432918771906393936314074164504375008477574656105727697880445527910219057133594637927080100600897841630227510464486144119039590400000000000000*sqrt(5*sqrt(10) + 15)*(sqrt(10) + 3)^3 - 88479788813675559140101689119876262370124674552441258721741278196880552775881107329414954676423607322776673860836587520246478485776517397944454201999360000000000000*sqrt(34)*sqrt(10)*sqrt(2)*sqrt(5*sqrt(10) + 15)*(sqrt(10) + 3) + 66613002760466694466283089112720351229008239164160516479902935949516319867791393771388674392587250859056061020242530181745529890039394135852873920020480000000000000*sqrt(34)*sqrt(10)*sqrt(2)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*sqrt(5*sqrt(10) + 15)*(sqrt(10) + 3)/(4*x - 3) - 22905774376811816957613998644203211801297496615249020440447828406889199897545645632555662837193691319944406078637801581748654537590410350630385850777600000000000000*sqrt(34)*sqrt(10)*(sqrt(10) + 3)^2 - 599409735998310779087828047188679645055370721554857127836908194431109011303457919344963832470065194035697837648947820214325330495385986767419520267059200000000000000*sqrt(34)*sqrt(2)*(sqrt(10) + 3)^2 - 110347927536172059208867528369595055489519860090152550491457703015420380881903632583724335879672393732855251941232797701119207354575807806829665399603200000000000000*sqrt(10)*sqrt(2)*(sqrt(10) + 3)^2 + 191948381881325797475244364164081022892686821750311763165698626674452177531310706577973049820391574959267047338178842149891238978699994380589325719961600000000000000*sqrt(34)*sqrt(10)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*(sqrt(10) + 3)^2/(4*x - 3) + 502107314611061956713510844435469117315209674690718872181269307996798538390709413737085061337705895546796804857148802878690029237136646183436852763033600000000000000*sqrt(34)*sqrt(2)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*(sqrt(10) + 3)^2/(4*x - 3) + 922356442394231566722612232170040479914866447955063805524820523296550890283637001346682644721257360412841114066515811526928361051209406577908171512217600000000000000*sqrt(10)*sqrt(2)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*(sqrt(10) + 3)^2/(4*x - 3) - 160759584663217135997219571360783242177047805187320653134803298723676860056991072522297586584820198591715919373359481319032490134882684410353130091315200000000000000*sqrt(34)*sqrt(5*sqrt(10) + 15)*(sqrt(10) + 3)^2 - 298702909695989839511073162502584268329787431402251685173982978004871239838764036749684268

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021411062782072455851551127196958179572834444299326153398681600000000000000*
sqrt(10)*sqrt(5*sqrt(10) + 15)*(sqrt(10) + 3)^2 - 6537327352401546707151186
799178447162666899035225370059378735580537750420267301178829972876214914126
88836584295348658732277986319611831173776373343846400000000000000*sqrt(2)*sq
rt(5*sqrt(10) + 15)*(sqrt(10) + 3)^2 + 1509311709472181669786901591236484 62
253661325444701424703956637123458669782967223137974363792009366074866895652
32920365200242471589486607139482424573952000000000000000*sqrt(34)*(2*sqrt(2)*
sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*sqrt(5*sqrt(10) + 15)*(sqrt(10) + 3)^2/(
4*x - 3) + 279627733625305396075880029108670360048173731451766606528518516 0
299947076441400596800642348264484091402841211774416273011617845512358939351
2026611908608000000000000000*sqrt(10)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqr
t(17))*sqrt(5*sqrt(10) + 15)*(sqrt(10) + 3)^2/(4*x - 3) + 62548596171745422
724436942203212169937072530535673554586251482084465446530979617989955253228
444968469011230444286975625227949793202607234210218536599552000000000000000*s
qrt(2)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*sqrt(5*sqrt(10) + 15)*
(sqrt(10) + 3)^2/(4*x - 3) + 6227349879797082767808522861738868912979567919
853051251825727475247045736553384455200151811436371484354195785220367944462
027908096497617206666921508864000000000000000*(2*sqrt(2)*sqrt(-2*x^2 + 3*x +
 1) - sqrt(17))*(sqrt(10) + 3)^3/(4*x - 3) - 5301909084591182909277073927 25
612769059871143641296329039549106240667122298676131815151676190839883569111
39680680436923011411056495132884669949594828800000000000000*(sqrt(10) + 3)^
3 + 53228287966042191782554998446929643285797108312564249228816029611464246
390134733082985139750584428262612287042604375899670423237396638209069857814
60992000000000000*sqrt(34)*sqrt(10)*sqrt(2)*(sqrt(10) + 3) - 5904036674984 2
559593870733612911981606475429110774041327131858262534452802921968022343565
877580267674135864819841823388963324531268283813629806614937600000000000000
*sqrt(34)*sqrt(10)*sqrt(2)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*(s
qrt(10) + 3)/(4*x - 3) - 6809006986639367400693407144485224916714071490365 8
900021351430898533090460262088275822010204129684657723867065880105963817512
01571823590637099546574848000000000000000*sqrt(34)*sqrt(10)*sqrt(5*sqrt(10) +
15)*(sqrt(10) + 3) - 2566512865353094836303043297323761122086040020155127 00
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(sqrt(10) + 3) - 4735144790584238038007082567534757581072410257651110574825
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20781271045945425920000000000000*sqrt(10)*sqrt(2)*sqrt(5*sqrt(10) + 15)*(sqr
t(10) + 3) + 4607596098240473624424888337448535616621104702088409114138585 0
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831537542561792000000000000000*sqrt(34)*sqrt(10)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x
+ 1) - sqrt(17))*sqrt(5*sqrt(10) + 15)*(sqrt(10) + 3)/(4*x - 3) + 180748506
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000000*sqrt(34)*sqrt(2)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*sqrt(
5*sqrt(10) + 15)*(sqrt(10) + 3)/(4*x - 3) + 3347762954124306332292195305012
648152899009217667803444151038221949204292917549336802218503734167651398455
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954755416061438086850960617085232758366540922880000000000000*sqrt(10)*sqrt(2
)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*sqrt(5*sqrt(10) + 15)*(sqrt
(10) + 3)/(4*x - 3) - 58825758257249702202169741056307062963122517761008060
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90262099187560041414656000000000000000*sqrt(34)*(sqrt(10) + 3)^2 - 1086864486
45299387838329896231106038174865552744495494853364865258046067208533876174 7
14159362576663101297523346014269578174707759549544102715883677286400000000 0
00000*sqrt(10)*(sqrt(10) + 3)^2 - 2939856900564346209021749181605802102989 2
69415598018061790586142589306371285586323841822960170821615233597976337013 9
675718871931816431966295194991591424000000000000000*sqrt(2)*(sqrt(10) + 3)^2
+ 4319559404313398457041135890823880732520052243617706954669479462695333301
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2624000000000000000*sqrt(34)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*(s
qrt(10) + 3)^2/(4*x - 3) + 797551958208975491917940936945535580345125660451
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773999854088282119597562265600000000000000000*sqrt(10)*(2*sqrt(2)*sqrt(-2*x^2
 + 3*x + 1) - sqrt(17))*(sqrt(10) + 3)^2/(4*x - 3) + 221197898897692441079 20
75602849719097919624648662753282720240343419331505977436065257590811419942 7
4994406346084375875358882980313409133444063724868468736000000000000000*sqrt(2
)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*(sqrt(10) + 3)^2/(4*x - 3)
+ 353267514008860810290266205292387391433195508814637473523763603189438761 8
90189843136578111936153860100372229815077888274607714303088430948161609971 7
939200000000000000*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*sqrt(5*sqrt
(10) + 15)*(sqrt(10) + 3)^2/(4*x - 3) - 52411687922773297196796556673232754
41586032097940466060554056264437639549214919300600342592609174362032183451 9
517036149694861245360130963917264262266880000000000000000*sqrt(5*sqrt(10) + 15
)*(sqrt(10) + 3)^2 + 1351487590630554960104653191198068226839770836408744 98
64581936680620420529201737599452343476791914854640745439351931668970910277 0
57980094875112217719603200000000000000*sqrt(34)*sqrt(10)*sqrt(2)*sqrt(5*sqrt(1
0) + 15) - 1057384188678503995629627379181601119855154171427134551207662272
72625519010577414642104139119718336204035056574455046204900970342134287988 9
72121741066240000000000000000*sqrt(34)*sqrt(10)*sqrt(2)*(2*sqrt(2)*sqrt(-2*x^2
 + 3*x + 1) - sqrt(17))*sqrt(5*sqrt(10) + 15)/(4*x - 3) + 19875452654329115
29549478049733820066925973211486563250191309728667406267093044650215818134 5
2319172563816914083637804848854926585196511682903651909632000000000000000*sq
rt(34)*sqrt(10)*(sqrt(10) + 3) + 1493407012155859125271862120220171516467 89
37424393919461443428642130522897608922162927068001710217964965131233202735 4
787472392334909597761144499973652480000000000000000*sqrt(34)*sqrt(2)*(sqrt(10)
+ 3) + 2702313754897341882629800818004135522694130195896472043194718094162 4
15342079434692097915976872031777463630228671590263905392141738778417894349 3
029232640000000000000000*sqrt(10)*sqrt(2)*(sqrt(10) + 3) - 165221082512883568 2
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1601157367736436793597599064534321926684621679126032836198400000000000000*sqr
t(34)*sqrt(10)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*(sqrt(10) + 3)
/(4*x - 3) - 1623884096224210195417282519474555116046069738376167641004947 5
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834183974024873035456198612050254045881223811952831947735203949455156137749
810854294978560000000000000*sqrt(34)*sqrt(2)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x
+ 1) - sqrt(17))*(sqrt(10) + 3)/(4*x - 3) - 2913864945499995074656761909520
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59161662325927884269015662914800719959058022400000000000000*sqrt(10)*sqrt(2
)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*(sqrt(10) + 3)/(4*x - 3) -
247257017745680540857351945388730981261524239056214355832163803655906306508
988911917001723241261327832767186657565136384161019138409023583281894916096
0000000000000*sqrt(34)*sqrt(5*sqrt(10) + 15)*(sqrt(10) + 3) - 455322878756
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00*sqrt(10)*sqrt(5*sqrt(10) + 15)*(sqrt(10) + 3) - 163188246880224749322156
717278554690483784979233823141923129319313755337517255371685275696643833601
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sqrt(5*sqrt(10) + 15)*(sqrt(10) + 3) + 186695768417757231934636575479319296
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sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*sqrt(5*sqrt(10) + 15)*(sqrt(10) + 3)/(4*
x - 3) + 344028090368790719873669907037209322832012641364674886839568280609
263776579231978995128748902381769180947067315171282045684581833029790245390
0651433820160000000000000*sqrt(10)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(
17))*sqrt(5*sqrt(10) + 15)*(sqrt(10) + 3)/(4*x - 3) + 123432109898879931059
724065664894298085479917521420669803013694919170420283707175004108659123058
3858990540451880853967478077315371744723082536129711308800000000000000*sqrt
(2)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*sqrt(5*sqrt(10) + 15)*(sq
rt(10) + 3)/(4*x - 3) + 354471537234651308904236028586778556690267432330148
338391363843034943558928156729340548486282569186750228296478171989150701038
38666602457539690045322035200000000000000*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1)
 - sqrt(17))*(sqrt(10) + 3)^2/(4*x - 3) - 4223973206381091919824700646441175
850125956224258393226748617254195059984847192495398908516336961736560376462
57264205334847311546398421586280733883760640000000000000*(sqrt(10) + 3)^2 +
 20318904860129330256671905589498117050387576382828207610228152524880107894
974687894011727824315919093962882880179871800086931826070773566508325190762
496000000000000*sqrt(34)*sqrt(10)*sqrt(2) - 1511832933122347862957310401023
852715296502957834982844763299946310777857086179998757800791269194356580750
342846969600145794203413691864686552334467072000000000000000*sqrt(34)*sqrt(1
0)*sqrt(2)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))/(4*x - 3) + 231090
818261198811699987150564944793581268740917187398753475976898350148143148686
775572146278734573944573854019099425323139521739067247804300544903741440000
00000000*sqrt(34)*sqrt(10)*sqrt(5*sqrt(10) + 15) + 429896310618006700561149
287145862835783820076218787403410305860000064434710029584586902156346865470
8543676799991880571102110002048796624800035980391219200000000000000*sqrt(34)*
sqrt(2)*sqrt(5*sqrt(10) + 15) + 7936146412160676231400872567228157540608924
774562552043041367521423667194518196652604877564799652700998082245324553128
8393012461567403244290169316048896000000000000000*sqrt(10)*sqrt(2)*sqrt(5*sqrt
```

(10) + 15) - 17748406984671196824385223932085285767487629022122842093601577
570754793653953317972946155732885544846842499208709704019273441446640570253
5541465114738688000000000000*sqrt(34)*sqrt(10)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x
 + 1) - sqrt(17))*sqrt(5*sqrt(10) + 15)/(4*x - 3) - 33766414851234154832199
628593758648141617066919258528941057974756501275657288936134102281127418114
61692172093149253588805272471696610262443925567648563200000000000000*sqrt(34
)*sqrt(2)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*sqrt(5*sqrt(10) + 1
5)/(4*x - 3) - 625855770742752278723327834088167322101527323147944718666299
394271501569825156627056255405918910706597068069609603811524660806490238181
4866381774247690240000000000000*sqrt(10)*sqrt(2)*(2*sqrt(2)*sqrt(-2*x^2 + 3*
x + 1) - sqrt(17))*sqrt(5*sqrt(10) + 15)/(4*x - 3) + 6546079101820069524382
886488745787926728577365252444878328029621560042652082170803004687438546673
28140125568446023587037214164780686926849963648142540800000000000000*sqrt(34
)*(sqrt(10) + 3) + 12090978298733502500274163185249182454210831013149361709
970404130508476590631425012161726183728947196970401608995078183958759488318
40365467198950473728000000000000*sqrt(10)*(sqrt(10) + 3) + 96336955028180
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sqrt(2)(sqrt(10) + 3) - 5560714512758985271139828756927946196648626953801
248159548479728588632824477239650785336694355414706643932517680316873810189
959811222344515676333408256000000000000000*sqrt(34)*(2*sqrt(2)*sqrt(-2*x^2 +
 3*x + 1) - sqrt(17))*(sqrt(10) + 3)/(4*x - 3) - 10299611118786605774078533
714541197480057128116762277325991095301586862957847793097515419409211486678
71290672756128955034103755477757372135421540736958464000000000000000*sqrt(10)*
(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*(sqrt(10) + 3)/(4*x - 3) - 10
643960238966268060830616114848023891243173852966658413266029075640230673990
616721705279380881980660571340676221772019858949178487581506152536738680012
80000000000000*sqrt(2)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*(sqrt(
10) + 3)/(4*x - 3) + 846790255792421303562325362070806079519854710824967254
164539133589079741064318968166560976157771556071228533637156928311975057027
5287468788456212261765120000000000000*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sq
rt(17))*sqrt(5*sqrt(10) + 15)*(sqrt(10) + 3)/(4*x - 3) - 125483269082963282
821710152213176787741937871803429330915757399098929742124753762712944511284
137752904070980180664662287510944887788900939659286151168000000000000000*sq
rt(5*sqrt(10) + 15)*(sqrt(10) + 3) + 31100888317790139125892598487276205795
618837261787831369420574561822191584866978391720779344525788176305228266460
1115468540549987716583453734994758860800000000000*sqrt(34)*sqrt(10) + 641
553686413224086488358943338057911056857802774926629090829104102511391259749
535917842864034571357121514975032695318628745629045494543342461677375324160
00000000000*sqrt(34)*sqrt(2) + 11853302229803625857828793183039208758275571
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2671361515789758039558278464143360000000000000*sqrt(10)*sqrt(2) - 229804194
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000000*sqrt(34)*sqrt(10)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))/(4*x

- 3) - 4768478227058167485591286118403988715750697203207631367428676093820
056537582560187320625443936273868122690429408786014869060147137400754862927
321759744000000000000*sqrt(34)*sqrt(2)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1)
- sqrt(17))/(4*x - 3) - 882583185245914957349231404605789849098340830055345
725382973969056215031410868890732784642382595004481057674167852801296541611
1814404616610113888505036800000000000000*sqrt(10)*sqrt(2)*(2*sqrt(2)*sqrt(-2
*x^2 + 3*x + 1) - sqrt(17))/(4*x - 3) + 72731902096351424426760401625218660
412461712887177392550953082967042633392666482499391797006007105627946417580
486781585855732795274680724062367131893760000000000000*sqrt(34)*sqrt(5*sqrt
(10) + 15) + 13416420365696244615415021432198170154806400395574519401257672
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566267290229145600000000000000*sqrt(10)*sqrt(5*sqrt(10) + 15) + 2494821203330
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7594763168348512599460226024467728808155996992496392118609117184000000000000
00*sqrt(2)*sqrt(5*sqrt(10) + 15) - 5568335259957117895150316183913914557399
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73595889671447463758524572437489647616000000000000*sqrt(34)*(2*sqrt(2)*sqrt
(-2*x^2 + 3*x + 1) - sqrt(17))*sqrt(5*sqrt(10) + 15)/(4*x - 3) - 1027389971
857366578574652888657754389735683213573836170724789346656379434892872563872
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000000*sqrt(10)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*sqrt(5*sqrt(1
0) + 15)/(4*x - 3) - 195965356029728298870294498184283955043511388016233569
147837225270357799000751250830266124679113478068921879110523181740691119599
48476502324445569456537600000000000000*sqrt(2)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x
+ 1) - sqrt(17))*sqrt(5*sqrt(10) + 15)/(4*x - 3) - 305952342948309193589795
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843228649920803718439466221099335253614698593605648384000000000000000*(2*sqrt
(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*(sqrt(10) + 3)/(4*x - 3) + 984651497
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894942993480904193955172925191518111086471918491855453009116921856000000000
00000*sqrt(34) + 5484319954069753848074868480702648504630589936414125964508
461091113282513705088692834902935111619078140039350086450328090998131272615
652618693236909670400000000000000*sqrt(10) + 3753649211064772821434627850901
925999083694917749762624090954158309237502137819146105761921264855707842713
479794477569630264520169527379168095387873443840000000000000*sqrt(2) - 72817
522578401224135859408527342461884967664896604034017353252274058818989737637
115285460731762847067369683639592067420075923301589093831013816835925606400
0000000000*sqrt(34)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))/(4*x - 3)
- 133315738377025384325978890232572073371490417671671476307465622126254722
087809922552709477555525075305992963118389696833974814120928660039941424497
34778880000000000000*sqrt(10)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))/
(4*x - 3) - 279823226880255772656132839248793419709466100817975516280631559
070687848040742745350933147639934606543784177441426458210575057493394248764
641363162562560000000000000*sqrt(2)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sq
rt(17))/(4*x - 3) - 3274862625175476142860698387497986568347072857792709318

```
9405812120011383477050449393679650447666696040130350067126185236569805198385
583139311905549079019520000000000000*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sq
rt(17))*sqrt(5*sqrt(10) + 15)/(4*x - 3) + 426227561753000005778667979155851
7276694080993278528780644809319029896869747378471025950392911106362981311978
844501742828695185678810809105465884687728640000000000000*sqrt(5*sqrt(10) +
 15) - 420711002275481853661928876771186640390501204050662920954822695990904
724981848491987344811291869616220414391568060499469521694557649694231899046
83381555200000000000000*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))/(4*x
- 3) + 167403619805344978685803887454997435446581313136330026285852999061024
375941284869868532384927513283001184707717091119358625063187457948905728311
585406976000000000000000)) - 9/10*sqrt(5*sqrt(10) + 15)*log(abs(-75268615523
266197367436685814547225335551637067879500577930479961191830350114410111887
1061646786340278224911811961858255307118137809599201280000000000000000000*sq
rt(34)*sqrt(10)*sqrt(2)*(sqrt(10) + 3)^6 + 420026894808371012506614343482800
7591534032817915731176810713829229167004868999127135539845129151554626257167
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(2)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*(sqrt(10) + 3)^6/(4*x - 3
) - 578989350178970748980282198573440194888858746675996158291772922778398695
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000*sqrt(34)*sqrt(2)*sqrt(5*sqrt(10) + 15)*(sqrt(10) + 3)^6 - 5789893501789
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10)*sqrt(2)*sqrt(5*sqrt(10) + 15)*(sqrt(10) + 3)^6 + 3230976113910546250050
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(10)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*sqrt(5*sqrt(10) + 15)*(s
qrt(10) + 3)^6/(4*x - 3) + 3230976113910546250050879565252313531949256013781
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35041176818155520000000000000000000*sqrt(34)*sqrt(2)*(2*sqrt(2)*sqrt(-2*x^2
 + 3*x + 1) - sqrt(17))*sqrt(5*sqrt(10) + 15)*(sqrt(10) + 3)^6/(4*x - 3) +
3230976113910546250050879565252313531949256013781331674469779868637820772976
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00000000*sqrt(10)*sqrt(2)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*sqr
t(5*sqrt(10) + 15)*(sqrt(10) + 3)^6/(4*x - 3) - 2067819107782038389215293566
333714981745924095271414851042046152779995339288857420656239180348314121643
4750324504446655365580168621142835200000000000000000000*sqrt(34)*(sqrt(10)
+ 3)^7 - 20678191077820383892152935663337149817459240952714148510420461527779
995339288857420656239180348314121643475032450444665536558016862114283520000
000000000000000*sqrt(10)*(sqrt(10) + 3)^7 - 20678191077820383892152935666333
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032450444665536558016862114283520000000000000000000*sqrt(2)*(sqrt(10) + 3)
^7 + 115392004068233794644674270187582626141044857635047559802492138165636
```

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561777197562399873583826689987534686034913490017566269768004202921984000000
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10) + 3)^7/(4*x - 3) + 1153920040682337946446742701875826261410448576350475
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- sqrt(17))*(sqrt(10) + 3)^7/(4*x - 3) + 1153920040682337946446742701875826
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2*x^2 + 3*x + 1) - sqrt(17))*(sqrt(10) + 3)^7/(4*x - 3) + 76928002712155863
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445999168979068994232667837751317866946861465600000000000000000000*(2*sqrt(2)
*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*sqrt(5*sqrt(10) + 15)*(sqrt(10) + 3)^7/
(4*x - 3) - 137854607185469225947686237755580998783061606351427656736136410
185333022619257161377082612023220941442898335496696311035770534457474285568
000000000000000000*sqrt(5*sqrt(10) + 15)*(sqrt(10) + 3)^7 - 631194446559137
376126598837272329024451732841184960455396103864098402326138927220023504357
338295075269910334874484867589887648115895963536588800000000000000000*sqrt(3
4)*sqrt(10)*sqrt(2)*sqrt(5*sqrt(10) + 15)*(sqrt(10) + 3)^5 + 15020184534041
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560851101499300282706587786552674032386614533700976640000000000000000000*sqrt
(34)*sqrt(10)*sqrt(2)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*sqrt(5*
sqrt(10) + 15)*(sqrt(10) + 3)^5/(4*x - 3) - 2629976860663072400527495155301
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t(10) + 3)^6 - 262997686066307240052749515530137093521555350493733523081709
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0)*sqrt(2)*(sqrt(10) + 3)^6 + 625841022251739504823990363645249793832501074
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418016108938904207360000000000000000000*sqrt(34)*sqrt(10)*(2*sqrt(2)*sqrt(-2
*x^2 + 3*x + 1) - sqrt(17))*(sqrt(10) + 3)^6/(4*x - 3) + 625841022251739504
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sqrt(2)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*(sqrt(10) + 3)^6/(4*x
 - 3) + 6258410222517395048239903636452497938325010749436539941181110648336
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0000000000000000*sqrt(10)*sqrt(2)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(
17))*(sqrt(10) + 3)^6/(4*x - 3) - 20230591235869787696365347348472084117042
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345829732311406921908224000000000000000000*sqrt(34)*sqrt(5*sqrt(10) + 15)*(s
qrt(10) + 3)^6 - 2023059123586978769636534734847208411704271926874873254474
691872110263865829894935972770376084279087403558765623348934582973231140692
1908224000000000000000000*sqrt(10)*sqrt(5*sqrt(10) + 15)*(sqrt(10) + 3)^6 -
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20230591235869787696365347348472084117042719268748732544746918721102638658298949359727703760842790874035587656233489345829732311406921908224000000000000000000*sqrt(2)*sqrt(5*sqrt(10) + 15)*(sqrt(10) + 3)^6 + 481416170962876542172300279727115226025000826879733841629316203718171629960811062399159761565740432669880855980377774124167704699183772467200000000000000000000*sqrt(34)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*sqrt(5*sqrt(10) + 15)*(sqrt(10) + 3)^6/(4*x - 3) + 481416170962876542172300279727115226025000826879733841629316203718171629960811062399159761565740432669880855980377774124167704699183772467200000000000000000000*sqrt(10)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*sqrt(5*sqrt(10) + 15)*(sqrt(10) + 3)^6/(4*x - 3) + 481416170962876542172300279727115226025000826879733841629316203718171629960811062399159761565740432669880855980377774124167704699183772467200000000000000000000*sqrt(2)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*sqrt(5*sqrt(10) + 15)*(sqrt(10) + 3)^6/(4*x - 3) + 1719343467724559079186786713311125807232145810284763720104700727564898678431468079996999148449072973821003057072777764729170373925656330240000000000000000000000*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*(sqrt(10) + 3)^7/(4*x - 3) - 7225211155667781320130481195882887184658114024553118765981042400393799520821053342759894200300996740726995591511960480653475825502472110080000000000000000000*(sqrt(10) + 3)^7 - 108280243136919949154619390964478001407423524199098241901650620898785255901164501143525936419577793896678190367333122178618344714023455295327436800000000000000000*sqrt(34)*sqrt(10)*sqrt(2)*(sqrt(10) + 3)^5 + 595803027071831967601472522397936121135463723142101038880511147616631994161220186534882051792439633933314718140792758323128565818210756914577408000000000000000000*sqrt(34)*sqrt(10)*sqrt(2)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*(sqrt(10) + 3)^5/(4*x - 3) - 110479563153649818082989316061671193696800556657313667275787623065413652766167852112016424640510212259613589832021500803556136357499003855699968000000000000000*sqrt(34)*sqrt(10)*sqrt(5*sqrt(10) + 15)*(sqrt(10) + 3)^5 - 158651477088540184398148794982981417911553604380756547645663130240576424190241074583624172585904538037419984187987059731895791918318818204581888000000000000000000*sqrt(34)*sqrt(2)*sqrt(5*sqrt(10) + 15)*(sqrt(10) + 3)^5 - 303167218893211283343627231746912090555812747551085188755289651766064738462460741998447416422087515370839167255883736516914758600778261251227648000000000000000000*sqrt(10)*sqrt(2)*sqrt(5*sqrt(10) + 15)*(sqrt(10) + 3)^5 + 608843418689186605275124224409917322496784273397652643265799737282786666769922337429515957333375182995215312983173257098630974759665466280509440000000000000000000*sqrt(34)*sqrt(10)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*sqrt(5*sqrt(10) + 15)*(sqrt(10) + 3)^5/(4*x - 3) + 877660631366544053279357404238909808354962373744259438581685422353453355081538281566190507421640882491295771676619891590869179913924578985902080000000000000000000*sqrt(34)*sqrt(2)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*sqrt(5*sqrt(10) + 15)*(sqrt(10) + 3)^5/(4*x - 3) + 168411226939861639729205694372588726592949667478407982452934247756545342001638611397621415768643798097953714775695979506758379537670191710208000000000000000000000*sqrt(10)*sqrt(2)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*sqrt(5*sqrt(10) + 15)*(sqrt(10) + 3)^5/(4*x - 3) - 7112268982177

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6156657057776963878739152184110963173861557555002930986298769274741750635878
98884387997841131594904076872766586274202182382465843200000000000000000*sqr
t(34)*(sqrt(10) + 3)^6 - 13133758224038911455100712561551651942062542061747
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sqrt(2)*(sqrt(10) + 3)^6 + 393693722723284756366839874665065804091461207846
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63800103723215093760000000000000000000*sqrt(34)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x
 + 1) - sqrt(17))*(sqrt(10) + 3)^6/(4*x - 3) + 7297152385699815663721313494
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2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*(sqrt(10) + 3)^6/(4*x - 3) + 84172241
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00*sqrt(2)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*(sqrt(10) + 3)^6/(
4*x - 3) + 6690186180816194132366688184196717931784499440076891081740932708
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qrt(10) + 15)*(sqrt(10) + 3)^6/(4*x - 3) - 12032855442164962590268155452848
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(sqrt(10) + 3)^6 - 58521403567890456309649901010773654448025270762028102490
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5)*(sqrt(10) + 3)^4 + 10141198101176128908775444790266835859400545062135782
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(-2*x^2 + 3*x + 1) - sqrt(17))*sqrt(5*sqrt(10) + 15)*(sqrt(10) + 3)^4/(4*x
- 3) - 33889461058712235579753341978717240069992009549703761251555969050963
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20000000000000000*sqrt(34)*sqrt(10)*(sqrt(10) + 3)^5 - 52404498157780265279
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sqrt(2)*(sqrt(10) + 3)^5 - 107949609454984354378607605552003845605661996248
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82815494024060983050240000000000000000000*sqrt(10)*sqrt(2)*(sqrt(10) + 3)^5 +
617207924975112598287794543514905756684547442296872252298036345264423686852
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0000000000*sqrt(34)*sqrt(10)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*
(sqrt(10) + 3)^5/(4*x - 3) + 105780004640337209683883759521161611542628199
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2*x^2 + 3*x + 1) - sqrt(17))*(sqrt(10) + 3)^5/(4*x - 3) + 23795762436360110
```

438721514075399291761168704693382018997346371038359063898132017073109585529
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10)*sqrt(2)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*(sqrt(10) + 3)^5/
(4*x - 3) - 518484158477702479130866357685175493635353458119566466206785855
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)*sqrt(5*sqrt(10) + 15)*(sqrt(10) + 3)^5 + 1061770931415130329575369148657
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qrt(-2*x^2 + 3*x + 1) - sqrt(17))*sqrt(5*sqrt(10) + 15)*(sqrt(10) + 3)^5/(4
*x - 3) + 2263385694138470178837430646855902854392173325934804547908567864
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93280000000000000000*sqrt(10)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))
*sqrt(5*sqrt(10) + 15)*(sqrt(10) + 3)^5/(4*x - 3) + 266392394837958346192478
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858479410190093691505287184422824055687610368000000000000000000*sqrt(2)*(2*s
qrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*sqrt(5*sqrt(10) + 15)*(sqrt(10) +
 3)^5/(4*x - 3) + 115169104664160907612359289073837612060757257745405322633
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61589788672000000000000000000*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*
(sqrt(10) + 3)^6/(4*x - 3) - 51409911024085757169764376663631182833916000477
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3199515301753808486400000000000000000*(sqrt(10) + 3)^6 - 269779645959594200
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07124167216653419282981207055494131337236250695303168000000000000000000*sqrt
(34)*sqrt(10)*sqrt(2)*(sqrt(10) + 3)^4 + 1449431124348468764032339810344159
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0828099287516015513869956757333842329600000000000000000*sqrt(34)*sqrt(10)*sq
rt(2)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*(sqrt(10) + 3)^4/(4*x -
 3) - 440596212671328194641320984633816701059527849479442736535924497846173
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8000000000000000*sqrt(34)*sqrt(10)*sqrt(5*sqrt(10) + 15)*(sqrt(10) + 3)^4
- 9333082742094942120846103868761933806144879478813738905604135833447420106
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318391062073590669703313039127546233564533114109159702603369881415485033409
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00000*sqrt(34)*sqrt(10)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*sqrt(
5*sqrt(10) + 15)*(sqrt(10) + 3)^4/(4*x - 3) + 508966765095757864492687799779
32647287852526080845579948027361682527984686091919143766588300532049967556
86184537088368642422119768591868063278694400000000000000000*sqrt(34)*sqrt(2)
*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*sqrt(5*sqrt(10) + 15)*(sqrt(
10) + 3)^4/(4*x - 3) + 94182148466732499331884839946709481407383441556478670
5957536766715080328947583022044875236902341519744460399448464565804274752107
35075567201157120000000000000000000*sqrt(10)*sqrt(2)*(2*sqrt(2)*sqrt(-2*x^2
 + 3*x + 1) - sqrt(17))*sqrt(5*sqrt(10) + 15)*(sqrt(10) + 3)^4/(4*x - 3) -
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)^5 - 1934047503478493156486253559256447691008945779511368850276693024305415
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40000000000000000*sqrt(2)*(sqrt(10) + 3)^5 + 317859499068275185510814587874
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(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*(sqrt(10) + 3)^5/(4*x - 3) + 5934266
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00000*sqrt(10)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*(sqrt(10) + 3)
^5/(4*x - 3) + 1063776996617730742932790370948715093340732996004783665735571
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754407936000000000000000000000*sqrt(2)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sq
rt(17))*(sqrt(10) + 3)^5/(4*x - 3) + 110084952523263945529567915149718014993
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3*x + 1) - sqrt(17))*sqrt(5*sqrt(10) + 15)*(sqrt(10) + 3)^5/(4*x - 3) - 199
887401357231965451113988241357838308381013004132823341460231689212694495423
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10)*sqrt(2)*sqrt(5*sqrt(10) + 15)*(sqrt(10) + 3)^3 - 2986067160406812142132
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sqrt(10)*sqrt(2)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*sqrt(5*sqrt(
10) + 15)*(sqrt(10) + 3)^3/(4*x - 3) - 401559551369916798715569549782973966
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782761351665597607784470864744939520000000000000000*sqrt(34)*sqrt(10)*(sqrt(
10) + 3)^4 - 153945670277270812247129182517770989312040408807641302837626505
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```
96470072959573410409033640862976568725231174157965721600000000000000*sqrt(3
4)*sqrt(10)*sqrt(5*sqrt(10) + 15)*(sqrt(10) + 3)^3 - 9458583411665409865070
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sqrt(2)*sqrt(5*sqrt(10) + 15)*(sqrt(10) + 3)^3 - 17203280069940223943446477
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t(2)*sqrt(5*sqrt(10) + 15)*(sqrt(10) + 3)^3 + 18829502706784771068460399603
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0)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*sqrt(5*sqrt(10) + 15)*(sqr
t(10) + 3)^3/(4*x - 3) + 50770521476040157229446961545243086343116725917597
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x^2 + 3*x + 1) - sqrt(17))*sqrt(5*sqrt(10) + 15)*(sqrt(10) + 3)^3/(4*x - 3)
 + 925205182240252342061401246961417583126043985976581922792869314603912208
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17))*sqrt(5*sqrt(10) + 15)*(sqrt(10) + 3)^3/(4*x - 3) - 1123829357562383050
254390588661347803082400978560839838110367805876023356912563911623498174536
54750604124140746234302679127646860770060259093485977600000000000000000*sqrt(
34)*(sqrt(10) + 3)^4 - 2062363509993681833505965451766842722717275878856879
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2 + 3*x + 1) - sqrt(17))*(sqrt(10) + 3)^4/(4*x - 3) + 111404369539793145340
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0)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*(sqrt(10) + 3)^4/(4*x - 3)
 + 270495099921604050647993827934305028244401590250776690751773925358356280
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rt(10) + 3)^4/(4*x - 3) + 4384741091306438086412446190189263450962086830712
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29524908258235370595942400000000000000000*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) -
 sqrt(17))*sqrt(5*sqrt(10) + 15)*(sqrt(10) + 3)^4/(4*x - 3) - 8082252263732
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sqrt(5*sqrt(10) + 15)*(sqrt(10) + 3)^4 + 1601721428966470263724164617130953
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rt(2)*sqrt(5*sqrt(10) + 15)*(sqrt(10) + 3)^2 - 8322443947005005295594734684
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10)*sqrt(2)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*sqrt(5*sqrt(10) +
 15)*(sqrt(10) + 3)^2/(4*x - 3) + 13388831299944365847428214376505402272011
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0362080092234581709792268559319040000000000000000*sqrt(34)*sqrt(10)*(sqrt(10)
 + 3)^3 + 21196568021788607521815509851411648841099071379496434897147187038
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8566497280000000000000000*sqrt(34)*sqrt(2)*(sqrt(10) + 3)^3 + 406394422714945
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7868959980021295055100641743866055923171798561478208493977600000000000000000*s
qrt(10)*sqrt(2)*(sqrt(10) + 3)^3 - 8414093520394173319320674240297413701929
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2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*(sqrt(10) + 3)^3/(4*x - 3) - 15469541
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000000*sqrt(34)*sqrt(2)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*(sqrt
(10) + 3)^3/(4*x - 3) - 291776103475152149800172424387283315945417048753975
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x^2 + 3*x + 1) - sqrt(17))*(sqrt(10) + 3)^3/(4*x - 3) + 1516763904845666973
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4)*sqrt(5*sqrt(10) + 15)*(sqrt(10) + 3)^3 + 3070369728254767881752122125440
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rt(10) + 15)*(sqrt(10) + 3)^3 - 2749942609544560599641181667168450835017183
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(sqrt(10) + 3)^3 - 21812559984213005929206850286041732348225736275327888118
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 - sqrt(17))*sqrt(5*sqrt(10) + 15)*(sqrt(10) + 3)^3/(4*x - 3) - 41389694485
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00*sqrt(10)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*sqrt(5*sqrt(10) +
 15)*(sqrt(10) + 3)^3/(4*x - 3) - 51435915903455278526703707891673533491343
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x^2 + 3*x + 1) - sqrt(17))*sqrt(5*sqrt(10) + 15)*(sqrt(10) + 3)^3/(4*x - 3)
 - 229749742039419532606875823393090424454618943548961651518062659961833751
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8000000000000000*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*(sqrt(10) +
```

3)^4/(4*x - 3) - 7524058615200123430513883738003162961852392753106552544962
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t(10)*sqrt(2)*(sqrt(10) + 3)^2 - 293891581524697438526151824504046585787033
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2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*(sqrt(10) + 3)^2/(4*x - 3) + 1
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0000*sqrt(34)*sqrt(10)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*sqrt(5
sqrt(10) + 15)(sqrt(10) + 3)^2/(4*x - 3) - 107875628477087793486349896345
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)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*sqrt(5*sqrt(10) + 15)*(sqrt
(10) + 3)^2/(4*x - 3) - 199498118156696035814085209879947471552134598021303
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84403006933108404476968960000000000000*sqrt(10)*sqrt(2)*(2*sqrt(2)*sqrt(-2*
x^2 + 3*x + 1) - sqrt(17))*sqrt(5*sqrt(10) + 15)*(sqrt(10) + 3)^2/(4*x - 3)
 - 134928509971466693312390727031202255131795011649590708452958876555588697
925750413123927757003750190113026902918641503811208326714989914615747276963
8400000000000000*sqrt(34)*(sqrt(10) + 3)^3 - 232733890736133920979807576729
706208105228018996422853603785971341309225960887862512152708402613851846618
631876474097391256363289773509856848437575680000000000000000*sqrt(10)*(sqrt(1
0) + 3)^3 - 104380913480380668672619448137073656015319434585336415040099728
723854239511417831307559158934720575451263810611486774899104577436306168601
526254174208000000000000000*sqrt(2)*(sqrt(10) + 3)^3 + 104648945194848641388
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403722457916720752949425853063292618417056004960157696000000000000000*sqrt(3
4)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*(sqrt(10) + 3)^3/(4*x - 3)
 + 187374362829090804891222008164389904261485085365245713529689831010593083
506885255879320251944711191873770133227056037437088561612093539206777899319
2960000000000000*sqrt(10)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*(s
qrt(10) + 3)^3/(4*x - 3) + 625981730115069834877638505809172830107139138433
474951347862459545219412225775536972464914827431219340549945607171104986885

3038948297817373114965688320000000000000*sqrt(2)*(2*sqrt(2)*sqrt(-2*x^2 +
3*x + 1) - sqrt(17))*(sqrt(10) + 3)^3/(4*x - 3) + 3517941240658657889591539
2933951061734896642798114887207546398119232143627273692704266565279485383555
781831775142828191288929593912943994254398041620480000000000000000*(2*sqrt(2)
*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*sqrt(5*sqrt(10) + 15)*(sqrt(10) + 3)^3/
(4*x - 3) - 6366291422872821139929087851884884936303073932618484114812497250
0466005315784106099813083256598294354816689915849027850498286654571964844361
111725670400000000000000*sqrt(5*sqrt(10) + 15)*(sqrt(10) + 3)^3 + 39667459
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00000*sqrt(34)*sqrt(10)*sqrt(2)*sqrt(5*sqrt(10) + 15)*(sqrt(10) + 3) - 1618
655731712367712981931593479423324465010769820913807761797768326707626109462
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000000000*sqrt(34)*sqrt(10)*sqrt(2)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sq
rt(17))*sqrt(5*sqrt(10) + 15)*(sqrt(10) + 3)/(4*x - 3) + 102771818469045615
9189998931947503318822065359584940254650551278360785662706030297383491193627
369014053314941676911667278894598777199566879197400924160000000000000000*sqr
t(34)*sqrt(10)*(sqrt(10) + 3)^2 + 2685949310108795916371265342791542390198 6
7954687439325471438446944923255900441935316380875832655438816353402426020110
950485843838133811977501072183787520000000000000*sqrt(34)*sqrt(2)*(sqrt(10)
 + 3)^2 + 494564326546598397966209963984661308925158995324544650951215382779
0533690729117357876831289588853613199766994101008933387360243577074423721 3
386748723200000000000000*sqrt(10)*sqrt(2)*(sqrt(10) + 3)^2 - 46762799688990
3518211991340747041025337827850663957297226708157572781517731654365909441284
692378229702657510178042526002405179100962478838511801729024000000000000000
*sqrt(34)*sqrt(10)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*(sqrt(10)
+ 3)^2/(4*x - 3) - 122319585994638482611790197605606152888465012014642369688
5715017119229306601777898554952955067418675241416186061829601752963859510 7
52828358543374548992000000000000000*sqrt(34)*sqrt(2)*(2*sqrt(2)*sqrt(-2*x^2
+ 3*x + 1) - sqrt(17))*(sqrt(10) + 3)^2/(4*x - 3) - 224694757010667860476633
910393621707808526717317742532127150934749946976764932316297680242768490062
441286600178879089602744140061777168984364435940311040000000000000000*sqrt(10
)*sqrt(2)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*(sqrt(10) + 3)^2/(4
*x - 3) + 723064172221284987139958069235710280744106169091233882107327041816
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31462963200000000000000000*sqrt(34)*sqrt(5*sqrt(10) + 15)*(sqrt(10) + 3)^2 + 1
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000000000000*sqrt(10)*sqrt(5*sqrt(10) + 15)*(sqrt(10) + 3)^2 + 29398576918
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000*sqrt(2)*sqrt(5*sqrt(10) + 15)*(sqrt(10) + 3)^2 - 3685912280275478977694
6671687590144665852745455985742068248301378417762850377495025702500514954575
748494604777276535233134608596809730250299831253729280000000000000000*sqrt(34
)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*sqrt(5*sqrt(10) + 15)*(sqrt

```
(10) + 3)^2/(4*x - 3) - 6828502248676703232049924076183604001935780685955598
7144902834416202655082418726716545615677403499229160400120063510075759928809
7684813870442819052634112000000000000000*sqrt(10)*(2*sqrt(2)*sqrt(-2*x^2 + 3*
x + 1) - sqrt(17))*sqrt(5*sqrt(10) + 15)*(sqrt(10) + 3)^2/(4*x - 3) - 15281
4317985032478812362380610064901048295175239445099433250821608769858879833324
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0000000000*sqrt(2)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*sqrt(5*sqr
t(10) + 15)*(sqrt(10) + 3)^2/(4*x - 3) - 1526888179448934566618799851888073
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*x^2 + 3*x + 1) - sqrt(17))*(sqrt(10) + 3)^3/(4*x - 3) + 240691730210844711
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rt(10) + 3)^3 - 23636071242933744021352568836360985312133000571419711727932
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4600194322163353840896266693572522855150734305733617523672351571600480193348
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00000000000*sqrt(34)*sqrt(10)*sqrt(2)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) -
sqrt(17))*(sqrt(10) + 3)/(4*x - 3) + 303006920987145347112403726931416524111
6164108662317341205140286789632444008022256180137227113250002833225733061411
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*sqrt(10) + 15)*(sqrt(10) + 3) + 114424926610131642550962506872333542272876
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t(10) + 15)*(sqrt(10) + 3) + 211118172130568562296365972053609689456589903311
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0199971765228289141124567662592000000000000000*sqrt(10)*sqrt(2)*sqrt(5*sqrt(10
) + 15)*(sqrt(10) + 3) - 111683723069097704285308917218649514037526393953557
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2*x^2 + 3*x + 1) - sqrt(17))*sqrt(5*sqrt(10) + 15)*(sqrt(10) + 3)/(4*x - 3)
 - 4385875288980458802183924260458641908435676751046727081318573943346444657
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(17))*sqrt(5*sqrt(10) + 15)*(sqrt(10) + 3)/(4*x - 3) - 812399222004744560444
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10)*sqrt(2)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*sqrt(5*sqrt(10) +
 15)*(sqrt(10) + 3)/(4*x - 3) + 260952027641734075965709898441342389531020077
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2244877465647151023652648580546560000000000000000*sqrt(34)*(sqrt(10) + 3)^2 +
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0000000000000*sqrt(10)*(sqrt(10) + 3)^2 + 130547696404621298855548860240003
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) + 3)^2 - 104910755427635220643757764057527580545416881425678022509463511З
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1566452424704000000000000000*sqrt(34)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqr
t(17))*(sqrt(10) + 3)^2/(4*x - 3) - 193695634933375281635314644456744997210
467357259171616428253612802456747354608363827405663192497197048389957191701
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t(-2*x^2 + 3*x + 1) - sqrt(17))*(sqrt(10) + 3)^2/(4*x - 3) - 53761228177221
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0*sqrt(2)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*(sqrt(10) + 3)^2/(4
*x - 3) - 855820384745422816900719751194313066376563040296572757828995247О1
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(5*sqrt(10) + 15)*(sqrt(10) + 3)^2/(4*x - 3) + 2299236544257897790866574417
770758535410913649518240271443901139396631277802704058759356761777434832671
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0) + 15)*(sqrt(10) + 3)^2 - 60326449851913844160634924769180064628298643809
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sqrt(10) + 15) + 2572185038166394047415045636751259262489330049707345402405
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4237374878190141440000000000000*sqrt(34)*sqrt(10)*sqrt(2)*(2*sqrt(2)*sqrt(-
2*x^2 + 3*x + 1) - sqrt(17))*sqrt(5*sqrt(10) + 15)/(4*x - 3) - 884604240530
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0*sqrt(34)*sqrt(10)*(sqrt(10) + 3) - 65584738212303644821858972607634747870
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0) + 3) - 11887143314439876410099548982233452087984209440004326510420228767
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qrt(34)*sqrt(10)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*(sqrt(10) +
3)/(4*x - 3) + 397681440228039941101696267157800430964944316605401610221773
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 + 1) - sqrt(17))*(sqrt(10) + 3)/(4*x - 3) + 713791396788513854767634786441
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262535112483928843802610315775565730404106240000000000000000*sqrt(10)*sqrt(2
)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*(sqrt(10) + 3)/(4*x - 3) +
111111056586594428589100227549755107409883015398579822591234983641742560837
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0000000000000*sqrt(34)*sqrt(5*sqrt(10) + 15)*(sqrt(10) + 3) + 204586444301
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00*sqrt(10)*sqrt(5*sqrt(10) + 15)*(sqrt(10) + 3) + 731566732675128616749433
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2272020001031254929286509851597384659591183338569728000000000000*sqrt(2)*s
qrt(5*sqrt(10) + 15)*(sqrt(10) + 3) - 4536740249689503259363271018276002144
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7474889811750660457504374774098221334528000000000000*sqrt(34)*(2*sqrt(2)*sq
rt(-2*x^2 + 3*x + 1) - sqrt(17))*sqrt(5*sqrt(10) + 15)*(sqrt(10) + 3)/(4*x
- 3) - 83595659458021112783400794617334866196585509422622442699491978378985
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)*sqrt(5*sqrt(10) + 15)*(sqrt(10) + 3)/(4*x - 3) - 299947899586725004534905
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616201328941672672615274091937012490088162913576550400000000000000*sqrt(2)*
(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*sqrt(5*sqrt(10) + 15)*(sqrt(1
0) + 3)/(4*x - 3) - 8635872578435880979538548057579424745967248160066837168
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t(17))*(sqrt(10) + 3)^2/(4*x - 3) + 189494649653165276357696071131450661787
995134880762581919285854270405870433520921547846102223593932335231496181956
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406803057074506242061559304789594552576190959340589675090127966750045831126
244277166757119329066219017483173673756328890393206857044059177978167296000
000000000*sqrt(34)*sqrt(10)*sqrt(2) + 3672796307452272642853949802435867486
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57927343986281016099355009057315356672000000000000000*sqrt(34)*sqrt(10)*sqrt
(2)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))/(4*x - 3) - 1032690335070
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0*sqrt(34)*sqrt(10)*sqrt(5*sqrt(10) + 15) - 1919768831391366789577736173283
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*sqrt(5*sqrt(10) + 15) - 35435450986069016860794106612315243198776734080919
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6217872410272149327640002560000000000000*sqrt(10)*sqrt(2)*sqrt(5*sqrt(10) +
15) + 431511722632392011277496092916761015484788637894778776889807631823500
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sqrt(17))*sqrt(5*sqrt(10) + 15)/(4*x - 3) + 8214517039920088983131158497040
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924861938996791600217038658661773902769815552000000000000000*sqrt(34)*sqrt(2)
*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*sqrt(5*sqrt(10) + 15)/(4*x -
 3) + 152257426185649782649242614370362731887852313487398881796438039141023
```

4935447566157135014305944736501993709951380543222813359657101212588574070197075312640000000000000*sqrt(10)*sqrt(2)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*sqrt(5*sqrt(10) + 15)/(4*x - 3) - 292255803145321371228534980567386207634617820709466190103303858566916784614197548985511861577117754249624506009432733526846757043899161548246903213260800000000000000*sqrt(34)*(sqrt(10) + 3) - 53967863633556647093410387835510828035655014898582086355356943310264820619544346976463423253508171702297287776203840838133746931815759266487873621196800000000000000000*sqrt(10)*(sqrt(10) + 3) - 4284897231461412014763107118050298321936641684989569957684971609436740010309893264499812071070890081017375204625202086162398176836437768319348204634112000000000000000*sqrt(2)*(sqrt(10) + 3) + 13550258488120320500418842154559388294153876774598551033096177061089232191799572815000648240614847203936731492391195583014301608942899352177703234043904000000000000000*sqrt(34)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*(sqrt(10) + 3)/(4*x - 3) + 2509678886009954328817152098278281615425812752608504099514163242411693717857983116006774692792944181307452469233061539042309596047390443944396565658992640000000000000*sqrt(10)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*(sqrt(10) + 3)/(4*x - 3) + 2607478184462274418715983029440591458532345858977579473213334559443759731376438785950474611176623661148494640461148476438691858767218467736665679776972800000000000000*sqrt(2)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*(sqrt(10) + 3)/(4*x - 3) - 2052382687134936684487792880207215580419696755930520162089995568649902379968328529538572018997115294466373106224145890496536712676678191996394007554949120000000000000*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*sqrt(5*sqrt(10) + 15)*(sqrt(10) + 3)/(4*x - 3) + 5583743961563013218960271145914742547765879534939771930995881317184285591122249369558774737924698742913088860572563157892301089498671199625606384844800000000000000000*sqrt(5*sqrt(10) + 15)*(sqrt(10) + 3) - 1390579759437870784497893792017989465986513080554774850986384926073354840636886927063193148062292422316882997897418405746741736958025126527892463812608000000000000000*sqrt(34)*sqrt(10) - 28684039881842979045242928560109384241837657177314586314998326227920255484955237363252676877069086732787241412202864802918052862990199882512344511676416000000000000000*sqrt(34)*sqrt(2) - 5298316203580683203124264557015231170042197212191874562561269023402886999091247893797363134662615534383304414658181323768903264730106325490409236594688000000000000000*sqrt(10)*sqrt(2) + 55810644728206653920906108682212823291387874411478484509409359048613047131816532653669653145053947663441447429056580195346700765666124816308325125718016000000000000000*sqrt(34)*sqrt(10)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))/(4*x - 3) + 115842093512713279795447240169971525943000449357506521380522908088357625096297654437427598119899503672849469989132074285363202966114474936747174553190400000000000000*sqrt(34)*sqrt(2)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))/(4*x - 3) + 2144018941703302015154108661605845921083677912519513941469810912844664248629971918524463760290151670875965469776023213158092832266181166744967045762252800000000000000*sqrt(10)*sqrt(2)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))/(4*x - 3) - 32488680564328000497835071776574235806303298482479668695739682882671451697307014366276172158349066676298965301087267113920865

57901498213880053834448896000000000000*sqrt(34)*sqrt(5*sqrt(10) + 15) - 59
923872866331316082935651705740471209978064271895489435588040717249795023368
868763185731458570437739108885444337776136981215877601822937577128172978176
000000000000*sqrt(10)*sqrt(5*sqrt(10) + 15) - 11134560517950710139151686818
149078549217293857116230419835005000098215135285572223812264409498843191147
79870740323873137982712216448968586913297675059200000000000000*sqrt(2)*sqrt(
5*sqrt(10) + 15) + 13537310422602299322458136543050382270861976150690100122
293370164273840228610588171090301190097873021850147472059226019840787665663
18636646931621992202240000000000000*sqrt(34)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x +
1) - sqrt(17))*sqrt(5*sqrt(10) + 15)/(4*x - 3) + 24976510590740763187671242
061699850218511316382453308361864430628841295336771414183284388652597888260
556639739695424607575179163040658855224298499892838400000000000000*sqrt(10)*(
2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*sqrt(5*sqrt(10) + 15)/(4*x - 3
) + 47671288719518521206169225363052105407772799651872798901533717685063035
584061253857922066375053920334577521686319405042161683794155230608622013938
4012800000000000000*sqrt(2)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*s
qrt(5*sqrt(10) + 15)/(4*x - 3) + 745378086821182996481645252311565937551400
248000070915628679631628781503977056157685851476563320049049110117750814053
64369538223367437033660431992881152000000000000000*(2*sqrt(2)*sqrt(-2*x^2 + 3
x + 1) - sqrt(17))(sqrt(10) + 3)/(4*x - 3) - 4403168228654525497501905489
166624159021960023174722136520653956109419544157054815960655449291860412535
21257137538356626220950908814342924400062365696000000000000000*sqrt(34) - 24
446848800860999960210298658284298772551688585671397420366469753219319015483
359452623854511903958187274804283873224277358693419233989798281883087273984
0000000000000*sqrt(10) - 16781514794635064528878078407429002280827743330378
073938095363437750769090386695794157113197041885820608746242392665148041738
1813136866513999513849430016000000000000*sqrt(2) + 176843243785222503523088
792973694984222812082146468747026695629775045810188229921598413727560723213
25302956379142805368870038079038250625111549672003993600000000000000*sqrt(34)
*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))/(4*x - 3) + 3237803603548719
855924624234441601159307091327619506546076171627266980630388226931396268900
02650761453699808678582414396830056867772863202518758292717568000000000000*
sqrt(10)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))/(4*x - 3) + 67977171
499296965604609576337610827940393660434827094962237632216830208649144843518
428830508104102831939219362382333650677876576459636053025196385239040000000
0000000*sqrt(2)*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))/(4*x - 3) + 7
961861272526344209484328834598173539365831908736413352031705125778843076764
957427370909400185102550210881510543568445014756192352100232154703867150336
00000000000000*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))*sqrt(5*sqrt(10
) + 15)/(4*x - 3) - 1904509687900912544092588108650782807443402821281436863
973431742938063277117532949516641956220513664777594780105691020994566778291
47339253740786354749440000000000000*sqrt(5*sqrt(10) + 15) + 102178490700035
521940611914193374916361392507603263609998612621883980460064097249339490966
861084275585433257354149516056151820329307048265947054396669952000000000000
0*(2*sqrt(2)*sqrt(-2*x^2 + 3*x + 1) - sqrt(17))/(4*x - 3) - 746227475210387

```
3926787676367245626936477058152452342765532474445619634463361002030059006897630834977571804809796306975249220765899564261897554917693849600000000000000
)) + 2/17*sqrt(-2*x^2 + 3*x + 1)*(14*x + 15)/(2*x^2 - 3*x - 1)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2+x}{(2+4x-3x^2)\,(1+3x-2x^2)^{3/2}}\,dx = \int \frac{x+2}{\left(-2\,x^2+3\,x+1\right)^{3/2}\,\left(-3\,x^2+4\,x+2\right)}\,dx$$

```
[In] int((x + 2)/((3*x - 2*x^2 + 1)^(3/2)*(4*x - 3*x^2 + 2)),x)
[Out] int((x + 2)/((3*x - 2*x^2 + 1)^(3/2)*(4*x - 3*x^2 + 2)), x)
```

3.27 $\int \frac{2+x}{(2+4x-3x^2)(1+3x-2x^2)^{5/2}} dx$

Optimal result

Integrand size = 30, antiderivative size = 193

$$\int \frac{2+x}{(2+4x-3x^2)(1+3x-2x^2)^{5/2}} dx = -\frac{2(15+14x)}{51(1+3x-2x^2)^{3/2}} - \frac{2(291+4814x)}{867\sqrt{1+3x-2x^2}} + \frac{9}{2}\sqrt{\frac{1}{5}\left(-53+17\sqrt{10}\right)}\arctan\left(\frac{3(4-\sqrt{10})+(1+4\sqrt{10})x}{2\sqrt{1+\sqrt{10}}\sqrt{1+3x-2x^2}}\right) + \frac{9}{2}\sqrt{\frac{1}{5}\left(53+17\sqrt{10}\right)}\operatorname{arctanh}\left(\frac{3(4+\sqrt{10})+(1-4\sqrt{10})x}{2\sqrt{-1+\sqrt{10}}\sqrt{1+3x-2x^2}}\right)$$

```
[Out] -2/51*(15+14*x)/(-2*x^2+3*x+1)^(3/2)-2/867*(291+4814*x)/(-2*x^2+3*x+1)^(1/2
)+9/10*arctan(1/2*(12-3*10^(1/2)+x*(1+4*10^(1/2)))/(-2*x^2+3*x+1)^(1/2)/(1+
10^(1/2))^(1/2))*(-265+85*10^(1/2))^(1/2)+9/10*arctanh(1/2*(x*(1-4*10^(1/2)
)+12+3*10^(1/2))/(-2*x^2+3*x+1)^(1/2)/(-1+10^(1/2))^(1/2))*(265+85*10^(1/2)
)^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1030, 1074, 1046, 738, 210, 212}

$$\int \frac{2+x}{(2+4x-3x^2)(1+3x-2x^2)^{5/2}} dx = \frac{9}{2}\sqrt{\frac{1}{5}\left(17\sqrt{10}-53\right)}\arctan\left(\frac{(1+4\sqrt{10})x+3(4-\sqrt{10})}{2\sqrt{1+\sqrt{10}}\sqrt{-2x^2+3x+1}}\right) + \frac{9}{2}\sqrt{\frac{1}{5}\left(53+17\sqrt{10}\right)}\operatorname{arctanh}\left(\frac{(1-4\sqrt{10})x+3(4+\sqrt{10})}{2\sqrt{\sqrt{10}-1}\sqrt{-2x^2+3x+1}}\right) - \frac{2(14x+15)}{51(-2x^2+3x+1)^{3/2}} - \frac{2(4814x+291)}{867\sqrt{-2x^2+3x+1}}$$

```
[In] Int[(2 + x)/((2 + 4*x - 3*x^2)*(1 + 3*x - 2*x^2)^(5/2)),x]
```

```
[Out] (-2*(15 + 14*x))/(51*(1 + 3*x - 2*x^2)^(3/2)) - (2*(291 + 4814*x))/(867*Sqr
t[1 + 3*x - 2*x^2]) + (9*Sqrt[(-53 + 17*Sqrt[10])/5]*ArcTan[(3*(4 - Sqrt[10
]) + (1 + 4*Sqrt[10])*x)/(2*Sqrt[1 + Sqrt[10]]*Sqrt[1 + 3*x - 2*x^2])])/2 +
 (9*Sqrt[(53 + 17*Sqrt[10])/5]*ArcTanh[(3*(4 + Sqrt[10]) + (1 - 4*Sqrt[10])
*x)/(2*Sqrt[-1 + Sqrt[10]]*Sqrt[1 + 3*x - 2*x^2])])/2
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1030

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x + c*x^2)^(p + 1)*
((d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
 - b*f))*(p + 1)))*(g*c*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d +
b^2*f - c*(b*e + 2*a*f)) + c*(g*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - h*(b*
c*d - 2*a*c*e + a*b*f))*x), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*
d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*
x^2)^q*Simp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)
 + (b^2*(g*f) - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*((-h)*c*
e)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((g*c)*(2*a*c*e - b*(c*d + a*f)) + (g*
b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((g*c)*(2*
a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(
p + q + 2) - (b^2*g*f - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*
((-h)*c*e)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(g*f) - b*(h*c
*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) + a*h*c*e))*(2*p + 2*q + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e
 - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
```

Rule 1046

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1074

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x +
c*x^2)^(p + 1)*((d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f))*(p + 1)))*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) +
(A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c
*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d
 - a*f)))*x), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] && !( !IntegerQ[p] && ILtQ[q, -1]) && !
IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2(15+14x)}{51\left(1+3x-2x^2\right)^{3/2}}+\frac{2}{51}\int\frac{-56+\frac{235x}{2}+84x^2}{\left(2+4x-3x^2\right)\left(1+3x-2x^2\right)^{3/2}}\,dx\\ &= -\frac{2(15+14x)}{51\left(1+3x-2x^2\right)^{3/2}}-\frac{2(291+4814x)}{867\sqrt{1+3x-2x^2}}+\frac{4}{867}\int\frac{\frac{7803}{2}+\frac{23409x}{4}}{\left(2+4x-3x^2\right)\sqrt{1+3x-2x^2}}\,dx\end{aligned}$$

$$= -\frac{2(15+14x)}{51\left(1+3x-2x^2\right)^{3/2}} - \frac{2(291+4814x)}{867\sqrt{1+3x-2x^2}} + \frac{1}{5}\left(27\left(5-2\sqrt{10}\right)\right)\int \frac{1}{\left(4-2\sqrt{10}-6x\right)\sqrt{1+3x-2x^2}}\,dx + \frac{1}{5}\left(27\left(5+2\sqrt{10}\right)\right)\int \frac{1}{\left(4+2\sqrt{10}-6x\right)\sqrt{1+3x-2x^2}}\,dx$$

$$= -\frac{2(15+14x)}{51\left(1+3x-2x^2\right)^{3/2}} - \frac{2(291+4814x)}{867\sqrt{1+3x-2x^2}} - \frac{1}{5}\left(54\left(5-2\sqrt{10}\right)\right)\text{Subst}\left(\int \frac{1}{144+72\left(4-2\sqrt{10}\right)-8\left(4-2\sqrt{10}\right)^2-x^2}\,dx, x, \frac{-12-3\left(4-2\sqrt{\right.}}{\sqrt{}}\right. - \frac{1}{5}\left(54\left(5+2\sqrt{10}\right)\right)\text{Subst}\left(\int \frac{1}{144+72\left(4+2\sqrt{10}\right)-8\left(4+2\sqrt{10}\right)^2-x^2}\,dx, x, \frac{-12-3\left(4+2\sqrt{\right.}}{\sqrt{}}\right.$$

$$= -\frac{2(15+14x)}{51\left(1+3x-2x^2\right)^{3/2}} - \frac{2(291+4814x)}{867\sqrt{1+3x-2x^2}} + \frac{9}{2}\sqrt{\frac{1}{5}\left(-53+17\sqrt{10}\right)}\tan^{-1}\left(\frac{3\left(4-\sqrt{10}\right)+\left(1+4\sqrt{10}\right)x}{2\sqrt{1+\sqrt{10}}\sqrt{1+3x-2x^2}}\right) + \frac{9}{2}\sqrt{\frac{1}{5}\left(53+17\sqrt{10}\right)}\tanh^{-1}\left(\frac{3\left(4+\sqrt{10}\right)+\left(1-4\sqrt{10}\right)x}{2\sqrt{-1+\sqrt{10}}\sqrt{1+3x-2x^2}}\right)$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.45 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.95

$$\int \frac{2+x}{\left(2+4x-3x^2\right)\left(1+3x-2x^2\right)^{5/2}}\,dx = -\frac{2(546+5925x+13860x^2-9628x^3)}{867\left(1+3x-2x^2\right)^{3/2}} - \frac{9}{2}\text{RootSum}\left[5+20\#1+8\#1^2-8\#1^3+2\#1^4\&, \frac{-13\log(x)+13\log\left(-1+\sqrt{1+3x-2x^2}-x\#1\right)+6\log(x}{}\right.$$

```
[In] Integrate[(2 + x)/((2 + 4*x - 3*x^2)*(1 + 3*x - 2*x^2)^(5/2)),x]
```

```
[Out] (-2*(546 + 5925*x + 13860*x^2 - 9628*x^3))/(867*(1 + 3*x - 2*x^2)^(3/2)) -
(9*RootSum[5 + 20*#1 + 8*#1^2 - 8*#1^3 + 2*#1^4 & , (-13*Log[x] + 13*Log[-1
 + Sqrt[1 + 3*x - 2*x^2] - x*#1] + 6*Log[x]*#1 - 6*Log[-1 + Sqrt[1 + 3*x -
2*x^2] - x*#1]*#1 - 2*Log[x]*#1^2 + 2*Log[-1 + Sqrt[1 + 3*x - 2*x^2] - x*#1
]*#1^2)/(5 + 4*#1 - 6*#1^2 + 2*#1^3) & ])/2
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.27 (sec) , antiderivative size = 484, normalized size of antiderivative = 2.51

method	result
trager	$\frac{2(9628x^3-13860x^2-5925x-546)\sqrt{-2x^2+3x+1}}{867(2x^2-3x-1)^2} - 18\,\mathrm{RootOf}\left(6400\_Z^4 - 8480\_Z^2 - 81\right)\ln\left(-\frac{-2105600x\,\mathrm{Roo}}{}\right.$
default	Expression too large to display

```
[In] int((2+x)/(-3*x^2+4*x+2)/(-2*x^2+3*x+1)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/867*(9628*x^3-13860*x^2-5925*x-546)/(2*x^2-3*x-1)^2*(-2*x^2+3*x+1)^(1/2)-
18*RootOf(6400*_Z^4-8480*_Z^2-81)*ln(-(-2105600*x*RootOf(6400*_Z^4-8480*_Z^
2-81)^5+5362400*RootOf(6400*_Z^4-8480*_Z^2-81)^3*x+74880*(-2*x^2+3*x+1)^(1/
2)*RootOf(6400*_Z^4-8480*_Z^2-81)^2+473760*RootOf(6400*_Z^4-8480*_Z^2-81)^3
-3406349*RootOf(6400*_Z^4-8480*_Z^2-81)*x-99945*(-2*x^2+3*x+1)^(1/2)-632106
*RootOf(6400*_Z^4-8480*_Z^2-81))/(80*x*RootOf(6400*_Z^4-8480*_Z^2-81)^2-87*
x-34))+9/10*RootOf(_Z^2+400*RootOf(6400*_Z^4-8480*_Z^2-81)^2-530)*ln(-(2105
600*x*RootOf(6400*_Z^4-8480*_Z^2-81)^4*RootOf(_Z^2+400*RootOf(6400*_Z^4-848
0*_Z^2-81)^2-530)-217440*RootOf(6400*_Z^4-8480*_Z^2-81)^2*RootOf(_Z^2+400*R
ootOf(6400*_Z^4-8480*_Z^2-81)^2-530)*x+473760*RootOf(6400*_Z^4-8480*_Z^2-81
)^2*RootOf(_Z^2+400*RootOf(6400*_Z^4-8480*_Z^2-81)^2-530)-1497600*(-2*x^2+3
*x+1)^(1/2)*RootOf(6400*_Z^4-8480*_Z^2-81)^2-2187*RootOf(_Z^2+400*RootOf(64
00*_Z^4-8480*_Z^2-81)^2-530)*x+4374*RootOf(_Z^2+400*RootOf(6400*_Z^4-8480*_
Z^2-81)^2-530)-14580*(-2*x^2+3*x+1)^(1/2))/(80*x*RootOf(6400*_Z^4-8480*_Z^2
-81)^2-19*x+34))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 439 vs. 2(137) = 274.

Time = 0.31 (sec) , antiderivative size = 439, normalized size of antiderivative = 2.27

$$\int \frac{2+x}{(2+4x-3x^2)\left(1+3x-2x^2\right)^{5/2}}\,dx =$$

$$-\frac{43680\,x^4 - 131040\,x^3 - 867\,\sqrt{5}(4\,x^4 - 12\,x^3 + 5\,x^2 + 6\,x + 1)\sqrt{-1377\,\sqrt{10} + 4293}\log\left(-\frac{405\,\sqrt{10}x + \left(13\,\sqrt{10}\right.}{}\right.}{}$$

```
[In] integrate((2+x)/(-3*x^2+4*x+2)/(-2*x^2+3*x+1)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/8670*(43680*x^4 - 131040*x^3 - 867*sqrt(5)*(4*x^4 - 12*x^3 + 5*x^2 + 6*x
 + 1)*sqrt(-1377*sqrt(10) + 4293)*log(-(405*sqrt(10)*x + (13*sqrt(10)*sqrt(
5)*x + 40*sqrt(5)*x)*sqrt(-1377*sqrt(10) + 4293) + 810*x - 810*sqrt(-2*x^2
+ 3*x + 1) + 810)/x) + 867*sqrt(5)*(4*x^4 - 12*x^3 + 5*x^2 + 6*x + 1)*sqrt(
-1377*sqrt(10) + 4293)*log(-(405*sqrt(10)*x - (13*sqrt(10)*sqrt(5)*x + 40*s
qrt(5)*x)*sqrt(-1377*sqrt(10) + 4293) + 810*x - 810*sqrt(-2*x^2 + 3*x + 1)
+ 810)/x) - 7803*sqrt(5)*(4*x^4 - 12*x^3 + 5*x^2 + 6*x + 1)*sqrt(17*sqrt(10
) + 53)*log(9*(45*sqrt(10)*x + (13*sqrt(10)*sqrt(5)*x - 40*sqrt(5)*x)*sqrt(
17*sqrt(10) + 53) - 90*x + 90*sqrt(-2*x^2 + 3*x + 1) - 90)/x) + 7803*sqrt(5
)*(4*x^4 - 12*x^3 + 5*x^2 + 6*x + 1)*sqrt(17*sqrt(10) + 53)*log(9*(45*sqrt(
10)*x - (13*sqrt(10)*sqrt(5)*x - 40*sqrt(5)*x)*sqrt(17*sqrt(10) + 53) - 90*
x + 90*sqrt(-2*x^2 + 3*x + 1) - 90)/x) + 54600*x^2 - 20*(9628*x^3 - 13860*x
^2 - 5925*x - 546)*sqrt(-2*x^2 + 3*x + 1) + 65520*x + 10920)/(4*x^4 - 12*x^
3 + 5*x^2 + 6*x + 1)
```

Sympy [F]

$$\int \frac{2+x}{(2+4x-3x^2)(1+3x-2x^2)^{5/2}}\,dx = -\int \frac{x}{12x^6\sqrt{-2x^2+3x+1}-52x^5\sqrt{-2x^2+3x+1}+55x^4\sqrt{-2x^2+3x+1}+22x^3\sqrt{-2x^2+3x+1}-31x^2\sqrt{\cdots}}$$
$$-\int \frac{2}{12x^6\sqrt{-2x^2+3x+1}-52x^5\sqrt{-2x^2+3x+1}+55x^4\sqrt{-2x^2+3x+1}+22x^3\sqrt{-2x^2+3x+1}-31x^2\sqrt{\cdots}}$$

```
[In] integrate((2+x)/(-3*x**2+4*x+2)/(-2*x**2+3*x+1)**(5/2),x)
```

```
[Out] -Integral(x/(12*x**6*sqrt(-2*x**2 + 3*x + 1) - 52*x**5*sqrt(-2*x**2 + 3*x +
 1) + 55*x**4*sqrt(-2*x**2 + 3*x + 1) + 22*x**3*sqrt(-2*x**2 + 3*x + 1) - 3
1*x**2*sqrt(-2*x**2 + 3*x + 1) - 16*x*sqrt(-2*x**2 + 3*x + 1) - 2*sqrt(-2*x
**2 + 3*x + 1)), x) - Integral(2/(12*x**6*sqrt(-2*x**2 + 3*x + 1) - 52*x**5
*sqrt(-2*x**2 + 3*x + 1) + 55*x**4*sqrt(-2*x**2 + 3*x + 1) + 22*x**3*sqrt(-
2*x**2 + 3*x + 1) - 31*x**2*sqrt(-2*x**2 + 3*x + 1) - 16*x*sqrt(-2*x**2 + 3
*x + 1) - 2*sqrt(-2*x**2 + 3*x + 1)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1276 vs. 2(137) = 274.

Time = 0.32 (sec) , antiderivative size = 1276, normalized size of antiderivative = 6.61

$$\int \frac{2+x}{(2+4x-3x^2)(1+3x-2x^2)^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((2+x)/(-3*x^2+4*x+2)/(-2*x^2+3*x+1)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/17340*sqrt(10)*(2108*sqrt(10)*x/(sqrt(10)*(-2*x^2 + 3*x + 1)^(3/2) + (-2*
x^2 + 3*x + 1)^(3/2)) - 2108*sqrt(10)*x/(sqrt(10)*(-2*x^2 + 3*x + 1)^(3/2)
- (-2*x^2 + 3*x + 1)^(3/2)) - 56916*sqrt(10)*x/(2*sqrt(10)*sqrt(-2*x^2 + 3*
x + 1) + 11*sqrt(-2*x^2 + 3*x + 1)) + 56916*sqrt(10)*x/(2*sqrt(10)*sqrt(-2*
x^2 + 3*x + 1) - 11*sqrt(-2*x^2 + 3*x + 1)) + 1984*sqrt(10)*x/(sqrt(10)*sqr
t(-2*x^2 + 3*x + 1) + sqrt(-2*x^2 + 3*x + 1)) - 1984*sqrt(10)*x/(sqrt(10)*s
qrt(-2*x^2 + 3*x + 1) - sqrt(-2*x^2 + 3*x + 1)) - 70227*sqrt(10)*arcsin(8/1
7*sqrt(17)*sqrt(10)*x/abs(6*x + 2*sqrt(10) - 4) + 2/17*sqrt(17)*x/abs(6*x +
 2*sqrt(10) - 4) - 6/17*sqrt(17)*sqrt(10)/abs(6*x + 2*sqrt(10) - 4) + 24/17
*sqrt(17)/abs(6*x + 2*sqrt(10) - 4))/(2*sqrt(10)*sqrt(sqrt(10) + 1) + 11*sq
rt(sqrt(10) + 1)) - 2176*x/(sqrt(10)*(-2*x^2 + 3*x + 1)^(3/2) + (-2*x^2 + 3
*x + 1)^(3/2)) - 2176*x/(sqrt(10)*(-2*x^2 + 3*x + 1)^(3/2) - (-2*x^2 + 3*x
+ 1)^(3/2)) + 58752*x/(2*sqrt(10)*sqrt(-2*x^2 + 3*x + 1) + 11*sqrt(-2*x^2 +
 3*x + 1)) + 58752*x/(2*sqrt(10)*sqrt(-2*x^2 + 3*x + 1) - 11*sqrt(-2*x^2 +
3*x + 1)) - 2048*x/(sqrt(10)*sqrt(-2*x^2 + 3*x + 1) + sqrt(-2*x^2 + 3*x + 1
)) - 2048*x/(sqrt(10)*sqrt(-2*x^2 + 3*x + 1) - sqrt(-2*x^2 + 3*x + 1)) + 56
1816*arcsin(8/17*sqrt(17)*sqrt(10)*x/abs(6*x + 2*sqrt(10) - 4) + 2/17*sqrt(
17)*x/abs(6*x + 2*sqrt(10) - 4) - 6/17*sqrt(17)*sqrt(10)/abs(6*x + 2*sqrt(1
0) - 4) + 24/17*sqrt(17)/abs(6*x + 2*sqrt(10) - 4))/(2*sqrt(10)*sqrt(sqrt(1
0) + 1) + 11*sqrt(sqrt(10) + 1)) - 714*sqrt(10)/(sqrt(10)*(-2*x^2 + 3*x + 1
)^(3/2) + (-2*x^2 + 3*x + 1)^(3/2)) + 714*sqrt(10)/(sqrt(10)*(-2*x^2 + 3*x
+ 1)^(3/2) - (-2*x^2 + 3*x + 1)^(3/2)) + 19278*sqrt(10)/(2*sqrt(10)*sqrt(-2
*x^2 + 3*x + 1) + 11*sqrt(-2*x^2 + 3*x + 1)) - 19278*sqrt(10)/(2*sqrt(10)*s
qrt(-2*x^2 + 3*x + 1) - 11*sqrt(-2*x^2 + 3*x + 1)) - 1488*sqrt(10)/(sqrt(10
)*sqrt(-2*x^2 + 3*x + 1) + sqrt(-2*x^2 + 3*x + 1)) + 1488*sqrt(10)/(sqrt(10
)*sqrt(-2*x^2 + 3*x + 1) - sqrt(-2*x^2 + 3*x + 1)) - 5304/(sqrt(10)*(-2*x^2
 + 3*x + 1)^(3/2) + (-2*x^2 + 3*x + 1)^(3/2)) - 5304/(sqrt(10)*(-2*x^2 + 3*
x + 1)^(3/2) - (-2*x^2 + 3*x + 1)^(3/2)) + 143208/(2*sqrt(10)*sqrt(-2*x^2 +
 3*x + 1) + 11*sqrt(-2*x^2 + 3*x + 1)) + 143208/(2*sqrt(10)*sqrt(-2*x^2 + 3
*x + 1) - 11*sqrt(-2*x^2 + 3*x + 1)) + 1536/(sqrt(10)*sqrt(-2*x^2 + 3*x + 1
) + sqrt(-2*x^2 + 3*x + 1)) + 1536/(sqrt(10)*sqrt(-2*x^2 + 3*x + 1) - sqrt(
-2*x^2 + 3*x + 1)) + 70227*sqrt(10)*log(-2/9*sqrt(10) + 2/3*sqrt(-2*x^2 + 3
*x + 1)*sqrt(sqrt(10) - 1)/abs(6*x - 2*sqrt(10) - 4) + 2/9*sqrt(10)/abs(6*x
 - 2*sqrt(10) - 4) - 2/9/abs(6*x - 2*sqrt(10) - 4) + 1/18)/(sqrt(10) - 1)^(
5/2) + 561816*log(-2/9*sqrt(10) + 2/3*sqrt(-2*x^2 + 3*x + 1)*sqrt(sqrt(10)
- 1)/abs(6*x - 2*sqrt(10) - 4) + 2/9*sqrt(10)/abs(6*x - 2*sqrt(10) - 4) - 2
/9/abs(6*x - 2*sqrt(10) - 4) + 1/18)/(sqrt(10) - 1)^(5/2))
```

Giac [F(-1)]

Timed out.

$$\int \frac{2+x}{(2+4x-3x^2)(1+3x-2x^2)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((2+x)/(-3*x^2+4*x+2)/(-2*x^2+3*x+1)^(5/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2+x}{(2+4x-3x^2)(1+3x-2x^2)^{5/2}}\,dx = \int \frac{x+2}{(-2\,x^2+3\,x+1)^{5/2}\,(-3\,x^2+4\,x+2)}\,dx$$

```
[In] int((x + 2)/((3*x - 2*x^2 + 1)^(5/2)*(4*x - 3*x^2 + 2)),x)
[Out] int((x + 2)/((3*x - 2*x^2 + 1)^(5/2)*(4*x - 3*x^2 + 2)), x)
```

3.28 $\int \frac{2+x}{(2+4x-3x^2)\sqrt{1+3x+2x^2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 151

$$\begin{aligned}\int \frac{2+x}{(2+4x-3x^2)\sqrt{1+3x+2x^2}}\,dx &= -\frac{1}{2}\sqrt{1+\frac{7\sqrt{\frac{2}{5}}}{5}}\operatorname{arctanh}\left(\frac{3(4-\sqrt{10})+(17-4\sqrt{10})\,x}{2\sqrt{55-17\sqrt{10}}\sqrt{1+3x+2x^2}}\right)\\ &\quad+\frac{1}{2}\sqrt{1-\frac{7\sqrt{\frac{2}{5}}}{5}}\operatorname{arctanh}\left(\frac{3(4+\sqrt{10})+(17+4\sqrt{10})\,x}{2\sqrt{55+17\sqrt{10}}\sqrt{1+3x+2x^2}}\right)\end{aligned}$$

```
[Out] 1/10*arctanh(1/2*(12+3*10^(1/2)+x*(17+4*10^(1/2)))/(2*x^2+3*x+1)^(1/2)/(55+
17*10^(1/2))^(1/2))*(25-7*10^(1/2))^(1/2)-1/10*arctanh(1/2*(x*(17-4*10^(1/2
))+12-3*10^(1/2))/(2*x^2+3*x+1)^(1/2)/(55-17*10^(1/2))^(1/2))*(25+7*10^(1/2
))^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used

$= \{1046, 738, 212\}$

$$\int \frac{2+x}{(2+4x-3x^2)\sqrt{1+3x+2x^2}}\,dx$$
$$= \frac{1}{2}\sqrt{1-\frac{7\sqrt{\frac{2}{5}}}{5}}\text{arctanh}\left(\frac{(17+4\sqrt{10})\,x+3(4+\sqrt{10})}{2\sqrt{55+17\sqrt{10}}\sqrt{2x^2+3x+1}}\right)$$
$$-\frac{1}{2}\sqrt{1+\frac{7\sqrt{\frac{2}{5}}}{5}}\text{arctanh}\left(\frac{(17-4\sqrt{10})\,x+3(4-\sqrt{10})}{2\sqrt{55-17\sqrt{10}}\sqrt{2x^2+3x+1}}\right)$$

```
[In] Int[(2 + x)/((2 + 4*x - 3*x^2)*Sqrt[1 + 3*x + 2*x^2]),x]
```

```
[Out] -1/2*(Sqrt[1 + (7*Sqrt[2/5])/5]*ArcTanh[(3*(4 - Sqrt[10]) + (17 - 4*Sqrt[10
])*x)/(2*Sqrt[55 - 17*Sqrt[10]]*Sqrt[1 + 3*x + 2*x^2])]) + (Sqrt[1 - (7*Sqr
t[2/5])/5]*ArcTanh[(3*(4 + Sqrt[10]) + (17 + 4*Sqrt[10])*x)/(2*Sqrt[55 + 17
*Sqrt[10]]*Sqrt[1 + 3*x + 2*x^2])])/2
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1046

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\text{integral} = \frac{1}{5}\left(5-4\sqrt{10}\right)\int \frac{1}{(4-2\sqrt{10}-6x)\sqrt{1+3x+2x^2}}\,dx$$
$$+\frac{1}{5}\left(5+4\sqrt{10}\right)\int \frac{1}{(4+2\sqrt{10}-6x)\sqrt{1+3x+2x^2}}\,dx$$

$$= -\left(\frac{1}{5}\left(2\left(5-4\sqrt{10}\right)\right)\text{Subst}\left(\int \frac{1}{144+72\left(4-2\sqrt{10}\right)+8\left(4-2\sqrt{10}\right)^2-x^2}\,dx, x, \frac{-12-3\left(4-2\right.}{}\right.\right.$$

$$-\frac{1}{5}\left(2\left(5+4\sqrt{10}\right)\right)\text{Subst}\left(\int \frac{1}{144+72\left(4+2\sqrt{10}\right)+8\left(4+2\sqrt{10}\right)^2-x^2}\,dx, x, \frac{-12-3\left(4+2\sqrt{10}\right)-(}{\sqrt{1+3x}}\right.$$

$$= -\frac{1}{10}\sqrt{25+7\sqrt{10}}\tanh^{-1}\left(\frac{3\left(4-\sqrt{10}\right)+\left(17-4\sqrt{10}\right)x}{2\sqrt{55-17\sqrt{10}}\sqrt{1+3x+2x^2}}\right) + \frac{1}{10}\sqrt{25-7\sqrt{10}}\tanh^{-1}\left(\frac{3\left(4+\sqrt{10}\right)+\left(17+4\sqrt{10}\right)x}{2\sqrt{55+17\sqrt{10}}\sqrt{1+3x+2x^2}}\right)$$

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.72

$$\int \frac{2+x}{(2+4x-3x^2)\sqrt{1+3x+2x^2}}\,dx = -\frac{1}{5}\sqrt{25+7\sqrt{10}}\,\text{arctanh}\left(\frac{\sqrt{1-\sqrt{\frac{2}{5}}}\sqrt{1+3x+2x^2}}{1+2x}\right) + \frac{1}{5}\sqrt{25-7\sqrt{10}}\,\text{arctanh}\left(\frac{\sqrt{1+\sqrt{\frac{2}{5}}}\sqrt{1+3x+2x^2}}{1+2x}\right)$$

```
[In] Integrate[(2 + x)/((2 + 4*x - 3*x^2)*Sqrt[1 + 3*x + 2*x^2]),x]
[Out] -1/5*(Sqrt[25 + 7*Sqrt[10]]*ArcTanh[(Sqrt[1 - Sqrt[2/5]]*Sqrt[1 + 3*x + 2*x
^2])/(1 + 2*x)]) + (Sqrt[25 - 7*Sqrt[10]]*ArcTanh[(Sqrt[1 + Sqrt[2/5]]*Sqrt
[1 + 3*x + 2*x^2])/(1 + 2*x)])/5
```

Maple [A] (verified)

Time = 2.32 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.23

method	result
default	$\frac{\left(8+\sqrt{10}\right)\sqrt{10}\,\operatorname{arctanh}\left(\frac{55+17\sqrt{10}+\frac{9\left(\frac{17}{3}+\frac{4\sqrt{10}}{3}\right)\left(x-\frac{2}{3}-\frac{\sqrt{10}}{3}\right)}{2}}{\sqrt{55+17\sqrt{10}}\sqrt{18\left(x-\frac{2}{3}-\frac{\sqrt{10}}{3}\right)^{2}+9\left(\frac{17}{3}+\frac{4\sqrt{10}}{3}\right)\left(x-\frac{2}{3}-\frac{\sqrt{10}}{3}\right)+55+17\sqrt{10}}}\right)}{20\sqrt{55+17\sqrt{10}}}+\frac{\left(-8+\sqrt{10}\right)\sqrt{10}\,\operatorname{arctanh}\left(\frac{}{\sqrt{55-}}\right.}{}$
trager	$\frac{\operatorname{RootOf}\left(\_Z^{2}+4\operatorname{RootOf}\left(2000\_Z^{4}-1000\_Z^{2}+27\right)^{2}-2\right)\ln\left(-\frac{10000x\operatorname{RootOf}\left(2000\_Z^{4}-1000\_Z^{2}+27\right)^{4}\operatorname{RootOf}\left(\_Z^{2}+4\operatorname{RootOf}\right.}{}\right.}{}$

```
[In] int((2+x)/(-3*x^2+4*x+2)/(2*x^2+3*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/20*(8+10^(1/2))*10^(1/2)/(55+17*10^(1/2))^(1/2)*arctanh(9/2*(110/9+34/9*1
0^(1/2)+(17/3+4/3*10^(1/2))*(x-2/3-1/3*10^(1/2)))/(55+17*10^(1/2))^(1/2)/(1
8*(x-2/3-1/3*10^(1/2))^2+9*(17/3+4/3*10^(1/2))*(x-2/3-1/3*10^(1/2))+55+17*1
0^(1/2))^(1/2))+1/20*(-8+10^(1/2))*10^(1/2)/(55-17*10^(1/2))^(1/2)*arctanh(
9/2*(110/9-34/9*10^(1/2)+(17/3-4/3*10^(1/2))*(x-2/3+1/3*10^(1/2)))/(55-17*1
0^(1/2))^(1/2)/(18*(x-2/3+1/3*10^(1/2))^2+9*(17/3-4/3*10^(1/2))*(x-2/3+1/3*
10^(1/2))+55-17*10^(1/2))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(103) = 206$.

Time = 0.31 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.62

$$
\begin{aligned}
&\int \frac{2+x}{(2+4x-3x^2)\sqrt{1+3x+2x^2}}\,dx \\
&= \frac{1}{10}\sqrt{7\sqrt{10}+25}\log\left(-\frac{3\sqrt{10}x+\left(\sqrt{10}x-4\,x\right)\sqrt{7\sqrt{10}+25}+6\,x-6\sqrt{2\,x^{2}+3\,x+1}+6}{x}\right) \\
&\quad -\frac{1}{10}\sqrt{7\sqrt{10}+25}\log\left(-\frac{3\sqrt{10}x-\left(\sqrt{10}x-4\,x\right)\sqrt{7\sqrt{10}+25}+6\,x-6\sqrt{2\,x^{2}+3\,x+1}+6}{x}\right) \\
&\quad +\frac{1}{10}\sqrt{-7\sqrt{10}+25}\log\left(\frac{3\sqrt{10}x+\left(\sqrt{10}x+4\,x\right)\sqrt{-7\sqrt{10}+25}-6\,x+6\sqrt{2\,x^{2}+3\,x+1}-6}{x}\right) \\
&\quad -\frac{1}{10}\sqrt{-7\sqrt{10}+25}\log\left(\frac{3\sqrt{10}x-\left(\sqrt{10}x+4\,x\right)\sqrt{-7\sqrt{10}+25}-6\,x+6\sqrt{2\,x^{2}+3\,x+1}-6}{x}\right)
\end{aligned}
$$

```
[In] integrate((2+x)/(-3*x^2+4*x+2)/(2*x^2+3*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/10*sqrt(7*sqrt(10) + 25)*log(-(3*sqrt(10)*x + (sqrt(10)*x - 4*x)*sqrt(7*s
qrt(10) + 25) + 6*x - 6*sqrt(2*x^2 + 3*x + 1) + 6)/x) - 1/10*sqrt(7*sqrt(10
) + 25)*log(-(3*sqrt(10)*x - (sqrt(10)*x - 4*x)*sqrt(7*sqrt(10) + 25) + 6*x
 - 6*sqrt(2*x^2 + 3*x + 1) + 6)/x) + 1/10*sqrt(-7*sqrt(10) + 25)*log((3*sqr
t(10)*x + (sqrt(10)*x + 4*x)*sqrt(-7*sqrt(10) + 25) - 6*x + 6*sqrt(2*x^2 +
3*x + 1) - 6)/x) - 1/10*sqrt(-7*sqrt(10) + 25)*log((3*sqrt(10)*x - (sqrt(10
)*x + 4*x)*sqrt(-7*sqrt(10) + 25) - 6*x + 6*sqrt(2*x^2 + 3*x + 1) - 6)/x)
```

Sympy [F]

$$\int \frac{2+x}{(2+4x-3x^2)\sqrt{1+3x+2x^2}}\,dx$$
$$= -\int \frac{x}{3x^2\sqrt{2x^2+3x+1}-4x\sqrt{2x^2+3x+1}-2\sqrt{2x^2+3x+1}}\,dx - \int \frac{2}{3x^2\sqrt{2x^2+3x+1}-4x\sqrt{2x^2+3x+1}-2\sqrt{2x^2+3x+1}}\,dx$$

```
[In] integrate((2+x)/(-3*x**2+4*x+2)/(2*x**2+3*x+1)**(1/2),x)
```

```
[Out] -Integral(x/(3*x**2*sqrt(2*x**2 + 3*x + 1) - 4*x*sqrt(2*x**2 + 3*x + 1) - 2
*sqrt(2*x**2 + 3*x + 1)), x) - Integral(2/(3*x**2*sqrt(2*x**2 + 3*x + 1) -
4*x*sqrt(2*x**2 + 3*x + 1) - 2*sqrt(2*x**2 + 3*x + 1)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 363 vs. $2(103) = 206$.

Time = 0.31 (sec) , antiderivative size = 363, normalized size of antiderivative = 2.40

$$\int \frac{2+x}{(2+4x-3x^2)\sqrt{1+3x+2x^2}}\,dx$$
$$= \frac{1}{60}\sqrt{10}\left(\frac{3\sqrt{10}\log\left(\frac{2}{9}\sqrt{10}+\frac{2\sqrt{2x^2+3x+1}\sqrt{17\sqrt{10}+55}}{3|6x-2\sqrt{10}-4|}+\frac{34\sqrt{10}}{9|6x-2\sqrt{10}-4|}+\frac{110}{9|6x-2\sqrt{10}-4|}+\frac{17}{18}\right)}{\sqrt{17\sqrt{10}+55}}+\frac{\sqrt{10}\log\left(-\right.}{}\right.$$

```
[In] integrate((2+x)/(-3*x^2+4*x+2)/(2*x^2+3*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/60*sqrt(10)*(3*sqrt(10)*log(2/9*sqrt(10) + 2/3*sqrt(2*x^2 + 3*x + 1)*sqrt
(17*sqrt(10) + 55)/abs(6*x - 2*sqrt(10) - 4) + 34/9*sqrt(10)/abs(6*x - 2*sq
rt(10) - 4) + 110/9/abs(6*x - 2*sqrt(10) - 4) + 17/18)/sqrt(17*sqrt(10) + 5
5) + sqrt(10)*log(-2/9*sqrt(10) + 2*sqrt(2*x^2 + 3*x + 1)*sqrt(-17/9*sqrt(1
0) + 55/9)/abs(6*x + 2*sqrt(10) - 4) - 34/9*sqrt(10)/abs(6*x + 2*sqrt(10) -
```

```
 4) + 110/9/abs(6*x + 2*sqrt(10) - 4) + 17/18)/sqrt(-17/9*sqrt(10) + 55/9)
+ 24*log(2/9*sqrt(10) + 2/3*sqrt(2*x^2 + 3*x + 1)*sqrt(17*sqrt(10) + 55)/ab
s(6*x - 2*sqrt(10) - 4) + 34/9*sqrt(10)/abs(6*x - 2*sqrt(10) - 4) + 110/9/a
bs(6*x - 2*sqrt(10) - 4) + 17/18)/sqrt(17*sqrt(10) + 55) - 8*log(-2/9*sqrt(
10) + 2*sqrt(2*x^2 + 3*x + 1)*sqrt(-17/9*sqrt(10) + 55/9)/abs(6*x + 2*sqrt(
10) - 4) - 34/9*sqrt(10)/abs(6*x + 2*sqrt(10) - 4) + 110/9/abs(6*x + 2*sqrt
(10) - 4) + 17/18)/sqrt(-17/9*sqrt(10) + 55/9))
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.62

$$\int \frac{2+x}{(2+4x-3x^2)\sqrt{1+3x+2x^2}}\,dx = 0.169235232112667 \log\left(-\sqrt{2}x+\sqrt{2\,x^2+3\,x+1}+5.90976932712000\right) - 0.686556214893333 \log\left(-\sqrt{2}x+\sqrt{2\,x^2+3\,x+1}-0.176527156327000\right) + 0.686556214893333 \log\left(-\sqrt{2}x+\sqrt{2\,x^2+3\,x+1}-0.919278730509000\right) - 0.169235232112667 \log\left(-\sqrt{2}x+\sqrt{2\,x^2+3\,x+1}-1.04272727395000\right)$$

```
[In] integrate((2+x)/(-3*x^2+4*x+2)/(2*x^2+3*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] 0.169235232112667*log(-sqrt(2)*x + sqrt(2*x^2 + 3*x + 1) + 5.90976932712000
) - 0.686556214893333*log(-sqrt(2)*x + sqrt(2*x^2 + 3*x + 1) - 0.1765271563
27000) + 0.686556214893333*log(-sqrt(2)*x + sqrt(2*x^2 + 3*x + 1) - 0.91927
8730509000) - 0.169235232112667*log(-sqrt(2)*x + sqrt(2*x^2 + 3*x + 1) - 1.
04272727395000)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2+x}{(2+4x-3x^2)\sqrt{1+3x+2x^2}}\,dx = \int \frac{x+2}{\sqrt{2\,x^2+3\,x+1}\,\left(-3\,x^2+4\,x+2\right)}\,dx$$

```
[In] int((x + 2)/((3*x + 2*x^2 + 1)^(1/2)*(4*x - 3*x^2 + 2)),x)
[Out] int((x + 2)/((3*x + 2*x^2 + 1)^(1/2)*(4*x - 3*x^2 + 2)), x)
```

3.29 $\int \frac{2+x}{(2+4x-3x^2)(1+3x+2x^2)^{3/2}}\, dx$

Optimal result

Integrand size = 30, antiderivative size = 174

$$\begin{aligned}\int \frac{2+x}{(2+4x-3x^2)(1+3x+2x^2)^{3/2}}\, dx &= \frac{2(21+22x)}{5\sqrt{1+3x+2x^2}} \\ &- \frac{1}{10}\sqrt{\frac{3}{5}\left(2065+653\sqrt{10}\right)}\operatorname{arctanh}\left(\frac{3(4-\sqrt{10})+(17-4\sqrt{10})\,x}{2\sqrt{55-17\sqrt{10}}\sqrt{1+3x+2x^2}}\right) \\ &+ \frac{1}{10}\sqrt{\frac{3}{5}\left(2065-653\sqrt{10}\right)}\operatorname{arctanh}\left(\frac{3(4+\sqrt{10})+(17+4\sqrt{10})\,x}{2\sqrt{55+17\sqrt{10}}\sqrt{1+3x+2x^2}}\right)\end{aligned}$$

```
[Out] 2/5*(21+22*x)/(2*x^2+3*x+1)^(1/2)+1/50*arctanh(1/2*(12+3*10^(1/2)+x*(17+4*1
0^(1/2)))/(2*x^2+3*x+1)^(1/2)/(55+17*10^(1/2))^(1/2))*(30975-9795*10^(1/2))
^(1/2)-1/50*arctanh(1/2*(x*(17-4*10^(1/2))+12-3*10^(1/2))/(2*x^2+3*x+1)^(1/
2)/(55-17*10^(1/2))^(1/2))*(30975+9795*10^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1030, 1046, 738, 212}

$$\int \frac{2+x}{(2+4x-3x^2)(1+3x+2x^2)^{3/2}}\,dx = -\frac{1}{10}\sqrt{\frac{3}{5}\left(2065+653\sqrt{10}\right)}\text{arctanh}\left(\frac{\left(17-4\sqrt{10}\right)x+3\left(4-\sqrt{10}\right)}{2\sqrt{55-17\sqrt{10}}\sqrt{2x^2+3x+1}}\right)+\frac{1}{10}\sqrt{\frac{3}{5}\left(2065-653\sqrt{10}\right)}\text{arctanh}\left(\frac{\left(17+4\sqrt{10}\right)x+3\left(4+\sqrt{10}\right)}{2\sqrt{55+17\sqrt{10}}\sqrt{2x^2+3x+1}}\right)+\frac{2(22x+21)}{5\sqrt{2x^2+3x+1}}$$

```
[In] Int[(2 + x)/((2 + 4*x - 3*x^2)*(1 + 3*x + 2*x^2)^(3/2)),x]
```

```
[Out] (2*(21 + 22*x))/(5*Sqrt[1 + 3*x + 2*x^2]) - (Sqrt[(3*(2065 + 653*Sqrt[10]))
/5]*ArcTanh[(3*(4 - Sqrt[10]) + (17 - 4*Sqrt[10])*x)/(2*Sqrt[55 - 17*Sqrt[1
0]]*Sqrt[1 + 3*x + 2*x^2])])/10 + (Sqrt[(3*(2065 - 653*Sqrt[10]))/5]*ArcTan
h[(3*(4 + Sqrt[10]) + (17 + 4*Sqrt[10])*x)/(2*Sqrt[55 + 17*Sqrt[10]]*Sqrt[1
 + 3*x + 2*x^2])])/10
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1030

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x + c*x^2)^(p + 1)*
((d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
 - b*f))*(p + 1)))*(g*c*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d +
b^2*f - c*(b*e + 2*a*f)) + c*(g*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - h*(b*
c*d - 2*a*c*e + a*b*f))*x), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*
d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*
```

```
x^2)^q*Simp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)
 + (b^2*(g*f) - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*((-h)*c*
e)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((g*c)*(2*a*c*e - b*(c*d + a*f)) + (g*
b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((g*c)*(2*
a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(
p + q + 2) - (b^2*g*f - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*
((-h)*c*e)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(g*f) - b*(h*c
*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) + a*h*c*e))*(2*p + 2*q + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e
 - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
```

Rule 1046

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\text{integral} = \frac{2(21+22x)}{5\sqrt{1+3x+2x^2}} - \frac{2}{15}\int \frac{-72+\frac{81x}{2}}{(2+4x-3x^2)\sqrt{1+3x+2x^2}}\,dx$$

$$= \frac{2(21+22x)}{5\sqrt{1+3x+2x^2}} - \frac{1}{5}\left(9\left(3-\sqrt{10}\right)\right)\int \frac{1}{\left(4+2\sqrt{10}-6x\right)\sqrt{1+3x+2x^2}}\,dx$$
$$- \frac{1}{5}\left(9\left(3+\sqrt{10}\right)\right)\int \frac{1}{\left(4-2\sqrt{10}-6x\right)\sqrt{1+3x+2x^2}}\,dx$$

$$= \frac{2(21+22x)}{5\sqrt{1+3x+2x^2}} + \frac{1}{5}\Big(18\Big(3$$
$$-\sqrt{10}\Big)\Big)\,\text{Subst}\left(\int \frac{1}{144+72\left(4+2\sqrt{10}\right)+8\left(4+2\sqrt{10}\right)^2-x^2}\,dx, x, \frac{-12-3\left(4+2\sqrt{10}\right)-(18}{\sqrt{1+3x+2}}\right.$$
$$+\frac{1}{5}\Big(18\Big(3$$
$$+\sqrt{10}\Big)\Big)\,\text{Subst}\left(\int \frac{1}{144+72\left(4-2\sqrt{10}\right)+8\left(4-2\sqrt{10}\right)^2-x^2}\,dx, x, \frac{-12-3\left(4-2\sqrt{10}\right)-(18}{\sqrt{1+3x+2}}\right.$$

$$= \frac{2(21+22x)}{5\sqrt{1+3x+2x^2}} - \frac{1}{10}\sqrt{\frac{3}{5}\left(2065+653\sqrt{10}\right)}\tanh^{-1}\left(\frac{3\left(4-\sqrt{10}\right)+\left(17-4\sqrt{10}\right)x}{2\sqrt{55-17\sqrt{10}}\sqrt{1+3x+2x^2}}\right)$$
$$+\frac{1}{10}\sqrt{\frac{3}{5}\left(2065-653\sqrt{10}\right)}\tanh^{-1}\left(\frac{3\left(4+\sqrt{10}\right)+\left(17+4\sqrt{10}\right)x}{2\sqrt{55+17\sqrt{10}}\sqrt{1+3x+2x^2}}\right)$$

Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.75

$$\int \frac{2+x}{(2+4x-3x^2)(1+3x+2x^2)^{3/2}}\,dx = \frac{1}{25}\left(\frac{5(42+44x)}{\sqrt{1+3x+2x^2}} - \sqrt{30975+9795\sqrt{10}}\operatorname{arctanh}\left(\frac{\sqrt{1-\sqrt{\frac{2}{5}}}\sqrt{1+3x+2x^2}}{1+2x}\right) + \frac{45\operatorname{arctanh}\left(\frac{\sqrt{1+\sqrt{\frac{2}{5}}}\sqrt{1+3x+2x^2}}{1+2x}\right)}{\sqrt{2065+653\sqrt{10}}}\right)$$

```
[In] Integrate[(2 + x)/((2 + 4*x - 3*x^2)*(1 + 3*x + 2*x^2)^(3/2)),x]
```

```
[Out] ((5*(42 + 44*x))/Sqrt[1 + 3*x + 2*x^2] - Sqrt[30975 + 9795*Sqrt[10]]*ArcTan
h[(Sqrt[1 - Sqrt[2/5]]*Sqrt[1 + 3*x + 2*x^2])/(1 + 2*x)] + (45*ArcTanh[(Sqr
t[1 + Sqrt[2/5]]*Sqrt[1 + 3*x + 2*x^2])/(1 + 2*x)])/Sqrt[2065 + 653*Sqrt[10
]])/25
```

Maple [A] (verified)

Time = 2.46 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.20

method	result
risch	$\frac{\frac{42}{5}+\frac{44x}{5}}{\sqrt{2x^2+3x+1}} - \frac{9\left(10+3\sqrt{10}\right)\sqrt{10}\,\mathrm{arctanh}\left(\frac{55-17\sqrt{10}+\frac{9\left(\frac{17}{3}-\frac{4\sqrt{10}}{3}\right)\left(x-\frac{2}{3}+\frac{\sqrt{10}}{3}\right)}{2}}{\sqrt{55-17\sqrt{10}}\sqrt{18\left(x-\frac{2}{3}+\frac{\sqrt{10}}{3}\right)^2+9\left(\frac{17}{3}-\frac{4\sqrt{10}}{3}\right)\left(x-\frac{2}{3}+\frac{\sqrt{10}}{3}\right)+55-17\sqrt{10}}}\right)}{100\sqrt{55-17\sqrt{10}}} - 9\left(-10+3\sqrt{\cdots}\right.$
trager	$\frac{\frac{42}{5}+\frac{44x}{5}}{\sqrt{2x^2+3x+1}} - \frac{\mathrm{RootOf}\left(\_Z^2+144\,\mathrm{RootOf}\left(3840\_Z^4-66080\_Z^2+9\right)^2-2478\right)\ln\left(\frac{928000x\,\mathrm{RootOf}\left(3840\_Z^4-66080\_Z^2+9\right)^4\cdots}{\cdots}\right.}{\cdots}$
default	$-\frac{\left(8+\sqrt{10}\right)\sqrt{10}\left(\frac{1}{3\left(\frac{55}{9}+\frac{17\sqrt{10}}{9}\right)\sqrt{2\left(x-\frac{2}{3}-\frac{\sqrt{10}}{3}\right)^2+\left(\frac{17}{3}+\frac{4\sqrt{10}}{3}\right)\left(x-\frac{2}{3}-\frac{\sqrt{10}}{3}\right)+\frac{55}{9}+\frac{17\sqrt{10}}{9}}}-\frac{\cdots}{3\left(\frac{55}{9}+\frac{17\sqrt{10}}{9}\right)\left(\frac{440}{9}+\frac{136\sqrt{10}}{9}-\left(\frac{17}{3}+\frac{4\sqrt{10}}{3}\cdots\right.\right.}\right.}{\cdots}$

```
[In] int((2+x)/(-3*x^2+4*x+2)/(2*x^2+3*x+1)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/5*(21+22*x)/(2*x^2+3*x+1)^(1/2)-9/100*(10+3*10^(1/2))*10^(1/2)/(55-17*10^
(1/2))^(1/2)*arctanh(9/2*(110/9-34/9*10^(1/2)+(17/3-4/3*10^(1/2))*(x-2/3+1/
3*10^(1/2)))/(55-17*10^(1/2))^(1/2)/(18*(x-2/3+1/3*10^(1/2))^2+9*(17/3-4/3*
10^(1/2))*(x-2/3+1/3*10^(1/2))+55-17*10^(1/2))^(1/2))-9/100*(-10+3*10^(1/2)
)*10^(1/2)/(55+17*10^(1/2))^(1/2)*arctanh(9/2*(110/9+34/9*10^(1/2)+(17/3+4/
3*10^(1/2))*(x-2/3-1/3*10^(1/2)))/(55+17*10^(1/2))^(1/2)/(18*(x-2/3-1/3*10^
(1/2))^2+9*(17/3+4/3*10^(1/2))*(x-2/3-1/3*10^(1/2))+55+17*10^(1/2))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 365 vs. $2(122) = 244$.

Time = 0.32 (sec) , antiderivative size = 365, normalized size of antiderivative = 2.10

$$\int \frac{2+x}{\left(2+4x-3x^2\right)\left(1+3x+2x^2\right)^{3/2}}\,dx = \frac{\sqrt{5}\left(2\,x^2+3\,x+1\right)\sqrt{1959\,\sqrt{10}+6195}\,\log\left(-\frac{45\,\sqrt{10}x+\left(41\,\sqrt{10}\sqrt{5}x-130\cdots\right.}{\cdots}\right.}{\cdots}$$

```
[In] integrate((2+x)/(-3*x^2+4*x+2)/(2*x^2+3*x+1)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/50*(sqrt(5)*(2*x^2 + 3*x + 1)*sqrt(1959*sqrt(10) + 6195)*log(-(45*sqrt(10
)*x + (41*sqrt(10)*sqrt(5)*x - 130*sqrt(5)*x)*sqrt(1959*sqrt(10) + 6195) +
90*x - 90*sqrt(2*x^2 + 3*x + 1) + 90)/x) - sqrt(5)*(2*x^2 + 3*x + 1)*sqrt(1
959*sqrt(10) + 6195)*log(-(45*sqrt(10)*x - (41*sqrt(10)*sqrt(5)*x - 130*sqr
t(5)*x)*sqrt(1959*sqrt(10) + 6195) + 90*x - 90*sqrt(2*x^2 + 3*x + 1) + 90)/
x) + sqrt(5)*(2*x^2 + 3*x + 1)*sqrt(-1959*sqrt(10) + 6195)*log((45*sqrt(10)
*x + (41*sqrt(10)*sqrt(5)*x + 130*sqrt(5)*x)*sqrt(-1959*sqrt(10) + 6195) -
```

```
90*x + 90*sqrt(2*x^2 + 3*x + 1) - 90)/x) - sqrt(5)*(2*x^2 + 3*x + 1)*sqrt(-
1959*sqrt(10) + 6195)*log((45*sqrt(10)*x - (41*sqrt(10)*sqrt(5)*x + 130*sqr
t(5)*x)*sqrt(-1959*sqrt(10) + 6195) - 90*x + 90*sqrt(2*x^2 + 3*x + 1) - 90)
/x) + 840*x^2 + 20*sqrt(2*x^2 + 3*x + 1)*(22*x + 21) + 1260*x + 420)/(2*x^2
 + 3*x + 1)
```

Sympy [F]

$$\int \frac{2+x}{(2+4x-3x^2)(1+3x+2x^2)^{3/2}}\,dx = -\int \frac{x}{6x^4\sqrt{2x^2+3x+1}+x^3\sqrt{2x^2+3x+1}-13x^2\sqrt{2x^2+3x+1}-10x\sqrt{2x^2+3x+1}-2\sqrt{2x^2+3x+1}} - \int \frac{2}{6x^4\sqrt{2x^2+3x+1}+x^3\sqrt{2x^2+3x+1}-13x^2\sqrt{2x^2+3x+1}-10x\sqrt{2x^2+3x+1}-2\sqrt{2x^2+3x+1}}$$

```
[In] integrate((2+x)/(-3*x**2+4*x+2)/(2*x**2+3*x+1)**(3/2),x)
```

```
[Out] -Integral(x/(6*x**4*sqrt(2*x**2 + 3*x + 1) + x**3*sqrt(2*x**2 + 3*x + 1) -
13*x**2*sqrt(2*x**2 + 3*x + 1) - 10*x*sqrt(2*x**2 + 3*x + 1) - 2*sqrt(2*x**
2 + 3*x + 1)), x) - Integral(2/(6*x**4*sqrt(2*x**2 + 3*x + 1) + x**3*sqrt(2
*x**2 + 3*x + 1) - 13*x**2*sqrt(2*x**2 + 3*x + 1) - 10*x*sqrt(2*x**2 + 3*x
+ 1) - 2*sqrt(2*x**2 + 3*x + 1)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 668 vs. $2(122) = 244$.

Time = 0.30 (sec) , antiderivative size = 668, normalized size of antiderivative = 3.84

$$\int \frac{2+x}{(2+4x-3x^2)(1+3x+2x^2)^{3/2}}\,dx = -\frac{1}{60}\sqrt{10}\left(\frac{588\sqrt{10}x}{17\sqrt{10}\sqrt{2x^2+3x+1}+55\sqrt{2x^2+3x+1}} - \frac{588\sqrt{10}x}{17\sqrt{10}\sqrt{2x^2+3x+1}-55\sqrt{2x^2+3x+1}} + \right.$$

```
[In] integrate((2+x)/(-3*x^2+4*x+2)/(2*x^2+3*x+1)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/60*sqrt(10)*(588*sqrt(10)*x/(17*sqrt(10)*sqrt(2*x^2 + 3*x + 1) + 55*sqrt
(2*x^2 + 3*x + 1)) - 588*sqrt(10)*x/(17*sqrt(10)*sqrt(2*x^2 + 3*x + 1) - 55
*sqrt(2*x^2 + 3*x + 1)) + 2112*x/(17*sqrt(10)*sqrt(2*x^2 + 3*x + 1) + 55*sq
rt(2*x^2 + 3*x + 1)) + 2112*x/(17*sqrt(10)*sqrt(2*x^2 + 3*x + 1) - 55*sqrt(
2*x^2 + 3*x + 1)) - 27*sqrt(10)*log(2/9*sqrt(10) + 2/3*sqrt(2*x^2 + 3*x + 1
```

```
)*sqrt(17*sqrt(10) + 55)/abs(6*x - 2*sqrt(10) - 4) + 34/9*sqrt(10)/abs(6*x
- 2*sqrt(10) - 4) + 110/9/abs(6*x - 2*sqrt(10) - 4) + 17/18)/(17*sqrt(10) +
 55)^(3/2) - sqrt(10)*log(-2/9*sqrt(10) + 2*sqrt(2*x^2 + 3*x + 1)*sqrt(-17/
9*sqrt(10) + 55/9)/abs(6*x + 2*sqrt(10) - 4) - 34/9*sqrt(10)/abs(6*x + 2*sq
rt(10) - 4) + 110/9/abs(6*x + 2*sqrt(10) - 4) + 17/18)/(-17/9*sqrt(10) + 55
/9)^(3/2) + 450*sqrt(10)/(17*sqrt(10)*sqrt(2*x^2 + 3*x + 1) + 55*sqrt(2*x^2
 + 3*x + 1)) - 450*sqrt(10)/(17*sqrt(10)*sqrt(2*x^2 + 3*x + 1) - 55*sqrt(2*
x^2 + 3*x + 1)) - 216*log(2/9*sqrt(10) + 2/3*sqrt(2*x^2 + 3*x + 1)*sqrt(17*
sqrt(10) + 55)/abs(6*x - 2*sqrt(10) - 4) + 34/9*sqrt(10)/abs(6*x - 2*sqrt(1
0) - 4) + 110/9/abs(6*x - 2*sqrt(10) - 4) + 17/18)/(17*sqrt(10) + 55)^(3/2)
 + 8*log(-2/9*sqrt(10) + 2*sqrt(2*x^2 + 3*x + 1)*sqrt(-17/9*sqrt(10) + 55/9
)/abs(6*x + 2*sqrt(10) - 4) - 34/9*sqrt(10)/abs(6*x + 2*sqrt(10) - 4) + 110
/9/abs(6*x + 2*sqrt(10) - 4) + 17/18)/(-17/9*sqrt(10) + 55/9)^(3/2) + 1656/
(17*sqrt(10)*sqrt(2*x^2 + 3*x + 1) + 55*sqrt(2*x^2 + 3*x + 1)) + 1656/(17*s
qrt(10)*sqrt(2*x^2 + 3*x + 1) - 55*sqrt(2*x^2 + 3*x + 1)))
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.64

$$\int \frac{2+x}{(2+4x-3x^2)\,(1+3x+2x^2)^{3/2}}\,dx = \frac{2\,(22\,x+21)}{5\,\sqrt{2\,x^2+3\,x+1}} + 0.0140045514133333\,\log\left(-\sqrt{2}x+\sqrt{2\,x^2+3\,x+1}+5.90976932712000\right) - 4.97793168620000\,\log\left(-\sqrt{2}x+\sqrt{2\,x^2+3\,x+1}-0.176527156327000\right) + 4.97793168620000\,\log\left(-\sqrt{2}x+\sqrt{2\,x^2+3\,x+1}-0.919278730509000\right) - 0.0140045514125333\,\log\left(-\sqrt{2}x+\sqrt{2\,x^2+3\,x+1}-1.04272727395000\right)$$

```
[In] integrate((2+x)/(-3*x^2+4*x+2)/(2*x^2+3*x+1)^(3/2),x, algorithm="giac")
```

```
[Out] 2/5*(22*x + 21)/sqrt(2*x^2 + 3*x + 1) + 0.0140045514133333*log(-sqrt(2)*x +
 sqrt(2*x^2 + 3*x + 1) + 5.90976932712000) - 4.97793168620000*log(-sqrt(2)*
x + sqrt(2*x^2 + 3*x + 1) - 0.176527156327000) + 4.97793168620000*log(-sqrt
(2)*x + sqrt(2*x^2 + 3*x + 1) - 0.919278730509000) - 0.0140045514125333*log
(-sqrt(2)*x + sqrt(2*x^2 + 3*x + 1) - 1.04272727395000)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2+x}{(2+4x-3x^2)\,(1+3x+2x^2)^{3/2}}\,dx = \int \frac{x+2}{\left(2\,x^2+3\,x+1\right)^{3/2}\,\left(-3\,x^2+4\,x+2\right)}\,dx$$

```
[In] int((x + 2)/((3*x + 2*x^2 + 1)^(3/2)*(4*x - 3*x^2 + 2)),x)
[Out] int((x + 2)/((3*x + 2*x^2 + 1)^(3/2)*(4*x - 3*x^2 + 2)), x)
```

3.30 $\int \frac{2+x}{(2+4x-3x^2)(1+3x+2x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 197

$$\int \frac{2+x}{(2+4x-3x^2)(1+3x+2x^2)^{5/2}}\,dx = \frac{2(21+22x)}{15(1+3x+2x^2)^{3/2}} + \frac{2(273+230x)}{15\sqrt{1+3x+2x^2}} - \frac{1}{50}\sqrt{\frac{1}{3}\left(4885115+1544809\sqrt{10}\right)}\,\mathrm{arctanh}\left(\frac{3(4-\sqrt{10})+(17-4\sqrt{10})\,x}{2\sqrt{55-17\sqrt{10}}\sqrt{1+3x+2x^2}}\right) + \frac{1}{50}\sqrt{\frac{1}{3}\left(4885115-1544809\sqrt{10}\right)}\,\mathrm{arctanh}\left(\frac{3(4+\sqrt{10})+(17+4\sqrt{10})\,x}{2\sqrt{55+17\sqrt{10}}\sqrt{1+3x+2x^2}}\right)$$

```
[Out] 2/15*(21+22*x)/(2*x^2+3*x+1)^(3/2)+2/15*(273+230*x)/(2*x^2+3*x+1)^(1/2)+1/1
50*arctanh(1/2*(12+3*10^(1/2)+x*(17+4*10^(1/2)))/(2*x^2+3*x+1)^(1/2)/(55+17
*10^(1/2))^(1/2))*(14655345-4634427*10^(1/2))^(1/2)-1/150*arctanh(1/2*(x*(1
7-4*10^(1/2))+12-3*10^(1/2))/(2*x^2+3*x+1)^(1/2)/(55-17*10^(1/2))^(1/2))*(1
4655345+4634427*10^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used

= {1030, 1074, 1046, 738, 212}

$$\int \frac{2+x}{(2+4x-3x^2)(1+3x+2x^2)^{5/2}}\,dx = -\frac{1}{50}\sqrt{\frac{1}{3}\left(4885115+1544809\sqrt{10}\right)}\operatorname{arctanh}\left(\frac{(17-4\sqrt{10})\,x+3(4-\sqrt{10})}{2\sqrt{55-17\sqrt{10}}\sqrt{2x^2+3x+1}}\right) + \frac{1}{50}\sqrt{\frac{1}{3}\left(4885115-1544809\sqrt{10}\right)}\operatorname{arctanh}\left(\frac{(17+4\sqrt{10})\,x+3(4+\sqrt{10})}{2\sqrt{55+17\sqrt{10}}\sqrt{2x^2+3x+1}}\right) + \frac{2(22x+21)}{15\,(2x^2+3x+1)^{3/2}} + \frac{2(230x+273)}{15\sqrt{2x^2+3x+1}}$$

```
[In] Int[(2 + x)/((2 + 4*x - 3*x^2)*(1 + 3*x + 2*x^2)^(5/2)),x]
```

```
[Out] (2*(21 + 22*x))/(15*(1 + 3*x + 2*x^2)^(3/2)) + (2*(273 + 230*x))/(15*Sqrt[1
 + 3*x + 2*x^2]) - (Sqrt[(4885115 + 1544809*Sqrt[10])/3]*ArcTanh[(3*(4 - Sq
rt[10]) + (17 - 4*Sqrt[10])*x)/(2*Sqrt[55 - 17*Sqrt[10]]*Sqrt[1 + 3*x + 2*x
^2])])/50 + (Sqrt[(4885115 - 1544809*Sqrt[10])/3]*ArcTanh[(3*(4 + Sqrt[10])
 + (17 + 4*Sqrt[10])*x)/(2*Sqrt[55 + 17*Sqrt[10]]*Sqrt[1 + 3*x + 2*x^2])])/
50
```

<u>Rule 212</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

<u>Rule 738</u>

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

<u>Rule 1030</u>

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x + c*x^2)^(p + 1)*
((d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
 - b*f))*(p + 1)))*(g*c*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d +
b^2*f - c*(b*e + 2*a*f)) + c*(g*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - h*(b*
c*d - 2*a*c*e + a*b*f))*x), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*
d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*
x^2)^q*Simp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)
 + (b^2*(g*f) - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*((-h)*c*
e)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((g*c)*(2*a*c*e - b*(c*d + a*f)) + (g*
```

```
b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((g*c)*(2*
a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(
p + q + 2) - (b^2*g*f - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*
((-h)*c*e)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(g*f) - b*(h*c
*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) + a*h*c*e))*(2*p + 2*q + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e
 - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
```

Rule 1046

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1074

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x +
c*x^2)^(p + 1)*((d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f))*(p + 1)))*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) +
(A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c
*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d
 - a*f)))*x), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rubi steps

$$\text{integral} = \frac{2(21+22x)}{15\left(1+3x+2x^2\right)^{3/2}} - \frac{2}{45}\int \frac{-480-\frac{813x}{2}+396x^2}{\left(2+4x-3x^2\right)\left(1+3x+2x^2\right)^{3/2}}\,dx$$

$$= \frac{2(21+22x)}{15\left(1+3x+2x^2\right)^{3/2}} + \frac{2(273+230x)}{15\sqrt{1+3x+2x^2}} + \frac{4}{675}\int \frac{\frac{23355}{2}-\frac{27135x}{4}}{\left(2+4x-3x^2\right)\sqrt{1+3x+2x^2}}\,dx$$

$$= \frac{2(21+22x)}{15\left(1+3x+2x^2\right)^{3/2}} + \frac{2(273+230x)}{15\sqrt{1+3x+2x^2}} - \frac{1}{25}\left(3\left(335-106\sqrt{10}\right)\right)\int \frac{1}{\left(4+2\sqrt{10}-6x\right)\sqrt{1+3x+2x^2}}\,dx - \frac{1}{25}\left(3\left(335+106\sqrt{10}\right)\right)\int \frac{1}{\left(4-2\sqrt{10}-6x\right)\sqrt{1+3x+2x^2}}\,dx$$

$$= \frac{2(21+22x)}{15\left(1+3x+2x^2\right)^{3/2}} + \frac{2(273+230x)}{15\sqrt{1+3x+2x^2}} + \frac{1}{25}\left(6\left(335-106\sqrt{10}\right)\right)\text{Subst}\left(\int \frac{1}{144+72\left(4+2\sqrt{10}\right)+8\left(4+2\sqrt{10}\right)^2-x^2}\,dx, x, \frac{-12-3(4}{}\right.$$
$$+ \frac{1}{25}\left(6\left(335+106\sqrt{10}\right)\right)\text{Subst}\left(\int \frac{1}{144+72\left(4-2\sqrt{10}\right)+8\left(4-2\sqrt{10}\right)^2-x^2}\,dx, x, \frac{-12-3(4}{}\right.$$

$$= \frac{2(21+22x)}{15\left(1+3x+2x^2\right)^{3/2}} + \frac{2(273+230x)}{15\sqrt{1+3x+2x^2}} - \frac{1}{50}\sqrt{\frac{1}{3}\left(4885115+1544809\sqrt{10}\right)}\tanh^{-1}\left(\frac{3\left(4-\sqrt{10}\right)+\left(17-4\sqrt{10}\right)x}{2\sqrt{55-17\sqrt{10}}\sqrt{1+3x+2x^2}}\right) + \frac{1}{50}\sqrt{\frac{1}{3}\left(4885115-1544809\sqrt{10}\right)}\tanh^{-1}\left(\frac{3\left(4+\sqrt{10}\right)+\left(17+4\sqrt{10}\right)x}{2\sqrt{55+17\sqrt{10}}\sqrt{1+3x+2x^2}}\right)$$

Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.78

$$\int \frac{2+x}{\left(2+4x-3x^2\right)\left(1+3x+2x^2\right)^{5/2}}\,dx = \frac{2\sqrt{1+3x+2x^2}\left(294+1071x+1236x^2+460x^3\right)}{15(1+x)^2(1+2x)^2} - \frac{1}{75}\sqrt{14655345+4634427\sqrt{10}}\,\text{arctanh}\left(\frac{\sqrt{1-\sqrt{\frac{2}{5}}}\sqrt{1+3x+2x^2}}{1+2x}\right) + \frac{81\,\text{arctanh}\left(\frac{\sqrt{1+\sqrt{\frac{2}{5}}}\sqrt{1+3x+2x^2}}{1+2x}\right)}{5\sqrt{24425575+7724045\sqrt{10}}}$$

```
[In] Integrate[(2 + x)/((2 + 4*x - 3*x^2)*(1 + 3*x + 2*x^2)^(5/2)),x]
```

```
[Out] (2*Sqrt[1 + 3*x + 2*x^2]*(294 + 1071*x + 1236*x^2 + 460*x^3))/(15*(1 + x)^2
*(1 + 2*x)^2) - (Sqrt[14655345 + 4634427*Sqrt[10]]*ArcTanh[(Sqrt[1 - Sqrt[2
/5]]*Sqrt[1 + 3*x + 2*x^2])/(1 + 2*x)])/75 + (81*ArcTanh[(Sqrt[1 + Sqrt[2/5
]]*Sqrt[1 + 3*x + 2*x^2])/(1 + 2*x)])/(5*Sqrt[24425575 + 7724045*Sqrt[10]])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.52 (sec) , antiderivative size = 471, normalized size of antiderivative = 2.39

method	result
trager	$\frac{\frac{184}{3}x^3+\frac{824}{5}x^2+\frac{714}{5}x+\frac{196}{5}}{(2x^2+3x+1)^{\frac{3}{2}}}+\frac{\mathrm{RootOf}\left(\_Z^2+3600\,\mathrm{RootOf}\left(96000\_Z^4-781618400\_Z^2+6561\right)^2-29310690\right)\ln\left(\frac{-504352000x\,\mathrm{Ro}\ldots}{}\right.}{}$
default	Expression too large to display

```
[In] int((2+x)/(-3*x^2+4*x+2)/(2*x^2+3*x+1)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15*(460*x^3+1236*x^2+1071*x+294)/(2*x^2+3*x+1)^(3/2)+1/150*RootOf(_Z^2+36
00*RootOf(96000*_Z^4-781618400*_Z^2+6561)^2-29310690)*ln((-504352000*x*Root
Of(96000*_Z^4-781618400*_Z^2+6561)^4*RootOf(_Z^2+3600*RootOf(96000*_Z^4-781
618400*_Z^2+6561)^2-29310690)+20232850257120*RootOf(96000*_Z^4-781618400*_Z
^2+6561)^2*RootOf(_Z^2+3600*RootOf(96000*_Z^4-781618400*_Z^2+6561)^2-293106
90)*x+1601657971200*(2*x^2+3*x+1)^(1/2)*RootOf(96000*_Z^4-781618400*_Z^2+65
61)^2+11686912631520*RootOf(96000*_Z^4-781618400*_Z^2+6561)^2*RootOf(_Z^2+3
600*RootOf(96000*_Z^4-781618400*_Z^2+6561)^2-29310690)-1087910594433*RootOf
(_Z^2+3600*RootOf(96000*_Z^4-781618400*_Z^2+6561)^2-29310690)*x+30159574965
55836*(2*x^2+3*x+1)^(1/2)-628400494638*RootOf(_Z^2+3600*RootOf(96000*_Z^4-7
81618400*_Z^2+6561)^2-29310690))/(1200*x*RootOf(96000*_Z^4-781618400*_Z^2+6
561)^2-7974733*x-3089618))-2/5*RootOf(96000*_Z^4-781618400*_Z^2+6561)*ln(-(
-22695840000*RootOf(96000*_Z^4-781618400*_Z^2+6561)^5*x-540905633498400*Roo
tOf(96000*_Z^4-781618400*_Z^2+6561)^3*x+1201243478400*(2*x^2+3*x+1)^(1/2)*R
ootOf(96000*_Z^4-781618400*_Z^2+6561)^2-525911068418400*RootOf(96000*_Z^4-7
81618400*_Z^2+6561)^3+5908432074489101275*RootOf(96000*_Z^4-781618400*_Z^2+
6561)*x-12042322347390237*(2*x^2+3*x+1)^(1/2)+4281865136972328150*RootOf(96
000*_Z^4-781618400*_Z^2+6561))/(1200*x*RootOf(96000*_Z^4-781618400*_Z^2+656
1)^2-1795497*x+3089618))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 435 vs. $2(141) = 282$.

Time = 0.30 (sec) , antiderivative size = 435, normalized size of antiderivative = 2.21

$$\int \frac{2+x}{(2+4x-3x^2)(1+3x+2x^2)^{5/2}}\,dx = \frac{23520\,x^4+70560\,x^3+\sqrt{3}(4\,x^4+12\,x^3+13\,x^2+6\,x+1)\sqrt{15448}}{}$$

```
[In] integrate((2+x)/(-3*x^2+4*x+2)/(2*x^2+3*x+1)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/150*(23520*x^4 + 70560*x^3 + sqrt(3)*(4*x^4 + 12*x^3 + 13*x^2 + 6*x + 1)*
sqrt(1544809*sqrt(10) + 4885115)*log(-(243*sqrt(10)*x + (893*sqrt(10)*sqrt(
3)*x - 2824*sqrt(3)*x)*sqrt(1544809*sqrt(10) + 4885115) + 486*x - 486*sqrt(
2*x^2 + 3*x + 1) + 486)/x) - sqrt(3)*(4*x^4 + 12*x^3 + 13*x^2 + 6*x + 1)*sq
rt(1544809*sqrt(10) + 4885115)*log(-(243*sqrt(10)*x - (893*sqrt(10)*sqrt(3)
*x - 2824*sqrt(3)*x)*sqrt(1544809*sqrt(10) + 4885115) + 486*x - 486*sqrt(2*
x^2 + 3*x + 1) + 486)/x) + sqrt(3)*(4*x^4 + 12*x^3 + 13*x^2 + 6*x + 1)*sqrt
(-1544809*sqrt(10) + 4885115)*log((243*sqrt(10)*x + (893*sqrt(10)*sqrt(3)*x
 + 2824*sqrt(3)*x)*sqrt(-1544809*sqrt(10) + 4885115) - 486*x + 486*sqrt(2*x
^2 + 3*x + 1) - 486)/x) - sqrt(3)*(4*x^4 + 12*x^3 + 13*x^2 + 6*x + 1)*sqrt(
-1544809*sqrt(10) + 4885115)*log((243*sqrt(10)*x - (893*sqrt(10)*sqrt(3)*x
+ 2824*sqrt(3)*x)*sqrt(-1544809*sqrt(10) + 4885115) - 486*x + 486*sqrt(2*x^
2 + 3*x + 1) - 486)/x) + 76440*x^2 + 20*(460*x^3 + 1236*x^2 + 1071*x + 294)
*sqrt(2*x^2 + 3*x + 1) + 35280*x + 5880)/(4*x^4 + 12*x^3 + 13*x^2 + 6*x + 1
)
```

Sympy [F]

$$\int \frac{2+x}{(2+4x-3x^2)(1+3x+2x^2)^{5/2}}\,dx = -\int \frac{x}{12x^6\sqrt{2x^2+3x+1}+20x^5\sqrt{2x^2+3x+1}-17x^4\sqrt{2x^2+3x+1}-58x^3\sqrt{2x^2+3x+1}-47x^2\sqrt{2x^2+}} - \int \frac{2}{12x^6\sqrt{2x^2+3x+1}+20x^5\sqrt{2x^2+3x+1}-17x^4\sqrt{2x^2+3x+1}-58x^3\sqrt{2x^2+3x+1}-47x^2\sqrt{2x^2+}}$$

```
[In] integrate((2+x)/(-3*x**2+4*x+2)/(2*x**2+3*x+1)**(5/2),x)
```

```
[Out] -Integral(x/(12*x**6*sqrt(2*x**2 + 3*x + 1) + 20*x**5*sqrt(2*x**2 + 3*x + 1
) - 17*x**4*sqrt(2*x**2 + 3*x + 1) - 58*x**3*sqrt(2*x**2 + 3*x + 1) - 47*x*
*2*sqrt(2*x**2 + 3*x + 1) - 16*x*sqrt(2*x**2 + 3*x + 1) - 2*sqrt(2*x**2 + 3
*x + 1)), x) - Integral(2/(12*x**6*sqrt(2*x**2 + 3*x + 1) + 20*x**5*sqrt(2*
x**2 + 3*x + 1) - 17*x**4*sqrt(2*x**2 + 3*x + 1) - 58*x**3*sqrt(2*x**2 + 3*
x + 1) - 47*x**2*sqrt(2*x**2 + 3*x + 1) - 16*x*sqrt(2*x**2 + 3*x + 1) - 2*s
qrt(2*x**2 + 3*x + 1)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1276 vs. 2(141) = 282.

Time = 0.32 (sec) , antiderivative size = 1276, normalized size of antiderivative = 6.48

$$\int \frac{2+x}{(2+4x-3x^2)(1+3x+2x^2)^{5/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((2+x)/(-3*x^2+4*x+2)/(2*x^2+3*x+1)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/300*sqrt(10)*(980*sqrt(10)*x/(17*sqrt(10)*(2*x^2 + 3*x + 1)^(3/2) + 55*(
2*x^2 + 3*x + 1)^(3/2)) - 980*sqrt(10)*x/(17*sqrt(10)*(2*x^2 + 3*x + 1)^(3/
2) - 55*(2*x^2 + 3*x + 1)^(3/2)) + 5292*sqrt(10)*x/(374*sqrt(10)*sqrt(2*x^2
 + 3*x + 1) + 1183*sqrt(2*x^2 + 3*x + 1)) - 5292*sqrt(10)*x/(374*sqrt(10)*s
qrt(2*x^2 + 3*x + 1) - 1183*sqrt(2*x^2 + 3*x + 1)) - 15680*sqrt(10)*x/(17*s
qrt(10)*sqrt(2*x^2 + 3*x + 1) + 55*sqrt(2*x^2 + 3*x + 1)) + 15680*sqrt(10)*
x/(17*sqrt(10)*sqrt(2*x^2 + 3*x + 1) - 55*sqrt(2*x^2 + 3*x + 1)) + 3520*x/(
17*sqrt(10)*(2*x^2 + 3*x + 1)^(3/2) + 55*(2*x^2 + 3*x + 1)^(3/2)) + 3520*x/
(17*sqrt(10)*(2*x^2 + 3*x + 1)^(3/2) - 55*(2*x^2 + 3*x + 1)^(3/2)) + 19008*
x/(374*sqrt(10)*sqrt(2*x^2 + 3*x + 1) + 1183*sqrt(2*x^2 + 3*x + 1)) + 19008
*x/(374*sqrt(10)*sqrt(2*x^2 + 3*x + 1) - 1183*sqrt(2*x^2 + 3*x + 1)) - 5632
0*x/(17*sqrt(10)*sqrt(2*x^2 + 3*x + 1) + 55*sqrt(2*x^2 + 3*x + 1)) - 56320*
x/(17*sqrt(10)*sqrt(2*x^2 + 3*x + 1) - 55*sqrt(2*x^2 + 3*x + 1)) + 750*sqrt
(10)/(17*sqrt(10)*(2*x^2 + 3*x + 1)^(3/2) + 55*(2*x^2 + 3*x + 1)^(3/2)) - 7
50*sqrt(10)/(17*sqrt(10)*(2*x^2 + 3*x + 1)^(3/2) - 55*(2*x^2 + 3*x + 1)^(3/
2)) + 4050*sqrt(10)/(374*sqrt(10)*sqrt(2*x^2 + 3*x + 1) + 1183*sqrt(2*x^2 +
 3*x + 1)) - 4050*sqrt(10)/(374*sqrt(10)*sqrt(2*x^2 + 3*x + 1) - 1183*sqrt(
2*x^2 + 3*x + 1)) - 11760*sqrt(10)/(17*sqrt(10)*sqrt(2*x^2 + 3*x + 1) + 55*
sqrt(2*x^2 + 3*x + 1)) + 11760*sqrt(10)/(17*sqrt(10)*sqrt(2*x^2 + 3*x + 1)
- 55*sqrt(2*x^2 + 3*x + 1)) + 2760/(17*sqrt(10)*(2*x^2 + 3*x + 1)^(3/2) + 5
5*(2*x^2 + 3*x + 1)^(3/2)) + 2760/(17*sqrt(10)*(2*x^2 + 3*x + 1)^(3/2) - 55
*(2*x^2 + 3*x + 1)^(3/2)) + 14904/(374*sqrt(10)*sqrt(2*x^2 + 3*x + 1) + 118
3*sqrt(2*x^2 + 3*x + 1)) + 14904/(374*sqrt(10)*sqrt(2*x^2 + 3*x + 1) - 1183
*sqrt(2*x^2 + 3*x + 1)) - 42240/(17*sqrt(10)*sqrt(2*x^2 + 3*x + 1) + 55*sqr
t(2*x^2 + 3*x + 1)) - 42240/(17*sqrt(10)*sqrt(2*x^2 + 3*x + 1) - 55*sqrt(2*
x^2 + 3*x + 1)) - 1215*sqrt(10)*log(2/9*sqrt(10) + 2/3*sqrt(2*x^2 + 3*x + 1
)*sqrt(17*sqrt(10) + 55)/abs(6*x - 2*sqrt(10) - 4) + 34/9*sqrt(10)/abs(6*x
- 2*sqrt(10) - 4) + 110/9/abs(6*x - 2*sqrt(10) - 4) + 17/18)/(17*sqrt(10) +
 55)^(5/2) - 5*sqrt(10)*log(-2/9*sqrt(10) + 2*sqrt(2*x^2 + 3*x + 1)*sqrt(-1
7/9*sqrt(10) + 55/9)/abs(6*x + 2*sqrt(10) - 4) - 34/9*sqrt(10)/abs(6*x + 2*
sqrt(10) - 4) + 110/9/abs(6*x + 2*sqrt(10) - 4) + 17/18)/(-17/9*sqrt(10) +
55/9)^(5/2) - 9720*log(2/9*sqrt(10) + 2/3*sqrt(2*x^2 + 3*x + 1)*sqrt(17*sqr
t(10) + 55)/abs(6*x - 2*sqrt(10) - 4) + 34/9*sqrt(10)/abs(6*x - 2*sqrt(10)
- 4) + 110/9/abs(6*x - 2*sqrt(10) - 4) + 17/18)/(17*sqrt(10) + 55)^(5/2) +
40*log(-2/9*sqrt(10) + 2*sqrt(2*x^2 + 3*x + 1)*sqrt(-17/9*sqrt(10) + 55/9)/
```

```
abs(6*x + 2*sqrt(10) - 4) - 34/9*sqrt(10)/abs(6*x + 2*sqrt(10) - 4) + 110/9
/abs(6*x + 2*sqrt(10) - 4) + 17/18)/(-17/9*sqrt(10) + 55/9)^(5/2))
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.61

$$\int \frac{2+x}{(2+4x-3x^2)(1+3x+2x^2)^{5/2}}\,dx = \frac{2\,((4\,(115\,x+309)x+1071)x+294)}{15\,(2\,x^2+3\,x+1)^{\frac{3}{2}}} + 0.00115890443050800\,\log\left(-\sqrt{2}x+\sqrt{2\,x^2+3\,x+1}+5.90976932712000\right) - 36.0928986365333\,\log\left(-\sqrt{2}x+\sqrt{2\,x^2+3\,x+1}-0.176527156327000\right) + 36.0928986365333\,\log\left(-\sqrt{2}x+\sqrt{2\,x^2+3\,x+1}-0.919278730509000\right) - 0.00115890442528267\,\log\left(-\sqrt{2}x+\sqrt{2\,x^2+3\,x+1}-1.04272727395000\right)$$

```
[In] integrate((2+x)/(-3*x^2+4*x+2)/(2*x^2+3*x+1)^(5/2),x, algorithm="giac")
```

```
[Out] 2/15*((4*(115*x + 309)*x + 1071)*x + 294)/(2*x^2 + 3*x + 1)^(3/2) + 0.00115
890443050800*log(-sqrt(2)*x + sqrt(2*x^2 + 3*x + 1) + 5.90976932712000) - 3
6.0928986365333*log(-sqrt(2)*x + sqrt(2*x^2 + 3*x + 1) - 0.176527156327000)
 + 36.0928986365333*log(-sqrt(2)*x + sqrt(2*x^2 + 3*x + 1) - 0.919278730509
000) - 0.00115890442528267*log(-sqrt(2)*x + sqrt(2*x^2 + 3*x + 1) - 1.04272
727395000)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{2+x}{(2+4x-3x^2)(1+3x+2x^2)^{5/2}}\,dx = \int \frac{x+2}{(2\,x^2+3\,x+1)^{5/2}\,(-3\,x^2+4\,x+2)}\,dx$$

```
[In] int((x + 2)/((3*x + 2*x^2 + 1)^(5/2)*(4*x - 3*x^2 + 2)),x)
```

```
[Out] int((x + 2)/((3*x + 2*x^2 + 1)^(5/2)*(4*x - 3*x^2 + 2)), x)
```

3.31 $\int \frac{1+x}{(4+2x+x^2)\sqrt{5+2x+x^2}} \, dx$

Optimal result

Integrand size = 26, antiderivative size = 15

$$\int \frac{1+x}{(4+2x+x^2)\sqrt{5+2x+x^2}} \, dx = -\text{arctanh}\left(\sqrt{5+2x+x^2}\right)$$

```
[Out] -arctanh((x^2+2*x+5)^(1/2))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1038, 212}

$$\int \frac{1+x}{(4+2x+x^2)\sqrt{5+2x+x^2}} \, dx = -\text{arctanh}\left(\sqrt{x^2+2x+5}\right)$$

```
[In] Int[(1 + x)/((4 + 2*x + x^2)*Sqrt[5 + 2*x + x^2]),x]
```

```
[Out] -ArcTanh[Sqrt[5 + 2*x + x^2]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1038

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e
_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g, Subst[Int[1/(b*d - a*e -
 b*x^2), x], x, Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h
```

```
}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0] &&
 EqQ[h*e - 2*g*f, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\left(2\text{Subst}\left(\int \frac{1}{2-2x^2}\,dx, x, \sqrt{5+2x+x^2}\right)\right)\\ &= -\tanh^{-1}\left(\sqrt{5+2x+x^2}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1+x}{(4+2x+x^2)\sqrt{5+2x+x^2}}\,dx = -\text{arctanh}\left(\sqrt{5+2x+x^2}\right)$$

```
[In] Integrate[(1 + x)/((4 + 2*x + x^2)*Sqrt[5 + 2*x + x^2]),x]
[Out] -ArcTanh[Sqrt[5 + 2*x + x^2]]
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
default	$-\operatorname{arctanh}\left(\sqrt{x^2+2x+5}\right)$	14
pseudoelliptic	$-\operatorname{arctanh}\left(\sqrt{x^2+2x+5}\right)$	14
trager	$-\frac{\ln\left(\frac{x^2+2\sqrt{x^2+2x+5}+2x+6}{x^2+2x+4}\right)}{2}$	35

```
[In] int((1+x)/(x^2+2*x+4)/(x^2+2*x+5)^(1/2),x,method=_RETURNVERBOSE)
[Out] -arctanh((x^2+2*x+5)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(13) = 26$.

Time = 0.30 (sec) , antiderivative size = 49, normalized size of antiderivative = 3.27

$$\int \frac{1+x}{(4+2x+x^2)\sqrt{5+2x+x^2}}\,dx = \frac{1}{2}\log\left(x^2 - \sqrt{x^2+2\,x+5}(x+2) + 3\,x + 6\right) - \frac{1}{2}\log\left(x^2 - \sqrt{x^2+2\,x+5}x + x + 4\right)$$

```
[In] integrate((1+x)/(x^2+2*x+4)/(x^2+2*x+5)^(1/2),x, algorithm="fricas")
[Out] 1/2*log(x^2 - sqrt(x^2 + 2*x + 5)*(x + 2) + 3*x + 6) - 1/2*log(x^2 - sqrt(x
^2 + 2*x + 5)*x + x + 4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(14) = 28$.

Time = 1.96 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.13

$$\int \frac{1+x}{(4+2x+x^2)\sqrt{5+2x+x^2}}\,dx = \frac{\log\left(\sqrt{x^2+2x+5}-1\right)}{2} - \frac{\log\left(\sqrt{x^2+2x+5}+1\right)}{2}$$

```
[In] integrate((1+x)/(x**2+2*x+4)/(x**2+2*x+5)**(1/2),x)
[Out] log(sqrt(x**2 + 2*x + 5) - 1)/2 - log(sqrt(x**2 + 2*x + 5) + 1)/2
```

Maxima [F]

$$\int \frac{1+x}{(4+2x+x^2)\sqrt{5+2x+x^2}}\,dx = \int \frac{x+1}{\sqrt{x^2+2\,x+5}(x^2+2\,x+4)}\,dx$$

```
[In] integrate((1+x)/(x^2+2*x+4)/(x^2+2*x+5)^(1/2),x, algorithm="maxima")
[Out] integrate((x + 1)/(sqrt(x^2 + 2*x + 5)*(x^2 + 2*x + 4)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(13) = 26.

Time = 0.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.07

$$\int \frac{1+x}{(4+2x+x^2)\sqrt{5+2x+x^2}}\,dx = -\frac{1}{2}\log\left(\sqrt{x^2+2x+5}+1\right)+\frac{1}{2}\log\left(\sqrt{x^2+2x+5}-1\right)$$

```
[In] integrate((1+x)/(x^2+2*x+4)/(x^2+2*x+5)^(1/2),x, algorithm="giac")
[Out] -1/2*log(sqrt(x^2 + 2*x + 5) + 1) + 1/2*log(sqrt(x^2 + 2*x + 5) - 1)
```

Mupad [B] (verification not implemented)

Time = 12.56 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{1+x}{(4+2x+x^2)\sqrt{5+2x+x^2}}\,dx = -\text{atanh}\left(\sqrt{x^2+2x+5}\right)$$

```
[In] int((x + 1)/((2*x + x^2 + 4)*(2*x + x^2 + 5)^(1/2)),x)
[Out] -atanh((2*x + x^2 + 5)^(1/2))
```

3.32 $\int \frac{4+x}{(4+2x+x^2)\sqrt{5+2x+x^2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 44

$$\int \frac{4+x}{(4+2x+x^2)\sqrt{5+2x+x^2}}\,dx = \sqrt{3}\arctan\left(\frac{1+x}{\sqrt{3}\sqrt{5+2x+x^2}}\right)-\operatorname{arctanh}\left(\sqrt{5+2x+x^2}\right)$$

```
[Out] -arctanh((x^2+2*x+5)^(1/2))+arctan(1/3*(1+x)*3^(1/2)/(x^2+2*x+5)^(1/2))*3^(
1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1039, 996, 210, 1038, 212}

$$\int \frac{4+x}{(4+2x+x^2)\sqrt{5+2x+x^2}}\,dx = \sqrt{3}\arctan\left(\frac{x+1}{\sqrt{3}\sqrt{x^2+2x+5}}\right)-\operatorname{arctanh}\left(\sqrt{x^2+2x+5}\right)$$

```
[In] Int[(4 + x)/((4 + 2*x + x^2)*Sqrt[5 + 2*x + x^2]),x]
```

```
[Out] Sqrt[3]*ArcTan[(1 + x)/(Sqrt[3]*Sqrt[5 + 2*x + x^2])] - ArcTanh[Sqrt[5 + 2*
x + x^2]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 996

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e)
*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0
]
```

Rule 1038

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e
_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g, Subst[Int[1/(b*d - a*e -
 b*x^2), x], x, Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h
}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0] &&
 EqQ[h*e - 2*g*f, 0]
```

Rule 1039

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-(h*e - 2*g*f)/(2*f), Int[1/(
(a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/(2*f), Int[(e + 2
*f*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c,
d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e
 - b*f, 0] && NeQ[h*e - 2*g*f, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}\int \frac{2+2x}{(4+2x+x^2)\sqrt{5+2x+x^2}}\,dx + 3\int \frac{1}{(4+2x+x^2)\sqrt{5+2x+x^2}}\,dx \\
&= -\left(2\text{Subst}\left(\int \frac{1}{2-2x^2}\,dx, x, \sqrt{5+2x+x^2}\right)\right) \\
&\quad - 12\text{Subst}\left(\int \frac{1}{-24-2x^2}\,dx, x, \frac{2+2x}{\sqrt{5+2x+x^2}}\right) \\
&= \sqrt{3}\tan^{-1}\left(\frac{1+x}{\sqrt{3}\sqrt{5+2x+x^2}}\right) - \tanh^{-1}\left(\sqrt{5+2x+x^2}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.25

$$\int \frac{4+x}{(4+2x+x^2)\sqrt{5+2x+x^2}}\,dx = -\sqrt{3}\arctan\left(\frac{4+2x+x^2-(1+x)\sqrt{5+2x+x^2}}{\sqrt{3}}\right) - \operatorname{arctanh}\left(\sqrt{5+2x+x^2}\right)$$

```
[In] Integrate[(4 + x)/((4 + 2*x + x^2)*Sqrt[5 + 2*x + x^2]),x]
```

```
[Out] -(Sqrt[3]*ArcTan[(4 + 2*x + x^2 - (1 + x)*Sqrt[5 + 2*x + x^2])/Sqrt[3]]) -
ArcTanh[Sqrt[5 + 2*x + x^2]]
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.91

method	result
default	$-\operatorname{arctanh}\left(\sqrt{x^2+2x+5}\right)+\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(2x+2)}{6\sqrt{x^2+2x+5}}\right)$
trager	$\operatorname{RootOf}\left(\_Z^2+\_Z+1\right)\ln\left(\frac{40\operatorname{RootOf}\left(\_Z^2+\_Z+1\right)^2 x+21\sqrt{x^2+2x+5}\,\operatorname{RootOf}\left(\_Z^2+\_Z+1\right)+51\operatorname{RootOf}\left(\_Z^2+\right.}{\operatorname{RootOf}\left(\_Z^2+\_Z+1\right)x+x}\right.$

```
[In] int((4+x)/(x^2+2*x+4)/(x^2+2*x+5)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -arctanh((x^2+2*x+5)^(1/2))+3^(1/2)*arctan(1/6*3^(1/2)/(x^2+2*x+5)^(1/2)*(2
*x+2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 106 vs. 2(37) = 74.

Time = 0.32 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.41

$$\int \frac{4+x}{(4+2x+x^2)\sqrt{5+2x+x^2}}\,dx = -\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}(x+2)+\frac{1}{3}\sqrt{3}\sqrt{x^2+2x+5}\right) + \sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}x+\frac{1}{3}\sqrt{3}\sqrt{x^2+2x+5}\right) + \frac{1}{2}\log\left(x^2-\sqrt{x^2+2x+5}(x+2)+3x+6\right) - \frac{1}{2}\log\left(x^2-\sqrt{x^2+2x+5}x+x+4\right)$$

```
[In] integrate((4+x)/(x^2+2*x+4)/(x^2+2*x+5)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(3)*arctan(-1/3*sqrt(3)*(x + 2) + 1/3*sqrt(3)*sqrt(x^2 + 2*x + 5)) + s
qrt(3)*arctan(-1/3*sqrt(3)*x + 1/3*sqrt(3)*sqrt(x^2 + 2*x + 5)) + 1/2*log(x
^2 - sqrt(x^2 + 2*x + 5)*(x + 2) + 3*x + 6) - 1/2*log(x^2 - sqrt(x^2 + 2*x
+ 5)*x + x + 4)
```

Sympy [F]

$$\int \frac{4+x}{(4+2x+x^2)\sqrt{5+2x+x^2}}\,dx = \int \frac{x+4}{(x^2+2x+4)\sqrt{x^2+2x+5}}\,dx$$

```
[In] integrate((4+x)/(x**2+2*x+4)/(x**2+2*x+5)**(1/2),x)
[Out] Integral((x + 4)/((x**2 + 2*x + 4)*sqrt(x**2 + 2*x + 5)), x)
```

Maxima [F]

$$\int \frac{4+x}{(4+2x+x^2)\sqrt{5+2x+x^2}}\,dx = \int \frac{x+4}{\sqrt{x^2+2\,x+5}(x^2+2\,x+4)}\,dx$$

```
[In] integrate((4+x)/(x^2+2*x+4)/(x^2+2*x+5)^(1/2),x, algorithm="maxima")
[Out] integrate((x + 4)/(sqrt(x^2 + 2*x + 5)*(x^2 + 2*x + 4)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 108 vs. 2(37) = 74.

Time = 0.29 (sec) , antiderivative size = 108, normalized size of antiderivative = 2.45

$$\begin{aligned}\int \frac{4+x}{(4+2x+x^2)\sqrt{5+2x+x^2}}\,dx &= -\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}\left(x-\sqrt{x^2+2\,x+5}+2\right)\right) \\ &\quad+\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}\left(x-\sqrt{x^2+2\,x+5}\right)\right) \\ &\quad+\frac{1}{2}\log\left(\left(x-\sqrt{x^2+2\,x+5}\right)^2+4\,x-4\sqrt{x^2+2\,x+5}\right. \\ &\quad\left.+7\right)-\frac{1}{2}\log\left(\left(x-\sqrt{x^2+2\,x+5}\right)^2+3\right)\end{aligned}$$

```
[In] integrate((4+x)/(x^2+2*x+4)/(x^2+2*x+5)^(1/2),x, algorithm="giac")
[Out] -sqrt(3)*arctan(-1/3*sqrt(3)*(x - sqrt(x^2 + 2*x + 5) + 2)) + sqrt(3)*arcta
n(-1/3*sqrt(3)*(x - sqrt(x^2 + 2*x + 5))) + 1/2*log((x - sqrt(x^2 + 2*x + 5
))^2 + 4*x - 4*sqrt(x^2 + 2*x + 5) + 7) - 1/2*log((x - sqrt(x^2 + 2*x + 5))
^2 + 3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{4+x}{(4+2x+x^2)\sqrt{5+2x+x^2}}\,dx = \int \frac{x+4}{(x^2+2\,x+4)\,\sqrt{x^2+2\,x+5}}\,dx$$

```
[In] int((x + 4)/((2*x + x^2 + 4)*(2*x + x^2 + 5)^(1/2)),x)
[Out] int((x + 4)/((2*x + x^2 + 4)*(2*x + x^2 + 5)^(1/2)), x)
```

3.33 $\int \frac{1+2x}{(3+x+x^2)\sqrt{5+x+x^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1+2x}{(3+x+x^2)\sqrt{5+x+x^2}}\,dx = -\sqrt{2}\text{arctanh}\left(\frac{\sqrt{5+x+x^2}}{\sqrt{2}}\right)$$

```
[Out] -arctanh(1/2*(x^2+x+5)^(1/2)*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1038, 212}

$$\int \frac{1+2x}{(3+x+x^2)\sqrt{5+x+x^2}}\,dx = -\sqrt{2}\text{arctanh}\left(\frac{\sqrt{x^2+x+5}}{\sqrt{2}}\right)$$

```
[In] Int[(1 + 2*x)/((3 + x + x^2)*Sqrt[5 + x + x^2]),x]
```

```
[Out] -(Sqrt[2]*ArcTanh[Sqrt[5 + x + x^2]/Sqrt[2]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1038

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e
_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g, Subst[Int[1/(b*d - a*e -
```

```
 b*x^2), x], x, Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h
}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0] &&
 EqQ[h*e - 2*g*f, 0]
```

Rubi steps

$$\text{integral} = -\left(2\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \sqrt{5+x+x^2}\right)\right)$$
$$= -\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{5+x+x^2}}{\sqrt{2}}\right)$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1+2x}{(3+x+x^2)\sqrt{5+x+x^2}}\,dx = -\sqrt{2}\text{arctanh}\left(\frac{\sqrt{5+x+x^2}}{\sqrt{2}}\right)$$

```
[In] Integrate[(1 + 2*x)/((3 + x + x^2)*Sqrt[5 + x + x^2]),x]
[Out] -(Sqrt[2]*ArcTanh[Sqrt[5 + x + x^2]/Sqrt[2]])
```

Maple [A] (verified)

Time = 1.78 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

method	result	size
default	$-\operatorname{arctanh}\left(\frac{\sqrt{x^2+x+5}\,\sqrt{2}}{2}\right)\sqrt{2}$	20
pseudoelliptic	$-\operatorname{arctanh}\left(\frac{\sqrt{x^2+x+5}\,\sqrt{2}}{2}\right)\sqrt{2}$	20
trager	$-\frac{\operatorname{RootOf}\left(\_Z^2-2\right)\ln\left(\frac{\operatorname{RootOf}\left(\_Z^2-2\right)x^2+\operatorname{RootOf}\left(\_Z^2-2\right)x+4\sqrt{x^2+x+5}+7\operatorname{RootOf}\left(\_Z^2-2\right)}{x^2+x+3}\right)}{2}$	56

```
[In] int((1+2*x)/(x^2+x+3)/(x^2+x+5)^(1/2),x,method=_RETURNVERBOSE)
[Out] -arctanh(1/2*(x^2+x+5)^(1/2)*2^(1/2))*2^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.42

$$\int \frac{1+2x}{(3+x+x^2)\sqrt{5+x+x^2}}\,dx = \frac{1}{2}\sqrt{2}\log\left(\frac{x^2-2\sqrt{2}\sqrt{x^2+x+5}+x+7}{x^2+x+3}\right)$$

```
[In] integrate((1+2*x)/(x^2+x+3)/(x^2+x+5)^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(2)*log((x^2 - 2*sqrt(2)*sqrt(x^2 + x + 5) + x + 7)/(x^2 + x + 3))
```

Sympy [A] (verification not implemented)

Time = 1.74 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.62

$$\int \frac{1+2x}{(3+x+x^2)\sqrt{5+x+x^2}}\,dx = \frac{\sqrt{2}\left(\log\left(\sqrt{x^2+x+5}-\sqrt{2}\right)-\log\left(\sqrt{x^2+x+5}+\sqrt{2}\right)\right)}{2}$$

```
[In] integrate((1+2*x)/(x**2+x+3)/(x**2+x+5)**(1/2),x)
[Out] sqrt(2)*(log(sqrt(x**2 + x + 5) - sqrt(2)) - log(sqrt(x**2 + x + 5) + sqrt(
2)))/2
```

Maxima [F]

$$\int \frac{1+2x}{(3+x+x^2)\sqrt{5+x+x^2}}\,dx = \int \frac{2\,x+1}{\sqrt{x^2+x+5}(x^2+x+3)}\,dx$$

```
[In] integrate((1+2*x)/(x^2+x+3)/(x^2+x+5)^(1/2),x, algorithm="maxima")
[Out] integrate((2*x + 1)/(sqrt(x^2 + x + 5)*(x^2 + x + 3)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. 2(19) = 38.

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.62

$$\begin{aligned}\int \frac{1+2x}{(3+x+x^2)\sqrt{5+x+x^2}}\,dx = & -\frac{1}{2}\sqrt{2}\log\left(\sqrt{2}+\sqrt{x^2+x+5}\right)\\ & +\frac{1}{2}\sqrt{2}\log\left(-\sqrt{2}+\sqrt{x^2+x+5}\right)\end{aligned}$$

```
[In] integrate((1+2*x)/(x^2+x+3)/(x^2+x+5)^(1/2),x, algorithm="giac")
[Out] -1/2*sqrt(2)*log(sqrt(2) + sqrt(x^2 + x + 5)) + 1/2*sqrt(2)*log(-sqrt(2) +
sqrt(x^2 + x + 5))
```

Mupad [B] (verification not implemented)

Time = 12.55 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

$$\int \frac{1+2x}{(3+x+x^2)\sqrt{5+x+x^2}}\,dx = -\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{x^2+x+5}}{2}\right)$$

```
[In] int((2*x + 1)/((x + x^2 + 3)*(x + x^2 + 5)^(1/2)),x)
[Out] -2^(1/2)*atanh((2^(1/2)*(x + x^2 + 5)^(1/2))/2)
```

3.34 $\int \frac{x}{(3+x+x^2)\sqrt{5+x+x^2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 56

$$\int \frac{x}{(3+x+x^2)\sqrt{5+x+x^2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{\frac{2}{11}}(1+2x)}{\sqrt{5+x+x^2}}\right)}{\sqrt{22}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{5+x+x^2}}{\sqrt{2}}\right)}{\sqrt{2}}$$

```
[Out] -1/2*arctanh(1/2*(x^2+x+5)^(1/2)*2^(1/2))*2^(1/2)-1/22*arctan(1/11*(1+2*x)*
22^(1/2)/(x^2+x+5)^(1/2))*22^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1039, 996, 210, 1038, 212}

$$\int \frac{x}{(3+x+x^2)\sqrt{5+x+x^2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{\frac{2}{11}}(2x+1)}{\sqrt{x^2+x+5}}\right)}{\sqrt{22}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{x^2+x+5}}{\sqrt{2}}\right)}{\sqrt{2}}$$

```
[In] Int[x/((3 + x + x^2)*Sqrt[5 + x + x^2]),x]
```

```
[Out] -(ArcTan[(Sqrt[2/11]*(1 + 2*x))/Sqrt[5 + x + x^2]]/Sqrt[22]) - ArcTanh[Sqrt
[5 + x + x^2]/Sqrt[2]]/Sqrt[2]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 996

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e)
*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0
]
```

Rule 1038

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e
_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g, Subst[Int[1/(b*d - a*e -
 b*x^2), x], x, Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h
}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0] &&
 EqQ[h*e - 2*g*f, 0]
```

Rule 1039

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-(h*e - 2*g*f)/(2*f), Int[1/(
(a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/(2*f), Int[(e + 2
*f*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c,
d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e
 - b*f, 0] && NeQ[h*e - 2*g*f, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(\frac{1}{2}\int \frac{1}{(3+x+x^2)\sqrt{5+x+x^2}}\,dx\right)+\frac{1}{2}\int \frac{1+2x}{(3+x+x^2)\sqrt{5+x+x^2}}\,dx\\
&= \text{Subst}\left(\int \frac{1}{-11-2x^2}\,dx, x, \frac{1+2x}{\sqrt{5+x+x^2}}\right)-\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \sqrt{5+x+x^2}\right)\\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{\frac{2}{11}}(1+2x)}{\sqrt{5+x+x^2}}\right)}{\sqrt{22}}-\frac{\tanh^{-1}\left(\frac{\sqrt{5+x+x^2}}{\sqrt{2}}\right)}{\sqrt{2}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.12 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.64

$$\int \frac{x}{(3+x+x^2)\sqrt{5+x+x^2}}\,dx = \text{RootSum}\left[23-2\#1+3\#1^2-2\#1^3+\#1^4\&,\frac{-5\log\left(-x+\sqrt{5+x+x^2}-\#1\right)+\log\left(-x+\sqrt{5+x+x^2}-\#1\right)\#1^2}{-1+3\#1-3\#1^2+2\#1^3}\&\right]$$

```
[In] Integrate[x/((3 + x + x^2)*Sqrt[5 + x + x^2]),x]
[Out] RootSum[23 - 2*#1 + 3*#1^2 - 2*#1^3 + #1^4 & , (-5*Log[-x + Sqrt[5 + x + x^
2] - #1] + Log[-x + Sqrt[5 + x + x^2] - #1]*#1^2)/(-1 + 3*#1 - 3*#1^2 + 2*#
1^3) & ]
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.80

method	result
default	$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{x^2+x+5}\,\sqrt{2}}{2}\right)\sqrt{2}}{2}-\frac{\arctan\left(\frac{(1+2x)\sqrt{22}}{11\sqrt{x^2+x+5}}\right)\sqrt{22}}{22}$
trager	$-\operatorname{RootOf}\left(484\_Z^4-110\_Z^2+9\right)\ln\left(\frac{133342\operatorname{RootOf}\left(484\_Z^4-110\_Z^2+9\right)^5x-34298\operatorname{RootOf}\left(484\_Z^4-110\_\right.}{}\right.$

```
[In] int(x/(x^2+x+3)/(x^2+x+5)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/2*arctanh(1/2*(x^2+x+5)^(1/2)*2^(1/2))*2^(1/2)-1/22*arctan(1/11*(1+2*x)*
22^(1/2)/(x^2+x+5)^(1/2))*22^(1/2)
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 203, normalized size of antiderivative = 3.62

$$\int \frac{x}{(3+x+x^2)\sqrt{5+x+x^2}}\,dx = -\frac{1}{22}\sqrt{11}\sqrt{i\sqrt{11}+5}\log\left(\left(\sqrt{11}-i\right)\sqrt{i\sqrt{11}+5}-6x+3i\sqrt{11}+6\sqrt{x^2+x+5}-3\right)+\frac{1}{22}\sqrt{11}\sqrt{i\sqrt{11}+5}\log\left(-\left(\sqrt{11}-i\right)\sqrt{i\sqrt{11}+5}-6x+3i\sqrt{11}+6\sqrt{x^2+x+5}-3\right)-\frac{1}{22}\sqrt{11}\sqrt{-i\sqrt{11}+5}\log\left(\left(\sqrt{11}+i\right)\sqrt{-i\sqrt{11}+5}-6x-3i\sqrt{11}+6\sqrt{x^2+x+5}-3\right)+\frac{1}{22}\sqrt{11}\sqrt{-i\sqrt{11}+5}\log\left(-\left(\sqrt{11}+i\right)\sqrt{-i\sqrt{11}+5}-6x-3i\sqrt{11}+6\sqrt{x^2+x+5}-3\right)$$

```
[In] integrate(x/(x^2+x+3)/(x^2+x+5)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/22*sqrt(11)*sqrt(I*sqrt(11) + 5)*log((sqrt(11) - I)*sqrt(I*sqrt(11) + 5)
 - 6*x + 3*I*sqrt(11) + 6*sqrt(x^2 + x + 5) - 3) + 1/22*sqrt(11)*sqrt(I*sqr
t(11) + 5)*log(-(sqrt(11) - I)*sqrt(I*sqrt(11) + 5) - 6*x + 3*I*sqrt(11) +
6*sqrt(x^2 + x + 5) - 3) - 1/22*sqrt(11)*sqrt(-I*sqrt(11) + 5)*log((sqrt(11
) + I)*sqrt(-I*sqrt(11) + 5) - 6*x - 3*I*sqrt(11) + 6*sqrt(x^2 + x + 5) - 3
) + 1/22*sqrt(11)*sqrt(-I*sqrt(11) + 5)*log(-(sqrt(11) + I)*sqrt(-I*sqrt(11
) + 5) - 6*x - 3*I*sqrt(11) + 6*sqrt(x^2 + x + 5) - 3)
```

Sympy [F]

$$\int \frac{x}{(3+x+x^2)\sqrt{5+x+x^2}}\,dx = \int \frac{x}{(x^2+x+3)\sqrt{x^2+x+5}}\,dx$$

```
[In] integrate(x/(x**2+x+3)/(x**2+x+5)**(1/2),x)
```

```
[Out] Integral(x/((x**2 + x + 3)*sqrt(x**2 + x + 5)), x)
```

Maxima [F]

$$\int \frac{x}{(3+x+x^2)\sqrt{5+x+x^2}}\,dx = \int \frac{x}{\sqrt{x^2+x+5}(x^2+x+3)}\,dx$$

```
[In] integrate(x/(x^2+x+3)/(x^2+x+5)^(1/2),x, algorithm="maxima")
[Out] integrate(x/(sqrt(x^2 + x + 5)*(x^2 + x + 3)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(44) = 88$.

Time = 0.30 (sec) , antiderivative size = 133, normalized size of antiderivative = 2.38

$$\begin{aligned}
&\int \frac{x}{(3+x+x^2)\sqrt{5+x+x^2}}\,dx \\
&= \frac{1}{22}\sqrt{11}\sqrt{2}\arctan\left(-\frac{1}{11}\sqrt{11}\left(2x+2\sqrt{2}-2\sqrt{x^2+x+5}+1\right)\right) \\
&\quad - \frac{1}{22}\sqrt{11}\sqrt{2}\arctan\left(-\frac{1}{11}\sqrt{11}\left(2x-2\sqrt{2}-2\sqrt{x^2+x+5}+1\right)\right) \\
&\quad + \frac{1}{4}\sqrt{2}\log\left(324\left(2x+2\sqrt{2}-2\sqrt{x^2+x+5}+1\right)^2+3564\right) \\
&\quad - \frac{1}{4}\sqrt{2}\log\left(324\left(2x-2\sqrt{2}-2\sqrt{x^2+x+5}+1\right)^2+3564\right)
\end{aligned}$$

```
[In] integrate(x/(x^2+x+3)/(x^2+x+5)^(1/2),x, algorithm="giac")
[Out] 1/22*sqrt(11)*sqrt(2)*arctan(-1/11*sqrt(11)*(2*x + 2*sqrt(2) - 2*sqrt(x^2 +
 x + 5) + 1)) - 1/22*sqrt(11)*sqrt(2)*arctan(-1/11*sqrt(11)*(2*x - 2*sqrt(2
) - 2*sqrt(x^2 + x + 5) + 1)) + 1/4*sqrt(2)*log(324*(2*x + 2*sqrt(2) - 2*sq
rt(x^2 + x + 5) + 1)^2 + 3564) - 1/4*sqrt(2)*log(324*(2*x - 2*sqrt(2) - 2*s
qrt(x^2 + x + 5) + 1)^2 + 3564)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(3+x+x^2)\sqrt{5+x+x^2}}\,dx = \int \frac{x}{(x^2+x+3)\sqrt{x^2+x+5}}\,dx$$

```
[In] int(x/((x + x^2 + 3)*(x + x^2 + 5)^(1/2)),x)
[Out] int(x/((x + x^2 + 3)*(x + x^2 + 5)^(1/2)), x)
```

3.35 $\int \frac{A+Bx}{\sqrt{d+ex+fx^2}(ae+bex+bfx^2)^2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 249

$$
\begin{aligned}
\int \frac{A+Bx}{\sqrt{d+ex+fx^2}\,(ae+bex+bfx^2)^2}\,dx
&= -\frac{((Ab-2aB)e-b(Be-2Af)x)\sqrt{d+ex+fx^2}}{e(bd-ae)(be-4af)\,(ae+bex+bfx^2)} \\
&\quad+\frac{(Be-2Af)\,(8aef-b(e^2+4df))\,\mathrm{arctanh}\left(\frac{\sqrt{bd-ae}(e+2fx)}{\sqrt{e}\sqrt{be-4af}\sqrt{d+ex+fx^2}}\right)}{2e^{3/2}(bd-ae)^{3/2}f(be-4af)^{3/2}} \\
&\quad+\frac{B\,\mathrm{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex+fx^2}}{\sqrt{bd-ae}}\right)}{2\sqrt{b}(bd-ae)^{3/2}f}
\end{aligned}
$$

```
[Out] 1/2*(-2*A*f+B*e)*(8*a*e*f-b*(4*d*f+e^2))*arctanh((2*f*x+e)*(-a*e+b*d)^(1/2)
/e^(1/2)/(-4*a*f+b*e)^(1/2)/(f*x^2+e*x+d)^(1/2))/e^(3/2)/(-a*e+b*d)^(3/2)/f
/(-4*a*f+b*e)^(3/2)+1/2*B*arctanh(b^(1/2)*(f*x^2+e*x+d)^(1/2)/(-a*e+b*d)^(1
/2))/(-a*e+b*d)^(3/2)/f/b^(1/2)-((A*b-2*B*a)*e-b*(-2*A*f+B*e)*x)*(f*x^2+e*x
+d)^(1/2)/e/(-a*e+b*d)/(-4*a*f+b*e)/(b*f*x^2+b*e*x+a*e)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$, Rules used

= {1030, 1039, 996, 214, 1038}

$$\int \frac{A+Bx}{\sqrt{d+ex+fx^2}\,(ae+bex+bfx^2)^2}\,dx$$
$$= \frac{(Be-2Af)\,(8aef-b(4df+e^2))\,\text{arctanh}\left(\frac{(e+2fx)\sqrt{bd-ae}}{\sqrt{e}\sqrt{be-4af}\sqrt{d+ex+fx^2}}\right)}{2e^{3/2}f(bd-ae)^{3/2}(be-4af)^{3/2}}$$
$$-\frac{\sqrt{d+ex+fx^2}(e(Ab-2aB)-bx(Be-2Af))}{e(bd-ae)(be-4af)\,(ae+bex+bfx^2)}+\frac{B\text{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex+fx^2}}{\sqrt{bd-ae}}\right)}{2\sqrt{b}f(bd-ae)^{3/2}}$$

```
[In] Int[(A + B*x)/(Sqrt[d + e*x + f*x^2]*(a*e + b*e*x + b*f*x^2)^2),x]
```

```
[Out] -((((A*b - 2*a*B)*e - b*(B*e - 2*A*f)*x)*Sqrt[d + e*x + f*x^2])/(e*(b*d - a
*e)*(b*e - 4*a*f)*(a*e + b*e*x + b*f*x^2))) + ((B*e - 2*A*f)*(8*a*e*f - b*(
e^2 + 4*d*f))*ArcTanh[(Sqrt[b*d - a*e]*(e + 2*f*x))/(Sqrt[e]*Sqrt[b*e - 4*a
*f]*Sqrt[d + e*x + f*x^2])])/(2*e^(3/2)*(b*d - a*e)^(3/2)*f*(b*e - 4*a*f)^(
3/2)) + (B*ArcTanh[(Sqrt[b]*Sqrt[d + e*x + f*x^2])/Sqrt[b*d - a*e]])/(2*Sqr
t[b]*(b*d - a*e)^(3/2)*f)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 996

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e)
*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0
]
```

Rule 1030

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x + c*x^2)^(p + 1)*
((d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
 - b*f))*(p + 1)))*(g*c*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d +
b^2*f - c*(b*e + 2*a*f)) + c*(g*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - h*(b*
c*d - 2*a*c*e + a*b*f))*x), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*
d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*
x^2)^q*Simp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)
 + (b^2*(g*f) - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*((-h)*c*
e)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((g*c)*(2*a*c*e - b*(c*d + a*f)) + (g*
b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((g*c)*(2*
a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(
```

```
p + q + 2) - (b^2*g*f - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*
((-h)*c*e)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(g*f) - b*(h*c
*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) + a*h*c*e))*(2*p + 2*q + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e
 - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
```

Rule 1038

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e
_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g, Subst[Int[1/(b*d - a*e -
 b*x^2), x], x, Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h
}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0] &&
 EqQ[h*e - 2*g*f, 0]
```

Rule 1039

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-(h*e - 2*g*f)/(2*f), Int[1/(
(a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/(2*f), Int[(e + 2
*f*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c,
d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e
 - b*f, 0] && NeQ[h*e - 2*g*f, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{((Ab-2aB)e-b(Be-2Af)x)\sqrt{d+ex+fx^2}}{e(bd-ae)(be-4af)\left(ae+bex+bfx^2\right)} \\
&\quad -\frac{\int \frac{-\frac{1}{2}b(bd-ae)f^2\left(2bBde-2ae(Be-4Af)-Ab(e^2+4df)\right)+\frac{1}{2}bBe(bd-ae)f^2(be-4af)x}{\sqrt{d+ex+fx^2}\left(ae+bex+bfx^2\right)}\,dx}{be(bd-ae)^2f^2(be-4af)} \\
&= -\frac{((Ab-2aB)e-b(Be-2Af)x)\sqrt{d+ex+fx^2}}{e(bd-ae)(be-4af)\left(ae+bex+bfx^2\right)}-\frac{B\int \frac{e+2fx}{\sqrt{d+ex+fx^2}\left(ae+bex+bfx^2\right)}\,dx}{4(bd-ae)f} \\
&\quad -\frac{\left((Be-2Af)\left(8aef-b(e^2+4df)\right)\right)\int \frac{1}{\sqrt{d+ex+fx^2}\left(ae+bex+bfx^2\right)}\,dx}{4e(bd-ae)f(be-4af)} \\
&= -\frac{((Ab-2aB)e-b(Be-2Af)x)\sqrt{d+ex+fx^2}}{e(bd-ae)(be-4af)\left(ae+bex+bfx^2\right)} \\
&\quad +\frac{(Be)\text{Subst}\left(\int \frac{1}{bde-ae^2-bex^2}\,dx,x,\sqrt{d+ex+fx^2}\right)}{2(bd-ae)f} \\
&\quad +\frac{\left((Be-2Af)\left(8aef-b(e^2+4df)\right)\right)\text{Subst}\left(\int \frac{1}{e(be^2-4aef)-(bde-ae^2)x^2}\,dx,x,\frac{e+2fx}{\sqrt{d+ex+fx^2}}\right)}{2(bd-ae)f(be-4af)}
\end{aligned}$$

$$= -\frac{((Ab-2aB)e-b(Be-2Af)x)\sqrt{d+ex+fx^2}}{e(bd-ae)(be-4af)\,(ae+bex+bfx^2)} + \frac{(Be-2Af)\,(8aef-b(e^2+4df))\tanh^{-1}\left(\frac{\sqrt{bd-ae}(e+2fx)}{\sqrt{e}\sqrt{be-4af}\sqrt{d+ex+fx^2}}\right)}{2e^{3/2}(bd-ae)^{3/2}f(be-4af)^{3/2}} + \frac{B\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{d+ex+fx^2}}{\sqrt{bd-ae}}\right)}{2\sqrt{b}(bd-ae)^{3/2}f}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 3.53 (sec) , antiderivative size = 2374, normalized size of antiderivative = 9.53

$$\int \frac{A+Bx}{\sqrt{d+ex+fx^2}\,(ae+bex+bfx^2)^2}\,dx = \text{Result too large to show}$$

```
[In] Integrate[(A + B*x)/(Sqrt[d + e*x + f*x^2]*(a*e + b*e*x + b*f*x^2)^2),x]
```

```
[Out] ((2*e*Sqrt[d + x*(e + f*x)]*(B*e*(2*a + b*x) - A*b*(e + 2*f*x)))/((b*d - a*
e)*(b*e - 4*a*f)*(a*e + b*x*(e + f*x))) - (2*RootSum[a*e*f^2 - 2*b*Sqrt[d]*
e*f*#1 + b*e^2*#1^2 + 4*b*d*f*#1^2 - 2*a*e*f*#1^2 - 2*b*Sqrt[d]*e*#1^3 + a*
e*#1^4 & , (-4*A*b^2*d*e*Log[x] + 4*a*b*B*d*e*Log[x] + a*A*b*e^2*Log[x] - a
^2*B*e^2*Log[x] + 4*a*A*b*d*f*Log[x] + a^2*A*e*f*Log[x] + 4*A*b^2*d*e*Log[-
Sqrt[d] + Sqrt[d + e*x + f*x^2] - x*#1] - 4*a*b*B*d*e*Log[-Sqrt[d] + Sqrt[d
 + e*x + f*x^2] - x*#1] - a*A*b*e^2*Log[-Sqrt[d] + Sqrt[d + e*x + f*x^2] -
x*#1] + a^2*B*e^2*Log[-Sqrt[d] + Sqrt[d + e*x + f*x^2] - x*#1] - 4*a*A*b*d*
f*Log[-Sqrt[d] + Sqrt[d + e*x + f*x^2] - x*#1] - a^2*A*e*f*Log[-Sqrt[d] + S
qrt[d + e*x + f*x^2] - x*#1] - 2*a*A*b*Sqrt[d]*e*Log[x]*#1 + 2*a^2*B*Sqrt[d
]*e*Log[x]*#1 + 2*a*A*b*Sqrt[d]*e*Log[-Sqrt[d] + Sqrt[d + e*x + f*x^2] - x*
#1]*#1 - 2*a^2*B*Sqrt[d]*e*Log[-Sqrt[d] + Sqrt[d + e*x + f*x^2] - x*#1]*#1
- a^2*A*e*Log[x]*#1^2 + a^2*A*e*Log[-Sqrt[d] + Sqrt[d + e*x + f*x^2] - x*#1
]*#1^2)/(-(b*Sqrt[d]*e*f) + b*e^2*#1 + 4*b*d*f*#1 - 2*a*e*f*#1 - 3*b*Sqrt[d
]*e*#1^2 + 2*a*e*#1^3) & ])/a^3 + RootSum[a*e*f^2 - 2*b*Sqrt[d]*e*f*#1 + b*
e^2*#1^2 + 4*b*d*f*#1^2 - 2*a*e*f*#1^2 - 2*b*Sqrt[d]*e*#1^3 + a*e*#1^4 & ,
(-8*A*b^4*d^2*e^2*Log[x] + 8*a*b^3*B*d^2*e^2*Log[x] + 10*a*A*b^3*d*e^3*Log[
x] - 10*a^2*b^2*B*d*e^3*Log[x] - 2*a^2*A*b^2*e^4*Log[x] + a^3*b*B*e^4*Log[x
] + 40*a*A*b^3*d^2*e*f*Log[x] - 32*a^2*b^2*B*d^2*e*f*Log[x] - 46*a^2*A*b^2*
d*e^2*f*Log[x] + 38*a^3*b*B*d*e^2*f*Log[x] + 7*a^3*A*b*e^3*f*Log[x] - 2*a^4
*B*e^3*f*Log[x] - 32*a^2*A*b^2*d^2*f^2*Log[x] + 28*a^3*A*b*d*e*f^2*Log[x] +
 8*A*b^4*d^2*e^2*Log[-Sqrt[d] + Sqrt[d + e*x + f*x^2] - x*#1] - 8*a*b^3*B*d
^2*e^2*Log[-Sqrt[d] + Sqrt[d + e*x + f*x^2] - x*#1] - 10*a*A*b^3*d*e^3*Log[
-Sqrt[d] + Sqrt[d + e*x + f*x^2] - x*#1] + 10*a^2*b^2*B*d*e^3*Log[-Sqrt[d]
+ Sqrt[d + e*x + f*x^2] - x*#1] + 2*a^2*A*b^2*e^4*Log[-Sqrt[d] + Sqrt[d + e
*x + f*x^2] - x*#1] - a^3*b*B*e^4*Log[-Sqrt[d] + Sqrt[d + e*x + f*x^2] - x*
```

```
#1] - 40*a*A*b^3*d^2*e*f*Log[-Sqrt[d] + Sqrt[d + e*x + f*x^2] - x*#1] + 32*
a^2*b^2*B*d^2*e*f*Log[-Sqrt[d] + Sqrt[d + e*x + f*x^2] - x*#1] + 46*a^2*A*b
^2*d*e^2*f*Log[-Sqrt[d] + Sqrt[d + e*x + f*x^2] - x*#1] - 38*a^3*b*B*d*e^2*
f*Log[-Sqrt[d] + Sqrt[d + e*x + f*x^2] - x*#1] - 7*a^3*A*b*e^3*f*Log[-Sqrt[
d] + Sqrt[d + e*x + f*x^2] - x*#1] + 2*a^4*B*e^3*f*Log[-Sqrt[d] + Sqrt[d +
e*x + f*x^2] - x*#1] + 32*a^2*A*b^2*d^2*f^2*Log[-Sqrt[d] + Sqrt[d + e*x + f
*x^2] - x*#1] - 28*a^3*A*b*d*e*f^2*Log[-Sqrt[d] + Sqrt[d + e*x + f*x^2] - x
*#1] - 4*a*A*b^3*d^(3/2)*e^2*Log[x]*#1 + 4*a^2*b^2*B*d^(3/2)*e^2*Log[x]*#1
+ 4*a^2*A*b^2*Sqrt[d]*e^3*Log[x]*#1 - 2*a^3*b*B*Sqrt[d]*e^3*Log[x]*#1 + 16*
a^2*A*b^2*d^(3/2)*e*f*Log[x]*#1 - 16*a^3*b*B*d^(3/2)*e*f*Log[x]*#1 - 16*a^3
*A*b*Sqrt[d]*e^2*f*Log[x]*#1 + 8*a^4*B*Sqrt[d]*e^2*f*Log[x]*#1 + 4*a*A*b^3*
d^(3/2)*e^2*Log[-Sqrt[d] + Sqrt[d + e*x + f*x^2] - x*#1]*#1 - 4*a^2*b^2*B*d
^(3/2)*e^2*Log[-Sqrt[d] + Sqrt[d + e*x + f*x^2] - x*#1]*#1 - 4*a^2*A*b^2*Sq
rt[d]*e^3*Log[-Sqrt[d] + Sqrt[d + e*x + f*x^2] - x*#1]*#1 + 2*a^3*b*B*Sqrt[
d]*e^3*Log[-Sqrt[d] + Sqrt[d + e*x + f*x^2] - x*#1]*#1 - 16*a^2*A*b^2*d^(3/
2)*e*f*Log[-Sqrt[d] + Sqrt[d + e*x + f*x^2] - x*#1]*#1 + 16*a^3*b*B*d^(3/2)
*e*f*Log[-Sqrt[d] + Sqrt[d + e*x + f*x^2] - x*#1]*#1 + 16*a^3*A*b*Sqrt[d]*e
^2*f*Log[-Sqrt[d] + Sqrt[d + e*x + f*x^2] - x*#1]*#1 - 8*a^4*B*Sqrt[d]*e^2*
f*Log[-Sqrt[d] + Sqrt[d + e*x + f*x^2] - x*#1]*#1 - 2*a^2*A*b^2*d*e^2*Log[x
]*#1^2 + 2*a^3*b*B*d*e^2*Log[x]*#1^2 + a^3*A*b*e^3*Log[x]*#1^2 - 2*a^4*B*e^
3*Log[x]*#1^2 + 4*a^3*A*b*d*e*f*Log[x]*#1^2 + 2*a^2*A*b^2*d*e^2*Log[-Sqrt[d
] + Sqrt[d + e*x + f*x^2] - x*#1]*#1^2 - 2*a^3*b*B*d*e^2*Log[-Sqrt[d] + Sqr
t[d + e*x + f*x^2] - x*#1]*#1^2 - a^3*A*b*e^3*Log[-Sqrt[d] + Sqrt[d + e*x +
 f*x^2] - x*#1]*#1^2 + 2*a^4*B*e^3*Log[-Sqrt[d] + Sqrt[d + e*x + f*x^2] - x
*#1]*#1^2 - 4*a^3*A*b*d*e*f*Log[-Sqrt[d] + Sqrt[d + e*x + f*x^2] - x*#1]*#1
^2)/(-(b*Sqrt[d]*e*f) + b*e^2*#1 + 4*b*d*f*#1 - 2*a*e*f*#1 - 3*b*Sqrt[d]*e*
#1^2 + 2*a*e*#1^3) & ]/(a^3*(b*d - a*e)*(b*e - 4*a*f)))/(2*e^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1429 vs. $2(219) = 438$.

Time = 1.38 (sec) , antiderivative size = 1430, normalized size of antiderivative = 5.74

method	result	size
default	Expression too large to display	1430

```
[In] int((B*x+A)/(b*f*x^2+b*e*x+a*e)^2/(f*x^2+e*x+d)^(1/2),x,method=_RETURNVERBO
SE)
```

```
[Out] (2*A*f-B*e)/e/(4*a*f-b*e)/(-b*e*(4*a*f-b*e))^(1/2)/(-(a*e-b*d)/b)^(1/2)*ln(
(-2*(a*e-b*d)/b-(-b*e*(4*a*f-b*e))^(1/2)/b*(x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(
1/2))/b/f)+2*(-(a*e-b*d)/b)^(1/2)*((x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/
f)^2*f-(-b*e*(4*a*f-b*e))^(1/2)/b*(x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f
)-(a*e-b*d)/b)^(1/2))/(x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f))-(2*A*f-B*
e)/e/(4*a*f-b*e)/(-b*e*(4*a*f-b*e))^(1/2)/(-(a*e-b*d)/b)^(1/2)*ln((-2*(a*e-
```

```
b*d)/b+(-b*e*(4*a*f-b*e))^(1/2)/b*(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/
f)+2*(-(a*e-b*d)/b)^(1/2)*((x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)^2*f+
(-b*e*(4*a*f-b*e))^(1/2)/b*(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)-(a*e
-b*d)/b)^(1/2))/(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f))-1/2/f*(2*A*b*f
-B*b*e-B*(-b*e*(4*a*f-b*e))^(1/2))/e/(4*a*f-b*e)/b^2*(1/(a*e-b*d)*b/(x+1/2*
(b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)*((x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(1/2))/
b/f)^2*f-(-b*e*(4*a*f-b*e))^(1/2)/b*(x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(1/2))/b
/f)-(a*e-b*d)/b)^(1/2)+1/2*(-b*e*(4*a*f-b*e))^(1/2)/(a*e-b*d)/(-(a*e-b*d)/b
)^(1/2)*ln((-2*(a*e-b*d)/b-(-b*e*(4*a*f-b*e))^(1/2)/b*(x+1/2*(b*e+(-b*e*(4*
a*f-b*e))^(1/2))/b/f)+2*(-(a*e-b*d)/b)^(1/2)*((x+1/2*(b*e+(-b*e*(4*a*f-b*e)
)^(1/2))/b/f)^2*f-(-b*e*(4*a*f-b*e))^(1/2)/b*(x+1/2*(b*e+(-b*e*(4*a*f-b*e))
^(1/2))/b/f)-(a*e-b*d)/b)^(1/2))/(x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)
))-1/2/f*(2*A*b*f-B*b*e+B*(-b*e*(4*a*f-b*e))^(1/2))/e/(4*a*f-b*e)/b^2*(1/(a
*e-b*d)*b/(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)*((x-1/2*(-b*e+(-b*e*(
4*a*f-b*e))^(1/2))/b/f)^2*f+(-b*e*(4*a*f-b*e))^(1/2)/b*(x-1/2*(-b*e+(-b*e*(
4*a*f-b*e))^(1/2))/b/f)-(a*e-b*d)/b)^(1/2)-1/2*(-b*e*(4*a*f-b*e))^(1/2)/(a*
e-b*d)/(-(a*e-b*d)/b)^(1/2)*ln((-2*(a*e-b*d)/b+(-b*e*(4*a*f-b*e))^(1/2)/b*(
x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)+2*(-(a*e-b*d)/b)^(1/2)*((x-1/2*(
-b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)^2*f+(-b*e*(4*a*f-b*e))^(1/2)/b*(x-1/2*(
-b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)-(a*e-b*d)/b)^(1/2))/(x-1/2*(-b*e+(-b*e*
(4*a*f-b*e))^(1/2))/b/f)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx}{\sqrt{d+ex+fx^2}\,(ae+bex+bfx^2)^2}\,dx=\text{Timed out}$$

```
[In] integrate((B*x+A)/(b*f*x^2+b*e*x+a*e)^2/(f*x^2+e*x+d)^(1/2),x, algorithm="f
ricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx}{\sqrt{d+ex+fx^2}\,(ae+bex+bfx^2)^2}\,dx=\text{Timed out}$$

```
[In] integrate((B*x+A)/(b*f*x**2+b*e*x+a*e)**2/(f*x**2+e*x+d)**(1/2),x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int \frac{A+Bx}{\sqrt{d+ex+fx^2}\,(ae+bex+bfx^2)^2}\,dx=\int \frac{Bx+A}{(bfx^2+bex+ae)^2\sqrt{fx^2+ex+d}}\,dx$$

```
[In] integrate((B*x+A)/(b*f*x^2+b*e*x+a*e)^2/(f*x^2+e*x+d)^(1/2),x, algorithm="m
axima")
[Out] integrate((B*x + A)/((b*f*x^2 + b*e*x + a*e)^2*sqrt(f*x^2 + e*x + d)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5022 vs. $2(218) = 436$.

Time = 0.53 (sec) , antiderivative size = 5022, normalized size of antiderivative = 20.17

$$\int \frac{A+Bx}{\sqrt{d+ex+fx^2}\,(ae+bex+bfx^2)^2}\,dx=\text{Too large to display}$$

```
[In] integrate((B*x+A)/(b*f*x^2+b*e*x+a*e)^2/(f*x^2+e*x+d)^(1/2),x, algorithm="g
iac")
[Out] -1/2*((B*b*e^2*(e/sqrt(f) - sqrt((b*e^2*f + 4*b*d*f^2 - 8*a*e*f^2 + 4*sqrt(
b^2*d*e^2*f - a*b*e^3*f - 4*a*b*d*e*f^2 + 4*a^2*e^2*f^2)*f)/(b*f^2)))^2 - 4
*B*a*e*f*(e/sqrt(f) - sqrt((b*e^2*f + 4*b*d*f^2 - 8*a*e*f^2 + 4*sqrt(b^2*d*
e^2*f - a*b*e^3*f - 4*a*b*d*e*f^2 + 4*a^2*e^2*f^2)*f)/(b*f^2)))^2 + 8*B*b*d
*e*sqrt(f)*(e/sqrt(f) - sqrt((b*e^2*f + 4*b*d*f^2 - 8*a*e*f^2 + 4*sqrt(b^2*
d*e^2*f - a*b*e^3*f - 4*a*b*d*e*f^2 + 4*a^2*e^2*f^2)*f)/(b*f^2))) - 8*B*a*e
^2*sqrt(f)*(e/sqrt(f) - sqrt((b*e^2*f + 4*b*d*f^2 - 8*a*e*f^2 + 4*sqrt(b^2*
d*e^2*f - a*b*e^3*f - 4*a*b*d*e*f^2 + 4*a^2*e^2*f^2)*f)/(b*f^2))) - 4*A*b*e
^2*sqrt(f)*(e/sqrt(f) - sqrt((b*e^2*f + 4*b*d*f^2 - 8*a*e*f^2 + 4*sqrt(b^2*
d*e^2*f - a*b*e^3*f - 4*a*b*d*e*f^2 + 4*a^2*e^2*f^2)*f)/(b*f^2))) - 16*A*b*
d*f^(3/2)*(e/sqrt(f) - sqrt((b*e^2*f + 4*b*d*f^2 - 8*a*e*f^2 + 4*sqrt(b^2*d
*e^2*f - a*b*e^3*f - 4*a*b*d*e*f^2 + 4*a^2*e^2*f^2)*f)/(b*f^2))) + 32*A*a*e
*f^(3/2)*(e/sqrt(f) - sqrt((b*e^2*f + 4*b*d*f^2 - 8*a*e*f^2 + 4*sqrt(b^2*d*
e^2*f - a*b*e^3*f - 4*a*b*d*e*f^2 + 4*a^2*e^2*f^2)*f)/(b*f^2))) - 12*B*b*d*
e^2 + 8*B*a*e^3 + 4*A*b*e^3 + 16*B*a*d*e*f + 16*A*b*d*e*f - 32*A*a*e^2*f)*l
og(-sqrt(f)*x + sqrt(f*x^2 + e*x + d) - 1/2*e/sqrt(f) + 1/2*sqrt((b*e^2*f +
 4*b*d*f^2 - 8*a*e*f^2 + 4*sqrt(b^2*d*e^2*f - a*b*e^3*f - 4*a*b*d*e*f^2 + 4
*a^2*e^2*f^2)*f)/(b*f^2)))/(b*f*(e/sqrt(f) - sqrt((b*e^2*f + 4*b*d*f^2 - 8*
a*e*f^2 + 4*sqrt(b^2*d*e^2*f - a*b*e^3*f - 4*a*b*d*e*f^2 + 4*a^2*e^2*f^2)*f
)/(b*f^2)))^3 - 3*b*e*sqrt(f)*(e/sqrt(f) - sqrt((b*e^2*f + 4*b*d*f^2 - 8*a*
e*f^2 + 4*sqrt(b^2*d*e^2*f - a*b*e^3*f - 4*a*b*d*e*f^2 + 4*a^2*e^2*f^2)*f)/
(b*f^2)))^2 + 4*b*d*e*sqrt(f) - 8*a*e^2*sqrt(f) + 2*(b*e^2 - 2*b*d*f + 4*a*
e*f)*(e/sqrt(f) - sqrt((b*e^2*f + 4*b*d*f^2 - 8*a*e*f^2 + 4*sqrt(b^2*d*e^2*
```

f - a*b*e^3*f - 4*a*b*d*e*f^2 + 4*a^2*e^2*f^2)*f)/(b*f^2)))) + (B*b*e^2*(e/
sqrt(f) + sqrt((b*e^2*f + 4*b*d*f^2 - 8*a*e*f^2 + 4*sqrt(b^2*d*e^2*f - a*b*
e^3*f - 4*a*b*d*e*f^2 + 4*a^2*e^2*f^2)*f)/(b*f^2)))^2 - 4*B*a*e*f*(e/sqrt(f
) + sqrt((b*e^2*f + 4*b*d*f^2 - 8*a*e*f^2 + 4*sqrt(b^2*d*e^2*f - a*b*e^3*f
- 4*a*b*d*e*f^2 + 4*a^2*e^2*f^2)*f)/(b*f^2)))^2 + 8*B*b*d*e*sqrt(f)*(e/sqrt
(f) + sqrt((b*e^2*f + 4*b*d*f^2 - 8*a*e*f^2 + 4*sqrt(b^2*d*e^2*f - a*b*e^3*
f - 4*a*b*d*e*f^2 + 4*a^2*e^2*f^2)*f)/(b*f^2))) - 8*B*a*e^2*sqrt(f)*(e/sqrt
(f) + sqrt((b*e^2*f + 4*b*d*f^2 - 8*a*e*f^2 + 4*sqrt(b^2*d*e^2*f - a*b*e^3*
f - 4*a*b*d*e*f^2 + 4*a^2*e^2*f^2)*f)/(b*f^2))) - 4*A*b*e^2*sqrt(f)*(e/sqrt
(f) + sqrt((b*e^2*f + 4*b*d*f^2 - 8*a*e*f^2 + 4*sqrt(b^2*d*e^2*f - a*b*e^3*
f - 4*a*b*d*e*f^2 + 4*a^2*e^2*f^2)*f)/(b*f^2))) - 16*A*b*d*f^(3/2)*(e/sqrt(
f) + sqrt((b*e^2*f + 4*b*d*f^2 - 8*a*e*f^2 + 4*sqrt(b^2*d*e^2*f - a*b*e^3*f
 - 4*a*b*d*e*f^2 + 4*a^2*e^2*f^2)*f)/(b*f^2))) + 32*A*a*e*f^(3/2)*(e/sqrt(f
) + sqrt((b*e^2*f + 4*b*d*f^2 - 8*a*e*f^2 + 4*sqrt(b^2*d*e^2*f - a*b*e^3*f
- 4*a*b*d*e*f^2 + 4*a^2*e^2*f^2)*f)/(b*f^2))) - 12*B*b*d*e^2 + 8*B*a*e^3 +
4*A*b*e^3 + 16*B*a*d*e*f + 16*A*b*d*e*f - 32*A*a*e^2*f)*log(-sqrt(f)*x + sq
rt(f*x^2 + e*x + d) - 1/2*e/sqrt(f) - 1/2*sqrt((b*e^2*f + 4*b*d*f^2 - 8*a*e
*f^2 + 4*sqrt(b^2*d*e^2*f - a*b*e^3*f - 4*a*b*d*e*f^2 + 4*a^2*e^2*f^2)*f)/(
b*f^2)))/(b*f*(e/sqrt(f) + sqrt((b*e^2*f + 4*b*d*f^2 - 8*a*e*f^2 + 4*sqrt(b
^2*d*e^2*f - a*b*e^3*f - 4*a*b*d*e*f^2 + 4*a^2*e^2*f^2)*f)/(b*f^2)))^3 - 3*
b*e*sqrt(f)*(e/sqrt(f) + sqrt((b*e^2*f + 4*b*d*f^2 - 8*a*e*f^2 + 4*sqrt(b^2
*d*e^2*f - a*b*e^3*f - 4*a*b*d*e*f^2 + 4*a^2*e^2*f^2)*f)/(b*f^2)))^2 + 4*b*
d*e*sqrt(f) - 8*a*e^2*sqrt(f) + 2*(b*e^2 - 2*b*d*f + 4*a*e*f)*(e/sqrt(f) +
sqrt((b*e^2*f + 4*b*d*f^2 - 8*a*e*f^2 + 4*sqrt(b^2*d*e^2*f - a*b*e^3*f - 4*
a*b*d*e*f^2 + 4*a^2*e^2*f^2)*f)/(b*f^2)))) + (B*b*e^2*(e/sqrt(f) - sqrt((b*
e^2*f + 4*b*d*f^2 - 8*a*e*f^2 - 4*sqrt(b^2*d*e^2*f - a*b*e^3*f - 4*a*b*d*e*
f^2 + 4*a^2*e^2*f^2)*f)/(b*f^2)))^2 - 4*B*a*e*f*(e/sqrt(f) - sqrt((b*e^2*f
+ 4*b*d*f^2 - 8*a*e*f^2 - 4*sqrt(b^2*d*e^2*f - a*b*e^3*f - 4*a*b*d*e*f^2 +
4*a^2*e^2*f^2)*f)/(b*f^2)))^2 + 8*B*b*d*e*sqrt(f)*(e/sqrt(f) - sqrt((b*e^2*
f + 4*b*d*f^2 - 8*a*e*f^2 - 4*sqrt(b^2*d*e^2*f - a*b*e^3*f - 4*a*b*d*e*f^2
+ 4*a^2*e^2*f^2)*f)/(b*f^2))) - 8*B*a*e^2*sqrt(f)*(e/sqrt(f) - sqrt((b*e^2*
f + 4*b*d*f^2 - 8*a*e*f^2 - 4*sqrt(b^2*d*e^2*f - a*b*e^3*f - 4*a*b*d*e*f^2
+ 4*a^2*e^2*f^2)*f)/(b*f^2))) - 4*A*b*e^2*sqrt(f)*(e/sqrt(f) - sqrt((b*e^2*
f + 4*b*d*f^2 - 8*a*e*f^2 - 4*sqrt(b^2*d*e^2*f - a*b*e^3*f - 4*a*b*d*e*f^2
+ 4*a^2*e^2*f^2)*f)/(b*f^2))) - 16*A*b*d*f^(3/2)*(e/sqrt(f) - sqrt((b*e^2*f
 + 4*b*d*f^2 - 8*a*e*f^2 - 4*sqrt(b^2*d*e^2*f - a*b*e^3*f - 4*a*b*d*e*f^2 +
 4*a^2*e^2*f^2)*f)/(b*f^2))) + 32*A*a*e*f^(3/2)*(e/sqrt(f) - sqrt((b*e^2*f
+ 4*b*d*f^2 - 8*a*e*f^2 - 4*sqrt(b^2*d*e^2*f - a*b*e^3*f - 4*a*b*d*e*f^2 +
4*a^2*e^2*f^2)*f)/(b*f^2))) - 12*B*b*d*e^2 + 8*B*a*e^3 + 4*A*b*e^3 + 16*B*a
*d*e*f + 16*A*b*d*e*f - 32*A*a*e^2*f)*log(-sqrt(f)*x + sqrt(f*x^2 + e*x + d
) - 1/2*e/sqrt(f) + 1/2*sqrt((b*e^2*f + 4*b*d*f^2 - 8*a*e*f^2 - 4*sqrt(b^2*
d*e^2*f - a*b*e^3*f - 4*a*b*d*e*f^2 + 4*a^2*e^2*f^2)*f)/(b*f^2)))/(b*f*(e/s
qrt(f) - sqrt((b*e^2*f + 4*b*d*f^2 - 8*a*e*f^2 - 4*sqrt(b^2*d*e^2*f - a*b*e
^3*f - 4*a*b*d*e*f^2 + 4*a^2*e^2*f^2)*f)/(b*f^2)))^3 - 3*b*e*sqrt(f)*(e/sqr
t(f) - sqrt((b*e^2*f + 4*b*d*f^2 - 8*a*e*f^2 - 4*sqrt(b^2*d*e^2*f - a*b*e^3

```
*f - 4*a*b*d*e*f^2 + 4*a^2*e^2*f^2)*f)/(b*f^2)))^2 + 4*b*d*e*sqrt(f) - 8*a*
e^2*sqrt(f) + 2*(b*e^2 - 2*b*d*f + 4*a*e*f)*(e/sqrt(f) - sqrt((b*e^2*f + 4*
b*d*f^2 - 8*a*e*f^2 - 4*sqrt(b^2*d*e^2*f - a*b*e^3*f - 4*a*b*d*e*f^2 + 4*a^
2*e^2*f^2)*f)/(b*f^2)))) + (B*b*e^2*(e/sqrt(f) + sqrt((b*e^2*f + 4*b*d*f^2
- 8*a*e*f^2 - 4*sqrt(b^2*d*e^2*f - a*b*e^3*f - 4*a*b*d*e*f^2 + 4*a^2*e^2*f^
2)*f)/(b*f^2)))^2 - 4*B*a*e*f*(e/sqrt(f) + sqrt((b*e^2*f + 4*b*d*f^2 - 8*a*
e*f^2 - 4*sqrt(b^2*d*e^2*f - a*b*e^3*f - 4*a*b*d*e*f^2 + 4*a^2*e^2*f^2)*f)/
(b*f^2)))^2 + 8*B*b*d*e*sqrt(f)*(e/sqrt(f) + sqrt((b*e^2*f + 4*b*d*f^2 - 8*
a*e*f^2 - 4*sqrt(b^2*d*e^2*f - a*b*e^3*f - 4*a*b*d*e*f^2 + 4*a^2*e^2*f^2)*f
)/(b*f^2))) - 8*B*a*e^2*sqrt(f)*(e/sqrt(f) + sqrt((b*e^2*f + 4*b*d*f^2 - 8*
a*e*f^2 - 4*sqrt(b^2*d*e^2*f - a*b*e^3*f - 4*a*b*d*e*f^2 + 4*a^2*e^2*f^2)*f
)/(b*f^2))) - 4*A*b*e^2*sqrt(f)*(e/sqrt(f) + sqrt((b*e^2*f + 4*b*d*f^2 - 8*
a*e*f^2 - 4*sqrt(b^2*d*e^2*f - a*b*e^3*f - 4*a*b*d*e*f^2 + 4*a^2*e^2*f^2)*f
)/(b*f^2))) - 16*A*b*d*f^(3/2)*(e/sqrt(f) + sqrt((b*e^2*f + 4*b*d*f^2 - 8*a
*e*f^2 - 4*sqrt(b^2*d*e^2*f - a*b*e^3*f - 4*a*b*d*e*f^2 + 4*a^2*e^2*f^2)*f)
/(b*f^2))) + 32*A*a*e*f^(3/2)*(e/sqrt(f) + sqrt((b*e^2*f + 4*b*d*f^2 - 8*a*
e*f^2 - 4*sqrt(b^2*d*e^2*f - a*b*e^3*f - 4*a*b*d*e*f^2 + 4*a^2*e^2*f^2)*f)/
(b*f^2))) - 12*B*b*d*e^2 + 8*B*a*e^3 + 4*A*b*e^3 + 16*B*a*d*e*f + 16*A*b*d*
e*f - 32*A*a*e^2*f)*log(-sqrt(f)*x + sqrt(f*x^2 + e*x + d) - 1/2*e/sqrt(f)
- 1/2*sqrt((b*e^2*f + 4*b*d*f^2 - 8*a*e*f^2 - 4*sqrt(b^2*d*e^2*f - a*b*e^3*
f - 4*a*b*d*e*f^2 + 4*a^2*e^2*f^2)*f)/(b*f^2)))/(b*f*(e/sqrt(f) + sqrt((b*e
^2*f + 4*b*d*f^2 - 8*a*e*f^2 - 4*sqrt(b^2*d*e^2*f - a*b*e^3*f - 4*a*b*d*e*f
^2 + 4*a^2*e^2*f^2)*f)/(b*f^2)))^3 - 3*b*e*sqrt(f)*(e/sqrt(f) + sqrt((b*e^2
*f + 4*b*d*f^2 - 8*a*e*f^2 - 4*sqrt(b^2*d*e^2*f - a*b*e^3*f - 4*a*b*d*e*f^2
 + 4*a^2*e^2*f^2)*f)/(b*f^2)))^2 + 4*b*d*e*sqrt(f) - 8*a*e^2*sqrt(f) + 2*(b
*e^2 - 2*b*d*f + 4*a*e*f)*(e/sqrt(f) + sqrt((b*e^2*f + 4*b*d*f^2 - 8*a*e*f^
2 - 4*sqrt(b^2*d*e^2*f - a*b*e^3*f - 4*a*b*d*e*f^2 + 4*a^2*e^2*f^2)*f)/(b*f
^2)))))/(b^2*d*e^2 - a*b*e^3 - 4*a*b*d*e*f + 4*a^2*e^2*f) + ((sqrt(f)*x - s
qrt(f*x^2 + e*x + d))^3*B*b*e^2*f - 4*(sqrt(f)*x - sqrt(f*x^2 + e*x + d))^3
*B*a*e*f^2 + (sqrt(f)*x - sqrt(f*x^2 + e*x + d))^2*B*b*e^3*sqrt(f) - 2*(sqr
t(f)*x - sqrt(f*x^2 + e*x + d))^2*B*b*d*e*f^(3/2) - 2*(sqrt(f)*x - sqrt(f*x
^2 + e*x + d))^2*B*a*e^2*f^(3/2) + (sqrt(f)*x - sqrt(f*x^2 + e*x + d))^2*A*
b*e^2*f^(3/2) + 4*(sqrt(f)*x - sqrt(f*x^2 + e*x + d))^2*A*b*d*f^(5/2) - 8*(
sqrt(f)*x - sqrt(f*x^2 + e*x + d))^2*A*a*e*f^(5/2) - (sqrt(f)*x - sqrt(f*x^
2 + e*x + d))*B*b*d*e^2*f + 2*(sqrt(f)*x - sqrt(f*x^2 + e*x + d))*B*a*e^3*f
 + (sqrt(f)*x - sqrt(f*x^2 + e*x + d))*A*b*e^3*f - 4*(sqrt(f)*x - sqrt(f*x^
2 + e*x + d))*B*a*d*e*f^2 + 4*(sqrt(f)*x - sqrt(f*x^2 + e*x + d))*A*b*d*e*f
^2 - 8*(sqrt(f)*x - sqrt(f*x^2 + e*x + d))*A*a*e^2*f^2 - B*b*d*e^3*sqrt(f)
+ B*a*e^4*sqrt(f) + 2*B*b*d^2*e*f^(3/2) - 2*B*a*d*e^2*f^(3/2) + 3*A*b*d*e^2
*f^(3/2) - 2*A*a*e^3*f^(3/2) - 4*A*b*d^2*f^(5/2))/(((sqrt(f)*x - sqrt(f*x^2
 + e*x + d))^4*b*f + 2*(sqrt(f)*x - sqrt(f*x^2 + e*x + d))^3*b*e*sqrt(f) +
(sqrt(f)*x - sqrt(f*x^2 + e*x + d))^2*b*e^2 - 2*(sqrt(f)*x - sqrt(f*x^2 + e
*x + d))^2*b*d*f + 4*(sqrt(f)*x - sqrt(f*x^2 + e*x + d))^2*a*e*f - 2*(sqrt(
f)*x - sqrt(f*x^2 + e*x + d))*b*d*e*sqrt(f) + 4*(sqrt(f)*x - sqrt(f*x^2 + e
*x + d))*a*e^2*sqrt(f) - b*d*e^2 + a*e^3 + b*d^2*f)*(b^2*d*e^2*f - a*b*e^3*
```

```
f - 4*a*b*d*e*f^2 + 4*a^2*e^2*f^2))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{\sqrt{d+ex+fx^2}\,(ae+bex+bfx^2)^2}\,dx = \int \frac{A+B\,x}{\left(b\,f\,x^2+b\,e\,x+a\,e\right)^2\,\sqrt{f\,x^2+e\,x+d}}\,dx$$

```
[In] int((A + B*x)/((a*e + b*e*x + b*f*x^2)^2*(d + e*x + f*x^2)^(1/2)),x)
[Out] int((A + B*x)/((a*e + b*e*x + b*f*x^2)^2*(d + e*x + f*x^2)^(1/2)), x)
```

3.36 $\int \frac{(g+hx)\sqrt{a+bx+cx^2}}{(ad+bdx+cdx^2)^2} \, dx$

Optimal result

Integrand size = 36, antiderivative size = 48

$$\int \frac{(g+hx)\sqrt{a+bx+cx^2}}{(ad+bdx+cdx^2)^2} \, dx = -\frac{2(bg-2ah+(2cg-bh)x)}{(b^2-4ac)\, d^2\sqrt{a+bx+cx^2}}$$

```
[Out] -2*(b*g-2*a*h+(-b*h+2*c*g)*x)/(-4*a*c+b^2)/d^2/(c*x^2+b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {1012, 650}

$$\int \frac{(g+hx)\sqrt{a+bx+cx^2}}{(ad+bdx+cdx^2)^2} \, dx = -\frac{2(-2ah+x(2cg-bh)+bg)}{d^2\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

```
[In] Int[((g + h*x)*Sqrt[a + b*x + c*x^2])/(a*d + b*d*x + c*d*x^2)^2,x]
```

```
[Out] (-2*(b*g - 2*a*h + (2*c*g - b*h)*x))/((b^2 - 4*a*c)*d^2*Sqrt[a + b*x + c*x^
2])
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[-2*((b*d - 2*a*e + (2*c*d - b*e)*x)/((b^2 - 4*a*c)*Sqrt[a + b*x
+ c*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rule 1012

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_
) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Dist[(c/f)^p, Int[(g + h*
x)^m*(d + e*x + f*x^2)^(p + q), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, p,
 q}, x] && EqQ[c*d - a*f, 0] && EqQ[b*d - a*e, 0] && (IntegerQ[p] || GtQ[c/
f, 0]) && ( !IntegerQ[q] || LeafCount[d + e*x + f*x^2] <= LeafCount[a + b*x
 + c*x^2])
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\int \frac{g+hx}{(a+bx+cx^2)^{3/2}}\,dx}{d^2}\\ &= -\frac{2(bg-2ah+(2cg-bh)x)}{(b^2-4ac)\,d^2\sqrt{a+bx+cx^2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.96

$$\int \frac{(g+hx)\sqrt{a+bx+cx^2}}{(ad+bdx+cdx^2)^2}\,dx = \frac{-2bg+4ah-4cgx+2bhx}{(b^2-4ac)\,d^2\sqrt{a+x(b+cx)}}$$

```
[In] Integrate[((g + h*x)*Sqrt[a + b*x + c*x^2])/(a*d + b*d*x + c*d*x^2)^2,x]
[Out] (-2*b*g + 4*a*h - 4*c*g*x + 2*b*h*x)/((b^2 - 4*a*c)*d^2*Sqrt[a + x*(b + c*x
)])
```

Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

method	result	size
gosper	$-\frac{2(bhx-2cgx+2ah-bg)}{\sqrt{c\,x^2+bx+a}\,d^2(4ac-b^2)}$	48
trager	$-\frac{2(bhx-2cgx+2ah-bg)}{\sqrt{c\,x^2+bx+a}\,d^2(4ac-b^2)}$	48
default	$\frac{\frac{2g(2cx+b)}{\left(4ac-b^2\right)\sqrt{c\,x^2+bx+a}}+h\left(-\frac{1}{c\sqrt{c\,x^2+bx+a}}-\frac{b(2cx+b)}{c\left(4ac-b^2\right)\sqrt{c\,x^2+bx+a}}\right)}{d^2}$	95

```
[In] int((h*x+g)*(c*x^2+b*x+a)^(1/2)/(c*d*x^2+b*d*x+a*d)^2,x,method=_RETURNVERBO
SE)
[Out] -2/(c*x^2+b*x+a)^(1/2)*(b*h*x-2*c*g*x+2*a*h-b*g)/d^2/(4*a*c-b^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.52 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.77

$$\int \frac{(g+hx)\sqrt{a+bx+cx^2}}{(ad+bdx+cdx^2)^2}\,dx = -\frac{2\sqrt{cx^2+bx+a}(bg-2\,ah+(2\,cg-bh)x)}{(b^2c-4\,ac^2)d^2x^2+(b^3-4\,abc)d^2x+(ab^2-4\,a^2c)d^2}$$

```
[In] integrate((h*x+g)*(c*x^2+b*x+a)^(1/2)/(c*d*x^2+b*d*x+a*d)^2,x, algorithm="f
ricas")
```

```
[Out] -2*sqrt(c*x^2 + b*x + a)*(b*g - 2*a*h + (2*c*g - b*h)*x)/((b^2*c - 4*a*c^2)
*d^2*x^2 + (b^3 - 4*a*b*c)*d^2*x + (a*b^2 - 4*a^2*c)*d^2)
```

Sympy [F]

$$\int \frac{(g+hx)\sqrt{a+bx+cx^2}}{(ad+bdx+cdx^2)^2}\,dx = \frac{\int \frac{g}{a\sqrt{a+bx+cx^2}+bx\sqrt{a+bx+cx^2}+cx^2\sqrt{a+bx+cx^2}}\,dx + \int \frac{hx}{a\sqrt{a+bx+cx^2}+bx\sqrt{a+bx+cx^2}+cx^2\sqrt{a+bx+cx^2}}\,dx}{d^2}$$

```
[In] integrate((h*x+g)*(c*x**2+b*x+a)**(1/2)/(c*d*x**2+b*d*x+a*d)**2,x)
```

```
[Out] (Integral(g/(a*sqrt(a + b*x + c*x**2) + b*x*sqrt(a + b*x + c*x**2) + c*x**2
*sqrt(a + b*x + c*x**2)), x) + Integral(h*x/(a*sqrt(a + b*x + c*x**2) + b*x
*sqrt(a + b*x + c*x**2) + c*x**2*sqrt(a + b*x + c*x**2)), x))/d**2
```

Maxima [F]

$$\int \frac{(g+hx)\sqrt{a+bx+cx^2}}{(ad+bdx+cdx^2)^2}\,dx = \int \frac{\sqrt{cx^2+bx+a}(hx+g)}{(cdx^2+bdx+ad)^2}\,dx$$

```
[In] integrate((h*x+g)*(c*x^2+b*x+a)^(1/2)/(c*d*x^2+b*d*x+a*d)^2,x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)*(h*x + g)/(c*d*x^2 + b*d*x + a*d)^2, x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.69

$$\int \frac{(g+hx)\sqrt{a+bx+cx^2}}{(ad+bdx+cdx^2)^2}\,dx = -\frac{2\left(\frac{(2\,cd^2g-bd^2h)x}{b^2d^4-4\,acd^4}+\frac{bd^2g-2\,ad^2h}{b^2d^4-4\,acd^4}\right)}{\sqrt{cx^2+bx+a}}$$

```
[In] integrate((h*x+g)*(c*x^2+b*x+a)^(1/2)/(c*d*x^2+b*d*x+a*d)^2,x, algorithm="g
iac")
```

```
[Out] -2*((2*c*d^2*g - b*d^2*h)*x/(b^2*d^4 - 4*a*c*d^4) + (b*d^2*g - 2*a*d^2*h)/(
b^2*d^4 - 4*a*c*d^4))/sqrt(c*x^2 + b*x + a)
```

Mupad [B] (verification not implemented)

Time = 12.42 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.02

$$\int \frac{(g+hx)\sqrt{a+bx+cx^2}}{(ad+bdx+cdx^2)^2}\,dx = \frac{4\,a\,h-2\,b\,g+2\,b\,h\,x-4\,c\,g\,x}{(b^2\,d^2-4\,a\,c\,d^2)\,\sqrt{c\,x^2+b\,x+a}}$$

```
[In] int(((g + h*x)*(a + b*x + c*x^2)^(1/2))/(a*d + b*d*x + c*d*x^2)^2,x)
```

```
[Out] (4*a*h - 2*b*g + 2*b*h*x - 4*c*g*x)/((b^2*d^2 - 4*a*c*d^2)*(a + b*x + c*x^2
)^(1/2))
```

3.37 $\int \frac{3+2x}{\sqrt{-3-4x-x^2}(3+4x+2x^2)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 17

$$\int \frac{3+2x}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = \operatorname{arctanh}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right)$$

```
[Out] arctanh(x/(-x^2-4*x-3)^(1/2))
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1041, 212}

$$\int \frac{3+2x}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = \operatorname{arctanh}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)$$

```
[In] Int[(3 + 2*x)/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

```
[Out] ArcTanh[x/Sqrt[-3 - 4*x - x^2]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1041

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_
.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[g, Subst[Int[1/(a + (c*d - a*f)
*x^2), x], x, x/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h
```

```
}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0] &&
 EqQ[2*h*d - g*e, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= 3\text{Subst}\left(\int \frac{1}{3-3x^2}\,dx, x, \frac{x}{\sqrt{-3-4x-x^2}}\right) \\ &= \tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{3+2x}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = \operatorname{arctanh}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right)$$

```
[In] Integrate[(3 + 2*x)/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
[Out] ArcTanh[x/Sqrt[-3 - 4*x - x^2]]
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

method	result	size
pseudoelliptic	$\operatorname{arctanh}\left(\frac{\sqrt{-x^2-4x-3}}{x}\right)$	18
trager	$\frac{\ln\left(\frac{2x\sqrt{-x^2-4x-3}-4x-3}{2x^2+4x+3}\right)}{2}$	37
default	$-\frac{\sqrt{3}\,\sqrt{4}\,\sqrt{\frac{3x^2}{\left(-\frac{3}{2}-x\right)^2}-12}\,\operatorname{arctanh}\left(\frac{3x}{\left(-\frac{3}{2}-x\right)\sqrt{\frac{3x^2}{\left(-\frac{3}{2}-x\right)^2}-12}}\right)}{6\sqrt{\frac{\frac{x^2}{\left(-\frac{3}{2}-x\right)^2}-4}{\left(1+\frac{x}{-\frac{3}{2}-x}\right)^2}}\left(1+\frac{x}{-\frac{3}{2}-x}\right)}$	94

```
[In] int((3+2*x)/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x,method=_RETURNVERBOSE)
[Out] arctanh((-x^2-4*x-3)^(1/2)/x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. $2(15) = 30$.

Time = 0.31 (sec) , antiderivative size = 56, normalized size of antiderivative = 3.29

$$\int \frac{3+2x}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = -\frac{1}{4}\log\left(-\frac{2\sqrt{-x^2-4\,x-3}x+4\,x+3}{x^2}\right) + \frac{1}{4}\log\left(\frac{2\sqrt{-x^2-4\,x-3}x-4\,x-3}{x^2}\right)$$

```
[In] integrate((3+2*x)/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="fricas")
[Out] -1/4*log(-(2*sqrt(-x^2 - 4*x - 3)*x + 4*x + 3)/x^2) + 1/4*log((2*sqrt(-x^2
- 4*x - 3)*x - 4*x - 3)/x^2)
```

Sympy [F]

$$\int \frac{3+2x}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = \int \frac{2x+3}{\sqrt{-(x+1)(x+3)}\,(2x^2+4x+3)}\,dx$$

```
[In] integrate((3+2*x)/(2*x**2+4*x+3)/(-x**2-4*x-3)**(1/2),x)
[Out] Integral((2*x + 3)/(sqrt(-(x + 1)*(x + 3))*(2*x**2 + 4*x + 3)), x)
```

Maxima [F]

$$\int \frac{3+2x}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = \int \frac{2\,x+3}{(2\,x^2+4\,x+3)\sqrt{-x^2-4\,x-3}}\,dx$$

```
[In] integrate((3+2*x)/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="maxima")
[Out] integrate((2*x + 3)/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(15) = 30$.

Time = 0.28 (sec) , antiderivative size = 98, normalized size of antiderivative = 5.76

$$\int \frac{3+2x}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = \frac{1}{2}\log\left(\frac{2\left(\sqrt{-x^2-4\,x-3}-1\right)}{x+2} + \frac{3\left(\sqrt{-x^2-4\,x-3}-1\right)^2}{(x+2)^2}+1\right) - \frac{1}{2}\log\left(\frac{2\left(\sqrt{-x^2-4\,x-3}-1\right)}{x+2} + \frac{\left(\sqrt{-x^2-4\,x-3}-1\right)^2}{(x+2)^2}+3\right)$$

```
[In] integrate((3+2*x)/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="giac")
[Out] 1/2*log(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 3*(sqrt(-x^2 - 4*x - 3) - 1)
^2/(x + 2)^2 + 1) - 1/2*log(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + (sqrt(-x
^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{3+2x}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = \int \frac{2\,x+3}{\sqrt{-x^2-4\,x-3}\,\left(2\,x^2+4\,x+3\right)}\,dx$$

```
[In] int((2*x + 3)/((- 4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)),x)
[Out] int((2*x + 3)/((- 4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)), x)
```

3.38 $\int \frac{3+4x}{\sqrt{-3-4x-x^2}(3+4x+2x^2)}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 86

$$\int \frac{3+4x}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = \sqrt{2}\arctan\left(\frac{1-\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right) - \sqrt{2}\arctan\left(\frac{1+\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right) + \operatorname{arctanh}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right)$$

```
[Out] arctanh(x/(-x^2-4*x-3)^(1/2))+arctan(1/2*(1+(-3-x)/(-x^2-4*x-3)^(1/2))*2^(1
/2))*2^(1/2)-arctan(1/2*(1+(3+x)/(-x^2-4*x-3)^(1/2))*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.281, Rules used = {1042, 1000, 12, 1040, 1175, 632, 210, 1041, 212}

$$\int \frac{3+4x}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = \sqrt{2}\arctan\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right) - \sqrt{2}\arctan\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right) + \operatorname{arctanh}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)$$

```
[In] Int[(3 + 4*x)/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

```
[Out] Sqrt[2]*ArcTan[(1 - (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]] - Sqrt[2]*ArcTan
[(1 + (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]] + ArcTanh[x/Sqrt[-3 - 4*x - x^
2]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1000

```
Int[1/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*
(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f)
, 2]}, Dist[1/(2*q), Int[(c*d - a*f + q + (c*e - b*f)*x)/((a + b*x + c*x^2)
*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[(c*d - a*f - q + (c*e -
 b*f)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b,
 c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[c*e -
 b*f, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1040

```
Int[(x_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_.)*(x_) + (f_.)
*(x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[(1 - d*x^2)/(c*e - b*f - e*(2*
c*d - b*e + 2*a*f)*x^2 + d^2*(c*e - b*f)*x^4), x], x, (1 + (e + Sqrt[e^2 -
4*d*f])*(x/(2*d)))/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0]
```

Rule 1041

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_
.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[g, Subst[Int[1/(a + (c*d - a*f)
*x^2), x], x, x/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h
}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0] &&
 EqQ[2*h*d - g*e, 0]
```

Rule 1042

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_
.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-(2*h*d - g*e)/e, Int[1/((a + b
*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/e, Int[(2*d + e*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g
, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0]
 && NeQ[2*h*d - g*e, 0]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

integral

$$= -\left(3\int \frac{1}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx\right) - \int \frac{-6-4x}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx$$

$$= \frac{1}{2}\int \frac{-6-4x}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx - \frac{1}{2}\int -\frac{4x}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx + 6\text{Subst}\left(\int \frac{1}{3-3x^2}\,dx, x, \frac{x}{\sqrt{-3-4x-x^2}}\right)$$

$$= 2\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) + 2\int \frac{x}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx - 3\text{Subst}\left(\int \frac{1}{3-3x^2}\,dx, x, \frac{x}{\sqrt{-3-4x-x^2}}\right)$$

$$= \tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) + 16\text{Subst}\left(\int \frac{1+3x^2}{-4-8x^2-36x^4}\,dx, x, \frac{1+\frac{x}{3}}{\sqrt{-3-4x-x^2}}\right)$$

$$= \tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) - \frac{2}{3}\text{Subst}\left(\int \frac{1}{\frac{1}{3}-\frac{2x}{3}+x^2}\,dx, x, \frac{1+\frac{x}{3}}{\sqrt{-3-4x-x^2}}\right) - \frac{2}{3}\text{Subst}\left(\int \frac{1}{\frac{1}{3}+\frac{2x}{3}+x^2}\,dx, x, \frac{1+\frac{x}{3}}{\sqrt{-3-4x-x^2}}\right)$$

$$= \tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) + \frac{4}{3}\text{Subst}\left(\int \frac{1}{-\frac{8}{9}-x^2}\,dx, x, \frac{2}{3}\left(-1+\frac{3+x}{\sqrt{-3-4x-x^2}}\right)\right) + \frac{4}{3}\text{Subst}\left(\int \frac{1}{-\frac{8}{9}-x^2}\,dx, x, \frac{2}{3}\left(1+\frac{3+x}{\sqrt{-3-4x-x^2}}\right)\right)$$

$$= \sqrt{2}\tan^{-1}\left(\frac{1-\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right) - \sqrt{2}\tan^{-1}\left(\frac{1+\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right) + \tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right)$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.59

$$\int \frac{3+4x}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = -\sqrt{2}\arctan\left(\frac{3+2x}{\sqrt{2}\sqrt{-3-4x-x^2}}\right) + \operatorname{arctanh}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right)$$

```
[In] Integrate[(3 + 4*x)/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
[Out] -(Sqrt[2]*ArcTan[(3 + 2*x)/(Sqrt[2]*Sqrt[-3 - 4*x - x^2])]) + ArcTanh[x/Sqr
t[-3 - 4*x - x^2]]
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.43

method	result
default	$\frac{\sqrt{3}\,\sqrt{4}\,\sqrt{\frac{3x^2}{\left(-\frac{3}{2}-x\right)^2}-12}\left(\sqrt{2}\,\arctan\left(\frac{\sqrt{\frac{3x^2}{\left(-\frac{3}{2}-x\right)^2}-12}\,\sqrt{2}}{6}\right)-\operatorname{arctanh}\left(\frac{3x}{\left(-\frac{3}{2}-x\right)\sqrt{\frac{3x^2}{\left(-\frac{3}{2}-x\right)^2}-12}}\right)\right)}{6\sqrt{\frac{\frac{x^2}{\left(-\frac{3}{2}-x\right)^2}-4}{\left(1+\frac{x}{-\frac{3}{2}-x}\right)^2}}\left(1+\frac{x}{-\frac{3}{2}-x}\right)}$
trager	$-\ln\left(\frac{4\,\text{RootOf}\left(4\_Z^2-4\_Z+3\right)^2 x+4\,\text{RootOf}\left(4\_Z^2-4\_Z+3\right)x+12\,\text{RootOf}\left(4\_Z^2-4\_Z+3\right)-6\sqrt{-x^2-4x-3}-3x-6}{2\,\text{RootOf}\left(4\_Z^2-4\_Z+3\right)x-3x-3}\right)$

```
[In] int((3+4*x)/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/6*3^(1/2)*4^(1/2)*(3*x^2/(-3/2-x)^2-12)^(1/2)*(2^(1/2)*arctan(1/6*(3*x^2/
(-3/2-x)^2-12)^(1/2)*2^(1/2))-arctanh(3*x/(-3/2-x)/(3*x^2/(-3/2-x)^2-12)^(1
/2)))/((x^2/(-3/2-x)^2-4)/(1+x/(-3/2-x))^2)^(1/2)/(1+x/(-3/2-x))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.53

$$\int \frac{3+4x}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = \frac{1}{2}\,\sqrt{2}\arctan\left(\frac{\sqrt{2}x+3\,\sqrt{2}\sqrt{-x^2-4\,x-3}}{2\,(2\,x+3)}\right) + \frac{1}{2}\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}x-3\,\sqrt{2}\sqrt{-x^2-4\,x-3}}{2\,(2\,x+3)}\right) - \frac{1}{4}\,\log\left(-\frac{2\,\sqrt{-x^2-4\,x-3}x+4\,x+3}{x^2}\right) + \frac{1}{4}\,\log\left(\frac{2\,\sqrt{-x^2-4\,x-3}x-4\,x-3}{x^2}\right)$$

```
[In] integrate((3+4*x)/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*(sqrt(2)*x + 3*sqrt(2)*sqrt(-x^2 - 4*x - 3))/(2*x +
3)) + 1/2*sqrt(2)*arctan(-1/2*(sqrt(2)*x - 3*sqrt(2)*sqrt(-x^2 - 4*x - 3))/
(2*x + 3)) - 1/4*log(-(2*sqrt(-x^2 - 4*x - 3)*x + 4*x + 3)/x^2) + 1/4*log((
2*sqrt(-x^2 - 4*x - 3)*x - 4*x - 3)/x^2)
```

Sympy [F]

$$\int \frac{3+4x}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = \int \frac{4x+3}{\sqrt{-(x+1)\,(x+3)}\,(2x^2+4x+3)}\,dx$$

```
[In] integrate((3+4*x)/(2*x**2+4*x+3)/(-x**2-4*x-3)**(1/2),x)
```

```
[Out] Integral((4*x + 3)/(sqrt(-(x + 1)*(x + 3))*(2*x**2 + 4*x + 3)), x)
```

Maxima [F]

$$\int \frac{3+4x}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = \int \frac{4\,x+3}{(2\,x^2+4\,x+3)\sqrt{-x^2-4\,x-3}}\,dx$$

```
[In] integrate((3+4*x)/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((4*x + 3)/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(73) = 146$.

Time = 0.30 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.90

$$\int \frac{3+4x}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = \sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{3\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+1\right)\right) + \sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{\sqrt{-x^2-4x-3}-1}{x+2}+1\right)\right) + \frac{1}{2}\log\left(\frac{2\left(\sqrt{-x^2-4x-3}-1\right)}{x+2} + \frac{3\left(\sqrt{-x^2-4x-3}-1\right)^2}{(x+2)^2}+1\right) - \frac{1}{2}\log\left(\frac{2\left(\sqrt{-x^2-4x-3}-1\right)}{x+2} + \frac{\left(\sqrt{-x^2-4x-3}-1\right)^2}{(x+2)^2}+3\right)$$

```
[In] integrate((3+4*x)/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(2)*arctan(1/2*sqrt(2)*(3*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) + sq
rt(2)*arctan(1/2*sqrt(2)*((sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) + 1/2*lo
g(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 3*(sqrt(-x^2 - 4*x - 3) - 1)^2/(x
+ 2)^2 + 1) - 1/2*log(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + (sqrt(-x^2 - 4
*x - 3) - 1)^2/(x + 2)^2 + 3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{3+4x}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = \int \frac{4x+3}{\sqrt{-x^2-4x-3}\,(2x^2+4x+3)}\,dx$$

```
[In] int((4*x + 3)/((- 4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)),x)
```

```
[Out] int((4*x + 3)/((- 4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)), x)
```

3.39 $\int \frac{(g+hx)\sqrt{a+bx+cx^2}}{(ad+bdx+cdx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 136

$$\int \frac{(g+hx)\sqrt{a+bx+cx^2}}{(ad+bdx+cdx^2)^{3/2}}\,dx = -\frac{(2cg-bh)\sqrt{a+bx+cx^2}\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}d\sqrt{ad+bdx+cdx^2}} + \frac{h\sqrt{a+bx+cx^2}\log\left(a+bx+cx^2\right)}{2cd\sqrt{ad+bdx+cdx^2}}$$

```
[Out] 1/2*h*ln(c*x^2+b*x+a)*(c*x^2+b*x+a)^(1/2)/c/d/(c*d*x^2+b*d*x+a*d)^(1/2)-(-b
*h+2*c*g)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))*(c*x^2+b*x+a)^(1/2)/c/d/(-4
*a*c+b^2)^(1/2)/(c*d*x^2+b*d*x+a*d)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {1013, 648, 632, 212, 642}

$$\int \frac{(g+hx)\sqrt{a+bx+cx^2}}{(ad+bdx+cdx^2)^{3/2}}\,dx = \frac{h\sqrt{a+bx+cx^2}\log\left(a+bx+cx^2\right)}{2cd\sqrt{ad+bdx+cdx^2}} - \frac{\sqrt{a+bx+cx^2}(2cg-bh)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{cd\sqrt{b^2-4ac}\sqrt{ad+bdx+cdx^2}}$$

```
[In] Int[((g + h*x)*Sqrt[a + b*x + c*x^2])/(a*d + b*d*x + c*d*x^2)^(3/2),x]
```

```
[Out] -(((2*c*g - b*h)*Sqrt[a + b*x + c*x^2]*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c
]])/(c*Sqrt[b^2 - 4*a*c]*d*Sqrt[a*d + b*d*x + c*d*x^2])) + (h*Sqrt[a + b*x
+ c*x^2]*Log[a + b*x + c*x^2])/(2*c*d*Sqrt[a*d + b*d*x + c*d*x^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1013

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_
) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b
*x + c*x^2)^FracPart[p]/(d^IntPart[p]*(d + e*x + f*x^2)^FracPart[p])), Int[
(g + h*x)^m*(d + e*x + f*x^2)^(p + q), x], x] /; FreeQ[{a, b, c, d, e, f, g
, h, p, q}, x] && EqQ[c*d - a*f, 0] && EqQ[b*d - a*e, 0] &&  !IntegerQ[p] &
&  !IntegerQ[q] &&  !GtQ[c/f, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{a+bx+cx^2}\int \frac{g+hx}{ad+bdx+cdx^2}\,dx}{\sqrt{ad+bdx+cdx^2}}\\
&= \frac{\left(h\sqrt{a+bx+cx^2}\right)\int \frac{bd+2cdx}{ad+bdx+cdx^2}\,dx}{2cd\sqrt{ad+bdx+cdx^2}}+\frac{\left((2cdg-bdh)\sqrt{a+bx+cx^2}\right)\int \frac{1}{ad+bdx+cdx^2}\,dx}{2cd\sqrt{ad+bdx+cdx^2}}\\
&= \frac{h\sqrt{a+bx+cx^2}\log\left(a+bx+cx^2\right)}{2cd\sqrt{ad+bdx+cdx^2}}\\
&\quad -\frac{\left((2cdg-bdh)\sqrt{a+bx+cx^2}\right)\operatorname{Subst}\left(\int \frac{1}{(b^2-4ac)d^2-x^2}\,dx,x,bd+2cdx\right)}{cd\sqrt{ad+bdx+cdx^2}}
\end{aligned}
$$

$$= -\frac{(2cg - bh)\sqrt{a + bx + cx^2}\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2 - 4ac}d\sqrt{ad + bdx + cdx^2}} + \frac{h\sqrt{a + bx + cx^2}\log\left(a + bx + cx^2\right)}{2cd\sqrt{ad + bdx + cdx^2}}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.79

$$\int \frac{(g + hx)\sqrt{a + bx + cx^2}}{(ad + bdx + cdx^2)^{3/2}}\,dx = \frac{(a + x(b + cx))^{3/2}\left((4cg - 2bh)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right) + \sqrt{-b^2 + 4ac}h\log(a + x\right.}{2c\sqrt{-b^2 + 4ac}(d(a + x(b + cx)))^{3/2}}$$

```
[In] Integrate[((g + h*x)*Sqrt[a + b*x + c*x^2])/(a*d + b*d*x + c*d*x^2)^(3/2),x
]
```

```
[Out] ((a + x*(b + c*x))^(3/2)*((4*c*g - 2*b*h)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4*
a*c]] + Sqrt[-b^2 + 4*a*c]*h*Log[a + x*(b + c*x)]))/(2*c*Sqrt[-b^2 + 4*a*c]
*(d*(a + x*(b + c*x)))^(3/2))
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.89

method	result
default	$\frac{\sqrt{d(c\,x^2+bx+a)}\left(h\ln\left(c\,x^2+bx+a\right)\sqrt{4ac-b^2}-2\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)bh+4\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)cg\right)}{2\sqrt{c\,x^2+bx+a}\,d^2c\sqrt{4ac-b^2}}$
risch	$\frac{\sqrt{c\,x^2+bx+a}\left(4ach-b^2h+\sqrt{-(bh-2cg)^2(4ac-b^2)}\right)\ln\left(-4abch+8a\,c^2g+b^3h-2b^2cg-2\sqrt{-(bh-2cg)^2(4ac-b^2)}\,cx-\sqrt{-(bh-2cg)^2(4\right.}{2d\sqrt{d(c\,x^2+bx+a)}\,c(4ac-b^2)}$

```
[In] int((h*x+g)*(c*x^2+b*x+a)^(1/2)/(c*d*x^2+b*d*x+a*d)^(3/2),x,method=_RETURNV
ERBOSE)
```

```
[Out] 1/2/(c*x^2+b*x+a)^(1/2)*(d*(c*x^2+b*x+a))^(1/2)*(h*ln(c*x^2+b*x+a)*(4*a*c-b
^2)^(1/2)-2*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*h+4*arctan((2*c*x+b)/(4*a
*c-b^2)^(1/2))*c*g)/d^2/c/(4*a*c-b^2)^(1/2)
```

Fricas [F]

$$\int \frac{(g+hx)\sqrt{a+bx+cx^2}}{(ad+bdx+cdx^2)^{3/2}}\,dx = \int \frac{\sqrt{cx^2+bx+a}(hx+g)}{(cdx^2+bdx+ad)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((h*x+g)*(c*x^2+b*x+a)^(1/2)/(c*d*x^2+b*d*x+a*d)^(3/2),x, algorith
m="fricas")
```

```
[Out] integral(sqrt(c*d*x^2 + b*d*x + a*d)*sqrt(c*x^2 + b*x + a)*(h*x + g)/(c^2*d
^2*x^4 + 2*b*c*d^2*x^3 + 2*a*b*d^2*x + (b^2 + 2*a*c)*d^2*x^2 + a^2*d^2), x)
```

Sympy [F]

$$\int \frac{(g+hx)\sqrt{a+bx+cx^2}}{(ad+bdx+cdx^2)^{3/2}}\,dx = \int \frac{(g+hx)\sqrt{a+bx+cx^2}}{(d(a+bx+cx^2))^{\frac{3}{2}}}\,dx$$

```
[In] integrate((h*x+g)*(c*x**2+b*x+a)**(1/2)/(c*d*x**2+b*d*x+a*d)**(3/2),x)
```

```
[Out] Integral((g + h*x)*sqrt(a + b*x + c*x**2)/(d*(a + b*x + c*x**2))**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(g+hx)\sqrt{a+bx+cx^2}}{(ad+bdx+cdx^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((h*x+g)*(c*x^2+b*x+a)^(1/2)/(c*d*x^2+b*d*x+a*d)^(3/2),x, algorith
m="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Giac [F]

$$\int \frac{(g+hx)\sqrt{a+bx+cx^2}}{(ad+bdx+cdx^2)^{3/2}}\,dx = \int \frac{\sqrt{cx^2+bx+a}(hx+g)}{(cdx^2+bdx+ad)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((h*x+g)*(c*x^2+b*x+a)^(1/2)/(c*d*x^2+b*d*x+a*d)^(3/2),x, algorith
m="giac")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)*(h*x + g)/(c*d*x^2 + b*d*x + a*d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(g+hx)\sqrt{a+bx+cx^2}}{(ad+bdx+cdx^2)^{3/2}}\,dx = \int \frac{(g+h\,x)\,\sqrt{c\,x^2+b\,x+a}}{{(c\,d\,x^2+b\,d\,x+a\,d)}^{3/2}}\,dx$$

```
[In] int(((g + h*x)*(a + b*x + c*x^2)^(1/2))/(a*d + b*d*x + c*d*x^2)^(3/2),x)
```

```
[Out] int(((g + h*x)*(a + b*x + c*x^2)^(1/2))/(a*d + b*d*x + c*d*x^2)^(3/2), x)
```

3.40 $\int x^2\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 212

$$\begin{aligned}\int x^2\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}\,dx =& -\frac{acx\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{8d(a+bx)}\\&+\frac{bx^2\sqrt{a^2+2abx+b^2x^2}(c+dx^2)^{3/2}}{5d(a+bx)}\\&-\frac{(8bc-15adx)\sqrt{a^2+2abx+b^2x^2}(c+dx^2)^{3/2}}{60d^2(a+bx)}\\&-\frac{ac^2\sqrt{a^2+2abx+b^2x^2}\operatorname{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8d^{3/2}(a+bx)}\end{aligned}$$

```
[Out] 1/5*b*x^2*(d*x^2+c)^(3/2)*((b*x+a)^2)^(1/2)/d/(b*x+a)-1/60*(-15*a*d*x+8*b*c
)*(d*x^2+c)^(3/2)*((b*x+a)^2)^(1/2)/d^2/(b*x+a)-1/8*a*c^2*arctanh(x*d^(1/2)
/(d*x^2+c)^(1/2))*((b*x+a)^2)^(1/2)/d^(3/2)/(b*x+a)-1/8*a*c*x*((b*x+a)^2)^(
1/2)*(d*x^2+c)^(1/2)/d/(b*x+a)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used

= {1015, 847, 794, 201, 223, 212}

$$\int x^2\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}\,dx = -\frac{ac^2\sqrt{a^2+2abx+b^2x^2}\operatorname{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8d^{3/2}(a+bx)} - \frac{\sqrt{a^2+2abx+b^2x^2}(c+dx^2)^{3/2}(8bc-15adx)}{60d^2(a+bx)} - \frac{acx\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{8d(a+bx)} + \frac{bx^2\sqrt{a^2+2abx+b^2x^2}(c+dx^2)^{3/2}}{5d(a+bx)}$$

```
[In] Int[x^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2],x]
```

```
[Out] -1/8*(a*c*x*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/(d*(a + b*x)) +
(b*x^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(c + d*x^2)^(3/2))/(5*d*(a + b*x)) - (
(8*b*c - 15*a*d*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(c + d*x^2)^(3/2))/(60*d^2
*(a + b*x)) - (a*c^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[(Sqrt[d]*x)/Sqrt
[c + d*x^2]])/(8*d^(3/2)*(a + b*x))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1015

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_
) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((4
*c)^IntPart[p]*(b + 2*c*x)^(2*FracPart[p])), Int[(g + h*x)^m*(b + 2*c*x)^(2
*p)*(d + f*x^2)^q, x], x] /; FreeQ[{a, b, c, d, f, g, h, m, p, q}, x] && Eq
Q[b^2 - 4*a*c, 0]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{a^2+2abx+b^2x^2}\int x^2(2ab+2b^2x)\sqrt{c+dx^2}\,dx}{2ab+2b^2x}$$

$$= \frac{bx^2\sqrt{a^2+2abx+b^2x^2}(c+dx^2)^{3/2}}{5d(a+bx)} + \frac{\sqrt{a^2+2abx+b^2x^2}\int x(-4b^2c+10abdx)\sqrt{c+dx^2}\,dx}{5d\,(2ab+2b^2x)}$$

$$= \frac{bx^2\sqrt{a^2+2abx+b^2x^2}(c+dx^2)^{3/2}}{5d(a+bx)} - \frac{(8bc-15adx)\sqrt{a^2+2abx+b^2x^2}(c+dx^2)^{3/2}}{60d^2(a+bx)} - \frac{\left(abc\sqrt{a^2+2abx+b^2x^2}\right)\int\sqrt{c+dx^2}\,dx}{2d\,(2ab+2b^2x)}$$

$$= -\frac{acx\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{8d(a+bx)} + \frac{bx^2\sqrt{a^2+2abx+b^2x^2}(c+dx^2)^{3/2}}{5d(a+bx)} - \frac{(8bc-15adx)\sqrt{a^2+2abx+b^2x^2}(c+dx^2)^{3/2}}{60d^2(a+bx)} - \frac{\left(abc^2\sqrt{a^2+2abx+b^2x^2}\right)\int\frac{1}{\sqrt{c+dx^2}}\,dx}{4d\,(2ab+2b^2x)}$$

$$= -\frac{acx\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{8d(a+bx)} + \frac{bx^2\sqrt{a^2+2abx+b^2x^2}(c+dx^2)^{3/2}}{5d(a+bx)} - \frac{(8bc-15adx)\sqrt{a^2+2abx+b^2x^2}(c+dx^2)^{3/2}}{60d^2(a+bx)} - \frac{\left(abc^2\sqrt{a^2+2abx+b^2x^2}\right)\text{Subst}\left(\int\frac{1}{1-dx^2}\,dx,x,\frac{x}{\sqrt{c+dx^2}}\right)}{4d\,(2ab+2b^2x)}$$

$$= -\frac{acx\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{8d(a+bx)} + \frac{bx^2\sqrt{a^2+2abx+b^2x^2}\left(c+dx^2\right)^{3/2}}{5d(a+bx)} - \frac{(8bc-15adx)\sqrt{a^2+2abx+b^2x^2}\left(c+dx^2\right)^{3/2}}{60d^2(a+bx)} - \frac{ac^2\sqrt{a^2+2abx+b^2x^2}\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8d^{3/2}(a+bx)}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.50

$$\int x^2\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}\,dx = \frac{\sqrt{(a+bx)^2}\left(\sqrt{c+dx^2}(15adx(c+2dx^2)+8b(-2c^2+cdx^2+3d^2x^4))+15ac^2\sqrt{d}\log\left(-\sqrt{d}x+\sqrt{c+dx^2}\right)\right.}{120d^2(a+bx)}$$

```
[In] Integrate[x^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[(a + b*x)^2]*(Sqrt[c + d*x^2]*(15*a*d*x*(c + 2*d*x^2) + 8*b*(-2*c^2 +
 c*d*x^2 + 3*d^2*x^4)) + 15*a*c^2*Sqrt[d]*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2
]]))/(120*d^2*(a + b*x))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.45 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.49

method	result	siz
default	$\frac{\operatorname{csgn}(bx+a)\left(24(dx^2+c)^{\frac{3}{2}}d^{\frac{3}{2}}bx^2+30(dx^2+c)^{\frac{3}{2}}d^{\frac{3}{2}}ax-16(dx^2+c)^{\frac{3}{2}}\sqrt{d}\,bc-15\sqrt{dx^2+c}\,d^{\frac{3}{2}}acx-15\ln\left(\sqrt{d}\,x+\sqrt{dx^2+c}\right)ac^2d\right)}{120d^{\frac{5}{2}}}$	10
risch	$\frac{(24bx^4d^2+30ax^3d^2+8bcx^2d+15acxd-16bc^2)\sqrt{dx^2+c}\sqrt{(bx+a)^2}}{120d^2(bx+a)}-\frac{c^2a\ln\left(\sqrt{d}\,x+\sqrt{dx^2+c}\right)\sqrt{(bx+a)^2}}{8d^{\frac{3}{2}}(bx+a)}$	11

```
[In] int(x^2*((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/120*csgn(b*x+a)*(24*(d*x^2+c)^(3/2)*d^(3/2)*b*x^2+30*(d*x^2+c)^(3/2)*d^(3
/2)*a*x-16*(d*x^2+c)^(3/2)*d^(1/2)*b*c-15*(d*x^2+c)^(1/2)*d^(3/2)*a*c*x-15*
ln(d^(1/2)*x+(d*x^2+c)^(1/2))*a*c^2*d)/d^(5/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.83

$$\int x^2\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}\,dx$$
$$=\left[\frac{15\,ac^2\sqrt{d}\log\left(-2\,dx^2+2\,\sqrt{dx^2+c}\sqrt{d}x-c\right)+2\,(24\,bd^2x^4+30\,ad^2x^3+8\,bcdx^2+15\,acdx-16\,bc^2)\sqrt{a}}{240\,d^2}\right.$$

[In] integrate(x^2*((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2),x, algorithm="fricas")

[Out] [1/240*(15*a*c^2*sqrt(d)*log(-2*d*x^2 + 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*(24*b*d^2*x^4 + 30*a*d^2*x^3 + 8*b*c*d*x^2 + 15*a*c*d*x - 16*b*c^2)*sqrt(d*x^2 + c))/d^2, 1/120*(15*a*c^2*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (24*b*d^2*x^4 + 30*a*d^2*x^3 + 8*b*c*d*x^2 + 15*a*c*d*x - 16*b*c^2)*sqr t(d*x^2 + c))/d^2]

Sympy [F]

$$\int x^2\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}\,dx=\int x^2\sqrt{c+dx^2}\sqrt{(a+bx)^2}\,dx$$

[In] integrate(x**2*((b*x+a)**2)**(1/2)*(d*x**2+c)**(1/2),x)

[Out] Integral(x**2*sqrt(c + d*x**2)*sqrt((a + b*x)**2), x)

Maxima [F]

$$\int x^2\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}\,dx=\int\sqrt{dx^2+c}\sqrt{(bx+a)^2}x^2\,dx$$

[In] integrate(x^2*((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(d*x^2 + c)*sqrt((b*x + a)^2)*x^2, x)

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.55

$$\int x^2\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}\,dx=\frac{ac^2\log\left(\left|-\sqrt{d}x+\sqrt{dx^2+c}\right|\right)\operatorname{sgn}(bx+a)}{8\,d^{\frac{3}{2}}}+\frac{1}{120}\sqrt{dx^2+c}\left(\left(2\left(3\left(4\,bx\operatorname{sgn}(bx+a)+5\,a\operatorname{sgn}(bx+a)\right)x+\frac{4\,bc\operatorname{sgn}(bx+a)}{d}\right)x+\frac{15\,ac\operatorname{sgn}(bx+a)}{d}\right)x\right.$$

```
[In] integrate(x^2*((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*a*c^2*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))*sgn(b*x + a)/d^(3/2) + 1/1
20*sqrt(d*x^2 + c)*((2*(3*(4*b*x*sgn(b*x + a) + 5*a*sgn(b*x + a))*x + 4*b*c
*sgn(b*x + a)/d)*x + 15*a*c*sgn(b*x + a)/d)*x - 16*b*c^2*sgn(b*x + a)/d^2)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}\,dx=\int x^2\,\sqrt{(a+b\,x)^2}\,\sqrt{d\,x^2+c}\,dx$$

```
[In] int(x^2*((a + b*x)^2)^(1/2)*(c + d*x^2)^(1/2),x)
```

```
[Out] int(x^2*((a + b*x)^2)^(1/2)*(c + d*x^2)^(1/2), x)
```

3.41 $\int x\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 161

$$\int x\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}\,dx=-\frac{bcx\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{8d(a+bx)}+\frac{(4a+3bx)\sqrt{a^2+2abx+b^2x^2}\,(c+dx^2)^{3/2}}{12d(a+bx)}-\frac{bc^2\sqrt{a^2+2abx+b^2x^2}\,\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8d^{3/2}(a+bx)}$$

```
[Out] 1/12*(3*b*x+4*a)*(d*x^2+c)^(3/2)*((b*x+a)^2)^(1/2)/d/(b*x+a)-1/8*b*c^2*arct
anh(x*d^(1/2)/(d*x^2+c)^(1/2))*((b*x+a)^2)^(1/2)/d^(3/2)/(b*x+a)-1/8*b*c*x*
((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/d/(b*x+a)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {1015, 794, 201, 223, 212}

$$\int x\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}\,dx=-\frac{bc^2\sqrt{a^2+2abx+b^2x^2}\,\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8d^{3/2}(a+bx)}-\frac{bcx\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{8d(a+bx)}+\frac{(4a+3bx)\sqrt{a^2+2abx+b^2x^2}\,(c+dx^2)^{3/2}}{12d(a+bx)}$$

```
[In] Int[x*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2],x]
```

```
[Out] -1/8*(b*c*x*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/(d*(a + b*x)) +
((4*a + 3*b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(c + d*x^2)^(3/2))/(12*d*(a +
b*x)) - (b*c^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d
*x^2]])/(8*d^(3/2)*(a + b*x))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 1015

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_
) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((4
*c)^IntPart[p]*(b + 2*c*x)^(2*FracPart[p])), Int[(g + h*x)^m*(b + 2*c*x)^(2
*p)*(d + f*x^2)^q, x], x] /; FreeQ[{a, b, c, d, f, g, h, m, p, q}, x] && Eq
Q[b^2 - 4*a*c, 0]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{a^2+2abx+b^2x^2}\int x(2ab+2b^2x)\sqrt{c+dx^2}\,dx}{2ab+2b^2x}$$

$$= \frac{(4a+3bx)\sqrt{a^2+2abx+b^2x^2}(c+dx^2)^{3/2}}{12d(a+bx)} - \frac{\left(b^2c\sqrt{a^2+2abx+b^2x^2}\right)\int\sqrt{c+dx^2}\,dx}{2d\,(2ab+2b^2x)}$$

$$= -\frac{bcx\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{8d(a+bx)} + \frac{(4a+3bx)\sqrt{a^2+2abx+b^2x^2}(c+dx^2)^{3/2}}{12d(a+bx)} - \frac{\left(b^2c^2\sqrt{a^2+2abx+b^2x^2}\right)\int \frac{1}{\sqrt{c+dx^2}}\,dx}{4d\,(2ab+2b^2x)}$$

$$= -\frac{bcx\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{8d(a+bx)} + \frac{(4a+3bx)\sqrt{a^2+2abx+b^2x^2}(c+dx^2)^{3/2}}{12d(a+bx)} - \frac{\left(b^2c^2\sqrt{a^2+2abx+b^2x^2}\right)\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx, x, \frac{x}{\sqrt{c+dx^2}}\right)}{4d\,(2ab+2b^2x)}$$

$$= -\frac{bcx\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{8d(a+bx)} + \frac{(4a+3bx)\sqrt{a^2+2abx+b^2x^2}(c+dx^2)^{3/2}}{12d(a+bx)} - \frac{bc^2\sqrt{a^2+2abx+b^2x^2}\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8d^{3/2}(a+bx)}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.59

$$\int x\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}\,dx$$

$$= \frac{\sqrt{(a+bx)^2}\left(\sqrt{d}\sqrt{c+dx^2}(8a(c+dx^2)+3bx(c+2dx^2))+3bc^2\log\left(-\sqrt{d}x+\sqrt{c+dx^2}\right)\right)}{24d^{3/2}(a+bx)}$$

```
[In] Integrate[x*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2],x]
```

```
[Out] (Sqrt[(a + b*x)^2]*(Sqrt[d]*Sqrt[c + d*x^2]*(8*a*(c + d*x^2) + 3*b*x*(c + 2
*d*x^2)) + 3*b*c^2*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]]))/(24*d^(3/2)*(a + b
*x))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.51 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.52

method	result	size
default	$\frac{\operatorname{csgn}(bx+a)\left(6(d\,x^2+c)^{\frac{3}{2}}\sqrt{d}\,bx+8a(d\,x^2+c)^{\frac{3}{2}}\sqrt{d}-3\sqrt{d\,x^2+c}\,\sqrt{d}\,bcx-3\ln\left(\sqrt{d}\,x+\sqrt{d\,x^2+c}\right)b\,c^2\right)}{24d^{\frac{3}{2}}}$	83
risch	$\frac{(6bd\,x^3+8ad\,x^2+3bcx+8ac)\sqrt{d\,x^2+c}\,\sqrt{(bx+a)^2}}{24d(bx+a)}-\frac{c^2b\ln\left(\sqrt{d}\,x+\sqrt{d\,x^2+c}\right)\sqrt{(bx+a)^2}}{8d^{\frac{3}{2}}(bx+a)}$	97

[In] int(x*((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/24*csgn(b*x+a)*(6*(d*x^2+c)^(3/2)*d^(1/2)*b*x+8*a*(d*x^2+c)^(3/2)*d^(1/2)
-3*(d*x^2+c)^(1/2)*d^(1/2)*b*c*x-3*ln(d^(1/2)*x+(d*x^2+c)^(1/2))*b*c^2)/d^(
3/2)

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.98

$$\int x\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}\,dx$$
$$=\left[\frac{3\,bc^2\sqrt{d}\log\left(-2\,dx^2+2\,\sqrt{dx^2+c}\sqrt{d}x-c\right)+2\,(6\,bd^2x^3+8\,ad^2x^2+3\,bcdx+8\,acd)\sqrt{dx^2+c}}{48\,d^2},\frac{3\,bc^2\sqrt{-c}}{}\right.$$

[In] integrate(x*((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2),x, algorithm="fricas")

[Out] [1/48*(3*b*c^2*sqrt(d)*log(-2*d*x^2 + 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*
(6*b*d^2*x^3 + 8*a*d^2*x^2 + 3*b*c*d*x + 8*a*c*d)*sqrt(d*x^2 + c))/d^2, 1/2
4*(3*b*c^2*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (6*b*d^2*x^3 + 8*a
*d^2*x^2 + 3*b*c*d*x + 8*a*c*d)*sqrt(d*x^2 + c))/d^2]

Sympy [F]

$$\int x\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}\,dx=\int x\sqrt{c+dx^2}\sqrt{(a+bx)^2}\,dx$$

[In] integrate(x*((b*x+a)**2)**(1/2)*(d*x**2+c)**(1/2),x)

[Out] Integral(x*sqrt(c + d*x**2)*sqrt((a + b*x)**2), x)

Maxima [F]

$$\int x\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}\,dx=\int\sqrt{dx^2+c}\sqrt{(bx+a)^2}x\,dx$$

[In] integrate(x*((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(d*x^2 + c)*sqrt((b*x + a)^2)*x, x)

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.61

$$\int x\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}\,dx = \frac{bc^2\log\left(\left|-\sqrt{d}x+\sqrt{dx^2+c}\right|\right)\operatorname{sgn}(bx+a)}{8\,d^{\frac{3}{2}}} + \frac{1}{24}\sqrt{dx^2+c}\left(\left(2\,(3\,bx\operatorname{sgn}(bx+a)+4\,a\operatorname{sgn}(bx+a))x+\frac{3\,bc\operatorname{sgn}(bx+a)}{d}\right)x+\frac{8\,ac\operatorname{sgn}(bx+a)}{d}\right)$$

```
[In] integrate(x*((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*b*c^2*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))*sgn(b*x + a)/d^(3/2) + 1/2
4*sqrt(d*x^2 + c)*((2*(3*b*x*sgn(b*x + a) + 4*a*sgn(b*x + a))*x + 3*b*c*sgn
(b*x + a)/d)*x + 8*a*c*sgn(b*x + a)/d)
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}\,dx = \int x\,\sqrt{(a+b\,x)^2}\,\sqrt{d\,x^2+c}\,dx$$

```
[In] int(x*((a + b*x)^2)^(1/2)*(c + d*x^2)^(1/2),x)
```

```
[Out] int(x*((a + b*x)^2)^(1/2)*(c + d*x^2)^(1/2), x)
```

3.42 $\int \sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 148

$$\int \sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}\,dx = \frac{ax\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{2(a+bx)} + \frac{b\sqrt{a^2+2abx+b^2x^2}(c+dx^2)^{3/2}}{3d(a+bx)} + \frac{ac\sqrt{a^2+2abx+b^2x^2}\operatorname{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2\sqrt{d}(a+bx)}$$

```
[Out] 1/3*b*(d*x^2+c)^(3/2)*((b*x+a)^2)^(1/2)/d/(b*x+a)+1/2*a*c*arctanh(x*d^(1/2)
/(d*x^2+c)^(1/2))*((b*x+a)^2)^(1/2)/(b*x+a)/d^(1/2)+1/2*a*x*((b*x+a)^2)^(1/
2)*(d*x^2+c)^(1/2)/(b*x+a)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {984, 655, 201, 223, 212}

$$\int \sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}\,dx = \frac{ac\sqrt{a^2+2abx+b^2x^2}\operatorname{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2\sqrt{d}(a+bx)} + \frac{b\sqrt{a^2+2abx+b^2x^2}(c+dx^2)^{3/2}}{3d(a+bx)} + \frac{ax\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{2(a+bx)}$$

```
[In] Int[Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2],x]
```

```
[Out] (a*x*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/(2*(a + b*x)) + (b*Sqrt
[a^2 + 2*a*b*x + b^2*x^2]*(c + d*x^2)^(3/2))/(3*d*(a + b*x)) + (a*c*Sqrt[a^
2 + 2*a*b*x + b^2*x^2]*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(2*Sqrt[d]*(a
+ b*x))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 984

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q_.), x_
Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((4*c)^IntPart[p]*(b + 2*c*x)
^(2*FracPart[p])), Int[(b + 2*c*x)^(2*p)*(d + f*x^2)^q, x], x] /; FreeQ[{a,
 b, c, d, f, p, q}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{a^2+2abx+b^2x^2}\int (2ab+2b^2x)\sqrt{c+dx^2}\,dx}{2ab+2b^2x}\\
&= \frac{b\sqrt{a^2+2abx+b^2x^2}(c+dx^2)^{3/2}}{3d(a+bx)}+\frac{\left(2ab\sqrt{a^2+2abx+b^2x^2}\right)\int \sqrt{c+dx^2}\,dx}{2ab+2b^2x}
\end{aligned}$$

$$= \frac{ax\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{2(a+bx)} + \frac{b\sqrt{a^2+2abx+b^2x^2}(c+dx^2)^{3/2}}{3d(a+bx)} + \frac{\left(abc\sqrt{a^2+2abx+b^2x^2}\right)\int \frac{1}{\sqrt{c+dx^2}}\,dx}{2ab+2b^2x}$$

$$= \frac{ax\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{2(a+bx)} + \frac{b\sqrt{a^2+2abx+b^2x^2}(c+dx^2)^{3/2}}{3d(a+bx)} + \frac{\left(abc\sqrt{a^2+2abx+b^2x^2}\right)\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx, x, \frac{x}{\sqrt{c+dx^2}}\right)}{2ab+2b^2x}$$

$$= \frac{ax\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{2(a+bx)} + \frac{b\sqrt{a^2+2abx+b^2x^2}(c+dx^2)^{3/2}}{3d(a+bx)} + \frac{ac\sqrt{a^2+2abx+b^2x^2}\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2\sqrt{d}(a+bx)}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.57

$$\int \sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}\,dx = \frac{\sqrt{(a+bx)^2}\left(\sqrt{c+dx^2}(3adx+2b(c+dx^2)) - 3ac\sqrt{d}\log\left(-\sqrt{d}x+\sqrt{c+dx^2}\right)\right)}{6d(a+bx)}$$

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2],x]
[Out] (Sqrt[(a + b*x)^2]*(Sqrt[c + d*x^2]*(3*a*d*x + 2*b*(c + d*x^2)) - 3*a*c*Sqr
t[d]*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]]))/(6*d*(a + b*x))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.51 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.44

method	result	size
default	$\frac{\operatorname{csgn}(bx+a)\left(2\left(d\,x^2+c\right)^{\frac{3}{2}}\sqrt{d}\,b+3a\sqrt{d\,x^2+c}\,d^{\frac{3}{2}}x+3\ln\left(\sqrt{d}\,x+\sqrt{d\,x^2+c}\right)acd\right)}{6d^{\frac{3}{2}}}$	65
risch	$\frac{(2bd\,x^2+3adx+2bc)\sqrt{d\,x^2+c}\,\sqrt{(bx+a)^2}}{6d(bx+a)}+\frac{ac\ln\left(\sqrt{d}\,x+\sqrt{d\,x^2+c}\right)\sqrt{(bx+a)^2}}{2\sqrt{d}\,(bx+a)}$	88

```
[In] int(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*csgn(b*x+a)*(2*(d*x^2+c)^(3/2)*d^(1/2)*b+3*a*(d*x^2+c)^(1/2)*d^(3/2)*x+
3*ln(d^(1/2)*x+(d*x^2+c)^(1/2))*a*c*d)/d^(3/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.86

$$\int \sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}\,dx = \left[\frac{3\,ac\sqrt{d}\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\sqrt{d}x-c\right)+2\,(2\,bdx^2+3\,adx+2\,bc)\sqrt{dx^2+c}}{12\,d},\ -\frac{3\,ac\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2+c}}\right)-(2\,bdx^2+3\,adx+2\,bc)\sqrt{dx^2+c}}{6\,d}\right]$$

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*a*c*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*(2
*b*d*x^2 + 3*a*d*x + 2*b*c)*sqrt(d*x^2 + c))/d, -1/6*(3*a*c*sqrt(-d)*arctan
(sqrt(-d)*x/sqrt(d*x^2 + c)) - (2*b*d*x^2 + 3*a*d*x + 2*b*c)*sqrt(d*x^2 + c
))/d]
```

Sympy [F]

$$\int \sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}\,dx = \int \sqrt{c+dx^2}\sqrt{(a+bx)^2}\,dx$$

```
[In] integrate(((b*x+a)**2)**(1/2)*(d*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*x**2)*sqrt((a + b*x)**2), x)
```

Maxima [F]

$$\int \sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}\,dx = \int \sqrt{dx^2+c}\sqrt{(bx+a)^2}\,dx$$

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)*sqrt((b*x + a)^2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.53

$$\int \sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}\,dx$$
$$= -\frac{ac\log\left(\left|-\sqrt{d}x+\sqrt{dx^2+c}\right|\right)\operatorname{sgn}(bx+a)}{2\sqrt{d}} + \frac{1}{6}\sqrt{dx^2+c}\left((2\,bx\operatorname{sgn}(bx+a)+3\,a\operatorname{sgn}(bx+a))x+\frac{2\,bc\operatorname{sgn}(bx+a)}{d}\right)$$

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*a*c*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))*sgn(b*x + a)/sqrt(d) + 1/6*
sqrt(d*x^2 + c)*((2*b*x*sgn(b*x + a) + 3*a*sgn(b*x + a))*x + 2*b*c*sgn(b*x
+ a)/d)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}\,dx = \int \sqrt{(a+b\,x)^2}\,\sqrt{d\,x^2+c}\,dx$$

```
[In] int(((a + b*x)^2)^(1/2)*(c + d*x^2)^(1/2),x)
```

```
[Out] int(((a + b*x)^2)^(1/2)*(c + d*x^2)^(1/2), x)
```

3.43 $\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{x}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 160

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{x}\,dx = \frac{(2a+bx)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{2(a+bx)} + \frac{bc\sqrt{a^2+2abx+b^2x^2}\operatorname{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2\sqrt{d}(a+bx)} - \frac{a\sqrt{c}\sqrt{a^2+2abx+b^2x^2}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a+bx}$$

```
[Out] -a*arctanh((d*x^2+c)^(1/2)/c^(1/2))*c^(1/2)*((b*x+a)^2)^(1/2)/(b*x+a)+1/2*b
*c*arctanh(x*d^(1/2)/(d*x^2+c)^(1/2))*((b*x+a)^2)^(1/2)/(b*x+a)/d^(1/2)+1/2
*(b*x+2*a)*((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/(b*x+a)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {1015, 829, 858, 223, 212, 272, 65, 214}

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{x}\,dx = \frac{bc\sqrt{a^2+2abx+b^2x^2}\operatorname{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2\sqrt{d}(a+bx)} - \frac{a\sqrt{c}\sqrt{a^2+2abx+b^2x^2}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a+bx} + \frac{\sqrt{a^2+2abx+b^2x^2}(2a+bx)\sqrt{c+dx^2}}{2(a+bx)}$$

```
[In] Int[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/x,x]
```

```
[Out] ((2*a + b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/(2*(a + b*x)) +
 (b*c*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(
2*Sqrt[d]*(a + b*x)) - (a*Sqrt[c]*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[Sqr
t[c + d*x^2]/Sqrt[c]])/(a + b*x)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
```

```
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1015

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_
) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((4
*c)^IntPart[p]*(b + 2*c*x)^(2*FracPart[p])), Int[(g + h*x)^m*(b + 2*c*x)^(2
*p)*(d + f*x^2)^q, x], x] /; FreeQ[{a, b, c, d, f, g, h, m, p, q}, x] && Eq
Q[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(2ab+2b^2x)\sqrt{c+dx^2}}{x}\,dx}{2ab+2b^2x}\\
&= \frac{(2a+bx)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{2(a+bx)}+\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{4abcd+2b^2cdx}{x\sqrt{c+dx^2}}\,dx}{2d\,(2ab+2b^2x)}\\
&= \frac{(2a+bx)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{2(a+bx)}+\frac{\left(2abc\sqrt{a^2+2abx+b^2x^2}\right)\int \frac{1}{x\sqrt{c+dx^2}}\,dx}{2ab+2b^2x}\\
&\quad+\frac{\left(b^2c\sqrt{a^2+2abx+b^2x^2}\right)\int \frac{1}{\sqrt{c+dx^2}}\,dx}{2ab+2b^2x}\\
&= \frac{(2a+bx)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{2(a+bx)}\\
&\quad+\frac{\left(abc\sqrt{a^2+2abx+b^2x^2}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,x,x^2\right)}{2ab+2b^2x}\\
&\quad+\frac{\left(b^2c\sqrt{a^2+2abx+b^2x^2}\right)\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx,x,\frac{x}{\sqrt{c+dx^2}}\right)}{2ab+2b^2x}\\
&= \frac{(2a+bx)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{2(a+bx)}+\frac{bc\sqrt{a^2+2abx+b^2x^2}\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2\sqrt{d}(a+bx)}\\
&\quad+\frac{\left(2abc\sqrt{a^2+2abx+b^2x^2}\right)\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+dx^2}\right)}{d\,(2ab+2b^2x)}
\end{aligned}
$$

$$= \frac{(2a+bx)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{2(a+bx)} + \frac{bc\sqrt{a^2+2abx+b^2x^2}\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2\sqrt{d}(a+bx)} - \frac{a\sqrt{c}\sqrt{a^2+2abx+b^2x^2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a+bx}$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{x}\,dx = \frac{\sqrt{(a+bx)^2}\left(\sqrt{d}(2a+bx)\sqrt{c+dx^2}+4a\sqrt{c}\sqrt{d}\operatorname{arctanh}\left(\frac{\sqrt{d}x-\sqrt{c+dx^2}}{\sqrt{c}}\right)-bc\log\left(-\sqrt{d}x+\sqrt{c+dx^2}\right)\right)}{2\sqrt{d}(a+bx)}$$

```
[In] Integrate[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/x,x]
[Out] (Sqrt[(a + b*x)^2]*(Sqrt[d]*(2*a + b*x)*Sqrt[c + d*x^2] + 4*a*Sqrt[c]*Sqrt[
d]*ArcTanh[(Sqrt[d]*x - Sqrt[c + d*x^2])/Sqrt[c]] - b*c*Log[-(Sqrt[d]*x) +
Sqrt[c + d*x^2]]))/(2*Sqrt[d]*(a + b*x))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.49 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.58

method	result	size
default	$\frac{\operatorname{csgn}(bx+a)\left(\sqrt{d\,x^2+c}\,\sqrt{d}\,bx-2\sqrt{d}\,\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^2+c}}{x}\right)\sqrt{c}\,a+2\sqrt{d\,x^2+c}\,\sqrt{d}\,a+\ln\left(\sqrt{d}\,x+\sqrt{d\,x^2+c}\right)bc\right)}{2\sqrt{d}}$	92

```
[In] int(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x,x,method=_RETURNVERBOSE)
[Out] 1/2*csgn(b*x+a)*((d*x^2+c)^(1/2)*d^(1/2)*b*x-2*d^(1/2)*ln(2*(c^(1/2)*(d*x^2
+c)^(1/2)+c)/x)*c^(1/2)*a+2*(d*x^2+c)^(1/2)*d^(1/2)*a+ln(d^(1/2)*x+(d*x^2+c
)^(1/2))*b*c)/d^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 341, normalized size of antiderivative = 2.13

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{x}\,dx$$
$$= \left[\frac{bc\sqrt{d}\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\sqrt{d}x-c\right)+2\,a\sqrt{c}d\log\left(-\frac{dx^2-2\,\sqrt{dx^2+c}\sqrt{c}+2\,c}{x^2}\right)+2\,(bdx+2\,ad)\sqrt{dx^2+c}}{4\,d}\right.$$
$$-\frac{bc\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2+c}}\right)-a\sqrt{c}d\log\left(-\frac{dx^2-2\,\sqrt{dx^2+c}\sqrt{c}+2\,c}{x^2}\right)-(bdx+2\,ad)\sqrt{dx^2+c}}{2\,d},\ \frac{4\,a\sqrt{-c}d\arctan\left(\right.}{}$$
$$\left.-\frac{bc\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2+c}}\right)-2\,a\sqrt{-c}d\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)-(bdx+2\,ad)\sqrt{dx^2+c}}{2\,d}\right]$$

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/4*(b*c*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*a*sqr
t(c)*d*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 2*(b*d*x + 2*a
*d)*sqrt(d*x^2 + c))/d, -1/2*(b*c*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c
)) - a*sqrt(c)*d*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) - (b*d
*x + 2*a*d)*sqrt(d*x^2 + c))/d, 1/4*(4*a*sqrt(-c)*d*arctan(sqrt(-c)/sqrt(d*
x^2 + c)) + b*c*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2
*(b*d*x + 2*a*d)*sqrt(d*x^2 + c))/d, -1/2*(b*c*sqrt(-d)*arctan(sqrt(-d)*x/s
qrt(d*x^2 + c)) - 2*a*sqrt(-c)*d*arctan(sqrt(-c)/sqrt(d*x^2 + c)) - (b*d*x
+ 2*a*d)*sqrt(d*x^2 + c))/d]
```

Sympy [F]

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{x}\,dx = \int \frac{\sqrt{c+dx^2}\sqrt{(a+bx)^2}}{x}\,dx$$

```
[In] integrate(((b*x+a)**2)**(1/2)*(d*x**2+c)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(c + d*x**2)*sqrt((a + b*x)**2)/x, x)
```

Maxima [F]

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{x}\,dx = \int \frac{\sqrt{dx^2+c}\sqrt{(bx+a)^2}}{x}\,dx$$

[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x,x, algorithm="maxima")

[Out] integrate(sqrt(d*x^2 + c)*sqrt((b*x + a)^2)/x, x)

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{x}\,dx = \text{Exception raised: TypeError}$$

[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x,x, algorithm="giac")

[Out] Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Error: Bad Argument Value

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{x}\,dx = \int \frac{\sqrt{{(a+b\,x)}^2}\,\sqrt{d\,x^2+c}}{x}\,dx$$

[In] int((((a + b*x)^2)^(1/2)*(c + d*x^2)^(1/2))/x,x)

[Out] int((((a + b*x)^2)^(1/2)*(c + d*x^2)^(1/2))/x, x)

3.44 $\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{x^2} dx$

Optimal result

Integrand size = 35, antiderivative size = 156

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{x^2} dx = -\frac{(a-bx)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{x(a+bx)} + \frac{a\sqrt{d}\sqrt{a^2+2abx+b^2x^2}\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{a+bx} - \frac{b\sqrt{c}\sqrt{a^2+2abx+b^2x^2}\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a+bx}$$

```
[Out] -b*arctanh((d*x^2+c)^(1/2)/c^(1/2))*c^(1/2)*((b*x+a)^2)^(1/2)/(b*x+a)+a*arc
tanh(x*d^(1/2)/(d*x^2+c)^(1/2))*d^(1/2)*((b*x+a)^2)^(1/2)/(b*x+a)-(-b*x+a)*
((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x/(b*x+a)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.229, Rules used = {1015, 827, 858, 223, 212, 272, 65, 214}

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{x^2} dx = \frac{a\sqrt{d}\sqrt{a^2+2abx+b^2x^2}\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{a+bx} - \frac{b\sqrt{c}\sqrt{a^2+2abx+b^2x^2}\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a+bx} - \frac{\sqrt{a^2+2abx+b^2x^2}(a-bx)\sqrt{c+dx^2}}{x(a+bx)}$$

```
[In] Int[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/x^2,x]
```

```
[Out] -(((a - b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/(x*(a + b*x)))
+ (a*Sqrt[d]*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x
^2]])/(a + b*x) - (b*Sqrt[c]*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[Sqrt[c +
 d*x^2]/Sqrt[c]])/(a + b*x)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 827

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
```

```
p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1015

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_
) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((4
*c)^IntPart[p]*(b + 2*c*x)^(2*FracPart[p])), Int[(g + h*x)^m*(b + 2*c*x)^(2
*p)*(d + f*x^2)^q, x], x] /; FreeQ[{a, b, c, d, f, g, h, m, p, q}, x] && Eq
Q[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(2ab+2b^2x)\sqrt{c+dx^2}}{x^2}\,dx}{2ab+2b^2x}\\
&= -\frac{(a-bx)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{x(a+bx)}-\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{-4b^2c-4abdx}{x\sqrt{c+dx^2}}\,dx}{2\,(2ab+2b^2x)}\\
&= -\frac{(a-bx)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{x(a+bx)}+\frac{\left(2b^2c\sqrt{a^2+2abx+b^2x^2}\right)\int \frac{1}{x\sqrt{c+dx^2}}\,dx}{2ab+2b^2x}\\
&\quad+\frac{\left(2abd\sqrt{a^2+2abx+b^2x^2}\right)\int \frac{1}{\sqrt{c+dx^2}}\,dx}{2ab+2b^2x}\\
&= -\frac{(a-bx)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{x(a+bx)}\\
&\quad+\frac{\left(b^2c\sqrt{a^2+2abx+b^2x^2}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,x,x^2\right)}{2ab+2b^2x}\\
&\quad+\frac{\left(2abd\sqrt{a^2+2abx+b^2x^2}\right)\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx,x,\frac{x}{\sqrt{c+dx^2}}\right)}{2ab+2b^2x}\\
&= -\frac{(a-bx)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{x(a+bx)}+\frac{a\sqrt{d}\sqrt{a^2+2abx+b^2x^2}\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{a+bx}\\
&\quad+\frac{\left(2b^2c\sqrt{a^2+2abx+b^2x^2}\right)\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+dx^2}\right)}{d\,(2ab+2b^2x)}
\end{aligned}
$$

$$= -\frac{(a-bx)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{x(a+bx)} + \frac{a\sqrt{d}\sqrt{a^2+2abx+b^2x^2}\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{a+bx} - \frac{b\sqrt{c}\sqrt{a^2+2abx+b^2x^2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a+bx}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.70

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{x^2}\,dx = \frac{\sqrt{(a+bx)^2}\left((-a+bx)\sqrt{c+dx^2}+2b\sqrt{c}x\operatorname{arctanh}\left(\frac{\sqrt{d}x-\sqrt{c+dx^2}}{\sqrt{c}}\right)-a\sqrt{d}x\log\left(-\sqrt{d}x+\sqrt{c+dx^2}\right)\right)}{x(a+bx)}$$

```
[In] Integrate[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/x^2,x]
```

```
[Out] (Sqrt[(a + b*x)^2]*((-a + b*x)*Sqrt[c + d*x^2] + 2*b*Sqrt[c]*x*ArcTanh[(Sqr
t[d]*x - Sqrt[c + d*x^2])/Sqrt[c]] - a*Sqrt[d]*x*Log[-(Sqrt[d]*x) + Sqrt[c
+ d*x^2]]))/(x*(a + b*x))
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.72

method	result
risch	$-\frac{a\sqrt{d\,x^2+c}\,\sqrt{(bx+a)^2}}{x(bx+a)}+\frac{\left(a\sqrt{d}\,\ln\left(\sqrt{d}\,x+\sqrt{d\,x^2+c}\right)+\sqrt{d\,x^2+c}\,b-b\sqrt{c}\,\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^2+c}}{x}\right)\right)\sqrt{(bx+a)^2}}{bx+a}$
default	$-\frac{\operatorname{csgn}(bx+a)\left(-a\,d^{\frac{3}{2}}x^2\sqrt{d\,x^2+c}+c^{\frac{3}{2}}\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^2+c}}{x}\right)\sqrt{d}\,bx+a(d\,x^2+c)^{\frac{3}{2}}\sqrt{d}-\sqrt{d\,x^2+c}\,\sqrt{d}\,bcx-\ln\left(\sqrt{d}\,x+\sqrt{d\,x^2+c}\right)acdx\right)}{cx\sqrt{d}}$

```
[In] int(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a*(d*x^2+c)^(1/2)/x*((b*x+a)^2)^(1/2)/(b*x+a)+(a*d^(1/2)*ln(d^(1/2)*x+(d*x
^2+c)^(1/2))+(d*x^2+c)^(1/2)*b-b*c^(1/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2))
/x))*((b*x+a)^2)^(1/2)/(b*x+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 333, normalized size of antiderivative = 2.13

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{x^2}\,dx$$

$$= \left[\frac{a\sqrt{d}x\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\sqrt{d}x-c\right)+b\sqrt{c}x\log\left(-\frac{dx^2-2\,\sqrt{dx^2+c}\sqrt{c}+2\,c}{x^2}\right)+2\,\sqrt{dx^2+c}(bx-a)}{2\,x},\right.$$

$$-\frac{2\,a\sqrt{-d}x\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2+c}}\right)-b\sqrt{c}x\log\left(-\frac{dx^2-2\,\sqrt{dx^2+c}\sqrt{c}+2\,c}{x^2}\right)-2\,\sqrt{dx^2+c}(bx-a)}{2\,x}, \quad 2\,b\sqrt{-c}x\arctan\left(\right.$$

$$\left.-\frac{a\sqrt{-d}x\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2+c}}\right)-b\sqrt{-c}x\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)-\sqrt{dx^2+c}(bx-a)}{x}\right]$$

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/2*(a*sqrt(d)*x*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + b*sqrt(
c)*x*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 2*sqrt(d*x^2 + c
)*(b*x - a))/x, -1/2*(2*a*sqrt(-d)*x*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) - b
*sqrt(c)*x*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) - 2*sqrt(d*x
^2 + c)*(b*x - a))/x, 1/2*(2*b*sqrt(-c)*x*arctan(sqrt(-c)/sqrt(d*x^2 + c))
+ a*sqrt(d)*x*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*sqrt(d*x^
2 + c)*(b*x - a))/x, -(a*sqrt(-d)*x*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) - b*
sqrt(-c)*x*arctan(sqrt(-c)/sqrt(d*x^2 + c)) - sqrt(d*x^2 + c)*(b*x - a))/x]
```

Sympy [F]

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{x^2}\,dx = \int \frac{\sqrt{c+dx^2}\sqrt{(a+bx)^2}}{x^2}\,dx$$

```
[In] integrate(((b*x+a)**2)**(1/2)*(d*x**2+c)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(c + d*x**2)*sqrt((a + b*x)**2)/x**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{x^2}\,dx = \int \frac{\sqrt{dx^2+c}\sqrt{(bx+a)^2}}{x^2}\,dx$$

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x^2,x, algorithm="maxima")
[Out] integrate(sqrt(d*x^2 + c)*sqrt((b*x + a)^2)/x^2, x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{x^2}\,dx = \frac{2\,bc\arctan\left(-\frac{\sqrt{d}x-\sqrt{dx^2+c}}{\sqrt{-c}}\right)\mathrm{sgn}(bx+a)}{\sqrt{-c}} - a\sqrt{d}\log\left(\left|-\sqrt{d}x+\sqrt{dx^2+c}\right|\right)\mathrm{sgn}(bx+a) + \sqrt{dx^2+c}b\mathrm{sgn}(bx+a) + \frac{2\,ac\sqrt{d}\mathrm{sgn}(bx+a)}{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2-c}$$

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x^2,x, algorithm="giac")
[Out] 2*b*c*arctan(-(sqrt(d)*x - sqrt(d*x^2 + c))/sqrt(-c))*sgn(b*x + a)/sqrt(-c)
 - a*sqrt(d)*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))*sgn(b*x + a) + sqrt(d*x
^2 + c)*b*sgn(b*x + a) + 2*a*c*sqrt(d)*sgn(b*x + a)/((sqrt(d)*x - sqrt(d*x^
2 + c))^2 - c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{x^2}\,dx = \int \frac{\sqrt{(a+b\,x)^2}\,\sqrt{d\,x^2+c}}{x^2}\,dx$$

```
[In] int((((a + b*x)^2)^(1/2)*(c + d*x^2)^(1/2))/x^2,x)
[Out] int((((a + b*x)^2)^(1/2)*(c + d*x^2)^(1/2))/x^2, x)
```

3.45 $\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{x^3}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 161

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{x^3}\,dx = -\frac{(a+2bx)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{2x^2(a+bx)} + \frac{b\sqrt{d}\sqrt{a^2+2abx+b^2x^2}\operatorname{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{a+bx} - \frac{ad\sqrt{a^2+2abx+b^2x^2}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2\sqrt{c}(a+bx)}$$

```
[Out] -1/2*a*d*arctanh((d*x^2+c)^(1/2)/c^(1/2))*((b*x+a)^2)^(1/2)/(b*x+a)/c^(1/2)
+b*arctanh(x*d^(1/2)/(d*x^2+c)^(1/2))*d^(1/2)*((b*x+a)^2)^(1/2)/(b*x+a)-1/2
*(2*b*x+a)*((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x^2/(b*x+a)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {1015, 825, 858, 223, 212, 272, 65, 214}

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{x^3}\,dx = \frac{b\sqrt{d}\sqrt{a^2+2abx+b^2x^2}\operatorname{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{a+bx} - \frac{ad\sqrt{a^2+2abx+b^2x^2}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2\sqrt{c}(a+bx)} - \frac{\sqrt{a^2+2abx+b^2x^2}(a+2bx)\sqrt{c+dx^2}}{2x^2(a+bx)}$$

```
[In] Int[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/x^3,x]
```

```
[Out] -1/2*((a + 2*b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/(x^2*(a +
b*x)) + (b*Sqrt[d]*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[(Sqrt[d]*x)/Sqrt[c
 + d*x^2]])/(a + b*x) - (a*d*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[Sqrt[c +
 d*x^2]/Sqrt[c]])/(2*Sqrt[c]*(a + b*x))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 825

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + c*x^2)^p/(e^2*(m + 1)*(m
+ 2)*(c*d^2 + a*e^2)))*((d*g - e*f*(m + 2))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*
f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) + 2*c*d*p*(e*f - d*g))*x), x] - Dis
t[p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2
)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*
(m + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f,
g}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p,
```

```
0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1015

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_
) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((4
*c)^IntPart[p]*(b + 2*c*x)^(2*FracPart[p])), Int[(g + h*x)^m*(b + 2*c*x)^(2
*p)*(d + f*x^2)^q, x], x] /; FreeQ[{a, b, c, d, f, g, h, m, p, q}, x] && Eq
Q[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(2ab+2b^2x)\sqrt{c+dx^2}}{x^3}\,dx}{2ab+2b^2x}\\
&= -\frac{(a+2bx)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{2x^2(a+bx)}-\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{-4abcd-8b^2cdx}{x\sqrt{c+dx^2}}\,dx}{4c\,(2ab+2b^2x)}\\
&= -\frac{(a+2bx)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{2x^2(a+bx)}+\frac{\left(abd\sqrt{a^2+2abx+b^2x^2}\right)\int \frac{1}{x\sqrt{c+dx^2}}\,dx}{2ab+2b^2x}\\
&\quad+\frac{\left(2b^2d\sqrt{a^2+2abx+b^2x^2}\right)\int \frac{1}{\sqrt{c+dx^2}}\,dx}{2ab+2b^2x}\\
&= -\frac{(a+2bx)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{2x^2(a+bx)}\\
&\quad+\frac{\left(abd\sqrt{a^2+2abx+b^2x^2}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}}\,dx,x,x^2\right)}{2\,(2ab+2b^2x)}\\
&\quad+\frac{\left(2b^2d\sqrt{a^2+2abx+b^2x^2}\right)\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx,x,\frac{x}{\sqrt{c+dx^2}}\right)}{2ab+2b^2x}\\
&= -\frac{(a+2bx)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{2x^2(a+bx)}+\frac{b\sqrt{d}\sqrt{a^2+2abx+b^2x^2}\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{a+bx}\\
&\quad+\frac{\left(ab\sqrt{a^2+2abx+b^2x^2}\right)\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+dx^2}\right)}{2ab+2b^2x}
\end{aligned}$$

$$= -\frac{(a+2bx)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{2x^2(a+bx)} + \frac{b\sqrt{d}\sqrt{a^2+2abx+b^2x^2}\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{a+bx} - \frac{ad\sqrt{a^2+2abx+b^2x^2}\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2\sqrt{c}(a+bx)}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{x^3}\,dx = \frac{\sqrt{(a+bx)^2}\left(2adx^2\text{arctanh}\left(\frac{\sqrt{d}x-\sqrt{c+dx^2}}{\sqrt{c}}\right)-\sqrt{c}\left((a+2bx)\sqrt{c+dx^2}+2b\sqrt{d}x^2\log\left(-\sqrt{d}x+\sqrt{c+dx^2}\right)\right)\right)}{2\sqrt{c}x^2(a+bx)}$$

```
[In] Integrate[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/x^3,x]
```

```
[Out] (Sqrt[(a + b*x)^2]*(2*a*d*x^2*ArcTanh[(Sqrt[d]*x - Sqrt[c + d*x^2])/Sqrt[c]
] - Sqrt[c]*((a + 2*b*x)*Sqrt[c + d*x^2] + 2*b*Sqrt[d]*x^2*Log[-(Sqrt[d]*x)
 + Sqrt[c + d*x^2]])))/(2*Sqrt[c]*x^2*(a + b*x))
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.66

method	result
risch	$-\frac{(2bx+a)\sqrt{(bx+a)^{2}}\,\sqrt{d\,x^{2}+c}}{2x^{2}(bx+a)}+\frac{\left(\sqrt{d}\,b\ln\left(\sqrt{d}\,x+\sqrt{d\,x^{2}+c}\right)-\frac{da\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^{2}+c}}{x}\right)}{2\sqrt{c}}\right)\sqrt{(bx+a)^{2}}}{bx+a}$
default	$-\frac{\text{csgn}(bx+a)\left(-2b\,d^{\frac{3}{2}}\,x^{3}\sqrt{d\,x^{2}+c}+\sqrt{c}\,\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^{2}+c}}{x}\right)d^{\frac{3}{2}}\,a\,x^{2}+2\left(d\,x^{2}+c\right)^{\frac{3}{2}}\sqrt{d}\,bx-a\,d^{\frac{3}{2}}\,x^{2}\sqrt{d\,x^{2}+c}-2\ln\left(\sqrt{d}\,x+\sqrt{d\,x^{2}+c}\right)bcd\right.}{2c\,x^{2}\sqrt{d}}$

```
[In] int(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(2*b*x+a)*((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x^2/(b*x+a)+(d^(1/2)*b*ln(
d^(1/2)*x+(d*x^2+c)^(1/2))-1/2*d*a/c^(1/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2
))/x))*((b*x+a)^2)^(1/2)/(b*x+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 377, normalized size of antiderivative = 2.34

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{x^3}\,dx$$
$$= \left[\frac{2\,bc\sqrt{d}x^2\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\sqrt{d}x-c\right)+a\sqrt{c}dx^2\log\left(-\frac{dx^2-2\,\sqrt{dx^2+c}\sqrt{c}+2\,c}{x^2}\right)-2\,(2\,bcx+ac)\sqrt{dx^2}}{4\,cx^2}\right.$$
$$-\frac{4\,bc\sqrt{-d}x^2\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2+c}}\right)-a\sqrt{c}dx^2\log\left(-\frac{dx^2-2\,\sqrt{dx^2+c}\sqrt{c}+2\,c}{x^2}\right)+2\,(2\,bcx+ac)\sqrt{dx^2+c}}{4\,cx^2},\ a\sqrt{-c}dx^2$$
$$\left.-\frac{2\,bc\sqrt{-d}x^2\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2+c}}\right)-a\sqrt{-c}dx^2\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)+(2\,bcx+ac)\sqrt{dx^2+c}}{2\,cx^2}\right]$$

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/4*(2*b*c*sqrt(d)*x^2*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + a
*sqrt(c)*d*x^2*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) - 2*(2*b
*c*x + a*c)*sqrt(d*x^2 + c))/(c*x^2), -1/4*(4*b*c*sqrt(-d)*x^2*arctan(sqrt(
-d)*x/sqrt(d*x^2 + c)) - a*sqrt(c)*d*x^2*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sq
rt(c) + 2*c)/x^2) + 2*(2*b*c*x + a*c)*sqrt(d*x^2 + c))/(c*x^2), 1/2*(a*sqrt
(-c)*d*x^2*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + b*c*sqrt(d)*x^2*log(-2*d*x^2
- 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) - (2*b*c*x + a*c)*sqrt(d*x^2 + c))/(c*x^
2), -1/2*(2*b*c*sqrt(-d)*x^2*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) - a*sqrt(-c
)*d*x^2*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + (2*b*c*x + a*c)*sqrt(d*x^2 + c))
/(c*x^2)]
```

Sympy [F]

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{x^3}\,dx = \int \frac{\sqrt{c+dx^2}\sqrt{(a+bx)^2}}{x^3}\,dx$$

```
[In] integrate(((b*x+a)**2)**(1/2)*(d*x**2+c)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(c + d*x**2)*sqrt((a + b*x)**2)/x**3, x)
```

Maxima [F]

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{x^3}\,dx = \int \frac{\sqrt{dx^2+c}\sqrt{(bx+a)^2}}{x^3}\,dx$$

[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x^3,x, algorithm="maxima")

[Out] integrate(sqrt(d*x^2 + c)*sqrt((b*x + a)^2)/x^3, x)

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.24

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{x^3}\,dx = \frac{ad\arctan\left(-\frac{\sqrt{d}x-\sqrt{dx^2+c}}{\sqrt{-c}}\right)\mathrm{sgn}(bx+a)}{\sqrt{-c}} - b\sqrt{d}\log\left(\left|-\sqrt{d}x+\sqrt{dx^2+c}\right|\right)\mathrm{sgn}(bx+a) + \frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^3ad\mathrm{sgn}(bx+a)+2\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2bc\sqrt{d}\mathrm{sgn}(bx+a)+\left(\sqrt{d}x-\sqrt{dx^2+c}\right)acds}{\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2-c\right)^2}$$

[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x^3,x, algorithm="giac")

[Out] a*d*arctan(-(sqrt(d)*x - sqrt(d*x^2 + c))/sqrt(-c))*sgn(b*x + a)/sqrt(-c) - b*sqrt(d)*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))*sgn(b*x + a) + ((sqrt(d)*x - sqrt(d*x^2 + c))^3*a*d*sgn(b*x + a) + 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c*sqrt(d)*sgn(b*x + a) + (sqrt(d)*x - sqrt(d*x^2 + c))*a*c*d*sgn(b*x + a) - 2*b*c^2*sqrt(d)*sgn(b*x + a))/((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c)^2

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{x^3}\,dx = \int \frac{\sqrt{(a+b\,x)^2}\,\sqrt{d\,x^2+c}}{x^3}\,dx$$

[In] int((((a + b*x)^2)^(1/2)*(c + d*x^2)^(1/2))/x^3,x)

[Out] int((((a + b*x)^2)^(1/2)*(c + d*x^2)^(1/2))/x^3, x)

3.46 $\int x^2\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 317

$$\begin{aligned}&\int x^2\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}\,dx\\&=-\frac{(2ad(4cd-5e^2)-b(12cde-7e^3))\,(e+2dx)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{128d^4(a+bx)}\\&\quad+\frac{bx^2\sqrt{a^2+2abx+b^2x^2}(c+ex+dx^2)^{3/2}}{5d(a+bx)}\\&\quad-\frac{(32bcd+50ade-35be^2-6d(10ad-7be)x)\sqrt{a^2+2abx+b^2x^2}(c+ex+dx^2)^{3/2}}{240d^3(a+bx)}\\&\quad-\frac{(4cd-e^2)\,(8acd^2-12bcde-10ade^2+7be^3)\sqrt{a^2+2abx+b^2x^2}\,\mathrm{arctanh}\left(\frac{e+2dx}{2\sqrt{d}\sqrt{c+ex+dx^2}}\right)}{256d^{9/2}(a+bx)}\end{aligned}$$

```
[Out] 1/5*b*x^2*(d*x^2+e*x+c)^(3/2)*((b*x+a)^2)^(1/2)/d/(b*x+a)-1/240*(32*b*c*d+5
0*a*d*e-35*b*e^2-6*d*(10*a*d-7*b*e)*x)*(d*x^2+e*x+c)^(3/2)*((b*x+a)^2)^(1/2
)/d^3/(b*x+a)-1/256*(4*c*d-e^2)*(8*a*c*d^2-10*a*d*e^2-12*b*c*d*e+7*b*e^3)*a
rctanh(1/2*(2*d*x+e)/d^(1/2)/(d*x^2+e*x+c)^(1/2))*((b*x+a)^2)^(1/2)/d^(9/2)
/(b*x+a)-1/128*(2*a*d*(4*c*d-5*e^2)-b*(12*c*d*e-7*e^3))*(2*d*x+e)*((b*x+a)^
2)^(1/2)*(d*x^2+e*x+c)^(1/2)/d^4/(b*x+a)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1014, 846, 793, 626, 635, 212}

$$\int x^2\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}\,dx$$
$$= -\frac{\sqrt{a^2+2abx+b^2x^2}(4cd-e^2)\left(8acd^2-10ade^2-12bcde+7be^3\right)\operatorname{arctanh}\left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+dx^2+ex}}\right)}{256d^{9/2}(a+bx)}$$
$$-\frac{\sqrt{a^2+2abx+b^2x^2}(2dx+e)\sqrt{c+dx^2+ex}(2ad(4cd-5e^2)-b(12cde-7e^3))}{128d^4(a+bx)}$$
$$-\frac{\sqrt{a^2+2abx+b^2x^2}(c+dx^2+ex)^{3/2}\left(-6dx(10ad-7be)+50ade+32bcd-35be^2\right)}{240d^3(a+bx)}$$
$$+\frac{bx^2\sqrt{a^2+2abx+b^2x^2}(c+dx^2+ex)^{3/2}}{5d(a+bx)}$$

```
[In] Int[x^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2],x]
```

```
[Out] -1/128*((2*a*d*(4*c*d - 5*e^2) - b*(12*c*d*e - 7*e^3))*(e + 2*d*x)*Sqrt[a^2
 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2])/(d^4*(a + b*x)) + (b*x^2*Sqrt[
a^2 + 2*a*b*x + b^2*x^2]*(c + e*x + d*x^2)^(3/2))/(5*d*(a + b*x)) - ((32*b*
c*d + 50*a*d*e - 35*b*e^2 - 6*d*(10*a*d - 7*b*e)*x)*Sqrt[a^2 + 2*a*b*x + b^
2*x^2]*(c + e*x + d*x^2)^(3/2))/(240*d^3*(a + b*x)) - ((4*c*d - e^2)*(8*a*c
*d^2 - 12*b*c*d*e - 10*a*d*e^2 + 7*b*e^3)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Arc
Tanh[(e + 2*d*x)/(2*Sqrt[d]*Sqrt[c + e*x + d*x^2])])/(256*d^(9/2)*(a + b*x)
)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
```

```
b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 846

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1014

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_
) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Dist[(a + b*x + c*x^2)^Fr
acPart[p]/((4*c)^IntPart[p]*(b + 2*c*x)^(2*FracPart[p])), Int[(g + h*x)^m*(
b + 2*c*x)^(2*p)*(d + e*x + f*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 h, m, p, q}, x] && EqQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{a^2+2abx+b^2x^2}\int x^2(2ab+2b^2x)\sqrt{c+ex+dx^2}\,dx}{2ab+2b^2x} \\
&= \frac{bx^2\sqrt{a^2+2abx+b^2x^2}(c+ex+dx^2)^{3/2}}{5d(a+bx)} \\
&\quad+\frac{\sqrt{a^2+2abx+b^2x^2}\int x(-4b^2c+b(10ad-7be)x)\sqrt{c+ex+dx^2}\,dx}{5d\,(2ab+2b^2x)} \\
&= \frac{bx^2\sqrt{a^2+2abx+b^2x^2}(c+ex+dx^2)^{3/2}}{5d(a+bx)} \\
&\quad-\frac{(32bcd+50ade-35be^2-6d(10ad-7be)x)\sqrt{a^2+2abx+b^2x^2}(c+ex+dx^2)^{3/2}}{240d^3(a+bx)} \\
&\quad+\frac{\left(\left(16b^2cde-2bcd(10ad-7be)+\frac{5}{2}be^2(10ad-7be)\right)\sqrt{a^2+2abx+b^2x^2}\right)\int\sqrt{c+ex+dx^2}\,dx}{40d^3\,(2ab+2b^2x)}
\end{aligned}
$$

$$= -\frac{(8acd^2 - 12bcde - 10ade^2 + 7be^3)(e + 2dx)\sqrt{a^2 + 2abx + b^2x^2}\sqrt{c + ex + dx^2}}{128d^4(a + bx)}$$
$$+ \frac{bx^2\sqrt{a^2 + 2abx + b^2x^2}(c + ex + dx^2)^{3/2}}{5d(a + bx)}$$
$$- \frac{(32bcd + 50ade - 35be^2 - 6d(10ad - 7be)x)\sqrt{a^2 + 2abx + b^2x^2}(c + ex + dx^2)^{3/2}}{240d^3(a + bx)}$$
$$+ \frac{\left((4cd - e^2)\left(16b^2cde - 2bcd(10ad - 7be) + \frac{5}{2}be^2(10ad - 7be)\right)\sqrt{a^2 + 2abx + b^2x^2}\right)\int \frac{1}{\sqrt{c+ex+dx^2}}\, d}{320d^4(2ab + 2b^2x)}$$

$$= -\frac{(8acd^2 - 12bcde - 10ade^2 + 7be^3)(e + 2dx)\sqrt{a^2 + 2abx + b^2x^2}\sqrt{c + ex + dx^2}}{128d^4(a + bx)}$$
$$+ \frac{bx^2\sqrt{a^2 + 2abx + b^2x^2}(c + ex + dx^2)^{3/2}}{5d(a + bx)}$$
$$- \frac{(32bcd + 50ade - 35be^2 - 6d(10ad - 7be)x)\sqrt{a^2 + 2abx + b^2x^2}(c + ex + dx^2)^{3/2}}{240d^3(a + bx)}$$
$$+ \frac{\left((4cd - e^2)\left(16b^2cde - 2bcd(10ad - 7be) + \frac{5}{2}be^2(10ad - 7be)\right)\sqrt{a^2 + 2abx + b^2x^2}\right)\text{Subst}\left(\int \frac{1}{4d-x}\right.}{160d^4(2ab + 2b^2x)}$$

$$= -\frac{(8acd^2 - 12bcde - 10ade^2 + 7be^3)(e + 2dx)\sqrt{a^2 + 2abx + b^2x^2}\sqrt{c + ex + dx^2}}{128d^4(a + bx)}$$
$$+ \frac{bx^2\sqrt{a^2 + 2abx + b^2x^2}(c + ex + dx^2)^{3/2}}{5d(a + bx)}$$
$$- \frac{(32bcd + 50ade - 35be^2 - 6d(10ad - 7be)x)\sqrt{a^2 + 2abx + b^2x^2}(c + ex + dx^2)^{3/2}}{240d^3(a + bx)}$$
$$- \frac{(4cd - e^2)(8acd^2 - 12bcde - 10ade^2 + 7be^3)\sqrt{a^2 + 2abx + b^2x^2}\tanh^{-1}\left(\frac{e+2dx}{2\sqrt{d}\sqrt{c+ex+dx^2}}\right)}{256d^{9/2}(a + bx)}$$

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.74

$$\int x^2\sqrt{a^2 + 2abx + b^2x^2}\sqrt{c + ex + dx^2}\, dx$$
$$= \frac{\sqrt{(a + bx)^2}\left(2\sqrt{d}\sqrt{c + x(e + dx)}(10ad(15e^3 - 10de^2x + 8d^2ex^2 + 48d^3x^3 + 4cd(-13e + 6dx)) + b(-256c\right.}{}$$

```
[In] Integrate[x^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2],x]
```

```
[Out] (Sqrt[(a + b*x)^2]*(2*Sqrt[d]*Sqrt[c + x*(e + d*x)]*(10*a*d*(15*e^3 - 10*d*
e^2*x + 8*d^2*e*x^2 + 48*d^3*x^3 + 4*c*d*(-13*e + 6*d*x)) + b*(-256*c^2*d^2
```

```
 - 105*e^4 + 70*d*e^3*x - 56*d^2*e^2*x^2 + 48*d^3*e*x^3 + 384*d^4*x^4 + 4*c
*d*(115*e^2 - 58*d*e*x + 32*d^2*x^2))) + 15*(4*c*d - e^2)*(2*a*d*(4*c*d - 5
*e^2) + b*(-12*c*d*e + 7*e^3))*Log[e + 2*d*x - 2*Sqrt[d]*Sqrt[c + x*(e + d*
x)]]))/(3840*d^(9/2)*(a + b*x))
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.84

method	result
risch	$\frac{(384b\,x^4d^4+480a\,d^4x^3+48b\,d^3e\,x^3+80a\,d^3e\,x^2+128bc\,d^3x^2-56b\,d^2e^2x^2+240ac\,d^3x-100a\,d^2e^2x-232xbc\,d^2e+70bd\,e^3x-520ac\,d^2e-}{1920d^4(bx+a)}$
default	$\frac{\operatorname{csgn}(bx+a)\left(768(d\,x^2+ex+c)^{\frac{3}{2}}d^{\frac{9}{2}}b\,x^2+960(d\,x^2+ex+c)^{\frac{3}{2}}d^{\frac{9}{2}}ax-672(d\,x^2+ex+c)^{\frac{3}{2}}d^{\frac{7}{2}}bex-800(d\,x^2+ex+c)^{\frac{3}{2}}d^{\frac{7}{2}}ae-512(d\,x^2+e\right.}{}$

```
[In] int(x^2*((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/1920*(384*b*d^4*x^4+480*a*d^4*x^3+48*b*d^3*e*x^3+80*a*d^3*e*x^2+128*b*c*d
^3*x^2-56*b*d^2*e^2*x^2+240*a*c*d^3*x-100*a*d^2*e^2*x-232*b*c*d^2*e*x+70*b*
d*e^3*x-520*a*c*d^2*e+150*a*d*e^3-256*b*c^2*d^2+460*b*c*d*e^2-105*b*e^4)*(d
*x^2+e*x+c)^(1/2)/d^4*((b*x+a)^2)^(1/2)/(b*x+a)-1/256*(32*a*c^2*d^3-48*a*c*
d^2*e^2+10*a*d*e^4-48*b*c^2*d^2*e+40*b*c*d*e^3-7*b*e^5)/d^(9/2)*ln((1/2*e+d
*x)/d^(1/2)+(d*x^2+e*x+c)^(1/2))*((b*x+a)^2)^(1/2)/(b*x+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.35 (sec) , antiderivative size = 517, normalized size of antiderivative = 1.63

$$\int x^2\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}\,dx$$

$$= \left[-\frac{15\,(32\,ac^2d^3-48\,bc^2d^2e-48\,acd^2e^2+40\,bcde^3+10\,ade^4-7\,be^5)\sqrt{d}\log\left(8\,d^2x^2+8\,dex+4\,\sqrt{dx^2+}\right.}{}\right.$$

```
[In] integrate(x^2*((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/7680*(15*(32*a*c^2*d^3 - 48*b*c^2*d^2*e - 48*a*c*d^2*e^2 + 40*b*c*d*e^3
 + 10*a*d*e^4 - 7*b*e^5)*sqrt(d)*log(8*d^2*x^2 + 8*d*e*x + 4*sqrt(d*x^2 + e
*x + c)*(2*d*x + e)*sqrt(d) + 4*c*d + e^2) - 4*(384*b*d^5*x^4 - 256*b*c^2*d
^3 - 520*a*c*d^3*e + 460*b*c*d^2*e^2 + 150*a*d^2*e^3 - 105*b*d*e^4 + 48*(10
*a*d^5 + b*d^4*e)*x^3 + 8*(16*b*c*d^4 + 10*a*d^4*e - 7*b*d^3*e^2)*x^2 + 2*(
120*a*c*d^4 - 116*b*c*d^3*e - 50*a*d^3*e^2 + 35*b*d^2*e^3)*x)*sqrt(d*x^2 +
e*x + c))/d^5, 1/3840*(15*(32*a*c^2*d^3 - 48*b*c^2*d^2*e - 48*a*c*d^2*e^2 +
```

```
 40*b*c*d*e^3 + 10*a*d*e^4 - 7*b*e^5)*sqrt(-d)*arctan(1/2*sqrt(d*x^2 + e*x
+ c)*(2*d*x + e)*sqrt(-d)/(d^2*x^2 + d*e*x + c*d)) + 2*(384*b*d^5*x^4 - 256
*b*c^2*d^3 - 520*a*c*d^3*e + 460*b*c*d^2*e^2 + 150*a*d^2*e^3 - 105*b*d*e^4
+ 48*(10*a*d^5 + b*d^4*e)*x^3 + 8*(16*b*c*d^4 + 10*a*d^4*e - 7*b*d^3*e^2)*x
^2 + 2*(120*a*c*d^4 - 116*b*c*d^3*e - 50*a*d^3*e^2 + 35*b*d^2*e^3)*x)*sqrt(
d*x^2 + e*x + c))/d^5]
```

Sympy [F]

$$\int x^2\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}\,dx=\int x^2\sqrt{c+dx^2+ex}\sqrt{(a+bx)^2}\,dx$$

```
[In] integrate(x**2*((b*x+a)**2)**(1/2)*(d*x**2+e*x+c)**(1/2),x)
[Out] Integral(x**2*sqrt(c + d*x**2 + e*x)*sqrt((a + b*x)**2), x)
```

Maxima [F]

$$\int x^2\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}\,dx=\int\sqrt{dx^2+ex+c}\sqrt{(bx+a)^2}x^2\,dx$$

```
[In] integrate(x^2*((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(d*x^2 + e*x + c)*sqrt((b*x + a)^2)*x^2, x)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.16

$$\int x^2\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}\,dx$$
$$=\frac{1}{1920}\sqrt{dx^2+ex+c}\left(2\left(4\left(6\left(8\,bx\mathrm{sgn}(bx+a)+\frac{10\,ad^4\mathrm{sgn}(bx+a)+bd^3e\mathrm{sgn}(bx+a)}{d^4}\right)x+\frac{16\,bcd^3\mathrm{sgn}(b\right.\right.\right.$$
$$+\frac{(32\,ac^2d^3\mathrm{sgn}(bx+a)-48\,bc^2d^2e\mathrm{sgn}(bx+a)-48\,acd^2e^2\mathrm{sgn}(bx+a)+40\,bcde^3\mathrm{sgn}(bx+a)+10\,ade^4\mathrm{sgr}}{256\,d^{\frac{9}{2}}}$$

```
[In] integrate(x^2*((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2),x, algorithm="giac")
[Out] 1/1920*sqrt(d*x^2 + e*x + c)*(2*(4*(6*(8*b*x*sgn(b*x + a) + (10*a*d^4*sgn(b
*x + a) + b*d^3*e*sgn(b*x + a))/d^4)*x + (16*b*c*d^3*sgn(b*x + a) + 10*a*d^
3*e*sgn(b*x + a) - 7*b*d^2*e^2*sgn(b*x + a))/d^4)*x + (120*a*c*d^3*sgn(b*x
```

```
+ a) - 116*b*c*d^2*e*sgn(b*x + a) - 50*a*d^2*e^2*sgn(b*x + a) + 35*b*d*e^3*
sgn(b*x + a))/d^4)*x - (256*b*c^2*d^2*sgn(b*x + a) + 520*a*c*d^2*e*sgn(b*x
+ a) - 460*b*c*d*e^2*sgn(b*x + a) - 150*a*d*e^3*sgn(b*x + a) + 105*b*e^4*sg
n(b*x + a))/d^4) + 1/256*(32*a*c^2*d^3*sgn(b*x + a) - 48*b*c^2*d^2*e*sgn(b*
x + a) - 48*a*c*d^2*e^2*sgn(b*x + a) + 40*b*c*d*e^3*sgn(b*x + a) + 10*a*d*e
^4*sgn(b*x + a) - 7*b*e^5*sgn(b*x + a))*log(abs(2*(sqrt(d)*x - sqrt(d*x^2 +
 e*x + c))*sqrt(d) + e))/d^(9/2)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}\,dx=\int x^2\,\sqrt{{\left(a+b\,x\right)}^2}\,\sqrt{d\,x^2+e\,x+c}\,dx$$

```
[In] int(x^2*((a + b*x)^2)^(1/2)*(c + e*x + d*x^2)^(1/2),x)
[Out] int(x^2*((a + b*x)^2)^(1/2)*(c + e*x + d*x^2)^(1/2), x)
```

3.47 $\int x\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 227

$$\begin{aligned}&\int x\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}\,dx\\&=-\frac{(4bcd+8ade-5be^2)\,(e+2dx)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{64d^3(a+bx)}\\&\quad+\frac{(8ad-5be+6bdx)\sqrt{a^2+2abx+b^2x^2}\,(c+ex+dx^2)^{3/2}}{24d^2(a+bx)}\\&\quad-\frac{(4cd-e^2)\,(4bcd+8ade-5be^2)\,\sqrt{a^2+2abx+b^2x^2}\,\text{arctanh}\left(\frac{e+2dx}{2\sqrt{d}\sqrt{c+ex+dx^2}}\right)}{128d^{7/2}(a+bx)}\end{aligned}$$

```
[Out] 1/24*(6*b*d*x+8*a*d-5*b*e)*(d*x^2+e*x+c)^(3/2)*((b*x+a)^2)^(1/2)/d^2/(b*x+a
)-1/128*(4*c*d-e^2)*(8*a*d*e+4*b*c*d-5*b*e^2)*arctanh(1/2*(2*d*x+e)/d^(1/2)
/(d*x^2+e*x+c)^(1/2))*((b*x+a)^2)^(1/2)/d^(7/2)/(b*x+a)-1/64*(8*a*d*e+4*b*c
*d-5*b*e^2)*(2*d*x+e)*((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/d^3/(b*x+a)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used

= {1014, 793, 626, 635, 212}

$$\int x\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}\,dx$$
$$= -\frac{\sqrt{a^2+2abx+b^2x^2}(4cd-e^2)\left(8ade+4bcd-5be^2\right)\operatorname{arctanh}\left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+dx^2+ex}}\right)}{128d^{7/2}(a+bx)}$$
$$-\frac{\sqrt{a^2+2abx+b^2x^2}(2dx+e)\sqrt{c+dx^2+ex}(8ade+4bcd-5be^2)}{64d^3(a+bx)}$$
$$+\frac{\sqrt{a^2+2abx+b^2x^2}(c+dx^2+ex)^{3/2}\left(8ad+6bdx-5be\right)}{24d^2(a+bx)}$$

```
[In] Int[x*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2],x]
```

```
[Out] -1/64*((4*b*c*d + 8*a*d*e - 5*b*e^2)*(e + 2*d*x)*Sqrt[a^2 + 2*a*b*x + b^2*x
^2]*Sqrt[c + e*x + d*x^2])/(d^3*(a + b*x)) + ((8*a*d - 5*b*e + 6*b*d*x)*Sqr
t[a^2 + 2*a*b*x + b^2*x^2]*(c + e*x + d*x^2)^(3/2))/(24*d^2*(a + b*x)) - ((
4*c*d - e^2)*(4*b*c*d + 8*a*d*e - 5*b*e^2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Ar
cTanh[(e + 2*d*x)/(2*Sqrt[d]*Sqrt[c + e*x + d*x^2])])/(128*d^(7/2)*(a + b*x
))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
```

```
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1014

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_
) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Dist[(a + b*x + c*x^2)^Fr
acPart[p]/((4*c)^IntPart[p]*(b + 2*c*x)^(2*FracPart[p])), Int[(g + h*x)^m*(
b + 2*c*x)^(2*p)*(d + e*x + f*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 h, m, p, q}, x] && EqQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{a^2+2abx+b^2x^2}\int x(2ab+2b^2x)\sqrt{c+ex+dx^2}\,dx}{2ab+2b^2x}\\
&= \frac{(8ad-5be+6bdx)\sqrt{a^2+2abx+b^2x^2}(c+ex+dx^2)^{3/2}}{24d^2(a+bx)}\\
&\quad-\frac{\left(b(4bcd+8ade-5be^2)\sqrt{a^2+2abx+b^2x^2}\right)\int\sqrt{c+ex+dx^2}\,dx}{8d^2\left(2ab+2b^2x\right)}\\
&= -\frac{(4bcd+8ade-5be^2)\left(e+2dx\right)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{64d^3(a+bx)}\\
&\quad+\frac{(8ad-5be+6bdx)\sqrt{a^2+2abx+b^2x^2}(c+ex+dx^2)^{3/2}}{24d^2(a+bx)}\\
&\quad-\frac{\left(b(4cd-e^2)\left(4bcd+8ade-5be^2\right)\sqrt{a^2+2abx+b^2x^2}\right)\int\frac{1}{\sqrt{c+ex+dx^2}}\,dx}{64d^3\left(2ab+2b^2x\right)}\\
&= -\frac{(4bcd+8ade-5be^2)\left(e+2dx\right)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{64d^3(a+bx)}\\
&\quad+\frac{(8ad-5be+6bdx)\sqrt{a^2+2abx+b^2x^2}(c+ex+dx^2)^{3/2}}{24d^2(a+bx)}\\
&\quad-\frac{\left(b(4cd-e^2)\left(4bcd+8ade-5be^2\right)\sqrt{a^2+2abx+b^2x^2}\right)\text{Subst}\left(\int\frac{1}{4d-x^2}\,dx,x,\frac{e+2dx}{\sqrt{c+ex+dx^2}}\right)}{32d^3\left(2ab+2b^2x\right)}\\
&= -\frac{(4bcd+8ade-5be^2)\left(e+2dx\right)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{64d^3(a+bx)}\\
&\quad+\frac{(8ad-5be+6bdx)\sqrt{a^2+2abx+b^2x^2}(c+ex+dx^2)^{3/2}}{24d^2(a+bx)}\\
&\quad-\frac{(4cd-e^2)\left(4bcd+8ade-5be^2\right)\sqrt{a^2+2abx+b^2x^2}\tanh^{-1}\left(\frac{e+2dx}{2\sqrt{d}\sqrt{c+ex+dx^2}}\right)}{128d^{7/2}(a+bx)}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.78

$$\int x\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}\,dx$$

$$= \frac{\sqrt{(a+bx)^2}\Big(2\sqrt{d}\sqrt{c+x(e+dx)}(8ad(8cd-3e^2+2dex+8d^2x^2)+b(15e^3-10de^2x+8d^2ex^2+48d^3x^3}{384d^{7/2}(a+}$$

```
[In] Integrate[x*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2],x]
```

```
[Out] (Sqrt[(a + b*x)^2]*(2*Sqrt[d]*Sqrt[c + x*(e + d*x)]*(8*a*d*(8*c*d - 3*e^2 +
 2*d*e*x + 8*d^2*x^2) + b*(15*e^3 - 10*d*e^2*x + 8*d^2*e*x^2 + 48*d^3*x^3 +
 4*c*d*(-13*e + 6*d*x))) + 3*(4*c*d - e^2)*(4*b*c*d + 8*a*d*e - 5*b*e^2)*Lo
g[e + 2*d*x - 2*Sqrt[d]*Sqrt[c + x*(e + d*x)]]))/(384*d^(7/2)*(a + b*x))
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.88

method	result
risch	$\frac{\left(48b\,x^3d^3+64a\,d^3x^2+8b\,d^2e\,x^2+16a\,d^2ex+24bc\,d^2x-10bd\,e^2x+64c\,d^2a-24ad\,e^2-52bcde+15b\,e^3\right)\sqrt{d\,x^2+ex+c}\,\sqrt{(bx+a)^2}}{192d^3(bx+a)}-\frac{(32a}{}$
default	$\frac{\mathrm{csgn}(bx+a)\Big(96(d\,x^2+ex+c)^{\frac{3}{2}}d^{\frac{7}{2}}bx+128(d\,x^2+ex+c)^{\frac{3}{2}}d^{\frac{7}{2}}a-80(d\,x^2+ex+c)^{\frac{3}{2}}d^{\frac{5}{2}}be-96\sqrt{d\,x^2+ex+c}\,d^{\frac{7}{2}}aex-48\sqrt{d\,x^2+ex+c}\,d^{\frac{7}{2}}b}{}$

```
[In] int(x*((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/192*(48*b*d^3*x^3+64*a*d^3*x^2+8*b*d^2*e*x^2+16*a*d^2*e*x+24*b*c*d^2*x-10
*b*d*e^2*x+64*a*c*d^2-24*a*d*e^2-52*b*c*d*e+15*b*e^3)*(d*x^2+e*x+c)^(1/2)/d
^3*((b*x+a)^2)^(1/2)/(b*x+a)-1/128*(32*a*c*d^2*e-8*a*d*e^3+16*b*c^2*d^2-24*
b*c*d*e^2+5*b*e^4)/d^(7/2)*ln((1/2*e+d*x)/d^(1/2)+(d*x^2+e*x+c)^(1/2))*((b*
x+a)^2)^(1/2)/(b*x+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.72

$$\int x\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}\,dx$$
$$=\left[\frac{3\left(16\,bc^2d^2+32\,acd^2e-24\,bcde^2-8\,ade^3+5\,be^4\right)\sqrt{d}\log\left(8\,d^2x^2+8\,dex-4\,\sqrt{dx^2+ex+c}(2\,dx+e)\sqrt{d}\right.}{}\right.$$

```
[In] integrate(x*((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/768*(3*(16*b*c^2*d^2 + 32*a*c*d^2*e - 24*b*c*d*e^2 - 8*a*d*e^3 + 5*b*e^4
)*sqrt(d)*log(8*d^2*x^2 + 8*d*e*x - 4*sqrt(d*x^2 + e*x + c)*(2*d*x + e)*sqr
t(d) + 4*c*d + e^2) + 4*(48*b*d^4*x^3 + 64*a*c*d^3 - 52*b*c*d^2*e - 24*a*d^
2*e^2 + 15*b*d*e^3 + 8*(8*a*d^4 + b*d^3*e)*x^2 + 2*(12*b*c*d^3 + 8*a*d^3*e
- 5*b*d^2*e^2)*x)*sqrt(d*x^2 + e*x + c))/d^4, 1/384*(3*(16*b*c^2*d^2 + 32*a
*c*d^2*e - 24*b*c*d*e^2 - 8*a*d*e^3 + 5*b*e^4)*sqrt(-d)*arctan(1/2*sqrt(d*x
^2 + e*x + c)*(2*d*x + e)*sqrt(-d)/(d^2*x^2 + d*e*x + c*d)) + 2*(48*b*d^4*x
^3 + 64*a*c*d^3 - 52*b*c*d^2*e - 24*a*d^2*e^2 + 15*b*d*e^3 + 8*(8*a*d^4 + b
*d^3*e)*x^2 + 2*(12*b*c*d^3 + 8*a*d^3*e - 5*b*d^2*e^2)*x)*sqrt(d*x^2 + e*x
+ c))/d^4]
```

Sympy [F]

$$\int x\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}\,dx=\int x\sqrt{c+dx^2+ex}\sqrt{(a+bx)^2}\,dx$$

```
[In] integrate(x*((b*x+a)**2)**(1/2)*(d*x**2+e*x+c)**(1/2),x)
```

```
[Out] Integral(x*sqrt(c + d*x**2 + e*x)*sqrt((a + b*x)**2), x)
```

Maxima [F]

$$\int x\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}\,dx=\int\sqrt{dx^2+ex+c}\sqrt{(bx+a)^2}x\,dx$$

```
[In] integrate(x*((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + e*x + c)*sqrt((b*x + a)^2)*x, x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.17

$$\int x\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}\,dx$$
$$=\frac{1}{192}\sqrt{dx^2+ex+c}\left(2\left(4\left(6\,bx\text{sgn}(bx+a)+\frac{8\,ad^3\text{sgn}(bx+a)+bd^2e\text{sgn}(bx+a)}{d^3}\right)x+\frac{12\,bcd^2\text{sgn}(bx+\right.\right.$$
$$+\frac{(16\,bc^2d^2\text{sgn}(bx+a)+32\,acd^2e\text{sgn}(bx+a)-24\,bcde^2\text{sgn}(bx+a)-8\,ade^3\text{sgn}(bx+a)+5\,be^4\text{sgn}(bx-}{128\,d^{\frac{7}{2}}}$$

```
[In] integrate(x*((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/192*sqrt(d*x^2 + e*x + c)*(2*(4*(6*b*x*sgn(b*x + a) + (8*a*d^3*sgn(b*x +
a) + b*d^2*e*sgn(b*x + a))/d^3)*x + (12*b*c*d^2*sgn(b*x + a) + 8*a*d^2*e*sg
n(b*x + a) - 5*b*d*e^2*sgn(b*x + a))/d^3)*x + (64*a*c*d^2*sgn(b*x + a) - 52
*b*c*d*e*sgn(b*x + a) - 24*a*d*e^2*sgn(b*x + a) + 15*b*e^3*sgn(b*x + a))/d^
3) + 1/128*(16*b*c^2*d^2*sgn(b*x + a) + 32*a*c*d^2*e*sgn(b*x + a) - 24*b*c*
d*e^2*sgn(b*x + a) - 8*a*d*e^3*sgn(b*x + a) + 5*b*e^4*sgn(b*x + a))*log(abs
(2*(sqrt(d)*x - sqrt(d*x^2 + e*x + c))*sqrt(d) + e))/d^(7/2)
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}\,dx=\int x\,\sqrt{{\left(a+b\,x\right)}^2}\,\sqrt{d\,x^2+e\,x+c}\,dx$$

```
[In] int(x*((a + b*x)^2)^(1/2)*(c + e*x + d*x^2)^(1/2),x)
```

```
[Out] int(x*((a + b*x)^2)^(1/2)*(c + e*x + d*x^2)^(1/2), x)
```

3.48 $\int \sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}\,dx$

Optimal result

Integrand size = 35, antiderivative size = 198

$$\begin{aligned}
&\int \sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}\,dx \\
&= \frac{(2ad-be)(e+2dx)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{8d^2(a+bx)} \\
&\quad + \frac{b\sqrt{a^2+2abx+b^2x^2}(c+ex+dx^2)^{3/2}}{3d(a+bx)} \\
&\quad + \frac{(2ad-be)(4cd-e^2)\sqrt{a^2+2abx+b^2x^2}\,\text{arctanh}\left(\frac{e+2dx}{2\sqrt{d}\sqrt{c+ex+dx^2}}\right)}{16d^{5/2}(a+bx)}
\end{aligned}$$

```
[Out] 1/3*b*(d*x^2+e*x+c)^(3/2)*((b*x+a)^2)^(1/2)/d/(b*x+a)+1/16*(2*a*d-b*e)*(4*c
*d-e^2)*arctanh(1/2*(2*d*x+e)/d^(1/2)/(d*x^2+e*x+c)^(1/2))*((b*x+a)^2)^(1/2
)/d^(5/2)/(b*x+a)+1/8*(2*a*d-b*e)*(2*d*x+e)*((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)
^(1/2)/d^2/(b*x+a)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used

= {983, 654, 626, 635, 212}

$$\int \sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}\,dx$$
$$= \frac{\sqrt{a^2+2abx+b^2x^2}(4cd-e^2)\,(2ad-be)\text{arctanh}\left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+dx^2+ex}}\right)}{16d^{5/2}(a+bx)} + \frac{\sqrt{a^2+2abx+b^2x^2}(2dx+e)(2ad-be)\sqrt{c+dx^2+ex}}{8d^2(a+bx)} + \frac{b\sqrt{a^2+2abx+b^2x^2}(c+dx^2+ex)^{3/2}}{3d(a+bx)}$$

```
[In] Int[Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2],x]
```

```
[Out] ((2*a*d - b*e)*(e + 2*d*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x
^2])/(8*d^2*(a + b*x)) + (b*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(c + e*x + d*x^2)
^(3/2))/(3*d*(a + b*x)) + ((2*a*d - b*e)*(4*c*d - e^2)*Sqrt[a^2 + 2*a*b*x +
 b^2*x^2]*ArcTanh[(e + 2*d*x)/(2*Sqrt[d]*Sqrt[c + e*x + d*x^2])])/(16*d^(5/
2)*(a + b*x))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 983

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.)*(x_)
^2)^(q_.), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((4*c)^IntPart[p
]*(b + 2*c*x)^(2*FracPart[p])), Int[(b + 2*c*x)^(2*p)*(d + e*x + f*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, f, p, q}, x] && EqQ[b^2 - 4*a*c, 0] &&  !In
tegerQ[p]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{a^2+2abx+b^2x^2}\int(2ab+2b^2x)\sqrt{c+ex+dx^2}\,dx}{2ab+2b^2x}$$

$$= \frac{b\sqrt{a^2+2abx+b^2x^2}(c+ex+dx^2)^{3/2}}{3d(a+bx)} + \frac{\left(b(2ad-be)\sqrt{a^2+2abx+b^2x^2}\right)\int\sqrt{c+ex+dx^2}\,dx}{d\,(2ab+2b^2x)}$$

$$= \frac{(2ad-be)(e+2dx)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{8d^2(a+bx)} + \frac{b\sqrt{a^2+2abx+b^2x^2}(c+ex+dx^2)^{3/2}}{3d(a+bx)} + \frac{\left(b(2ad-be)\,(4cd-e^2)\sqrt{a^2+2abx+b^2x^2}\right)\int\frac{1}{\sqrt{c+ex+dx^2}}\,dx}{8d^2\,(2ab+2b^2x)}$$

$$= \frac{(2ad-be)(e+2dx)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{8d^2(a+bx)} + \frac{b\sqrt{a^2+2abx+b^2x^2}(c+ex+dx^2)^{3/2}}{3d(a+bx)} + \frac{\left(b(2ad-be)\,(4cd-e^2)\sqrt{a^2+2abx+b^2x^2}\right)\text{Subst}\left(\int\frac{1}{4d-x^2}\,dx, x, \frac{e+2dx}{\sqrt{c+ex+dx^2}}\right)}{4d^2\,(2ab+2b^2x)}$$

$$= \frac{(2ad-be)(e+2dx)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{8d^2(a+bx)} + \frac{b\sqrt{a^2+2abx+b^2x^2}(c+ex+dx^2)^{3/2}}{3d(a+bx)} + \frac{(2ad-be)\,(4cd-e^2)\sqrt{a^2+2abx+b^2x^2}\tanh^{-1}\left(\frac{e+2dx}{2\sqrt{d}\sqrt{c+ex+dx^2}}\right)}{16d^{5/2}(a+bx)}$$

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.87

$$\int \sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}\,dx$$
$$=\frac{\sqrt{(a+bx)^2}\left(\sqrt{d}\sqrt{c+x(e+dx)}(6ad(e+2dx)+b(8cd-3e^2+2dex+8d^2x^2))+6de(2bc+ae)\text{arctanh}\left(\right.\right.}{24d^{5/2}(a+bx)}$$

```
[In] Integrate[Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2],x]
```

```
[Out] (Sqrt[(a + b*x)^2]*(Sqrt[d]*Sqrt[c + x*(e + d*x)]*(6*a*d*(e + 2*d*x) + b*(8
*c*d - 3*e^2 + 2*d*e*x + 8*d^2*x^2)) + 6*d*e*(2*b*c + a*e)*ArcTanh[(Sqrt[d]
*x)/(Sqrt[c] - Sqrt[c + x*(e + d*x)])] + 3*(8*a*c*d^2 + b*e^3)*ArcTanh[(Sqr
t[d]*x)/(-Sqrt[c] + Sqrt[c + x*(e + d*x)])]))/(24*d^(5/2)*(a + b*x))
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.74

method	result
risch	$\frac{(8b\,x^2d^2+12a\,d^2x+2bdex+6ade+8bcd-3b\,e^2)\sqrt{d\,x^2+ex+c}\,\sqrt{(bx+a)^2}}{24d^2(bx+a)}+\frac{(8c\,d^2a-2ad\,e^2-4bcde+b\,e^3)\ln\left(\frac{\frac{e}{2}+dx}{\sqrt{d}}+\sqrt{d\,x^2+ex+c}\right)}{16d^{\frac{5}{2}}(bx+a)}$
default	$\frac{\text{csgn}(bx+a)\left(16(d\,x^2+ex+c)^{\frac{3}{2}}d^{\frac{5}{2}}b+24\sqrt{d\,x^2+ex+c}\,d^{\frac{7}{2}}ax-12\sqrt{d\,x^2+ex+c}\,d^{\frac{5}{2}}bex+12\sqrt{d\,x^2+ex+c}\,d^{\frac{5}{2}}ae-6\sqrt{d\,x^2+ex+c}\,d^{\frac{3}{2}}b\,e^2+2\right.}{}$

```
[In] int(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*(8*b*d^2*x^2+12*a*d^2*x+2*b*d*e*x+6*a*d*e+8*b*c*d-3*b*e^2)*(d*x^2+e*x+
c)^(1/2)/d^2*((b*x+a)^2)^(1/2)/(b*x+a)+1/16*(8*a*c*d^2-2*a*d*e^2-4*b*c*d*e+
b*e^3)/d^(5/2)*ln((1/2*e+d*x)/d^(1/2)+(d*x^2+e*x+c)^(1/2))*((b*x+a)^2)^(1/2
)/(b*x+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.45

$$\int \sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}\,dx$$
$$= \left[\frac{3\,(8\,acd^2-4\,bcde-2\,ade^2+be^3)\sqrt{d}\log\left(8\,d^2x^2+8\,dex+4\,\sqrt{dx^2+ex+c}(2\,dx+e)\sqrt{d}+4\,cd+e^2\right)+\cdots}{96\,d^3}\right.$$
$$-\frac{3\,(8\,acd^2-4\,bcde-2\,ade^2+be^3)\sqrt{-d}\arctan\left(\frac{\sqrt{dx^2+ex+c}(2\,dx+e)\sqrt{-d}}{2\,(d^2x^2+dex+cd)}\right)-2\,(8\,bd^3x^2+8\,bcd^2+6\,ad^2e-3\,b\cdots}{48\,d^3}$$

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/96*(3*(8*a*c*d^2 - 4*b*c*d*e - 2*a*d*e^2 + b*e^3)*sqrt(d)*log(8*d^2*x^2
+ 8*d*e*x + 4*sqrt(d*x^2 + e*x + c)*(2*d*x + e)*sqrt(d) + 4*c*d + e^2) + 4*
(8*b*d^3*x^2 + 8*b*c*d^2 + 6*a*d^2*e - 3*b*d*e^2 + 2*(6*a*d^3 + b*d^2*e)*x)
*sqrt(d*x^2 + e*x + c))/d^3, -1/48*(3*(8*a*c*d^2 - 4*b*c*d*e - 2*a*d*e^2 +
b*e^3)*sqrt(-d)*arctan(1/2*sqrt(d*x^2 + e*x + c)*(2*d*x + e)*sqrt(-d)/(d^2*
x^2 + d*e*x + c*d)) - 2*(8*b*d^3*x^2 + 8*b*c*d^2 + 6*a*d^2*e - 3*b*d*e^2 +
2*(6*a*d^3 + b*d^2*e)*x)*sqrt(d*x^2 + e*x + c))/d^3]
```

Sympy [F]

$$\int \sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}\,dx = \int \sqrt{c+dx^2+ex}\sqrt{(a+bx)^2}\,dx$$

```
[In] integrate(((b*x+a)**2)**(1/2)*(d*x**2+e*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*x**2 + e*x)*sqrt((a + b*x)**2), x)
```

Maxima [F]

$$\int \sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}\,dx = \int \sqrt{dx^2+ex+c}\sqrt{(bx+a)^2}\,dx$$

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + e*x + c)*sqrt((b*x + a)^2), x)
```

Giac [A] (verification not implemented)

none

Time = 0.50 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.91

$$\int \sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}\,dx$$
$$=\frac{1}{24}\sqrt{dx^2+ex+c}\left(2\left(4\,bx\mathrm{sgn}(bx+a)+\frac{6\,ad^2\mathrm{sgn}(bx+a)+bde\mathrm{sgn}(bx+a)}{d^2}\right)x+\frac{8\,bcd\mathrm{sgn}(bx+a)+6\,a\cdots}{\cdots}\right.$$
$$-\frac{\left(8\,acd^2\mathrm{sgn}(bx+a)-4\,bcde\mathrm{sgn}(bx+a)-2\,ade^2\mathrm{sgn}(bx+a)+be^3\mathrm{sgn}(bx+a)\right)\log\left(\left|2\left(\sqrt{d}x-\sqrt{dx^2+\cdots}\right.\right.\right.}{16\,d^{\frac{5}{2}}}$$

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*sqrt(d*x^2 + e*x + c)*(2*(4*b*x*sgn(b*x + a) + (6*a*d^2*sgn(b*x + a) +
 b*d*e*sgn(b*x + a))/d^2)*x + (8*b*c*d*sgn(b*x + a) + 6*a*d*e*sgn(b*x + a)
- 3*b*e^2*sgn(b*x + a))/d^2) - 1/16*(8*a*c*d^2*sgn(b*x + a) - 4*b*c*d*e*sgn
(b*x + a) - 2*a*d*e^2*sgn(b*x + a) + b*e^3*sgn(b*x + a))*log(abs(2*(sqrt(d)
*x - sqrt(d*x^2 + e*x + c))*sqrt(d) + e))/d^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}\,dx=\int \sqrt{{\left(a+b\,x\right)}^2}\,\sqrt{d\,x^2+e\,x+c}\,dx$$

```
[In] int(((a + b*x)^2)^(1/2)*(c + e*x + d*x^2)^(1/2),x)
```

```
[Out] int(((a + b*x)^2)^(1/2)*(c + e*x + d*x^2)^(1/2), x)
```

3.49 $\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{x} \, dx$

Optimal result

Integrand size = 38, antiderivative size = 211

$$\begin{aligned}
&\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{x} \, dx \\
&= \frac{(4ad+be+2bdx)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{4d(a+bx)} \\
&\quad + \frac{(4bcd+4ade-be^2)\sqrt{a^2+2abx+b^2x^2}\,\text{arctanh}\left(\frac{e+2dx}{2\sqrt{d}\sqrt{c+ex+dx^2}}\right)}{8d^{3/2}(a+bx)} \\
&\quad - \frac{a\sqrt{c}\sqrt{a^2+2abx+b^2x^2}\,\text{arctanh}\left(\frac{2c+ex}{2\sqrt{c}\sqrt{c+ex+dx^2}}\right)}{a+bx}
\end{aligned}$$

```
[Out] 1/8*(4*a*d*e+4*b*c*d-b*e^2)*arctanh(1/2*(2*d*x+e)/d^(1/2)/(d*x^2+e*x+c)^(1/
2))*((b*x+a)^2)^(1/2)/d^(3/2)/(b*x+a)-a*arctanh(1/2*(e*x+2*c)/c^(1/2)/(d*x^
2+e*x+c)^(1/2))*c^(1/2)*((b*x+a)^2)^(1/2)/(b*x+a)+1/4*(2*b*d*x+4*a*d+b*e)*(
(b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/d/(b*x+a)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used

= {1014, 828, 857, 635, 212, 738}

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{x}\,dx$$
$$= \frac{\sqrt{a^2+2abx+b^2x^2}(4ade+4bcd-be^2)\operatorname{arctanh}\left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+dx^2+ex}}\right)}{8d^{3/2}(a+bx)} - \frac{a\sqrt{c}\sqrt{a^2+2abx+b^2x^2}\operatorname{arctanh}\left(\frac{2c+ex}{2\sqrt{c}\sqrt{c+dx^2+ex}}\right)}{a+bx} + \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2+ex}(4ad+2bdx+be)}{4d(a+bx)}$$

```
[In] Int[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2])/x,x]
```

```
[Out] ((4*a*d + b*e + 2*b*d*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2
])/(4*d*(a + b*x)) + ((4*b*c*d + 4*a*d*e - b*e^2)*Sqrt[a^2 + 2*a*b*x + b^2*
x^2]*ArcTanh[(e + 2*d*x)/(2*Sqrt[d]*Sqrt[c + e*x + d*x^2])])/(8*d^(3/2)*(a
+ b*x)) - (a*Sqrt[c]*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[(2*c + e*x)/(2*S
qrt[c]*Sqrt[c + e*x + d*x^2])])/(a + b*x)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
```

```
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1014

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_
) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Dist[(a + b*x + c*x^2)^Fr
acPart[p]/((4*c)^IntPart[p]*(b + 2*c*x)^(2*FracPart[p])), Int[(g + h*x)^m*(
b + 2*c*x)^(2*p)*(d + e*x + f*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 h, m, p, q}, x] && EqQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(2ab+2b^2x)\sqrt{c+ex+dx^2}}{x}\,dx}{2ab+2b^2x}\\
&= \frac{(4ad+be+2bdx)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{4d(a+bx)}\\
&\quad - \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{-8abcd-b\left(4ade+b\left(4cd-e^2\right)\right)x}{x\sqrt{c+ex+dx^2}}\,dx}{4d\left(2ab+2b^2x\right)}\\
&= \frac{(4ad+be+2bdx)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{4d(a+bx)}\\
&\quad + \frac{\left(2abc\sqrt{a^2+2abx+b^2x^2}\right)\int \frac{1}{x\sqrt{c+ex+dx^2}}\,dx}{2ab+2b^2x}\\
&\quad + \frac{\left(b\left(4bcd+4ade-be^2\right)\sqrt{a^2+2abx+b^2x^2}\right)\int \frac{1}{\sqrt{c+ex+dx^2}}\,dx}{4d\left(2ab+2b^2x\right)}
\end{aligned}
$$

$$= \frac{(4ad+be+2bdx)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{4d(a+bx)} - \frac{\left(4abc\sqrt{a^2+2abx+b^2x^2}\right)\operatorname{Subst}\left(\int\frac{1}{4c-x^2}\,dx,x,\frac{2c+ex}{\sqrt{c+ex+dx^2}}\right)}{2ab+2b^2x} + \frac{\left(b(4bcd+4ade-be^2)\sqrt{a^2+2abx+b^2x^2}\right)\operatorname{Subst}\left(\int\frac{1}{4d-x^2}\,dx,x,\frac{e+2dx}{\sqrt{c+ex+dx^2}}\right)}{2d\,(2ab+2b^2x)}$$

$$= \frac{(4ad+be+2bdx)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{4d(a+bx)} + \frac{(4bcd+4ade-be^2)\sqrt{a^2+2abx+b^2x^2}\tanh^{-1}\left(\frac{e+2dx}{2\sqrt{d}\sqrt{c+ex+dx^2}}\right)}{8d^{3/2}(a+bx)} - \frac{a\sqrt{c}\sqrt{a^2+2abx+b^2x^2}\tanh^{-1}\left(\frac{2c+ex}{2\sqrt{c}\sqrt{c+ex+dx^2}}\right)}{a+bx}$$

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.71

$$\int\frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{x}\,dx = \frac{\sqrt{(a+bx)^2}\left((4bcd+4ade-be^2)\operatorname{arctanh}\left(\frac{e+2dx}{2\sqrt{d}\sqrt{c+x(e+dx)}}\right)+2\sqrt{d}\left(\sqrt{c+x(e+dx)}(4ad+b(e+2dx))+8\right.\right.}{8d^{3/2}(a+bx)}$$

```
[In] Integrate[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2])/x,x]
```

```
[Out] (Sqrt[(a + b*x)^2]*((4*b*c*d + 4*a*d*e - b*e^2)*ArcTanh[(e + 2*d*x)/(2*Sqrt
[d]*Sqrt[c + x*(e + d*x)])] + 2*Sqrt[d]*(Sqrt[c + x*(e + d*x)]*(4*a*d + b*(
e + 2*d*x)) + 8*a*Sqrt[c]*d*ArcTanh[(Sqrt[d]*x - Sqrt[c + x*(e + d*x)])/Sqr
t[c]])))/(8*d^(3/2)*(a + b*x))
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.70 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.02

method	result
default	$\frac{\operatorname{csgn}(bx+a)\left(4\sqrt{d\,x^2+ex+c}\,d^{\frac{5}{2}}bx-8\sqrt{c}\,d^{\frac{5}{2}}\ln\left(\frac{2c+ex+2\sqrt{c}\,\sqrt{d\,x^2+ex+c}}{x}\right)a+8\sqrt{d\,x^2+ex+c}\,d^{\frac{5}{2}}a+2\sqrt{d\,x^2+ex+c}\,d^{\frac{3}{2}}be+4d^2\ln\left(\frac{2\sqrt{d\,x^2}}{}\right.\right.}{8d^{\frac{5}{2}}}$

```
[In] int(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*csgn(b*x+a)*(4*(d*x^2+e*x+c)^(1/2)*d^(5/2)*b*x-8*c^(1/2)*d^(5/2)*ln((2*
c+e*x+2*c^(1/2)*(d*x^2+e*x+c)^(1/2))/x)*a+8*(d*x^2+e*x+c)^(1/2)*d^(5/2)*a+2
*(d*x^2+e*x+c)^(1/2)*d^(3/2)*b*e+4*d^2*ln(1/2*(2*(d*x^2+e*x+c)^(1/2)*d^(1/2
)+2*d*x+e)/d^(1/2))*a*e+4*ln(1/2*(2*(d*x^2+e*x+c)^(1/2)*d^(1/2)+2*d*x+e)/d^
(1/2))*b*c*d^2-ln(1/2*(2*(d*x^2+e*x+c)^(1/2)*d^(1/2)+2*d*x+e)/d^(1/2))*b*d*
e^2)/d^(5/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.78 (sec) , antiderivative size = 651, normalized size of antiderivative = 3.09

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{x}\,dx$$
$$= \left[\frac{8\,a\sqrt{c}d^2\log\left(\frac{8\,cex+(4\,cd+e^2)x^2-4\,\sqrt{dx^2+ex+c}(ex+2\,c)\sqrt{c}+8\,c^2}{x^2}\right)-(4\,bcd+4\,ade-be^2)\sqrt{d}\log\left(8\,d^2x^2+8\,dex-4\cdots\right.}{16\,d^2}\right.$$

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/16*(8*a*sqrt(c)*d^2*log((8*c*e*x + (4*c*d + e^2)*x^2 - 4*sqrt(d*x^2 + e*
x + c)*(e*x + 2*c)*sqrt(c) + 8*c^2)/x^2) - (4*b*c*d + 4*a*d*e - b*e^2)*sqrt
(d)*log(8*d^2*x^2 + 8*d*e*x - 4*sqrt(d*x^2 + e*x + c)*(2*d*x + e)*sqrt(d) +
 4*c*d + e^2) + 4*(2*b*d^2*x + 4*a*d^2 + b*d*e)*sqrt(d*x^2 + e*x + c))/d^2,
 1/8*(4*a*sqrt(c)*d^2*log((8*c*e*x + (4*c*d + e^2)*x^2 - 4*sqrt(d*x^2 + e*x
 + c)*(e*x + 2*c)*sqrt(c) + 8*c^2)/x^2) - (4*b*c*d + 4*a*d*e - b*e^2)*sqrt(
-d)*arctan(1/2*sqrt(d*x^2 + e*x + c)*(2*d*x + e)*sqrt(-d)/(d^2*x^2 + d*e*x
+ c*d)) + 2*(2*b*d^2*x + 4*a*d^2 + b*d*e)*sqrt(d*x^2 + e*x + c))/d^2, 1/16*
(16*a*sqrt(-c)*d^2*arctan(1/2*sqrt(d*x^2 + e*x + c)*(e*x + 2*c)*sqrt(-c)/(c
*d*x^2 + c*e*x + c^2)) - (4*b*c*d + 4*a*d*e - b*e^2)*sqrt(d)*log(8*d^2*x^2
+ 8*d*e*x - 4*sqrt(d*x^2 + e*x + c)*(2*d*x + e)*sqrt(d) + 4*c*d + e^2) + 4*
(2*b*d^2*x + 4*a*d^2 + b*d*e)*sqrt(d*x^2 + e*x + c))/d^2, 1/8*(8*a*sqrt(-c)
*d^2*arctan(1/2*sqrt(d*x^2 + e*x + c)*(e*x + 2*c)*sqrt(-c)/(c*d*x^2 + c*e*x
 + c^2)) - (4*b*c*d + 4*a*d*e - b*e^2)*sqrt(-d)*arctan(1/2*sqrt(d*x^2 + e*x
 + c)*(2*d*x + e)*sqrt(-d)/(d^2*x^2 + d*e*x + c*d)) + 2*(2*b*d^2*x + 4*a*d^
2 + b*d*e)*sqrt(d*x^2 + e*x + c))/d^2]
```

Sympy [F]

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{x}\,dx = \int \frac{\sqrt{c+dx^2+ex}\sqrt{(a+bx)^2}}{x}\,dx$$

```
[In] integrate(((b*x+a)**2)**(1/2)*(d*x**2+e*x+c)**(1/2)/x,x)
[Out] Integral(sqrt(c + d*x**2 + e*x)*sqrt((a + b*x)**2)/x, x)
```

Maxima [F]

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{x}\,dx = \int \frac{\sqrt{dx^2+ex+c}\sqrt{(bx+a)^2}}{x}\,dx$$

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/x,x, algorithm="maxima")
[Out] integrate(sqrt(d*x^2 + e*x + c)*sqrt((b*x + a)^2)/x, x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{x}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/x,x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{x}\,dx = \int \frac{\sqrt{(a+b\,x)^2}\,\sqrt{d\,x^2+e\,x+c}}{x}\,dx$$

```
[In] int((((a + b*x)^2)^(1/2)*(c + e*x + d*x^2)^(1/2))/x,x)
[Out] int((((a + b*x)^2)^(1/2)*(c + e*x + d*x^2)^(1/2))/x, x)
```

3.50 $\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{x^2} dx$

Optimal result

Integrand size = 38, antiderivative size = 202

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{x^2}\,dx = -\frac{(a-bx)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{x(a+bx)} + \frac{(2ad+be)\sqrt{a^2+2abx+b^2x^2}\,\text{arctanh}\left(\frac{e+2dx}{2\sqrt{d}\sqrt{c+ex+dx^2}}\right)}{2\sqrt{d}(a+bx)} - \frac{(2bc+ae)\sqrt{a^2+2abx+b^2x^2}\,\text{arctanh}\left(\frac{2c+ex}{2\sqrt{c}\sqrt{c+ex+dx^2}}\right)}{2\sqrt{c}(a+bx)}$$

```
[Out] -1/2*(a*e+2*b*c)*arctanh(1/2*(e*x+2*c)/c^(1/2)/(d*x^2+e*x+c)^(1/2))*((b*x+a
)^2)^(1/2)/(b*x+a)/c^(1/2)+1/2*(2*a*d+b*e)*arctanh(1/2*(2*d*x+e)/d^(1/2)/(d
*x^2+e*x+c)^(1/2))*((b*x+a)^2)^(1/2)/(b*x+a)/d^(1/2)-(-b*x+a)*((b*x+a)^2)^(
1/2)*(d*x^2+e*x+c)^(1/2)/x/(b*x+a)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used

= {1014, 826, 857, 635, 212, 738}

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{x^2}\,dx$$
$$= \frac{\sqrt{a^2+2abx+b^2x^2}(2ad+be)\text{arctanh}\left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+dx^2+ex}}\right)}{2\sqrt{d}(a+bx)} - \frac{\sqrt{a^2+2abx+b^2x^2}(ae+2bc)\text{arctanh}\left(\frac{2c+ex}{2\sqrt{c}\sqrt{c+dx^2+ex}}\right)}{2\sqrt{c}(a+bx)} - \frac{\sqrt{a^2+2abx+b^2x^2}(a-bx)\sqrt{c+dx^2+ex}}{x(a+bx)}$$

```
[In] Int[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2])/x^2,x]
```

```
[Out] -(((a - b*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2])/(x*(a + b
*x))) + ((2*a*d + b*e)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[(e + 2*d*x)/(2
*Sqrt[d]*Sqrt[c + e*x + d*x^2])])/(2*Sqrt[d]*(a + b*x)) - ((2*b*c + a*e)*Sq
rt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[(2*c + e*x)/(2*Sqrt[c]*Sqrt[c + e*x + d
*x^2])])/(2*Sqrt[c]*(a + b*x))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 826

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
```

```
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1014

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_
) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Dist[(a + b*x + c*x^2)^Fr
acPart[p]/((4*c)^IntPart[p]*(b + 2*c*x)^(2*FracPart[p])), Int[(g + h*x)^m*(
b + 2*c*x)^(2*p)*(d + e*x + f*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 h, m, p, q}, x] && EqQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(2ab+2b^2x)\sqrt{c+ex+dx^2}}{x^2}\,dx}{2ab+2b^2x} \\
&= -\frac{(a-bx)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{x(a+bx)} - \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{-2b(2bc+ae)-2b(2ad+be)x}{x\sqrt{c+ex+dx^2}}\,dx}{2\,(2ab+2b^2x)} \\
&= -\frac{(a-bx)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{x(a+bx)} \\
&\quad + \frac{\left(b(2bc+ae)\sqrt{a^2+2abx+b^2x^2}\right)\int \frac{1}{x\sqrt{c+ex+dx^2}}\,dx}{2ab+2b^2x} \\
&\quad + \frac{\left(b(2ad+be)\sqrt{a^2+2abx+b^2x^2}\right)\int \frac{1}{\sqrt{c+ex+dx^2}}\,dx}{2ab+2b^2x} \\
&= -\frac{(a-bx)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{x(a+bx)} \\
&\quad - \frac{\left(2b(2bc+ae)\sqrt{a^2+2abx+b^2x^2}\right)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{2c+ex}{\sqrt{c+ex+dx^2}}\right)}{2ab+2b^2x} \\
&\quad + \frac{\left(2b(2ad+be)\sqrt{a^2+2abx+b^2x^2}\right)\text{Subst}\left(\int \frac{1}{4d-x^2}\,dx, x, \frac{e+2dx}{\sqrt{c+ex+dx^2}}\right)}{2ab+2b^2x}
\end{aligned}
$$

$$= -\frac{(a-bx)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{x(a+bx)} + \frac{(2ad+be)\sqrt{a^2+2abx+b^2x^2}\tanh^{-1}\left(\frac{e+2dx}{2\sqrt{d}\sqrt{c+ex+dx^2}}\right)}{2\sqrt{d}(a+bx)} - \frac{(2bc+ae)\sqrt{a^2+2abx+b^2x^2}\tanh^{-1}\left(\frac{2c+ex}{2\sqrt{c}\sqrt{c+ex+dx^2}}\right)}{2\sqrt{c}(a+bx)}$$

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{x^2}\,dx =$$

$$-\frac{\sqrt{(a+bx)^2}\left(2\sqrt{d}(2bc+ae)x\operatorname{arctanh}\left(\frac{-\sqrt{d}x+\sqrt{c+x(e+dx)}}{\sqrt{c}}\right)+\sqrt{c}\left(2\sqrt{d}(a-bx)\sqrt{c+x(e+dx)}+(2ad+\right.\right.}{2\sqrt{c}\sqrt{d}x(a+bx)}$$

```
[In] Integrate[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2])/x^2,x]
```

```
[Out] -1/2*(Sqrt[(a + b*x)^2]*(2*Sqrt[d]*(2*b*c + a*e)*x*ArcTanh[(-(Sqrt[d]*x) +
Sqrt[c + x*(e + d*x)])/Sqrt[c]] + Sqrt[c]*(2*Sqrt[d]*(a - b*x)*Sqrt[c + x*(
e + d*x)] + (2*a*d + b*e)*x*Log[e + 2*d*x - 2*Sqrt[d]*Sqrt[c + x*(e + d*x)]
])))/(Sqrt[c]*Sqrt[d]*x*(a + b*x))
```

Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.00

method	result
risch	$-\frac{a\sqrt{d\,x^2+ex+c}\,\sqrt{(bx+a)^2}}{x(bx+a)}+\frac{\left(a\sqrt{d}\,\ln\left(\frac{\frac{e}{2}+dx}{\sqrt{d}}+\sqrt{d\,x^2+ex+c}\right)+\frac{be\ln\left(\frac{\frac{e}{2}+dx}{\sqrt{d}}+\sqrt{d\,x^2+ex+c}\right)}{2\sqrt{d}}+b\sqrt{d\,x^2+ex+c}-\frac{\ln\left(\frac{2c+ex+2\sqrt{c}\,\sqrt{d\,}}{x}\right.}{2\sqrt{c}}\right.}{bx+a}$
default	$-\frac{\operatorname{csgn}(bx+a)\left(-2\sqrt{d\,x^2+ex+c}\,d^{\frac{5}{2}}\,a\,x^2+2d^{\frac{3}{2}}\,c^{\frac{3}{2}}\,\ln\left(\frac{2c+ex+2\sqrt{c}\,\sqrt{d\,x^2+ex+c}}{x}\right)bx+d^{\frac{3}{2}}\sqrt{c}\,\ln\left(\frac{2c+ex+2\sqrt{c}\,\sqrt{d\,x^2+ex+c}}{x}\right)aex+2(d\,x^2+\right.}{}$

```
[In] int(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a*(d*x^2+e*x+c)^(1/2)/x*((b*x+a)^2)^(1/2)/(b*x+a)+(a*d^(1/2)*ln((1/2*e+d*x
)/d^(1/2)+(d*x^2+e*x+c)^(1/2))+1/2*b*e*ln((1/2*e+d*x)/d^(1/2)+(d*x^2+e*x+c)
^(1/2))/d^(1/2)+b*(d*x^2+e*x+c)^(1/2)-1/2/c^(1/2)*ln((2*c+e*x+2*c^(1/2)*(d*
x^2+e*x+c)^(1/2))/x)*a*e-c^(1/2)*ln((2*c+e*x+2*c^(1/2)*(d*x^2+e*x+c)^(1/2))
/x)*b)*((b*x+a)^2)^(1/2)/(b*x+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.45 (sec) , antiderivative size = 647, normalized size of antiderivative = 3.20

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{x^2}\,dx$$
$$= \left[\frac{(2\,acd+bce)\sqrt{d}x\log\left(8\,d^2x^2+8\,dex+4\,\sqrt{dx^2+ex+c}(2\,dx+e)\sqrt{d}+4\,cd+e^2\right)+(2\,bcd+ade)\sqrt{c}x\,\mathrm{lo}}{4\,cdx}\right.$$
$$-\frac{2\,(2\,acd+bce)\sqrt{-d}x\arctan\left(\frac{\sqrt{dx^2+ex+c}(2\,dx+e)\sqrt{-d}}{2\,(d^2x^2+dex+cd)}\right)-(2\,bcd+ade)\sqrt{c}x\log\left(\frac{8\,cex+(4\,cd+e^2)x^2-4\,\sqrt{dx^2+ex+c}(}{x^2}\right.}{4\,cdx}$$

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/4*((2*a*c*d + b*c*e)*sqrt(d)*x*log(8*d^2*x^2 + 8*d*e*x + 4*sqrt(d*x^2 +
e*x + c)*(2*d*x + e)*sqrt(d) + 4*c*d + e^2) + (2*b*c*d + a*d*e)*sqrt(c)*x*l
og((8*c*e*x + (4*c*d + e^2)*x^2 - 4*sqrt(d*x^2 + e*x + c)*(e*x + 2*c)*sqrt(
c) + 8*c^2)/x^2) + 4*(b*c*d*x - a*c*d)*sqrt(d*x^2 + e*x + c))/(c*d*x), -1/4
*(2*(2*a*c*d + b*c*e)*sqrt(-d)*x*arctan(1/2*sqrt(d*x^2 + e*x + c)*(2*d*x +
e)*sqrt(-d)/(d^2*x^2 + d*e*x + c*d)) - (2*b*c*d + a*d*e)*sqrt(c)*x*log((8*c
*e*x + (4*c*d + e^2)*x^2 - 4*sqrt(d*x^2 + e*x + c)*(e*x + 2*c)*sqrt(c) + 8*
c^2)/x^2) - 4*(b*c*d*x - a*c*d)*sqrt(d*x^2 + e*x + c))/(c*d*x), 1/4*(2*(2*b
*c*d + a*d*e)*sqrt(-c)*x*arctan(1/2*sqrt(d*x^2 + e*x + c)*(e*x + 2*c)*sqrt(
-c)/(c*d*x^2 + c*e*x + c^2)) + (2*a*c*d + b*c*e)*sqrt(d)*x*log(8*d^2*x^2 +
8*d*e*x + 4*sqrt(d*x^2 + e*x + c)*(2*d*x + e)*sqrt(d) + 4*c*d + e^2) + 4*(b
*c*d*x - a*c*d)*sqrt(d*x^2 + e*x + c))/(c*d*x), 1/2*((2*b*c*d + a*d*e)*sqrt
(-c)*x*arctan(1/2*sqrt(d*x^2 + e*x + c)*(e*x + 2*c)*sqrt(-c)/(c*d*x^2 + c*e
*x + c^2)) - (2*a*c*d + b*c*e)*sqrt(-d)*x*arctan(1/2*sqrt(d*x^2 + e*x + c)*
(2*d*x + e)*sqrt(-d)/(d^2*x^2 + d*e*x + c*d)) + 2*(b*c*d*x - a*c*d)*sqrt(d*
x^2 + e*x + c))/(c*d*x)]
```

Sympy [F]

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{x^2}\,dx=\int \frac{\sqrt{c+dx^2+ex}\sqrt{(a+bx)^2}}{x^2}\,dx$$

```
[In] integrate(((b*x+a)**2)**(1/2)*(d*x**2+e*x+c)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(c + d*x**2 + e*x)*sqrt((a + b*x)**2)/x**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{x^2}\,dx = \int \frac{\sqrt{dx^2+ex+c}\sqrt{(bx+a)^2}}{x^2}\,dx$$

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/x^2,x, algorithm="maxima")
[Out] integrate(sqrt(d*x^2 + e*x + c)*sqrt((b*x + a)^2)/x^2, x)
```

Giac [A] (verification not implemented)

none

Time = 0.66 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.00

$$\begin{aligned}
&\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{x^2}\,dx \\
&= \sqrt{dx^2+ex+c}\,b\mathrm{sgn}(bx+a) \\
&\quad + \frac{(2\,bc\mathrm{sgn}(bx+a)+ae\mathrm{sgn}(bx+a))\arctan\left(-\frac{\sqrt{d}x-\sqrt{dx^2+ex+c}}{\sqrt{-c}}\right)}{\sqrt{-c}} \\
&\quad - \frac{(2\,ad\mathrm{sgn}(bx+a)+be\mathrm{sgn}(bx+a))\log\left(\left|-2\left(\sqrt{d}x-\sqrt{dx^2+ex+c}\right)\sqrt{d}-e\right|\right)}{2\sqrt{d}} \\
&\quad + \frac{\left(\sqrt{d}x-\sqrt{dx^2+ex+c}\right)ae\mathrm{sgn}(bx+a)+2\,ac\sqrt{d}\mathrm{sgn}(bx+a)}{\left(\sqrt{d}x-\sqrt{dx^2+ex+c}\right)^2-c}
\end{aligned}$$

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/x^2,x, algorithm="giac")
[Out] sqrt(d*x^2 + e*x + c)*b*sgn(b*x + a) + (2*b*c*sgn(b*x + a) + a*e*sgn(b*x +
a))*arctan(-(sqrt(d)*x - sqrt(d*x^2 + e*x + c))/sqrt(-c))/sqrt(-c) - 1/2*(2
*a*d*sgn(b*x + a) + b*e*sgn(b*x + a))*log(abs(-2*(sqrt(d)*x - sqrt(d*x^2 +
e*x + c))*sqrt(d) - e))/sqrt(d) + ((sqrt(d)*x - sqrt(d*x^2 + e*x + c))*a*e*
sgn(b*x + a) + 2*a*c*sqrt(d)*sgn(b*x + a))/((sqrt(d)*x - sqrt(d*x^2 + e*x +
 c))^2 - c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2}\sqrt{c + ex + dx^2}}{x^2} \, dx = \int \frac{\sqrt{(a + b\,x)^2}\,\sqrt{d\,x^2 + e\,x + c}}{x^2} \, dx$$

```
[In] int((((a + b*x)^2)^(1/2)*(c + e*x + d*x^2)^(1/2))/x^2,x)
[Out] int((((a + b*x)^2)^(1/2)*(c + e*x + d*x^2)^(1/2))/x^2, x)
```

3.51 $\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{x^3}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 215

$$\begin{aligned}\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{x^3}\,dx &= -\frac{(2ac+(4bc+ae)x)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{4cx^2(a+bx)}\\ &+\frac{b\sqrt{d}\sqrt{a^2+2abx+b^2x^2}\operatorname{arctanh}\left(\frac{e+2dx}{2\sqrt{d}\sqrt{c+ex+dx^2}}\right)}{a+bx}\\ &-\frac{(4acd+4bce-ae^2)\sqrt{a^2+2abx+b^2x^2}\operatorname{arctanh}\left(\frac{2c+ex}{2\sqrt{c}\sqrt{c+ex+dx^2}}\right)}{8c^{3/2}(a+bx)}\end{aligned}$$

```
[Out] -1/8*(4*a*c*d-a*e^2+4*b*c*e)*arctanh(1/2*(e*x+2*c)/c^(1/2)/(d*x^2+e*x+c)^(1
/2))*((b*x+a)^2)^(1/2)/c^(3/2)/(b*x+a)+b*arctanh(1/2*(2*d*x+e)/d^(1/2)/(d*x
^2+e*x+c)^(1/2))*d^(1/2)*((b*x+a)^2)^(1/2)/(b*x+a)-1/4*(2*a*c+(a*e+4*b*c)*x
)*((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/c/x^2/(b*x+a)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used

$= \{1014, 824, 857, 635, 212, 738\}$

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{x^3}\,dx$$
$$= -\frac{\sqrt{a^2+2abx+b^2x^2}(4acd-ae^2+4bce)\operatorname{arctanh}\left(\frac{2c+ex}{2\sqrt{c}\sqrt{c+dx^2+ex}}\right)}{8c^{3/2}(a+bx)}$$
$$+\frac{b\sqrt{d}\sqrt{a^2+2abx+b^2x^2}\operatorname{arctanh}\left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+dx^2+ex}}\right)}{a+bx}$$
$$-\frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2+ex}(x(ae+4bc)+2ac)}{4cx^2(a+bx)}$$

```
[In] Int[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2])/x^3,x]
```

```
[Out] -1/4*((2*a*c + (4*b*c + a*e)*x)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x
+ d*x^2])/(c*x^2*(a + b*x)) + (b*Sqrt[d]*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcT
anh[(e + 2*d*x)/(2*Sqrt[d]*Sqrt[c + e*x + d*x^2])])/(a + b*x) - ((4*a*c*d +
 4*b*c*e - a*e^2)*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*ArcTanh[(2*c + e*x)/(2*Sqrt
[c]*Sqrt[c + e*x + d*x^2])])/(8*c^(3/2)*(a + b*x))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 824

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Dist[p/(e^2*(m + 1
```

```
)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)
^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p
 + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*
(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m
 - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &
& LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1014

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_
) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Dist[(a + b*x + c*x^2)^Fr
acPart[p]/((4*c)^IntPart[p]*(b + 2*c*x)^(2*FracPart[p])), Int[(g + h*x)^m*(
b + 2*c*x)^(2*p)*(d + e*x + f*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g,
 h, m, p, q}, x] && EqQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(2ab+2b^2x)\sqrt{c+ex+dx^2}}{x^3}\,dx}{2ab+2b^2x}\\
&= -\frac{(2ac+(4bc+ae)x)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{4cx^2(a+bx)}\\
&\quad-\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{-b\left(4bce+a\left(4cd-e^2\right)\right)-8b^2cdx}{x\sqrt{c+ex+dx^2}}\,dx}{4c\left(2ab+2b^2x\right)}\\
&= -\frac{(2ac+(4bc+ae)x)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{4cx^2(a+bx)}\\
&\quad+\frac{\left(2b^2d\sqrt{a^2+2abx+b^2x^2}\right)\int \frac{1}{\sqrt{c+ex+dx^2}}\,dx}{2ab+2b^2x}\\
&\quad+\frac{\left(b(4acd+4bce-ae^2)\sqrt{a^2+2abx+b^2x^2}\right)\int \frac{1}{x\sqrt{c+ex+dx^2}}\,dx}{4c\left(2ab+2b^2x\right)}
\end{aligned}$$

$$= -\frac{(2ac + (4bc + ae)x)\sqrt{a^2 + 2abx + b^2x^2}\sqrt{c + ex + dx^2}}{4cx^2(a + bx)} + \frac{\left(4b^2d\sqrt{a^2 + 2abx + b^2x^2}\right) \text{Subst}\left(\int \frac{1}{4d-x^2}\,dx, x, \frac{e+2dx}{\sqrt{c+ex+dx^2}}\right)}{2ab + 2b^2x} - \frac{\left(b(4acd + 4bce - ae^2)\sqrt{a^2 + 2abx + b^2x^2}\right) \text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{2c+ex}{\sqrt{c+ex+dx^2}}\right)}{2c\,(2ab + 2b^2x)}$$

$$= -\frac{(2ac + (4bc + ae)x)\sqrt{a^2 + 2abx + b^2x^2}\sqrt{c + ex + dx^2}}{4cx^2(a + bx)} + \frac{b\sqrt{d}\sqrt{a^2 + 2abx + b^2x^2}\tanh^{-1}\left(\frac{e+2dx}{2\sqrt{d}\sqrt{c+ex+dx^2}}\right)}{a + bx} - \frac{(4acd + 4bce - ae^2)\sqrt{a^2 + 2abx + b^2x^2}\tanh^{-1}\left(\frac{2c+ex}{2\sqrt{c}\sqrt{c+ex+dx^2}}\right)}{8c^{3/2}(a + bx)}$$

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.72

$$\int \frac{\sqrt{a^2 + 2abx + b^2x^2}\sqrt{c + ex + dx^2}}{x^3}\,dx = -\frac{\sqrt{(a+bx)^2}\left((4acd + 4bce - ae^2)\,x^2\text{arctanh}\left(\frac{-\sqrt{d}x+\sqrt{c+x(e+dx)}}{\sqrt{c}}\right) + \sqrt{c}\left((2ac + 4bcx + aex)\sqrt{c + x(e + d\right)\right)}{4c^{3/2}x^2(a + bx)}$$

```
[In] Integrate[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2])/x^3,x]
```

```
[Out] -1/4*(Sqrt[(a + b*x)^2]*((4*a*c*d + 4*b*c*e - a*e^2)*x^2*ArcTanh[(-(Sqrt[d]
*x) + Sqrt[c + x*(e + d*x)])/Sqrt[c]] + Sqrt[c]*((2*a*c + 4*b*c*x + a*e*x)*
Sqrt[c + x*(e + d*x)] + 4*b*c*Sqrt[d]*x^2*Log[e + 2*d*x - 2*Sqrt[d]*Sqrt[c
+ x*(e + d*x)]])))/(c^(3/2)*x^2*(a + b*x))
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.73

method	result
risch	$-\frac{\sqrt{d\,x^2+ex+c}\,(aex+4bcx+2ac)\sqrt{(bx+a)^2}}{4x^2c(bx+a)} + \frac{\left(8bc\sqrt{d}\,\ln\left(\frac{\frac{e}{2}+dx}{\sqrt{d}}+\sqrt{d\,x^2+ex+c}\right)-\frac{\left(4acd-e^2a+4bce\right)\ln\left(\frac{2c+ex+2\sqrt{c}\,\sqrt{d\,x^2+ex+c}}{x}\right)}{\sqrt{c}}\right)}{8c(bx+a)}$
default	$-\frac{\text{csgn}(bx+a)\left(4d^{\frac{5}{2}}c^{\frac{3}{2}}\ln\left(\frac{2c+ex+2\sqrt{c}\,\sqrt{d\,x^2+ex+c}}{x}\right)a\,x^2+2\sqrt{d\,x^2+ex+c}\,d^{\frac{5}{2}}ae\,x^3-8\sqrt{d\,x^2+ex+c}\,d^{\frac{5}{2}}bc\,x^3+4d^{\frac{3}{2}}c^{\frac{3}{2}}\ln\left(\frac{2c+ex+2\sqrt{c}\,\sqrt{d}}{x}\right)\right)}{}$

```
[In] int(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(d*x^2+e*x+c)^(1/2)*(a*e*x+4*b*c*x+2*a*c)/x^2/c*((b*x+a)^2)^(1/2)/(b*x
+a)+1/8/c*(8*b*c*d^(1/2)*ln((1/2*e+d*x)/d^(1/2)+(d*x^2+e*x+c)^(1/2))-(4*a*c
*d-a*e^2+4*b*c*e)/c^(1/2)*ln((2*c+e*x+2*c^(1/2)*(d*x^2+e*x+c)^(1/2))/x))*((
b*x+a)^2)^(1/2)/(b*x+a)
```

Fricas [A] (verification not implemented)

none

Time = 0.53 (sec) , antiderivative size = 693, normalized size of antiderivative = 3.22

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{x^3}\,dx$$

$$= \left[\frac{8\,bc^2\sqrt{d}x^2\log\left(8\,d^2x^2+8\,dex+4\,\sqrt{dx^2+ex+c}(2\,dx+e)\sqrt{d}+4\,cd+e^2\right)-(4\,acd+4\,bce-ae^2)\sqrt{c}x}{16\,c^2x^2}\right.$$

$$-\frac{16\,bc^2\sqrt{-d}x^2\arctan\left(\frac{\sqrt{dx^2+ex+c}(2\,dx+e)\sqrt{-d}}{2\,(d^2x^2+dex+cd)}\right)+(4\,acd+4\,bce-ae^2)\sqrt{c}x^2\log\left(\frac{8\,cex+(4\,cd+e^2)x^2+4\,\sqrt{dx^2+ex+}}{x^2}\right.}{16\,c^2x^2}$$

$$-\frac{8\,bc^2\sqrt{-d}x^2\arctan\left(\frac{\sqrt{dx^2+ex+c}(2\,dx+e)\sqrt{-d}}{2\,(d^2x^2+dex+cd)}\right)-(4\,acd+4\,bce-ae^2)\sqrt{-c}x^2\arctan\left(\frac{\sqrt{dx^2+ex+c}(ex+2\,c)\sqrt{-c}}{2\,(cdx^2+cex+c^2)}\right)}{8\,c^2x^2}$$

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/16*(8*b*c^2*sqrt(d)*x^2*log(8*d^2*x^2 + 8*d*e*x + 4*sqrt(d*x^2 + e*x + c
)*(2*d*x + e)*sqrt(d) + 4*c*d + e^2) - (4*a*c*d + 4*b*c*e - a*e^2)*sqrt(c)*
x^2*log((8*c*e*x + (4*c*d + e^2)*x^2 + 4*sqrt(d*x^2 + e*x + c)*(e*x + 2*c)*
sqrt(c) + 8*c^2)/x^2) - 4*(2*a*c^2 + (4*b*c^2 + a*c*e)*x)*sqrt(d*x^2 + e*x
+ c))/(c^2*x^2), -1/16*(16*b*c^2*sqrt(-d)*x^2*arctan(1/2*sqrt(d*x^2 + e*x +
 c)*(2*d*x + e)*sqrt(-d)/(d^2*x^2 + d*e*x + c*d)) + (4*a*c*d + 4*b*c*e - a*
e^2)*sqrt(c)*x^2*log((8*c*e*x + (4*c*d + e^2)*x^2 + 4*sqrt(d*x^2 + e*x + c)
*(e*x + 2*c)*sqrt(c) + 8*c^2)/x^2) + 4*(2*a*c^2 + (4*b*c^2 + a*c*e)*x)*sqrt
(d*x^2 + e*x + c))/(c^2*x^2), 1/8*(4*b*c^2*sqrt(d)*x^2*log(8*d^2*x^2 + 8*d*
e*x + 4*sqrt(d*x^2 + e*x + c)*(2*d*x + e)*sqrt(d) + 4*c*d + e^2) + (4*a*c*d
 + 4*b*c*e - a*e^2)*sqrt(-c)*x^2*arctan(1/2*sqrt(d*x^2 + e*x + c)*(e*x + 2*
c)*sqrt(-c)/(c*d*x^2 + c*e*x + c^2)) - 2*(2*a*c^2 + (4*b*c^2 + a*c*e)*x)*sq
rt(d*x^2 + e*x + c))/(c^2*x^2), -1/8*(8*b*c^2*sqrt(-d)*x^2*arctan(1/2*sqrt(
d*x^2 + e*x + c)*(2*d*x + e)*sqrt(-d)/(d^2*x^2 + d*e*x + c*d)) - (4*a*c*d +
 4*b*c*e - a*e^2)*sqrt(-c)*x^2*arctan(1/2*sqrt(d*x^2 + e*x + c)*(e*x + 2*c)
*sqrt(-c)/(c*d*x^2 + c*e*x + c^2)) + 2*(2*a*c^2 + (4*b*c^2 + a*c*e)*x)*sqrt
(d*x^2 + e*x + c))/(c^2*x^2)]
```

Sympy [F]

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{x^3}\,dx = \int \frac{\sqrt{c+dx^2+ex}\sqrt{(a+bx)^2}}{x^3}\,dx$$

```
[In] integrate(((b*x+a)**2)**(1/2)*(d*x**2+e*x+c)**(1/2)/x**3,x)
[Out] Integral(sqrt(c + d*x**2 + e*x)*sqrt((a + b*x)**2)/x**3, x)
```

Maxima [F]

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{x^3}\,dx = \int \frac{\sqrt{dx^2+ex+c}\sqrt{(bx+a)^2}}{x^3}\,dx$$

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/x^3,x, algorithm="maxima")
[Out] integrate(sqrt(d*x^2 + e*x + c)*sqrt((b*x + a)^2)/x^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 434 vs. 2(160) = 320.

Time = 0.34 (sec) , antiderivative size = 434, normalized size of antiderivative = 2.02

$$\begin{aligned}&\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{x^3}\,dx\\&= -b\sqrt{d}\log\left(\left|2\left(\sqrt{d}x-\sqrt{dx^2+ex+c}\right)\sqrt{d}+e\right|\right)\mathrm{sgn}(bx+a)\\&\quad+\frac{(4\,acd\mathrm{sgn}(bx+a)+4\,bce\mathrm{sgn}(bx+a)-ae^2\mathrm{sgn}(bx+a))\arctan\left(-\frac{\sqrt{d}x-\sqrt{dx^2+ex+c}}{\sqrt{-c}}\right)}{4\sqrt{-c}c}\\&\quad+\frac{4\left(\sqrt{d}x-\sqrt{dx^2+ex+c}\right)^3acd\mathrm{sgn}(bx+a)+4\left(\sqrt{d}x-\sqrt{dx^2+ex+c}\right)^3bce\mathrm{sgn}(bx+a)+\left(\sqrt{d}x-\sqrt{dx}\right.}{}\end{aligned}$$

```
[In] integrate(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/x^3,x, algorithm="giac")
[Out] -b*sqrt(d)*log(abs(2*(sqrt(d)*x - sqrt(d*x^2 + e*x + c))*sqrt(d) + e))*sgn(
b*x + a) + 1/4*(4*a*c*d*sgn(b*x + a) + 4*b*c*e*sgn(b*x + a) - a*e^2*sgn(b*x
 + a))*arctan(-(sqrt(d)*x - sqrt(d*x^2 + e*x + c))/sqrt(-c))/(sqrt(-c)*c) +
 1/4*(4*(sqrt(d)*x - sqrt(d*x^2 + e*x + c))^3*a*c*d*sgn(b*x + a) + 4*(sqrt(
d)*x - sqrt(d*x^2 + e*x + c))^3*b*c*e*sgn(b*x + a) + (sqrt(d)*x - sqrt(d*x^
```

```
2 + e*x + c))^3*a*e^2*sgn(b*x + a) + 8*(sqrt(d)*x - sqrt(d*x^2 + e*x + c))^
2*b*c^2*sqrt(d)*sgn(b*x + a) + 8*(sqrt(d)*x - sqrt(d*x^2 + e*x + c))^2*a*c*
sqrt(d)*e*sgn(b*x + a) + 4*(sqrt(d)*x - sqrt(d*x^2 + e*x + c))*a*c^2*d*sgn(
b*x + a) - 4*(sqrt(d)*x - sqrt(d*x^2 + e*x + c))*b*c^2*e*sgn(b*x + a) + (sq
rt(d)*x - sqrt(d*x^2 + e*x + c))*a*c*e^2*sgn(b*x + a) - 8*b*c^3*sqrt(d)*sgn
(b*x + a))/(((sqrt(d)*x - sqrt(d*x^2 + e*x + c))^2 - c)^2*c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{x^3}\,dx=\int \frac{\sqrt{(a+b\,x)^2}\,\sqrt{d\,x^2+e\,x+c}}{x^3}\,dx$$

```
[In] int((((a + b*x)^2)^(1/2)*(c + e*x + d*x^2)^(1/2))/x^3,x)
[Out] int((((a + b*x)^2)^(1/2)*(c + e*x + d*x^2)^(1/2))/x^3, x)
```

3.52 $\int \frac{x^2\sqrt{a+cx^2}}{d+ex+fx^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 452

$$\int \frac{x^2\sqrt{a+cx^2}}{d+ex+fx^2}\,dx = -\frac{(2e-fx)\sqrt{a+cx^2}}{2f^2}+\frac{(af^2+2c(e^2-df))\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2\sqrt{c}f^3}$$
$$-\frac{\left(e\left(e-\sqrt{e^2-4df}\right)(af^2+c(e^2-2df))-2df(af^2+c(e^2-df))\right)\text{arctanh}\left(\frac{2af-c\left(e-\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}\sqrt{a}\cdots}\right)}{\sqrt{2}f^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}}$$
$$+\frac{\left(e\left(e+\sqrt{e^2-4df}\right)(af^2+c(e^2-2df))-2df(af^2+c(e^2-df))\right)\text{arctanh}\left(\frac{2af-c\left(e+\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}\sqrt{a}\cdots}\right)}{\sqrt{2}f^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}}$$

```
[Out] 1/2*(a*f^2+2*c*(-d*f+e^2))*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/f^3/c^(1/2)-1
/2*(-f*x+2*e)*(c*x^2+a)^(1/2)/f^2-1/2*arctanh(1/2*(2*a*f-c*x*(e-(-4*d*f+e^2
)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+c*(e^2-2*d*f-e*(-4*d*f+e^2)^(1/2
)))^(1/2))*(-2*d*f*(a*f^2+c*(-d*f+e^2))+e*(a*f^2+c*(-2*d*f+e^2))*(e-(-4*d*f
+e^2)^(1/2)))/f^3*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+c*(e^2-2*d*f-e*(-4*d*
f+e^2)^(1/2)))^(1/2)+1/2*arctanh(1/2*(2*a*f-c*x*(e+(-4*d*f+e^2)^(1/2)))*2^(
1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+c*(e^2-2*d*f+e*(-4*d*f+e^2)^(1/2)))^(1/2))*(-
2*d*f*(a*f^2+c*(-d*f+e^2))+e*(a*f^2+c*(-2*d*f+e^2))*(e+(-4*d*f+e^2)^(1/2)))
/f^3*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+c*(e^2-2*d*f+e*(-4*d*f+e^2)^(1/2))
)^(1/2)
```

Rubi [A] (verified)

Time = 1.30 (sec) , antiderivative size = 452, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1083, 1094, 223, 212, 1048, 739}

$$\int \frac{x^2\sqrt{a+cx^2}}{d+ex+fx^2}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(af^2+2c(e^2-df)\right)}{2\sqrt{c}f^3}$$

$$-\frac{\left(e\left(e-\sqrt{e^2-4df}\right)\left(af^2+c(e^2-2df)\right)-2df\left(af^2+c(e^2-df)\right)\right)\operatorname{arctanh}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-\cdots\right.}}\cdots\right.}{\sqrt{2}f^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}$$

$$+\frac{\left(e\left(\sqrt{e^2-4df}+e\right)\left(af^2+c(e^2-2df)\right)-2df\left(af^2+c(e^2-df)\right)\right)\operatorname{arctanh}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2\cdots\right.}}\cdots\right.}{\sqrt{2}f^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}$$

$$-\frac{\sqrt{a+cx^2}(2e-fx)}{2f^2}$$

```
[In] Int[(x^2*Sqrt[a + c*x^2])/(d + e*x + f*x^2),x]
```

```
[Out] -1/2*((2*e - f*x)*Sqrt[a + c*x^2])/f^2 + ((a*f^2 + 2*c*(e^2 - d*f))*ArcTanh
[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*Sqrt[c]*f^3) - ((e*(e - Sqrt[e^2 - 4*d*f]
)*(a*f^2 + c*(e^2 - 2*d*f)) - 2*d*f*(a*f^2 + c*(e^2 - d*f)))*ArcTanh[(2*a*f
 - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*
Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*f^3*Sqrt[e^2 - 4*d*f]*Sqrt[
2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) + ((e*(e + Sqrt[e^2 - 4*d
*f])*(a*f^2 + c*(e^2 - 2*d*f)) - 2*d*f*(a*f^2 + c*(e^2 - d*f)))*ArcTanh[(2*
a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f +
 e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*f^3*Sqrt[e^2 - 4*d*f]*Sq
rt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1048

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1083

```
Int[((a_) + (c_.)*(x_)^2)^(p_)*((A_.) + (C_.)*(x_)^2)*((d_) + (e_.)*(x_) +
(f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(C*((-c)*e*(2*p + q + 2)) + 2*c*C*f*(
p + q + 1)*x)*(a + c*x^2)^p*((d + e*x + f*x^2)^(q + 1)/(2*c*f^2*(p + q + 1)
*(2*p + 2*q + 3))), x] - Dist[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), Int[
(a + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[p*((-a)*e)*(C*(c*e)*(q + 1) -
c*(C*e)*(2*p + 2*q + 3)) + (p + q + 1)*(a*c*(C*(2*d*f - e^2*(2*p + q + 2))
+ f*(-2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)*(C*(c*e)*(q + 1) - c*(C*e
)*(2*p + 2*q + 3)) + (p + q + 1)*(C*e*f*p*(-4*a*c)))*x + (p*(c*e)*(C*(c*e)*
(q + 1) - c*(C*e)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(-4*a*c) - c^2*(C
*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d + 2*A*f)*(2*p + 2*q + 3))))*x^2, x]
, x], x] /; FreeQ[{a, c, d, e, f, A, C, q}, x] && NeQ[e^2 - 4*d*f, 0] && Gt
Q[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*q + 3, 0] &&  !IGtQ[p, 0] &&  !
IGtQ[q, 0]
```

Rule 1094

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + f*x^2],
 x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt
[d + f*x^2]), x], x] /; FreeQ[{a, b, c, d, f, A, B, C}, x] && NeQ[b^2 - 4*a
*c, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(2e-fx)\sqrt{a+cx^2}}{2f^2} - \frac{\int \frac{acdf-ce(2cd-af)x-c\left(af^2+2c\left(e^2-df\right)\right)x^2}{\sqrt{a+cx^2}(d+ex+fx^2)}\,dx}{2cf^2} \\
&= -\frac{(2e-fx)\sqrt{a+cx^2}}{2f^2} - \frac{\int \frac{acdf^2+cd\left(af^2+2c\left(e^2-df\right)\right)+\left(-cef(2cd-af)+ce\left(af^2+2c\left(e^2-df\right)\right)\right)x}{\sqrt{a+cx^2}(d+ex+fx^2)}\,dx}{2cf^3} \\
&\quad + \frac{\left(af^2+2c\left(e^2-df\right)\right)\int \frac{1}{\sqrt{a+cx^2}}\,dx}{2f^3}
\end{aligned}$$

$$= -\frac{(2e - fx)\sqrt{a+cx^2}}{2f^2} + \frac{(af^2 + 2c(e^2 - df))\,\mathrm{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{2f^3}$$
$$+ \frac{\left(e\left(e - \sqrt{e^2-4df}\right)(af^2 + c(e^2 - 2df)) - 2df(af^2 + c(e^2 - df))\right)\int \frac{1}{\left(e-\sqrt{e^2-4df}+2fx\right)\sqrt{a+cx^2}}\,dx}{f^3\sqrt{e^2-4df}}$$
$$- \frac{\left(e\left(e + \sqrt{e^2-4df}\right)(af^2 + c(e^2 - 2df)) - 2df(af^2 + c(e^2 - df))\right)\int \frac{1}{\left(e+\sqrt{e^2-4df}+2fx\right)\sqrt{a+cx^2}}\,dx}{f^3\sqrt{e^2-4df}}$$

$$= -\frac{(2e - fx)\sqrt{a+cx^2}}{2f^2} + \frac{(af^2 + 2c(e^2 - df))\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2\sqrt{c}f^3}$$
$$- \frac{\left(e\left(e - \sqrt{e^2-4df}\right)(af^2 + c(e^2 - 2df)) - 2df(af^2 + c(e^2 - df))\right)\mathrm{Subst}\left(\int \frac{1}{4af^2+c\left(e-\sqrt{e^2-4df}\right)^2-x}\right.}{f^3\sqrt{e^2-4df}}$$
$$+ \frac{\left(e\left(e + \sqrt{e^2-4df}\right)(af^2 + c(e^2 - 2df)) - 2df(af^2 + c(e^2 - df))\right)\mathrm{Subst}\left(\int \frac{1}{4af^2+c\left(e+\sqrt{e^2-4df}\right)^2-x}\right.}{f^3\sqrt{e^2-4df}}$$

$$= -\frac{(2e - fx)\sqrt{a+cx^2}}{2f^2} + \frac{(af^2 + 2c(e^2 - df))\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2\sqrt{c}f^3}$$
$$- \frac{\left(e\left(e - \sqrt{e^2-4df}\right)(af^2 + c(e^2 - 2df)) - 2df(af^2 + c(e^2 - df))\right)\tanh^{-1}\left(\frac{2af-c\left(e-\sqrt{e^2-\right.}}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{\right.}}}\right.}{\sqrt{2}f^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}}$$
$$+ \frac{\left(e\left(e + \sqrt{e^2-4df}\right)(af^2 + c(e^2 - 2df)) - 2df(af^2 + c(e^2 - df))\right)\tanh^{-1}\left(\frac{2af-c\left(e+\sqrt{e^2-\right.}}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{\right.}}}\right.}{\sqrt{2}f^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.69 (sec) , antiderivative size = 615, normalized size of antiderivative = 1.36

$$\int \frac{x^2\sqrt{a+cx^2}}{d+ex+fx^2}\,dx$$
$$= \frac{f(-2e+fx)\sqrt{a+cx^2} + \frac{2(af^2+2c(e^2-df))\mathrm{arctanh}\left(\frac{\sqrt{c}x}{-\sqrt{a}+\sqrt{a+cx^2}}\right)}{\sqrt{c}} - 2\mathrm{RootSum}\left[c^2d + 2\sqrt{a}ce\#1 - 2cd\#1^2 + \right.}{}$$

```
[In] Integrate[(x^2*Sqrt[a + c*x^2])/(d + e*x + f*x^2),x]
```

```
[Out] (f*(-2*e + f*x)*Sqrt[a + c*x^2] + (2*(a*f^2 + 2*c*(e^2 - d*f))*ArcTanh[(Sqr
t[c]*x)/(-Sqrt[a] + Sqrt[a + c*x^2])])/Sqrt[c] - 2*RootSum[c^2*d + 2*Sqrt[a
]*c*e*#1 - 2*c*d*#1^2 + 4*a*f*#1^2 - 2*Sqrt[a]*e*#1^3 + d*#1^4 & , (c^2*d*e
^2*Log[x] - c^2*d^2*f*Log[x] + a*c*d*f^2*Log[x] - c^2*d*e^2*Log[-Sqrt[a] +
Sqrt[a + c*x^2] - x*#1] + c^2*d^2*f*Log[-Sqrt[a] + Sqrt[a + c*x^2] - x*#1]
- a*c*d*f^2*Log[-Sqrt[a] + Sqrt[a + c*x^2] - x*#1] + 2*Sqrt[a]*c*e^3*Log[x]
*#1 - 4*Sqrt[a]*c*d*e*f*Log[x]*#1 + 2*a^(3/2)*e*f^2*Log[x]*#1 - 2*Sqrt[a]*c
*e^3*Log[-Sqrt[a] + Sqrt[a + c*x^2] - x*#1]*#1 + 4*Sqrt[a]*c*d*e*f*Log[-Sqr
t[a] + Sqrt[a + c*x^2] - x*#1]*#1 - 2*a^(3/2)*e*f^2*Log[-Sqrt[a] + Sqrt[a +
 c*x^2] - x*#1]*#1 - c*d*e^2*Log[x]*#1^2 + c*d^2*f*Log[x]*#1^2 - a*d*f^2*Lo
g[x]*#1^2 + c*d*e^2*Log[-Sqrt[a] + Sqrt[a + c*x^2] - x*#1]*#1^2 - c*d^2*f*L
og[-Sqrt[a] + Sqrt[a + c*x^2] - x*#1]*#1^2 + a*d*f^2*Log[-Sqrt[a] + Sqrt[a
+ c*x^2] - x*#1]*#1^2)/(-(Sqrt[a]*c*e) + 2*c*d*#1 - 4*a*f*#1 + 3*Sqrt[a]*e*
#1^2 - 2*d*#1^3) & ])/(2*f^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 841 vs. $2(403) = 806$.

Time = 0.82 (sec) , antiderivative size = 842, normalized size of antiderivative = 1.86

method	result
risch	$-\frac{(-fx+2e)\sqrt{c\,x^2+a}}{2f^2}+\frac{\frac{\left(a\,f^2-2cdf+2c\,e^2\right)\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{f\sqrt{c}}-\frac{\left(-2ae\,f^2\sqrt{-4df+e^2}+4cdef\sqrt{-4df+e^2}-2c\,e^3\sqrt{-4df+e^2}-4ad\,f^3+2a\,e^2f^2+\cdots\right.}{\cdots}}{\cdots}$
default	Expression too large to display

```
[In] int(x^2*(c*x^2+a)^(1/2)/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(-f*x+2*e)*(c*x^2+a)^(1/2)/f^2+1/2/f^2*(1/f*(a*f^2-2*c*d*f+2*c*e^2)*ln
(x*c^(1/2)+(c*x^2+a)^(1/2))/c^(1/2)-1/2*(-2*a*e*f^2*(-4*d*f+e^2)^(1/2)+4*c*
d*e*f*(-4*d*f+e^2)^(1/2)-2*c*e^3*(-4*d*f+e^2)^(1/2)-4*a*d*f^3+2*a*e^2*f^2+4
*c*d^2*f^2-8*c*d*e^2*f+2*c*e^4)/f^2/(-4*d*f+e^2)^(1/2)*2^(1/2)/((-(-4*d*f+e
^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*
e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2/f*(-e+(-4*d*
f+e^2)^(1/2)))+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)
/f^2)^(1/2)*(4*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2*c-4*c*(e-(-4*d*f+e^2)^(1
/2))/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2
-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2))))-1/2*(-2*a*e*
```

```
f^2*(-4*d*f+e^2)^(1/2)+4*c*d*e*f*(-4*d*f+e^2)^(1/2)-2*c*e^3*(-4*d*f+e^2)^(1
/2)+4*a*d*f^3-2*a*e^2*f^2-4*c*d^2*f^2+8*c*d*e^2*f-2*c*e^4)/f^2/(-4*d*f+e^2)
^(1/2)*2^(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*l
n((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(1/
2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e
+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-
4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^
2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2
))/f)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{a+cx^2}}{d+ex+fx^2}\,dx = \text{Timed out}$$

```
[In] integrate(x^2*(c*x^2+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{x^2\sqrt{a+cx^2}}{d+ex+fx^2}\,dx = \int \frac{x^2\sqrt{a+cx^2}}{d+ex+fx^2}\,dx$$

```
[In] integrate(x**2*(c*x**2+a)**(1/2)/(f*x**2+e*x+d),x)
[Out] Integral(x**2*sqrt(a + c*x**2)/(d + e*x + f*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2\sqrt{a+cx^2}}{d+ex+fx^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2*(c*x^2+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re deta
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^2\sqrt{a+cx^2}}{d+ex+fx^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^2*(c*x^2+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{a+cx^2}}{d+ex+fx^2}\,dx = \int \frac{x^2\,\sqrt{c\,x^2+a}}{f\,x^2+e\,x+d}\,dx$$

```
[In] int((x^2*(a + c*x^2)^(1/2))/(d + e*x + f*x^2),x)
```

```
[Out] int((x^2*(a + c*x^2)^(1/2))/(d + e*x + f*x^2), x)
```

3.53 $\int \frac{x\sqrt{a+cx^2}}{d+ex+fx^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 395

$$\int \frac{x\sqrt{a+cx^2}}{d+ex+fx^2}\,dx = \frac{\sqrt{a+cx^2}}{f}-\frac{\sqrt{c}e\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{f^2}-\frac{\left(2cdef-\left(e-\sqrt{e^2-4df}\right)\left(af^2+c(e^2-df)\right)\right)\operatorname{arctanh}\left(\frac{2af-c\left(e-\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}}+\frac{\left(2cdef-\left(e+\sqrt{e^2-4df}\right)\left(af^2+c(e^2-df)\right)\right)\operatorname{arctanh}\left(\frac{2af-c\left(e+\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}}$$

```
[Out] -e*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))*c^(1/2)/f^2+(c*x^2+a)^(1/2)/f-1/2*arc
tanh(1/2*(2*a*f-c*x*(e-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^
2+c*(e^2-2*d*f-e*(-4*d*f+e^2)^(1/2)))^(1/2))*(2*c*d*e*f-(a*f^2+c*(-d*f+e^2)
)*(e-(-4*d*f+e^2)^(1/2)))/f^2*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+c*(e^2-2*
d*f-e*(-4*d*f+e^2)^(1/2)))^(1/2)+1/2*arctanh(1/2*(2*a*f-c*x*(e+(-4*d*f+e^2)
^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+c*(e^2-2*d*f+e*(-4*d*f+e^2)^(1/2)
))^(1/2))*(2*c*d*e*f-(a*f^2+c*(-d*f+e^2))*(e+(-4*d*f+e^2)^(1/2)))/f^2*2^(1/
2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+c*(e^2-2*d*f+e*(-4*d*f+e^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1034, 1094, 223, 212, 1048, 739}

$$\int \frac{x\sqrt{a+cx^2}}{d+ex+fx^2}\,dx = -\frac{\left(2cdef-\left(e-\sqrt{e^2-4df}\right)\left(af^2+c(e^2-df)\right)\right)\text{arctanh}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{\left(2cdef-\left(\sqrt{e^2-4df}+e\right)\left(af^2+c(e^2-df)\right)\right)\text{arctanh}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{\sqrt{c}e\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{f^2}+\frac{\sqrt{a+cx^2}}{f}$$

```
[In] Int[(x*Sqrt[a + c*x^2])/(d + e*x + f*x^2),x]
```

```
[Out] Sqrt[a + c*x^2]/f - (Sqrt[c]*e*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/f^2 -
((2*c*d*e*f - (e - Sqrt[e^2 - 4*d*f])*(a*f^2 + c*(e^2 - d*f)))*ArcTanh[(2*a
*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f -
e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])]])/(Sqrt[2]*f^2*Sqrt[e^2 - 4*d*f]*Sqr
t[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) + ((2*c*d*e*f - (e + Sq
rt[e^2 - 4*d*f])*(a*f^2 + c*(e^2 - d*f)))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2
- 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]
*Sqrt[a + c*x^2])]])/(Sqrt[2]*f^2*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 -
2*d*f + e*Sqrt[e^2 - 4*d*f])])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[h*(a + c*x^2)^p*((d + e*x + f*x^2)^(q +
 1)/(2*f*(p + q + 1))), x] + Dist[1/(2*f*(p + q + 1)), Int[(a + c*x^2)^(p -
 1)*(d + e*x + f*x^2)^q*Simp[a*h*e*p - a*(h*e - 2*g*f)*(p + q + 1) - 2*h*p*
(c*d - a*f)*x - (h*c*e*p + c*(h*e - 2*g*f)*(p + q + 1))*x^2, x], x], x] /;
FreeQ[{a, c, d, e, f, g, h, q}, x] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && N
eQ[p + q + 1, 0]
```

Rule 1048

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1094

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + f*x^2],
 x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt
[d + f*x^2]), x], x] /; FreeQ[{a, b, c, d, f, A, B, C}, x] && NeQ[b^2 - 4*a
*c, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{a+cx^2}}{f}+\frac{\int \frac{-((cd-af)x)-cex^2}{\sqrt{a+cx^2}(d+ex+fx^2)}\,dx}{f}\\
&= \frac{\sqrt{a+cx^2}}{f}+\frac{\int \frac{cde+(ce^2+f(-cd+af))x}{\sqrt{a+cx^2}(d+ex+fx^2)}\,dx}{f^2}-\frac{(ce)\int \frac{1}{\sqrt{a+cx^2}}\,dx}{f^2}\\
&= \frac{\sqrt{a+cx^2}}{f}-\frac{(ce)\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx,x,\frac{x}{\sqrt{a+cx^2}}\right)}{f^2}\\
&\quad+\frac{\left(2cdef-\left(e-\sqrt{e^2-4df}\right)\left(af^2+c(e^2-df)\right)\right)\int \frac{1}{\left(e-\sqrt{e^2-4df}+2fx\right)\sqrt{a+cx^2}}\,dx}{f^2\sqrt{e^2-4df}}\\
&\quad-\frac{\left(2cdef-\left(e+\sqrt{e^2-4df}\right)\left(af^2+c(e^2-df)\right)\right)\int \frac{1}{\left(e+\sqrt{e^2-4df}+2fx\right)\sqrt{a+cx^2}}\,dx}{f^2\sqrt{e^2-4df}}
\end{aligned}$$

$$= \frac{\sqrt{a+cx^2}}{f} - \frac{\sqrt{c}e\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{f^2} - \frac{\left(2cdef-\left(e-\sqrt{e^2-4df}\right)\left(af^2+c(e^2-df)\right)\right)\text{Subst}\left(\int \frac{1}{4af^2+c\left(e-\sqrt{e^2-4df}\right)^2-x^2}\,dx, x, \frac{2af-c\left(e-\sqrt{e^2-\right.}}{\sqrt{a+cx^2}}\right.}{f^2\sqrt{e^2-4df}} + \frac{\left(2cdef-\left(e+\sqrt{e^2-4df}\right)\left(af^2+c(e^2-df)\right)\right)\text{Subst}\left(\int \frac{1}{4af^2+c\left(e+\sqrt{e^2-4df}\right)^2-x^2}\,dx, x, \frac{2af-c\left(e+\sqrt{e^2-\right.}}{\sqrt{a+cx^2}}\right.}{f^2\sqrt{e^2-4df}}$$

$$= \frac{\sqrt{a+cx^2}}{f} - \frac{\sqrt{c}e\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{f^2} - \frac{\left(2cdef-\left(e-\sqrt{e^2-4df}\right)\left(af^2+c(e^2-df)\right)\right)\tanh^{-1}\left(\frac{2af-c\left(e-\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}} + \frac{\left(2cdef-\left(e+\sqrt{e^2-4df}\right)\left(af^2+c(e^2-df)\right)\right)\tanh^{-1}\left(\frac{2af-c\left(e+\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.34 (sec) , antiderivative size = 379, normalized size of antiderivative = 0.96

$$\int \frac{x\sqrt{a+cx^2}}{d+ex+fx^2}\,dx$$

$$= \frac{f\sqrt{a+cx^2} + \sqrt{c}e\log\left(-\sqrt{c}x+\sqrt{a+cx^2}\right) - \text{RootSum}\left[a^2f + 2a\sqrt{c}e\#1 + 4cd\#1^2 - 2af\#1^2 - 2\sqrt{c}e\#1^3\right.}{}$$

```
[In] Integrate[(x*Sqrt[a + c*x^2])/(d + e*x + f*x^2),x]
```

```
[Out] (f*Sqrt[a + c*x^2] + Sqrt[c]*e*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]] - RootSu
m[a^2*f + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 + f
*#1^4 & , (a*c*e^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] - a*c*d*f*Log[-
(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] + a^2*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c
*x^2] - #1] + 2*c^(3/2)*d*e*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 - c
*e^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2 + c*d*f*Log[-(Sqrt[c]*x)
 + Sqrt[a + c*x^2] - #1]*#1^2 - a*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] -
#1]*#1^2)/(a*Sqrt[c]*e + 4*c*d*#1 - 2*a*f*#1 - 3*Sqrt[c]*e*#1^2 + 2*f*#1^3)
 & ])/f^2
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 768 vs. 2(350) = 700.

Time = 0.79 (sec) , antiderivative size = 769, normalized size of antiderivative = 1.95

method	result
risch	$\frac{\sqrt{c x^2+a}}{f}-\frac{\frac{\sqrt{c}\, e \ln\left(x\sqrt{c}+\sqrt{c x^2+a}\right)}{f}-\frac{\left(-a f^2\sqrt{-4df+e^2}+cdf\sqrt{-4df+e^2}-c e^2\sqrt{-4df+e^2}+ae f^2-3cdef+c e^3\right)\sqrt{2}\,\ln\left(\frac{-\sqrt{-4df+e^2}\, ce+2a f^2}{f^2}\right.}{}}{}$
default	Expression too large to display

```
[In] int(x*(c*x^2+a)^(1/2)/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] (c*x^2+a)^(1/2)/f-1/f*(c^(1/2)*e/f*ln(x*c^(1/2)+(c*x^2+a)^(1/2))-1/2*(-a*f^
2*(-4*d*f+e^2)^(1/2)+c*d*f*(-4*d*f+e^2)^(1/2)-c*e^2*(-4*d*f+e^2)^(1/2)+a*e*
f^2-3*c*d*e*f+c*e^3)/f^2/(-4*d*f+e^2)^(1/2)*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c
*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2
*c*d*f+c*e^2)/f^2-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2
)))+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)
*(4*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1
/2/f*(-e+(-4*d*f+e^2)^(1/2)))+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*
e^2)/f^2)^(1/2))/(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2))))-1/2*(-a*f^2*(-4*d*f+e^2
)^(1/2)+c*d*f*(-4*d*f+e^2)^(1/2)-c*e^2*(-4*d*f+e^2)^(1/2)-a*e*f^2+3*c*d*e*f
-c*e^3)/f^2/(-4*d*f+e^2)^(1/2)*2^(1/2)/((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c
*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f
^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*
(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4
*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)
^(1/2))/f)+2*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+
1/2*(e+(-4*d*f+e^2)^(1/2))/f)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x\sqrt{a+cx^2}}{d+ex+fx^2}\,dx = \text{Timed out}$$

```
[In] integrate(x*(c*x^2+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{x\sqrt{a+cx^2}}{d+ex+fx^2}\,dx = \int \frac{x\sqrt{a+cx^2}}{d+ex+fx^2}\,dx$$

```
[In] integrate(x*(c*x**2+a)**(1/2)/(f*x**2+e*x+d),x)
[Out] Integral(x*sqrt(a + c*x**2)/(d + e*x + f*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x\sqrt{a+cx^2}}{d+ex+fx^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x*(c*x^2+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re deta
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x\sqrt{a+cx^2}}{d+ex+fx^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x*(c*x^2+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt{a+cx^2}}{d+ex+fx^2}\,dx=\int \frac{x\,\sqrt{c\,x^2+a}}{f\,x^2+e\,x+d}\,dx$$

```
[In] int((x*(a + c*x^2)^(1/2))/(d + e*x + f*x^2),x)
[Out] int((x*(a + c*x^2)^(1/2))/(d + e*x + f*x^2), x)
```

3.54 $\int \frac{\sqrt{a+cx^2}}{d+ex+fx^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 298

$$\int \frac{\sqrt{a+cx^2}}{d+ex+fx^2}\,dx = \frac{\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{f} - \frac{\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}\,\text{arctanh}\left(\frac{2af-c\left(e-\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}f\sqrt{e^2-4df}} + \frac{\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}\,\text{arctanh}\left(\frac{2af-c\left(e+\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}f\sqrt{e^2-4df}}$$

```
[Out] arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))*c^(1/2)/f-1/2*arctanh(1/2*(2*a*f-c*x*(e-
(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+c*(e^2-2*d*f-e*(-4*d*
f+e^2)^(1/2)))^(1/2))*(2*a*f^2+c*(e^2-2*d*f-e*(-4*d*f+e^2)^(1/2)))^(1/2)/f*
2^(1/2)/(-4*d*f+e^2)^(1/2)+1/2*arctanh(1/2*(2*a*f-c*x*(e+(-4*d*f+e^2)^(1/2)
))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+c*(e^2-2*d*f+e*(-4*d*f+e^2)^(1/2)))^(1/
2))*(2*a*f^2+c*(e^2-2*d*f+e*(-4*d*f+e^2)^(1/2)))^(1/2)/f*2^(1/2)/(-4*d*f+e^
2)^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1005, 223, 212, 1048, 739}

$$\int \frac{\sqrt{a+cx^2}}{d+ex+fx^2}\,dx$$
$$= -\frac{\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}\operatorname{arctanh}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}f\sqrt{e^2-4df}}$$
$$+\frac{\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}\operatorname{arctanh}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}f\sqrt{e^2-4df}}$$
$$+\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{f}$$

```
[In] Int[Sqrt[a + c*x^2]/(d + e*x + f*x^2),x]
```

```
[Out] (Sqrt[c]*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/f - (Sqrt[2*a*f^2 + c*(e^2 -
 2*d*f - e*Sqrt[e^2 - 4*d*f])]*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x
)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c
*x^2])])/(Sqrt[2]*f*Sqrt[e^2 - 4*d*f]) + (Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e
*Sqrt[e^2 - 4*d*f])]*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]
*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(
Sqrt[2]*f*Sqrt[e^2 - 4*d*f])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1005

```
Int[Sqrt[(a_) + (c_.)*(x_)^2]/((d_) + (e_.)*(x_) + (f_.)*(x_)^2), x_Symbol]
 :> Dist[c/f, Int[1/Sqrt[a + c*x^2], x], x] - Dist[1/f, Int[(c*d - a*f + c*
e*x)/(Sqrt[a + c*x^2]*(d + e*x + f*x^2)), x], x] /; FreeQ[{a, c, d, e, f},
x] && NeQ[e^2 - 4*d*f, 0]
```

Rule 1048

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\text{integral} = -\frac{\int \frac{cd-af+cex}{\sqrt{a+cx^2}(d+ex+fx^2)}\,dx}{f} + \frac{c\int \frac{1}{\sqrt{a+cx^2}}\,dx}{f}$$

$$= \frac{c\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{f} + \frac{\left(2f(cd-af)-ce\left(e+\sqrt{e^2-4df}\right)\right)\int \frac{1}{\left(e+\sqrt{e^2-4df}+2fx\right)\sqrt{a+cx^2}}\,dx}{f\sqrt{e^2-4df}} + \frac{\left(2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)\right)\int \frac{1}{\left(e-\sqrt{e^2-4df}+2fx\right)\sqrt{a+cx^2}}\,dx}{f\sqrt{e^2-4df}}$$

$$= \frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{f} - \frac{\left(2f(cd-af)-ce\left(e+\sqrt{e^2-4df}\right)\right)\text{Subst}\left(\int \frac{1}{4af^2+c\left(e+\sqrt{e^2-4df}\right)^2-x^2}\,dx, x, \frac{2af-c\left(e+\sqrt{e^2-4df}\right)x}{\sqrt{a+cx^2}}\right)}{f\sqrt{e^2-4df}} - \frac{\left(2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)\right)\text{Subst}\left(\int \frac{1}{4af^2+c\left(e-\sqrt{e^2-4df}\right)^2-x^2}\,dx, x, \frac{2af-c\left(e-\sqrt{e^2-4df}\right)x}{\sqrt{a+cx^2}}\right)}{f\sqrt{e^2-4df}}$$

$$= \frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{f} - \frac{\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}\tanh^{-1}\left(\frac{2af-c\left(e-\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}f\sqrt{e^2-4df}} + \frac{\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}\tanh^{-1}\left(\frac{2af-c\left(e+\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}f\sqrt{e^2-4df}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.29 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{a+cx^2}}{d+ex+fx^2}\,dx$$

$$= \frac{-\sqrt{c}\log\left(-\sqrt{c}x+\sqrt{a+cx^2}\right) + \text{RootSum}\left[a^2f + 2a\sqrt{c}e\#1 + 4cd\#1^2 - 2af\#1^2 - 2\sqrt{c}e\#1^3 + f\#1^4\&,\right.}{}$$

```
[In] Integrate[Sqrt[a + c*x^2]/(d + e*x + f*x^2),x]
```

```
[Out] (-(Sqrt[c]*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]]) + RootSum[a^2*f + 2*a*Sqrt[
c]*e*#1 + 4*c*d*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (a*c*e*Lo
g[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] + 2*c^(3/2)*d*Log[-(Sqrt[c]*x) + Sqr
t[a + c*x^2] - #1]*#1 - 2*a*Sqrt[c]*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] -
#1]*#1 - c*e*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2)/(a*Sqrt[c]*e +
4*c*d*#1 - 2*a*f*#1 - 3*Sqrt[c]*e*#1^2 + 2*f*#1^3) & ])/f
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1211 vs. 2(259) = 518.

Time = 0.74 (sec) , antiderivative size = 1212, normalized size of antiderivative = 4.07

method	result	size
default	Expression too large to display	1212

```
[In] int((c*x^2+a)^(1/2)/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/(-4*d*f+e^2)^(1/2)*(1/2*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(
-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)
```

```
*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)-1/2*c^(1/2)*(e+(-4*d*f+e^2)^(1/2))/f
*ln((-1/2*c*(e+(-4*d*f+e^2)^(1/2))/f+c*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))/c^
(1/2)+((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1
/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*
e^2)/f^2)^(1/2))-1/2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2*2^(
1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d*
f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1
/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2
*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4
*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*c
*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))+1/
(-4*d*f+e^2)^(1/2)*(1/2*(4*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2*c-4*c*(e-(-4
*d*f+e^2)^(1/2))/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+2*(-(-4*d*f+e^2)^(1/2)
*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)-1/2*c^(1/2)*(e-(-4*d*f+e^2)^(1/2))/f
*ln((-1/2*c*(e-(-4*d*f+e^2)^(1/2))/f+c*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2))))/c
^(1/2)+((x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2*c-c*(e-(-4*d*f+e^2)^(1/2))/f*(x
-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+1/2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*
f+c*e^2)/f^2)^(1/2))-1/2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^
2*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((
-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e-(-4*d*f+e^2)^(1/2))
/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+
2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2*c-
4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+2*(-(-4*d*f+
e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2/f*(-e+(-4*d*f+e^2)
^(1/2)))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1188 vs. $2(257) = 514$.

Time = 138.40 (sec) , antiderivative size = 2384, normalized size of antiderivative = 8.00

$$\int \frac{\sqrt{a+cx^2}}{d+ex+fx^2}\,dx = \text{Too large to display}$$

```
[In] integrate((c*x^2+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(2)*f*sqrt((c*e^2 - 2*c*d*f + 2*a*f^2 + (e^2*f^2 - 4*d*f^3)*sqrt(
c^2*e^2/(e^2*f^4 - 4*d*f^5)))/(e^2*f^2 - 4*d*f^3))*log((4*c^2*d*e*x - 2*a*c
*e^2 + sqrt(2)*(c*e^3 - 4*c*d*e*f - (e^3*f^2 - 4*d*e*f^3)*sqrt(c^2*e^2/(e^2
*f^4 - 4*d*f^5)))*sqrt(c*x^2 + a)*sqrt((c*e^2 - 2*c*d*f + 2*a*f^2 + (e^2*f^
2 - 4*d*f^3)*sqrt(c^2*e^2/(e^2*f^4 - 4*d*f^5)))/(e^2*f^2 - 4*d*f^3)) + 2*(a
*e^2*f^2 - 4*a*d*f^3)*sqrt(c^2*e^2/(e^2*f^4 - 4*d*f^5)))/x) - sqrt(2)*f*sqr
t((c*e^2 - 2*c*d*f + 2*a*f^2 + (e^2*f^2 - 4*d*f^3)*sqrt(c^2*e^2/(e^2*f^4 -
4*d*f^5)))/(e^2*f^2 - 4*d*f^3))*log((4*c^2*d*e*x - 2*a*c*e^2 - sqrt(2)*(c*e
```

```
^3 - 4*c*d*e*f - (e^3*f^2 - 4*d*e*f^3)*sqrt(c^2*e^2/(e^2*f^4 - 4*d*f^5)))*s
qrt(c*x^2 + a)*sqrt((c*e^2 - 2*c*d*f + 2*a*f^2 + (e^2*f^2 - 4*d*f^3)*sqrt(c
^2*e^2/(e^2*f^4 - 4*d*f^5)))/(e^2*f^2 - 4*d*f^3)) + 2*(a*e^2*f^2 - 4*a*d*f^
3)*sqrt(c^2*e^2/(e^2*f^4 - 4*d*f^5)))/x) + sqrt(2)*f*sqrt((c*e^2 - 2*c*d*f
 + 2*a*f^2 - (e^2*f^2 - 4*d*f^3)*sqrt(c^2*e^2/(e^2*f^4 - 4*d*f^5)))/(e^2*f^2
 - 4*d*f^3))*log((4*c^2*d*e*x - 2*a*c*e^2 + sqrt(2)*(c*e^3 - 4*c*d*e*f + (e
^3*f^2 - 4*d*e*f^3)*sqrt(c^2*e^2/(e^2*f^4 - 4*d*f^5)))*sqrt(c*x^2 + a)*sqrt
((c*e^2 - 2*c*d*f + 2*a*f^2 - (e^2*f^2 - 4*d*f^3)*sqrt(c^2*e^2/(e^2*f^4 - 4
*d*f^5)))/(e^2*f^2 - 4*d*f^3)) - 2*(a*e^2*f^2 - 4*a*d*f^3)*sqrt(c^2*e^2/(e^
2*f^4 - 4*d*f^5)))/x) - sqrt(2)*f*sqrt((c*e^2 - 2*c*d*f + 2*a*f^2 - (e^2*f^
2 - 4*d*f^3)*sqrt(c^2*e^2/(e^2*f^4 - 4*d*f^5)))/(e^2*f^2 - 4*d*f^3))*log((4
*c^2*d*e*x - 2*a*c*e^2 - sqrt(2)*(c*e^3 - 4*c*d*e*f + (e^3*f^2 - 4*d*e*f^3)
*sqrt(c^2*e^2/(e^2*f^4 - 4*d*f^5)))*sqrt(c*x^2 + a)*sqrt((c*e^2 - 2*c*d*f +
 2*a*f^2 - (e^2*f^2 - 4*d*f^3)*sqrt(c^2*e^2/(e^2*f^4 - 4*d*f^5)))/(e^2*f^2
- 4*d*f^3)) - 2*(a*e^2*f^2 - 4*a*d*f^3)*sqrt(c^2*e^2/(e^2*f^4 - 4*d*f^5)))/
x) + 2*sqrt(c)*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a))/f, 1/4*(sqr
t(2)*f*sqrt((c*e^2 - 2*c*d*f + 2*a*f^2 + (e^2*f^2 - 4*d*f^3)*sqrt(c^2*e^2/(
e^2*f^4 - 4*d*f^5)))/(e^2*f^2 - 4*d*f^3))*log((4*c^2*d*e*x - 2*a*c*e^2 + sq
rt(2)*(c*e^3 - 4*c*d*e*f - (e^3*f^2 - 4*d*e*f^3)*sqrt(c^2*e^2/(e^2*f^4 - 4*
d*f^5)))*sqrt(c*x^2 + a)*sqrt((c*e^2 - 2*c*d*f + 2*a*f^2 + (e^2*f^2 - 4*d*f
^3)*sqrt(c^2*e^2/(e^2*f^4 - 4*d*f^5)))/(e^2*f^2 - 4*d*f^3)) + 2*(a*e^2*f^2
- 4*a*d*f^3)*sqrt(c^2*e^2/(e^2*f^4 - 4*d*f^5)))/x) - sqrt(2)*f*sqrt((c*e^2
- 2*c*d*f + 2*a*f^2 + (e^2*f^2 - 4*d*f^3)*sqrt(c^2*e^2/(e^2*f^4 - 4*d*f^5))
)/(e^2*f^2 - 4*d*f^3))*log((4*c^2*d*e*x - 2*a*c*e^2 - sqrt(2)*(c*e^3 - 4*c*
d*e*f - (e^3*f^2 - 4*d*e*f^3)*sqrt(c^2*e^2/(e^2*f^4 - 4*d*f^5)))*sqrt(c*x^2
 + a)*sqrt((c*e^2 - 2*c*d*f + 2*a*f^2 + (e^2*f^2 - 4*d*f^3)*sqrt(c^2*e^2/(e
^2*f^4 - 4*d*f^5)))/(e^2*f^2 - 4*d*f^3)) + 2*(a*e^2*f^2 - 4*a*d*f^3)*sqrt(c
^2*e^2/(e^2*f^4 - 4*d*f^5)))/x) + sqrt(2)*f*sqrt((c*e^2 - 2*c*d*f + 2*a*f^2
 - (e^2*f^2 - 4*d*f^3)*sqrt(c^2*e^2/(e^2*f^4 - 4*d*f^5)))/(e^2*f^2 - 4*d*f^
3))*log((4*c^2*d*e*x - 2*a*c*e^2 + sqrt(2)*(c*e^3 - 4*c*d*e*f + (e^3*f^2 -
4*d*e*f^3)*sqrt(c^2*e^2/(e^2*f^4 - 4*d*f^5)))*sqrt(c*x^2 + a)*sqrt((c*e^2 -
 2*c*d*f + 2*a*f^2 - (e^2*f^2 - 4*d*f^3)*sqrt(c^2*e^2/(e^2*f^4 - 4*d*f^5)))
/(e^2*f^2 - 4*d*f^3)) - 2*(a*e^2*f^2 - 4*a*d*f^3)*sqrt(c^2*e^2/(e^2*f^4 - 4
*d*f^5)))/x) - sqrt(2)*f*sqrt((c*e^2 - 2*c*d*f + 2*a*f^2 - (e^2*f^2 - 4*d*f
^3)*sqrt(c^2*e^2/(e^2*f^4 - 4*d*f^5)))/(e^2*f^2 - 4*d*f^3))*log((4*c^2*d*e*
x - 2*a*c*e^2 - sqrt(2)*(c*e^3 - 4*c*d*e*f + (e^3*f^2 - 4*d*e*f^3)*sqrt(c^2
*e^2/(e^2*f^4 - 4*d*f^5)))*sqrt(c*x^2 + a)*sqrt((c*e^2 - 2*c*d*f + 2*a*f^2
- (e^2*f^2 - 4*d*f^3)*sqrt(c^2*e^2/(e^2*f^4 - 4*d*f^5)))/(e^2*f^2 - 4*d*f^3
)) - 2*(a*e^2*f^2 - 4*a*d*f^3)*sqrt(c^2*e^2/(e^2*f^4 - 4*d*f^5)))/x) - 4*sq
rt(-c)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)))/f]
```

Sympy [F]

$$\int \frac{\sqrt{a+cx^2}}{d+ex+fx^2}\,dx=\int \frac{\sqrt{a+cx^2}}{d+ex+fx^2}\,dx$$

```
[In] integrate((c*x**2+a)**(1/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral(sqrt(a + c*x**2)/(d + e*x + f*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+cx^2}}{d+ex+fx^2}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate((c*x^2+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re deta
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+cx^2}}{d+ex+fx^2}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((c*x^2+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+cx^2}}{d+ex+fx^2}\,dx=\int \frac{\sqrt{c\,x^2+a}}{f\,x^2+e\,x+d}\,dx$$

```
[In] int((a + c*x^2)^(1/2)/(d + e*x + f*x^2),x)
[Out] int((a + c*x^2)^(1/2)/(d + e*x + f*x^2), x)
```

3.55 $\int \frac{\sqrt{a+cx^2}}{x(d+ex+fx^2)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 358

$$\int \frac{\sqrt{a+cx^2}}{x(d+ex+fx^2)}\,dx = \frac{\left(2aef+(cd-af)\left(e-\sqrt{e^2-4df}\right)\right)\operatorname{arctanh}\left(\frac{2af-c\left(e-\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}} - \frac{\left(2aef+(cd-af)\left(e+\sqrt{e^2-4df}\right)\right)\operatorname{arctanh}\left(\frac{2af-c\left(e+\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}} - \frac{\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d}$$

```
[Out] -arctanh((c*x^2+a)^(1/2)/a^(1/2))*a^(1/2)/d+1/2*arctanh(1/2*(2*a*f-c*x*(e-(
-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+c*(e^2-2*d*f-e*(-4*d*f
+e^2)^(1/2)))^(1/2))*(2*a*e*f+(-a*f+c*d)*(e-(-4*d*f+e^2)^(1/2)))/d*2^(1/2)/
(-4*d*f+e^2)^(1/2)/(2*a*f^2+c*(e^2-2*d*f-e*(-4*d*f+e^2)^(1/2)))^(1/2)-1/2*a
rctanh(1/2*(2*a*f-c*x*(e+(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*
f^2+c*(e^2-2*d*f+e*(-4*d*f+e^2)^(1/2)))^(1/2))*(2*a*e*f+(-a*f+c*d)*(e+(-4*d
*f+e^2)^(1/2)))/d*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+c*(e^2-2*d*f+e*(-4*d*
f+e^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6860, 272, 52, 65, 214, 1034, 1048, 739, 212}

$$\int \frac{\sqrt{a+cx^2}}{x(d+ex+fx^2)}\,dx$$
$$= \frac{\left(\left(e-\sqrt{e^2-4df}\right)(cd-af)+2aef\right)\operatorname{arctanh}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}$$
$$-\frac{\left(\left(\sqrt{e^2-4df}+e\right)(cd-af)+2aef\right)\operatorname{arctanh}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}$$
$$-\frac{\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d}$$

```
[In] Int[Sqrt[a + c*x^2]/(x*(d + e*x + f*x^2)),x]
```

```
[Out] ((2*a*e*f + (c*d - a*f)*(e - Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e - Sq
rt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4
*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*d*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e
^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) - ((2*a*e*f + (c*d - a*f)*(e + Sqrt[e^2
 - 4*d*f]))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a
*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*d
*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]) -
 (Sqrt[a]*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/d
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[h*(a + c*x^2)^p*((d + e*x + f*x^2)^(q +
 1)/(2*f*(p + q + 1))), x] + Dist[1/(2*f*(p + q + 1)), Int[(a + c*x^2)^(p -
 1)*(d + e*x + f*x^2)^q*Simp[a*h*e*p - a*(h*e - 2*g*f)*(p + q + 1) - 2*h*p*
(c*d - a*f)*x - (h*c*e*p + c*(h*e - 2*g*f)*(p + q + 1))*x^2, x], x], x] /;
FreeQ[{a, c, d, e, f, g, h, q}, x] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && N
eQ[p + q + 1, 0]
```

Rule 1048

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 6860

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{\sqrt{a+cx^2}}{dx} + \frac{(-e-fx)\sqrt{a+cx^2}}{d\,(d+ex+fx^2)} \right) dx$$

$$= \frac{\int \frac{\sqrt{a+cx^2}}{x}\,dx}{d} + \frac{\int \frac{(-e-fx)\sqrt{a+cx^2}}{d+ex+fx^2}\,dx}{d}$$

$$= -\frac{\sqrt{a+cx^2}}{d} + \frac{\text{Subst}\left(\int \frac{\sqrt{a+cx}}{x}\,dx, x, x^2\right)}{2d} + \frac{\int \frac{-aef+f(cd-af)x}{\sqrt{a+cx^2}(d+ex+fx^2)}\,dx}{df}$$

$$= \frac{a\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx, x, x^2\right)}{2d} - \frac{\left(2aef+(cd-af)\left(e-\sqrt{e^2-4df}\right)\right)\int \frac{1}{\left(e-\sqrt{e^2-4df}+2fx\right)\sqrt{a+cx^2}}\,dx}{d\sqrt{e^2-4df}} + \frac{\left(2aef+(cd-af)\left(e+\sqrt{e^2-4df}\right)\right)\int \frac{1}{\left(e+\sqrt{e^2-4df}+2fx\right)\sqrt{a+cx^2}}\,dx}{d\sqrt{e^2-4df}}$$

$$= \frac{a\text{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx, x, \sqrt{a+cx^2}\right)}{cd} + \frac{\left(2aef+(cd-af)\left(e-\sqrt{e^2-4df}\right)\right)\text{Subst}\left(\int \frac{1}{4af^2+c\left(e-\sqrt{e^2-4df}\right)^2-x^2}\,dx, x, \frac{2af-c\left(e-\sqrt{e^2-4df}\right)x}{\sqrt{a+cx^2}}\right)}{d\sqrt{e^2-4df}} - \frac{\left(2aef+(cd-af)\left(e+\sqrt{e^2-4df}\right)\right)\text{Subst}\left(\int \frac{1}{4af^2+c\left(e+\sqrt{e^2-4df}\right)^2-x^2}\,dx, x, \frac{2af-c\left(e+\sqrt{e^2-4df}\right)x}{\sqrt{a+cx^2}}\right)}{d\sqrt{e^2-4df}}$$

$$= \frac{(2aef + (cd - af)(e - \sqrt{e^2 - 4df}))\tanh^{-1}\left(\frac{2af - c(e - \sqrt{e^2 - 4df})x}{\sqrt{2}\sqrt{2af^2 + c(e^2 - 2df - e\sqrt{e^2 - 4df})}\sqrt{a + cx^2}}\right)}{\sqrt{2}d\sqrt{e^2 - 4df}\sqrt{2af^2 + c(e^2 - 2df - e\sqrt{e^2 - 4df})}}$$
$$- \frac{(2aef + (cd - af)(e + \sqrt{e^2 - 4df}))\tanh^{-1}\left(\frac{2af - c(e + \sqrt{e^2 - 4df})x}{\sqrt{2}\sqrt{2af^2 + c(e^2 - 2df + e\sqrt{e^2 - 4df})}\sqrt{a + cx^2}}\right)}{\sqrt{2}d\sqrt{e^2 - 4df}\sqrt{2af^2 + c(e^2 - 2df + e\sqrt{e^2 - 4df})}}$$
$$- \frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a + cx^2}}{\sqrt{a}}\right)}{d}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.36 (sec) , antiderivative size = 310, normalized size of antiderivative = 0.87

$$\int \frac{\sqrt{a + cx^2}}{x(d + ex + fx^2)}\,dx$$
$$= \frac{\sqrt{a}(-\log(x) + \log(-\sqrt{a} + \sqrt{a + cx^2})) - \text{RootSum}\left[c^2d + 2\sqrt{a}ce\#1 - 2cd\#1^2 + 4af\#1^2 - 2\sqrt{a}e\#1^3 + \right.}{}$$

```
[In] Integrate[Sqrt[a + c*x^2]/(x*(d + e*x + f*x^2)),x]
```

```
[Out] (Sqrt[a]*(-Log[x] + Log[-Sqrt[a] + Sqrt[a + c*x^2]]) - RootSum[c^2*d + 2*Sq
rt[a]*c*e*#1 - 2*c*d*#1^2 + 4*a*f*#1^2 - 2*Sqrt[a]*e*#1^3 + d*#1^4 & , (-(a
*c*e*Log[x]) + a*c*e*Log[-Sqrt[a] + Sqrt[a + c*x^2] - x*#1] + 2*Sqrt[a]*c*d
*Log[x]*#1 - 2*a^(3/2)*f*Log[x]*#1 - 2*Sqrt[a]*c*d*Log[-Sqrt[a] + Sqrt[a +
c*x^2] - x*#1]*#1 + 2*a^(3/2)*f*Log[-Sqrt[a] + Sqrt[a + c*x^2] - x*#1]*#1 +
 a*e*Log[x]*#1^2 - a*e*Log[-Sqrt[a] + Sqrt[a + c*x^2] - x*#1]*#1^2)/(Sqrt[a
]*c*e - 2*c*d*#1 + 4*a*f*#1 - 3*Sqrt[a]*e*#1^2 + 2*d*#1^3) & ])/d
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1315 vs. 2(315) = 630.

Time = 0.68 (sec) , antiderivative size = 1316, normalized size of antiderivative = 3.68

method	result	size
default	Expression too large to display	1316

```
[In] int((c*x^2+a)^(1/2)/x/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -4*f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))*((c*x^2+a)^(1/2)-a^(1/2
)*ln((2*a+2*a^(1/2)*(c*x^2+a)^(1/2))/x))+2*f/(e+(-4*d*f+e^2)^(1/2))/(-4*d*f
+e^2)^(1/2)*(1/2*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2
)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f
^2-2*c*d*f+c*e^2)/f^2)^(1/2)-1/2*c^(1/2)*(e+(-4*d*f+e^2)^(1/2))/f*ln((-1/2*
c*(e+(-4*d*f+e^2)^(1/2))/f+c*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))/c^(1/2)+((x+
1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*
d*f+e^2)^(1/2))/f)+1/2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^
(1/2))-1/2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2*2^(1/2)/(((-4
*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/
2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*
d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e
^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^
(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2
-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)))+2*f/(-e+(-4*
d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)*(1/2*(4*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2))
)^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+2*(-(-
4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)-1/2*c^(1/2)*(e-(-4*d
*f+e^2)^(1/2))/f*ln((-1/2*c*(e-(-4*d*f+e^2)^(1/2))/f+c*(x-1/2/f*(-e+(-4*d*f
+e^2)^(1/2))))/c^(1/2)+((x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2*c-c*(e-(-4*d*f+
e^2)^(1/2))/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+1/2*(-(-4*d*f+e^2)^(1/2)*c*
e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))-1/2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2
*c*d*f+c*e^2)/f^2*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/
f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e-(-4
*d*f+e^2)^(1/2))/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+1/2*2^(1/2)*((-(-4*d*f
+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2/f*(-e+(-4*d*f+e
^2)^(1/2)))^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2
)))+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2/f*
(-e+(-4*d*f+e^2)^(1/2)))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1127 vs. $2(313) = 626$.

Time = 24.10 (sec) , antiderivative size = 2266, normalized size of antiderivative = 6.33

$$\int \frac{\sqrt{a+cx^2}}{x\left(d+ex+fx^2\right)}\,dx = \text{Too large to display}$$

```
[In] integrate((c*x^2+a)^(1/2)/x/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] [-1/4*(sqrt(2)*d*sqrt((2*c*d^2 + a*e^2 - 2*a*d*f + (d^2*e^2 - 4*d^3*f)*sqrt
(a^2*e^2/(d^4*e^2 - 4*d^5*f)))/(d^2*e^2 - 4*d^3*f))*log((2*a*c*d*e*x - a^2*
e^2 + sqrt(2)*(d^3*e^2 - 4*d^4*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f))*sqrt(c*
x^2 + a)*sqrt((2*c*d^2 + a*e^2 - 2*a*d*f + (d^2*e^2 - 4*d^3*f)*sqrt(a^2*e^2
```

```
/(d^4*e^2 - 4*d^5*f)))/(d^2*e^2 - 4*d^3*f)) - (a*d^2*e^2 - 4*a*d^3*f)*sqrt(
a^2*e^2/(d^4*e^2 - 4*d^5*f)))/x) - sqrt(2)*d*sqrt((2*c*d^2 + a*e^2 - 2*a*d*
f + (d^2*e^2 - 4*d^3*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f)))/(d^2*e^2 - 4*d^3
*f))*log((2*a*c*d*e*x - a^2*e^2 - sqrt(2)*(d^3*e^2 - 4*d^4*f)*sqrt(a^2*e^2/
(d^4*e^2 - 4*d^5*f))*sqrt(c*x^2 + a)*sqrt((2*c*d^2 + a*e^2 - 2*a*d*f + (d^2
*e^2 - 4*d^3*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f)))/(d^2*e^2 - 4*d^3*f)) - (
a*d^2*e^2 - 4*a*d^3*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f)))/x) - sqrt(2)*d*sq
rt((2*c*d^2 + a*e^2 - 2*a*d*f - (d^2*e^2 - 4*d^3*f)*sqrt(a^2*e^2/(d^4*e^2 -
 4*d^5*f)))/(d^2*e^2 - 4*d^3*f))*log((2*a*c*d*e*x - a^2*e^2 + sqrt(2)*(d^3*
e^2 - 4*d^4*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f))*sqrt(c*x^2 + a)*sqrt((2*c*
d^2 + a*e^2 - 2*a*d*f - (d^2*e^2 - 4*d^3*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f
)))/(d^2*e^2 - 4*d^3*f)) + (a*d^2*e^2 - 4*a*d^3*f)*sqrt(a^2*e^2/(d^4*e^2 -
4*d^5*f)))/x) + sqrt(2)*d*sqrt((2*c*d^2 + a*e^2 - 2*a*d*f - (d^2*e^2 - 4*d^
3*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f)))/(d^2*e^2 - 4*d^3*f))*log((2*a*c*d*e
*x - a^2*e^2 - sqrt(2)*(d^3*e^2 - 4*d^4*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f)
)*sqrt(c*x^2 + a)*sqrt((2*c*d^2 + a*e^2 - 2*a*d*f - (d^2*e^2 - 4*d^3*f)*sqr
t(a^2*e^2/(d^4*e^2 - 4*d^5*f)))/(d^2*e^2 - 4*d^3*f)) + (a*d^2*e^2 - 4*a*d^3
*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f)))/x) - 2*sqrt(a)*log(-(c*x^2 - 2*sqrt(
c*x^2 + a)*sqrt(a) + 2*a)/x^2))/d, -1/4*(sqrt(2)*d*sqrt((2*c*d^2 + a*e^2 -
2*a*d*f + (d^2*e^2 - 4*d^3*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f)))/(d^2*e^2 -
 4*d^3*f))*log((2*a*c*d*e*x - a^2*e^2 + sqrt(2)*(d^3*e^2 - 4*d^4*f)*sqrt(a^
2*e^2/(d^4*e^2 - 4*d^5*f))*sqrt(c*x^2 + a)*sqrt((2*c*d^2 + a*e^2 - 2*a*d*f
+ (d^2*e^2 - 4*d^3*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f)))/(d^2*e^2 - 4*d^3*f
)) - (a*d^2*e^2 - 4*a*d^3*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f)))/x) - sqrt(2
)*d*sqrt((2*c*d^2 + a*e^2 - 2*a*d*f + (d^2*e^2 - 4*d^3*f)*sqrt(a^2*e^2/(d^4
*e^2 - 4*d^5*f)))/(d^2*e^2 - 4*d^3*f))*log((2*a*c*d*e*x - a^2*e^2 - sqrt(2)
*(d^3*e^2 - 4*d^4*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f))*sqrt(c*x^2 + a)*sqrt
((2*c*d^2 + a*e^2 - 2*a*d*f + (d^2*e^2 - 4*d^3*f)*sqrt(a^2*e^2/(d^4*e^2 - 4
*d^5*f)))/(d^2*e^2 - 4*d^3*f)) - (a*d^2*e^2 - 4*a*d^3*f)*sqrt(a^2*e^2/(d^4*
e^2 - 4*d^5*f)))/x) - sqrt(2)*d*sqrt((2*c*d^2 + a*e^2 - 2*a*d*f - (d^2*e^2
- 4*d^3*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f)))/(d^2*e^2 - 4*d^3*f))*log((2*a
*c*d*e*x - a^2*e^2 + sqrt(2)*(d^3*e^2 - 4*d^4*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*
d^5*f))*sqrt(c*x^2 + a)*sqrt((2*c*d^2 + a*e^2 - 2*a*d*f - (d^2*e^2 - 4*d^3*
f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f)))/(d^2*e^2 - 4*d^3*f)) + (a*d^2*e^2 - 4
*a*d^3*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f)))/x) + sqrt(2)*d*sqrt((2*c*d^2 +
 a*e^2 - 2*a*d*f - (d^2*e^2 - 4*d^3*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f)))/(
d^2*e^2 - 4*d^3*f))*log((2*a*c*d*e*x - a^2*e^2 - sqrt(2)*(d^3*e^2 - 4*d^4*f
)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f))*sqrt(c*x^2 + a)*sqrt((2*c*d^2 + a*e^2 -
 2*a*d*f - (d^2*e^2 - 4*d^3*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f)))/(d^2*e^2
- 4*d^3*f)) + (a*d^2*e^2 - 4*a*d^3*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f)))/x)
 - 4*sqrt(-a)*arctan(sqrt(-a)/sqrt(c*x^2 + a)))/d]
```

Sympy [F]

$$\int \frac{\sqrt{a+cx^2}}{x\left(d+ex+fx^2\right)}\,dx=\int \frac{\sqrt{a+cx^2}}{x\left(d+ex+fx^2\right)}\,dx$$

```
[In] integrate((c*x**2+a)**(1/2)/x/(f*x**2+e*x+d),x)
[Out] Integral(sqrt(a + c*x**2)/(x*(d + e*x + f*x**2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+cx^2}}{x\left(d+ex+fx^2\right)}\,dx=\int \frac{\sqrt{cx^2+a}}{(fx^2+ex+d)x}\,dx$$

```
[In] integrate((c*x^2+a)^(1/2)/x/(f*x^2+e*x+d),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^2 + a)/((f*x^2 + e*x + d)*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+cx^2}}{x\left(d+ex+fx^2\right)}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((c*x^2+a)^(1/2)/x/(f*x^2+e*x+d),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+cx^2}}{x\left(d+ex+fx^2\right)}\,dx=\int \frac{\sqrt{c\,x^2+a}}{x\,\left(f\,x^2+e\,x+d\right)}\,dx$$

```
[In] int((a + c*x^2)^(1/2)/(x*(d + e*x + f*x^2)),x)
[Out] int((a + c*x^2)^(1/2)/(x*(d + e*x + f*x^2)), x)
```

3.56 $\int \frac{\sqrt{a+cx^2}}{x^2(d+ex+fx^2)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 382

$$\begin{aligned}
\int \frac{\sqrt{a+cx^2}}{x^2(d+ex+fx^2)}\,dx
&= -\frac{\sqrt{a+cx^2}}{dx} \\
&\quad - \frac{f\left(2cd^2+a\left(e^2-2df+e\sqrt{e^2-4df}\right)\right)\operatorname{arctanh}\left(\frac{2af-c\left(e-\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}} \\
&\quad + \frac{f\left(2cd^2+a\left(e^2-2df-e\sqrt{e^2-4df}\right)\right)\operatorname{arctanh}\left(\frac{2af-c\left(e+\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}} \\
&\quad + \frac{\sqrt{a}e\operatorname{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^2}
\end{aligned}$$

```
[Out] e*arctanh((c*x^2+a)^(1/2)/a^(1/2))*a^(1/2)/d^2-(c*x^2+a)^(1/2)/d/x-1/2*f*ar
ctanh(1/2*(2*a*f-c*x*(e-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f
^2+c*(e^2-2*d*f-e*(-4*d*f+e^2)^(1/2)))^(1/2))*(2*c*d^2+a*(e^2-2*d*f+e*(-4*d
*f+e^2)^(1/2)))/d^2*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+c*(e^2-2*d*f-e*(-4*
d*f+e^2)^(1/2)))^(1/2)+1/2*f*arctanh(1/2*(2*a*f-c*x*(e+(-4*d*f+e^2)^(1/2)))
*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+c*(e^2-2*d*f+e*(-4*d*f+e^2)^(1/2)))^(1/2)
)*(2*c*d^2+a*(e^2-2*d*f-e*(-4*d*f+e^2)^(1/2)))/d^2*2^(1/2)/(-4*d*f+e^2)^(1/
2)/(2*a*f^2+c*(e^2-2*d*f+e*(-4*d*f+e^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {6860, 283, 223, 212, 272, 52, 65, 214, 1034, 1094, 1048, 739}

$$\int \frac{\sqrt{a+cx^2}}{x^2\left(d+ex+fx^2\right)}\,dx$$
$$= -\frac{f\left(a\left(e\sqrt{e^2-4df}-2df+e^2\right)+2cd^2\right)\operatorname{arctanh}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}$$
$$+\frac{f\left(a\left(-e\sqrt{e^2-4df}-2df+e^2\right)+2cd^2\right)\operatorname{arctanh}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}$$
$$+\frac{\sqrt{a}e\operatorname{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^2}-\frac{\sqrt{a+cx^2}}{dx}$$

```
[In] Int[Sqrt[a + c*x^2]/(x^2*(d + e*x + f*x^2)),x]
```

```
[Out] -(Sqrt[a + c*x^2]/(d*x)) - (f*(2*c*d^2 + a*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*
f]))*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 +
c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*d^2*Sqrt
[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) + (f*(
2*c*d^2 + a*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e + Sq
rt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4
*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*d^2*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*
(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]) + (Sqrt[a]*e*ArcTanh[Sqrt[a + c*x^2]/
Sqrt[a]])/d^2
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[h*(a + c*x^2)^p*((d + e*x + f*x^2)^(q +
 1)/(2*f*(p + q + 1))), x] + Dist[1/(2*f*(p + q + 1)), Int[(a + c*x^2)^(p -
 1)*(d + e*x + f*x^2)^q*Simp[a*h*e*p - a*(h*e - 2*g*f)*(p + q + 1) - 2*h*p*
```

```
(c*d - a*f)*x - (h*c*e*p + c*(h*e - 2*g*f)*(p + q + 1))*x^2, x], x], x] /;
FreeQ[{a, c, d, e, f, g, h, q}, x] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && N
eQ[p + q + 1, 0]
```

Rule 1048

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1094

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + f*x^2],
 x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt
[d + f*x^2]), x], x] /; FreeQ[{a, b, c, d, f, A, B, C}, x] && NeQ[b^2 - 4*a
*c, 0]
```

Rule 6860

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{\sqrt{a+cx^2}}{dx^2} - \frac{e\sqrt{a+cx^2}}{d^2x} + \frac{(e^2-df+efx)\sqrt{a+cx^2}}{d^2(d+ex+fx^2)}\right) dx$$

$$= \frac{\int \frac{(e^2-df+efx)\sqrt{a+cx^2}}{d+ex+fx^2}\,dx}{d^2} + \frac{\int \frac{\sqrt{a+cx^2}}{x^2}\,dx}{d} - \frac{e\int \frac{\sqrt{a+cx^2}}{x}\,dx}{d^2}$$

$$= \frac{e\sqrt{a+cx^2}}{d^2} - \frac{\sqrt{a+cx^2}}{dx} + \frac{c\int \frac{1}{\sqrt{a+cx^2}}\,dx}{d} - \frac{e\text{Subst}\left(\int \frac{\sqrt{a+cx}}{x}\,dx, x, x^2\right)}{2d^2} + \frac{\int \frac{af(e^2-df)-ef(cd-af)x-cdf^2x^2}{\sqrt{a+cx^2}(d+ex+fx^2)}\,dx}{d^2f}$$

$$= -\frac{\sqrt{a+cx^2}}{dx} - \frac{c\int \frac{1}{\sqrt{a+cx^2}}\,dx}{d} + \frac{c\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{d} - \frac{(ae)\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx, x, x^2\right)}{2d^2} + \frac{\int \frac{cd^2f^2+af^2(e^2-df)+(cdef^2-ef^2(cd-af))x}{\sqrt{a+cx^2}(d+ex+fx^2)}\,dx}{d^2f^2}$$

$$= -\frac{\sqrt{a+cx^2}}{dx} + \frac{\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{d} - \frac{c\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{d} - \frac{(ae)\text{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx, x, \sqrt{a+cx^2}\right)}{cd^2} - \frac{\left(f\left(2cd^2+a\left(e^2-2df-e\sqrt{e^2-4df}\right)\right)\right)\int \frac{1}{\left(e+\sqrt{e^2-4df}+2fx\right)\sqrt{a+cx^2}}\,dx}{d^2\sqrt{e^2-4df}} + \frac{\left(f\left(2cd^2+a\left(e^2-2df+e\sqrt{e^2-4df}\right)\right)\right)\int \frac{1}{\left(e-\sqrt{e^2-4df}+2fx\right)\sqrt{a+cx^2}}\,dx}{d^2\sqrt{e^2-4df}}$$

$$= -\frac{\sqrt{a+cx^2}}{dx} + \frac{\sqrt{a}e\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^2} + \frac{\left(f\left(2cd^2+a\left(e^2-2df-e\sqrt{e^2-4df}\right)\right)\right)\text{Subst}\left(\int \frac{1}{4af^2+c\left(e+\sqrt{e^2-4df}\right)^2-x^2}\,dx, x, \frac{2af-c\left(e+\sqrt{e^2-4df}\right)x}{\sqrt{a+cx^2}}\right)}{d^2\sqrt{e^2-4df}} - \frac{\left(f\left(2cd^2+a\left(e^2-2df+e\sqrt{e^2-4df}\right)\right)\right)\text{Subst}\left(\int \frac{1}{4af^2+c\left(e-\sqrt{e^2-4df}\right)^2-x^2}\,dx, x, \frac{2af-c\left(e-\sqrt{e^2-4df}\right)x}{\sqrt{a+cx^2}}\right)}{d^2\sqrt{e^2-4df}}$$

$$= -\frac{\sqrt{a+cx^2}}{dx} - \frac{f\left(2cd^2+a\left(e^2-2df+e\sqrt{e^2-4df}\right)\right)\tanh^{-1}\left(\frac{2af-c\left(e-\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}} + \frac{f\left(2cd^2+a\left(e^2-2df-e\sqrt{e^2-4df}\right)\right)\tanh^{-1}\left(\frac{2af-c\left(e+\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}} + \frac{\sqrt{a}e\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^2}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.38 (sec) , antiderivative size = 467, normalized size of antiderivative = 1.22

$$\int \frac{\sqrt{a+cx^2}}{x^2\left(d+ex+fx^2\right)}\,dx =$$

$$-\frac{d\sqrt{a+cx^2}-\sqrt{a}ex\log(x)+\sqrt{a}ex\log\left(-\sqrt{a}+\sqrt{a+cx^2}\right)+x\text{RootSum}\left[c^2d+2\sqrt{a}ce\#1-2cd\#1^2+\right.}{}$$

```
[In] Integrate[Sqrt[a + c*x^2]/(x^2*(d + e*x + f*x^2)),x]
```

```
[Out] -((d*Sqrt[a + c*x^2] - Sqrt[a]*e*x*Log[x] + Sqrt[a]*e*x*Log[-Sqrt[a] + Sqrt
[a + c*x^2]] + x*RootSum[c^2*d + 2*Sqrt[a]*c*e*#1 - 2*c*d*#1^2 + 4*a*f*#1^2
 - 2*Sqrt[a]*e*#1^3 + d*#1^4 & , (-(c^2*d^2*Log[x]) - a*c*e^2*Log[x] + a*c*
d*f*Log[x] + c^2*d^2*Log[-Sqrt[a] + Sqrt[a + c*x^2] - x*#1] + a*c*e^2*Log[-
Sqrt[a] + Sqrt[a + c*x^2] - x*#1] - a*c*d*f*Log[-Sqrt[a] + Sqrt[a + c*x^2]
- x*#1] - 2*a^(3/2)*e*f*Log[x]*#1 + 2*a^(3/2)*e*f*Log[-Sqrt[a] + Sqrt[a + c
*x^2] - x*#1]*#1 + c*d^2*Log[x]*#1^2 + a*e^2*Log[x]*#1^2 - a*d*f*Log[x]*#1^
2 - c*d^2*Log[-Sqrt[a] + Sqrt[a + c*x^2] - x*#1]*#1^2 - a*e^2*Log[-Sqrt[a]
+ Sqrt[a + c*x^2] - x*#1]*#1^2 + a*d*f*Log[-Sqrt[a] + Sqrt[a + c*x^2] - x*#
1]*#1^2)/(-(Sqrt[a]*c*e) + 2*c*d*#1 - 4*a*f*#1 + 3*Sqrt[a]*e*#1^2 - 2*d*#1^
3) & ])/(d^2*x))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 775 vs. $2(337) = 674$.

Time = 0.76 (sec) , antiderivative size = 776, normalized size of antiderivative = 2.03

method	result
risch	$-\frac{\sqrt{c\,x^2+a}}{dx}-\frac{\frac{4f\sqrt{a}\,e\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c\,x^2+a}}{x}\right)}{\left(-e+\sqrt{-4df+e^2}\right)\left(e+\sqrt{-4df+e^2}\right)}-\frac{\left(fa\sqrt{-4df+e^2}-cd\sqrt{-4df+e^2}+aef+cde\right)\sqrt{2}\ln\left(\frac{-\sqrt{-4df+e^2}\,ce+2a\,f^2-2cdf+c\,e^2}{f^2}-\ldots\right.}{}}{}$
default	Expression too large to display

```
[In] int((c*x^2+a)^(1/2)/x^2/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -(c*x^2+a)^(1/2)/d/x-1/d*(4*f*a^(1/2)*e/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+
e^2)^(1/2))*ln((2*a+2*a^(1/2)*(c*x^2+a)^(1/2))/x)-(f*a*(-4*d*f+e^2)^(1/2)-c
*d*(-4*d*f+e^2)^(1/2)+a*e*f+c*d*e)/(-4*d*f+e^2)^(1/2)/(-e+(-4*d*f+e^2)^(1/2
))*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((
(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e-(-4*d*f+e^2)^(1/2)
)/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e
+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2*c
-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+2*(-(-4*d*f
+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2/f*(-e+(-4*d*f+e^2
)^(1/2))))+(f*a*(-4*d*f+e^2)^(1/2)-c*d*(-4*d*f+e^2)^(1/2)-a*e*f-c*d*e)/(-4*
d*f+e^2)^(1/2)/(e+(-4*d*f+e^2)^(1/2))*2^(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*
f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c
*e^2)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2
^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2
*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d
*f+e^2)^(1/2))/f)+2*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/
2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2656 vs. $2(335) = 670$.

Time = 176.36 (sec) , antiderivative size = 5324, normalized size of antiderivative = 13.94

$$\int \frac{\sqrt{a+cx^2}}{x^2\left(d+ex+fx^2\right)}\,dx = \text{Too large to display}$$

```
[In] integrate((c*x^2+a)^(1/2)/x^2/(f*x^2+e*x+d),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F]

$$\int \frac{\sqrt{a+cx^2}}{x^2\left(d+ex+fx^2\right)}\,dx = \int \frac{\sqrt{a+cx^2}}{x^2\left(d+ex+fx^2\right)}\,dx$$

```
[In] integrate((c*x**2+a)**(1/2)/x**2/(f*x**2+e*x+d),x)
[Out] Integral(sqrt(a + c*x**2)/(x**2*(d + e*x + f*x**2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+cx^2}}{x^2\,(d+ex+fx^2)}\,dx = \int \frac{\sqrt{cx^2+a}}{(fx^2+ex+d)x^2}\,dx$$

```
[In] integrate((c*x^2+a)^(1/2)/x^2/(f*x^2+e*x+d),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^2 + a)/((f*x^2 + e*x + d)*x^2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+cx^2}}{x^2\,(d+ex+fx^2)}\,dx = \text{Exception raised: AttributeError}$$

```
[In] integrate((c*x^2+a)^(1/2)/x^2/(f*x^2+e*x+d),x, algorithm="giac")
[Out] Exception raised: AttributeError >> type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+cx^2}}{x^2\,(d+ex+fx^2)}\,dx = \int \frac{\sqrt{c\,x^2+a}}{x^2\,\left(f\,x^2+e\,x+d\right)}\,dx$$

```
[In] int((a + c*x^2)^(1/2)/(x^2*(d + e*x + f*x^2)),x)
[Out] int((a + c*x^2)^(1/2)/(x^2*(d + e*x + f*x^2)), x)
```

3.57 $\int \frac{\sqrt{a+cx^2}}{x^3(d+ex+fx^2)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 507

$$\int \frac{\sqrt{a+cx^2}}{x^3(d+ex+fx^2)}\,dx = -\frac{\sqrt{a+cx^2}}{2dx^2}+\frac{e\sqrt{a+cx^2}}{d^2x} + \frac{f\left(cd^2\left(e+\sqrt{e^2-4df}\right)+a\left(e^3-3def+e^2\sqrt{e^2-4df}-df\sqrt{e^2-4df}\right)\right)\operatorname{arctanh}\left(\frac{2af-c\left(e-\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}d^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}} - \frac{f\left(cd^2\left(e-\sqrt{e^2-4df}\right)+a\left(e^3-3def-e^2\sqrt{e^2-4df}+df\sqrt{e^2-4df}\right)\right)\operatorname{arctanh}\left(\frac{2af-c\left(e+\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}d^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}} - \frac{c\operatorname{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2\sqrt{a}d} - \frac{\sqrt{a}(e^2-df)\operatorname{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^3}$$

```
[Out] -1/2*c*arctanh((c*x^2+a)^(1/2)/a^(1/2))/d/a^(1/2)-(-d*f+e^2)*arctanh((c*x^2
+a)^(1/2)/a^(1/2))*a^(1/2)/d^3-1/2*(c*x^2+a)^(1/2)/d/x^2+e*(c*x^2+a)^(1/2)/
d^2/x+1/2*f*arctanh(1/2*(2*a*f-c*x*(e-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a
)^(1/2)/(2*a*f^2+c*(e^2-2*d*f-e*(-4*d*f+e^2)^(1/2)))^(1/2))*(c*d^2*(e+(-4*d
*f+e^2)^(1/2))+a*(e^3-3*d*e*f+e^2*(-4*d*f+e^2)^(1/2)-d*f*(-4*d*f+e^2)^(1/2)
))/d^3*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+c*(e^2-2*d*f-e*(-4*d*f+e^2)^(1/2
)))^(1/2)-1/2*f*arctanh(1/2*(2*a*f-c*x*(e+(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x
^2+a)^(1/2)/(2*a*f^2+c*(e^2-2*d*f+e*(-4*d*f+e^2)^(1/2)))^(1/2))*(c*d^2*(e-(
-4*d*f+e^2)^(1/2))+a*(e^3-3*d*e*f-e^2*(-4*d*f+e^2)^(1/2)+d*f*(-4*d*f+e^2)^(
1/2)))/d^3*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+c*(e^2-2*d*f+e*(-4*d*f+e^2)^
(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 507, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.481$, Rules used = {6860, 272, 43, 65, 214, 283, 223, 212, 52, 1034, 1094, 1048, 739}

$$\int \frac{\sqrt{a+cx^2}}{x^3\left(d+ex+fx^2\right)}\,dx = -\frac{\sqrt{a}(e^2-df)\operatorname{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^3}$$

$$+\frac{f\left(a\left(e^2\sqrt{e^2-4df}-df\sqrt{e^2-4df}-3def+e^3\right)+cd^2\left(\sqrt{e^2-4df}+e\right)\right)\operatorname{arctanh}\left(\frac{2af-cx\left(e-\sqrt{e^2-\cdots}\right.}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e\cdots}\right.}}\right.}{\sqrt{2}d^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}$$

$$-\frac{f\left(a\left(-e^2\sqrt{e^2-4df}+df\sqrt{e^2-4df}-3def+e^3\right)+cd^2\left(e-\sqrt{e^2-4df}\right)\right)\operatorname{arctanh}\left(\frac{2af-cx\left(\sqrt{e^2-\cdots}\right.}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{\cdots}\right.}}\right.}{\sqrt{2}d^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}$$

$$-\frac{c\operatorname{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2\sqrt{a}d}+\frac{e\sqrt{a+cx^2}}{d^2x}-\frac{\sqrt{a+cx^2}}{2dx^2}$$

```
[In] Int[Sqrt[a + c*x^2]/(x^3*(d + e*x + f*x^2)),x]
```

```
[Out] -1/2*Sqrt[a + c*x^2]/(d*x^2) + (e*Sqrt[a + c*x^2])/(d^2*x) + (f*(c*d^2*(e +
 Sqrt[e^2 - 4*d*f]) + a*(e^3 - 3*d*e*f + e^2*Sqrt[e^2 - 4*d*f] - d*f*Sqrt[e
^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2
*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])]])/(Sqrt[2]
*d^3*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])
]) - (f*(c*d^2*(e - Sqrt[e^2 - 4*d*f]) + a*(e^3 - 3*d*e*f - e^2*Sqrt[e^2 -
4*d*f] + d*f*Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])
*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a +
 c*x^2])]])/(Sqrt[2]*d^3*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e
*Sqrt[e^2 - 4*d*f])]) - (c*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(2*Sqrt[a]*d)
- (Sqrt[a]*(e^2 - d*f)*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/d^3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
```

```
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
```

```
[{a, c, d, e}, x]
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[h*(a + c*x^2)^p*((d + e*x + f*x^2)^(q +
 1)/(2*f*(p + q + 1))), x] + Dist[1/(2*f*(p + q + 1)), Int[(a + c*x^2)^(p -
 1)*(d + e*x + f*x^2)^q*Simp[a*h*e*p - a*(h*e - 2*g*f)*(p + q + 1) - 2*h*p*
(c*d - a*f)*x - (h*c*e*p + c*(h*e - 2*g*f)*(p + q + 1))*x^2, x], x], x] /;
FreeQ[{a, c, d, e, f, g, h, q}, x] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && N
eQ[p + q + 1, 0]
```

Rule 1048

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1094

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + f*x^2],
 x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt
[d + f*x^2]), x], x] /; FreeQ[{a, b, c, d, f, A, B, C}, x] && NeQ[b^2 - 4*a
*c, 0]
```

Rule 6860

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{\sqrt{a+cx^2}}{dx^3} - \frac{e\sqrt{a+cx^2}}{d^2x^2} + \frac{(e^2-df)\sqrt{a+cx^2}}{d^3x} + \frac{(-e(e^2-2df)-f(e^2-df)x)\sqrt{a+cx^2}}{d^3(d+ex+fx^2)}\right) dx$$

$$= \frac{\int \frac{(-e(e^2-2df)-f(e^2-df)x)\sqrt{a+cx^2}}{d+ex+fx^2}\,dx}{d^3} + \frac{\int \frac{\sqrt{a+cx^2}}{x^3}\,dx}{d} - \frac{e\int \frac{\sqrt{a+cx^2}}{x^2}\,dx}{d^2} + \frac{(e^2-df)\int \frac{\sqrt{a+cx^2}}{x}\,dx}{d^3}$$

$$= -\frac{(e^2-df)\sqrt{a+cx^2}}{d^3} + \frac{e\sqrt{a+cx^2}}{d^2x} + \frac{\text{Subst}\left(\int \frac{\sqrt{a+cx}}{x^2}\,dx, x, x^2\right)}{2d} - \frac{(ce)\int \frac{1}{\sqrt{a+cx^2}}\,dx}{d^2} + \frac{\int \frac{-aef(e^2-2df)+f(cd-af)(e^2-df)x+cdef^2x^2}{\sqrt{a+cx^2}(d+ex+fx^2)}\,dx}{d^3f} + \frac{(e^2-df)\,\text{Subst}\left(\int \frac{\sqrt{a+cx}}{x}\,dx, x, x^2\right)}{2d^3}$$

$$= -\frac{\sqrt{a+cx^2}}{2dx^2} + \frac{e\sqrt{a+cx^2}}{d^2x} + \frac{c\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx, x, x^2\right)}{4d} + \frac{(ce)\int \frac{1}{\sqrt{a+cx^2}}\,dx}{d^2} - \frac{(ce)\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{d^2} + \frac{\int \frac{-cd^2ef^2-aef^2(e^2-2df)+(-cde^2f^2+f^2(cd-af)(e^2-df))x}{\sqrt{a+cx^2}(d+ex+fx^2)}\,dx}{d^3f^2} + \frac{(a(e^2-df))\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx, x, x^2\right)}{2d^3}$$

$$= -\frac{\sqrt{a+cx^2}}{2dx^2} + \frac{e\sqrt{a+cx^2}}{d^2x} - \frac{\sqrt{c}e\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{d^2} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx, x, \sqrt{a+cx^2}\right)}{2d} + \frac{(ce)\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{d^2} + \frac{(a(e^2-df))\,\text{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx, x, \sqrt{a+cx^2}\right)}{cd^3} - \frac{\left(f\left(cd^2\left(e+\sqrt{e^2-4df}\right)+a\left(e^3-3def+e^2\sqrt{e^2-4df}-df\sqrt{e^2-4df}\right)\right)\right)\int \frac{1}{\left(e-\sqrt{e^2-4df}+2fx\right)\sqrt{a+cx}}}{d^3\sqrt{e^2-4df}} + \frac{\left(f\left(cd^2\left(e-\sqrt{e^2-4df}\right)+a\left(e^3-3def-e^2\sqrt{e^2-4df}+df\sqrt{e^2-4df}\right)\right)\right)\int \frac{1}{\left(e+\sqrt{e^2-4df}+2fx\right)\sqrt{a+cx}}}{d^3\sqrt{e^2-4df}}$$

$$= -\frac{\sqrt{a+cx^2}}{2dx^2} + \frac{e\sqrt{a+cx^2}}{d^2x} - \frac{c\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2\sqrt{a}d} - \frac{\sqrt{a}(e^2-df)\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^3} + \frac{\left(f\left(cd^2\left(e+\sqrt{e^2-4df}\right)+a\left(e^3-3def+e^2\sqrt{e^2-4df}-df\sqrt{e^2-4df}\right)\right)\right)\text{Subst}\left(\int \frac{1}{4af^2+c\left(e-\sqrt{e^2-\right.}}\right.}{d^3\sqrt{e^2-4df}} - \frac{\left(f\left(cd^2\left(e-\sqrt{e^2-4df}\right)+a\left(e^3-3def-e^2\sqrt{e^2-4df}+df\sqrt{e^2-4df}\right)\right)\right)\text{Subst}\left(\int \frac{1}{4af^2+c\left(e+\sqrt{e^2-\right.}}\right.}{d^3\sqrt{e^2-4df}}$$

$$= -\frac{\sqrt{a+cx^2}}{2dx^2} + \frac{e\sqrt{a+cx^2}}{d^2x}$$

$$+ \frac{f\left(cd^2\left(e+\sqrt{e^2-4df}\right) + a\left(e^3-3def+e^2\sqrt{e^2-4df}-df\sqrt{e^2-4df}\right)\right)\tanh^{-1}\left(\frac{2af-c\left(e\right.}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2\right.}}\right.}{\sqrt{2}d^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}}$$

$$- \frac{f\left(cd^2\left(e-\sqrt{e^2-4df}\right) + a\left(e^3-3def-e^2\sqrt{e^2-4df}+df\sqrt{e^2-4df}\right)\right)\tanh^{-1}\left(\frac{2af-c\left(e\right.}{\sqrt{2}\sqrt{2af^2+c\left(e^2-\right.}}\right.}{\sqrt{2}d^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}}$$

$$- \frac{c\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2\sqrt{a}d} - \frac{\sqrt{a}(e^2-df)\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^3}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.79 (sec) , antiderivative size = 533, normalized size of antiderivative = 1.05

$$\int \frac{\sqrt{a+cx^2}}{x^3\left(d+ex+fx^2\right)}\,dx$$

$$= \frac{\frac{d(-d+2ex)\sqrt{a+cx^2}}{x^2} + \frac{2cd^2\operatorname{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}} - 4\sqrt{a}(e^2-df)\operatorname{arctanh}\left(\frac{-\sqrt{c}x+\sqrt{a+cx^2}}{\sqrt{a}}\right) - 2\text{RootSum}\left[a^2f+2\right.}{}$$

```
[In] Integrate[Sqrt[a + c*x^2]/(x^3*(d + e*x + f*x^2)),x]
```

```
[Out] ((d*(-d + 2*e*x)*Sqrt[a + c*x^2])/x^2 + (2*c*d^2*ArcTanh[(Sqrt[c]*x - Sqrt[
a + c*x^2])/Sqrt[a]])/Sqrt[a] - 4*Sqrt[a]*(e^2 - d*f)*ArcTanh[(-(Sqrt[c]*x)
 + Sqrt[a + c*x^2])/Sqrt[a]] - 2*RootSum[a^2*f + 2*a*Sqrt[c]*e*#1 + 4*c*d*#
1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (-(a*c*d^2*f*Log[-(Sqrt[c]
*x) + Sqrt[a + c*x^2] - #1]) - a^2*e^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]
 - #1] + a^2*d*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] - 2*c^(3/2)*d^2
*e*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 - 2*a*Sqrt[c]*e^3*Log[-(Sqrt
[c]*x) + Sqrt[a + c*x^2] - #1]*#1 + 4*a*Sqrt[c]*d*e*f*Log[-(Sqrt[c]*x) + Sq
rt[a + c*x^2] - #1]*#1 + c*d^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#
1^2 + a*e^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2 - a*d*f^2*Log[-
(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2)/(a*Sqrt[c]*e + 4*c*d*#1 - 2*a*f*#
1 - 3*Sqrt[c]*e*#1^2 + 2*f*#1^3) & ])/(2*d^3)
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 789, normalized size of antiderivative = 1.56

method	result
risch	$-\frac{\sqrt{c\,x^2+a}\,(-2ex+d)}{2d^2x^2}-\frac{\frac{4f\left(2adf-2e^2a-c\,d^2\right)\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c\,x^2+a}}{x}\right)}{\left(-e+\sqrt{-4df+e^2}\right)\left(e+\sqrt{-4df+e^2}\right)\sqrt{a}}+\frac{2f\left(\sqrt{-4df+e^2}\,ae-2adf+e^2a+2c\,d^2\right)\sqrt{2}\,\ln\left(\frac{-\sqrt{-4df+e^2}\,ce+2a\,f^2-\cdots}{f^2}\cdots\right.}{\cdots}}{\cdots}$
default	Expression too large to display

```
[In] int((c*x^2+a)^(1/2)/x^3/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(c*x^2+a)^(1/2)*(-2*e*x+d)/d^2/x^2-1/2/d^2*(4*f*(2*a*d*f-2*a*e^2-c*d^2
)/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))/a^(1/2)*ln((2*a+2*a^(1/2)*
(c*x^2+a)^(1/2))/x)+2*f*((-4*d*f+e^2)^(1/2)*a*e-2*a*d*f+e^2*a+2*c*d^2)/(-4*
d*f+e^2)^(1/2)/(-e+(-4*d*f+e^2)^(1/2))*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*
a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*
f+c*e^2)/f^2-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+1
/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(
x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2/f*
(-e+(-4*d*f+e^2)^(1/2)))+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/
f^2)^(1/2))/(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2))))-2*f*((-4*d*f+e^2)^(1/2)*a*e+
2*a*d*f-e^2*a-2*c*d^2)/(-4*d*f+e^2)^(1/2)/(e+(-4*d*f+e^2)^(1/2))*2^(1/2)/((
(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f+e^2)^
(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(
-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+
c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^
2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*(((-4*d*f+e^2)^(1/2)*c*e+2*a*
f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+cx^2}}{x^3\,(d+ex+fx^2)}\,dx = \text{Timed out}$$

```
[In] integrate((c*x^2+a)^(1/2)/x^3/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+cx^2}}{x^3(d+ex+fx^2)}\,dx = \int \frac{\sqrt{a+cx^2}}{x^3(d+ex+fx^2)}\,dx$$

```
[In] integrate((c*x**2+a)**(1/2)/x**3/(f*x**2+e*x+d),x)
[Out] Integral(sqrt(a + c*x**2)/(x**3*(d + e*x + f*x**2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+cx^2}}{x^3(d+ex+fx^2)}\,dx = \int \frac{\sqrt{cx^2+a}}{(fx^2+ex+d)x^3}\,dx$$

```
[In] integrate((c*x^2+a)^(1/2)/x^3/(f*x^2+e*x+d),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^2 + a)/((f*x^2 + e*x + d)*x^3), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+cx^2}}{x^3(d+ex+fx^2)}\,dx = \text{Exception raised: AttributeError}$$

```
[In] integrate((c*x^2+a)^(1/2)/x^3/(f*x^2+e*x+d),x, algorithm="giac")
[Out] Exception raised: AttributeError >> type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+cx^2}}{x^3(d+ex+fx^2)}\,dx = \int \frac{\sqrt{c\,x^2+a}}{x^3\,\left(f\,x^2+e\,x+d\right)}\,dx$$

```
[In] int((a + c*x^2)^(1/2)/(x^3*(d + e*x + f*x^2)),x)
[Out] int((a + c*x^2)^(1/2)/(x^3*(d + e*x + f*x^2)), x)
```

3.58 $\int \frac{x^2(a+cx^2)^{3/2}}{d+ex+fx^2}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 795

$$\int \frac{x^2(a+cx^2)^{3/2}}{d+ex+fx^2}\,dx = -\frac{(8e(af^2+c(e^2-2df))-f(3af^2+4c(e^2-df))x)\sqrt{a+cx^2}}{8f^4}$$
$$-\frac{(4e-3fx)(a+cx^2)^{3/2}}{12f^2}$$
$$+\frac{(3a^2f^4+12acf^2(e^2-df)+8c^2(e^4-3de^2f+d^2f^2))\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{8\sqrt{c}f^5}$$
$$-\frac{\left(a^2f^4\left(e^2-2df-e\sqrt{e^2-4df}\right)+2acf^2\left(e^4-4de^2f+2d^2f^2-e^3\sqrt{e^2-4df}+2def\sqrt{e^2-4df}\right)+c^2\left(e^6-6\right.\cdots}{\sqrt{2}f^5\sqrt{e^2-4df}\cdots}$$
$$+\frac{\left(a^2f^4\left(e^2-2df+e\sqrt{e^2-4df}\right)+2acf^2\left(e^4-4de^2f+2d^2f^2+e^3\sqrt{e^2-4df}-2def\sqrt{e^2-4df}\right)+c^2\left(e^6-6\right.\cdots}{\sqrt{2}f^5\sqrt{e^2-4df}\cdots}$$

```
[Out] -1/12*(-3*f*x+4*e)*(c*x^2+a)^(3/2)/f^2+1/8*(3*a^2*f^4+12*a*c*f^2*(-d*f+e^2)
+8*c^2*(d^2*f^2-3*d*e^2*f+e^4))*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/f^5/c^(1
/2)-1/8*(8*e*(a*f^2+c*(-2*d*f+e^2))-f*(3*a*f^2+4*c*(-d*f+e^2))*x)*(c*x^2+a)
^(1/2)/f^4-1/2*arctanh(1/2*(2*a*f-c*x*(e-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^
2+a)^(1/2)/(2*a*f^2+c*(e^2-2*d*f-e*(-4*d*f+e^2)^(1/2)))^(1/2))*(a^2*f^4*(e^
2-2*d*f-e*(-4*d*f+e^2)^(1/2))+2*a*c*f^2*(e^4-4*d*e^2*f+2*d^2*f^2-e^3*(-4*d*
f+e^2)^(1/2)+2*d*e*f*(-4*d*f+e^2)^(1/2))+c^2*(e^6-6*d*e^4*f+9*d^2*e^2*f^2-2
*d^3*f^3-e^5*(-4*d*f+e^2)^(1/2)+4*d*e^3*f*(-4*d*f+e^2)^(1/2)-3*d^2*e*f^2*(-
```

```
4*d*f+e^2)^(1/2)))/f^5*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+c*(e^2-2*d*f-e*(
-4*d*f+e^2)^(1/2)))^(1/2)+1/2*arctanh(1/2*(2*a*f-c*x*(e+(-4*d*f+e^2)^(1/2))
)*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+c*(e^2-2*d*f+e*(-4*d*f+e^2)^(1/2)))^(1/2
))*(a^2*f^4*(e^2-2*d*f+e*(-4*d*f+e^2)^(1/2))+2*a*c*f^2*(e^4-4*d*e^2*f+2*d^2
*f^2+e^3*(-4*d*f+e^2)^(1/2)-2*d*e*f*(-4*d*f+e^2)^(1/2))+c^2*(e^6-6*d*e^4*f+
9*d^2*e^2*f^2-2*d^3*f^3+e^5*(-4*d*f+e^2)^(1/2)-4*d*e^3*f*(-4*d*f+e^2)^(1/2)
+3*d^2*e*f^2*(-4*d*f+e^2)^(1/2)))/f^5*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+c
*(e^2-2*d*f+e*(-4*d*f+e^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 2.50 (sec) , antiderivative size = 795, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {1083, 1082, 1094, 223, 212, 1048, 739}

$$\int \frac{x^2(a+cx^2)^{3/2}}{d+ex+fx^2}\,dx = -\frac{(4e-3fx)\,(cx^2+a)^{3/2}}{12f^2}$$

$$-\frac{(8e(af^2+c(e^2-2df))-f(3af^2+4c(e^2-df))\,x)\sqrt{cx^2+a}}{8f^4}$$

$$+\frac{(3a^2f^4+12ac(e^2-df)\,f^2+8c^2(e^4-3dfe^2+d^2f^2))\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{cx^2+a}}\right)}{8\sqrt{c}f^5}$$

$$-\frac{\left(a^2\left(e^2-\sqrt{e^2-4df}e-2df\right)f^4+2ac\left(e^4-\sqrt{e^2-4df}e^3-4dfe^2+2df\sqrt{e^2-4df}e+2d^2f^2\right)f^2+c^2\left(e^6-\right.\right.}{\sqrt{2}f^5\sqrt{e^2-4d}}$$

$$+\frac{\left(a^2\left(e^2+\sqrt{e^2-4df}e-2df\right)f^4+2ac\left(e^4+\sqrt{e^2-4df}e^3-4dfe^2-2df\sqrt{e^2-4df}e+2d^2f^2\right)f^2+c^2\left(e^6+\right.\right.}{\sqrt{2}f^5\sqrt{e^2-4df}}$$

```
[In] Int[(x^2*(a + c*x^2)^(3/2))/(d + e*x + f*x^2),x]
```

```
[Out] -1/8*((8*e*(a*f^2 + c*(e^2 - 2*d*f)) - f*(3*a*f^2 + 4*c*(e^2 - d*f))*x)*Sqr
t[a + c*x^2])/f^4 - ((4*e - 3*f*x)*(a + c*x^2)^(3/2))/(12*f^2) + ((3*a^2*f^
4 + 12*a*c*f^2*(e^2 - d*f) + 8*c^2*(e^4 - 3*d*e^2*f + d^2*f^2))*ArcTanh[(Sq
rt[c]*x)/Sqrt[a + c*x^2]])/(8*Sqrt[c]*f^5) - ((a^2*f^4*(e^2 - 2*d*f - e*Sqr
t[e^2 - 4*d*f]) + 2*a*c*f^2*(e^4 - 4*d*e^2*f + 2*d^2*f^2 - e^3*Sqrt[e^2 - 4
*d*f] + 2*d*e*f*Sqrt[e^2 - 4*d*f]) + c^2*(e^6 - 6*d*e^4*f + 9*d^2*e^2*f^2 -
 2*d^3*f^3 - e^5*Sqrt[e^2 - 4*d*f] + 4*d*e^3*f*Sqrt[e^2 - 4*d*f] - 3*d^2*e*
f^2*Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt
[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])]
)/(Sqrt[2]*f^5*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2
```

```
 - 4*d*f])]) + ((a^2*f^4*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + 2*a*c*f^2*(e
^4 - 4*d*e^2*f + 2*d^2*f^2 + e^3*Sqrt[e^2 - 4*d*f] - 2*d*e*f*Sqrt[e^2 - 4*d
*f]) + c^2*(e^6 - 6*d*e^4*f + 9*d^2*e^2*f^2 - 2*d^3*f^3 + e^5*Sqrt[e^2 - 4*
d*f] - 4*d*e^3*f*Sqrt[e^2 - 4*d*f] + 3*d^2*e*f^2*Sqrt[e^2 - 4*d*f]))*ArcTan
h[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*
d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*f^5*Sqrt[e^2 - 4*d*
f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1048

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1082

```
Int[((a_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)*((d_) +
(e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(B*c*f*(2*p + 2*q + 3) +
 C*((-c)*e*(2*p + q + 2)) + 2*c*C*f*(p + q + 1)*x)*(a + c*x^2)^p*((d + e*x
+ f*x^2)^(q + 1)/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3))), x] - Dist[1/(2*c*f
^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + c*x^2)^(p - 1)*(d + e*x + f*x^2)^
q*Simp[p*((-a)*e)*(C*(c*e)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p +
q + 1)*(a*c*(C*(2*d*f - e^2*(2*p + q + 2)) + f*(B*e - 2*A*f)*(2*p + 2*q + 3
))) + (2*p*(c*d - a*f)*(C*(c*e)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) +
(p + q + 1)*(C*e*f*p*(-4*a*c)))*x + (p*(c*e)*(C*(c*e)*(q + 1) - c*(C*e - B*
f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(-4*a*c) - c^2*(C*(e^2 - 4*d*f)*
(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3))))*x^2, x], x], x]
/; FreeQ[{a, c, d, e, f, A, B, C, q}, x] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0
```

```
] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*q + 3, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q
, 0]
```

Rule 1083

```
Int[((a_) + (c_.)*(x_)^2)^(p_)*((A_.) + (C_.)*(x_)^2)*((d_) + (e_.)*(x_) +
(f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(C*((-c)*e*(2*p + q + 2)) + 2*c*C*f*(
p + q + 1)*x)*(a + c*x^2)^p*((d + e*x + f*x^2)^(q + 1)/(2*c*f^2*(p + q + 1)
*(2*p + 2*q + 3))), x] - Dist[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), Int[
(a + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[p*((-a)*e)*(C*(c*e)*(q + 1) -
c*(C*e)*(2*p + 2*q + 3)) + (p + q + 1)*(a*c*(C*(2*d*f - e^2*(2*p + q + 2))
+ f*(-2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)*(C*(c*e)*(q + 1) - c*(C*e
)*(2*p + 2*q + 3)) + (p + q + 1)*(C*e*f*p*(-4*a*c)))*x + (p*(c*e)*(C*(c*e)*
(q + 1) - c*(C*e)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(-4*a*c) - c^2*(C
*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d + 2*A*f)*(2*p + 2*q + 3))))*x^2, x]
, x], x] /; FreeQ[{a, c, d, e, f, A, C, q}, x] && NeQ[e^2 - 4*d*f, 0] && Gt
Q[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*q + 3, 0] &&  !IGtQ[p, 0] &&  !
IGtQ[q, 0]
```

Rule 1094

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + f*x^2],
 x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt
[d + f*x^2]), x], x] /; FreeQ[{a, b, c, d, f, A, B, C}, x] && NeQ[b^2 - 4*a
*c, 0]
```

Rubi steps

$$\text{integral} = -\frac{(4e-3fx)\left(a+cx^2\right)^{3/2}}{12f^2} - \frac{\int \frac{\sqrt{a+cx^2}\left(3acdf-3ce(4cd-af)x-3c\left(3af^2+4c\left(e^2-df\right)\right)x^2\right)}{d+ex+fx^2}\,dx}{12cf^2}$$

$$= -\frac{\left(8e\left(af^2+c\left(e^2-2df\right)\right)-f\left(3af^2+4c\left(e^2-df\right)\right)x\right)\sqrt{a+cx^2}}{8f^4} - \frac{(4e-3fx)\left(a+cx^2\right)^{3/2}}{12f^2} + \frac{\int \frac{-3ac^2df\left(5af^2+4c\left(e^2-df\right)\right)-3c^2e\left(5a^2f^3+4acf\left(e^2-5df\right)-8c^2d\left(e^2-2df\right)\right)x+3c^2\left(3a^2f^4+12acf^2\left(e^2-df\right)+8c^2\left(e^4-3de^2f+d^2f\right.\right.}{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}}{24c^2f^4}$$

$$= -\frac{(8e(af^2+c(e^2-2df)) - f(3af^2+4c(e^2-df))x)\sqrt{a+cx^2}}{8f^4}$$
$$-\frac{(4e-3fx)(a+cx^2)^{3/2}}{12f^2}$$
$$+\frac{\int \frac{-3ac^2df^2(5af^2+4c(e^2-df))-3c^2d(3a^2f^4+12acf^2(e^2-df)+8c^2(e^4-3de^2f+d^2f^2))+(-3c^2ef(5a^2f^3+4acf(e^2-5df)-8c^2d(e^2\cdots}{\sqrt{a+cx^2}(d+ex+fx^2)}}{24c^2f^5}$$
$$+\frac{(3a^2f^4+12acf^2(e^2-df)+8c^2(e^4-3de^2f+d^2f^2))\int \frac{1}{\sqrt{a+cx^2}}\,dx}{8f^5}$$

$$= -\frac{(8e(af^2+c(e^2-2df)) - f(3af^2+4c(e^2-df))x)\sqrt{a+cx^2}}{8f^4} - \frac{(4e-3fx)(a+cx^2)^{3/2}}{12f^2}$$
$$+\frac{(3a^2f^4+12acf^2(e^2-df)+8c^2(e^4-3de^2f+d^2f^2))\,\mathrm{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{8f^5}$$
$$+\frac{\left(a^2f^4\left(e^2-2df-e\sqrt{e^2-4df}\right)+2acf^2\left(e^4-4de^2f+2d^2f^2-e^3\sqrt{e^2-4df}+2def\sqrt{e^2-4df}\right)+\cdots\right.}{\cdots}$$
$$-\frac{\left(a^2f^4\left(e^2-2df+e\sqrt{e^2-4df}\right)+2acf^2\left(e^4-4de^2f+2d^2f^2+e^3\sqrt{e^2-4df}-2def\sqrt{e^2-4df}\right)-\cdots\right.}{\cdots}$$

$$= -\frac{(8e(af^2+c(e^2-2df)) - f(3af^2+4c(e^2-df))x)\sqrt{a+cx^2}}{8f^4}$$
$$-\frac{(4e-3fx)(a+cx^2)^{3/2}}{12f^2}+\frac{(3a^2f^4+12acf^2(e^2-df)+8c^2(e^4-3de^2f+d^2f^2))\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{8\sqrt{c}f^5}$$
$$-\frac{\left(a^2f^4\left(e^2-2df-e\sqrt{e^2-4df}\right)+2acf^2\left(e^4-4de^2f+2d^2f^2-e^3\sqrt{e^2-4df}+2def\sqrt{e^2-4df}\right)-\cdots\right.}{\cdots}$$
$$+\frac{\left(a^2f^4\left(e^2-2df+e\sqrt{e^2-4df}\right)+2acf^2\left(e^4-4de^2f+2d^2f^2+e^3\sqrt{e^2-4df}-2def\sqrt{e^2-4df}\right)+\cdots\right.}{\cdots}$$

$$= -\frac{(8e(af^2+c(e^2-2df))-f(3af^2+4c(e^2-df))x)\sqrt{a+cx^2}}{8f^4} - \frac{(4e-3fx)(a+cx^2)^{3/2}}{12f^2} + \frac{(3a^2f^4+12acf^2(e^2-df)+8c^2(e^4-3de^2f+d^2f^2))\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right.}{8\sqrt{c}f^5}$$

$$-\frac{\left(a^2f^4\left(e^2-2df-e\sqrt{e^2-4df}\right)+2acf^2\left(e^4-4de^2f+2d^2f^2-e^3\sqrt{e^2-4df}+2def\sqrt{e^2-4df}\right)\right.}{\sqrt{2}f}$$

$$+\frac{\left(a^2f^4\left(e^2-2df+e\sqrt{e^2-4df}\right)+2acf^2\left(e^4-4de^2f+2d^2f^2+e^3\sqrt{e^2-4df}-2def\sqrt{e^2-4df}\right)\right.}{\sqrt{2}f}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 1.82 (sec) , antiderivative size = 1239, normalized size of antiderivative = 1.56

$$\int \frac{x^2(a+cx^2)^{3/2}}{d+ex+fx^2}\,dx = \frac{f\sqrt{a+cx^2}(af^2(-32e+15fx)-2c(12e^3-6e^2fx+4ef(-6d+fx^2)-3f^2x(-2d+}{}$$

```
[In] Integrate[(x^2*(a + c*x^2)^(3/2))/(d + e*x + f*x^2),x]
```

```
[Out] (f*Sqrt[a + c*x^2]*(a*f^2*(-32*e + 15*f*x) - 2*c*(12*e^3 - 6*e^2*f*x + 4*e*
f*(-6*d + f*x^2) - 3*f^2*x*(-2*d + f*x^2))) + (6*(3*a^2*f^4 + 12*a*c*f^2*(e
^2 - d*f) + 8*c^2*(e^4 - 3*d*e^2*f + d^2*f^2))*ArcTanh[(Sqrt[c]*x)/(-Sqrt[a
] + Sqrt[a + c*x^2])])/Sqrt[c] + 24*RootSum[c^2*d + 2*Sqrt[a]*c*e*#1 - 2*c*
d*#1^2 + 4*a*f*#1^2 - 2*Sqrt[a]*e*#1^3 + d*#1^4 & , (-(c^3*d*e^4*Log[x]) +
3*c^3*d^2*e^2*f*Log[x] - c^3*d^3*f^2*Log[x] - 2*a*c^2*d*e^2*f^2*Log[x] + 2*
a*c^2*d^2*f^3*Log[x] - a^2*c*d*f^4*Log[x] + c^3*d*e^4*Log[-Sqrt[a] + Sqrt[a
 + c*x^2] - x*#1] - 3*c^3*d^2*e^2*f*Log[-Sqrt[a] + Sqrt[a + c*x^2] - x*#1]
+ c^3*d^3*f^2*Log[-Sqrt[a] + Sqrt[a + c*x^2] - x*#1] + 2*a*c^2*d*e^2*f^2*Lo
g[-Sqrt[a] + Sqrt[a + c*x^2] - x*#1] - 2*a*c^2*d^2*f^3*Log[-Sqrt[a] + Sqrt[
a + c*x^2] - x*#1] + a^2*c*d*f^4*Log[-Sqrt[a] + Sqrt[a + c*x^2] - x*#1] - 2
*Sqrt[a]*c^2*e^5*Log[x]*#1 + 8*Sqrt[a]*c^2*d*e^3*f*Log[x]*#1 - 6*Sqrt[a]*c^
2*d^2*e*f^2*Log[x]*#1 - 4*a^(3/2)*c*e^3*f^2*Log[x]*#1 + 8*a^(3/2)*c*d*e*f^3
*Log[x]*#1 - 2*a^(5/2)*e*f^4*Log[x]*#1 + 2*Sqrt[a]*c^2*e^5*Log[-Sqrt[a] + S
qrt[a + c*x^2] - x*#1]*#1 - 8*Sqrt[a]*c^2*d*e^3*f*Log[-Sqrt[a] + Sqrt[a + c
*x^2] - x*#1]*#1 + 6*Sqrt[a]*c^2*d^2*e*f^2*Log[-Sqrt[a] + Sqrt[a + c*x^2] -
 x*#1]*#1 + 4*a^(3/2)*c*e^3*f^2*Log[-Sqrt[a] + Sqrt[a + c*x^2] - x*#1]*#1 -
```

```
 8*a^(3/2)*c*d*e*f^3*Log[-Sqrt[a] + Sqrt[a + c*x^2] - x*#1]*#1 + 2*a^(5/2)*
e*f^4*Log[-Sqrt[a] + Sqrt[a + c*x^2] - x*#1]*#1 + c^2*d*e^4*Log[x]*#1^2 - 3
*c^2*d^2*e^2*f*Log[x]*#1^2 + c^2*d^3*f^2*Log[x]*#1^2 + 2*a*c*d*e^2*f^2*Log[
x]*#1^2 - 2*a*c*d^2*f^3*Log[x]*#1^2 + a^2*d*f^4*Log[x]*#1^2 - c^2*d*e^4*Log
[-Sqrt[a] + Sqrt[a + c*x^2] - x*#1]*#1^2 + 3*c^2*d^2*e^2*f*Log[-Sqrt[a] + S
qrt[a + c*x^2] - x*#1]*#1^2 - c^2*d^3*f^2*Log[-Sqrt[a] + Sqrt[a + c*x^2] -
x*#1]*#1^2 - 2*a*c*d*e^2*f^2*Log[-Sqrt[a] + Sqrt[a + c*x^2] - x*#1]*#1^2 +
2*a*c*d^2*f^3*Log[-Sqrt[a] + Sqrt[a + c*x^2] - x*#1]*#1^2 - a^2*d*f^4*Log[-
Sqrt[a] + Sqrt[a + c*x^2] - x*#1]*#1^2)/(-(Sqrt[a]*c*e) + 2*c*d*#1 - 4*a*f*
#1 + 3*Sqrt[a]*e*#1^2 - 2*d*#1^3) & ])/(24*f^5)
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 1183, normalized size of antiderivative = 1.49

method	result	size
risch	Expression too large to display	1183
default	Expression too large to display	2367

```
[In] int(x^2*(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*(-6*c*f^3*x^3+8*c*e*f^2*x^2-15*a*f^3*x+12*c*d*f^2*x-12*c*e^2*f*x+32*a
*e*f^2-48*c*d*e*f+24*c*e^3)*(c*x^2+a)^(1/2)/f^4+1/8/f^4*(1/f*(3*a^2*f^4-12*
a*c*d*f^3+12*a*c*e^2*f^2+8*c^2*d^2*f^2-24*c^2*d*e^2*f+8*c^2*e^4)*ln(x*c^(1/
2)+(c*x^2+a)^(1/2))/c^(1/2)-1/2*(-8*e*f^4*a^2*(-4*d*f+e^2)^(1/2)+32*a*c*d*e
*f^3*(-4*d*f+e^2)^(1/2)-16*a*c*e^3*f^2*(-4*d*f+e^2)^(1/2)-24*c^2*d^2*e*f^2*
(-4*d*f+e^2)^(1/2)+32*c^2*d*e^3*f*(-4*d*f+e^2)^(1/2)-8*c^2*e^5*(-4*d*f+e^2)
^(1/2)+16*a^2*d*f^5-8*a^2*e^2*f^4-32*a*c*d^2*f^4+64*a*c*d*e^2*f^3-16*a*c*e^
4*f^2+16*c^2*d^3*f^3-72*c^2*d^2*e^2*f^2+48*c^2*d*e^4*f-8*c^2*e^6)/f^2/(-4*d
*f+e^2)^(1/2)*2^(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^
(1/2)*ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e
^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1
/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/
f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4
*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^
2)^(1/2))/f))-1/2*(-8*e*f^4*a^2*(-4*d*f+e^2)^(1/2)+32*a*c*d*e*f^3*(-4*d*f+e
^2)^(1/2)-16*a*c*e^3*f^2*(-4*d*f+e^2)^(1/2)-24*c^2*d^2*e*f^2*(-4*d*f+e^2)^(
1/2)+32*c^2*d*e^3*f*(-4*d*f+e^2)^(1/2)-8*c^2*e^5*(-4*d*f+e^2)^(1/2)-16*a^2*
d*f^5+8*a^2*e^2*f^4+32*a*c*d^2*f^4-64*a*c*d*e^2*f^3+16*a*c*e^4*f^2-16*c^2*d
^3*f^3+72*c^2*d^2*e^2*f^2-48*c^2*d*e^4*f+8*c^2*e^6)/f^2/(-4*d*f+e^2)^(1/2)*
2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-(
-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e-(-4*d*f+e^2)^(1/2))/f
*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+2*
a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2*c-4*
c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+2*(-(-4*d*f+e^
```

```
2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2/f*(-e+(-4*d*f+e^2)^(
1/2)))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^2(a+cx^2)^{3/2}}{d+ex+fx^2}\,dx = \text{Timed out}$$

```
[In] integrate(x^2*(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(a+cx^2)^{3/2}}{d+ex+fx^2}\,dx = \text{Timed out}$$

```
[In] integrate(x**2*(c*x**2+a)**(3/2)/(f*x**2+e*x+d),x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a+cx^2)^{3/2}}{d+ex+fx^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2*(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re deta
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^2(a+cx^2)^{3/2}}{d+ex+fx^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^2*(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+cx^2)^{3/2}}{d+ex+fx^2}\,dx = \int \frac{x^2\,{\left(c\,x^2+a\right)}^{3/2}}{f\,x^2+e\,x+d}\,dx$$

```
[In] int((x^2*(a + c*x^2)^(3/2))/(d + e*x + f*x^2),x)
```

```
[Out] int((x^2*(a + c*x^2)^(3/2))/(d + e*x + f*x^2), x)
```

3.59 $\int \frac{x\left(a+cx^2\right)^{3/2}}{d+ex+fx^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 553

$$\int \frac{x(a+cx^2)^{3/2}}{d+ex+fx^2}\,dx = \frac{(2(af^2+c(e^2-df))-cefx)\sqrt{a+cx^2}}{2f^3}$$
$$+\frac{(a+cx^2)^{3/2}}{3f}-\frac{\sqrt{c}e(3af^2+2c(e^2-2df))\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2f^4}$$
$$-\frac{\left(2cdef(2af^2+c(e^2-2df))-\left(e-\sqrt{e^2-4df}\right)\left(a^2f^4+2acf^2(e^2-df)+c^2(e^4-3de^2f+d^2f^2)\right)\right)\operatorname{arctan}}{\sqrt{2}f^4\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}}$$
$$+\frac{\left(2cdef(2af^2+c(e^2-2df))-\left(e+\sqrt{e^2-4df}\right)\left(a^2f^4+2acf^2(e^2-df)+c^2(e^4-3de^2f+d^2f^2)\right)\right)\operatorname{arctanh}}{\sqrt{2}f^4\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}}$$

```
[Out] 1/3*(c*x^2+a)^(3/2)/f-1/2*e*(3*a*f^2+2*c*(-2*d*f+e^2))*arctanh(x*c^(1/2)/(c
*x^2+a)^(1/2))*c^(1/2)/f^4+1/2*(2*a*f^2+2*c*(-d*f+e^2)-c*e*f*x)*(c*x^2+a)^(
1/2)/f^3-1/2*arctanh(1/2*(2*a*f-c*x*(e-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+
a)^(1/2)/(2*a*f^2+c*(e^2-2*d*f-e*(-4*d*f+e^2)^(1/2)))^(1/2))*(2*c*d*e*f*(2*
a*f^2+c*(-2*d*f+e^2))-(a^2*f^4+2*a*c*f^2*(-d*f+e^2)+c^2*(d^2*f^2-3*d*e^2*f+
e^4))*(e-(-4*d*f+e^2)^(1/2)))/f^4*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+c*(e^
2-2*d*f-e*(-4*d*f+e^2)^(1/2)))^(1/2)+1/2*arctanh(1/2*(2*a*f-c*x*(e+(-4*d*f+
e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+c*(e^2-2*d*f+e*(-4*d*f+e^2)^(
1/2)))^(1/2))*(2*c*d*e*f*(2*a*f^2+c*(-2*d*f+e^2))-(a^2*f^4+2*a*c*f^2*(-d*f+
e^2)+c^2*(d^2*f^2-3*d*e^2*f+e^4))*(e+(-4*d*f+e^2)^(1/2)))/f^4*2^(1/2)/(-4*d
*f+e^2)^(1/2)/(2*a*f^2+c*(e^2-2*d*f+e*(-4*d*f+e^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 1.47 (sec) , antiderivative size = 553, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {1034, 1082, 1094, 223, 212, 1048, 739}

$$\int \frac{x(a+cx^2)^{3/2}}{d+ex+fx^2}\,dx =$$

$$-\frac{\left(2cdef(2af^2+c(e^2-2df))-\left(e-\sqrt{e^2-4df}\right)(a^2f^4+2acf^2(e^2-df)+c^2(d^2f^2-3de^2f+e^4))\right)\text{arctanh}}{\sqrt{2}f^4\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}$$

$$+\frac{\left(2cdef(2af^2+c(e^2-2df))-\left(\sqrt{e^2-4df}+e\right)(a^2f^4+2acf^2(e^2-df)+c^2(d^2f^2-3de^2f+e^4))\right)\text{arctanh}}{\sqrt{2}f^4\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}$$

$$-\frac{\sqrt{c}e\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)(3af^2+2c(e^2-2df))}{2f^4}$$

$$+\frac{\sqrt{a+cx^2}(2(af^2+c(e^2-df))-cefx)}{2f^3}+\frac{(a+cx^2)^{3/2}}{3f}$$

```
[In] Int[(x*(a + c*x^2)^(3/2))/(d + e*x + f*x^2),x]
```

```
[Out] ((2*(a*f^2 + c*(e^2 - d*f)) - c*e*f*x)*Sqrt[a + c*x^2])/(2*f^3) + (a + c*x^
2)^(3/2)/(3*f) - (Sqrt[c]*e*(3*a*f^2 + 2*c*(e^2 - 2*d*f))*ArcTanh[(Sqrt[c]*
x)/Sqrt[a + c*x^2]])/(2*f^4) - ((2*c*d*e*f*(2*a*f^2 + c*(e^2 - 2*d*f)) - (e
 - Sqrt[e^2 - 4*d*f])*(a^2*f^4 + 2*a*c*f^2*(e^2 - d*f) + c^2*(e^4 - 3*d*e^2
*f + d^2*f^2)))*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt
[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[
2]*f^4*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f
])]) + ((2*c*d*e*f*(2*a*f^2 + c*(e^2 - 2*d*f)) - (e + Sqrt[e^2 - 4*d*f])*(a
^2*f^4 + 2*a*c*f^2*(e^2 - d*f) + c^2*(e^4 - 3*d*e^2*f + d^2*f^2)))*ArcTanh[
(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*
f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*f^4*Sqrt[e^2 - 4*d*f]
*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[h*(a + c*x^2)^p*((d + e*x + f*x^2)^(q +
 1)/(2*f*(p + q + 1))), x] + Dist[1/(2*f*(p + q + 1)), Int[(a + c*x^2)^(p -
 1)*(d + e*x + f*x^2)^q*Simp[a*h*e*p - a*(h*e - 2*g*f)*(p + q + 1) - 2*h*p*
(c*d - a*f)*x - (h*c*e*p + c*(h*e - 2*g*f)*(p + q + 1))*x^2, x], x], x] /;
FreeQ[{a, c, d, e, f, g, h, q}, x] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && N
eQ[p + q + 1, 0]
```

Rule 1048

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1082

```
Int[((a_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)*((d_) +
(e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(B*c*f*(2*p + 2*q + 3) +
 C*((-c)*e*(2*p + q + 2)) + 2*c*C*f*(p + q + 1)*x)*(a + c*x^2)^p*((d + e*x
+ f*x^2)^(q + 1)/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3))), x] - Dist[1/(2*c*f
^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + c*x^2)^(p - 1)*(d + e*x + f*x^2)^
q*Simp[p*((-a)*e)*(C*(c*e)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p +
q + 1)*(a*c*(C*(2*d*f - e^2*(2*p + q + 2)) + f*(B*e - 2*A*f)*(2*p + 2*q + 3
))) + (2*p*(c*d - a*f)*(C*(c*e)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) +
(p + q + 1)*(C*e*f*p*(-4*a*c)))*x + (p*(c*e)*(C*(c*e)*(q + 1) - c*(C*e - B*
f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(-4*a*c) - c^2*(C*(e^2 - 4*d*f)*
(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3))))*x^2, x], x], x]
/; FreeQ[{a, c, d, e, f, A, B, C, q}, x] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0
] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*q + 3, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q
, 0]
```

Rule 1094

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + f*x^2],
 x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt
[d + f*x^2]), x], x] /; FreeQ[{a, b, c, d, f, A, B, C}, x] && NeQ[b^2 - 4*a
*c, 0]
```

Rubi steps

$$\text{integral} = \frac{(a+cx^2)^{3/2}}{3f} + \frac{\int \frac{\sqrt{a+cx^2}(-3(cd-af)x-3cex^2)}{d+ex+fx^2}\,dx}{3f}$$

$$= \frac{(2(af^2+c(e^2-df))-cefx)\sqrt{a+cx^2}}{2f^3} + \frac{(a+cx^2)^{3/2}}{3f} - \frac{\int \frac{-3ac^2def+3c(ace^2f+2(cd-af)(ce^2-cdf+af^2))x+3c^2e(3af^2+2c(e^2-2df))x^2}{\sqrt{a+cx^2}(d+ex+fx^2)}\,dx}{6cf^3}$$

$$= \frac{(2(af^2+c(e^2-df))-cefx)\sqrt{a+cx^2}}{2f^3} + \frac{(a+cx^2)^{3/2}}{3f} - \frac{\int \frac{-3ac^2def^2-3c^2de(3af^2+2c(e^2-2df))+(-3c^2e^2(3af^2+2c(e^2-2df))+3cf(ace^2f+2(cd-af)(ce^2-cdf+af^2)))x}{\sqrt{a+cx^2}(d+ex+fx^2)}\,dx}{6cf^4} - \frac{(ce(3af^2+2c(e^2-2df)))\int \frac{1}{\sqrt{a+cx^2}}\,dx}{2f^4}$$

$$= \frac{(2(af^2+c(e^2-df))-cefx)\sqrt{a+cx^2}}{2f^3} + \frac{(a+cx^2)^{3/2}}{3f} - \frac{(ce(3af^2+2c(e^2-2df)))\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{2f^4}$$

$$+ \frac{\left(2cdef(2af^2+c(e^2-2df))-\left(e-\sqrt{e^2-4df}\right)\left(a^2f^4+2acf^2(e^2-df)+c^2(e^4-3de^2f+d^2f^2)\right.}{f^4\sqrt{e^2-4df}}$$

$$- \frac{\left(2cdef(2af^2+c(e^2-2df))-\left(e+\sqrt{e^2-4df}\right)\left(a^2f^4+2acf^2(e^2-df)+c^2(e^4-3de^2f+d^2f^2)\right.}{f^4\sqrt{e^2-4df}}$$

$$= \frac{(2(af^2 + c(e^2 - df)) - cefx)\sqrt{a + cx^2}}{2f^3} + \frac{(a + cx^2)^{3/2}}{3f}$$

$$- \frac{\sqrt{c}e(3af^2 + 2c(e^2 - 2df))\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2f^4}$$

$$- \frac{\left(2cdef(2af^2 + c(e^2 - 2df)) - \left(e - \sqrt{e^2 - 4df}\right)(a^2f^4 + 2acf^2(e^2 - df) + c^2(e^4 - 3de^2f + d^2f^2}{f^4\sqrt{e^2 - 4df}}$$

$$+ \frac{\left(2cdef(2af^2 + c(e^2 - 2df)) - \left(e + \sqrt{e^2 - 4df}\right)(a^2f^4 + 2acf^2(e^2 - df) + c^2(e^4 - 3de^2f + d^2f^2}{f^4\sqrt{e^2 - 4df}}$$

$$= \frac{(2(af^2 + c(e^2 - df)) - cefx)\sqrt{a + cx^2}}{2f^3} + \frac{(a + cx^2)^{3/2}}{3f}$$

$$- \frac{\sqrt{c}e(3af^2 + 2c(e^2 - 2df))\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2f^4}$$

$$- \frac{\left(2cdef(2af^2 + c(e^2 - 2df)) - \left(e - \sqrt{e^2 - 4df}\right)(a^2f^4 + 2acf^2(e^2 - df) + c^2(e^4 - 3de^2f + d^2f^2}{\sqrt{2}f^4\sqrt{e^2 - 4df}\sqrt{2af^2 + c\left(e^2 - 2df - e\sqrt{e^2 - 4}\right.}}$$

$$+ \frac{\left(2cdef(2af^2 + c(e^2 - 2df)) - \left(e + \sqrt{e^2 - 4df}\right)(a^2f^4 + 2acf^2(e^2 - df) + c^2(e^4 - 3de^2f + d^2f^2}{\sqrt{2}f^4\sqrt{e^2 - 4df}\sqrt{2af^2 + c\left(e^2 - 2df + e\sqrt{e^2 - 4}\right.}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.68 (sec) , antiderivative size = 755, normalized size of antiderivative = 1.37

$$\int \frac{x(a + cx^2)^{3/2}}{d + ex + fx^2}\,dx = \frac{f\sqrt{a + cx^2}(8af^2 + c(6e^2 - 6df - 3efx + 2f^2x^2)) + 3\sqrt{c}e(3af^2 + 2c(e^2 - 2df))\log\left(\right.}{}$$

```
[In] Integrate[(x*(a + c*x^2)^(3/2))/(d + e*x + f*x^2),x]
```

```
[Out] (f*Sqrt[a + c*x^2]*(8*a*f^2 + c*(6*e^2 - 6*d*f - 3*e*f*x + 2*f^2*x^2)) + 3*
Sqrt[c]*e*(3*a*f^2 + 2*c*(e^2 - 2*d*f))*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]]
 - 6*RootSum[a^2*f + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]
*e*#1^3 + f*#1^4 & , (a*c^2*e^4*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] -
3*a*c^2*d*e^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] + a*c^2*d^2*f^2*Lo
```

```
g[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] + 2*a^2*c*e^2*f^2*Log[-(Sqrt[c]*x) +
 Sqrt[a + c*x^2] - #1] - 2*a^2*c*d*f^3*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] -
 #1] + a^3*f^4*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] + 2*c^(5/2)*d*e^3*L
og[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 - 4*c^(5/2)*d^2*e*f*Log[-(Sqrt[c
]*x) + Sqrt[a + c*x^2] - #1]*#1 + 4*a*c^(3/2)*d*e*f^2*Log[-(Sqrt[c]*x) + Sq
rt[a + c*x^2] - #1]*#1 - c^2*e^4*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#
1^2 + 3*c^2*d*e^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2 - c^2*d^2
*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2 - 2*a*c*e^2*f^2*Log[-(Sq
rt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2 + 2*a*c*d*f^3*Log[-(Sqrt[c]*x) + Sqrt
[a + c*x^2] - #1]*#1^2 - a^2*f^4*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#
1^2)/(a*Sqrt[c]*e + 4*c*d*#1 - 2*a*f*#1 - 3*Sqrt[c]*e*#1^2 + 2*f*#1^3) & ])
/(6*f^4)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1038 vs. 2(499) = 998.

Time = 0.76 (sec) , antiderivative size = 1039, normalized size of antiderivative = 1.88

method	result	size
risch	Expression too large to display	1039
default	Expression too large to display	2294

```
[In] int(x*(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*(2*c*f^2*x^2-3*c*e*f*x+8*a*f^2-6*c*d*f+6*c*e^2)*(c*x^2+a)^(1/2)/f^3-1/2
/f^3*(1/f*c^(1/2)*e*(3*a*f^2-4*c*d*f+2*c*e^2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))
-1/2*(-2*a^2*f^4*(-4*d*f+e^2)^(1/2)+4*a*c*d*f^3*(-4*d*f+e^2)^(1/2)-4*a*c*e^
2*f^2*(-4*d*f+e^2)^(1/2)-2*c^2*d^2*f^2*(-4*d*f+e^2)^(1/2)+6*c^2*d*e^2*f*(-4
*d*f+e^2)^(1/2)-2*c^2*e^4*(-4*d*f+e^2)^(1/2)+2*e*f^4*a^2-12*f^3*a*c*d*e+4*a
*c*e^3*f^2+10*c^2*d^2*e*f^2-10*c^2*d*e^3*f+2*c^2*e^5)/f^2/(-4*d*f+e^2)^(1/2
)*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((
-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e-(-4*d*f+e^2)^(1/2))
/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+
2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2*c-
4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+2*(-(-4*d*f+
e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2/f*(-e+(-4*d*f+e^2)
^(1/2))))-1/2*(-2*a^2*f^4*(-4*d*f+e^2)^(1/2)+4*a*c*d*f^3*(-4*d*f+e^2)^(1/2)
-4*a*c*e^2*f^2*(-4*d*f+e^2)^(1/2)-2*c^2*d^2*f^2*(-4*d*f+e^2)^(1/2)+6*c^2*d*
e^2*f*(-4*d*f+e^2)^(1/2)-2*c^2*e^4*(-4*d*f+e^2)^(1/2)-2*e*f^4*a^2+12*f^3*a*
c*d*e-4*a*c*e^3*f^2-10*c^2*d^2*e*f^2+10*c^2*d*e^3*f-2*c^2*e^5)/f^2/(-4*d*f+
e^2)^(1/2)*2^(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/
2)*ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)
^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)
*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^
```

```
2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*
f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^
(1/2))/f)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x(a+cx^2)^{3/2}}{d+ex+fx^2}\,dx = \text{Timed out}$$

```
[In] integrate(x*(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{x(a+cx^2)^{3/2}}{d+ex+fx^2}\,dx = \int \frac{x(a+cx^2)^{\frac{3}{2}}}{d+ex+fx^2}\,dx$$

```
[In] integrate(x*(c*x**2+a)**(3/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral(x*(a + c*x**2)**(3/2)/(d + e*x + f*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x(a+cx^2)^{3/2}}{d+ex+fx^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x*(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re deta
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x(a+cx^2)^{3/2}}{d+ex+fx^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x*(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+cx^2)^{3/2}}{d+ex+fx^2}\,dx = \int \frac{x\,(c\,x^2+a)^{3/2}}{f\,x^2+e\,x+d}\,dx$$

```
[In] int((x*(a + c*x^2)^(3/2))/(d + e*x + f*x^2),x)
```

```
[Out] int((x*(a + c*x^2)^(3/2))/(d + e*x + f*x^2), x)
```

3.60 $\int \frac{(a+cx^2)^{3/2}}{d+ex+fx^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 484

$$\int \frac{(a+cx^2)^{3/2}}{d+ex+fx^2}\,dx = -\frac{c(2e-fx)\sqrt{a+cx^2}}{2f^2} + \frac{\sqrt{c}(3af^2+2c(e^2-df))\,\mathrm{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2f^3}$$

$$-\frac{\left(ce\left(e-\sqrt{e^2-4df}\right)(2af^2+c(e^2-2df))-2f(2acdf^2-a^2f^3+c^2d(e^2-df))\right)\mathrm{arctanh}\left(\frac{2af-c\left(e-\right.}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2d\right.}}\right.}{\sqrt{2}f^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}}$$

$$+\frac{\left(ce\left(e+\sqrt{e^2-4df}\right)(2af^2+c(e^2-2df))-2f(2acdf^2-a^2f^3+c^2d(e^2-df))\right)\mathrm{arctanh}\left(\frac{2af-c\left(e+\right.}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df\right.}}\right.}{\sqrt{2}f^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}}$$

```
[Out] 1/2*(3*a*f^2+2*c*(-d*f+e^2))*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))*c^(1/2)/f^3
-1/2*c*(-f*x+2*e)*(c*x^2+a)^(1/2)/f^2-1/2*arctanh(1/2*(2*a*f-c*x*(e-(-4*d*f
+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+c*(e^2-2*d*f-e*(-4*d*f+e^2)^
(1/2)))^(1/2))*(-2*f*(2*a*c*d*f^2-a^2*f^3+c^2*d*(-d*f+e^2))+c*e*(2*a*f^2+c*
(-2*d*f+e^2))*(e-(-4*d*f+e^2)^(1/2)))/f^3*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f
^2+c*(e^2-2*d*f-e*(-4*d*f+e^2)^(1/2)))^(1/2)+1/2*arctanh(1/2*(2*a*f-c*x*(e+
(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+c*(e^2-2*d*f+e*(-4*d*
f+e^2)^(1/2)))^(1/2))*(-2*f*(2*a*c*d*f^2-a^2*f^3+c^2*d*(-d*f+e^2))+c*e*(2*a
*f^2+c*(-2*d*f+e^2))*(e+(-4*d*f+e^2)^(1/2)))/f^3*2^(1/2)/(-4*d*f+e^2)^(1/2)
/(2*a*f^2+c*(e^2-2*d*f+e*(-4*d*f+e^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 2.74 (sec) , antiderivative size = 482, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {993, 1094, 223, 212, 1048, 739}

$$\int \frac{(a+cx^2)^{3/2}}{d+ex+fx^2}\,dx = \frac{\left(-2a^2f^4 - ce\left(e-\sqrt{e^2-4df}\right)\left(2af^2+c(e^2-2df)\right)+4acdf^3+2c^2df(e^2-df)\right)\text{arctan}}{\sqrt{2}f^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+\right.}}$$

$$-\frac{\left(-2a^2f^4 - ce\left(\sqrt{e^2-4df}+e\right)\left(2af^2+c(e^2-2df)\right)+4acdf^3+2c^2df(e^2-df)\right)\text{arctanh}\left(\frac{2af-cx\left(\sqrt{\right.}}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c(}}\right.}{\sqrt{2}f^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}$$

$$+\frac{\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)(3af^2+2c(e^2-df))}{2f^3}-\frac{c\sqrt{a+cx^2}(2e-fx)}{2f^2}$$

```
[In] Int[(a + c*x^2)^(3/2)/(d + e*x + f*x^2),x]
```

```
[Out] -1/2*(c*(2*e - f*x)*Sqrt[a + c*x^2])/f^2 + (Sqrt[c]*(3*a*f^2 + 2*c*(e^2 - d
*f))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*f^3) + ((4*a*c*d*f^3 - 2*a^2*
f^4 + 2*c^2*d*f*(e^2 - d*f) - c*e*(e - Sqrt[e^2 - 4*d*f])*(2*a*f^2 + c*(e^2
 - 2*d*f)))*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a
*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*f
^3*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])])
 - ((4*a*c*d*f^3 - 2*a^2*f^4 + 2*c^2*d*f*(e^2 - d*f) - c*e*(e + Sqrt[e^2 -
4*d*f])*(2*a*f^2 + c*(e^2 - 2*d*f)))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d
*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt
[a + c*x^2])])/(Sqrt[2]*f^3*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f
 + e*Sqrt[e^2 - 4*d*f])])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 993

```
Int[((a_.) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[(-c)*(e*(2*p + q) - 2*f*(p + q)*x)*(a + c*x^2)^(p - 1)*((d
 + e*x + f*x^2)^(q + 1)/(2*f^2*(p + q)*(2*p + 2*q + 1))), x] - Dist[1/(2*f^
2*(p + q)*(2*p + 2*q + 1)), Int[(a + c*x^2)^(p - 2)*(d + e*x + f*x^2)^q*Sim
p[(-a)*c*e^2*(1 - p)*(2*p + q) + a*(p + q)*(-2*a*f^2*(2*p + 2*q + 1) + c*(2
*d*f - e^2*(2*p + q))) + (2*(c*d - a*f)*(c*e)*(1 - p)*(2*p + q) + 4*a*c*e*f
*(1 - p)*(p + q))*x + (p*c^2*e^2*(1 - p) - c*(p + q)*(2*a*f^2*(4*p + 2*q -
1) + c*(2*d*f*(1 - 2*p) + e^2*(3*p + q - 1))))*x^2, x], x], x] /; FreeQ[{a,
 c, d, e, f, q}, x] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 1] && NeQ[p + q, 0] &&
 NeQ[2*p + 2*q + 1, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1048

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1094

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + f*x^2],
 x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt
[d + f*x^2]), x], x] /; FreeQ[{a, b, c, d, f, A, B, C}, x] && NeQ[b^2 - 4*a
*c, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{c(2e-fx)\sqrt{a+cx^2}}{2f^2} - \frac{\int \frac{af(cd-2af)-ce(2cd-af)x-c(3af^2+2c(e^2-df))x^2}{\sqrt{a+cx^2}(d+ex+fx^2)}\,dx}{2f^2}\\ &= -\frac{c(2e-fx)\sqrt{a+cx^2}}{2f^2}\\ &\quad -\frac{\int \frac{af^2(cd-2af)+cd(3af^2+2c(e^2-df))+(-cef(2cd-af)+ce(3af^2+2c(e^2-df)))x}{\sqrt{a+cx^2}(d+ex+fx^2)}\,dx}{2f^3}\\ &\quad +\frac{(c(3af^2+2c(e^2-df)))\int \frac{1}{\sqrt{a+cx^2}}\,dx}{2f^3}\end{aligned}$$

$$= -\frac{c(2e-fx)\sqrt{a+cx^2}}{2f^2} + \frac{(c(3af^2+2c(e^2-df)))\,\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{2f^3}$$
$$- \frac{\left(2f(af^2(cd-2af)+cd(3af^2+2c(e^2-df)))-\left(e-\sqrt{e^2-4df}\right)\left(-cef(2cd-af)+ce(3af^2+\right.\right.}{2f^3\sqrt{e^2-4df}}$$
$$+ \frac{\left(2f(af^2(cd-2af)+cd(3af^2+2c(e^2-df)))-\left(e+\sqrt{e^2-4df}\right)\left(-cef(2cd-af)+ce(3af^2+\right.\right.}{2f^3\sqrt{e^2-4df}}$$

$$= -\frac{c(2e-fx)\sqrt{a+cx^2}}{2f^2} + \frac{\sqrt{c}(3af^2+2c(e^2-df))\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2f^3}$$
$$+ \frac{\left(2f(af^2(cd-2af)+cd(3af^2+2c(e^2-df)))-\left(e-\sqrt{e^2-4df}\right)\left(-cef(2cd-af)+ce(3af^2+\right.\right.}{2f^3\sqrt{e^2-4df}}$$
$$- \frac{\left(2f(af^2(cd-2af)+cd(3af^2+2c(e^2-df)))-\left(e+\sqrt{e^2-4df}\right)\left(-cef(2cd-af)+ce(3af^2+\right.\right.}{2f^3\sqrt{e^2-4df}}$$

$$= -\frac{c(2e-fx)\sqrt{a+cx^2}}{2f^2} + \frac{\sqrt{c}(3af^2+2c(e^2-df))\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2f^3}$$
$$- \frac{\left(ce\left(e-\sqrt{e^2-4df}\right)(2af^2+c(e^2-2df))-2f(2acdf^2-a^2f^3+c^2d(e^2-df))\right)\tanh^{-1}\left(\frac{}{\sqrt{2}\sqrt{2af^2}}\right.}{\sqrt{2}f^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}}$$
$$+ \frac{\left(ce\left(e+\sqrt{e^2-4df}\right)(2af^2+c(e^2-2df))-2f(2acdf^2-a^2f^3+c^2d(e^2-df))\right)\tanh^{-1}\left(\frac{}{\sqrt{2}\sqrt{2af^2}}\right.}{\sqrt{2}f^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.55 (sec) , antiderivative size = 542, normalized size of antiderivative = 1.12

$$\int \frac{(a+cx^2)^{3/2}}{d+ex+fx^2}\,dx = \frac{cf(-2e+fx)\sqrt{a+cx^2}+\sqrt{c}(-2ce^2+2cdf-3af^2)\log\left(-\sqrt{c}x+\sqrt{a+cx^2}\right)+2\text{Root}}{}$$

```
[In] Integrate[(a + c*x^2)^(3/2)/(d + e*x + f*x^2),x]
```

[Out] (c*f*(-2*e + f*x)*Sqrt[a + c*x^2] + Sqrt[c]*(-2*c*e^2 + 2*c*d*f - 3*a*f^2)*
Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]] + 2*RootSum[a^2*f + 2*a*Sqrt[c]*e*#1 +
4*c*d*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (a*c^2*e^3*Log[-(Sq
rt[c]*x) + Sqrt[a + c*x^2] - #1] - 2*a*c^2*d*e*f*Log[-(Sqrt[c]*x) + Sqrt[a
+ c*x^2] - #1] + 2*a^2*c*e*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] + 2
*c^(5/2)*d*e^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 - 2*c^(5/2)*d^2*
f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 + 4*a*c^(3/2)*d*f^2*Log[-(Sqr
t[c]*x) + Sqrt[a + c*x^2] - #1]*#1 - 2*a^2*Sqrt[c]*f^3*Log[-(Sqrt[c]*x) + S
qrt[a + c*x^2] - #1]*#1 - c^2*e^3*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*
#1^2 + 2*c^2*d*e*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2 - 2*a*c*e*
f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2)/(a*Sqrt[c]*e + 4*c*d*#1
- 2*a*f*#1 - 3*Sqrt[c]*e*#1^2 + 2*f*#1^3) &])/(2*f^3)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 885 vs. $2(435) = 870$.

Time = 0.78 (sec) , antiderivative size = 886, normalized size of antiderivative = 1.83

method	result
risch	$-\frac{c(-fx+2e)\sqrt{c\,x^2+a}}{2f^2}+\frac{\frac{\sqrt{c}\left(3a\,f^2-2cdf+2c\,e^2\right)\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{f}-\frac{\left(-4ace\,f^2\sqrt{-4df+e^2}+4c^2def\sqrt{-4df+e^2}-2c^2e^3\sqrt{-4df+e^2}+4a^2f^4\right.}{}}{}$
default	Expression too large to display

[In] int((c*x^2+a)^(3/2)/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)

[Out] -1/2*c*(-f*x+2*e)*(c*x^2+a)^(1/2)/f^2+1/2/f^2*(1/f*c^(1/2)*(3*a*f^2-2*c*d*f
+2*c*e^2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))-1/2*(-4*a*c*e*f^2*(-4*d*f+e^2)^(1/2
)+4*c^2*d*e*f*(-4*d*f+e^2)^(1/2)-2*c^2*e^3*(-4*d*f+e^2)^(1/2)+4*a^2*f^4-8*a
*c*d*f^3+4*a*c*e^2*f^2+4*c^2*d^2*f^2-8*c^2*d*e^2*f+2*c^2*e^4)/f^2/(-4*d*f+e
^2)^(1/2)*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/
2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e-(-4*d*f+e^2
)^(1/2))/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1
/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2
)))^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+2*(-
(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2/f*(-e+(-4*
d*f+e^2)^(1/2))))-1/2*(-4*a*c*e*f^2*(-4*d*f+e^2)^(1/2)+4*c^2*d*e*f*(-4*d*f+
e^2)^(1/2)-2*c^2*e^3*(-4*d*f+e^2)^(1/2)-4*a^2*f^4+8*a*c*d*f^3-4*a*c*e^2*f^2
-4*c^2*d^2*f^2+8*c^2*d*e^2*f-2*c^2*e^4)/f^2/(-4*d*f+e^2)^(1/2)*2^(1/2)/(((-

```
4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1
/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4
*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*
e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)
^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^
2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+cx^2)^{3/2}}{d+ex+fx^2}\,dx = \text{Timed out}$$

```
[In] integrate((c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{(a+cx^2)^{3/2}}{d+ex+fx^2}\,dx = \int \frac{(a+cx^2)^{\frac{3}{2}}}{d+ex+fx^2}\,dx$$

```
[In] integrate((c*x**2+a)**(3/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral((a + c*x**2)**(3/2)/(d + e*x + f*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+cx^2)^{3/2}}{d+ex+fx^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re deta
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+cx^2)^{3/2}}{d+ex+fx^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+cx^2)^{3/2}}{d+ex+fx^2}\,dx = \int \frac{(c\,x^2+a)^{3/2}}{f\,x^2+e\,x+d}\,dx$$

```
[In] int((a + c*x^2)^(3/2)/(d + e*x + f*x^2),x)
```

```
[Out] int((a + c*x^2)^(3/2)/(d + e*x + f*x^2), x)
```

3.61 $\int \frac{(a+cx^2)^{3/2}}{x(d+ex+fx^2)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 496

$$\int \frac{(a+cx^2)^{3/2}}{x(d+ex+fx^2)}\,dx = \frac{a\sqrt{a+cx^2}}{d} + \frac{(cd-af)\sqrt{a+cx^2}}{df} - \frac{c^{3/2}e\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{f^2}$$
$$- \frac{\left(2ef(c^2d^2-a^2f^2)-(c^2de^2-f(cd-af)^2)\left(e-\sqrt{e^2-4df}\right)\right)\text{arctanh}\left(\frac{2af-c\left(e-\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}df^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}}$$
$$+ \frac{\left(2ef(c^2d^2-a^2f^2)-(c^2de^2-f(cd-af)^2)\left(e+\sqrt{e^2-4df}\right)\right)\text{arctanh}\left(\frac{2af-c\left(e+\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}df^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}}$$
$$- \frac{a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d}$$

```
[Out] -c^(3/2)*e*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/f^2-a^(3/2)*arctanh((c*x^2+a)
^(1/2)/a^(1/2))/d+a*(c*x^2+a)^(1/2)/d+(-a*f+c*d)*(c*x^2+a)^(1/2)/d/f-1/2*ar
ctanh(1/2*(2*a*f-c*x*(e-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f
^2+c*(e^2-2*d*f-e*(-4*d*f+e^2)^(1/2)))^(1/2))*(2*e*f*(-a^2*f^2+c^2*d^2)-(c^
2*d*e^2-f*(-a*f+c*d)^2)*(e-(-4*d*f+e^2)^(1/2)))/d/f^2*2^(1/2)/(-4*d*f+e^2)^
(1/2)/(2*a*f^2+c*(e^2-2*d*f-e*(-4*d*f+e^2)^(1/2)))^(1/2)+1/2*arctanh(1/2*(2
*a*f-c*x*(e+(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+c*(e^2-2*
d*f+e*(-4*d*f+e^2)^(1/2)))^(1/2))*(2*e*f*(-a^2*f^2+c^2*d^2)-(c^2*d*e^2-f*(-
a*f+c*d)^2)*(e+(-4*d*f+e^2)^(1/2)))/d/f^2*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f
^2+c*(e^2-2*d*f+e*(-4*d*f+e^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 1.43 (sec) , antiderivative size = 496, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.407, Rules used = {6860, 272, 52, 65, 214, 1034, 1094, 223, 212, 1048, 739}

$$\int \frac{(a+cx^2)^{3/2}}{x(d+ex+fx^2)}\,dx = -\frac{a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d}$$

$$-\frac{\left(2ef(c^2d^2-a^2f^2)-\left(e-\sqrt{e^2-4df}\right)\left(c^2de^2-f(cd-af)^2\right)\right)\text{arctanh}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}df^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}$$

$$+\frac{\left(2ef(c^2d^2-a^2f^2)-\left(\sqrt{e^2-4df}+e\right)\left(c^2de^2-f(cd-af)^2\right)\right)\text{arctanh}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}df^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}$$

$$-\frac{c^{3/2}e\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{f^2}+\frac{\sqrt{a+cx^2}(cd-af)}{df}+\frac{a\sqrt{a+cx^2}}{d}$$

```
[In] Int[(a + c*x^2)^(3/2)/(x*(d + e*x + f*x^2)),x]
```

```
[Out] (a*Sqrt[a + c*x^2])/d + ((c*d - a*f)*Sqrt[a + c*x^2])/(d*f) - (c^(3/2)*e*Ar
cTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/f^2 - ((2*e*f*(c^2*d^2 - a^2*f^2) - (c^
2*d*e^2 - f*(c*d - a*f)^2)*(e - Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e -
 Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2
- 4*d*f])]*Sqrt[a + c*x^2])]])/(Sqrt[2]*d*f^2*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2
 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) + ((2*e*f*(c^2*d^2 - a^2*f^2) -
(c^2*d*e^2 - f*(c*d - a*f)^2)*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(
e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e
^2 - 4*d*f])]*Sqrt[a + c*x^2])]])/(Sqrt[2]*d*f^2*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*
f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]) - (a^(3/2)*ArcTanh[Sqrt[a + c
*x^2]/Sqrt[a]])/d
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[h*(a + c*x^2)^p*((d + e*x + f*x^2)^(q +
 1)/(2*f*(p + q + 1))), x] + Dist[1/(2*f*(p + q + 1)), Int[(a + c*x^2)^(p -
 1)*(d + e*x + f*x^2)^q*Simp[a*h*e*p - a*(h*e - 2*g*f)*(p + q + 1) - 2*h*p*
(c*d - a*f)*x - (h*c*e*p + c*(h*e - 2*g*f)*(p + q + 1))*x^2, x], x], x] /;
FreeQ[{a, c, d, e, f, g, h, q}, x] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && N
eQ[p + q + 1, 0]
```

Rule 1048

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1094

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + f*x^2],
 x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt
[d + f*x^2]), x], x] /; FreeQ[{a, b, c, d, f, A, B, C}, x] && NeQ[b^2 - 4*a
*c, 0]
```

Rule 6860

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{(a+cx^2)^{3/2}}{dx} + \frac{(-e-fx)(a+cx^2)^{3/2}}{d(d+ex+fx^2)} \right) dx$$

$$= \frac{\int \frac{(a+cx^2)^{3/2}}{x} \, dx}{d} + \frac{\int \frac{(-e-fx)(a+cx^2)^{3/2}}{d+ex+fx^2} \, dx}{d}$$

$$= -\frac{(a+cx^2)^{3/2}}{3d} + \frac{\text{Subst}\left(\int \frac{(a+cx)^{3/2}}{x} \, dx, x, x^2\right)}{2d} + \frac{\int \frac{(-3aef+3f(cd-af)x)\sqrt{a+cx^2}}{d+ex+fx^2} \, dx}{3df}$$

$$= \frac{(cd-af)\sqrt{a+cx^2}}{df} + \frac{a\text{Subst}\left(\int \frac{\sqrt{a+cx}}{x} \, dx, x, x^2\right)}{2d} + \frac{\int \frac{-3a^2ef^2-3f(cd-af)^2x-3c^2defx^2}{\sqrt{a+cx^2}(d+ex+fx^2)} \, dx}{3df^2}$$

$$= \frac{a\sqrt{a+cx^2}}{d} + \frac{(cd-af)\sqrt{a+cx^2}}{df} + \frac{a^2\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}} \, dx, x, x^2\right)}{2d} + \frac{\int \frac{3c^2d^2ef-3a^2ef^3+(3c^2de^2f-3f^2(cd-af)^2)x}{\sqrt{a+cx^2}(d+ex+fx^2)} \, dx}{3df^3} - \frac{(c^2e)\int \frac{1}{\sqrt{a+cx^2}} \, dx}{f^2}$$

$$=\frac{a\sqrt{a+cx^2}}{d}+\frac{(cd-af)\sqrt{a+cx^2}}{df}+\frac{a^2\text{Subst}\left(\int\frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx,x,\sqrt{a+cx^2}\right)}{cd}$$

$$-\frac{(c^2e)\,\text{Subst}\left(\int\frac{1}{1-cx^2}\,dx,x,\frac{x}{\sqrt{a+cx^2}}\right)}{f^2}$$

$$+\frac{\left(2ef(c^2d^2-a^2f^2)-(c^2de^2-f(cd-af)^2)\left(e-\sqrt{e^2-4df}\right)\right)\int\frac{1}{\left(e-\sqrt{e^2-4df}+2fx\right)\sqrt{a+cx^2}}\,dx}{df^2\sqrt{e^2-4df}}$$

$$-\frac{\left(2ef(c^2d^2-a^2f^2)-(c^2de^2-f(cd-af)^2)\left(e+\sqrt{e^2-4df}\right)\right)\int\frac{1}{\left(e+\sqrt{e^2-4df}+2fx\right)\sqrt{a+cx^2}}\,dx}{df^2\sqrt{e^2-4df}}$$

$$=\frac{a\sqrt{a+cx^2}}{d}+\frac{(cd-af)\sqrt{a+cx^2}}{df}-\frac{c^{3/2}e\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{f^2}-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d}$$

$$-\frac{\left(2ef(c^2d^2-a^2f^2)-(c^2de^2-f(cd-af)^2)\left(e-\sqrt{e^2-4df}\right)\right)\text{Subst}\left(\int\frac{1}{4af^2+c\left(e-\sqrt{e^2-4df}\right)^2-x^2}\,dx,\right.}{df^2\sqrt{e^2-4df}}$$

$$+\frac{\left(2ef(c^2d^2-a^2f^2)-(c^2de^2-f(cd-af)^2)\left(e+\sqrt{e^2-4df}\right)\right)\text{Subst}\left(\int\frac{1}{4af^2+c\left(e+\sqrt{e^2-4df}\right)^2-x^2}\,dx,\right.}{df^2\sqrt{e^2-4df}}$$

$$=\frac{a\sqrt{a+cx^2}}{d}+\frac{(cd-af)\sqrt{a+cx^2}}{df}-\frac{c^{3/2}e\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{f^2}$$

$$-\frac{\left(2ef(c^2d^2-a^2f^2)-(c^2de^2-f(cd-af)^2)\left(e-\sqrt{e^2-4df}\right)\right)\tanh^{-1}\left(\frac{2af-c\left(e-\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4d\right.}}}\right.}{\sqrt{2}df^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}}$$

$$+\frac{\left(2ef(c^2d^2-a^2f^2)-(c^2de^2-f(cd-af)^2)\left(e+\sqrt{e^2-4df}\right)\right)\tanh^{-1}\left(\frac{2af-c\left(e+\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4d\right.}}}\right.}{\sqrt{2}df^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}}$$

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.55 (sec) , antiderivative size = 552, normalized size of antiderivative = 1.11

$$\int \frac{(a+cx^2)^{3/2}}{x(d+ex+fx^2)}\,dx = \frac{cdf\sqrt{a+cx^2}+2a^{3/2}f^2\operatorname{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+cx^2}}{\sqrt{a}}\right)+c^{3/2}de\log\left(-\sqrt{c}x+\sqrt{a+cx^2}\right)+}{}$$

```
[In] Integrate[(a + c*x^2)^(3/2)/(x*(d + e*x + f*x^2)),x]
```

```
[Out] (c*d*f*Sqrt[a + c*x^2] + 2*a^(3/2)*f^2*ArcTanh[(Sqrt[c]*x - Sqrt[a + c*x^2]
)/Sqrt[a]] + c^(3/2)*d*e*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]] + RootSum[a^2*
f + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4
& , (-(a*c^2*d*e^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]) + a*c^2*d^2*f*
Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] - 2*a^2*c*d*f^2*Log[-(Sqrt[c]*x) +
 Sqrt[a + c*x^2] - #1] + a^3*f^3*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] -
 2*c^(5/2)*d^2*e*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 + 2*a^2*Sqrt[c
]*e*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 + c^2*d*e^2*Log[-(Sqrt[
c]*x) + Sqrt[a + c*x^2] - #1]*#1^2 - c^2*d^2*f*Log[-(Sqrt[c]*x) + Sqrt[a +
c*x^2] - #1]*#1^2 + 2*a*c*d*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1
^2 - a^2*f^3*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2)/(a*Sqrt[c]*e +
4*c*d*#1 - 2*a*f*#1 - 3*Sqrt[c]*e*#1^2 + 2*f*#1^3) & ])/(d*f^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2378 vs. 2(443) = 886.

Time = 0.65 (sec) , antiderivative size = 2379, normalized size of antiderivative = 4.80

method	result	size
default	Expression too large to display	2379

```
[In] int((c*x^2+a)^(3/2)/x/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -4*f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))*(1/3*(c*x^2+a)^(3/2)+a*
((c*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(c*x^2+a)^(1/2))/x)))+2*f/(e+(-4
*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)*(1/3*((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^
2*c-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*((-4*d*
f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(3/2)-1/2*c*(e+(-4*d*f+e^2)^(1
/2))/f*(1/4*(2*c*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)-c*(e+(-4*d*f+e^2)^(1/2))/
f)/c*((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/
2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e
^2)/f^2)^(1/2)+1/8*(2*c*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-
c^2*(e+(-4*d*f+e^2)^(1/2))^2/f^2)/c^(3/2)*ln((-1/2*c*(e+(-4*d*f+e^2)^(1/2))
```

```
/f+c*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))/c^(1/2)+((x+1/2*(e+(-4*d*f+e^2)^(1/2
))/f)^2*c-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*(
(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)))+1/2*((-4*d*f+e^2
)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2*(1/2*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2)
)/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((
-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)-1/2*c^(1/2)*(e+(-4*
d*f+e^2)^(1/2))/f*ln((-1/2*c*(e+(-4*d*f+e^2)^(1/2))/f+c*(x+1/2*(e+(-4*d*f+e
^2)^(1/2))/f))/c^(1/2)+((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e+(-4*d*f+e
^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*((-4*d*f+e^2)^(1/2)*c*e+2
*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))-1/2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d
*f+c*e^2)/f^2*2^(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^
(1/2)*ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e
^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1
/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/
f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4
*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^
2)^(1/2))/f))))+2*f/(-e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)*(1/3*((x-1/2
/f*(-e+(-4*d*f+e^2)^(1/2)))^2*c-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2/f*(-e+(-4
*d*f+e^2)^(1/2)))+1/2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^
(3/2)-1/2*c*(e-(-4*d*f+e^2)^(1/2))/f*(1/4*(2*c*(x-1/2/f*(-e+(-4*d*f+e^2)^(1
/2)))-c*(e-(-4*d*f+e^2)^(1/2))/f)/c*((x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2*c-
c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+1/2*(-(-4*d*f+
e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)+1/8*(2*c*(-(-4*d*f+e^2)^(1
/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c^2*(e-(-4*d*f+e^2)^(1/2))^2/f^2)/c^(3/2
)*ln((-1/2*c*(e-(-4*d*f+e^2)^(1/2))/f+c*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2))))/
c^(1/2)+((x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2*c-c*(e-(-4*d*f+e^2)^(1/2))/f*(
x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+1/2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d
*f+c*e^2)/f^2)^(1/2)))+1/2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/
f^2*(1/2*(4*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2*c-4*c*(e-(-4*d*f+e^2)^(1/2)
)/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*
c*d*f+c*e^2)/f^2)^(1/2)-1/2*c^(1/2)*(e-(-4*d*f+e^2)^(1/2))/f*ln((-1/2*c*(e-
(-4*d*f+e^2)^(1/2))/f+c*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2))))/c^(1/2)+((x-1/2/
f*(-e+(-4*d*f+e^2)^(1/2)))^2*c-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2/f*(-e+(-4*
d*f+e^2)^(1/2)))+1/2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(
1/2))-1/2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2*2^(1/2)/((-(-
4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(
1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2/f*(-e
+(-4*d*f+e^2)^(1/2)))+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f
+c*e^2)/f^2)^(1/2)*(4*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2*c-4*c*(e-(-4*d*f+
e^2)^(1/2))/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+2*(-(-4*d*f+e^2)^(1/2)*c*e+
2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2))))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+cx^2)^{3/2}}{x(d+ex+fx^2)}\,dx = \text{Timed out}$$

```
[In] integrate((c*x^2+a)^(3/2)/x/(f*x^2+e*x+d),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{(a+cx^2)^{3/2}}{x(d+ex+fx^2)}\,dx = \int \frac{(a+cx^2)^{\frac{3}{2}}}{x(d+ex+fx^2)}\,dx$$

```
[In] integrate((c*x**2+a)**(3/2)/x/(f*x**2+e*x+d),x)
[Out] Integral((a + c*x**2)**(3/2)/(x*(d + e*x + f*x**2)), x)
```

Maxima [F]

$$\int \frac{(a+cx^2)^{3/2}}{x(d+ex+fx^2)}\,dx = \int \frac{(cx^2+a)^{\frac{3}{2}}}{(fx^2+ex+d)x}\,dx$$

```
[In] integrate((c*x^2+a)^(3/2)/x/(f*x^2+e*x+d),x, algorithm="maxima")
[Out] integrate((c*x^2 + a)^(3/2)/((f*x^2 + e*x + d)*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+cx^2)^{3/2}}{x(d+ex+fx^2)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c*x^2+a)^(3/2)/x/(f*x^2+e*x+d),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+cx^2)^{3/2}}{x(d+ex+fx^2)}\,dx = \int \frac{(c\,x^2+a)^{3/2}}{x\,(f\,x^2+e\,x+d)}\,dx$$

```
[In] int((a + c*x^2)^(3/2)/(x*(d + e*x + f*x^2)),x)
[Out] int((a + c*x^2)^(3/2)/(x*(d + e*x + f*x^2)), x)
```

3.62 $\int \frac{(a+cx^2)^{3/2}}{x^2(d+ex+fx^2)} \, dx$

Optimal result

Integrand size = 27, antiderivative size = 604

$$\int \frac{(a+cx^2)^{3/2}}{x^2(d+ex+fx^2)} \, dx = -\frac{ae\sqrt{a+cx^2}}{d^2} + \frac{3cx\sqrt{a+cx^2}}{2d} + \frac{(2ae-cdx)\sqrt{a+cx^2}}{2d^2}$$

$$-\frac{(a+cx^2)^{3/2}}{dx} + \frac{3a\sqrt{c}\,\mathrm{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2d} + \frac{\sqrt{c}(2cd-3af)\mathrm{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2df}$$

$$-\frac{\left(4acd^2f^2+c^2d^2\left(e^2-2df-e\sqrt{e^2-4df}\right)+a^2f^2\left(e^2-2df+e\sqrt{e^2-4df}\right)\right)\mathrm{arctanh}\left(\frac{2af-c\left(e-\sqrt{e^2-\cdots}\right.}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{\cdots}\right.}}\right.}{\sqrt{2}d^2f\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}}$$

$$+\frac{\left(4acd^2f^2+a^2f^2\left(e^2-2df-e\sqrt{e^2-4df}\right)+c^2d^2\left(e^2-2df+e\sqrt{e^2-4df}\right)\right)\mathrm{arctanh}\left(\frac{2af-c\left(e+\sqrt{e^2-\cdots}\right.}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{\cdots}\right.}}\right.}{\sqrt{2}d^2f\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}}$$

$$+\frac{a^{3/2}e\,\mathrm{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^2}$$

```
[Out] -(c*x^2+a)^(3/2)/d/x+a^(3/2)*e*arctanh((c*x^2+a)^(1/2)/a^(1/2))/d^2+3/2*a*a
rctanh(x*c^(1/2)/(c*x^2+a)^(1/2))*c^(1/2)/d+1/2*(-3*a*f+2*c*d)*arctanh(x*c^
(1/2)/(c*x^2+a)^(1/2))*c^(1/2)/d/f-a*e*(c*x^2+a)^(1/2)/d^2+3/2*c*x*(c*x^2+a
)^(1/2)/d+1/2*(-c*d*x+2*a*e)*(c*x^2+a)^(1/2)/d^2-1/2*arctanh(1/2*(2*a*f-c*x
*(e-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+c*(e^2-2*d*f-e*(-
4*d*f+e^2)^(1/2)))^(1/2))*(4*a*c*d^2*f^2+c^2*d^2*(e^2-2*d*f-e*(-4*d*f+e^2)^
(1/2))+a^2*f^2*(e^2-2*d*f+e*(-4*d*f+e^2)^(1/2)))/d^2/f*2^(1/2)/(-4*d*f+e^2)
^(1/2)/(2*a*f^2+c*(e^2-2*d*f-e*(-4*d*f+e^2)^(1/2)))^(1/2)+1/2*arctanh(1/2*(
```

```
2*a*f-c*x*(e+(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+c*(e^2-2
*d*f+e*(-4*d*f+e^2)^(1/2)))^(1/2))*(4*a*c*d^2*f^2+a^2*f^2*(e^2-2*d*f-e*(-4*
d*f+e^2)^(1/2))+c^2*d^2*(e^2-2*d*f+e*(-4*d*f+e^2)^(1/2)))/d^2/f*2^(1/2)/(-4
*d*f+e^2)^(1/2)/(2*a*f^2+c*(e^2-2*d*f+e*(-4*d*f+e^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 1.98 (sec) , antiderivative size = 604, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.519$, Rules used = {6860, 283, 201, 223, 212, 272, 52, 65, 214, 1034, 1082, 1094, 1048, 739}

$$\int \frac{(a+cx^2)^{3/2}}{x^2(d+ex+fx^2)}\,dx = \frac{a^{3/2}e\,\text{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^2}$$

$$-\frac{\left(a^2f^2\left(e\sqrt{e^2-4df}-2df+e^2\right)+4acd^2f^2+c^2d^2\left(-e\sqrt{e^2-4df}-2df+e^2\right)\right)\text{arctanh}\left(\frac{2af-cx\left(e-\sqrt{\ldots}\right.}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\ldots\right.}}\right.}{\sqrt{2}d^2f\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}$$

$$+\frac{\left(a^2f^2\left(-e\sqrt{e^2-4df}-2df+e^2\right)+4acd^2f^2+c^2d^2\left(e\sqrt{e^2-4df}-2df+e^2\right)\right)\text{arctanh}\left(\frac{2af-cx\left(\sqrt{e^2-4\ldots}\right.}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{\ldots}\right.}}\right.}{\sqrt{2}d^2f\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}$$

$$+\frac{\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)(2cd-3af)}{2df}+\frac{3a\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2d}$$

$$-\frac{ae\sqrt{a+cx^2}}{d^2}+\frac{\sqrt{a+cx^2}(2ae-cdx)}{2d^2}-\frac{(a+cx^2)^{3/2}}{dx}+\frac{3cx\sqrt{a+cx^2}}{2d}$$

```
[In] Int[(a + c*x^2)^(3/2)/(x^2*(d + e*x + f*x^2)),x]
```

```
[Out] -((a*e*Sqrt[a + c*x^2])/d^2) + (3*c*x*Sqrt[a + c*x^2])/(2*d) + ((2*a*e - c*
d*x)*Sqrt[a + c*x^2])/(2*d^2) - (a + c*x^2)^(3/2)/(d*x) + (3*a*Sqrt[c]*ArcT
anh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*d) + (Sqrt[c]*(2*c*d - 3*a*f)*ArcTanh[
(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*d*f) - ((4*a*c*d^2*f^2 + c^2*d^2*(e^2 - 2*
d*f - e*Sqrt[e^2 - 4*d*f]) + a^2*f^2*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]))*A
rcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2
 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*d^2*f*Sqrt[e^2
 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) + ((4*a*c*
d^2*f^2 + a^2*f^2*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + c^2*d^2*(e^2 - 2*d*
f + e*Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sq
rt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2]
)])/(Sqrt[2]*d^2*f*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt
[e^2 - 4*d*f])]) + (a^(3/2)*e*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/d^2
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[h*(a + c*x^2)^p*((d + e*x + f*x^2)^(q +
 1)/(2*f*(p + q + 1))), x] + Dist[1/(2*f*(p + q + 1)), Int[(a + c*x^2)^(p -
 1)*(d + e*x + f*x^2)^q*Simp[a*h*e*p - a*(h*e - 2*g*f)*(p + q + 1) - 2*h*p*
(c*d - a*f)*x - (h*c*e*p + c*(h*e - 2*g*f)*(p + q + 1))*x^2, x], x], x] /;
FreeQ[{a, c, d, e, f, g, h, q}, x] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && N
eQ[p + q + 1, 0]
```

Rule 1048

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1082

```
Int[((a_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)*((d_) +
(e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(B*c*f*(2*p + 2*q + 3) +
 C*((-c)*e*(2*p + q + 2)) + 2*c*C*f*(p + q + 1)*x)*(a + c*x^2)^p*((d + e*x
+ f*x^2)^(q + 1)/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3))), x] - Dist[1/(2*c*f
^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + c*x^2)^(p - 1)*(d + e*x + f*x^2)^
q*Simp[p*((-a)*e)*(C*(c*e)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p +
q + 1)*(a*c*(C*(2*d*f - e^2*(2*p + q + 2)) + f*(B*e - 2*A*f)*(2*p + 2*q + 3
))) + (2*p*(c*d - a*f)*(C*(c*e)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) +
(p + q + 1)*(C*e*f*p*(-4*a*c)))*x + (p*(c*e)*(C*(c*e)*(q + 1) - c*(C*e - B*
f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(-4*a*c) - c^2*(C*(e^2 - 4*d*f)*
(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3))))*x^2, x], x], x]
/; FreeQ[{a, c, d, e, f, A, B, C, q}, x] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0
```

```
] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*q + 3, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q
, 0]
```

Rule 1094

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + f*x^2],
 x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt
[d + f*x^2]), x], x] /; FreeQ[{a, b, c, d, f, A, B, C}, x] && NeQ[b^2 - 4*a
*c, 0]
```

Rule 6860

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{(a+cx^2)^{3/2}}{dx^2} - \frac{e(a+cx^2)^{3/2}}{d^2x} + \frac{(e^2-df+efx)(a+cx^2)^{3/2}}{d^2(d+ex+fx^2)}\right) dx$$

$$= \frac{\int \frac{(e^2-df+efx)(a+cx^2)^{3/2}}{d+ex+fx^2}\,dx}{d^2} + \frac{\int \frac{(a+cx^2)^{3/2}}{x^2}\,dx}{d} - \frac{e\int \frac{(a+cx^2)^{3/2}}{x}\,dx}{d^2}$$

$$= \frac{e(a+cx^2)^{3/2}}{3d^2} - \frac{(a+cx^2)^{3/2}}{dx} + \frac{(3c)\int \sqrt{a+cx^2}\,dx}{d} - \frac{e\text{Subst}\left(\int \frac{(a+cx)^{3/2}}{x}\,dx, x, x^2\right)}{2d^2} + \frac{\int \frac{\sqrt{a+cx^2}(3af(e^2-df)-3ef(cd-af)x-3cdf^2x^2)}{d+ex+fx^2}\,dx}{3d^2f}$$

$$= \frac{3cx\sqrt{a+cx^2}}{2d} + \frac{(2ae-cdx)\sqrt{a+cx^2}}{2d^2} - \frac{(a+cx^2)^{3/2}}{dx} + \frac{(3ac)\int \frac{1}{\sqrt{a+cx^2}}\,dx}{2d} - \frac{(ae)\text{Subst}\left(\int \frac{\sqrt{a+cx}}{x}\,dx, x, x^2\right)}{2d^2} - \frac{\int \frac{-3acf^3(cd^2+2ae^2-2adf)+3acef^3(3cd-2af)x-3c^2df^3(2cd-3af)x^2}{\sqrt{a+cx^2}(d+ex+fx^2)}\,dx}{6cd^2f^3}$$

$$= -\frac{ae\sqrt{a+cx^2}}{d^2} + \frac{3cx\sqrt{a+cx^2}}{2d} + \frac{(2ae-cdx)\sqrt{a+cx^2}}{2d^2} - \frac{(a+cx^2)^{3/2}}{dx} + \frac{(3ac)\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{2d} - \frac{(a^2e)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx, x, x^2\right)}{2d^2} - \frac{\int \frac{3c^2d^2f^3(2cd-3af)-3acf^4(cd^2+2ae^2-2adf)+(3c^2def^3(2cd-3af)+3acef^4(3cd-2af))x}{\sqrt{a+cx^2}(d+ex+fx^2)}\,dx}{6cd^2f^4} + \frac{(c(2cd-3af))\int \frac{1}{\sqrt{a+cx^2}}\,dx}{2df}$$

$$= -\frac{ae\sqrt{a+cx^2}}{d^2} + \frac{3cx\sqrt{a+cx^2}}{2d} + \frac{(2ae-cdx)\sqrt{a+cx^2}}{2d^2} - \frac{(a+cx^2)^{3/2}}{dx} + \frac{3a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2d} - \frac{(a^2e)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx, x, \sqrt{a+cx^2}\right)}{cd^2} + \frac{(c(2cd-3af))\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{2df} - \frac{\left(4acd^2f^2+a^2f^2\left(e^2-2df-e\sqrt{e^2-4df}\right)+c^2d^2\left(e^2-2df+e\sqrt{e^2-4df}\right)\right)\int \frac{1}{\left(e+\sqrt{e^2-4df}+2fx\right)\sqrt{a+}}}{d^2f\sqrt{e^2-4df}} + \frac{\left(4acd^2f^2+c^2d^2\left(e^2-2df-e\sqrt{e^2-4df}\right)+a^2f^2\left(e^2-2df+e\sqrt{e^2-4df}\right)\right)\int \frac{1}{\left(e-\sqrt{e^2-4df}+2fx\right)\sqrt{a+}}}{d^2f\sqrt{e^2-4df}}$$

$$= -\frac{ae\sqrt{a+cx^2}}{d^2} + \frac{3cx\sqrt{a+cx^2}}{2d} + \frac{(2ae-cdx)\sqrt{a+cx^2}}{2d^2} - \frac{(a+cx^2)^{3/2}}{dx} + \frac{3a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2d} + \frac{\sqrt{c}(2cd-3af)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2df} + \frac{a^{3/2}e\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^2} + \frac{\left(4acd^2f^2+a^2f^2\left(e^2-2df-e\sqrt{e^2-4df}\right)+c^2d^2\left(e^2-2df+e\sqrt{e^2-4df}\right)\right)\text{Subst}\left(\int \frac{1}{4af^2+c\left(e+\sqrt{e}\right.}\right.}{d^2f\sqrt{e^2-4df}} - \frac{\left(4acd^2f^2+c^2d^2\left(e^2-2df-e\sqrt{e^2-4df}\right)+a^2f^2\left(e^2-2df+e\sqrt{e^2-4df}\right)\right)\text{Subst}\left(\int \frac{1}{4af^2+c\left(e-\sqrt{e}\right.}\right.}{d^2f\sqrt{e^2-4df}}$$

$$= -\frac{ae\sqrt{a+cx^2}}{d^2} + \frac{3cx\sqrt{a+cx^2}}{2d} + \frac{(2ae-cdx)\sqrt{a+cx^2}}{2d^2} - \frac{(a+cx^2)^{3/2}}{dx} + \frac{3a\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2d} + \frac{\sqrt{c}(2cd-3af)\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2df}$$

$$- \frac{\left(4acd^2f^2+c^2d^2\left(e^2-2df-e\sqrt{e^2-4df}\right)+a^2f^2\left(e^2-2df+e\sqrt{e^2-4df}\right)\right)\tanh^{-1}\left(\frac{2a\ldots}{\sqrt{2}\sqrt{2af^2+c\ldots}}\right)}{\sqrt{2}d^2f\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}}$$

$$+ \frac{\left(4acd^2f^2+a^2f^2\left(e^2-2df-e\sqrt{e^2-4df}\right)+c^2d^2\left(e^2-2df+e\sqrt{e^2-4df}\right)\right)\tanh^{-1}\left(\frac{2a\ldots}{\sqrt{2}\sqrt{2af^2+c\ldots}}\right)}{\sqrt{2}d^2f\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}}$$

$$+ \frac{a^{3/2}e\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^2}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.60 (sec) , antiderivative size = 497, normalized size of antiderivative = 0.82

$$\int \frac{(a+cx^2)^{3/2}}{x^2(d+ex+fx^2)}\,dx =$$

$$-\frac{2a^{3/2}efx\operatorname{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+cx^2}}{\sqrt{a}}\right)+d\left(af\sqrt{a+cx^2}+c^{3/2}dx\log\left(-\sqrt{c}x+\sqrt{a+cx^2}\right)\right)+x\text{RootSum}\left[a^2f+2\ldots\right.}{}$$

```
[In] Integrate[(a + c*x^2)^(3/2)/(x^2*(d + e*x + f*x^2)),x]
```

```
[Out] -((2*a^(3/2)*e*f*x*ArcTanh[(Sqrt[c]*x - Sqrt[a + c*x^2])/Sqrt[a]] + d*(a*f*
Sqrt[a + c*x^2] + c^(3/2)*d*x*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]]) + x*Root
Sum[a^2*f + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 +
 f*#1^4 & , (-(a*c^2*d^2*e*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]) + a^3*
e*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] - 2*c^(5/2)*d^3*Log[-(Sqrt[c
]*x) + Sqrt[a + c*x^2] - #1]*#1 + 4*a*c^(3/2)*d^2*f*Log[-(Sqrt[c]*x) + Sqrt
[a + c*x^2] - #1]*#1 + 2*a^2*Sqrt[c]*e^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^
2] - #1]*#1 - 2*a^2*Sqrt[c]*d*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*
#1 + c^2*d^2*e*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2 - a^2*e*f^2*Lo
g[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2)/(a*Sqrt[c]*e + 4*c*d*#1 - 2*a*
f*#1 - 3*Sqrt[c]*e*#1^2 + 2*f*#1^3) & ])/(d^2*f*x))
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 957, normalized size of antiderivative = 1.58

method	result
risch	$-\frac{a\sqrt{c\,x^{2}+a}}{dx}+\frac{\frac{c^{\frac{3}{2}}\,d\ln\left(x\sqrt{c}+\sqrt{c\,x^{2}+a}\right)}{f}-\frac{4f\,a^{\frac{3}{2}}\,e\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c\,x^{2}+a}}{x}\right)}{\left(-e+\sqrt{-4df+e^{2}}\right)\left(e+\sqrt{-4df+e^{2}}\right)}-\frac{\left(2a^{2}f^{3}\sqrt{-4df+e^{2}}-4acd\,f^{2}\sqrt{-4df+e^{2}}+2c^{2}d^{2}f\sqrt{-4df+e^{2}}-2\cdots\right.}{\cdots}}{\cdots}$
default	Expression too large to display

```
[In] int((c*x^2+a)^(3/2)/x^2/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -a/d*(c*x^2+a)^(1/2)/x+1/d*(c^(3/2)*d/f*ln(x*c^(1/2)+(c*x^2+a)^(1/2))-4*f*a
^(3/2)*e/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))*ln((2*a+2*a^(1/2)*(
c*x^2+a)^(1/2))/x)-1/2*(2*a^2*f^3*(-4*d*f+e^2)^(1/2)-4*a*c*d*f^2*(-4*d*f+e^
2)^(1/2)+2*c^2*d^2*f*(-4*d*f+e^2)^(1/2)-2*(-4*d*f+e^2)^(1/2)*c^2*d*e^2-2*a^
2*e*f^3-4*a*c*d*e*f^2+6*c^2*d^2*e*f-2*c^2*d*e^3)/f^2/(-4*d*f+e^2)^(1/2)/(e+
(-4*d*f+e^2)^(1/2))*2^(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)
/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4
*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e
^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(
1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+
2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*
d*f+e^2)^(1/2))/f))-1/2*(-2*a^2*f^3*(-4*d*f+e^2)^(1/2)+4*a*c*d*f^2*(-4*d*f+
e^2)^(1/2)-2*c^2*d^2*f*(-4*d*f+e^2)^(1/2)+2*(-4*d*f+e^2)^(1/2)*c^2*d*e^2-2*
a^2*e*f^3-4*a*c*d*e*f^2+6*c^2*d^2*e*f-2*c^2*d*e^3)/f^2/(-4*d*f+e^2)^(1/2)/(
-e+(-4*d*f+e^2)^(1/2))*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*
e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(
e-(-4*d*f+e^2)^(1/2))/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+1/2*2^(1/2)*((-(-
4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2/f*(-e+(-4*
d*f+e^2)^(1/2)))^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2/f*(-e+(-4*d*f+e^2)
^(1/2)))+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1
/2/f*(-e+(-4*d*f+e^2)^(1/2)))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+cx^2)^{3/2}}{x^2(d+ex+fx^2)}\,dx = \text{Timed out}$$

```
[In] integrate((c*x^2+a)^(3/2)/x^2/(f*x^2+e*x+d),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+cx^2)^{3/2}}{x^2(d+ex+fx^2)}\,dx = \text{Timed out}$$

```
[In] integrate((c*x**2+a)**(3/2)/x**2/(f*x**2+e*x+d),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+cx^2)^{3/2}}{x^2(d+ex+fx^2)}\,dx = \int \frac{(cx^2+a)^{\frac{3}{2}}}{(fx^2+ex+d)x^2}\,dx$$

```
[In] integrate((c*x^2+a)^(3/2)/x^2/(f*x^2+e*x+d),x, algorithm="maxima")
[Out] integrate((c*x^2 + a)^(3/2)/((f*x^2 + e*x + d)*x^2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+cx^2)^{3/2}}{x^2(d+ex+fx^2)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c*x^2+a)^(3/2)/x^2/(f*x^2+e*x+d),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+cx^2)^{3/2}}{x^2(d+ex+fx^2)}\,dx = \int \frac{(c\,x^2+a)^{3/2}}{x^2\,(f\,x^2+e\,x+d)}\,dx$$

```
[In] int((a + c*x^2)^(3/2)/(x^2*(d + e*x + f*x^2)),x)
[Out] int((a + c*x^2)^(3/2)/(x^2*(d + e*x + f*x^2)), x)
```

3.63 $\int \frac{(a+cx^2)^{3/2}}{x^3(d+ex+fx^2)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 668

$$\int \frac{(a+cx^2)^{3/2}}{x^3(d+ex+fx^2)}\,dx = \frac{3c\sqrt{a+cx^2}}{2d} + \frac{a(e^2-df)\sqrt{a+cx^2}}{d^3} - \frac{3cex\sqrt{a+cx^2}}{2d^2} - \frac{(2(cd^2+a(e^2-df))-cdex)\sqrt{a+cx^2}}{2d^3} - \frac{(a+cx^2)^{3/2}}{2dx^2} + \frac{e(a+cx^2)^{3/2}}{d^2x}$$

$$+ \frac{\left(c^2d^3\left(e-\sqrt{e^2-4df}\right)+2acd^2f\left(e+\sqrt{e^2-4df}\right)+a^2f\left(e^3-3def+e^2\sqrt{e^2-4df}-df\sqrt{e^2-4df}\right)\right)\text{arctan}}{\sqrt{2}d^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}}$$

$$- \frac{\left(2acd^2f\left(e-\sqrt{e^2-4df}\right)+c^2d^3\left(e+\sqrt{e^2-4df}\right)+a^2f\left(e^3-3def-e^2\sqrt{e^2-4df}+df\sqrt{e^2-4df}\right)\right)\text{arctan}}{\sqrt{2}d^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}}$$

$$- \frac{3\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2d} - \frac{a^{3/2}(e^2-df)\,\text{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^3}$$

```
[Out] -1/2*(c*x^2+a)^(3/2)/d/x^2+e*(c*x^2+a)^(3/2)/d^2/x-a^(3/2)*(-d*f+e^2)*arcta
nh((c*x^2+a)^(1/2)/a^(1/2))/d^3-3/2*c*arctanh((c*x^2+a)^(1/2)/a^(1/2))*a^(1
/2)/d+3/2*c*(c*x^2+a)^(1/2)/d+a*(-d*f+e^2)*(c*x^2+a)^(1/2)/d^3-3/2*c*e*x*(c
*x^2+a)^(1/2)/d^2-1/2*(2*c*d^2+2*a*(-d*f+e^2)-c*d*e*x)*(c*x^2+a)^(1/2)/d^3+
1/2*arctanh(1/2*(2*a*f-c*x*(e-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/
(2*a*f^2+c*(e^2-2*d*f-e*(-4*d*f+e^2)^(1/2)))^(1/2))*(c^2*d^3*(e-(-4*d*f+e^2
)^(1/2))+2*a*c*d^2*f*(e+(-4*d*f+e^2)^(1/2))+a^2*f*(e^3-3*d*e*f+e^2*(-4*d*f+
e^2)^(1/2)-d*f*(-4*d*f+e^2)^(1/2)))/d^3*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2
```

```
+c*(e^2-2*d*f-e*(-4*d*f+e^2)^(1/2)))^(1/2)-1/2*arctanh(1/2*(2*a*f-c*x*(e+(-
4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+c*(e^2-2*d*f+e*(-4*d*f+
e^2)^(1/2)))^(1/2))*(2*a*c*d^2*f*(e-(-4*d*f+e^2)^(1/2))+c^2*d^3*(e+(-4*d*f+
e^2)^(1/2))+a^2*f*(e^3-3*d*e*f-e^2*(-4*d*f+e^2)^(1/2)+d*f*(-4*d*f+e^2)^(1/2
)))/d^3*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+c*(e^2-2*d*f+e*(-4*d*f+e^2)^(1/
2)))^(1/2)
```

Rubi [A] (verified)

Time = 2.22 (sec) , antiderivative size = 668, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {6860, 272, 43, 52, 65, 214, 283, 201, 223, 212, 1034, 1082, 1094, 1048, 739}

$$\int \frac{(a+cx^2)^{3/2}}{x^3(d+ex+fx^2)}\,dx = -\frac{a^{3/2}(e^2-df)\,\text{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^3}$$

$$+\frac{\left(a^2f\left(e^2\sqrt{e^2-4df}-df\sqrt{e^2-4df}-3def+e^3\right)+2acd^2f\left(\sqrt{e^2-4df}+e\right)+c^2d^3\left(e-\sqrt{e^2-4df}\right)\right)\text{arctanh}}{\sqrt{2}d^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}$$

$$-\frac{\left(a^2f\left(-e^2\sqrt{e^2-4df}+df\sqrt{e^2-4df}-3def+e^3\right)+2acd^2f\left(e-\sqrt{e^2-4df}\right)+c^2d^3\left(\sqrt{e^2-4df}+e\right)\right)\text{arcta}}{\sqrt{2}d^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}$$

$$-\frac{3\sqrt{a}c\,\text{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2d}+\frac{a\sqrt{a+cx^2}(e^2-df)}{d^3}+\frac{e(a+cx^2)^{3/2}}{d^2x}-\frac{3cex\sqrt{a+cx^2}}{2d^2}$$
$$-\frac{\sqrt{a+cx^2}(2(a(e^2-df)+cd^2)-cdex)}{2d^3}-\frac{(a+cx^2)^{3/2}}{2dx^2}+\frac{3c\sqrt{a+cx^2}}{2d}$$

```
[In] Int[(a + c*x^2)^(3/2)/(x^3*(d + e*x + f*x^2)),x]
```

```
[Out] (3*c*Sqrt[a + c*x^2])/(2*d) + (a*(e^2 - d*f)*Sqrt[a + c*x^2])/d^3 - (3*c*e*
x*Sqrt[a + c*x^2])/(2*d^2) - ((2*(c*d^2 + a*(e^2 - d*f)) - c*d*e*x)*Sqrt[a
+ c*x^2])/(2*d^3) - (a + c*x^2)^(3/2)/(2*d*x^2) + (e*(a + c*x^2)^(3/2))/(d^
2*x) + ((c^2*d^3*(e - Sqrt[e^2 - 4*d*f]) + 2*a*c*d^2*f*(e + Sqrt[e^2 - 4*d*
f]) + a^2*f*(e^3 - 3*d*e*f + e^2*Sqrt[e^2 - 4*d*f] - d*f*Sqrt[e^2 - 4*d*f])
)*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(
e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])]])/(Sqrt[2]*d^3*Sqrt[e^
2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) - ((2*a*c
*d^2*f*(e - Sqrt[e^2 - 4*d*f]) + c^2*d^3*(e + Sqrt[e^2 - 4*d*f]) + a^2*f*(e
^3 - 3*d*e*f - e^2*Sqrt[e^2 - 4*d*f] + d*f*Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a
*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f +
e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])]])/(Sqrt[2]*d^3*Sqrt[e^2 - 4*d*f]*Sqr
```

```
t[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]) - (3*Sqrt[a]*c*ArcTanh[
Sqrt[a + c*x^2]/Sqrt[a]])/(2*d) - (a^(3/2)*(e^2 - d*f)*ArcTanh[Sqrt[a + c*x
^2]/Sqrt[a]])/d^3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1034

```
Int[((g_.) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[h*(a + c*x^2)^p*((d + e*x + f*x^2)^(q +
 1)/(2*f*(p + q + 1))), x] + Dist[1/(2*f*(p + q + 1)), Int[(a + c*x^2)^(p -
 1)*(d + e*x + f*x^2)^q*Simp[a*h*e*p - a*(h*e - 2*g*f)*(p + q + 1) - 2*h*p*
(c*d - a*f)*x - (h*c*e*p + c*(h*e - 2*g*f)*(p + q + 1))*x^2, x], x], x] /;
FreeQ[{a, c, d, e, f, g, h, q}, x] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && N
eQ[p + q + 1, 0]
```

Rule 1048

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1082

```
Int[((a_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)*((d_) +
(e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(B*c*f*(2*p + 2*q + 3) +
```

```
 C*((-c)*e*(2*p + q + 2)) + 2*c*C*f*(p + q + 1)*x)*(a + c*x^2)^p*((d + e*x
+ f*x^2)^(q + 1)/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3))), x] - Dist[1/(2*c*f
^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + c*x^2)^(p - 1)*(d + e*x + f*x^2)^
q*Simp[p*((-a)*e)*(C*(c*e)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p +
q + 1)*(a*c*(C*(2*d*f - e^2*(2*p + q + 2)) + f*(B*e - 2*A*f)*(2*p + 2*q + 3
))) + (2*p*(c*d - a*f)*(C*(c*e)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) +
(p + q + 1)*(C*e*f*p*(-4*a*c)))*x + (p*(c*e)*(C*(c*e)*(q + 1) - c*(C*e - B*
f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(-4*a*c) - c^2*(C*(e^2 - 4*d*f)*
(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3))))*x^2, x], x], x]
/; FreeQ[{a, c, d, e, f, A, B, C, q}, x] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0
] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*q + 3, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q
, 0]
```

Rule 1094

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + f*x^2],
 x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt
[d + f*x^2]), x], x] /; FreeQ[{a, b, c, d, f, A, B, C}, x] && NeQ[b^2 - 4*a
*c, 0]
```

Rule 6860

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{(a+cx^2)^{3/2}}{dx^3} - \frac{e(a+cx^2)^{3/2}}{d^2x^2} + \frac{(e^2-df)(a+cx^2)^{3/2}}{d^3x} + \frac{(-e(e^2-2df)-f(e^2-df)x)(a+cx^2)^{3/2}}{d^3(d+ex+fx^2)}\right) dx$$

$$= \frac{\int \frac{(-e(e^2-2df)-f(e^2-df)x)(a+cx^2)^{3/2}}{d+ex+fx^2}\,dx}{d^3} + \frac{\int \frac{(a+cx^2)^{3/2}}{x^3}\,dx}{d} - \frac{e\int \frac{(a+cx^2)^{3/2}}{x^2}\,dx}{d^2} + \frac{(e^2-df)\int \frac{(a+cx^2)^{3/2}}{x}\,dx}{d^3}$$

$$= -\frac{(e^2-df)\,(a+cx^2)^{3/2}}{3d^3} + \frac{e(a+cx^2)^{3/2}}{d^2x} + \frac{\text{Subst}\left(\int \frac{(a+cx)^{3/2}}{x^2}\,dx, x, x^2\right)}{2d}$$
$$- \frac{(3ce)\int \sqrt{a+cx^2}\,dx}{d^2} + \frac{\int \frac{\sqrt{a+cx^2}\left(-3aef\left(e^2-2df\right)+3f(cd-af)\left(e^2-df\right)x+3cdef^2x^2\right)}{d+ex+fx^2}\,dx}{3d^3f}$$
$$+ \frac{(e^2-df)\,\text{Subst}\left(\int \frac{(a+cx)^{3/2}}{x}\,dx, x, x^2\right)}{2d^3}$$

$$= -\frac{3cex\sqrt{a+cx^2}}{2d^2} - \frac{(2(cd^2+a(e^2-df))-cdex)\sqrt{a+cx^2}}{2d^3} - \frac{(a+cx^2)^{3/2}}{2dx^2}$$
$$+ \frac{e(a+cx^2)^{3/2}}{d^2x} + \frac{(3c)\text{Subst}\left(\int \frac{\sqrt{a+cx}}{x}\,dx, x, x^2\right)}{4d} - \frac{(3ace)\int \frac{1}{\sqrt{a+cx^2}}\,dx}{2d^2}$$
$$- \frac{\int \frac{3acef^3\left(cd^2+2a\left(e^2-2df\right)\right)-3cf^3\left(2c^2d^3+acd\left(3e^2-4df\right)-2a^2f\left(e^2-df\right)\right)x-9ac^2def^4x^2}{\sqrt{a+cx^2}(d+ex+fx^2)}\,dx}{6cd^3f^3}$$
$$+ \frac{(a(e^2-df))\,\text{Subst}\left(\int \frac{\sqrt{a+cx}}{x}\,dx, x, x^2\right)}{2d^3}$$

$$= \frac{3c\sqrt{a+cx^2}}{2d} + \frac{a(e^2-df)\sqrt{a+cx^2}}{d^3} - \frac{3cex\sqrt{a+cx^2}}{2d^2}$$
$$- \frac{(2(cd^2+a(e^2-df))-cdex)\sqrt{a+cx^2}}{2d^3} - \frac{(a+cx^2)^{3/2}}{2dx^2}$$
$$+ \frac{e(a+cx^2)^{3/2}}{d^2x} + \frac{(3ac)\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx, x, x^2\right)}{4d}$$
$$+ \frac{(3ace)\int \frac{1}{\sqrt{a+cx^2}}\,dx}{2d^2} - \frac{(3ace)\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{2d^2}$$
$$- \frac{\int \frac{9ac^2d^2ef^4+3acef^4\left(cd^2+2a\left(e^2-2df\right)\right)+\left(9ac^2de^2f^4-3cf^4\left(2c^2d^3+acd\left(3e^2-4df\right)-2a^2f\left(e^2-df\right)\right)\right)x}{\sqrt{a+cx^2}(d+ex+fx^2)}\,dx}{6cd^3f^4}$$
$$+ \frac{(a^2(e^2-df))\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx, x, x^2\right)}{2d^3}$$

$$= \frac{3c\sqrt{a+cx^2}}{2d} + \frac{a(e^2-df)\sqrt{a+cx^2}}{d^3} - \frac{3cex\sqrt{a+cx^2}}{2d^2} - \frac{(2(cd^2+a(e^2-df))-cdex)\sqrt{a+cx^2}}{2d^3} - \frac{(a+cx^2)^{3/2}}{2dx^2} + \frac{e(a+cx^2)^{3/2}}{d^2x} - \frac{3a\sqrt{c}e\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2d^2} + \frac{(3a)\text{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx, x, \sqrt{a+cx^2}\right)}{2d} + \frac{(3ace)\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx, x, \frac{x}{\sqrt{a+cx^2}}\right)}{2d^2} + \frac{(a^2(e^2-df))\,\text{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx, x, \sqrt{a+cx^2}\right)}{cd^3} - \frac{\left(c^2d^3\left(e-\sqrt{e^2-4df}\right)+2acd^2f\left(e+\sqrt{e^2-4df}\right)+a^2f\left(e^3-3def+e^2\sqrt{e^2-4df}-df\sqrt{e^2-4\right.}{d^3\sqrt{e^2-4df}} + \frac{\left(2acd^2f\left(e-\sqrt{e^2-4df}\right)+c^2d^3\left(e+\sqrt{e^2-4df}\right)+a^2f\left(e^3-3def-e^2\sqrt{e^2-4df}+df\sqrt{e^2-4\right.}{d^3\sqrt{e^2-4df}}$$

$$= \frac{3c\sqrt{a+cx^2}}{2d} + \frac{a(e^2-df)\sqrt{a+cx^2}}{d^3} - \frac{3cex\sqrt{a+cx^2}}{2d^2} - \frac{(2(cd^2+a(e^2-df))-cdex)\sqrt{a+cx^2}}{2d^3} - \frac{(a+cx^2)^{3/2}}{2dx^2} + \frac{e(a+cx^2)^{3/2}}{d^2x} - \frac{3\sqrt{a}c\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2d} - \frac{a^{3/2}(e^2-df)\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^3} + \frac{\left(c^2d^3\left(e-\sqrt{e^2-4df}\right)+2acd^2f\left(e+\sqrt{e^2-4df}\right)+a^2f\left(e^3-3def+e^2\sqrt{e^2-4df}-df\sqrt{e^2-4\right.}{d^3\sqrt{e^2-4df}} - \frac{\left(2acd^2f\left(e-\sqrt{e^2-4df}\right)+c^2d^3\left(e+\sqrt{e^2-4df}\right)+a^2f\left(e^3-3def-e^2\sqrt{e^2-4df}+df\sqrt{e^2-4\right.}{d^3\sqrt{e^2-4df}}$$

$$= \frac{3c\sqrt{a+cx^2}}{2d} + \frac{a(e^2-df)\sqrt{a+cx^2}}{d^3} - \frac{3cex\sqrt{a+cx^2}}{2d^2}$$
$$- \frac{(2(cd^2+a(e^2-df))-cdex)\sqrt{a+cx^2}}{2d^3} - \frac{(a+cx^2)^{3/2}}{2dx^2} + \frac{e(a+cx^2)^{3/2}}{d^2x}$$
$$+ \frac{\left(c^2d^3\left(e-\sqrt{e^2-4df}\right)+2acd^2f\left(e+\sqrt{e^2-4df}\right)+a^2f\left(e^3-3def+e^2\sqrt{e^2-4df}-df\sqrt{e^2-4d\right.\right.}{\sqrt{2}d^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4d\right.}}$$
$$- \frac{\left(2acd^2f\left(e-\sqrt{e^2-4df}\right)+c^2d^3\left(e+\sqrt{e^2-4df}\right)+a^2f\left(e^3-3def-e^2\sqrt{e^2-4df}+df\sqrt{e^2-4d\right.\right.}{\sqrt{2}d^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4d\right.}}$$
$$- \frac{3\sqrt{a}c\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2d} - \frac{a^{3/2}(e^2-df)\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^3}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.78 (sec) , antiderivative size = 617, normalized size of antiderivative = 0.92

$$\int \frac{(a+cx^2)^{3/2}}{x^3(d+ex+fx^2)}\,dx = \frac{\frac{ad(-d+2ex)\sqrt{a+cx^2}}{x^2} + 6\sqrt{a}cd^2\text{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+cx^2}}{\sqrt{a}}\right) - 4a^{3/2}(e^2-df)\,\text{arctanh}\left(\frac{-\sqrt{c}x+\sqrt{}}{\sqrt{}}\right.}{}$$

```
[In] Integrate[(a + c*x^2)^(3/2)/(x^3*(d + e*x + f*x^2)),x]
```

```
[Out] ((a*d*(-d + 2*e*x)*Sqrt[a + c*x^2])/x^2 + 6*Sqrt[a]*c*d^2*ArcTanh[(Sqrt[c]*
x - Sqrt[a + c*x^2])/Sqrt[a]] - 4*a^(3/2)*(e^2 - d*f)*ArcTanh[(-(Sqrt[c]*x)
 + Sqrt[a + c*x^2])/Sqrt[a]] - 2*RootSum[a^2*f + 2*a*Sqrt[c]*e*#1 + 4*c*d*#
1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (a*c^2*d^3*Log[-(Sqrt[c]*x
) + Sqrt[a + c*x^2] - #1] - 2*a^2*c*d^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2
] - #1] - a^3*e^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] + a^3*d*f^2*Lo
g[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] - 4*a*c^(3/2)*d^2*e*Log[-(Sqrt[c]*x)
 + Sqrt[a + c*x^2] - #1]*#1 - 2*a^2*Sqrt[c]*e^3*Log[-(Sqrt[c]*x) + Sqrt[a +
 c*x^2] - #1]*#1 + 4*a^2*Sqrt[c]*d*e*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] -
 #1]*#1 - c^2*d^3*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2 + 2*a*c*d^2
*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2 + a^2*e^2*f*Log[-(Sqrt[c]*
x) + Sqrt[a + c*x^2] - #1]*#1^2 - a^2*d*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x
^2] - #1]*#1^2)/(a*Sqrt[c]*e + 4*c*d*#1 - 2*a*f*#1 - 3*Sqrt[c]*e*#1^2 + 2*f
*#1^3) & ])/(2*d^3)
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 911, normalized size of antiderivative = 1.36

method	result
risch	$-\frac{a\sqrt{c\,x^2+a}\,(-2ex+d)}{2d^2x^2}-\frac{\frac{4f\sqrt{a}\left(2adf-2e^2a-3c\,d^2\right)\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c\,x^2+a}}{x}\right)}{\left(-e+\sqrt{-4df+e^2}\right)\left(e+\sqrt{-4df+e^2}\right)}-\frac{\left(4a^2e\,f^2\sqrt{-4df+e^2}-4\sqrt{-4df+e^2}\,c^2d^2e+8a^2d\,f^3-4a^2e^2f^2\right.}{}}{}$
default	Expression too large to display

[In] int((c*x^2+a)^(3/2)/x^3/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)

[Out] -1/2*a*(c*x^2+a)^(1/2)*(-2*e*x+d)/d^2/x^2-1/2/d^2*(4*f*a^(1/2)*(2*a*d*f-2*a*e^2-3*c*d^2)/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))*ln((2*a+2*a^(1/2)*(c*x^2+a)^(1/2))/x)-1/2*(4*a^2*e*f^2*(-4*d*f+e^2)^(1/2)-4*(-4*d*f+e^2)^(1/2)*c^2*d^2*e+8*a^2*d*f^3-4*a^2*e^2*f^2-16*a*c*d^2*f^2+8*d^3*f*c^2-4*c^2*d^2*e^2)/(-4*d*f+e^2)^(1/2)/(e+(-4*d*f+e^2)^(1/2))/f*2^(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))-1/2*(-4*a^2*e*f^2*(-4*d*f+e^2)^(1/2)+4*(-4*d*f+e^2)^(1/2)*c^2*d^2*e+8*a^2*d*f^3-4*a^2*e^2*f^2-16*a*c*d^2*f^2+8*d^3*f*c^2-4*c^2*d^2*e^2)/(-4*d*f+e^2)^(1/2)/(-e+(-4*d*f+e^2)^(1/2))/f*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))))

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+cx^2)^{3/2}}{x^3(d+ex+fx^2)}\,dx = \text{Timed out}$$

```
[In] integrate((c*x^2+a)^(3/2)/x^3/(f*x^2+e*x+d),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+cx^2)^{3/2}}{x^3(d+ex+fx^2)}\,dx = \text{Timed out}$$

```
[In] integrate((c*x**2+a)**(3/2)/x**3/(f*x**2+e*x+d),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+cx^2)^{3/2}}{x^3(d+ex+fx^2)}\,dx = \int \frac{(cx^2+a)^{\frac{3}{2}}}{(fx^2+ex+d)x^3}\,dx$$

```
[In] integrate((c*x^2+a)^(3/2)/x^3/(f*x^2+e*x+d),x, algorithm="maxima")
[Out] integrate((c*x^2 + a)^(3/2)/((f*x^2 + e*x + d)*x^3), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+cx^2)^{3/2}}{x^3(d+ex+fx^2)}\,dx = \text{Exception raised: AttributeError}$$

```
[In] integrate((c*x^2+a)^(3/2)/x^3/(f*x^2+e*x+d),x, algorithm="giac")
[Out] Exception raised: AttributeError >> type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+cx^2)^{3/2}}{x^3(d+ex+fx^2)}\,dx=\int \frac{\left(c\,x^2+a\right)^{3/2}}{x^3\,\left(f\,x^2+e\,x+d\right)}\,dx$$

```
[In] int((a + c*x^2)^(3/2)/(x^3*(d + e*x + f*x^2)),x)
[Out] int((a + c*x^2)^(3/2)/(x^3*(d + e*x + f*x^2)), x)
```

3.64 $\int \frac{x^3}{\sqrt{a+cx^2}(d+ex+fx^2)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 380

$$
\begin{aligned}
&\int \frac{x^3}{\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx \\
&= \frac{\sqrt{a+cx^2}}{cf} - \frac{e\,\mathrm{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}f^2} \\
&\quad - \frac{\left(2def-(e^2-df)\left(e-\sqrt{e^2-4df}\right)\right)\mathrm{arctanh}\left(\frac{2af-c\left(e-\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}} \\
&\quad + \frac{\left(2def-(e^2-df)\left(e+\sqrt{e^2-4df}\right)\right)\mathrm{arctanh}\left(\frac{2af-c\left(e+\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}}
\end{aligned}
$$

```
[Out] -e*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/f^2/c^(1/2)+(c*x^2+a)^(1/2)/c/f-1/2*a
rctanh(1/2*(2*a*f-c*x*(e-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*
f^2+c*(e^2-2*d*f-e*(-4*d*f+e^2)^(1/2)))^(1/2))*(2*d*e*f-(-d*f+e^2)*(e-(-4*d
*f+e^2)^(1/2)))/f^2*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+c*(e^2-2*d*f-e*(-4*
d*f+e^2)^(1/2)))^(1/2)+1/2*arctanh(1/2*(2*a*f-c*x*(e+(-4*d*f+e^2)^(1/2)))*2
^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+c*(e^2-2*d*f+e*(-4*d*f+e^2)^(1/2)))^(1/2))*
(2*d*e*f-(-d*f+e^2)*(e+(-4*d*f+e^2)^(1/2)))/f^2*2^(1/2)/(-4*d*f+e^2)^(1/2)/
(2*a*f^2+c*(e^2-2*d*f+e*(-4*d*f+e^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 380, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6860, 223, 212, 267, 1048, 739}

$$\int \frac{x^3}{\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx$$
$$= -\frac{\left(2def-(e^2-df)\left(e-\sqrt{e^2-4df}\right)\right)\operatorname{arctanh}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}$$
$$+\frac{\left(2def-(e^2-df)\left(\sqrt{e^2-4df}+e\right)\right)\operatorname{arctanh}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}$$
$$-\frac{e\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}f^2}+\frac{\sqrt{a+cx^2}}{cf}$$

```
[In] Int[x^3/(Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] Sqrt[a + c*x^2]/(c*f) - (e*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(Sqrt[c]*f
^2) - ((2*d*e*f - (e^2 - d*f)*(e - Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(
e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e
^2 - 4*d*f])]*Sqrt[a + c*x^2])]])/(Sqrt[2]*f^2*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^
2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) + ((2*d*e*f - (e^2 - d*f)*(e +
Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*
Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])]])/(S
qrt[2]*f^2*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4
*d*f])])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1048

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 6860

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{e}{f^2\sqrt{a+cx^2}}+\frac{x}{f\sqrt{a+cx^2}}+\frac{de+(e^2-df)\,x}{f^2\sqrt{a+cx^2}\,(d+ex+fx^2)}\right)dx \\
&= \frac{\int \frac{de+(e^2-df)x}{\sqrt{a+cx^2}(d+ex+fx^2)}\,dx}{f^2}-\frac{e\int \frac{1}{\sqrt{a+cx^2}}\,dx}{f^2}+\frac{\int \frac{x}{\sqrt{a+cx^2}}\,dx}{f} \\
&= \frac{\sqrt{a+cx^2}}{cf}-\frac{e\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx,x,\frac{x}{\sqrt{a+cx^2}}\right)}{f^2} \\
&\quad+\frac{\left(2def-(e^2-df)\left(e-\sqrt{e^2-4df}\right)\right)\int \frac{1}{\left(e-\sqrt{e^2-4df}+2fx\right)\sqrt{a+cx^2}}\,dx}{f^2\sqrt{e^2-4df}} \\
&\quad-\frac{\left(2def-(e^2-df)\left(e+\sqrt{e^2-4df}\right)\right)\int \frac{1}{\left(e+\sqrt{e^2-4df}+2fx\right)\sqrt{a+cx^2}}\,dx}{f^2\sqrt{e^2-4df}}
\end{aligned}$$

$$= \frac{\sqrt{a+cx^2}}{cf} - \frac{e\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}f^2} - \frac{(2def-(e^2-df))\left(e-\sqrt{e^2-4df}\right)\text{Subst}\left(\int \frac{1}{4af^2+c\left(e-\sqrt{e^2-4df}\right)^2-x^2}dx, x, \frac{2af-c\left(e-\sqrt{e^2-4df}\right)x}{\sqrt{a+cx^2}}\right)}{f^2\sqrt{e^2-4df}} + \frac{(2def-(e^2-df))\left(e+\sqrt{e^2-4df}\right)\text{Subst}\left(\int \frac{1}{4af^2+c\left(e+\sqrt{e^2-4df}\right)^2-x^2}dx, x, \frac{2af-c\left(e+\sqrt{e^2-4df}\right)x}{\sqrt{a+cx^2}}\right)}{f^2\sqrt{e^2-4df}}$$

$$= \frac{\sqrt{a+cx^2}}{cf} - \frac{e\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}f^2} - \frac{(2def-(e^2-df))\left(e-\sqrt{e^2-4df}\right)\tanh^{-1}\left(\frac{2af-c\left(e-\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}} + \frac{(2def-(e^2-df))\left(e+\sqrt{e^2-4df}\right)\tanh^{-1}\left(\frac{2af-c\left(e+\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.36 (sec) , antiderivative size = 312, normalized size of antiderivative = 0.82

$$\int \frac{x^3}{\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx$$

$$= \frac{f\sqrt{a+cx^2}+\sqrt{c}e\log\left(-\sqrt{c}x+\sqrt{a+cx^2}\right)-c\text{RootSum}\left[a^2f+2a\sqrt{c}e\#1+4cd\#1^2-2af\#1^2-2\sqrt{c}e\#\right.}{}$$

```
[In] Integrate[x^3/(Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] (f*Sqrt[a + c*x^2] + Sqrt[c]*e*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]] - c*Root
Sum[a^2*f + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 +
 f*#1^4 & , (a*e^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] - a*d*f*Log[-(S
qrt[c]*x) + Sqrt[a + c*x^2] - #1] + 2*Sqrt[c]*d*e*Log[-(Sqrt[c]*x) + Sqrt[a
 + c*x^2] - #1]*#1 - e^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2 + d*
f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2)/(a*Sqrt[c]*e + 4*c*d*#1 -
2*a*f*#1 - 3*Sqrt[c]*e*#1^2 + 2*f*#1^3) & ])/(c*f^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 712 vs. 2(335) = 670.

Time = 0.78 (sec) , antiderivative size = 713, normalized size of antiderivative = 1.88

method	result
default	$\frac{\sqrt{c\,x^2+a}}{cf}-\frac{e\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{f^2\sqrt{c}}-\frac{\left(e^3-3def+e^2\sqrt{-4df+e^2}-df\sqrt{-4df+e^2}\right)\sqrt{2}\,\ln\left(\frac{\frac{\sqrt{-4df+e^2}\,ce+2a\,f^2-2cdf+c\,e^2}{f^2}-\frac{c\left(e+\sqrt{-4df\right.}}{}}{}\right.}{}$
	$\frac{\left(df\sqrt{-4df+e^2}-e^2\sqrt{-4df+e^2}+3def-e^3\right)\sqrt{2}\,\ln\left(\frac{\frac{\sqrt{-4df+e^2}\,ce+2a\,f^2-2cdf+c\,e^2}{f^2}-\frac{c\left(e+\sqrt{-4df+e^2}\right)\left(x+\right.}{f}}{}\right.}{}$
risch	$\frac{\sqrt{c\,x^2+a}}{cf}-\frac{\frac{e\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{f\sqrt{c}}-}{}$

```
[In] int(x^3/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (c*x^2+a)^(1/2)/c/f-e/f^2*ln(x*c^(1/2)+(c*x^2+a)^(1/2))/c^(1/2)-1/2*(e^3-3*
d*e*f+e^2*(-4*d*f+e^2)^(1/2)-d*f*(-4*d*f+e^2)^(1/2))/f^3/(-4*d*f+e^2)^(1/2)
*2^(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-
4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*
(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f
^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e
+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/
2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))
-1/2*(-d*f*(-4*d*f+e^2)^(1/2)+e^2*(-4*d*f+e^2)^(1/2)+3*d*e*f-e^3)/f^3/(-4*d
*f+e^2)^(1/2)*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)
^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e-(-4*d*f
+e^2)^(1/2))/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+1/2*2^(1/2)*((-(-4*d*f+e^2
)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2/f*(-e+(-4*d*f+e^2)^
(1/2)))^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+
2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2/f*(-e+
(-4*d*f+e^2)^(1/2))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx = \text{Timed out}$$

```
[In] integrate(x^3/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{x^3}{\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx = \int \frac{x^3}{\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx$$

```
[In] integrate(x**3/(f*x**2+e*x+d)/(c*x**2+a)**(1/2),x)
[Out] Integral(x**3/(sqrt(a + c*x**2)*(d + e*x + f*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^3/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re deta
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\,dx=\int \frac{x^3}{\sqrt{c\,x^2+a}\,\left(f\,x^2+e\,x+d\right)} \,dx$$

```
[In] int(x^3/((a + c*x^2)^(1/2)*(d + e*x + f*x^2)),x)
[Out] int(x^3/((a + c*x^2)^(1/2)*(d + e*x + f*x^2)), x)
```

3.65 $\int \frac{x^2}{\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 344

$$\int \frac{x^2}{\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}f} - \frac{\left(e^2-2df-e\sqrt{e^2-4df}\right)\operatorname{arctanh}\left(\frac{2af-c\left(e-\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}f\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}} - \frac{\left(2df-e\left(e+\sqrt{e^2-4df}\right)\right)\operatorname{arctanh}\left(\frac{2af-c\left(e+\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}f\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}}$$

```
[Out] arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/f/c^(1/2)-1/2*arctanh(1/2*(2*a*f-c*x*(e-
(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+c*(e^2-2*d*f-e*(-4*d*
f+e^2)^(1/2)))^(1/2))*(e^2-2*d*f-e*(-4*d*f+e^2)^(1/2))/f*2^(1/2)/(-4*d*f+e^
2)^(1/2)/(2*a*f^2+c*(e^2-2*d*f-e*(-4*d*f+e^2)^(1/2)))^(1/2)-1/2*arctanh(1/2
*(2*a*f-c*x*(e+(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+c*(e^2
-2*d*f+e*(-4*d*f+e^2)^(1/2)))^(1/2))*(2*d*f-e*(e+(-4*d*f+e^2)^(1/2)))/f*2^(
1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+c*(e^2-2*d*f+e*(-4*d*f+e^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1095, 223, 212, 1048, 739}

$$\int \frac{x^2}{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\,dx$$
$$= -\frac{\left(-e\sqrt{e^2-4df}-2df+e^2\right)\operatorname{arctanh}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}f\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}$$
$$-\frac{\left(2df-e\left(\sqrt{e^2-4df}+e\right)\right)\operatorname{arctanh}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}f\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}$$
$$+\frac{\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}f}$$

```
[In] Int[x^2/(Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]]/(Sqrt[c]*f) - ((e^2 - 2*d*f - e*Sqrt[e
^2 - 4*d*f])*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*
a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*
f*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])])
- ((2*d*f - e*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4
*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sq
rt[a + c*x^2])])/(Sqrt[2]*f*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f
 + e*Sqrt[e^2 - 4*d*f])])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
```

```
[{a, c, d, e}, x]
```

Rule 1048

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1095

```
Int[((A_.) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
 (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + f*x^2], x], x] + Dis
t[1/c, Int[(A*c - a*C - b*C*x)/((a + b*x + c*x^2)*Sqrt[d + f*x^2]), x], x]
/; FreeQ[{a, b, c, d, f, A, C}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \frac{1}{\sqrt{a+cx^2}}\,dx}{f}+\frac{\int \frac{-d-ex}{\sqrt{a+cx^2}(d+ex+fx^2)}\,dx}{f}\\
&= \frac{\text{Subst}\left(\int \frac{1}{1-cx^2}\,dx,x,\frac{x}{\sqrt{a+cx^2}}\right)}{f}\\
&\quad+\frac{\left(e^2-2df-e\sqrt{e^2-4df}\right)\int \frac{1}{\left(e-\sqrt{e^2-4df}+2fx\right)\sqrt{a+cx^2}}\,dx}{f\sqrt{e^2-4df}}\\
&\quad-\frac{\left(-2df+e\left(e+\sqrt{e^2-4df}\right)\right)\int \frac{1}{\left(e+\sqrt{e^2-4df}+2fx\right)\sqrt{a+cx^2}}\,dx}{f\sqrt{e^2-4df}}\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}f}\\
&\quad-\frac{\left(e^2-2df-e\sqrt{e^2-4df}\right)\text{Subst}\left(\int \frac{1}{4af^2+c\left(e-\sqrt{e^2-4df}\right)^2-x^2}\,dx,x,\frac{2af-c\left(e-\sqrt{e^2-4df}\right)x}{\sqrt{a+cx^2}}\right)}{f\sqrt{e^2-4df}}\\
&\quad+\frac{\left(-2df+e\left(e+\sqrt{e^2-4df}\right)\right)\text{Subst}\left(\int \frac{1}{4af^2+c\left(e+\sqrt{e^2-4df}\right)^2-x^2}\,dx,x,\frac{2af-c\left(e+\sqrt{e^2-4df}\right)x}{\sqrt{a+cx^2}}\right)}{f\sqrt{e^2-4df}}
\end{aligned}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}f} - \frac{\left(e^2-2df-e\sqrt{e^2-4df}\right)\tanh^{-1}\left(\frac{2af-c\left(e-\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}f\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}}$$
$$- \frac{\left(2df-e\left(e+\sqrt{e^2-4df}\right)\right)\tanh^{-1}\left(\frac{2af-c\left(e+\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}f\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.36 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.76

$$\int \frac{x^2}{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\,dx$$
$$= \frac{\frac{2\operatorname{arctanh}\left(\frac{\sqrt{c}x}{-\sqrt{a}+\sqrt{a+cx^2}}\right)}{\sqrt{c}} + \operatorname{RootSum}\left[c^2d+2\sqrt{a}ce\#1-2cd\#1^2+4af\#1^2-2\sqrt{a}e\#1^3+d\#1^4\&, \frac{-cd\log(x)+c\ldots}{\ldots}\right.}{\ldots}$$

```
[In] Integrate[x^2/(Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] ((2*ArcTanh[(Sqrt[c]*x)/(-Sqrt[a] + Sqrt[a + c*x^2])])/Sqrt[c] + RootSum[c^
2*d + 2*Sqrt[a]*c*e*#1 - 2*c*d*#1^2 + 4*a*f*#1^2 - 2*Sqrt[a]*e*#1^3 + d*#1^
4 & , (-(c*d*Log[x]) + c*d*Log[-Sqrt[a] + Sqrt[a + c*x^2] - x*#1] - 2*Sqrt[
a]*e*Log[x]*#1 + 2*Sqrt[a]*e*Log[-Sqrt[a] + Sqrt[a + c*x^2] - x*#1]*#1 + d*
Log[x]*#1^2 - d*Log[-Sqrt[a] + Sqrt[a + c*x^2] - x*#1]*#1^2)/(-(Sqrt[a]*c*e
) + 2*c*d*#1 - 4*a*f*#1 + 3*Sqrt[a]*e*#1^2 - 2*d*#1^3) & ])/f
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 662 vs. 2(300) = 600.

Time = 0.74 (sec) , antiderivative size = 663, normalized size of antiderivative = 1.93

method	result
default	$\frac{\ln\left(x\sqrt{c}+\sqrt{c\,x^{2}+a}\right)}{f\sqrt{c}}-\frac{\left(-e\sqrt{-4df+e^{2}}+2df-e^{2}\right)\sqrt{2}\,\ln\left(\frac{\sqrt{-4df+e^{2}}\,ce+2a\,f^{2}-2cdf+c\,e^{2}}{f^{2}}-\frac{c\left(e+\sqrt{-4df+e^{2}}\right)\left(x+\frac{e+\sqrt{-4df+e^{2}}}{2f}\right)}{f}+\frac{\sqrt{2}}{}\right.}{2f^{2}\sqrt{-4df}}$

```
[In] int(x^2/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*ln(x*c^(1/2)+(c*x^2+a)^(1/2))/c^(1/2)-1/2*(-e*(-4*d*f+e^2)^(1/2)+2*d*f-
e^2)/f^2/(-4*d*f+e^2)^(1/2)*2^(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*
f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-
c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((
-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*
f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1
/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2
*(e+(-4*d*f+e^2)^(1/2))/f))-1/2*(e^2-2*d*f-e*(-4*d*f+e^2)^(1/2))/f^2/(-4*d*
f+e^2)^(1/2)*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^
(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e-(-4*d*f+
e^2)^(1/2))/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+1/2*2^(1/2)*((-(-4*d*f+e^2)
^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2/f*(-e+(-4*d*f+e^2)^(
1/2)))^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+2
*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2/f*(-e+(
-4*d*f+e^2)^(1/2))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^{2}}{\sqrt{a+cx^{2}}\,(d+ex+fx^{2})}\,dx=\text{Timed out}$$

```
[In] integrate(x^2/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx = \int \frac{x^2}{\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx$$

```
[In] integrate(x**2/(f*x**2+e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(a + c*x**2)*(d + e*x + f*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re deta
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^2}{\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^2/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx = \int \frac{x^2}{\sqrt{c\,x^2+a}\,\left(f\,x^2+e\,x+d\right)}\,dx$$

```
[In] int(x^2/((a + c*x^2)^(1/2)*(d + e*x + f*x^2)),x)
[Out] int(x^2/((a + c*x^2)^(1/2)*(d + e*x + f*x^2)), x)
```

3.66 $\int \frac{x}{\sqrt{a+cx^2}(d+ex+fx^2)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 294

$$\int \frac{x}{\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx = \frac{\left(e-\sqrt{e^2-4df}\right)\operatorname{arctanh}\left(\frac{2af-c\left(e-\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}} - \frac{\left(e+\sqrt{e^2-4df}\right)\operatorname{arctanh}\left(\frac{2af-c\left(e+\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}}$$

```
[Out] 1/2*arctanh(1/2*(2*a*f-c*x*(e-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/
(2*a*f^2+c*(e^2-2*d*f-e*(-4*d*f+e^2)^(1/2)))^(1/2))*(e-(-4*d*f+e^2)^(1/2))*
2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+c*(e^2-2*d*f-e*(-4*d*f+e^2)^(1/2)))^(1/
2)-1/2*arctanh(1/2*(2*a*f-c*x*(e+(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/
2)/(2*a*f^2+c*(e^2-2*d*f+e*(-4*d*f+e^2)^(1/2)))^(1/2))*(e+(-4*d*f+e^2)^(1/2
))*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+c*(e^2-2*d*f+e*(-4*d*f+e^2)^(1/2)))^
(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1048, 739, 212}

$$\int \frac{x}{\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx = \frac{\left(e-\sqrt{e^2-4df}\right)\operatorname{arctanh}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}} - \frac{\left(\sqrt{e^2-4df}+e\right)\operatorname{arctanh}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}$$

```
[In] Int[x/(Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] ((e - Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqr
t[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])
])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 -
4*d*f])]) - ((e + Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d
*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt
[a + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e
*Sqrt[e^2 - 4*d*f])])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1048

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
```

```
b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(\left(-1-\frac{e}{\sqrt{e^2-4df}}\right)\int\frac{1}{\left(e+\sqrt{e^2-4df}+2fx\right)\sqrt{a+cx^2}}\,dx\right) \\
&\quad+\left(1-\frac{e}{\sqrt{e^2-4df}}\right)\int\frac{1}{\left(e-\sqrt{e^2-4df}+2fx\right)\sqrt{a+cx^2}}\,dx \\
&= \left(-1+\frac{e}{\sqrt{e^2-4df}}\right)\text{Subst}\left(\int\frac{1}{4af^2+c\left(e-\sqrt{e^2-4df}\right)^2-x^2}\,dx,x,\frac{2af-c\left(e-\sqrt{e^2-4df}\right)x}{\sqrt{a+cx^2}}\right) \\
&\quad-\left(1+\frac{e}{\sqrt{e^2-4df}}\right)\text{Subst}\left(\int\frac{1}{4af^2+c\left(e+\sqrt{e^2-4df}\right)^2-x^2}\,dx,x,\frac{2af-c\left(e+\sqrt{e^2-4df}\right)x}{\sqrt{a+cx^2}}\right) \\
&= -\frac{\left(1-\frac{e}{\sqrt{e^2-4df}}\right)\tanh^{-1}\left(\frac{2af-c\left(e-\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}} \\
&\quad-\frac{\left(1+\frac{e}{\sqrt{e^2-4df}}\right)\tanh^{-1}\left(\frac{2af-c\left(e+\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.28 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.53

$$\begin{aligned}
&\int\frac{x}{\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx \\
&= \text{RootSum}\Big[a^2f+2a\sqrt{c}e\#1+4cd\#1^2-2af\#1^2-2\sqrt{c}e\#1^3 \\
&\quad+f\#1^4\&,\frac{-a\log\left(-\sqrt{c}x+\sqrt{a+cx^2}-\#1\right)+\log\left(-\sqrt{c}x+\sqrt{a+cx^2}-\#1\right)\#1^2}{a\sqrt{c}e+4cd\#1-2af\#1-3\sqrt{c}e\#1^2+2f\#1^3}\&\Big]
\end{aligned}$$

```
[In] Integrate[x/(Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] RootSum[a^2*f + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1
^3 + f*#1^4 & , (-(a*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]) + Log[-(Sqrt
[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2)/(a*Sqrt[c]*e + 4*c*d*#1 - 2*a*f*#1 - 3
*Sqrt[c]*e*#1^2 + 2*f*#1^3) & ]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 621 vs. $2(257) = 514$.

Time = 0.74 (sec) , antiderivative size = 622, normalized size of antiderivative = 2.12

method	result
default	$-\frac{\left(e+\sqrt{-4df+e^2}\right)\sqrt{2}\,\ln\left(\frac{\sqrt{-4df+e^2}\,ce+2a\,f^2-2cdf+c\,e^2}{f^2}-\frac{c\left(e+\sqrt{-4df+e^2}\right)\left(x+\frac{e+\sqrt{-4df+e^2}}{2f}\right)}{f}+\frac{\sqrt{2}\sqrt{\frac{\sqrt{-4df+e^2}\,ce+2a\,f^2-2cdf+c\,e^2}{f^2}}}{x+\frac{e+\sqrt{-4d}}{2f}}\right.}{2\sqrt{-4df+e^2}\,f\sqrt{\frac{\sqrt{-4df+e^2}\,ce+2a\,f^2-}{f^2}}}$

```
[In] int(x/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)/f*2^(1/2)/(((-4*d*f+e^2)^(1/
2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^
2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2
))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2
)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1
/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^
2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))-1/2*(-e+(-4*d*f+e^2)^(1/2)
)/(-4*d*f+e^2)^(1/2)/f*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*
e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(
e-(-4*d*f+e^2)^(1/2))/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+1/2*2^(1/2)*((-(-
4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2/f*(-e+(-4*
d*f+e^2)^(1/2)))^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2/f*(-e+(-4*d*f+e^2)
^(1/2)))+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1
/2/f*(-e+(-4*d*f+e^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5085 vs. $2(255) = 510$.

Time = 1.03 (sec) , antiderivative size = 5085, normalized size of antiderivative = 17.30

$$\int \frac{x}{\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx = \text{Too large to display}$$

```
[In] integrate(x/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F]

$$\int \frac{x}{\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx = \int \frac{x}{\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx$$

```
[In] integrate(x/(f*x**2+e*x+d)/(c*x**2+a)**(1/2),x)
[Out] Integral(x/(sqrt(a + c*x**2)*(d + e*x + f*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re deta
```

Giac [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx = \text{Timed out}$$

```
[In] integrate(x/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx = \int \frac{x}{\sqrt{c\,x^2+a}\,\left(f\,x^2+e\,x+d\right)}\,dx$$

```
[In] int(x/((a + c*x^2)^(1/2)*(d + e*x + f*x^2)),x)
[Out] int(x/((a + c*x^2)^(1/2)*(d + e*x + f*x^2)), x)
```

3.67 $\int \frac{1}{\sqrt{a+cx^2}(d+ex+fx^2)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 266

$$\int \frac{1}{\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx = -\frac{\sqrt{2}f\operatorname{arctanh}\left(\frac{2af-c\left(e-\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}} + \frac{\sqrt{2}f\operatorname{arctanh}\left(\frac{2af-c\left(e+\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}}$$

```
[Out] -f*arctanh(1/2*(2*a*f-c*x*(e-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(
2*a*f^2+c*(e^2-2*d*f-e*(-4*d*f+e^2)^(1/2)))^(1/2))*2^(1/2)/(-4*d*f+e^2)^(1/
2)/(2*a*f^2+c*(e^2-2*d*f-e*(-4*d*f+e^2)^(1/2)))^(1/2)+f*arctanh(1/2*(2*a*f-
c*x*(e+(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+c*(e^2-2*d*f+e
*(-4*d*f+e^2)^(1/2)))^(1/2))*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+c*(e^2-2*d
*f+e*(-4*d*f+e^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used

= {999, 739, 212}

$$\int \frac{1}{\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx = \frac{\sqrt{2}f\operatorname{arctanh}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}} - \frac{\sqrt{2}f\operatorname{arctanh}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}$$

```
[In] Int[1/(Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] -((Sqrt[2]*f*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*
a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])]])/(Sqrt[e^2
 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])])) + (Sqrt[2
]*f*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c
*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])]])/(Sqrt[e^2 - 4*d*f]
*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 999

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f_.)*(x_)^2]), x_Sym
bol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[1/((b - q + 2*c*x)
*Sqrt[d + f*x^2]), x], x] - Dist[2*(c/q), Int[1/((b + q + 2*c*x)*Sqrt[d + f
*x^2]), x], x]] /; FreeQ[{a, b, c, d, f}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ
[b^2 - 4*a*c]
```

Rubi steps

$$\text{integral} = \frac{(2f)\int \frac{1}{\left(e-\sqrt{e^2-4df}+2fx\right)\sqrt{a+cx^2}}\,dx}{\sqrt{e^2-4df}} - \frac{(2f)\int \frac{1}{\left(e+\sqrt{e^2-4df}+2fx\right)\sqrt{a+cx^2}}\,dx}{\sqrt{e^2-4df}}$$

$$= -\frac{(2f)\text{Subst}\left(\int \frac{1}{4af^2+c\left(e-\sqrt{e^2-4df}\right)^2-x^2}\,dx, x, \frac{2af-c\left(e-\sqrt{e^2-4df}\right)x}{\sqrt{a+cx^2}}\right)}{\sqrt{e^2-4df}} + \frac{(2f)\text{Subst}\left(\int \frac{1}{4af^2+c\left(e+\sqrt{e^2-4df}\right)^2-x^2}\,dx, x, \frac{2af-c\left(e+\sqrt{e^2-4df}\right)x}{\sqrt{a+cx^2}}\right)}{\sqrt{e^2-4df}}$$

$$= -\frac{\sqrt{2}f\tanh^{-1}\left(\frac{2af-c\left(e-\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}} + \frac{\sqrt{2}f\tanh^{-1}\left(\frac{2af-c\left(e+\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.28 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.49

$$\int \frac{1}{\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx$$
$$= -2\sqrt{c}\text{RootSum}\left[a^2f+2a\sqrt{c}e\#1+4cd\#1^2-2af\#1^2-2\sqrt{c}e\#1^3 + f\#1^4\&, \frac{\log\left(-\sqrt{c}x+\sqrt{a+cx^2}-\#1\right)\#1}{a\sqrt{c}e+4cd\#1-2af\#1-3\sqrt{c}e\#1^2+2f\#1^3}\&\right]$$

```
[In] Integrate[1/(Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] -2*Sqrt[c]*RootSum[a^2*f + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 - 2*a*f*#1^2 - 2*S
qrt[c]*e*#1^3 + f*#1^4 & , (Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1)/(a
*Sqrt[c]*e + 4*c*d*#1 - 2*a*f*#1 - 3*Sqrt[c]*e*#1^2 + 2*f*#1^3) & ]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 588 vs. 2(232) = 464.

Time = 0.63 (sec) , antiderivative size = 589, normalized size of antiderivative = 2.21

method	result
default	$\frac{\sqrt{2}\,\ln\left(\frac{\frac{\sqrt{-4df+e^2}\,ce+2a\,f^2-2cdf+c\,e^2}{f^2}-\frac{c\left(e+\sqrt{-4df+e^2}\right)\left(x+\frac{e+\sqrt{-4df+e^2}}{2f}\right)}{f}+\frac{\sqrt{2}\,\sqrt{\frac{\sqrt{-4df+e^2}\,ce+2a\,f^2-2cdf+c\,e^2}{f^2}}\,\sqrt{4\left(x+\frac{e+\sqrt{-4df+e}}{2f}\right.}}{x+\frac{e+\sqrt{-4df+e^2}}{2f}}\right.}{\sqrt{-4df+e^2}\,\sqrt{\frac{\sqrt{-4df+e^2}\,ce+2a\,f^2-2cdf+c\,e^2}{f^2}}}$

```
[In] int(1/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(-4*d*f+e^2)^(1/2)*2^(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2
)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-
4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+
e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^
(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)
+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4
*d*f+e^2)^(1/2))/f))-1/(-4*d*f+e^2)^(1/2)*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e
+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c
*d*f+c*e^2)/f^2-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2))
)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(
4*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2
/f*(-e+(-4*d*f+e^2)^(1/2)))+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^
2)/f^2)^(1/2))/(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5073 vs. 2(230) = 460.

Time = 1.03 (sec) , antiderivative size = 5073, normalized size of antiderivative = 19.07

$$\int \frac{1}{\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx = \int \frac{1}{\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx$$

```
[In] integrate(1/(f*x**2+e*x+d)/(c*x**2+a)**(1/2),x)
[Out] Integral(1/(sqrt(a + c*x**2)*(d + e*x + f*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re deta
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx = \text{Timed out}$$

```
[In] integrate(1/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx = \int \frac{1}{\sqrt{c\,x^2+a}\,\left(f\,x^2+e\,x+d\right)}\,dx$$

```
[In] int(1/((a + c*x^2)^(1/2)*(d + e*x + f*x^2)),x)
[Out] int(1/((a + c*x^2)^(1/2)*(d + e*x + f*x^2)), x)
```

3.68 $\int \frac{1}{x\sqrt{a+cx^2}(d+ex+fx^2)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 330

$$\int \frac{1}{x\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx = \frac{f\left(e+\sqrt{e^2-4df}\right)\operatorname{arctanh}\left(\frac{2af-c\left(e-\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}} - \frac{f\left(e-\sqrt{e^2-4df}\right)\operatorname{arctanh}\left(\frac{2af-c\left(e+\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}d}$$

```
[Out] -arctanh((c*x^2+a)^(1/2)/a^(1/2))/d/a^(1/2)+1/2*f*arctanh(1/2*(2*a*f-c*x*(e
-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+c*(e^2-2*d*f-e*(-4*d
*f+e^2)^(1/2)))^(1/2))*(e+(-4*d*f+e^2)^(1/2))/d*2^(1/2)/(-4*d*f+e^2)^(1/2)/
(2*a*f^2+c*(e^2-2*d*f-e*(-4*d*f+e^2)^(1/2)))^(1/2)-1/2*f*arctanh(1/2*(2*a*f
-c*x*(e+(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+c*(e^2-2*d*f+
e*(-4*d*f+e^2)^(1/2)))^(1/2))*(e-(-4*d*f+e^2)^(1/2))/d*2^(1/2)/(-4*d*f+e^2)
^(1/2)/(2*a*f^2+c*(e^2-2*d*f+e*(-4*d*f+e^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {6860, 272, 65, 214, 1048, 739, 212}

$$\int \frac{1}{x\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx$$
$$= \frac{f\left(\sqrt{e^2-4df}+e\right)\operatorname{arctanh}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}} - \frac{f\left(e-\sqrt{e^2-4df}\right)\operatorname{arctanh}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}d}$$

```
[In] Int[1/(x*Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] (f*(e + Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(S
qrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2
])])/(Sqrt[2]*d*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^
2 - 4*d*f])]) - (f*(e - Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f - c*(e + Sqrt[e^2
 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])
]*Sqrt[a + c*x^2])])/(Sqrt[2]*d*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2
*d*f + e*Sqrt[e^2 - 4*d*f])]) - ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]]/(Sqrt[a]*d
)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1048

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 6860

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{1}{dx\sqrt{a+cx^2}} + \frac{-e-fx}{d\sqrt{a+cx^2}\,(d+ex+fx^2)}\right) dx \\ &= \frac{\int \frac{1}{x\sqrt{a+cx^2}}\,dx}{d} + \frac{\int \frac{-e-fx}{\sqrt{a+cx^2}(d+ex+fx^2)}\,dx}{d} \\ &= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx, x, x^2\right)}{2d} - \frac{\left(f\left(1-\frac{e}{\sqrt{e^2-4df}}\right)\right)\int \frac{1}{\left(e+\sqrt{e^2-4df}+2fx\right)\sqrt{a+cx^2}}\,dx}{d} \\ &\quad - \frac{\left(f\left(1+\frac{e}{\sqrt{e^2-4df}}\right)\right)\int \frac{1}{\left(e-\sqrt{e^2-4df}+2fx\right)\sqrt{a+cx^2}}\,dx}{d}\end{aligned}$$

$$=\frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx,x,\sqrt{a+cx^2}\right)}{cd}+\frac{\left(f\left(1-\frac{e}{\sqrt{e^2-4df}}\right)\right)\text{Subst}\left(\int \frac{1}{4af^2+c\left(e+\sqrt{e^2-4df}\right)^2-x^2}\,dx,x,\frac{2af-c\left(e+\sqrt{e^2-4df}\right)x}{\sqrt{a+cx^2}}\right)}{d}+\frac{\left(f\left(1+\frac{e}{\sqrt{e^2-4df}}\right)\right)\text{Subst}\left(\int \frac{1}{4af^2+c\left(e-\sqrt{e^2-4df}\right)^2-x^2}\,dx,x,\frac{2af-c\left(e-\sqrt{e^2-4df}\right)x}{\sqrt{a+cx^2}}\right)}{d}$$

$$=\frac{f\left(1+\frac{e}{\sqrt{e^2-4df}}\right)\tanh^{-1}\left(\frac{2af-c\left(e-\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}d\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}}+\frac{f\left(1-\frac{e}{\sqrt{e^2-4df}}\right)\tanh^{-1}\left(\frac{2af-c\left(e+\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}d\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}d}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.31 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.72

$$\int \frac{1}{x\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx$$

$$=\frac{\frac{2\text{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}}-\text{RootSum}\left[a^2f+2a\sqrt{c}e\#1+4cd\#1^2-2af\#1^2-2\sqrt{c}e\#1^3+f\#1^4\&,\frac{af\log\left(-\sqrt{c}x\right.}{}\right.}{d}$$

```
[In] Integrate[1/(x*Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] ((2*ArcTanh[(Sqrt[c]*x - Sqrt[a + c*x^2])/Sqrt[a]])/Sqrt[a] - RootSum[a^2*f
 + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 &
 , (a*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] + 2*Sqrt[c]*e*Log[-(Sqrt[c
]*x) + Sqrt[a + c*x^2] - #1]*#1 - f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1
]*#1^2)/(-(a*Sqrt[c]*e) - 4*c*d*#1 + 2*a*f*#1 + 3*Sqrt[c]*e*#1^2 - 2*f*#1^3
) & ])/d
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 680 vs. $2(287) = 574$.

Time = 0.67 (sec) , antiderivative size = 681, normalized size of antiderivative = 2.06

method	result
default	$\frac{4f\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c\,x^{2}+a}}{x}\right)}{\left(-e+\sqrt{-4df+e^{2}}\right)\left(e+\sqrt{-4df+e^{2}}\right)\sqrt{a}}-\frac{2f\sqrt{2}\,\ln\left(\frac{\frac{\sqrt{-4df+e^{2}}\,ce+2a\,f^{2}-2cdf+c\,e^{2}}{f^{2}}-\frac{c\left(e+\sqrt{-4df+e^{2}}\right)\left(x+\frac{e+\sqrt{-4df+e^{2}}}{2f}\right)}{f}+\frac{\sqrt{2}\,\sqrt{\sqrt{\cdots}}}{\cdots}}{\cdots}\right)}{\left(e+\sqrt{-4d\cdots}\right.}$

```
[In] int(1/x/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 4*f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))/a^(1/2)*ln((2*a+2*a^(1/2
)*(c*x^2+a)^(1/2))/x)-2*f/(e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)*2^(1/2)
/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f+e^
2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(
e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d
*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f
+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2
*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))-2*f/(-e
+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2
*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d
*f+c*e^2)/f^2-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+
1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*
(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2/f
*(-e+(-4*d*f+e^2)^(1/2)))+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)
/f^2)^(1/2))/(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx = \text{Timed out}$$

```
[In] integrate(1/x/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{x\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx = \int \frac{1}{x\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx$$

```
[In] integrate(1/x/(f*x**2+e*x+d)/(c*x**2+a)**(1/2),x)
[Out] Integral(1/(x*sqrt(a + c*x**2)*(d + e*x + f*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{x\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx = \int \frac{1}{\sqrt{cx^2+a}(fx^2+ex+d)x}\,dx$$

```
[In] integrate(1/x/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(c*x^2 + a)*(f*x^2 + e*x + d)*x), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx = \text{Timed out}$$

```
[In] integrate(1/x/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx = \int \frac{1}{x\,\sqrt{c\,x^2+a}\,\left(f\,x^2+e\,x+d\right)}\,dx$$

```
[In] int(1/(x*(a + c*x^2)^(1/2)*(d + e*x + f*x^2)),x)
[Out] int(1/(x*(a + c*x^2)^(1/2)*(d + e*x + f*x^2)), x)
```

3.69 $\int \frac{1}{x^2\sqrt{a+cx^2}(d+ex+fx^2)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 367

$$\int \frac{1}{x^2\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx = -\frac{\sqrt{a+cx^2}}{adx} - \frac{f\left(e^2-2df+e\sqrt{e^2-4df}\right)\operatorname{arctanh}\left(\frac{2af-c\left(e-\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}} + \frac{f\left(e^2-2df-e\sqrt{e^2-4df}\right)\operatorname{arctanh}\left(\frac{2af-c\left(e+\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}} + \frac{e\operatorname{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}d^2}$$

```
[Out] e*arctanh((c*x^2+a)^(1/2)/a^(1/2))/d^2/a^(1/2)-(c*x^2+a)^(1/2)/a/d/x-1/2*f*
arctanh(1/2*(2*a*f-c*x*(e-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a
*f^2+c*(e^2-2*d*f-e*(-4*d*f+e^2)^(1/2)))^(1/2))*(e^2-2*d*f+e*(-4*d*f+e^2)^(
1/2))/d^2*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+c*(e^2-2*d*f-e*(-4*d*f+e^2)^(
1/2)))^(1/2)+1/2*f*arctanh(1/2*(2*a*f-c*x*(e+(-4*d*f+e^2)^(1/2)))*2^(1/2)/(
c*x^2+a)^(1/2)/(2*a*f^2+c*(e^2-2*d*f+e*(-4*d*f+e^2)^(1/2)))^(1/2))*(e^2-2*d
*f-e*(-4*d*f+e^2)^(1/2))/d^2*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+c*(e^2-2*d
*f+e*(-4*d*f+e^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {6860, 270, 272, 65, 214, 1048, 739, 212}

$$\int \frac{1}{x^2\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx$$
$$= -\frac{f\left(e\sqrt{e^2-4df}-2df+e^2\right)\operatorname{arctanh}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}$$
$$+\frac{f\left(-e\sqrt{e^2-4df}-2df+e^2\right)\operatorname{arctanh}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}$$
$$+\frac{e\operatorname{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}d^2}-\frac{\sqrt{a+cx^2}}{adx}$$

```
[In] Int[1/(x^2*Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] -(Sqrt[a + c*x^2]/(a*d*x)) - (f*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])*ArcTanh
[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d
*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*d^2*Sqrt[e^2 - 4*d*f
]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) + (f*(e^2 - 2*d*f
- e*Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[
2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])
/(Sqrt[2]*d^2*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2
- 4*d*f])]) + (e*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(Sqrt[a]*d^2)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1048

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 6860

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(\frac{1}{dx^2\sqrt{a+cx^2}} - \frac{e}{d^2x\sqrt{a+cx^2}} + \frac{e^2-df+efx}{d^2\sqrt{a+cx^2}\,(d+ex+fx^2)}\right) dx \\ &= \frac{\int \frac{e^2-df+efx}{\sqrt{a+cx^2}(d+ex+fx^2)}\, dx}{d^2} + \frac{\int \frac{1}{x^2\sqrt{a+cx^2}}\, dx}{d} - \frac{e\int \frac{1}{x\sqrt{a+cx^2}}\, dx}{d^2}\end{aligned}$$

$$= -\frac{\sqrt{a+cx^2}}{adx} - \frac{e\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx, x, x^2\right)}{2d^2} - \frac{\left(f\left(e^2-2df-e\sqrt{e^2-4df}\right)\right)\int \frac{1}{\left(e+\sqrt{e^2-4df}+2fx\right)\sqrt{a+cx^2}}\,dx}{d^2\sqrt{e^2-4df}} + \frac{\left(f\left(e^2-2df+e\sqrt{e^2-4df}\right)\right)\int \frac{1}{\left(e-\sqrt{e^2-4df}+2fx\right)\sqrt{a+cx^2}}\,dx}{d^2\sqrt{e^2-4df}}$$

$$= -\frac{\sqrt{a+cx^2}}{adx} - \frac{e\text{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx, x, \sqrt{a+cx^2}\right)}{cd^2} + \frac{\left(f\left(e^2-2df-e\sqrt{e^2-4df}\right)\right)\text{Subst}\left(\int \frac{1}{4af^2+c\left(e+\sqrt{e^2-4df}\right)^2-x^2}\,dx, x, \frac{2af-c\left(e+\sqrt{e^2-4df}\right)x}{\sqrt{a+cx^2}}\right)}{d^2\sqrt{e^2-4df}} - \frac{\left(f\left(e^2-2df+e\sqrt{e^2-4df}\right)\right)\text{Subst}\left(\int \frac{1}{4af^2+c\left(e-\sqrt{e^2-4df}\right)^2-x^2}\,dx, x, \frac{2af-c\left(e-\sqrt{e^2-4df}\right)x}{\sqrt{a+cx^2}}\right)}{d^2\sqrt{e^2-4df}}$$

$$= -\frac{\sqrt{a+cx^2}}{adx} - \frac{f\left(e^2-2df+e\sqrt{e^2-4df}\right)\tanh^{-1}\left(\frac{2af-c\left(e-\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}} + \frac{f\left(e^2-2df-e\sqrt{e^2-4df}\right)\tanh^{-1}\left(\frac{2af-c\left(e+\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}} + \frac{e\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}d^2}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.39 (sec) , antiderivative size = 298, normalized size of antiderivative = 0.81

$$\int \frac{1}{x^2\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx =$$
$$-\frac{d\sqrt{a+cx^2}+2\sqrt{a}ex\,\mathrm{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+cx^2}}{\sqrt{a}}\right)+ax\,\mathrm{RootSum}\left[a^2f+2a\sqrt{c}e\#1+4cd\#1^2-2af\#1^2-2\sqrt{}\right.}{}$$

```
[In] Integrate[1/(x^2*Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] -((d*Sqrt[a + c*x^2] + 2*Sqrt[a]*e*x*ArcTanh[(Sqrt[c]*x - Sqrt[a + c*x^2])/
Sqrt[a]] + a*x*RootSum[a^2*f + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 - 2*a*f*#1^2 -
 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (a*e*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] -
#1] + 2*Sqrt[c]*e^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 - 2*Sqrt[c]
*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 - e*f*Log[-(Sqrt[c]*x) + S
qrt[a + c*x^2] - #1]*#1^2)/(a*Sqrt[c]*e + 4*c*d*#1 - 2*a*f*#1 - 3*Sqrt[c]*e
*#1^2 + 2*f*#1^3) & ])/(a*d^2*x))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 732 vs. 2(322) = 644.

Time = 0.81 (sec) , antiderivative size = 733, normalized size of antiderivative = 2.00

method	result
risch	$-\frac{\sqrt{c\,x^2+a}}{adx}-\frac{\frac{4fe\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c\,x^2+a}}{x}\right)}{\left(-e+\sqrt{-4df+e^2}\right)\left(e+\sqrt{-4df+e^2}\right)\sqrt{a}}+\frac{f\left(-e+\sqrt{-4df+e^2}\right)\sqrt{2}\,\ln\left(\frac{\sqrt{-4df+e^2}\,ce+2a\,f^2-2cdf+c\,e^2}{f^2}-\frac{c\left(e+\sqrt{-4df+e^2}\right)\left(x+\right.}{f}\right.}{}}{}$
default	$\frac{4f\sqrt{c\,x^2+a}}{\left(-e+\sqrt{-4df+e^2}\right)\left(e+\sqrt{-4df+e^2}\right)ax}+\frac{16f^2e\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c\,x^2+a}}{x}\right)}{\left(-e+\sqrt{-4df+e^2}\right)^2\left(e+\sqrt{-4df+e^2}\right)^2\sqrt{a}}+\frac{4f^2\sqrt{2}\,\ln\left(\frac{\sqrt{-4df+e^2}\,ce+2a\,f^2-2cdf+c\,e^2}{f^2}\right.}{}$

```
[In] int(1/x^2/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(c*x^2+a)^(1/2)/a/d/x-1/d*(4*f*e/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(
1/2))/a^(1/2)*ln((2*a+2*a^(1/2)*(c*x^2+a)^(1/2))/x)+f*(-e+(-4*d*f+e^2)^(1/2
))/(-4*d*f+e^2)^(1/2)/(e+(-4*d*f+e^2)^(1/2))*2^(1/2)/(((-4*d*f+e^2)^(1/2)*c
*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*
c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f
)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4
*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(
e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f
^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))-f*(e+(-4*d*f+e^2)^(1/2))/(-4*d
*f+e^2)^(1/2)/(-e+(-4*d*f+e^2)^(1/2))*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a
*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f
+c*e^2)/f^2-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+1/
2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x
-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2/f*(
-e+(-4*d*f+e^2)^(1/2)))+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f
^2)^(1/2))/(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx = \text{Timed out}$$

```
[In] integrate(1/x^2/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{x^2\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx = \int \frac{1}{x^2\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx$$

```
[In] integrate(1/x**2/(f*x**2+e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(a + c*x**2)*(d + e*x + f*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\,dx=\int \frac{1}{\sqrt{cx^2+a}(fx^2+ex+d)x^2}\,dx$$

```
[In] integrate(1/x^2/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(c*x^2 + a)*(f*x^2 + e*x + d)*x^2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x^2\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\,dx=\text{Exception raised: AttributeError}$$

```
[In] integrate(1/x^2/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
[Out] Exception raised: AttributeError >> type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\,dx=\int \frac{1}{x^2\,\sqrt{c\,x^2+a}\,\left(f\,x^2+e\,x+d\right)}\,dx$$

```
[In] int(1/(x^2*(a + c*x^2)^(1/2)*(d + e*x + f*x^2)),x)
[Out] int(1/(x^2*(a + c*x^2)^(1/2)*(d + e*x + f*x^2)), x)
```

3.70 $\int \frac{1}{x^3\sqrt{a+cx^2}(d+ex+fx^2)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 457

$$
\begin{aligned}
\int \frac{1}{x^3\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx = &-\frac{\sqrt{a+cx^2}}{2adx^2}+\frac{e\sqrt{a+cx^2}}{ad^2x} \\
&+\frac{f\left(2e^3-4def-(e^2-df)\left(e-\sqrt{e^2-4df}\right)\right)\operatorname{arctanh}\left(\frac{2af-c\left(e-\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}d^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}} \\
&-\frac{f\left(2e^3-4def-(e^2-df)\left(e+\sqrt{e^2-4df}\right)\right)\operatorname{arctanh}\left(\frac{2af-c\left(e+\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}d^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}} \\
&+\frac{c\operatorname{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2a^{3/2}d}-\frac{(e^2-df)\operatorname{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}d^3}
\end{aligned}
$$

```
[Out] 1/2*c*arctanh((c*x^2+a)^(1/2)/a^(1/2))/a^(3/2)/d-(-d*f+e^2)*arctanh((c*x^2+
a)^(1/2)/a^(1/2))/d^3/a^(1/2)-1/2*(c*x^2+a)^(1/2)/a/d/x^2+e*(c*x^2+a)^(1/2)
/a/d^2/x+1/2*f*arctanh(1/2*(2*a*f-c*x*(e-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^
2+a)^(1/2)/(2*a*f^2+c*(e^2-2*d*f-e*(-4*d*f+e^2)^(1/2)))^(1/2))*(2*e^3-4*d*e
*f-(-d*f+e^2)*(e-(-4*d*f+e^2)^(1/2)))/d^3*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f
^2+c*(e^2-2*d*f-e*(-4*d*f+e^2)^(1/2)))^(1/2)-1/2*f*arctanh(1/2*(2*a*f-c*x*(
e+(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+c*(e^2-2*d*f+e*(-4*
d*f+e^2)^(1/2)))^(1/2))*(2*e^3-4*d*e*f-(-d*f+e^2)*(e+(-4*d*f+e^2)^(1/2)))/d
^3*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+c*(e^2-2*d*f+e*(-4*d*f+e^2)^(1/2)))^
(1/2)
```

Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 457, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6860, 272, 44, 65, 214, 270, 1048, 739, 212}

$$\int \frac{1}{x^3\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx = \frac{c\,\text{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2a^{3/2}d} - \frac{(e^2-df)\,\text{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}d^3} + \frac{f\left(-(e^2-df)\left(e-\sqrt{e^2-4df}\right)-4def+2e^3\right)\text{arctanh}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}d^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}} - \frac{f\left(-(e^2-df)\left(\sqrt{e^2-4df}+e\right)-4def+2e^3\right)\text{arctanh}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}d^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}} + \frac{e\sqrt{a+cx^2}}{ad^2x} - \frac{\sqrt{a+cx^2}}{2adx^2}$$

```
[In] Int[1/(x^3*Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] -1/2*Sqrt[a + c*x^2]/(a*d*x^2) + (e*Sqrt[a + c*x^2])/(a*d^2*x) + (f*(2*e^3
- 4*d*e*f - (e^2 - d*f)*(e - Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e - Sq
rt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4
*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*d^3*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*
(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) - (f*(2*e^3 - 4*d*e*f - (e^2 - d*f)*(
e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt
[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])]
)/(Sqrt[2]*d^3*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2
 - 4*d*f])]) + (c*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(2*a^(3/2)*d) - ((e^2 -
 d*f)*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(Sqrt[a]*d^3)
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1048

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 6860

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{1}{dx^3\sqrt{a+cx^2}} - \frac{e}{d^2x^2\sqrt{a+cx^2}} + \frac{e^2-df}{d^3x\sqrt{a+cx^2}} + \frac{-e(e^2-2df)-f(e^2-df)\,x}{d^3\sqrt{a+cx^2}\,(d+ex+fx^2)} \right) dx$$

$$= \frac{\int \frac{-e(e^2-2df)-f(e^2-df)x}{\sqrt{a+cx^2}(d+ex+fx^2)}\,dx}{d^3} + \frac{\int \frac{1}{x^3\sqrt{a+cx^2}}\,dx}{d} - \frac{e\int \frac{1}{x^2\sqrt{a+cx^2}}\,dx}{d^2} + \frac{(e^2-df)\int \frac{1}{x\sqrt{a+cx^2}}\,dx}{d^3}$$

$$= \frac{e\sqrt{a+cx^2}}{ad^2x} + \frac{\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+cx}}\,dx, x, x^2\right)}{2d} + \frac{(e^2-df)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx, x, x^2\right)}{2d^3} + \frac{\left(-2ef(e^2-2df)+f(e^2-df)\left(e-\sqrt{e^2-4df}\right)\right)\int \frac{1}{\left(e-\sqrt{e^2-4df}+2fx\right)\sqrt{a+cx^2}}\,dx}{d^3\sqrt{e^2-4df}} - \frac{\left(-2ef(e^2-2df)+f(e^2-df)\left(e+\sqrt{e^2-4df}\right)\right)\int \frac{1}{\left(e+\sqrt{e^2-4df}+2fx\right)\sqrt{a+cx^2}}\,dx}{d^3\sqrt{e^2-4df}}$$

$$= -\frac{\sqrt{a+cx^2}}{2adx^2} + \frac{e\sqrt{a+cx^2}}{ad^2x} - \frac{c\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx, x, x^2\right)}{4ad} + \frac{(e^2-df)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx, x, \sqrt{a+cx^2}\right)}{cd^3} - \frac{\left(-2ef(e^2-2df)+f(e^2-df)\left(e-\sqrt{e^2-4df}\right)\right)\text{Subst}\left(\int \frac{1}{4af^2+c\left(e-\sqrt{e^2-4df}\right)^2-x^2}\,dx, x, \frac{2af-c(e-}{\sqrt{a-}}\right.}{d^3\sqrt{e^2-4df}} + \frac{\left(-2ef(e^2-2df)+f(e^2-df)\left(e+\sqrt{e^2-4df}\right)\right)\text{Subst}\left(\int \frac{1}{4af^2+c\left(e+\sqrt{e^2-4df}\right)^2-x^2}\,dx, x, \frac{2af-c(e+}{\sqrt{a-}}\right.}{d^3\sqrt{e^2-4df}}$$

$$= -\frac{\sqrt{a+cx^2}}{2adx^2} + \frac{e\sqrt{a+cx^2}}{ad^2x}$$
$$+ \frac{f\left(2e^3 - 4def - (e^2 - df)\left(e - \sqrt{e^2-4df}\right)\right)\tanh^{-1}\left(\frac{2af - c\left(e-\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}d^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}}$$
$$- \frac{f\left(2e^3 - 4def - (e^2 - df)\left(e + \sqrt{e^2-4df}\right)\right)\tanh^{-1}\left(\frac{2af - c\left(e+\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}d^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}}$$
$$- \frac{(e^2-df)\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}d^3} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx, x, \sqrt{a+cx^2}\right)}{2ad}$$
$$= -\frac{\sqrt{a+cx^2}}{2adx^2} + \frac{e\sqrt{a+cx^2}}{ad^2x}$$
$$+ \frac{f\left(2e^3 - 4def - (e^2 - df)\left(e - \sqrt{e^2-4df}\right)\right)\tanh^{-1}\left(\frac{2af - c\left(e-\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}d^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}}$$
$$- \frac{f\left(2e^3 - 4def - (e^2 - df)\left(e + \sqrt{e^2-4df}\right)\right)\tanh^{-1}\left(\frac{2af - c\left(e+\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}d^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}}$$
$$+ \frac{c\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2a^{3/2}d} - \frac{(e^2-df)\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}d^3}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.61 (sec) , antiderivative size = 422, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^3\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx$$
$$= \frac{\frac{d(-d+2ex)\sqrt{a+cx^2}}{ax^2} - \frac{2cd^2\text{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{4(e^2-df)\text{arctanh}\left(\frac{-\sqrt{c}x+\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}} + 2\text{RootSum}\left[a^2f + 2a\sqrt{c}e\#1 \right.}{}$$

```
[In] Integrate[1/(x^3*Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
```

[Out] ((d*(-d + 2*e*x)*Sqrt[a + c*x^2])/(a*x^2) - (2*c*d^2*ArcTanh[(Sqrt[c]*x - S
qrt[a + c*x^2])/Sqrt[a]])/a^(3/2) - (4*(e^2 - d*f)*ArcTanh[(-(Sqrt[c]*x) +
Sqrt[a + c*x^2])/Sqrt[a]])/Sqrt[a] + 2*RootSum[a^2*f + 2*a*Sqrt[c]*e*#1 + 4
*c*d*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (a*e^2*f*Log[-(Sqrt[
c]*x) + Sqrt[a + c*x^2] - #1] - a*d*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]
- #1] + 2*Sqrt[c]*e^3*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 - 4*Sqrt[
c]*d*e*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 - e^2*f*Log[-(Sqrt[c]*
x) + Sqrt[a + c*x^2] - #1]*#1^2 + d*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]
- #1]*#1^2)/(a*Sqrt[c]*e + 4*c*d*#1 - 2*a*f*#1 - 3*Sqrt[c]*e*#1^2 + 2*f*#1^
3) &])/(2*d^3)

Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 777, normalized size of antiderivative = 1.70

method	result
risch	$-\frac{\sqrt{c x^2+a}\,(-2ex+d)}{2a\,d^2x^2} - \frac{\frac{4f\left(2adf-2e^2a+c\,d^2\right)\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c\,x^2+a}}{x}\right)}{\left(-e+\sqrt{-4df+e^2}\right)\left(e+\sqrt{-4df+e^2}\right)\sqrt{a}} - \frac{2fa\left(e\sqrt{-4df+e^2}+2df-e^2\right)\sqrt{2}\ln\left(\frac{\sqrt{-4df+e^2}\,ce+2a\,f^2-2cdf+c\,e}{f^2}\right.}{}}{}$
default	$-\frac{4f\left(-\frac{\sqrt{c\,x^2+a}}{2a\,x^2}+\frac{c\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c\,x^2+a}}{x}\right)}{2a^{\frac{3}{2}}}\right)}{\left(-e+\sqrt{-4df+e^2}\right)\left(e+\sqrt{-4df+e^2}\right)} + \frac{16f^2e\sqrt{c\,x^2+a}}{\left(-e+\sqrt{-4df+e^2}\right)^2\left(e+\sqrt{-4df+e^2}\right)^2ax} - \frac{64f^3\left(df-e^2\right)\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c\,x^2+}}{x}\right.}{\left(-e+\sqrt{-4df+e^2}\right)^3\left(e+\sqrt{-4df+e^2}\right.}$

[In] int(1/x^3/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/2*(c*x^2+a)^(1/2)*(-2*e*x+d)/a/d^2/x^2-1/2/a/d^2*(4*f*(2*a*d*f-2*a*e^2+c
*d^2)/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))/a^(1/2)*ln((2*a+2*a^(1
/2)*(c*x^2+a)^(1/2))/x)-2*f*a*(e*(-4*d*f+e^2)^(1/2)+2*d*f-e^2)/(-4*d*f+e^2)
^(1/2)/(e+(-4*d*f+e^2)^(1/2))*2^(1/2)/((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*
d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^
2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(
((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*
d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^
(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1

```
/2*(e+(-4*d*f+e^2)^(1/2))/f))+2*f*a*(e^2-2*d*f+e*(-4*d*f+e^2)^(1/2))/(-4*d*
f+e^2)^(1/2)/(-e+(-4*d*f+e^2)^(1/2))*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a*
f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+
c*e^2)/f^2-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+1/2
*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-
1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2/f*(-
e+(-4*d*f+e^2)^(1/2)))+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^
2)^(1/2))/(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx = \text{Timed out}$$

```
[In] integrate(1/x^3/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{x^3\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx = \int \frac{1}{x^3\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx$$

```
[In] integrate(1/x**3/(f*x**2+e*x+d)/(c*x**2+a)**(1/2),x)
[Out] Integral(1/(x**3*sqrt(a + c*x**2)*(d + e*x + f*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{x^3\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx = \int \frac{1}{\sqrt{cx^2+a}(fx^2+ex+d)x^3}\,dx$$

```
[In] integrate(1/x^3/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(c*x^2 + a)*(f*x^2 + e*x + d)*x^3), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx = \text{Timed out}$$

```
[In] integrate(1/x^3/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx = \int \frac{1}{x^3\,\sqrt{c\,x^2+a}\,\left(f\,x^2+e\,x+d\right)}\,dx$$

```
[In] int(1/(x^3*(a + c*x^2)^(1/2)*(d + e*x + f*x^2)),x)
[Out] int(1/(x^3*(a + c*x^2)^(1/2)*(d + e*x + f*x^2)), x)
```

3.71 $\int \frac{x^3}{(a+cx^2)^{3/2}(d+ex+fx^2)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 499

$$\int \frac{x^3}{(a+cx^2)^{3/2}(d+ex+fx^2)}\,dx = -\frac{1}{cf\sqrt{a+cx^2}} - \frac{ex}{af^2\sqrt{a+cx^2}} + \frac{af(cd^2+a(e^2-df))+ce(cd^2+a(e^2-2df))x}{af^2(ace^2+(cd-af)^2)\sqrt{a+cx^2}} - \frac{\left(2adef-\left(e-\sqrt{e^2-4df}\right)(cd^2+a(e^2-df))\right)\operatorname{arctanh}\left(\frac{2af-c\left(e-\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}\sqrt{e^2-4df}\,(ace^2+(cd-af)^2)\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}} + \frac{\left(2adef-\left(e+\sqrt{e^2-4df}\right)(cd^2+a(e^2-df))\right)\operatorname{arctanh}\left(\frac{2af-c\left(e+\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}\sqrt{e^2-4df}\,(ace^2+(cd-af)^2)\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}}$$

```
[Out] -1/c/f/(c*x^2+a)^(1/2)-e*x/a/f^2/(c*x^2+a)^(1/2)+(a*f*(c*d^2+a*(-d*f+e^2))+
c*e*(c*d^2+a*(-2*d*f+e^2))*x)/a/f^2/(a*c*e^2+(-a*f+c*d)^2)/(c*x^2+a)^(1/2)-
1/2*arctanh(1/2*(2*a*f-c*x*(e-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/
(2*a*f^2+c*(e^2-2*d*f-e*(-4*d*f+e^2)^(1/2)))^(1/2))*(2*a*d*e*f-(c*d^2+a*(-d
*f+e^2))*(e-(-4*d*f+e^2)^(1/2)))/(a*c*e^2+(-a*f+c*d)^2)*2^(1/2)/(-4*d*f+e^2
)^(1/2)/(2*a*f^2+c*(e^2-2*d*f-e*(-4*d*f+e^2)^(1/2)))^(1/2)+1/2*arctanh(1/2*
(2*a*f-c*x*(e+(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+c*(e^2-
2*d*f+e*(-4*d*f+e^2)^(1/2)))^(1/2))*(2*a*d*e*f-(c*d^2+a*(-d*f+e^2))*(e+(-4*
d*f+e^2)^(1/2)))/(a*c*e^2+(-a*f+c*d)^2)*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2
+c*(e^2-2*d*f+e*(-4*d*f+e^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 1.53 (sec) , antiderivative size = 499, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {6860, 197, 267, 1031, 1048, 739, 212}

$$\int \frac{x^3}{(a+cx^2)^{3/2}(d+ex+fx^2)}\,dx = -\frac{\left(2adef-\left(e-\sqrt{e^2-4df}\right)\left(a(e^2-df)+cd^2\right)\right)\text{arctanh}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2+ace^2\right)\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}} + \frac{\left(2adef-\left(\sqrt{e^2-4df}+e\right)\left(a(e^2-df)+cd^2\right)\right)\text{arctanh}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2+ace^2\right)\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}} + \frac{cex(a(e^2-2df)+cd^2)+af(a(e^2-df)+cd^2)}{af^2\sqrt{a+cx^2}\left((cd-af)^2+ace^2\right)} - \frac{ex}{af^2\sqrt{a+cx^2}} - \frac{1}{cf\sqrt{a+cx^2}}$$

```
[In] Int[x^3/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] -(1/(c*f*Sqrt[a + c*x^2])) - (e*x)/(a*f^2*Sqrt[a + c*x^2]) + (a*f*(c*d^2 +
a*(e^2 - d*f)) + c*e*(c*d^2 + a*(e^2 - 2*d*f))*x)/(a*f^2*(a*c*e^2 + (c*d -
a*f)^2)*Sqrt[a + c*x^2]) - ((2*a*d*e*f - (e - Sqrt[e^2 - 4*d*f])*(c*d^2 + a
*(e^2 - d*f)))*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[
2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2
]*Sqrt[e^2 - 4*d*f]*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f
 - e*Sqrt[e^2 - 4*d*f])]) + ((2*a*d*e*f - (e + Sqrt[e^2 - 4*d*f])*(c*d^2 +
a*(e^2 - d*f)))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt
[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[
2]*Sqrt[e^2 - 4*d*f]*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[2*a*f^2 + c*(e^2 - 2*d*
f + e*Sqrt[e^2 - 4*d*f])])
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1031

```
Int[((g_.) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + c*x^2)^(p + 1)*((d + e*x + f*x^2)^
(q + 1)/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)))*(g*c*(2*a*c*e) + ((-a
)*h)*(2*c^2*d - c*(2*a*f)) + c*(g*(2*c^2*d - c*(2*a*f)) - h*(-2*a*c*e))*x),
 x] + Dist[1/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)), Int[(a + c*x^2)^
(p + 1)*(d + e*x + f*x^2)^q*Simp[(-2*g*c)*((c*d - a*f)^2 - ((-a)*e)*(c*e))*
(p + 1) + (2*(g*c*(c*d - a*f) - a*((-h)*c*e)))*(a*f*(p + 1) - c*d*(p + 2))
- e*((g*c)*(2*a*c*e) + ((-a)*h)*(2*c^2*d - c*((Plus[2])*a*f)))*(p + q + 2)
- (2*f*((g*c)*(2*a*c*e) + ((-a)*h)*(2*c^2*d + (-c)*((Plus[2])*a*f)))*(p + q
 + 2) - (2*(g*c*(c*d - a*f) - a*((-h)*c*e)))*((-c)*e*(2*p + q + 4)))*x - c*
f*(2*(g*c*(c*d - a*f) - a*((-h)*c*e)))*(2*p + 2*q + 5)*x^2, x], x], x] /; F
reeQ[{a, c, d, e, f, g, h, q}, x] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && N
eQ[a*c*e^2 + (c*d - a*f)^2, 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
```

Rule 1048

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 6860

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \int \left(-\frac{e}{f^2 (a+cx^2)^{3/2}}+\frac{x}{f (a+cx^2)^{3/2}}+\frac{de+(e^2-df)\,x}{f^2 (a+cx^2)^{3/2} (d+ex+fx^2)}\right) dx$$

$$= \frac{\int \frac{de+(e^2-df)x}{(a+cx^2)^{3/2}(d+ex+fx^2)}\,dx}{f^2} - \frac{e\int \frac{1}{(a+cx^2)^{3/2}}\,dx}{f^2} + \frac{\int \frac{x}{(a+cx^2)^{3/2}}\,dx}{f}$$

$$= -\frac{1}{cf\sqrt{a+cx^2}} - \frac{ex}{af^2\sqrt{a+cx^2}} + \frac{af(cd^2+a(e^2-df))+ce(cd^2+a(e^2-2df))\,x}{af^2\left(ace^2+(cd-af)^2\right)\sqrt{a+cx^2}} + \frac{\int \frac{2a^2cdef^2+2acf^2(cd^2+ae^2-adf)x}{\sqrt{a+cx^2}(d+ex+fx^2)}\,dx}{2acf^2\left(ace^2+(cd-af)^2\right)}$$

$$= -\frac{1}{cf\sqrt{a+cx^2}} - \frac{ex}{af^2\sqrt{a+cx^2}} + \frac{af(cd^2+a(e^2-df))+ce(cd^2+a(e^2-2df))\,x}{af^2\left(ace^2+(cd-af)^2\right)\sqrt{a+cx^2}} + \frac{\left(2adef-\left(e-\sqrt{e^2-4df}\right)(cd^2+a(e^2-df))\right)\int \frac{1}{\left(e-\sqrt{e^2-4df}+2fx\right)\sqrt{a+cx^2}}\,dx}{\sqrt{e^2-4df}\left(ace^2+(cd-af)^2\right)} - \frac{\left(2adef-\left(e+\sqrt{e^2-4df}\right)(cd^2+a(e^2-df))\right)\int \frac{1}{\left(e+\sqrt{e^2-4df}+2fx\right)\sqrt{a+cx^2}}\,dx}{\sqrt{e^2-4df}\left(ace^2+(cd-af)^2\right)}$$

$$= -\frac{1}{cf\sqrt{a+cx^2}} - \frac{ex}{af^2\sqrt{a+cx^2}} + \frac{af(cd^2+a(e^2-df))+ce(cd^2+a(e^2-2df))\,x}{af^2\left(ace^2+(cd-af)^2\right)\sqrt{a+cx^2}} - \frac{\left(2adef-\left(e-\sqrt{e^2-4df}\right)(cd^2+a(e^2-df))\right)\text{Subst}\left(\int \frac{1}{4af^2+c\left(e-\sqrt{e^2-4df}\right)^2-x^2}\,dx, x, \frac{2af-c\left(e-\sqrt{e\cdots\right.}{\sqrt{a+c\cdots}}\right.}{\sqrt{e^2-4df}\left(ace^2+(cd-af)^2\right)} + \frac{\left(2adef-\left(e+\sqrt{e^2-4df}\right)(cd^2+a(e^2-df))\right)\text{Subst}\left(\int \frac{1}{4af^2+c\left(e+\sqrt{e^2-4df}\right)^2-x^2}\,dx, x, \frac{2af-c\left(e+\sqrt{e\cdots\right.}{\sqrt{a+c\cdots}}\right.}{\sqrt{e^2-4df}\left(ace^2+(cd-af)^2\right)}$$

$$= -\frac{1}{cf\sqrt{a+cx^2}} - \frac{ex}{af^2\sqrt{a+cx^2}} + \frac{af(cd^2+a(e^2-df))+ce(cd^2+a(e^2-2df))\,x}{af^2\left(ace^2+(cd-af)^2\right)\sqrt{a+cx^2}} - \frac{\left(2adef-\left(e-\sqrt{e^2-4df}\right)(cd^2+a(e^2-df))\right)\tanh^{-1}\left(\frac{2af-c\left(e-\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}\sqrt{e^2-4df}\left(ace^2+(cd-af)^2\right)\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}} + \frac{\left(2adef-\left(e+\sqrt{e^2-4df}\right)(cd^2+a(e^2-df))\right)\tanh^{-1}\left(\frac{2af-c\left(e+\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}\sqrt{e^2-4df}\left(ace^2+(cd-af)^2\right)\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.59 (sec) , antiderivative size = 401, normalized size of antiderivative = 0.80

$$\int \frac{x^3}{(a+cx^2)^{3/2}(d+ex+fx^2)}\,dx = \frac{-a(-cd+af+cex)-\sqrt{a}c\sqrt{a+cx^2}\text{RootSum}\Big[c^2d+2\sqrt{a}ce\#1-2cd\#}{}$$

```
[In] Integrate[x^3/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] (-(a*(-(c*d) + a*f + c*e*x)) - Sqrt[a]*c*Sqrt[a + c*x^2]*RootSum[c^2*d + 2*
Sqrt[a]*c*e*#1 - 2*c*d*#1^2 + 4*a*f*#1^2 - 2*Sqrt[a]*e*#1^3 + d*#1^4 & , (S
qrt[a]*c*d*e*Log[x] - Sqrt[a]*c*d*e*Log[-Sqrt[a] + Sqrt[a + c*x^2] - x*#1]
+ 2*c*d^2*Log[x]*#1 + 2*a*e^2*Log[x]*#1 - 2*a*d*f*Log[x]*#1 - 2*c*d^2*Log[-
Sqrt[a] + Sqrt[a + c*x^2] - x*#1]*#1 - 2*a*e^2*Log[-Sqrt[a] + Sqrt[a + c*x^
2] - x*#1]*#1 + 2*a*d*f*Log[-Sqrt[a] + Sqrt[a + c*x^2] - x*#1]*#1 - Sqrt[a]
*d*e*Log[x]*#1^2 + Sqrt[a]*d*e*Log[-Sqrt[a] + Sqrt[a + c*x^2] - x*#1]*#1^2)
/(Sqrt[a]*c*e - 2*c*d*#1 + 4*a*f*#1 - 3*Sqrt[a]*e*#1^2 + 2*d*#1^3) & ])/(c*
(c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f))*Sqrt[a + c*x^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1575 vs. 2(456) = 912.

Time = 0.64 (sec) , antiderivative size = 1576, normalized size of antiderivative = 3.16

method	result	size
default	Expression too large to display	1576

```
[In] int(x^3/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/c/f/(c*x^2+a)^(1/2)-e*x/a/f^2/(c*x^2+a)^(1/2)+1/2*(e^3-3*d*e*f+e^2*(-4*d
*f+e^2)^(1/2)-d*f*(-4*d*f+e^2)^(1/2))/f^3/(-4*d*f+e^2)^(1/2)*(2/((-4*d*f+e^
2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)*f^2/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2
*c-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*((-4*d*f
+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)+2*c*(e+(-4*d*f+e^2)^(1/2)
)*f/((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)*(2*c*(x+1/2*(e+(-4*d*f+e
^2)^(1/2))/f)-c*(e+(-4*d*f+e^2)^(1/2))/f)/(2*c*((-4*d*f+e^2)^(1/2)*c*e+2*a*
f^2-2*c*d*f+c*e^2)/f^2-c^2*(e+(-4*d*f+e^2)^(1/2))^2/f^2)/((x+1/2*(e+(-4*d*f
+e^2)^(1/2))/f)^2*c-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2)
)/f)+1/2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)-2/((-4*d
*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)*f^2*2^(1/2)/(((-4*d*f+e^2)^(1/2)*c
*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*
c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f
```

```
)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4
*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(
e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f
^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)))+1/2*(-d*f*(-4*d*f+e^2)^(1/2)+
e^2*(-4*d*f+e^2)^(1/2)+3*d*e*f-e^3)/f^3/(-4*d*f+e^2)^(1/2)*(2/(-(-4*d*f+e^2
)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)*f^2/((x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2
*c-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+1/2*(-(-4*d
*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)+2*c*(e-(-4*d*f+e^2)^(1/
2))*f/(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)*(2*c*(x-1/2/f*(-e+(-4
*d*f+e^2)^(1/2)))-c*(e-(-4*d*f+e^2)^(1/2))/f)/(2*c*(-(-4*d*f+e^2)^(1/2)*c*e
+2*a*f^2-2*c*d*f+c*e^2)/f^2-c^2*(e-(-4*d*f+e^2)^(1/2))^2/f^2)/((x-1/2/f*(-e
+(-4*d*f+e^2)^(1/2)))^2*c-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2/f*(-e+(-4*d*f+e
^2)^(1/2)))+1/2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)-
2/(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)*f^2*2^(1/2)/((-(-4*d*f+e^
2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e
+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2/f*(-e+(-4*d*f
+e^2)^(1/2)))+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/
f^2)^(1/2)*(4*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2*c-4*c*(e-(-4*d*f+e^2)^(1/
2))/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-
2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27621 vs. $2(454) = 908$.

Time = 121.37 (sec) , antiderivative size = 27621, normalized size of antiderivative = 55.35

$$\int \frac{x^3}{(a+cx^2)^{3/2}(d+ex+fx^2)}\,dx = \text{Too large to display}$$

```
[In] integrate(x^3/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F]

$$\int \frac{x^3}{(a+cx^2)^{3/2}(d+ex+fx^2)}\,dx = \int \frac{x^3}{(a+cx^2)^{\frac{3}{2}}(d+ex+fx^2)}\,dx$$

```
[In] integrate(x**3/(c*x**2+a)**(3/2)/(f*x**2+e*x+d),x)
[Out] Integral(x**3/((a + c*x**2)**(3/2)*(d + e*x + f*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{(a+cx^2)^{3/2}(d+ex+fx^2)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^3/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re deta
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{(a+cx^2)^{3/2}(d+ex+fx^2)}\,dx = \text{Exception raised: AttributeError}$$

```
[In] integrate(x^3/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: AttributeError >> type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(a+cx^2)^{3/2}(d+ex+fx^2)}\,dx = \int \frac{x^3}{(c\,x^2+a)^{3/2}\,(f\,x^2+e\,x+d)}\,dx$$

```
[In] int(x^3/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x)
```

```
[Out] int(x^3/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)), x)
```

3.72 $\int \frac{x^2}{(a+cx^2)^{3/2}(d+ex+fx^2)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 410

$$\int \frac{x^2}{(a+cx^2)^{3/2}(d+ex+fx^2)}\,dx = -\frac{ae+(cd-af)x}{(ace^2+(cd-af)^2)\sqrt{a+cx^2}} - \frac{f\left(2d(cd-af)+ae\left(e-\sqrt{e^2-4df}\right)\right)\operatorname{arctanh}\left(\frac{2af-c\left(e-\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}\sqrt{e^2-4df}\,(ace^2+(cd-af)^2)\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}} + \frac{f\left(2d(cd-af)+ae\left(e+\sqrt{e^2-4df}\right)\right)\operatorname{arctanh}\left(\frac{2af-c\left(e+\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}\sqrt{e^2-4df}\,(ace^2+(cd-af)^2)\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}}$$

```
[Out] (-a*e-(-a*f+c*d)*x)/(a*c*e^2+(-a*f+c*d)^2)/(c*x^2+a)^(1/2)-1/2*f*arctanh(1/
2*(2*a*f-c*x*(e-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+c*(e^
2-2*d*f-e*(-4*d*f+e^2)^(1/2)))^(1/2))*(2*d*(-a*f+c*d)+a*e*(e-(-4*d*f+e^2)^(
1/2)))/(a*c*e^2+(-a*f+c*d)^2)*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+c*(e^2-2*
d*f-e*(-4*d*f+e^2)^(1/2)))^(1/2)+1/2*f*arctanh(1/2*(2*a*f-c*x*(e+(-4*d*f+e^
2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+c*(e^2-2*d*f+e*(-4*d*f+e^2)^(1/
2)))^(1/2))*(2*d*(-a*f+c*d)+a*e*(e+(-4*d*f+e^2)^(1/2)))/(a*c*e^2+(-a*f+c*d)
^2)*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+c*(e^2-2*d*f+e*(-4*d*f+e^2)^(1/2)))
^(1/2)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1077, 1048, 739, 212}

$$\int \frac{x^2}{(a+cx^2)^{3/2}(d+ex+fx^2)}\,dx = -\frac{f\left(2d(cd-af)+ae\left(e-\sqrt{e^2-4df}\right)\right)\operatorname{arctanh}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2+ace^2\right)\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}} + \frac{f\left(2d(cd-af)+ae\left(\sqrt{e^2-4df}+e\right)\right)\operatorname{arctanh}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2+ace^2\right)\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}} - \frac{x(cd-af)+ae}{\sqrt{a+cx^2}\left((cd-af)^2+ace^2\right)}$$

```
[In] Int[x^2/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] -((a*e + (c*d - a*f)*x)/((a*c*e^2 + (c*d - a*f)^2)*Sqrt[a + c*x^2])) - (f*(
2*d*(c*d - a*f) + a*e*(e - Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e - Sqrt
[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d
*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*(a*c*e^2 + (c*d - a*f)^
2)*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) + (f*(2*d*(c*d -
a*f) + a*e*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*
f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[
a + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[2*a
*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1048

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1077

```
Int[((a_) + (c_.)*(x_)^2)^(p_)*((A_.) + (C_.)*(x_)^2)*((d_) + (e_.)*(x_) +
(f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + c*x^2)^(p + 1)*((d + e*x + f*x^2
)^(q + 1)/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)))*((A*c - a*C)*(2*a*c
*e) + c*(A*(2*c^2*d - c*(2*a*f)) + C*(-2*a*(c*d - a*f)))*x), x] + Dist[1/((
-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)), Int[(a + c*x^2)^(p + 1)*(d + e*
x + f*x^2)^q*Simp[(-2*A*c - 2*a*C)*((c*d - a*f)^2 - ((-a)*e)*(c*e))*(p + 1)
 + (2*(A*c*(c*d - a*f) - a*(c*C*d - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) -
e*((A*c - a*C)*(2*a*c*e))*(p + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e))*(p + q
 + 2) - (2*(A*c*(c*d - a*f) - a*(c*C*d - a*C*f)))*((-c)*e*(2*p + q + 4)))*x
 - c*f*(2*(A*c*(c*d - a*f) - a*(c*C*d - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x
], x] /; FreeQ[{a, c, d, e, f, A, C, q}, x] && NeQ[e^2 - 4*d*f, 0] && LtQ[p
, -1] && NeQ[a*c*e^2 + (c*d - a*f)^2, 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]
) &&  !IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{ae+(cd-af)x}{(ace^2+(cd-af)^2)\sqrt{a+cx^2}}+\frac{\int \frac{2acd(cd-af)-2a^2cefx}{\sqrt{a+cx^2}(d+ex+fx^2)}\,dx}{2ac\,(ace^2+(cd-af)^2)}\\
&= -\frac{ae+(cd-af)x}{(ace^2+(cd-af)^2)\sqrt{a+cx^2}}\\
&\quad+\frac{\left(f\left(2d(cd-af)+ae\left(e-\sqrt{e^2-4df}\right)\right)\right)\int \frac{1}{\left(e-\sqrt{e^2-4df}+2fx\right)\sqrt{a+cx^2}}\,dx}{\sqrt{e^2-4df}\,(ace^2+(cd-af)^2)}\\
&\quad-\frac{\left(f\left(2d(cd-af)+ae\left(e+\sqrt{e^2-4df}\right)\right)\right)\int \frac{1}{\left(e+\sqrt{e^2-4df}+2fx\right)\sqrt{a+cx^2}}\,dx}{\sqrt{e^2-4df}\,(ace^2+(cd-af)^2)}\\
&= -\frac{ae+(cd-af)x}{(ace^2+(cd-af)^2)\sqrt{a+cx^2}}\\
&\quad-\frac{\left(f\left(2d(cd-af)+ae\left(e-\sqrt{e^2-4df}\right)\right)\right)\text{Subst}\left(\int \frac{1}{4af^2+c\left(e-\sqrt{e^2-4df}\right)^2-x^2}\,dx,x,\frac{2af-c\left(e-\sqrt{e^2-4df}\right)x}{\sqrt{a+cx^2}}\right.}{\sqrt{e^2-4df}\,(ace^2+(cd-af)^2)}\\
&\quad+\frac{\left(f\left(2d(cd-af)+ae\left(e+\sqrt{e^2-4df}\right)\right)\right)\text{Subst}\left(\int \frac{1}{4af^2+c\left(e+\sqrt{e^2-4df}\right)^2-x^2}\,dx,x,\frac{2af-c\left(e+\sqrt{e^2-4df}\right)x}{\sqrt{a+cx^2}}\right.}{\sqrt{e^2-4df}\,(ace^2+(cd-af)^2)}
\end{aligned}$$

$$= -\frac{ae + (cd - af)x}{(ace^2 + (cd - af)^2)\sqrt{a + cx^2}}$$
$$- \frac{f\left(2d(cd - af) + ae\left(e - \sqrt{e^2 - 4df}\right)\right)\tanh^{-1}\left(\frac{2af - c\left(e - \sqrt{e^2 - 4df}\right)x}{\sqrt{2}\sqrt{2af^2 + c\left(e^2 - 2df - e\sqrt{e^2 - 4df}\right)}\sqrt{a + cx^2}}\right)}{\sqrt{2}\sqrt{e^2 - 4df}\,(ace^2 + (cd - af)^2)\sqrt{2af^2 + c\left(e^2 - 2df - e\sqrt{e^2 - 4df}\right)}}$$
$$+ \frac{f\left(2d(cd - af) + ae\left(e + \sqrt{e^2 - 4df}\right)\right)\tanh^{-1}\left(\frac{2af - c\left(e + \sqrt{e^2 - 4df}\right)x}{\sqrt{2}\sqrt{2af^2 + c\left(e^2 - 2df + e\sqrt{e^2 - 4df}\right)}\sqrt{a + cx^2}}\right)}{\sqrt{2}\sqrt{e^2 - 4df}\,(ace^2 + (cd - af)^2)\sqrt{2af^2 + c\left(e^2 - 2df + e\sqrt{e^2 - 4df}\right)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.55 (sec) , antiderivative size = 385, normalized size of antiderivative = 0.94

$$\int \frac{x^2}{(a + cx^2)^{3/2}(d + ex + fx^2)}\,dx = \frac{-ae - cdx + afx - \sqrt{a + cx^2}\text{RootSum}\left[c^2d + 2\sqrt{a}ce\#1 - 2cd\#1^2 + 4a\right.}{}$$

```
[In] Integrate[x^2/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] (-(a*e) - c*d*x + a*f*x - Sqrt[a + c*x^2]*RootSum[c^2*d + 2*Sqrt[a]*c*e*#1
- 2*c*d*#1^2 + 4*a*f*#1^2 - 2*Sqrt[a]*e*#1^3 + d*#1^4 & , (-(c^2*d^2*Log[x]
) + a*c*d*f*Log[x] + c^2*d^2*Log[-Sqrt[a] + Sqrt[a + c*x^2] - x*#1] - a*c*d
*f*Log[-Sqrt[a] + Sqrt[a + c*x^2] - x*#1] + 2*a^(3/2)*e*f*Log[x]*#1 - 2*a^(
3/2)*e*f*Log[-Sqrt[a] + Sqrt[a + c*x^2] - x*#1]*#1 + c*d^2*Log[x]*#1^2 - a*
d*f*Log[x]*#1^2 - c*d^2*Log[-Sqrt[a] + Sqrt[a + c*x^2] - x*#1]*#1^2 + a*d*f
*Log[-Sqrt[a] + Sqrt[a + c*x^2] - x*#1]*#1^2)/(-(Sqrt[a]*c*e) + 2*c*d*#1 -
4*a*f*#1 + 3*Sqrt[a]*e*#1^2 - 2*d*#1^3) & ])/((c^2*d^2 + a^2*f^2 + a*c*(e^2
 - 2*d*f))*Sqrt[a + c*x^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1524 vs. 2(372) = 744.

Time = 0.65 (sec) , antiderivative size = 1525, normalized size of antiderivative = 3.72

method	result	size
default	Expression too large to display	1525

```
[In] int(x^2/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*x/a/(c*x^2+a)^(1/2)+1/2*(-e*(-4*d*f+e^2)^(1/2)+2*d*f-e^2)/f^2/(-4*d*f+e
^2)^(1/2)*(2/((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)*f^2/((x+1/2*(e+
(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2
)^(1/2))/f)+1/2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)+2
*c*(e+(-4*d*f+e^2)^(1/2))*f/((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)*
(2*c*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)-c*(e+(-4*d*f+e^2)^(1/2))/f)/(2*c*((-4
*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c^2*(e+(-4*d*f+e^2)^(1/2))^2
/f^2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1
/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*
e^2)/f^2)^(1/2)-2/((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)*f^2*2^(1/2
)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f+e
^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*
(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*
d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*
f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*e+
2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)))+1/2*(
e^2-2*d*f-e*(-4*d*f+e^2)^(1/2))/f^2/(-4*d*f+e^2)^(1/2)*(2/(-(-4*d*f+e^2)^(1
/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)*f^2/((x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2*c-c
*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+1/2*(-(-4*d*f+e
^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)+2*c*(e-(-4*d*f+e^2)^(1/2))*
f/(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)*(2*c*(x-1/2/f*(-e+(-4*d*f
+e^2)^(1/2)))-c*(e-(-4*d*f+e^2)^(1/2))/f)/(2*c*(-(-4*d*f+e^2)^(1/2)*c*e+2*a
*f^2-2*c*d*f+c*e^2)/f^2-c^2*(e-(-4*d*f+e^2)^(1/2))^2/f^2)/((x-1/2/f*(-e+(-4
*d*f+e^2)^(1/2)))^2*c-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2/f*(-e+(-4*d*f+e^2)^
(1/2)))+1/2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)-2/(-
(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)*f^2*2^(1/2)/((-(-4*d*f+e^2)^(
1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a
*f^2-2*c*d*f+c*e^2)/f^2-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2/f*(-e+(-4*d*f+e^2
)^(1/2)))+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)
^(1/2)*(4*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/
f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*
d*f+c*e^2)/f^2)^(1/2))/(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26116 vs. $2(369) = 738$.

Time = 125.13 (sec) , antiderivative size = 26116, normalized size of antiderivative = 63.70

$$\int \frac{x^2}{(a+cx^2)^{3/2}(d+ex+fx^2)}\,dx = \text{Too large to display}$$

```
[In] integrate(x^2/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F]

$$\int \frac{x^2}{(a+cx^2)^{3/2}\,(d+ex+fx^2)}\,dx = \int \frac{x^2}{(a+cx^2)^{\frac{3}{2}}\,(d+ex+fx^2)}\,dx$$

```
[In] integrate(x**2/(c*x**2+a)**(3/2)/(f*x**2+e*x+d),x)
[Out] Integral(x**2/((a + c*x**2)**(3/2)*(d + e*x + f*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{(a+cx^2)^{3/2}\,(d+ex+fx^2)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re deta
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^2}{(a+cx^2)^{3/2}\,(d+ex+fx^2)}\,dx = \text{Exception raised: AttributeError}$$

```
[In] integrate(x^2/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
[Out] Exception raised: AttributeError >> type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+cx^2)^{3/2}\,(d+ex+fx^2)}\,dx = \int \frac{x^2}{(c\,x^2+a)^{3/2}\,(f\,x^2+e\,x+d)}\,dx$$

```
[In] int(x^2/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x)
[Out] int(x^2/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)), x)
```

3.73 $\int \frac{x}{(a+cx^2)^{3/2}(d+ex+fx^2)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 411

$$\int \frac{x}{(a+cx^2)^{3/2}(d+ex+fx^2)}\,dx = -\frac{cd-af-cex}{(ace^2+(cd-af)^2)\sqrt{a+cx^2}} + \frac{f\left(2cde-(cd-af)\left(e-\sqrt{e^2-4df}\right)\right)\operatorname{arctanh}\left(\frac{2af-c\left(e-\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}\sqrt{e^2-4df}\,(ace^2+(cd-af)^2)\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}} - \frac{f\left(2cde-(cd-af)\left(e+\sqrt{e^2-4df}\right)\right)\operatorname{arctanh}\left(\frac{2af-c\left(e+\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}\sqrt{e^2-4df}\,(ace^2+(cd-af)^2)\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}}$$

```
[Out] (c*e*x+a*f-c*d)/(a*c*e^2+(-a*f+c*d)^2)/(c*x^2+a)^(1/2)+1/2*f*arctanh(1/2*(2
*a*f-c*x*(e-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+c*(e^2-2*
d*f-e*(-4*d*f+e^2)^(1/2)))^(1/2))*(2*c*d*e-(-a*f+c*d)*(e-(-4*d*f+e^2)^(1/2)
))/(a*c*e^2+(-a*f+c*d)^2)*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+c*(e^2-2*d*f-
e*(-4*d*f+e^2)^(1/2)))^(1/2)-1/2*f*arctanh(1/2*(2*a*f-c*x*(e+(-4*d*f+e^2)^(
1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+c*(e^2-2*d*f+e*(-4*d*f+e^2)^(1/2)))
^(1/2))*(2*c*d*e-(-a*f+c*d)*(e+(-4*d*f+e^2)^(1/2)))/(a*c*e^2+(-a*f+c*d)^2)*
2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+c*(e^2-2*d*f+e*(-4*d*f+e^2)^(1/2)))^(1/
2)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1031, 1048, 739, 212}

$$\int \frac{x}{(a+cx^2)^{3/2}(d+ex+fx^2)}\,dx = \frac{f\left(2cde-\left(e-\sqrt{e^2-4df}\right)(cd-af)\right)\operatorname{arctanh}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2+ace^2\right)\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}$$
$$-\frac{f\left(2cde-\left(\sqrt{e^2-4df}+e\right)(cd-af)\right)\operatorname{arctanh}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2+ace^2\right)\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}$$
$$-\frac{-af+cd-cex}{\sqrt{a+cx^2}\left((cd-af)^2+ace^2\right)}$$

```
[In] Int[x/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] -((c*d - a*f - c*e*x)/((a*c*e^2 + (c*d - a*f)^2)*Sqrt[a + c*x^2])) + (f*(2*
c*d*e - (c*d - a*f)*(e - Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e - Sqrt[e
^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f
])]*Sqrt[a + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*(a*c*e^2 + (c*d - a*f)^2)
*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) - (f*(2*c*d*e - (c*
d - a*f)*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f]
)*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a
+ c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[2*a*f
^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1031

```
Int[((g_.) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + c*x^2)^(p + 1)*((d + e*x + f*x^2)^
```

```
(q + 1)/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)))*(g*c*(2*a*c*e) + ((-a
)*h)*(2*c^2*d - c*(2*a*f)) + c*(g*(2*c^2*d - c*(2*a*f)) - h*(-2*a*c*e))*x),
 x] + Dist[1/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)), Int[(a + c*x^2)^
(p + 1)*(d + e*x + f*x^2)^q*Simp[(-2*g*c)*((c*d - a*f)^2 - ((-a)*e)*(c*e))*
(p + 1) + (2*(g*c*(c*d - a*f) - a*((-h)*c*e)))*(a*f*(p + 1) - c*d*(p + 2))
- e*((g*c)*(2*a*c*e) + ((-a)*h)*(2*c^2*d - c*((Plus[2])*a*f)))*(p + q + 2)
- (2*f*((g*c)*(2*a*c*e) + ((-a)*h)*(2*c^2*d + (-c)*((Plus[2])*a*f)))*(p + q
 + 2) - (2*(g*c*(c*d - a*f) - a*((-h)*c*e)))*((-c)*e*(2*p + q + 4)))*x - c*
f*(2*(g*c*(c*d - a*f) - a*((-h)*c*e)))*(2*p + 2*q + 5)*x^2, x], x], x] /; F
reeQ[{a, c, d, e, f, g, h, q}, x] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && N
eQ[a*c*e^2 + (c*d - a*f)^2, 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
```

Rule 1048

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\text{integral} = -\frac{cd-af-cex}{\left(ace^2+(cd-af)^2\right)\sqrt{a+cx^2}}+\frac{\int \frac{-2ac^2de-2acf(cd-af)x}{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\,dx}{2ac\left(ace^2+(cd-af)^2\right)}$$

$$= -\frac{cd-af-cex}{\left(ace^2+(cd-af)^2\right)\sqrt{a+cx^2}} - \frac{\left(f\left(2cde-(cd-af)\left(e-\sqrt{e^2-4df}\right)\right)\right)\int \frac{1}{\left(e-\sqrt{e^2-4df}+2fx\right)\sqrt{a+cx^2}}\,dx}{\sqrt{e^2-4df}\left(ace^2+(cd-af)^2\right)} + \frac{\left(f\left(2cde-(cd-af)\left(e+\sqrt{e^2-4df}\right)\right)\right)\int \frac{1}{\left(e+\sqrt{e^2-4df}+2fx\right)\sqrt{a+cx^2}}\,dx}{\sqrt{e^2-4df}\left(ace^2+(cd-af)^2\right)}$$

$$= -\frac{cd-af-cex}{\left(ace^2+(cd-af)^2\right)\sqrt{a+cx^2}} + \frac{\left(f\left(2cde-(cd-af)\left(e-\sqrt{e^2-4df}\right)\right)\right)\text{Subst}\left(\int \frac{1}{4af^2+c\left(e-\sqrt{e^2-4df}\right)^2-x^2}\,dx, x, \frac{2af-c\left(e-\sqrt{e^2-4df}\right)x}{\sqrt{a+cx^2}}\right.}{\sqrt{e^2-4df}\left(ace^2+(cd-af)^2\right)} - \frac{\left(f\left(2cde-(cd-af)\left(e+\sqrt{e^2-4df}\right)\right)\right)\text{Subst}\left(\int \frac{1}{4af^2+c\left(e+\sqrt{e^2-4df}\right)^2-x^2}\,dx, x, \frac{2af-c\left(e+\sqrt{e^2-4df}\right)x}{\sqrt{a+cx^2}}\right.}{\sqrt{e^2-4df}\left(ace^2+(cd-af)^2\right)}$$

$$= -\frac{cd - af - cex}{(ace^2 + (cd - af)^2)\sqrt{a + cx^2}} + \frac{f\left(2cde - (cd - af)\left(e - \sqrt{e^2 - 4df}\right)\right)\tanh^{-1}\left(\frac{2af - c\left(e - \sqrt{e^2 - 4df}\right)x}{\sqrt{2}\sqrt{2af^2 + c\left(e^2 - 2df - e\sqrt{e^2 - 4df}\right)}\sqrt{a + cx^2}}\right)}{\sqrt{2}\sqrt{e^2 - 4df}\,(ace^2 + (cd - af)^2)\sqrt{2af^2 + c\left(e^2 - 2df - e\sqrt{e^2 - 4df}\right)}} - \frac{f\left(2cde - (cd - af)\left(e + \sqrt{e^2 - 4df}\right)\right)\tanh^{-1}\left(\frac{2af - c\left(e + \sqrt{e^2 - 4df}\right)x}{\sqrt{2}\sqrt{2af^2 + c\left(e^2 - 2df + e\sqrt{e^2 - 4df}\right)}\sqrt{a + cx^2}}\right)}{\sqrt{2}\sqrt{e^2 - 4df}\,(ace^2 + (cd - af)^2)\sqrt{2af^2 + c\left(e^2 - 2df + e\sqrt{e^2 - 4df}\right)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.50 (sec) , antiderivative size = 330, normalized size of antiderivative = 0.80

$$\int \frac{x}{(a + cx^2)^{3/2}(d + ex + fx^2)}\,dx = \frac{-cd + af + cex - \sqrt{a + cx^2}\,\text{RootSum}\left[a^2f + 2a\sqrt{c}e\#1 + 4cd\#1^2 - 2af\right.}{}$$

```
[In] Integrate[x/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] (-(c*d) + a*f + c*e*x - Sqrt[a + c*x^2]*RootSum[a^2*f + 2*a*Sqrt[c]*e*#1 +
4*c*d*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (-(a*c*d*f*Log[-(Sq
rt[c]*x) + Sqrt[a + c*x^2] - #1]) + a^2*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x
^2] - #1] - 2*c^(3/2)*d*e*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 + c*d
*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2 - a*f^2*Log[-(Sqrt[c]*x) +
 Sqrt[a + c*x^2] - #1]*#1^2)/(a*Sqrt[c]*e + 4*c*d*#1 - 2*a*f*#1 - 3*Sqrt[c]
*e*#1^2 + 2*f*#1^3) & ])/((c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f))*Sqrt[a +
c*x^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1489 vs. 2(370) = 740.

Time = 0.75 (sec) , antiderivative size = 1490, normalized size of antiderivative = 3.63

method	result	size
default	Expression too large to display	1490

```
[In] int(x/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)/f*(2/((-4*d*f+e^2)^(1/2)*c*e+
2*a*f^2-2*c*d*f+c*e^2)*f^2/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e+(-4*d
*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*((-4*d*f+e^2)^(1/2)*c
*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)+2*c*(e+(-4*d*f+e^2)^(1/2))*f/((-4*d*f+
e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)*(2*c*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)
-c*(e+(-4*d*f+e^2)^(1/2))/f)/(2*c*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c
*e^2)/f^2-c^2*(e+(-4*d*f+e^2)^(1/2))^2/f^2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/
f)^2*c-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*((-4
*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)-2/((-4*d*f+e^2)^(1/2)
*c*e+2*a*f^2-2*c*d*f+c*e^2)*f^2*2^(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*
c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/
f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)
*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-
4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2
)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x
+1/2*(e+(-4*d*f+e^2)^(1/2))/f)))+1/2*(-e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(
1/2)/f*(2/(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)*f^2/((x-1/2/f*(-e
+(-4*d*f+e^2)^(1/2)))^2*c-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2/f*(-e+(-4*d*f+e
^2)^(1/2)))+1/2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)+
2*c*(e-(-4*d*f+e^2)^(1/2))*f/(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2
)*(2*c*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))-c*(e-(-4*d*f+e^2)^(1/2))/f)/(2*c*(
-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c^2*(e-(-4*d*f+e^2)^(1/2
))^2/f^2)/((x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2*c-c*(e-(-4*d*f+e^2)^(1/2))/f
*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+1/2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c
*d*f+c*e^2)/f^2)^(1/2)-2/(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)*f^
2*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((
-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e-(-4*d*f+e^2)^(1/2))
/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+
2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2*c-
4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+2*(-(-4*d*f+
e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2/f*(-e+(-4*d*f+e^2)
^(1/2)))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26234 vs. $2(368) = 736$.

Time = 105.28 (sec) , antiderivative size = 26234, normalized size of antiderivative = 63.83

$$\int \frac{x}{(a+cx^2)^{3/2}\,(d+ex+fx^2)}\,dx = \text{Too large to display}$$

```
[In] integrate(x/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F]

$$\int \frac{x}{(a+cx^2)^{3/2}\,(d+ex+fx^2)}\,dx=\int \frac{x}{(a+cx^2)^{\frac{3}{2}}\,(d+ex+fx^2)}\,dx$$

```
[In] integrate(x/(c*x**2+a)**(3/2)/(f*x**2+e*x+d),x)
[Out] Integral(x/((a + c*x**2)**(3/2)*(d + e*x + f*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{(a+cx^2)^{3/2}\,(d+ex+fx^2)}\,dx=\text{Exception raised: ValueError}$$

```
[In] integrate(x/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re deta
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x}{(a+cx^2)^{3/2}\,(d+ex+fx^2)}\,dx=\text{Exception raised: AttributeError}$$

```
[In] integrate(x/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
[Out] Exception raised: AttributeError >> type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+cx^2)^{3/2}\,(d+ex+fx^2)}\,dx=\int \frac{x}{{\left(c\,x^2+a\right)}^{3/2}\,\left(f\,x^2+e\,x+d\right)}\,dx$$

```
[In] int(x/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x)
[Out] int(x/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)), x)
```

3.74 $\int \frac{1}{(a+cx^2)^{3/2}(d+ex+fx^2)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 416

$$\int \frac{1}{(a+cx^2)^{3/2}(d+ex+fx^2)}\,dx = \frac{c(ae+(cd-af)x)}{a(ace^2+(cd-af)^2)\sqrt{a+cx^2}} - \frac{f\left(2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)\right)\operatorname{arctanh}\left(\frac{2af-c\left(e-\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}\sqrt{e^2-4df}\,(ace^2+(cd-af)^2)\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}} + \frac{f\left(2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)\right)\operatorname{arctanh}\left(\frac{2af-c\left(e+\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}\sqrt{e^2-4df}\,(ace^2+(cd-af)^2)\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}}$$

```
[Out] c*(a*e+(-a*f+c*d)*x)/a/(a*c*e^2+(-a*f+c*d)^2)/(c*x^2+a)^(1/2)-1/2*f*arctanh
(1/2*(2*a*f-c*x*(e-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+c*
(e^2-2*d*f-e*(-4*d*f+e^2)^(1/2)))^(1/2))*(2*a*f^2+c*(e^2-2*d*f+e*(-4*d*f+e^
2)^(1/2)))/(a*c*e^2+(-a*f+c*d)^2)*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+c*(e^
2-2*d*f-e*(-4*d*f+e^2)^(1/2)))^(1/2)+1/2*f*arctanh(1/2*(2*a*f-c*x*(e+(-4*d*
f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+c*(e^2-2*d*f+e*(-4*d*f+e^2)
^(1/2)))^(1/2))*(2*a*f^2+c*(e^2-2*d*f-e*(-4*d*f+e^2)^(1/2)))/(a*c*e^2+(-a*f
+c*d)^2)*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+c*(e^2-2*d*f+e*(-4*d*f+e^2)^(1
/2)))^(1/2)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {990, 1048, 739, 212}

$$\int \frac{1}{(a+cx^2)^{3/2}(d+ex+fx^2)}\,dx = -\frac{f\left(2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)\right)\operatorname{arctanh}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2+ace^2\right)\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}} + \frac{f\left(2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)\right)\operatorname{arctanh}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2+ace^2\right)\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}} + \frac{c(x(cd-af)+ae)}{a\sqrt{a+cx^2}\left((cd-af)^2+ace^2\right)}$$

```
[In] Int[1/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] (c*(a*e + (c*d - a*f)*x))/(a*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[a + c*x^2]) - (
f*(2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e -
 Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2
- 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*(a*c*e^2 + (c*d -
a*f)^2)*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) + (f*(2*a*f^
2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2
 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])
]*Sqrt[a + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*(a*c*e^2 + (c*d - a*f)^2)*S
qrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 990

```
Int[((a_.) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[(2*a*c^2*e + c*(2*c^2*d - c*(2*a*f))*x)*(a + c*x^2)^(p + 1
)*((d + e*x + f*x^2)^(q + 1)/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1))),
 x] - Dist[1/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)), Int[(a + c*x^2)^
(p + 1)*(d + e*x + f*x^2)^q*Simp[2*c*((c*d - a*f)^2 - ((-a)*e)*(c*e))*(p +
1) - (2*c^2*d - c*(2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(-2*a*c^2*e)*(p
+ q + 2) + (2*f*(2*a*c^2*e)*(p + q + 2) - (2*c^2*d - c*(2*a*f))*((-c)*e*(2*
p + q + 4)))*x + c*f*(2*c^2*d - c*(2*a*f))*(2*p + 2*q + 5)*x^2, x], x], x]
/; FreeQ[{a, c, d, e, f, q}, x] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ
[a*c*e^2 + (c*d - a*f)^2, 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[
q, 0]
```

Rule 1048

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\text{integral} = \frac{c(ae+(cd-af)x)}{a\left(ace^2+(cd-af)^2\right)\sqrt{a+cx^2}} - \frac{\int \frac{-2ac\left(af^2+c\left(e^2-df\right)\right)-2ac^2efx}{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\,dx}{2ac\left(ace^2+(cd-af)^2\right)}$$

$$= \frac{c(ae+(cd-af)x)}{a\left(ace^2+(cd-af)^2\right)\sqrt{a+cx^2}} - \frac{\left(f\left(2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)\right)\right)\int \frac{1}{\left(e+\sqrt{e^2-4df}+2fx\right)\sqrt{a+cx^2}}\,dx}{\sqrt{e^2-4df}\left(ace^2+(cd-af)^2\right)} + \frac{\left(f\left(2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)\right)\right)\int \frac{1}{\left(e-\sqrt{e^2-4df}+2fx\right)\sqrt{a+cx^2}}\,dx}{\sqrt{e^2-4df}\left(ace^2+(cd-af)^2\right)}$$

$$= \frac{c(ae+(cd-af)x)}{a\left(ace^2+(cd-af)^2\right)\sqrt{a+cx^2}} + \frac{\left(f\left(2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)\right)\right)\text{Subst}\left(\int \frac{1}{4af^2+c\left(e+\sqrt{e^2-4df}\right)^2-x^2}\,dx, x, \frac{2af-c\left(e+\sqrt{e^2-4df}\right)x}{\sqrt{a+cx^2}}\right.}{\sqrt{e^2-4df}\left(ace^2+(cd-af)^2\right)} - \frac{\left(f\left(2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)\right)\right)\text{Subst}\left(\int \frac{1}{4af^2+c\left(e-\sqrt{e^2-4df}\right)^2-x^2}\,dx, x, \frac{2af-c\left(e-\sqrt{e^2-4df}\right)x}{\sqrt{a+cx^2}}\right.}{\sqrt{e^2-4df}\left(ace^2+(cd-af)^2\right)}$$

$$= \frac{c(ae+(cd-af)x)}{a\left(ace^2+(cd-af)^2\right)\sqrt{a+cx^2}} - \frac{f\left(2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)\right)\tanh^{-1}\left(\frac{2af-c\left(e-\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}\sqrt{e^2-4df}\left(ace^2+(cd-af)^2\right)\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}} + \frac{f\left(2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)\right)\tanh^{-1}\left(\frac{2af-c\left(e+\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}\sqrt{e^2-4df}\left(ace^2+(cd-af)^2\right)\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.50 (sec) , antiderivative size = 346, normalized size of antiderivative = 0.83

$$\int \frac{1}{(a+cx^2)^{3/2}(d+ex+fx^2)}\,dx = \frac{c(cdx+a(e-fx)) - a\sqrt{a+cx^2}\text{RootSum}\left[a^2f+2a\sqrt{c}e\#1+4cd\#1^2-\right.}{}$$

```
[In] Integrate[1/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] (c*(c*d*x + a*(e - f*x)) - a*Sqrt[a + c*x^2]*RootSum[a^2*f + 2*a*Sqrt[c]*e*
#1 + 4*c*d*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (a*c*e*f*Log[-
(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] + 2*c^(3/2)*e^2*Log[-(Sqrt[c]*x) + Sqrt
[a + c*x^2] - #1]*#1 - 2*c^(3/2)*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #
1]*#1 + 2*a*Sqrt[c]*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 - c*e*f
*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2)/(a*Sqrt[c]*e + 4*c*d*#1 - 2
*a*f*#1 - 3*Sqrt[c]*e*#1^2 + 2*f*#1^3) & ])/(a*(c^2*d^2 + a^2*f^2 + a*c*(e^
2 - 2*d*f))*Sqrt[a + c*x^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1456 vs. 2(377) = 754.

Time = 0.79 (sec) , antiderivative size = 1457, normalized size of antiderivative = 3.50

method	result	size
default	Expression too large to display	1457

```
[In] int(1/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/(-4*d*f+e^2)^(1/2)*(2/((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)*f^2
/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e
+(-4*d*f+e^2)^(1/2))/f)+1/2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/
f^2)^(1/2)+2*c*(e+(-4*d*f+e^2)^(1/2))*f/((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c
*d*f+c*e^2)*(2*c*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)-c*(e+(-4*d*f+e^2)^(1/2))/
f)/(2*c*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c^2*(e+(-4*d*f+e
^2)^(1/2))^2/f^2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e+(-4*d*f+e^2)^(
1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^
2-2*c*d*f+c*e^2)/f^2)^(1/2)-2/((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2
)*f^2*2^(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln
((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(1/2
))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+
2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4
*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2
)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2)
)/f)))+1/(-4*d*f+e^2)^(1/2)*(2/(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e
^2)*f^2/((x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2*c-c*(e-(-4*d*f+e^2)^(1/2))/f*(
x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+1/2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d
*f+c*e^2)/f^2)^(1/2)+2*c*(e-(-4*d*f+e^2)^(1/2))*f/(-(-4*d*f+e^2)^(1/2)*c*e+
2*a*f^2-2*c*d*f+c*e^2)*(2*c*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))-c*(e-(-4*d*f+
e^2)^(1/2))/f)/(2*c*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c^2
*(e-(-4*d*f+e^2)^(1/2))^2/f^2)/((x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2*c-c*(e-
(-4*d*f+e^2)^(1/2))/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+1/2*(-(-4*d*f+e^2)^
(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)-2/(-(-4*d*f+e^2)^(1/2)*c*e+2*a*
f^2-2*c*d*f+c*e^2)*f^2*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*
e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(
e-(-4*d*f+e^2)^(1/2))/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+1/2*2^(1/2)*((-(-
4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2/f*(-e+(-4*
d*f+e^2)^(1/2)))^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2/f*(-e+(-4*d*f+e^2)
^(1/2)))+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1
/2/f*(-e+(-4*d*f+e^2)^(1/2)))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27447 vs. $2(375) = 750$.

Time = 74.11 (sec) , antiderivative size = 27447, normalized size of antiderivative = 65.98

$$\int \frac{1}{(a+cx^2)^{3/2}(d+ex+fx^2)}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F]

$$\int \frac{1}{(a+cx^2)^{3/2}(d+ex+fx^2)}\,dx = \int \frac{1}{(a+cx^2)^{\frac{3}{2}}(d+ex+fx^2)}\,dx$$

```
[In] integrate(1/(c*x**2+a)**(3/2)/(f*x**2+e*x+d),x)
[Out] Integral(1/((a + c*x**2)**(3/2)*(d + e*x + f*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a+cx^2)^{3/2}(d+ex+fx^2)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re deta
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a+cx^2)^{3/2}(d+ex+fx^2)}\,dx = \text{Exception raised: AttributeError}$$

```
[In] integrate(1/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
[Out] Exception raised: AttributeError >> type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+cx^2)^{3/2}(d+ex+fx^2)}\,dx = \int \frac{1}{(c\,x^2+a)^{3/2}\,(f\,x^2+e\,x+d)}\,dx$$

```
[In] int(1/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x)
[Out] int(1/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)), x)
```

3.75 $\int \frac{1}{x(a+cx^2)^{3/2}(d+ex+fx^2)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 526

$$\int \frac{1}{x(a+cx^2)^{3/2}(d+ex+fx^2)}\,dx = \frac{1}{ad\sqrt{a+cx^2}} - \frac{a(af^2+c(e^2-df))+c^2dex}{ad(ace^2+(cd-af)^2)\sqrt{a+cx^2}}$$
$$+\frac{f\left(2e(af^2+c(e^2-2df))-\left(e-\sqrt{e^2-4df}\right)(af^2+c(e^2-df))\right)\operatorname{arctanh}\left(\frac{2af-c\left(e-\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}d\sqrt{e^2-4df}\,(ace^2+(cd-af)^2)\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}}$$
$$-\frac{f\left(2e(af^2+c(e^2-2df))-\left(e+\sqrt{e^2-4df}\right)(af^2+c(e^2-df))\right)\operatorname{arctanh}\left(\frac{2af-c\left(e+\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}d\sqrt{e^2-4df}\,(ace^2+(cd-af)^2)\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}}$$
$$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d}$$

```
[Out] -arctanh((c*x^2+a)^(1/2)/a^(1/2))/a^(3/2)/d+1/a/d/(c*x^2+a)^(1/2)+(-a*(a*f^
2+c*(-d*f+e^2))-c^2*d*e*x)/a/d/(a*c*e^2+(-a*f+c*d)^2)/(c*x^2+a)^(1/2)+1/2*f
*arctanh(1/2*(2*a*f-c*x*(e-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*
a*f^2+c*(e^2-2*d*f-e*(-4*d*f+e^2)^(1/2)))^(1/2))*(2*e*(a*f^2+c*(-2*d*f+e^2)
)-(a*f^2+c*(-d*f+e^2))*(e-(-4*d*f+e^2)^(1/2)))/d/(a*c*e^2+(-a*f+c*d)^2)*2^(
1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+c*(e^2-2*d*f-e*(-4*d*f+e^2)^(1/2)))^(1/2)-
1/2*f*arctanh(1/2*(2*a*f-c*x*(e+(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2
)/(2*a*f^2+c*(e^2-2*d*f+e*(-4*d*f+e^2)^(1/2)))^(1/2))*(2*e*(a*f^2+c*(-2*d*f
+e^2))-(a*f^2+c*(-d*f+e^2))*(e+(-4*d*f+e^2)^(1/2)))/d/(a*c*e^2+(-a*f+c*d)^2
)*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+c*(e^2-2*d*f+e*(-4*d*f+e^2)^(1/2)))^(
1/2)
```

Rubi [A] (verified)

Time = 1.32 (sec) , antiderivative size = 526, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6860, 272, 53, 65, 214, 1031, 1048, 739, 212}

$$\int \frac{1}{x(a+cx^2)^{3/2}(d+ex+fx^2)}dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d}$$
$$+\frac{f\left(2e(af^2+c(e^2-2df))-\left(e-\sqrt{e^2-4df}\right)(af^2+c(e^2-df))\right)\operatorname{arctanh}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}d\sqrt{e^2-4df}\left((cd-af)^2+ace^2\right)\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}$$
$$-\frac{f\left(2e(af^2+c(e^2-2df))-\left(\sqrt{e^2-4df}+e\right)(af^2+c(e^2-df))\right)\operatorname{arctanh}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}d\sqrt{e^2-4df}\left((cd-af)^2+ace^2\right)\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}$$
$$-\frac{a(af^2+c(e^2-df))+c^2dex}{ad\sqrt{a+cx^2}\left((cd-af)^2+ace^2\right)}+\frac{1}{ad\sqrt{a+cx^2}}$$

```
[In] Int[1/(x*(a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] 1/(a*d*Sqrt[a + c*x^2]) - (a*(a*f^2 + c*(e^2 - d*f)) + c^2*d*e*x)/(a*d*(a*c
*e^2 + (c*d - a*f)^2)*Sqrt[a + c*x^2]) + (f*(2*e*(a*f^2 + c*(e^2 - 2*d*f))
- (e - Sqrt[e^2 - 4*d*f])*(a*f^2 + c*(e^2 - d*f)))*ArcTanh[(2*a*f - c*(e -
Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 -
 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*d*Sqrt[e^2 - 4*d*f]*(a*c*e^2 + (c*d -
 a*f)^2)*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) - (f*(2*e*(
a*f^2 + c*(e^2 - 2*d*f)) - (e + Sqrt[e^2 - 4*d*f])*(a*f^2 + c*(e^2 - d*f)))
*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e
^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*d*Sqrt[e^2 -
 4*d*f]*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^
2 - 4*d*f])]) - ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]]/(a^(3/2)*d)
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 1031

```
Int[((g_.) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + c*x^2)^(p + 1)*((d + e*x + f*x^2)^
(q + 1)/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)))*(g*c*(2*a*c*e) + ((-a
)*h)*(2*c^2*d - c*(2*a*f)) + c*(g*(2*c^2*d - c*(2*a*f)) - h*(-2*a*c*e))*x),
 x] + Dist[1/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)), Int[(a + c*x^2)^
(p + 1)*(d + e*x + f*x^2)^q*Simp[(-2*g*c)*((c*d - a*f)^2 - ((-a)*e)*(c*e))*
(p + 1) + (2*(g*c*(c*d - a*f) - a*((-h)*c*e)))*(a*f*(p + 1) - c*d*(p + 2))
- e*((g*c)*(2*a*c*e) + ((-a)*h)*(2*c^2*d - c*((Plus[2])*a*f)))*(p + q + 2)
- (2*f*((g*c)*(2*a*c*e) + ((-a)*h)*(2*c^2*d + (-c)*((Plus[2])*a*f)))*(p + q
 + 2) - (2*(g*c*(c*d - a*f) - a*((-h)*c*e)))*((-c)*e*(2*p + q + 4)))*x - c*
f*(2*(g*c*(c*d - a*f) - a*((-h)*c*e)))*(2*p + 2*q + 5)*x^2, x], x], x] /; F
reeQ[{a, c, d, e, f, g, h, q}, x] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && N
eQ[a*c*e^2 + (c*d - a*f)^2, 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
```

Rule 1048

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 6860

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{1}{dx\,(a+cx^2)^{3/2}} + \frac{-e-fx}{d\,(a+cx^2)^{3/2}\,(d+ex+fx^2)}\right) dx$$

$$= \frac{\int \frac{1}{x(a+cx^2)^{3/2}}\,dx}{d} + \frac{\int \frac{-e-fx}{(a+cx^2)^{3/2}(d+ex+fx^2)}\,dx}{d}$$

$$= -\frac{a(af^2+c(e^2-df))+c^2dex}{ad\,(ace^2+(cd-af)^2)\sqrt{a+cx^2}} + \frac{\text{Subst}\left(\int \frac{1}{x(a+cx)^{3/2}}\,dx, x, x^2\right)}{2d} + \frac{\int \frac{-2ace(af^2+c(e^2-2df))-2acf(af^2+c(e^2-df))x}{\sqrt{a+cx^2}(d+ex+fx^2)}\,dx}{2acd\,(ace^2+(cd-af)^2)}$$

$$= \frac{1}{ad\sqrt{a+cx^2}} - \frac{a(af^2+c(e^2-df))+c^2dex}{ad\,(ace^2+(cd-af)^2)\sqrt{a+cx^2}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx, x, x^2\right)}{2ad} - \frac{\left(f\left(2e(af^2+c(e^2-2df))-\left(e-\sqrt{e^2-4df}\right)(af^2+c(e^2-df))\right)\right)\int \frac{1}{\left(e-\sqrt{e^2-4df}+2fx\right)\sqrt{a+cx^2}}\,dx}{d\sqrt{e^2-4df}\,(ace^2+(cd-af)^2)} + \frac{\left(f\left(2e(af^2+c(e^2-2df))-\left(e+\sqrt{e^2-4df}\right)(af^2+c(e^2-df))\right)\right)\int \frac{1}{\left(e+\sqrt{e^2-4df}+2fx\right)\sqrt{a+cx^2}}\,dx}{d\sqrt{e^2-4df}\,(ace^2+(cd-af)^2)}$$

$$= \frac{1}{ad\sqrt{a+cx^2}} - \frac{a(af^2+c(e^2-df))+c^2dex}{ad\,(ace^2+(cd-af)^2)\sqrt{a+cx^2}} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx, x, \sqrt{a+cx^2}\right)}{acd} + \frac{\left(f\left(2e(af^2+c(e^2-2df))-\left(e-\sqrt{e^2-4df}\right)(af^2+c(e^2-df))\right)\right)\text{Subst}\left(\int \frac{1}{4af^2+c\left(e-\sqrt{e^2-4df}\right)^2-a\right.}{d\sqrt{e^2-4df}\,(ace^2+(cd-af)^2)} - \frac{\left(f\left(2e(af^2+c(e^2-2df))-\left(e+\sqrt{e^2-4df}\right)(af^2+c(e^2-df))\right)\right)\text{Subst}\left(\int \frac{1}{4af^2+c\left(e+\sqrt{e^2-4df}\right)^2-a\right.}{d\sqrt{e^2-4df}\,(ace^2+(cd-af)^2)}$$

$$= \frac{1}{ad\sqrt{a+cx^2}} - \frac{a(af^2+c(e^2-df))+c^2dex}{ad\,(ace^2+(cd-af)^2)\sqrt{a+cx^2}}$$
$$+ \frac{f\left(2e(af^2+c(e^2-2df)) - \left(e-\sqrt{e^2-4df}\right)(af^2+c(e^2-df))\right)\tanh^{-1}\left(\frac{2af-c\left(e-\sqrt{e^2-4df}\right)\ldots}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}\ldots}\right)}{\sqrt{2}d\sqrt{e^2-4df}\,(ace^2+(cd-af)^2)\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}}$$
$$- \frac{f\left(2e(af^2+c(e^2-2df)) - \left(e+\sqrt{e^2-4df}\right)(af^2+c(e^2-df))\right)\tanh^{-1}\left(\frac{2af-c\left(e+\sqrt{e^2-4df}\right)\ldots}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}\ldots}\right)}{\sqrt{2}d\sqrt{e^2-4df}\,(ace^2+(cd-af)^2)\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}}$$
$$- \frac{\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.82 (sec) , antiderivative size = 542, normalized size of antiderivative = 1.03

$$\int \frac{1}{x\,(a+cx^2)^{3/2}\,(d+ex+fx^2)}\,dx =$$
$$-\frac{c(-cd+af+cex)}{a\,(c^2d^2+a^2f^2+ac\,(e^2-2df))\sqrt{a+cx^2}} + \frac{2\text{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d}$$
$$+ \frac{\text{RootSum}\left[a^2f+2a\sqrt{c}e\#1+4cd\#1^2-2af\#1^2-2\sqrt{c}e\#1^3+f\#1^4\&, \frac{ace^2f\log\left(-\sqrt{c}x+\sqrt{a+cx^2}-\#1\right)-acdf^2\log\ldots}{\ldots}\right.}{\ldots}$$

```
[In] Integrate[1/(x*(a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] -((c*(-(c*d) + a*f + c*e*x))/(a*(c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f))*Sqr
t[a + c*x^2])) + (2*ArcTanh[(Sqrt[c]*x - Sqrt[a + c*x^2])/Sqrt[a]])/(a^(3/2
)*d) + RootSum[a^2*f + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 - 2*a*f*#1^2 - 2*Sqrt[
c]*e*#1^3 + f*#1^4 & , (a*c*e^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]
 - a*c*d*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] + a^2*f^3*Log[-(Sqrt[c
]*x) + Sqrt[a + c*x^2] - #1] + 2*c^(3/2)*e^3*Log[-(Sqrt[c]*x) + Sqrt[a + c*
x^2] - #1]*#1 - 4*c^(3/2)*d*e*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1
 + 2*a*Sqrt[c]*e*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 - c*e^2*f*
Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2 + c*d*f^2*Log[-(Sqrt[c]*x) +
Sqrt[a + c*x^2] - #1]*#1^2 - a*f^3*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]
*#1^2)/(a*Sqrt[c]*e + 4*c*d*#1 - 2*a*f*#1 - 3*Sqrt[c]*e*#1^2 + 2*f*#1^3) &
]/(d*(c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f)))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1564 vs. $2(480) = 960$.

Time = 0.68 (sec) , antiderivative size = 1565, normalized size of antiderivative = 2.98

method	result	size
default	Expression too large to display	1565

```
[In] int(1/x/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -4*f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))*(1/a/(c*x^2+a)^(1/2)-1/
a^(3/2)*ln((2*a+2*a^(1/2)*(c*x^2+a)^(1/2))/x))+2*f/(e+(-4*d*f+e^2)^(1/2))/(
-4*d*f+e^2)^(1/2)*(2/((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)*f^2/((x
+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4
*d*f+e^2)^(1/2))/f)+1/2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)
^(1/2)+2*c*(e+(-4*d*f+e^2)^(1/2))*f/((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f
+c*e^2)*(2*c*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)-c*(e+(-4*d*f+e^2)^(1/2))/f)/(
2*c*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c^2*(e+(-4*d*f+e^2)^
(1/2))^2/f^2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e+(-4*d*f+e^2)^(1/2)
)/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*
c*d*f+c*e^2)/f^2)^(1/2)-2/((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)*f^
2*2^(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((
-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f
*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2*a*
f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(
e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1
/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)
))+2*f/(-e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)*(2/(-(-4*d*f+e^2)^(1/2)*c
*e+2*a*f^2-2*c*d*f+c*e^2)*f^2/((x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2*c-c*(e-(
-4*d*f+e^2)^(1/2))/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+1/2*(-(-4*d*f+e^2)^(
1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)+2*c*(e-(-4*d*f+e^2)^(1/2))*f/(-(
-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)*(2*c*(x-1/2/f*(-e+(-4*d*f+e^2)
^(1/2)))-c*(e-(-4*d*f+e^2)^(1/2))/f)/(2*c*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-
2*c*d*f+c*e^2)/f^2-c^2*(e-(-4*d*f+e^2)^(1/2))^2/f^2)/((x-1/2/f*(-e+(-4*d*f+
e^2)^(1/2)))^2*c-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)
))+1/2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)-2/(-(-4*d
*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)*f^2*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*
c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-
2*c*d*f+c*e^2)/f^2-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/
2)))+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2
)*(4*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-
1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c
*e^2)/f^2)^(1/2))/(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x(a+cx^2)^{3/2}(d+ex+fx^2)}\,dx = \text{Timed out}$$

```
[In] integrate(1/x/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{x(a+cx^2)^{3/2}(d+ex+fx^2)}\,dx = \int \frac{1}{x(a+cx^2)^{\frac{3}{2}}(d+ex+fx^2)}\,dx$$

```
[In] integrate(1/x/(c*x**2+a)**(3/2)/(f*x**2+e*x+d),x)
[Out] Integral(1/(x*(a + c*x**2)**(3/2)*(d + e*x + f*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{x(a+cx^2)^{3/2}(d+ex+fx^2)}\,dx = \int \frac{1}{(cx^2+a)^{\frac{3}{2}}(fx^2+ex+d)x}\,dx$$

```
[In] integrate(1/x/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")
[Out] integrate(1/((c*x^2 + a)^(3/2)*(f*x^2 + e*x + d)*x), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{x(a+cx^2)^{3/2}(d+ex+fx^2)}\,dx = \text{Timed out}$$

```
[In] integrate(1/x/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx=\int \frac{1}{x\,{\left(c\,x^2+a\right)}^{3/2}\,\left(f\,x^2+e\,x+d\right)} \,dx$$

```
[In] int(1/(x*(a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x)
[Out] int(1/(x*(a + c*x^2)^(3/2)*(d + e*x + f*x^2)), x)
```

3.76 $\int \frac{1}{x^2\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 618

$$\int \frac{1}{x^2\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx = -\frac{e}{ad^2\sqrt{a+cx^2}}-\frac{1}{adx\sqrt{a+cx^2}}$$

$$-\frac{2cx}{a^2d\sqrt{a+cx^2}}+\frac{ae(af^2+c(e^2-2df))+cd(af^2+c(e^2-df))\,x}{ad^2\left(ace^2+(cd-af)^2\right)\sqrt{a+cx^2}}$$

$$+\frac{f\left(e\left(e-\sqrt{e^2-4df}\right)\left(af^2+c(e^2-2df)\right)-2(af^2(e^2-df)+c(e^4-3de^2f+d^2f^2))\right)\operatorname{arctanh}\left(\frac{2af-\cdots}{\sqrt{2}\sqrt{2af^2+c(\cdots}}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\left(ace^2+(cd-af)^2\right)\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}}$$

$$-\frac{f\left(e\left(e+\sqrt{e^2-4df}\right)\left(af^2+c(e^2-2df)\right)-2(af^2(e^2-df)+c(e^4-3de^2f+d^2f^2))\right)\operatorname{arctanh}\left(\frac{2af\cdots}{\sqrt{2}\sqrt{2af^2+c(\cdots}}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\left(ace^2+(cd-af)^2\right)\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}}$$

$$+\frac{e\operatorname{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d^2}$$

```
[Out] e*arctanh((c*x^2+a)^(1/2)/a^(1/2))/a^(3/2)/d^2-e/a/d^2/(c*x^2+a)^(1/2)-1/a/
d/x/(c*x^2+a)^(1/2)-2*c*x/a^2/d/(c*x^2+a)^(1/2)+(a*e*(a*f^2+c*(-2*d*f+e^2))
+c*d*(a*f^2+c*(-d*f+e^2))*x)/a/d^2/(a*c*e^2+(-a*f+c*d)^2)/(c*x^2+a)^(1/2)+1
/2*f*arctanh(1/2*(2*a*f-c*x*(e-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)
/(2*a*f^2+c*(e^2-2*d*f-e*(-4*d*f+e^2)^(1/2)))^(1/2))*(-2*a*f^2*(-d*f+e^2)-2
*c*(d^2*f^2-3*d*e^2*f+e^4)+e*(a*f^2+c*(-2*d*f+e^2))*(e-(-4*d*f+e^2)^(1/2)))
/d^2/(a*c*e^2+(-a*f+c*d)^2)*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+c*(e^2-2*d*
f-e*(-4*d*f+e^2)^(1/2)))^(1/2)-1/2*f*arctanh(1/2*(2*a*f-c*x*(e+(-4*d*f+e^2)
^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+c*(e^2-2*d*f+e*(-4*d*f+e^2)^(1/2)
```

```
))^(1/2))*(-2*a*f^2*(-d*f+e^2)-2*c*(d^2*f^2-3*d*e^2*f+e^4)+e*(a*f^2+c*(-2*d
*f+e^2))*(e+(-4*d*f+e^2)^(1/2)))/d^2/(a*c*e^2+(-a*f+c*d)^2)*2^(1/2)/(-4*d*f
+e^2)^(1/2)/(2*a*f^2+c*(e^2-2*d*f+e*(-4*d*f+e^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 1.47 (sec) , antiderivative size = 618, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.407$, Rules used = {6860, 277, 197, 272, 53, 65, 214, 1031, 1048, 739, 212}

$$\int \frac{1}{x^2 (a+cx^2)^{3/2} (d+ex+fx^2)} \, dx = \frac{e \text{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d^2} - \frac{2cx}{a^2 d\sqrt{a+cx^2}}$$
$$+ \frac{f\left(e\left(e-\sqrt{e^2-4df}\right)(af^2+c(e^2-2df)) - 2(af^2(e^2-df)+c(d^2f^2-3de^2f+e^4))\right)\text{arctanh}\left(\frac{2af-\cdots}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2a\cdots}}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\left((cd-af)^2+ace^2\right)\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}$$
$$- \frac{f\left(e\left(\sqrt{e^2-4df}+e\right)(af^2+c(e^2-2df)) - 2(af^2(e^2-df)+c(d^2f^2-3de^2f+e^4))\right)\text{arctanh}\left(\frac{2af-\cdots}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2a\cdots}}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\left((cd-af)^2+ace^2\right)\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}$$
$$+ \frac{cdx(af^2+c(e^2-df))+ae(af^2+c(e^2-2df))}{ad^2\sqrt{a+cx^2}\left((cd-af)^2+ace^2\right)} - \frac{e}{ad^2\sqrt{a+cx^2}} - \frac{1}{adx\sqrt{a+cx^2}}$$

```
[In] Int[1/(x^2*(a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] -(e/(a*d^2*Sqrt[a + c*x^2])) - 1/(a*d*x*Sqrt[a + c*x^2]) - (2*c*x)/(a^2*d*S
qrt[a + c*x^2]) + (a*e*(a*f^2 + c*(e^2 - 2*d*f)) + c*d*(a*f^2 + c*(e^2 - d*
f))*x)/(a*d^2*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[a + c*x^2]) + (f*(e*(e - Sqrt[
e^2 - 4*d*f])*(a*f^2 + c*(e^2 - 2*d*f)) - 2*(a*f^2*(e^2 - d*f) + c*(e^4 - 3
*d*e^2*f + d^2*f^2)))*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2
]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/
(Sqrt[2]*d^2*Sqrt[e^2 - 4*d*f]*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[2*a*f^2 + c*(
e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) - (f*(e*(e + Sqrt[e^2 - 4*d*f])*(a*f^2
 + c*(e^2 - 2*d*f)) - 2*(a*f^2*(e^2 - d*f) + c*(e^4 - 3*d*e^2*f + d^2*f^2))
)*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(
e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*d^2*Sqrt[e^
2 - 4*d*f]*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt
[e^2 - 4*d*f])]) + (e*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(a^(3/2)*d^2)
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
```

```
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
```

```
[{a, c, d, e}, x]
```

Rule 1031

```
Int[((g_.) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + c*x^2)^(p + 1)*((d + e*x + f*x^2)^
(q + 1)/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)))*(g*c*(2*a*c*e) + ((-a
)*h)*(2*c^2*d - c*(2*a*f)) + c*(g*(2*c^2*d - c*(2*a*f)) - h*(-2*a*c*e))*x),
 x] + Dist[1/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)), Int[(a + c*x^2)^
(p + 1)*(d + e*x + f*x^2)^q*Simp[(-2*g*c)*((c*d - a*f)^2 - ((-a)*e)*(c*e))*
(p + 1) + (2*(g*c*(c*d - a*f) - a*((-h)*c*e)))*(a*f*(p + 1) - c*d*(p + 2))
- e*((g*c)*(2*a*c*e) + ((-a)*h)*(2*c^2*d - c*((Plus[2])*a*f)))*(p + q + 2)
- (2*f*((g*c)*(2*a*c*e) + ((-a)*h)*(2*c^2*d + (-c)*((Plus[2])*a*f)))*(p + q
 + 2) - (2*(g*c*(c*d - a*f) - a*((-h)*c*e)))*((-c)*e*(2*p + q + 4)))*x - c*
f*(2*(g*c*(c*d - a*f) - a*((-h)*c*e)))*(2*p + 2*q + 5)*x^2, x], x], x] /; F
reeQ[{a, c, d, e, f, g, h, q}, x] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && N
eQ[a*c*e^2 + (c*d - a*f)^2, 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
```

Rule 1048

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c*g - h*(
b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Dist[(2*c*g -
h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a,
 b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 6860

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{1}{dx^2\left(a+cx^2\right)^{3/2}}-\frac{e}{d^2x\left(a+cx^2\right)^{3/2}}+\frac{e^2-df+efx}{d^2\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\right)dx\\
&= \frac{\int \frac{e^2-df+efx}{\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx}{d^2}+\frac{\int \frac{1}{x^2\left(a+cx^2\right)^{3/2}}\,dx}{d}-\frac{e\int \frac{1}{x\left(a+cx^2\right)^{3/2}}\,dx}{d^2}\\
&= -\frac{1}{adx\sqrt{a+cx^2}}+\frac{ae\left(af^2+c\left(e^2-2df\right)\right)+cd\left(af^2+c\left(e^2-df\right)\right)x}{ad^2\left(ace^2+(cd-af)^2\right)\sqrt{a+cx^2}}-\frac{(2c)\int \frac{1}{\left(a+cx^2\right)^{3/2}}\,dx}{ad}\\
&\quad -\frac{e\text{Subst}\left(\int \frac{1}{x(a+cx)^{3/2}}\,dx,x,x^2\right)}{2d^2}+\frac{\int \frac{2ac\left(af^2\left(e^2-df\right)+c\left(e^4-3de^2f+d^2f^2\right)\right)+2acef\left(af^2+c\left(e^2-2df\right)\right)x}{\sqrt{a+cx^2}\left(d+ex+fx^2\right)}\,dx}{2acd^2\left(ace^2+(cd-af)^2\right)}
\end{aligned}$$

$$= -\frac{e}{ad^2\sqrt{a+cx^2}} - \frac{1}{adx\sqrt{a+cx^2}} - \frac{2cx}{a^2d\sqrt{a+cx^2}}$$
$$+ \frac{ae(af^2+c(e^2-2df))+cd(af^2+c(e^2-df))\,x}{ad^2\left(ace^2+(cd-af)^2\right)\sqrt{a+cx^2}} - \frac{e\text{Subst}\left(\int \frac{1}{x\sqrt{a+cx}}\,dx, x, x^2\right)}{2ad^2}$$
$$- \frac{\left(f\left(e\left(e-\sqrt{e^2-4df}\right)(af^2+c(e^2-2df))-2(af^2(e^2-df)+c(e^4-3de^2f+d^2f^2))\right)\right)\int \frac{}{\left(e-\sqrt{e\cdots}\right.}}{d^2\sqrt{e^2-4df}\left(ace^2+(cd-af)^2\right)}$$
$$+ \frac{\left(f\left(e\left(e+\sqrt{e^2-4df}\right)(af^2+c(e^2-2df))-2(af^2(e^2-df)+c(e^4-3de^2f+d^2f^2))\right)\right)\int \frac{}{\left(e+\sqrt{e\cdots}\right.}}{d^2\sqrt{e^2-4df}\left(ace^2+(cd-af)^2\right)}$$

$$= -\frac{e}{ad^2\sqrt{a+cx^2}} - \frac{1}{adx\sqrt{a+cx^2}} - \frac{2cx}{a^2d\sqrt{a+cx^2}}$$
$$+ \frac{ae(af^2+c(e^2-2df))+cd(af^2+c(e^2-df))\,x}{ad^2\left(ace^2+(cd-af)^2\right)\sqrt{a+cx^2}}$$
$$- \frac{e\text{Subst}\left(\int \frac{1}{-\frac{a}{c}+\frac{x^2}{c}}\,dx, x, \sqrt{a+cx^2}\right)}{acd^2}$$
$$+ \frac{\left(f\left(e\left(e-\sqrt{e^2-4df}\right)(af^2+c(e^2-2df))-2(af^2(e^2-df)+c(e^4-3de^2f+d^2f^2))\right)\right)\text{Subst}\left(\cdots\right.}{d^2\sqrt{e^2-4df}\left(ace^2+(cd-af)^2\right)}$$
$$- \frac{\left(f\left(e\left(e+\sqrt{e^2-4df}\right)(af^2+c(e^2-2df))-2(af^2(e^2-df)+c(e^4-3de^2f+d^2f^2))\right)\right)\text{Subst}\left(\cdots\right.}{d^2\sqrt{e^2-4df}\left(ace^2+(cd-af)^2\right)}$$

$$= -\frac{e}{ad^2\sqrt{a+cx^2}} - \frac{1}{adx\sqrt{a+cx^2}} - \frac{2cx}{a^2d\sqrt{a+cx^2}}$$
$$+ \frac{ae(af^2+c(e^2-2df))+cd(af^2+c(e^2-df))\,x}{ad^2\left(ace^2+(cd-af)^2\right)\sqrt{a+cx^2}}$$
$$+ \frac{f\left(e\left(e-\sqrt{e^2-4df}\right)(af^2+c(e^2-2df))-2(af^2(e^2-df)+c(e^4-3de^2f+d^2f^2))\right)\tanh^{-1}\left(\cdots\right.}{\sqrt{2}d^2\sqrt{e^2-4df}\left(ace^2+(cd-af)^2\right)\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-\cdots}\right.}}$$
$$- \frac{f\left(e\left(e+\sqrt{e^2-4df}\right)(af^2+c(e^2-2df))-2(af^2(e^2-df)+c(e^4-3de^2f+d^2f^2))\right)\tanh^{-1}\left(\cdots\right.}{\sqrt{2}d^2\sqrt{e^2-4df}\left(ace^2+(cd-af)^2\right)\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-\cdots}\right.}}$$
$$+ \frac{e\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d^2}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 1.19 (sec) , antiderivative size = 684, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2 (a+cx^2)^{3/2} (d+ex+fx^2)} \, dx =$$

$$-\frac{\frac{d(a^3f^2+2c^3d^2x^2+ac^2(d^2+e^2x^2+dx(e-3fx))+a^2c(e^2+f(-2d+fx^2)))}{a^2(c^2d^2+a^2f^2+ac(e^2-2df))x\sqrt{a+cx^2}}+\frac{2e\text{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}}+\frac{\text{RootSum}\left[a^2f+2a\sqrt{c}e\#1+4\dots\right.}{}}{}$$

```
[In] Integrate[1/(x^2*(a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] -(((d*(a^3*f^2 + 2*c^3*d^2*x^2 + a*c^2*(d^2 + e^2*x^2 + d*x*(e - 3*f*x)) +
a^2*c*(e^2 + f*(-2*d + f*x^2))))/(a^2*(c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f
))*x*Sqrt[a + c*x^2]) + (2*e*ArcTanh[(Sqrt[c]*x - Sqrt[a + c*x^2])/Sqrt[a]]
)/a^(3/2) + RootSum[a^2*f + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 - 2*a*f*#1^2 - 2*
Sqrt[c]*e*#1^3 + f*#1^4 & , (a*c*e^3*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] -
 #1] - 2*a*c*d*e*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] + a^2*e*f^3*L
og[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] + 2*c^(3/2)*e^4*Log[-(Sqrt[c]*x) +
Sqrt[a + c*x^2] - #1]*#1 - 6*c^(3/2)*d*e^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*
x^2] - #1]*#1 + 2*c^(3/2)*d^2*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*
#1 + 2*a*Sqrt[c]*e^2*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 - 2*a*
Sqrt[c]*d*f^3*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 - c*e^3*f*Log[-(S
qrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2 + 2*c*d*e*f^2*Log[-(Sqrt[c]*x) + Sqr
t[a + c*x^2] - #1]*#1^2 - a*e*f^3*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*
#1^2)/(a*Sqrt[c]*e + 4*c*d*#1 - 2*a*f*#1 - 3*Sqrt[c]*e*#1^2 + 2*f*#1^3) & ]
/(c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f)))/d^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1638 vs. 2(565) = 1130.

Time = 0.79 (sec) , antiderivative size = 1639, normalized size of antiderivative = 2.65

method	result	size
default	Expression too large to display	1639
risch	Expression too large to display	1788

```
[In] int(1/x^2/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -4*f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))*(-1/a/x/(c*x^2+a)^(1/2)
-2*c/a^2*x/(c*x^2+a)^(1/2))-16*f^2*e/(-e+(-4*d*f+e^2)^(1/2))^2/(e+(-4*d*f+e
^2)^(1/2))^2*(1/a/(c*x^2+a)^(1/2)-1/a^(3/2)*ln((2*a+2*a^(1/2)*(c*x^2+a)^(1/
```

```
2))/x))-4*f^2/(e+(-4*d*f+e^2)^(1/2))^2/(-4*d*f+e^2)^(1/2)*(2/((-4*d*f+e^2)^
(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)*f^2/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-
c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*((-4*d*f+e^
2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)+2*c*(e+(-4*d*f+e^2)^(1/2))*f
/((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)*(2*c*(x+1/2*(e+(-4*d*f+e^2)
^(1/2))/f)-c*(e+(-4*d*f+e^2)^(1/2))/f)/(2*c*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2
-2*c*d*f+c*e^2)/f^2-c^2*(e+(-4*d*f+e^2)^(1/2))^2/f^2)/((x+1/2*(e+(-4*d*f+e^
2)^(1/2))/f)^2*c-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f
)+1/2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)-2/((-4*d*f+
e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)*f^2*2^(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+
2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d
*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1
/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x
+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(
-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)
^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)))+4*f^2/(-e+(-4*d*f+e^2)^(1/2))^2/
(-4*d*f+e^2)^(1/2)*(2/(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)*f^2/(
(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2*c-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2/f*(
-e+(-4*d*f+e^2)^(1/2)))+1/2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)
/f^2)^(1/2)+2*c*(e-(-4*d*f+e^2)^(1/2))*f/(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2
*c*d*f+c*e^2)*(2*c*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))-c*(e-(-4*d*f+e^2)^(1/2
))/f)/(2*c*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c^2*(e-(-4*d
*f+e^2)^(1/2))^2/f^2)/((x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2*c-c*(e-(-4*d*f+e
^2)^(1/2))/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+1/2*(-(-4*d*f+e^2)^(1/2)*c*e
+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)-2/(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d
*f+c*e^2)*f^2*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)
^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e-(-4*d*f
+e^2)^(1/2))/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+1/2*2^(1/2)*((-(-4*d*f+e^2
)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2/f*(-e+(-4*d*f+e^2)^
(1/2)))^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+
2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2/f*(-e+
(-4*d*f+e^2)^(1/2)))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a+cx^2)^{3/2} (d+ex+fx^2)} dx = \text{Timed out}$$

```
[In] integrate(1/x^2/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{x^2 (a + cx^2)^{3/2} (d + ex + fx^2)} dx = \int \frac{1}{x^2 (a + cx^2)^{\frac{3}{2}} (d + ex + fx^2)} dx$$

```
[In] integrate(1/x**2/(c*x**2+a)**(3/2)/(f*x**2+e*x+d),x)
[Out] Integral(1/(x**2*(a + c*x**2)**(3/2)*(d + e*x + f*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{x^2 (a + cx^2)^{3/2} (d + ex + fx^2)} dx = \int \frac{1}{(cx^2 + a)^{\frac{3}{2}} (fx^2 + ex + d)x^2} dx$$

```
[In] integrate(1/x^2/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")
[Out] integrate(1/((c*x^2 + a)^(3/2)*(f*x^2 + e*x + d)*x^2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x^2 (a + cx^2)^{3/2} (d + ex + fx^2)} dx = \text{Exception raised: AttributeError}$$

```
[In] integrate(1/x^2/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
[Out] Exception raised: AttributeError >> type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a + cx^2)^{3/2} (d + ex + fx^2)} dx = \int \frac{1}{x^2 (c\,x^2 + a)^{3/2} (f\,x^2 + e\,x + d)} dx$$

```
[In] int(1/(x^2*(a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x)
[Out] int(1/(x^2*(a + c*x^2)^(3/2)*(d + e*x + f*x^2)), x)
```

3.77 $\int \frac{x^3\sqrt{a+bx+cx^2}}{d-fx^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 392

$$\begin{aligned}\int \frac{x^3\sqrt{a+bx+cx^2}}{d-fx^2}\,dx &= -\frac{d\sqrt{a+bx+cx^2}}{f^2}+\frac{b(b+2cx)\sqrt{a+bx+cx^2}}{8c^2f}-\frac{(a+bx+cx^2)^{3/2}}{3cf}\\ &\quad-\frac{bd\,\mathrm{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}f^2}-\frac{b(b^2-4ac)\,\mathrm{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{5/2}f}\\ &\quad-\frac{d\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\,\mathrm{arctanh}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2f^{5/2}}\\ &\quad+\frac{d\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\,\mathrm{arctanh}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2f^{5/2}}\end{aligned}$$

```
[Out] -1/3*(c*x^2+b*x+a)^(3/2)/c/f-1/16*b*(-4*a*c+b^2)*arctanh(1/2*(2*c*x+b)/c^(1
/2)/(c*x^2+b*x+a)^(1/2))/c^(5/2)/f-1/2*b*d*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c
*x^2+b*x+a)^(1/2))/f^2/c^(1/2)-d*(c*x^2+b*x+a)^(1/2)/f^2+1/8*b*(2*c*x+b)*(c
*x^2+b*x+a)^(1/2)/c^2/f-1/2*d*arctanh(1/2*(b*d^(1/2)-2*a*f^(1/2)+x*(2*c*d^(
1/2)-b*f^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2))*(c*
d+a*f-b*d^(1/2)*f^(1/2))^(1/2)/f^(5/2)+1/2*d*arctanh(1/2*(b*d^(1/2)+2*a*f^(
1/2)+x*(2*c*d^(1/2)+b*f^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f+b*d^(1/2)*f^(1
/2))^(1/2))*(c*d+a*f+b*d^(1/2)*f^(1/2))^(1/2)/f^(5/2)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {6857, 654, 626, 635, 212, 1035, 1092, 1047, 738}

$$\int \frac{x^3\sqrt{a+bx+cx^2}}{d-fx^2}\,dx$$
$$= -\frac{b(b^2-4ac)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{5/2}f}$$
$$-\frac{d\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\operatorname{arctanh}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2f^{5/2}}$$
$$+\frac{d\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\operatorname{arctanh}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2f^{5/2}}-\frac{bd\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}f^2}$$
$$+\frac{b(b+2cx)\sqrt{a+bx+cx^2}}{8c^2f}-\frac{d\sqrt{a+bx+cx^2}}{f^2}-\frac{\left(a+bx+cx^2\right)^{3/2}}{3cf}$$

```
[In] Int[(x^3*Sqrt[a + b*x + c*x^2])/(d - f*x^2),x]
```

```
[Out] -((d*Sqrt[a + b*x + c*x^2])/f^2) + (b*(b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(8
*c^2*f) - (a + b*x + c*x^2)^(3/2)/(3*c*f) - (b*d*ArcTanh[(b + 2*c*x)/(2*Sqr
t[c]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[c]*f^2) - (b*(b^2 - 4*a*c)*ArcTanh[(b
 + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(5/2)*f) - (d*Sqrt[c*d
- b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d]
- b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2
])])/(2*f^(5/2)) + (d*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d
] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt
[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*f^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1035

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[h*(a + b*x + c*x^2)^p*((d + f*x^2)^(q +
 1)/(2*f*(p + q + 1))), x] - Dist[1/(2*f*(p + q + 1)), Int[(a + b*x + c*x^2
)^(p - 1)*(d + f*x^2)^q*Simp[h*p*(b*d) + a*(-2*g*f)*(p + q + 1) + (2*h*p*(c
*d - a*f) + b*(-2*g*f)*(p + q + 1))*x + (h*p*((-b)*f) + c*(-2*g*f)*(p + q +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*a
*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]
```

Rule 1047

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(-a)*c, 2]}, Dist[h/2 + c*(g/(2*q)
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - c*(g/(2*q)
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[(-a)*c]
```

Rule 1092

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
 (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + e*x + f
*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x
 + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f
, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{x\sqrt{a+bx+cx^2}}{f}+\frac{dx\sqrt{a+bx+cx^2}}{f\,(d-fx^2)}\right)dx\\
&= -\frac{\int x\sqrt{a+bx+cx^2}\,dx}{f}+\frac{d\int \frac{x\sqrt{a+bx+cx^2}}{d-fx^2}\,dx}{f}\\
&= -\frac{d\sqrt{a+bx+cx^2}}{f^2}-\frac{(a+bx+cx^2)^{3/2}}{3cf}+\frac{d\int \frac{\frac{bd}{2}+(cd+af)x+\frac{1}{2}bfx^2}{\sqrt{a+bx+cx^2}(d-fx^2)}\,dx}{f^2}+\frac{b\int \sqrt{a+bx+cx^2}\,dx}{2cf}\\
&= -\frac{d\sqrt{a+bx+cx^2}}{f^2}+\frac{b(b+2cx)\sqrt{a+bx+cx^2}}{8c^2f}-\frac{(a+bx+cx^2)^{3/2}}{3cf}\\
&\quad-\frac{d\int \frac{-bdf-f(cd+af)x}{\sqrt{a+bx+cx^2}(d-fx^2)}\,dx}{f^3}-\frac{(bd)\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{2f^2}-\frac{(b(b^2-4ac))\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{16c^2f}\\
&= -\frac{d\sqrt{a+bx+cx^2}}{f^2}+\frac{b(b+2cx)\sqrt{a+bx+cx^2}}{8c^2f}\\
&\quad-\frac{(a+bx+cx^2)^{3/2}}{3cf}-\frac{(bd)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx,x,\frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{f^2}\\
&\quad-\frac{(b(b^2-4ac))\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx,x,\frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{8c^2f}\\
&\quad+\frac{\left(d\left(cd-b\sqrt{d}\sqrt{f}+af\right)\right)\int \frac{1}{\left(-\sqrt{d}\sqrt{f}-fx\right)\sqrt{a+bx+cx^2}}\,dx}{2f^2}\\
&\quad+\frac{\left(d\left(cd+b\sqrt{d}\sqrt{f}+af\right)\right)\int \frac{1}{\left(\sqrt{d}\sqrt{f}-fx\right)\sqrt{a+bx+cx^2}}\,dx}{2f^2}
\end{aligned}$$

$$= -\frac{d\sqrt{a+bx+cx^2}}{f^2} + \frac{b(b+2cx)\sqrt{a+bx+cx^2}}{8c^2f} - \frac{(a+bx+cx^2)^{3/2}}{3cf} - \frac{bd\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}f^2} - \frac{b(b^2-4ac)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{5/2}f} - \frac{\left(d\left(cd-b\sqrt{d}\sqrt{f}+af\right)\right)\operatorname{Subst}\left(\int \frac{1}{4cdf-4b\sqrt{d}f^{3/2}+4af^2-x^2}\,dx, x, \frac{b\sqrt{d}\sqrt{f}-2af-\left(-2c\sqrt{d}\sqrt{f}+bf\right)x}{\sqrt{a+bx+cx^2}}\right)}{f^2} - \frac{\left(d\left(cd+b\sqrt{d}\sqrt{f}+af\right)\right)\operatorname{Subst}\left(\int \frac{1}{4cdf+4b\sqrt{d}f^{3/2}+4af^2-x^2}\,dx, x, \frac{-b\sqrt{d}\sqrt{f}-2af-\left(2c\sqrt{d}\sqrt{f}+bf\right)x}{\sqrt{a+bx+cx^2}}\right)}{f^2}$$

$$= -\frac{d\sqrt{a+bx+cx^2}}{f^2} + \frac{b(b+2cx)\sqrt{a+bx+cx^2}}{8c^2f} - \frac{(a+bx+cx^2)^{3/2}}{3cf} - \frac{bd\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}f^2} - \frac{b(b^2-4ac)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{5/2}f} - \frac{d\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\tanh^{-1}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2f^{5/2}} + \frac{d\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\tanh^{-1}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2f^{5/2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.69 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.04

$$\int \frac{x^3\sqrt{a+bx+cx^2}}{d-fx^2}\,dx$$

$$= \frac{-2\sqrt{c}\sqrt{a+x(b+cx)}(-3b^2f+2cf(4a+bx)+8c^2(3d+fx^2))+3b(8c^2d+b^2f-4acf)\log\left(b+2cx-2\right.}{}$$

```
[In] Integrate[(x^3*Sqrt[a + b*x + c*x^2])/(d - f*x^2),x]
```

```
[Out] (-2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(-3*b^2*f + 2*c*f*(4*a + b*x) + 8*c^2*(3*
d + f*x^2)) + 3*b*(8*c^2*d + b^2*f - 4*a*c*f)*Log[b + 2*c*x - 2*Sqrt[c]*Sqr
t[a + x*(b + c*x)]] - 24*c^(5/2)*d*RootSum[b^2*d - a^2*f - 4*b*Sqrt[c]*d*#1
 + 4*c*d*#1^2 + 2*a*f*#1^2 - f*#1^4 & , (b^2*d*Log[-(Sqrt[c]*x) + Sqrt[a +
b*x + c*x^2] - #1] - a*c*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] -
 a^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*b*Sqrt[c]*d*Log[-
```

```
(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + c*d*Log[-(Sqrt[c]*x) + Sqrt[
a + b*x + c*x^2] - #1]*#1^2 + a*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2]
- #1]*#1^2)/(b*Sqrt[c]*d - 2*c*d*#1 - a*f*#1 + f*#1^3) & ])/(48*c^(5/2)*f^2
)
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 525, normalized size of antiderivative = 1.34

method	result
risch	$-\frac{\left(8f\,c^{2}x^{2}+2bcfx+8acf-3b^{2}f+24c^{2}d\right)\sqrt{c\,x^{2}+bx+a}}{24c^{2}f^{2}}+\frac{\frac{b\left(4acf-b^{2}f-8c^{2}d\right)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx+a}\right)}{\sqrt{c}}+\frac{8c^{2}d\left(\sqrt{df}\,af+\sqrt{df}\,cd+bdf\right)\ln\left(\frac{2b \cdots}{\cdots}\right.}{\cdots}}{\cdots}$
default	$-\frac{\frac{\left(c\,x^{2}+bx+a\right)^{\frac{3}{2}}}{3c}-\frac{b\left(\frac{(2cx+b)\sqrt{c\,x^{2}+bx+a}}{4c}+\frac{\left(4ac-b^{2}\right)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx+a}\right)}{8c^{\frac{3}{2}}}\right)}{2c}}{f}-\frac{d\left(\sqrt{\left(x-\frac{\sqrt{df}}{f}\right)^{2}c+\frac{\left(2c\sqrt{df}+bf\right)\left(x-\frac{\sqrt{df}}{f}\right)}{f}+\frac{b\sqrt{df}+\cdots}{f}}\cdots\right.}{\cdots}$

```
[In] int(x^3*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*(8*c^2*f*x^2+2*b*c*f*x+8*a*c*f-3*b^2*f+24*c^2*d)*(c*x^2+b*x+a)^(1/2)/
c^2/f^2+1/16/f^2/c^2*(b*(4*a*c*f-b^2*f-8*c^2*d)*ln((1/2*b+c*x)/c^(1/2)+(c*x
^2+b*x+a)^(1/2))/c^(1/2)+8*c^2*d*((d*f)^(1/2)*a*f+(d*f)^(1/2)*c*d+b*d*f)/(d
*f)^(1/2)/f/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+f*a+c*d)
/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f*a+c*d)/f)^
(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(
d*f)^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))+8*c^2*d*((d*f)^(1/2)*a*f+(
d*f)^(1/2)*c*d-b*d*f)/(d*f)^(1/2)/f/(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*ln
((2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)
+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d
*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/(x+(d
*f)^(1/2)/f)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^3\sqrt{a+bx+cx^2}}{d-fx^2}\,dx = \text{Timed out}$$

```
[In] integrate(x^3*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{x^3\sqrt{a+bx+cx^2}}{d-fx^2}\,dx = -\int \frac{x^3\sqrt{a+bx+cx^2}}{-d+fx^2}\,dx$$

```
[In] integrate(x**3*(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
[Out] -Integral(x**3*sqrt(a + b*x + c*x**2)/(-d + f*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3\sqrt{a+bx+cx^2}}{d-fx^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^3*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see `
assume?
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3\sqrt{a+bx+cx^2}}{d-fx^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3\sqrt{a+bx+cx^2}}{d-fx^2}\,dx = \int \frac{x^3\,\sqrt{c\,x^2+b\,x+a}}{d-f\,x^2}\,dx$$

```
[In] int((x^3*(a + b*x + c*x^2)^(1/2))/(d - f*x^2),x)
[Out] int((x^3*(a + b*x + c*x^2)^(1/2))/(d - f*x^2), x)
```

3.78 $\int \frac{x^2\sqrt{a+bx+cx^2}}{d-fx^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 316

$$\begin{aligned}\int \frac{x^2\sqrt{a+bx+cx^2}}{d-fx^2}\,dx =& -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4cf} \\ &- \frac{(8c^2d-b^2f+4acf)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}f^2} \\ &+ \frac{\sqrt{d}\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\operatorname{arctanh}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2f^2} \\ &+ \frac{\sqrt{d}\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\operatorname{arctanh}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2f^2}\end{aligned}$$

```
[Out] -1/8*(4*a*c*f-b^2*f+8*c^2*d)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1
/2))/c^(3/2)/f^2-1/4*(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/c/f+1/2*arctanh(1/2*(b*d
^(1/2)-2*a*f^(1/2)+x*(2*c*d^(1/2)-b*f^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f-
b*d^(1/2)*f^(1/2))^(1/2))*d^(1/2)*(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2)/f^2+1/2
*arctanh(1/2*(b*d^(1/2)+2*a*f^(1/2)+x*(2*c*d^(1/2)+b*f^(1/2)))/(c*x^2+b*x+a
)^(1/2)/(c*d+a*f+b*d^(1/2)*f^(1/2))^(1/2))*d^(1/2)*(c*d+a*f+b*d^(1/2)*f^(1/
2))^(1/2)/f^2
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1085, 1092, 635, 212, 1047, 738}

$$\int \frac{x^2\sqrt{a+bx+cx^2}}{d-fx^2}\,dx$$
$$= -\frac{\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(4acf+b^2(-f)+8c^2d\right)}{8c^{3/2}f^2}$$
$$+\frac{\sqrt{d}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\operatorname{arctanh}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2f^2}$$
$$+\frac{\sqrt{d}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\operatorname{arctanh}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2f^2}-\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4cf}$$

```
[In] Int[(x^2*Sqrt[a + b*x + c*x^2])/(d - f*x^2),x]
```

```
[Out] -1/4*((b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(c*f) - ((8*c^2*d - b^2*f + 4*a*c*
f)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(3/2)*f^2)
+ (Sqrt[d]*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] - 2*a*Sqr
t[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*
Sqrt[a + b*x + c*x^2])]])/(2*f^2) + (Sqrt[d]*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] +
a*f]*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqr
t[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])]])/(2*f^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
```

```
d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1047

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(-a)*c, 2]}, Dist[h/2 + c*(g/(2*q)
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - c*(g/(2*q)
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[(-a)*c]
```

Rule 1085

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (C_.)*(x_)^2)*((d_) +
(f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(C*(b*f*p) + 2*c*C*f*(p + q + 1)*x)*(
a + b*x + c*x^2)^p*((d + f*x^2)^(q + 1)/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3
))), x] - Dist[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + b*x + c*x^
2)^(p - 1)*(d + f*x^2)^q*Simp[p*(b*d)*(C*((-b)*f)*(q + 1)) + (p + q + 1)*(b
^2*C*d*f*p + a*c*(C*(2*d*f) + f*(-2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*
f)*(C*((-b)*f)*(q + 1)) + (p + q + 1)*((-b)*c*(C*(-4*d*f)*(2*p + q + 2) + f
*(2*C*d + 2*A*f)*(2*p + 2*q + 3))))*x + (p*((-b)*f)*(C*((-b)*f)*(q + 1)) +
(p + q + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(-4*d*f)*(2*p + q + 2) + f*(2*C
*d + 2*A*f)*(2*p + 2*q + 3))))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, A,
C, q}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2
*p + 2*q + 3, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1092

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
 (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + e*x + f
*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x
 + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f
, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4cf} - \frac{\int \frac{-\frac{1}{4}(b^2+4ac)df-2bcdfx-\frac{1}{4}f(8c^2d-b^2f+4acf)x^2}{\sqrt{a+bx+cx^2}(d-fx^2)}\,dx}{2cf^2} \\
&= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4cf} + \frac{\int \frac{\frac{1}{4}(b^2+4ac)df^2+\frac{1}{4}df(8c^2d-b^2f+4acf)+2bcdf^2x}{\sqrt{a+bx+cx^2}(d-fx^2)}\,dx}{2cf^3} \\
&\quad - \frac{(8c^2d-b^2f+4acf)\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{8cf^2}
\end{aligned}$$

$$= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4cf} - \frac{\left(\sqrt{d}\left(cd-b\sqrt{d}\sqrt{f}+af\right)\right)\int \frac{1}{\left(-\sqrt{d}\sqrt{f}-fx\right)\sqrt{a+bx+cx^2}}\,dx}{2f^{3/2}} + \frac{\left(\sqrt{d}\left(cd+b\sqrt{d}\sqrt{f}+af\right)\right)\int \frac{1}{\left(\sqrt{d}\sqrt{f}-fx\right)\sqrt{a+bx+cx^2}}\,dx}{2f^{3/2}} - \frac{(8c^2d-b^2f+4acf)\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{4cf^2}$$

$$= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4cf} - \frac{(8c^2d-b^2f+4acf)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}f^2} + \frac{\left(\sqrt{d}\left(cd-b\sqrt{d}\sqrt{f}+af\right)\right)\text{Subst}\left(\int \frac{1}{4cdf-4b\sqrt{d}f^{3/2}+4af^2-x^2}\,dx, x, \frac{b\sqrt{d}\sqrt{f}-2af-\left(-2c\sqrt{d}\sqrt{f}+bf\right)x}{\sqrt{a+bx+cx^2}}\right)}{f^{3/2}} - \frac{\left(\sqrt{d}\left(cd+b\sqrt{d}\sqrt{f}+af\right)\right)\text{Subst}\left(\int \frac{1}{4cdf+4b\sqrt{d}f^{3/2}+4af^2-x^2}\,dx, x, \frac{-b\sqrt{d}\sqrt{f}-2af-\left(2c\sqrt{d}\sqrt{f}+bf\right)x}{\sqrt{a+bx+cx^2}}\right)}{f^{3/2}}$$

$$= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4cf} - \frac{(8c^2d-b^2f+4acf)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}f^2} + \frac{\sqrt{d}\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\tanh^{-1}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2f^2} + \frac{\sqrt{d}\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\tanh^{-1}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2f^2}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.54 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.01

$$\int \frac{x^2\sqrt{a+bx+cx^2}}{d-fx^2}\,dx$$

$$= \frac{-2\sqrt{c}f(b+2cx)\sqrt{a+x(b+cx)} + (-8c^2d+b^2f-4acf)\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right) - 4c^{3/2}d\text{RootSum}\left[b^2d\right.}{}$$

```
[In] Integrate[(x^2*Sqrt[a + b*x + c*x^2])/(d - f*x^2),x]
```

```
[Out] (-2*Sqrt[c]*f*(b + 2*c*x)*Sqrt[a + x*(b + c*x)] + (-8*c^2*d + b^2*f - 4*a*c
*f)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])] - 4*c^(3/2)*d*Ro
```

```
otSum[b^2*d - a^2*f - 4*b*Sqrt[c]*d*#1 + 4*c*d*#1^2 + 2*a*f*#1^2 - f*#1^4 &
 , (b*c*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*c^(3/2)*d*Log[
-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - 2*a*Sqrt[c]*f*Log[-(Sqrt[c]
*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + b*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x
+ c*x^2] - #1]*#1^2)/(b*Sqrt[c]*d - 2*c*d*#1 - a*f*#1 + f*#1^3) & ])/(8*c^(
3/2)*f^2)
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 483, normalized size of antiderivative = 1.53

method	result
risch	$-\frac{(2cx+b)\sqrt{c\,x^2+bx+a}}{4cf}-\frac{\frac{\left(4acf-b^2f+8c^2d\right)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{f\sqrt{c}}-\frac{4cd\left(b\sqrt{df}+fa+cd\right)\ln\left(\frac{\frac{2b\sqrt{df}+2fa+2cd}{f}+\frac{\left(2c\sqrt{df}+bf\right)\left(x-\frac{\sqrt{df}}{f}\right)}{f}}{\quad}\right.}{\sqrt{df}}}{\quad}$
default	$-\frac{\frac{(2cx+b)\sqrt{c\,x^2+bx+a}}{4c}+\frac{\left(4ac-b^2\right)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{8c^{\frac{3}{2}}}}{f}-\frac{d\left(\sqrt{\left(x-\frac{\sqrt{df}}{f}\right)^2c+\frac{\left(2c\sqrt{df}+bf\right)\left(x-\frac{\sqrt{df}}{f}\right)}{f}+\frac{b\sqrt{df}+fa+cd}{f}}+\frac{\left(2c\sqrt{df}+bf\right)}{\quad}\right.}{\quad}$

```
[In] int(x^2*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/c/f-1/8/f/c*(1/f*(4*a*c*f-b^2*f+8*c^2*d)
*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-4*c*d*(b*(d*f)^(1/2)+f
*a+c*d)/(d*f)^(1/2)/f/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2
)+f*a+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f*
a+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/
2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))-4*c*d*(b*(d*f)^(
1/2)-f*a-c*d)/(d*f)^(1/2)/f/(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*ln((2/f*(-
b*(d*f)^(1/2)+f*a+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*
(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2
)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1/2
)/f)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{a+bx+cx^2}}{d-fx^2}\,dx = \text{Timed out}$$

```
[In] integrate(x^2*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{x^2\sqrt{a+bx+cx^2}}{d-fx^2}\,dx = -\int \frac{x^2\sqrt{a+bx+cx^2}}{-d+fx^2}\,dx$$

```
[In] integrate(x**2*(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
[Out] -Integral(x**2*sqrt(a + b*x + c*x**2)/(-d + f*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2\sqrt{a+bx+cx^2}}{d-fx^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see `
assume?
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^2\sqrt{a+bx+cx^2}}{d-fx^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^2*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{a+bx+cx^2}}{d-fx^2}\,dx=\int \frac{x^2\,\sqrt{c\,x^2+b\,x+a}}{d-f\,x^2}\,dx$$

```
[In] int((x^2*(a + b*x + c*x^2)^(1/2))/(d - f*x^2),x)
[Out] int((x^2*(a + b*x + c*x^2)^(1/2))/(d - f*x^2), x)
```

3.79 $\int \frac{x\sqrt{a+bx+cx^2}}{d-fx^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 282

$$\int \frac{x\sqrt{a+bx+cx^2}}{d-fx^2}\,dx = -\frac{\sqrt{a+bx+cx^2}}{f} - \frac{b\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}f} - \frac{\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\,\text{arctanh}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2f^{3/2}} + \frac{\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\,\text{arctanh}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2f^{3/2}}$$

```
[Out] -1/2*b*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/f/c^(1/2)-(c*x^2+
b*x+a)^(1/2)/f-1/2*arctanh(1/2*(b*d^(1/2)-2*a*f^(1/2)+x*(2*c*d^(1/2)-b*f^(1
/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2))*(c*d+a*f-b*d^(
1/2)*f^(1/2))^(1/2)/f^(3/2)+1/2*arctanh(1/2*(b*d^(1/2)+2*a*f^(1/2)+x*(2*c*d
^(1/2)+b*f^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f+b*d^(1/2)*f^(1/2))^(1/2))*(
c*d+a*f+b*d^(1/2)*f^(1/2))^(1/2)/f^(3/2)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used

$= \{1035, 1092, 635, 212, 1047, 738\}$

$$\int \frac{x\sqrt{a+bx+cx^2}}{d-fx^2}\,dx = -\frac{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\,\text{arctanh}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2f^{3/2}} + \frac{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\,\text{arctanh}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2f^{3/2}} - \frac{b\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}f} - \frac{\sqrt{a+bx+cx^2}}{f}$$

```
[In] Int[(x*Sqrt[a + b*x + c*x^2])/(d - f*x^2),x]
```

```
[Out] -(Sqrt[a + b*x + c*x^2]/f) - (b*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x
 + c*x^2])])/(2*Sqrt[c]*f) - (Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(
b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt
[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*f^(3/2)) + (Sqrt[c*d + b*Sqr
t[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqr
t[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2
*f^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1035

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[h*(a + b*x + c*x^2)^p*((d + f*x^2)^(q +
 1)/(2*f*(p + q + 1))), x] - Dist[1/(2*f*(p + q + 1)), Int[(a + b*x + c*x^2
)^(p - 1)*(d + f*x^2)^q*Simp[h*p*(b*d) + a*(-2*g*f)*(p + q + 1) + (2*h*p*(c
*d - a*f) + b*(-2*g*f)*(p + q + 1))*x + (h*p*((-b)*f) + c*(-2*g*f)*(p + q +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*a
*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]
```

Rule 1047

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(-a)*c, 2]}, Dist[h/2 + c*(g/(2*q)
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - c*(g/(2*q)
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[(-a)*c]
```

Rule 1092

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
 (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + e*x + f
*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x
 + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f
, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\sqrt{a+bx+cx^2}}{f}+\frac{\int \frac{\frac{bd}{2}+(cd+af)x+\frac{1}{2}bfx^2}{\sqrt{a+bx+cx^2}(d-fx^2)}\,dx}{f}\\
&= -\frac{\sqrt{a+bx+cx^2}}{f}-\frac{\int \frac{-bdf-f(cd+af)x}{\sqrt{a+bx+cx^2}(d-fx^2)}\,dx}{f^2}-\frac{b\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{2f}\\
&= -\frac{\sqrt{a+bx+cx^2}}{f}-\frac{b\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx,x,\frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{f}\\
&\quad+\frac{\left(cd-b\sqrt{d}\sqrt{f}+af\right)\int \frac{1}{\left(-\sqrt{d}\sqrt{f}-fx\right)\sqrt{a+bx+cx^2}}\,dx}{2f}\\
&\quad+\frac{\left(cd+b\sqrt{d}\sqrt{f}+af\right)\int \frac{1}{\left(\sqrt{d}\sqrt{f}-fx\right)\sqrt{a+bx+cx^2}}\,dx}{2f}
\end{aligned}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{f} - \frac{b\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}f} - \frac{\left(cd-b\sqrt{d}\sqrt{f}+af\right)\text{Subst}\left(\int \frac{1}{4cdf-4b\sqrt{d}f^{3/2}+4af^2-x^2}\,dx, x, \frac{b\sqrt{d}\sqrt{f}-2af-\left(-2c\sqrt{d}\sqrt{f}+bf\right)x}{\sqrt{a+bx+cx^2}}\right)}{f} - \frac{\left(cd+b\sqrt{d}\sqrt{f}+af\right)\text{Subst}\left(\int \frac{1}{4cdf+4b\sqrt{d}f^{3/2}+4af^2-x^2}\,dx, x, \frac{-b\sqrt{d}\sqrt{f}-2af-\left(2c\sqrt{d}\sqrt{f}+bf\right)x}{\sqrt{a+bx+cx^2}}\right)}{f}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{f} - \frac{b\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}f} - \frac{\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\tanh^{-1}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2f^{3/2}} + \frac{\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\tanh^{-1}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2f^{3/2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.37 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.24

$$\int \frac{x\sqrt{a+bx+cx^2}}{d-fx^2}\,dx = -\frac{2\sqrt{a+x(b+cx)} - \frac{b\log\left(f\left(b+2cx-2\sqrt{c}\sqrt{a+x(b+cx)}\right)\right)}{\sqrt{c}} + \text{RootSum}\left[b^2d-a^2f-4b\sqrt{c}d\#1+4cd\#1^2+2af\#\right.}{}$$

```
[In] Integrate[(x*Sqrt[a + b*x + c*x^2])/(d - f*x^2),x]
```

```
[Out] -1/2*(2*Sqrt[a + x*(b + c*x)] - (b*Log[f*(b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*
(b + c*x)])])/Sqrt[c] + RootSum[b^2*d - a^2*f - 4*b*Sqrt[c]*d*#1 + 4*c*d*#1
^2 + 2*a*f*#1^2 - f*#1^4 & , (b^2*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2
] - #1] - a*c*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - a^2*f*Log[
-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*b*Sqrt[c]*d*Log[-(Sqrt[c]*x)
 + Sqrt[a + b*x + c*x^2] - #1]*#1 + c*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c
*x^2] - #1]*#1^2 + a*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2)
/(b*Sqrt[c]*d - 2*c*d*#1 - a*f*#1 + f*#1^3) & ])/f
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 459 vs. $2(214) = 428$.

Time = 0.75 (sec) , antiderivative size = 460, normalized size of antiderivative = 1.63

method	result
risch	$-\frac{\sqrt{c x^2+bx+a}}{f}-\frac{\frac{b \ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c x^2+bx+a}\right)}{\sqrt{c}}-\frac{(\sqrt{df}\, af+\sqrt{df}\, cd+bdf)\ln\left(\frac{\frac{2b\sqrt{df}+2fa+2cd}{f}+\frac{(2c\sqrt{df}+bf)\left(x-\frac{\sqrt{df}}{f}\right)}{f}+2\sqrt{\frac{b\sqrt{df}+fa+cd}{f}}\sqrt{(x- \cdots}}{x-\frac{\sqrt{df}}{f}}\cdots\right.}{\sqrt{df}\, f\sqrt{\frac{b\sqrt{df}+fa+cd}{f}}}\cdots}{\cdots}$
default	$-\frac{\sqrt{\left(x-\frac{\sqrt{df}}{f}\right)^2 c+\frac{(2c\sqrt{df}+bf)\left(x-\frac{\sqrt{df}}{f}\right)}{f}+\frac{b\sqrt{df}+fa+cd}{f}}+\frac{(2c\sqrt{df}+bf)\ln\left(\frac{\frac{2c\sqrt{df}+bf}{2f}+c\left(x-\frac{\sqrt{df}}{f}\right)}{\sqrt{c}}+\sqrt{\left(x-\frac{\sqrt{df}}{f}\right)^2 c+\frac{(2c\sqrt{df}+bf)\left(x-\frac{\sqrt{df}}{f}\right)}{f}\cdots}\right.}{2f\sqrt{c}}\cdots}{\cdots}$

```
[In] int(x*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] -(c*x^2+b*x+a)^(1/2)/f-1/2/f*(b*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
/c^(1/2)-((d*f)^(1/2)*a*f+(d*f)^(1/2)*c*d+b*d*f)/(d*f)^(1/2)/f/((b*(d*f)^(1
/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+f*a+c*d)/f+(2*c*(d*f)^(1/2)+b*f)
/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)
^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(
1/2))/(x-(d*f)^(1/2)/f))-((d*f)^(1/2)*a*f+(d*f)^(1/2)*c*d-b*d*f)/(d*f)^(1/2
)/f/(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1
/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d)
)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)
+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1/2)/f)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 595 vs. $2(214) = 428$.

Time = 186.76 (sec) , antiderivative size = 1192, normalized size of antiderivative = 4.23

$$\int \frac{x\sqrt{a+bx+cx^2}}{d-fx^2}\,dx = \text{Too large to display}$$

```
[In] integrate(x*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] [1/4*(c*f*sqrt((f^3*sqrt(b^2*d/f^5) + c*d + a*f)/f^3)*log((2*sqrt(c*x^2 + b
*x + a)*f^4*sqrt(b^2*d/f^5)*sqrt((f^3*sqrt(b^2*d/f^5) + c*d + a*f)/f^3) + 2
*b*c*d*x + b^2*d + (b*f^3*x + 2*a*f^3)*sqrt(b^2*d/f^5))/x) - c*f*sqrt((f^3*
```

```
sqrt(b^2*d/f^5) + c*d + a*f)/f^3)*log(-(2*sqrt(c*x^2 + b*x + a)*f^4*sqrt(b^
2*d/f^5)*sqrt((f^3*sqrt(b^2*d/f^5) + c*d + a*f)/f^3) - 2*b*c*d*x - b^2*d -
(b*f^3*x + 2*a*f^3)*sqrt(b^2*d/f^5))/x) - c*f*sqrt(-(f^3*sqrt(b^2*d/f^5) -
c*d - a*f)/f^3)*log((2*sqrt(c*x^2 + b*x + a)*f^4*sqrt(b^2*d/f^5)*sqrt(-(f^3
*sqrt(b^2*d/f^5) - c*d - a*f)/f^3) + 2*b*c*d*x + b^2*d - (b*f^3*x + 2*a*f^3
)*sqrt(b^2*d/f^5))/x) + c*f*sqrt(-(f^3*sqrt(b^2*d/f^5) - c*d - a*f)/f^3)*lo
g(-(2*sqrt(c*x^2 + b*x + a)*f^4*sqrt(b^2*d/f^5)*sqrt(-(f^3*sqrt(b^2*d/f^5)
- c*d - a*f)/f^3) - 2*b*c*d*x - b^2*d + (b*f^3*x + 2*a*f^3)*sqrt(b^2*d/f^5)
)/x) + b*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(
2*c*x + b)*sqrt(c) - 4*a*c) - 4*sqrt(c*x^2 + b*x + a)*c)/(c*f), 1/4*(c*f*sq
rt((f^3*sqrt(b^2*d/f^5) + c*d + a*f)/f^3)*log((2*sqrt(c*x^2 + b*x + a)*f^4*
sqrt(b^2*d/f^5)*sqrt((f^3*sqrt(b^2*d/f^5) + c*d + a*f)/f^3) + 2*b*c*d*x + b
^2*d + (b*f^3*x + 2*a*f^3)*sqrt(b^2*d/f^5))/x) - c*f*sqrt((f^3*sqrt(b^2*d/f
^5) + c*d + a*f)/f^3)*log(-(2*sqrt(c*x^2 + b*x + a)*f^4*sqrt(b^2*d/f^5)*sqr
t((f^3*sqrt(b^2*d/f^5) + c*d + a*f)/f^3) - 2*b*c*d*x - b^2*d - (b*f^3*x + 2
*a*f^3)*sqrt(b^2*d/f^5))/x) - c*f*sqrt(-(f^3*sqrt(b^2*d/f^5) - c*d - a*f)/f
^3)*log((2*sqrt(c*x^2 + b*x + a)*f^4*sqrt(b^2*d/f^5)*sqrt(-(f^3*sqrt(b^2*d/
f^5) - c*d - a*f)/f^3) + 2*b*c*d*x + b^2*d - (b*f^3*x + 2*a*f^3)*sqrt(b^2*d
/f^5))/x) + c*f*sqrt(-(f^3*sqrt(b^2*d/f^5) - c*d - a*f)/f^3)*log(-(2*sqrt(c
*x^2 + b*x + a)*f^4*sqrt(b^2*d/f^5)*sqrt(-(f^3*sqrt(b^2*d/f^5) - c*d - a*f)
/f^3) - 2*b*c*d*x - b^2*d + (b*f^3*x + 2*a*f^3)*sqrt(b^2*d/f^5))/x) + 2*b*s
qrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*
c*x + a*c)) - 4*sqrt(c*x^2 + b*x + a)*c)/(c*f)]
```

Sympy [F]

$$\int \frac{x\sqrt{a+bx+cx^2}}{d-fx^2}\,dx = -\int \frac{x\sqrt{a+bx+cx^2}}{-d+fx^2}\,dx$$

```
[In] integrate(x*(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
[Out] -Integral(x*sqrt(a + b*x + c*x**2)/(-d + f*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x\sqrt{a+bx+cx^2}}{d-fx^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see `
assume?
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x\sqrt{a+bx+cx^2}}{d-fx^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt{a+bx+cx^2}}{d-fx^2}\,dx = \int \frac{x\,\sqrt{c\,x^2+b\,x+a}}{d-f\,x^2}\,dx$$

```
[In] int((x*(a + b*x + c*x^2)^(1/2))/(d - f*x^2),x)
```

```
[Out] int((x*(a + b*x + c*x^2)^(1/2))/(d - f*x^2), x)
```

3.80 $\int \frac{\sqrt{a+bx+cx^2}}{d-fx^2} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 266

$$\int \frac{\sqrt{a+bx+cx^2}}{d-fx^2} \, dx = -\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{f} + \frac{\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\operatorname{arctanh}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{d}f} + \frac{\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\operatorname{arctanh}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{d}f}$$

```
[Out] -arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))*c^(1/2)/f+1/2*arctanh(1
/2*(b*d^(1/2)-2*a*f^(1/2)+x*(2*c*d^(1/2)-b*f^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c
*d+a*f-b*d^(1/2)*f^(1/2))^(1/2))*(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2)/f/d^(1/2
)+1/2*arctanh(1/2*(b*d^(1/2)+2*a*f^(1/2)+x*(2*c*d^(1/2)+b*f^(1/2)))/(c*x^2+
b*x+a)^(1/2)/(c*d+a*f+b*d^(1/2)*f^(1/2))^(1/2))*(c*d+a*f+b*d^(1/2)*f^(1/2))
^(1/2)/f/d^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used

$= \{1004, 635, 212, 1047, 738\}$

$$\int \frac{\sqrt{a+bx+cx^2}}{d-fx^2}\,dx = \frac{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\,\text{arctanh}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\sqrt{d}f} + \frac{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\,\text{arctanh}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\sqrt{d}f} - \frac{\sqrt{c}\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{f}$$

```
[In] Int[Sqrt[a + b*x + c*x^2]/(d - f*x^2),x]
```

```
[Out] -((Sqrt[c]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/f) + (Sq
rt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*S
qrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x
+ c*x^2])])/(2*Sqrt[d]*f) + (Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b
*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[
d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[d]*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1004

```
Int[Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]/((d_) + (f_.)*(x_)^2), x_Symbol]
 :> Dist[c/f, Int[1/Sqrt[a + b*x + c*x^2], x], x] - Dist[1/f, Int[(c*d - a*
f - b*f*x)/(Sqrt[a + b*x + c*x^2]*(d + f*x^2)), x], x] /; FreeQ[{a, b, c, d
```

```
, f}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1047

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(-a)*c, 2]}, Dist[h/2 + c*(g/(2*q)
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - c*(g/(2*q)
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[(-a)*c]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{cd+af+bfx}{\sqrt{a+bx+cx^2}(d-fx^2)}\,dx}{f} - \frac{c\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{f}$$

$$= -\frac{(2c)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{f} + \frac{1}{2}\left(b-\frac{cd+af}{\sqrt{d}\sqrt{f}}\right)\int \frac{1}{\left(-\sqrt{d}\sqrt{f}-fx\right)\sqrt{a+bx+cx^2}}\,dx + \frac{1}{2}\left(b+\frac{cd+af}{\sqrt{d}\sqrt{f}}\right)\int \frac{1}{\left(\sqrt{d}\sqrt{f}-fx\right)\sqrt{a+bx+cx^2}}\,dx$$

$$= -\frac{\sqrt{c}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{f} + \left(-b-\frac{cd+af}{\sqrt{d}\sqrt{f}}\right)\text{Subst}\left(\int \frac{1}{4cdf+4b\sqrt{d}f^{3/2}+4af^2-x^2}\,dx, x, \frac{-b\sqrt{d}\sqrt{f}-2af-\left(2c\sqrt{d}\sqrt{f}+bf\right)}{\sqrt{a+bx+cx^2}}\right. + \left(-b+\frac{cd+af}{\sqrt{d}\sqrt{f}}\right)\text{Subst}\left(\int \frac{1}{4cdf-4b\sqrt{d}f^{3/2}+4af^2-x^2}\,dx, x, \frac{b\sqrt{d}\sqrt{f}-2af-\left(-2c\sqrt{d}\sqrt{f}+bf\right)}{\sqrt{a+bx+cx^2}}\right.$$

$$= -\frac{\sqrt{c}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{f} + \frac{\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\tanh^{-1}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{d}f} + \frac{\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\tanh^{-1}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{d}f}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.36 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.46

$$\int \frac{\sqrt{a+bx+cx^2}}{d-fx^2}\,dx = -\frac{-4\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a}-\sqrt{a+x(b+cx)}}\right)+\operatorname{RootSum}\left[c^2d-b^2f+4\sqrt{a}bf\#1-2cd\#1^2-4af\#1^2+d\#1^4\&,\frac{-c^2d}{}\right.}{}$$

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/(d - f*x^2),x]
```

```
[Out] -1/2*(-4*Sqrt[c]*ArcTanh[(Sqrt[c]*x)/(Sqrt[a] - Sqrt[a + x*(b + c*x)])] + R
ootSum[c^2*d - b^2*f + 4*Sqrt[a]*b*f*#1 - 2*c*d*#1^2 - 4*a*f*#1^2 + d*#1^4
& , (-(c^2*d*Log[x]) + b^2*f*Log[x] - a*c*f*Log[x] + c^2*d*Log[-Sqrt[a] + S
qrt[a + b*x + c*x^2] - x*#1] - b^2*f*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] -
 x*#1] + a*c*f*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] - 2*Sqrt[a]*b*f
*Log[x]*#1 + 2*Sqrt[a]*b*f*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1]*#1
+ c*d*Log[x]*#1^2 + a*f*Log[x]*#1^2 - c*d*Log[-Sqrt[a] + Sqrt[a + b*x + c*x
^2] - x*#1]*#1^2 - a*f*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1]*#1^2)/(
-(Sqrt[a]*b*f) + c*d*#1 + 2*a*f*#1 - d*#1^3) & ])/f
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 771 vs. 2(202) = 404.

Time = 0.68 (sec) , antiderivative size = 772, normalized size of antiderivative = 2.90

method	result
default	$-\frac{\sqrt{\left(x-\frac{\sqrt{df}}{f}\right)^2 c+\frac{(2c\sqrt{df}+bf)\left(x-\frac{\sqrt{df}}{f}\right)}{f}+\frac{b\sqrt{df}+fa+cd}{f}}+\frac{(2c\sqrt{df}+bf)\ln\left(\frac{\frac{2c\sqrt{df}+bf}{2f}+c\left(x-\frac{\sqrt{df}}{f}\right)}{\sqrt{c}}+\sqrt{\left(x-\frac{\sqrt{df}}{f}\right)^2 c+\frac{(2c\sqrt{df}+bf)\left(x-\frac{\sqrt{df}}{f}\right)}{f}}\right.}{2f\sqrt{c}}}{}$

```
[In] int((c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/(d*f)^(1/2)*(((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(
1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)+1/2*(2*c*(d*f)^(1/2)+b*f)/f*ln((1/
2*(2*c*(d*f)^(1/2)+b*f)/f+c*(x-(d*f)^(1/2)/f))/c^(1/2)+((x-(d*f)^(1/2)/f)^2
*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/
2))/c^(1/2)-(b*(d*f)^(1/2)+f*a+c*d)/f/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln(
(2*(b*(d*f)^(1/2)+f*a+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((
```

```
b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f
)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f)))
+1/2/(d*f)^(1/2)*(((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f
)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)+1/2/f*(-2*c*(d*f)^(1/2)+b*f)
*ln((1/2/f*(-2*c*(d*f)^(1/2)+b*f)+c*(x+(d*f)^(1/2)/f))/c^(1/2)+((x+(d*f)^(1
/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)
+f*a+c*d))^(1/2))/c^(1/2)-1/f*(-b*(d*f)^(1/2)+f*a+c*d)/(1/f*(-b*(d*f)^(1/2)
+f*a+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f
)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*((x+(d*f)^(1/2)/
f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a
+c*d))^(1/2))/(x+(d*f)^(1/2)/f)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 568 vs. $2(202) = 404$.

Time = 65.90 (sec) , antiderivative size = 1139, normalized size of antiderivative = 4.28

$$\int \frac{\sqrt{a+bx+cx^2}}{d-fx^2}\,dx = \left[\frac{f\sqrt{\frac{df^2\sqrt{\frac{b^2}{df^3}}+cd+af}{df^2}}\log\left(\frac{2\,bcx+2\,\sqrt{cx^2+bx+a}bf\sqrt{\frac{df^2\sqrt{\frac{b^2}{df^3}}+cd+af}{df^2}}+b^2+\left(bf^2x+2\,af^2\right)\sqrt{\frac{b^2}{df^3}}}{x}\right)-f\sqrt{\frac{df^2\sqrt{\frac{b^2}{df^3}}+cd+af}{df^2}}\log\left(2\right.}{} \right.$$

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] [1/4*(f*sqrt((d*f^2*sqrt(b^2/(d*f^3)) + c*d + a*f)/(d*f^2))*log((2*b*c*x +
2*sqrt(c*x^2 + b*x + a)*b*f*sqrt((d*f^2*sqrt(b^2/(d*f^3)) + c*d + a*f)/(d*f
^2)) + b^2 + (b*f^2*x + 2*a*f^2)*sqrt(b^2/(d*f^3)))/x) - f*sqrt((d*f^2*sqrt
(b^2/(d*f^3)) + c*d + a*f)/(d*f^2))*log((2*b*c*x - 2*sqrt(c*x^2 + b*x + a)*
b*f*sqrt((d*f^2*sqrt(b^2/(d*f^3)) + c*d + a*f)/(d*f^2)) + b^2 + (b*f^2*x +
2*a*f^2)*sqrt(b^2/(d*f^3)))/x) + f*sqrt(-(d*f^2*sqrt(b^2/(d*f^3)) - c*d - a
*f)/(d*f^2))*log((2*b*c*x + 2*sqrt(c*x^2 + b*x + a)*b*f*sqrt(-(d*f^2*sqrt(b
^2/(d*f^3)) - c*d - a*f)/(d*f^2)) + b^2 - (b*f^2*x + 2*a*f^2)*sqrt(b^2/(d*f
^3)))/x) - f*sqrt(-(d*f^2*sqrt(b^2/(d*f^3)) - c*d - a*f)/(d*f^2))*log((2*b*
c*x - 2*sqrt(c*x^2 + b*x + a)*b*f*sqrt(-(d*f^2*sqrt(b^2/(d*f^3)) - c*d - a*
f)/(d*f^2)) + b^2 - (b*f^2*x + 2*a*f^2)*sqrt(b^2/(d*f^3)))/x) + 2*sqrt(c)*l
og(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c)
 - 4*a*c))/f, 1/4*(f*sqrt((d*f^2*sqrt(b^2/(d*f^3)) + c*d + a*f)/(d*f^2))*lo
g((2*b*c*x + 2*sqrt(c*x^2 + b*x + a)*b*f*sqrt((d*f^2*sqrt(b^2/(d*f^3)) + c*
d + a*f)/(d*f^2)) + b^2 + (b*f^2*x + 2*a*f^2)*sqrt(b^2/(d*f^3)))/x) - f*sqr
```

```
t((d*f^2*sqrt(b^2/(d*f^3)) + c*d + a*f)/(d*f^2))*log((2*b*c*x - 2*sqrt(c*x^
2 + b*x + a)*b*f*sqrt((d*f^2*sqrt(b^2/(d*f^3)) + c*d + a*f)/(d*f^2)) + b^2
+ (b*f^2*x + 2*a*f^2)*sqrt(b^2/(d*f^3)))/x) + f*sqrt(-(d*f^2*sqrt(b^2/(d*f^
3)) - c*d - a*f)/(d*f^2))*log((2*b*c*x + 2*sqrt(c*x^2 + b*x + a)*b*f*sqrt(-
(d*f^2*sqrt(b^2/(d*f^3)) - c*d - a*f)/(d*f^2)) + b^2 - (b*f^2*x + 2*a*f^2)*
sqrt(b^2/(d*f^3)))/x) - f*sqrt(-(d*f^2*sqrt(b^2/(d*f^3)) - c*d - a*f)/(d*f^
2))*log((2*b*c*x - 2*sqrt(c*x^2 + b*x + a)*b*f*sqrt(-(d*f^2*sqrt(b^2/(d*f^3
)) - c*d - a*f)/(d*f^2)) + b^2 - (b*f^2*x + 2*a*f^2)*sqrt(b^2/(d*f^3)))/x)
+ 4*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2
 + b*c*x + a*c)))/f]
```

Sympy [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{d-fx^2}\,dx = -\int \frac{\sqrt{a+bx+cx^2}}{-d+fx^2}\,dx$$

```
[In] integrate((c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
[Out] -Integral(sqrt(a + b*x + c*x**2)/(-d + f*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx+cx^2}}{d-fx^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see `
assume?
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx+cx^2}}{d-fx^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}}{d-fx^2}\,dx=\int \frac{\sqrt{c\,x^2+b\,x+a}}{d-f\,x^2}\,dx$$

```
[In] int((a + b*x + c*x^2)^(1/2)/(d - f*x^2),x)
[Out] int((a + b*x + c*x^2)^(1/2)/(d - f*x^2), x)
```

3.81 $\int \frac{\sqrt{a+bx+cx^2}}{x(d-fx^2)} \, dx$

Optimal result

Integrand size = 28, antiderivative size = 267

$$\int \frac{\sqrt{a+bx+cx^2}}{x(d-fx^2)} \, dx = -\frac{\sqrt{a}\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d} - \frac{\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\operatorname{arctanh}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2d\sqrt{f}} + \frac{\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\operatorname{arctanh}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2d\sqrt{f}}$$

```
[Out] -arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))*a^(1/2)/d-1/2*arctanh(1
/2*(b*d^(1/2)-2*a*f^(1/2)+x*(2*c*d^(1/2)-b*f^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c
*d+a*f-b*d^(1/2)*f^(1/2))^(1/2))*(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2)/d/f^(1/2
)+1/2*arctanh(1/2*(b*d^(1/2)+2*a*f^(1/2)+x*(2*c*d^(1/2)+b*f^(1/2)))/(c*x^2+
b*x+a)^(1/2)/(c*d+a*f+b*d^(1/2)*f^(1/2))^(1/2))*(c*d+a*f+b*d^(1/2)*f^(1/2))
^(1/2)/d/f^(1/2)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used

= {6857, 748, 857, 635, 212, 738, 1035, 1092, 1047}

$$\int \frac{\sqrt{a+bx+cx^2}}{x\left(d-fx^2\right)}\,dx = -\frac{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\,\operatorname{arctanh}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2d\sqrt{f}} + \frac{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\,\operatorname{arctanh}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2d\sqrt{f}} - \frac{\sqrt{a}\,\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d}$$

```
[In] Int[Sqrt[a + b*x + c*x^2]/(x*(d - f*x^2)),x]
```

```
[Out] -((Sqrt[a]*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/d) - (Sq
rt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*S
qrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x
+ c*x^2])])/(2*d*Sqrt[f]) + (Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b
*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[
d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d*Sqrt[f])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 748

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
```

```
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1035

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[h*(a + b*x + c*x^2)^p*((d + f*x^2)^(q +
 1)/(2*f*(p + q + 1))), x] - Dist[1/(2*f*(p + q + 1)), Int[(a + b*x + c*x^2
)^(p - 1)*(d + f*x^2)^q*Simp[h*p*(b*d) + a*(-2*g*f)*(p + q + 1) + (2*h*p*(c
*d - a*f) + b*(-2*g*f)*(p + q + 1))*x + (h*p*((-b)*f) + c*(-2*g*f)*(p + q +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*a
*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]
```

Rule 1047

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(-a)*c, 2]}, Dist[h/2 + c*(g/(2*q)
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - c*(g/(2*q)
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[(-a)*c]
```

Rule 1092

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
 (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + e*x + f
*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x
 + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f
, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{\sqrt{a+bx+cx^2}}{dx} - \frac{fx\sqrt{a+bx+cx^2}}{d(-d+fx^2)}\right) dx$$

$$= \frac{\int \frac{\sqrt{a+bx+cx^2}}{x}\, dx}{d} - \frac{f\int \frac{x\sqrt{a+bx+cx^2}}{-d+fx^2}\, dx}{d}$$

$$= -\frac{\int \frac{-2a-bx}{x\sqrt{a+bx+cx^2}}\, dx}{2d} + \frac{\int \frac{-\frac{bd}{2}-(cd+af)x-\frac{1}{2}bfx^2}{\sqrt{a+bx+cx^2}(-d+fx^2)}\, dx}{d}$$

$$= \frac{a\int \frac{1}{x\sqrt{a+bx+cx^2}}\, dx}{d} + \frac{\int \frac{-bdf+f(-cd-af)x}{\sqrt{a+bx+cx^2}(-d+fx^2)}\, dx}{df}$$

$$= -\frac{(2a)\text{Subst}\left(\int \frac{1}{4a-x^2}\, dx, x, \frac{2a+bx}{\sqrt{a+bx+cx^2}}\right)}{d} - \frac{\left(cd-b\sqrt{d}\sqrt{f}+af\right)\int \frac{1}{\left(\sqrt{d}\sqrt{f}+fx\right)\sqrt{a+bx+cx^2}}\, dx}{2d} - \frac{\left(cd+b\sqrt{d}\sqrt{f}+af\right)\int \frac{1}{\left(-\sqrt{d}\sqrt{f}+fx\right)\sqrt{a+bx+cx^2}}\, dx}{2d}$$

$$= -\frac{\sqrt{a}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d} + \frac{\left(cd-b\sqrt{d}\sqrt{f}+af\right)\text{Subst}\left(\int \frac{1}{4cdf-4b\sqrt{d}f^{3/2}+4af^2-x^2}\, dx, x, \frac{-b\sqrt{d}\sqrt{f}+2af-\left(2c\sqrt{d}\sqrt{f}-bf\right)x}{\sqrt{a+bx+cx^2}}\right)}{d} + \frac{\left(cd+b\sqrt{d}\sqrt{f}+af\right)\text{Subst}\left(\int \frac{1}{4cdf+4b\sqrt{d}f^{3/2}+4af^2-x^2}\, dx, x, \frac{b\sqrt{d}\sqrt{f}+2af-\left(-2c\sqrt{d}\sqrt{f}-bf\right)x}{\sqrt{a+bx+cx^2}}\right)}{d}$$

$$= -\frac{\sqrt{a}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d} - \frac{\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\tanh^{-1}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2d\sqrt{f}} + \frac{\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\tanh^{-1}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2d\sqrt{f}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.24 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.25

$$\int \frac{\sqrt{a+bx+cx^2}}{x(d-fx^2)}\,dx =$$

$$-\frac{-4\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)+\operatorname{RootSum}\left[b^2d-a^2f-4b\sqrt{c}d\#1+4cd\#1^2+2af\#1^2-f\#1^4\&,\ \frac{b^2d\ldots}{\ldots}\right.}{\ldots}$$

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/(x*(d - f*x^2)),x]
```

```
[Out] -1/2*(-4*Sqrt[a]*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)])/Sqrt[a]] + Roo
tSum[b^2*d - a^2*f - 4*b*Sqrt[c]*d*#1 + 4*c*d*#1^2 + 2*a*f*#1^2 - f*#1^4 &
, (b^2*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - a*c*d*Log[-(Sqrt[
c]*x) + Sqrt[a + b*x + c*x^2] - #1] - a^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x
 + c*x^2] - #1] - 2*b*Sqrt[c]*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] -
#1]*#1 + c*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 + a*f*Log[
-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2)/(b*Sqrt[c]*d - 2*c*d*#1 -
a*f*#1 + f*#1^3) & ])/d
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 849 vs. 2(203) = 406.

Time = 0.73 (sec) , antiderivative size = 850, normalized size of antiderivative = 3.18

method	result
default	$\frac{\sqrt{cx^2+bx+a}+\frac{b\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{cx^2+bx+a}\right)}{2\sqrt{c}}-\sqrt{a}\ln\left(\frac{2a+bx+2\sqrt{a}\sqrt{cx^2+bx+a}}{x}\right)}{d}-\frac{\sqrt{\left(x-\frac{\sqrt{df}}{f}\right)^2c+\frac{(2c\sqrt{df}+bf)\left(x-\frac{\sqrt{df}}{f}\right)}{f}+\frac{b\sqrt{df}+fa+cd}{f}}\ldots}{\ldots}$

```
[In] int((c*x^2+b*x+a)^(1/2)/x/(-f*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*((c*x^2+b*x+a)^(1/2)+1/2*b*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/
c^(1/2)-a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x))-1/2/d*(((x-(
d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+
f*a+c*d)/f)^(1/2)+1/2*(2*c*(d*f)^(1/2)+b*f)/f*ln((1/2*(2*c*(d*f)^(1/2)+b*f)
/f+c*(x-(d*f)^(1/2)/f))/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f
)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/c^(1/2)-(b*(d*f)^(1
/2)+f*a+c*d)/f/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+f*a+c
*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f*a+c*d)/
```

```
f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(
b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f)))-1/2/d*(((x+(d*f)^(1/2)
/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*
a+c*d))^(1/2)+1/2/f*(-2*c*(d*f)^(1/2)+b*f)*ln((1/2/f*(-2*c*(d*f)^(1/2)+b*f)
+c*(x+(d*f)^(1/2)/f))/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+
b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/c^(1/2)-1/f*(-b
*(d*f)^(1/2)+f*a+c*d)/(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*ln((2/f*(-b*(d*f
)^(1/2)+f*a+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d
*f)^(1/2)+f*a+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)
*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1/2)/f)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 622 vs. $2(203) = 406$.

Time = 10.14 (sec) , antiderivative size = 1253, normalized size of antiderivative = 4.69

$$\int \frac{\sqrt{a+bx+cx^2}}{x\,(d-fx^2)}\,dx = \text{Too large to display}$$

```
[In] integrate((c*x^2+b*x+a)^(1/2)/x/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] [1/4*(d*sqrt((d^2*f*sqrt(b^2/(d^3*f)) + c*d + a*f)/(d^2*f))*log((2*sqrt(c*x
^2 + b*x + a)*d^2*f*sqrt(b^2/(d^3*f))*sqrt((d^2*f*sqrt(b^2/(d^3*f)) + c*d +
 a*f)/(d^2*f)) + 2*b*c*x + b^2 + (b*d*f*x + 2*a*d*f)*sqrt(b^2/(d^3*f)))/x)
- d*sqrt((d^2*f*sqrt(b^2/(d^3*f)) + c*d + a*f)/(d^2*f))*log(-(2*sqrt(c*x^2
+ b*x + a)*d^2*f*sqrt(b^2/(d^3*f))*sqrt((d^2*f*sqrt(b^2/(d^3*f)) + c*d + a*
f)/(d^2*f)) - 2*b*c*x - b^2 - (b*d*f*x + 2*a*d*f)*sqrt(b^2/(d^3*f)))/x) - d
*sqrt(-(d^2*f*sqrt(b^2/(d^3*f)) - c*d - a*f)/(d^2*f))*log((2*sqrt(c*x^2 + b
*x + a)*d^2*f*sqrt(b^2/(d^3*f))*sqrt(-(d^2*f*sqrt(b^2/(d^3*f)) - c*d - a*f)
/(d^2*f)) + 2*b*c*x + b^2 - (b*d*f*x + 2*a*d*f)*sqrt(b^2/(d^3*f)))/x) + d*s
qrt(-(d^2*f*sqrt(b^2/(d^3*f)) - c*d - a*f)/(d^2*f))*log(-(2*sqrt(c*x^2 + b*
x + a)*d^2*f*sqrt(b^2/(d^3*f))*sqrt(-(d^2*f*sqrt(b^2/(d^3*f)) - c*d - a*f)/
(d^2*f)) - 2*b*c*x - b^2 + (b*d*f*x + 2*a*d*f)*sqrt(b^2/(d^3*f)))/x) + 2*sq
rt(a)*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 + b*x + a)*(b*x + 2*
a)*sqrt(a) + 8*a^2)/x^2))/d, 1/4*(d*sqrt((d^2*f*sqrt(b^2/(d^3*f)) + c*d + a
*f)/(d^2*f))*log((2*sqrt(c*x^2 + b*x + a)*d^2*f*sqrt(b^2/(d^3*f))*sqrt((d^2
*f*sqrt(b^2/(d^3*f)) + c*d + a*f)/(d^2*f)) + 2*b*c*x + b^2 + (b*d*f*x + 2*a
*d*f)*sqrt(b^2/(d^3*f)))/x) - d*sqrt((d^2*f*sqrt(b^2/(d^3*f)) + c*d + a*f)/
(d^2*f))*log(-(2*sqrt(c*x^2 + b*x + a)*d^2*f*sqrt(b^2/(d^3*f))*sqrt((d^2*f*
sqrt(b^2/(d^3*f)) + c*d + a*f)/(d^2*f)) - 2*b*c*x - b^2 - (b*d*f*x + 2*a*d*
f)*sqrt(b^2/(d^3*f)))/x) - d*sqrt(-(d^2*f*sqrt(b^2/(d^3*f)) - c*d - a*f)/(d
^2*f))*log((2*sqrt(c*x^2 + b*x + a)*d^2*f*sqrt(b^2/(d^3*f))*sqrt(-(d^2*f*sq
rt(b^2/(d^3*f)) - c*d - a*f)/(d^2*f)) + 2*b*c*x + b^2 - (b*d*f*x + 2*a*d*f)
*sqrt(b^2/(d^3*f)))/x) + d*sqrt(-(d^2*f*sqrt(b^2/(d^3*f)) - c*d - a*f)/(d^2
```

```
*f))*log(-(2*sqrt(c*x^2 + b*x + a)*d^2*f*sqrt(b^2/(d^3*f))*sqrt(-(d^2*f*sqr
t(b^2/(d^3*f)) - c*d - a*f)/(d^2*f)) - 2*b*c*x - b^2 + (b*d*f*x + 2*a*d*f)*
sqrt(b^2/(d^3*f)))/x) + 4*sqrt(-a)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(b*x +
2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2)))/d]
```

Sympy [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{x\left(d-fx^2\right)}\,dx=-\int \frac{\sqrt{a+bx+cx^2}}{-dx+fx^3}\,dx$$

```
[In] integrate((c*x**2+b*x+a)**(1/2)/x/(-f*x**2+d),x)
[Out] -Integral(sqrt(a + b*x + c*x**2)/(-d*x + f*x**3), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{x\left(d-fx^2\right)}\,dx=\int -\frac{\sqrt{cx^2+bx+a}}{(fx^2-d)x}\,dx$$

```
[In] integrate((c*x^2+b*x+a)^(1/2)/x/(-f*x^2+d),x, algorithm="maxima")
[Out] -integrate(sqrt(c*x^2 + b*x + a)/((f*x^2 - d)*x), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{x\left(d-fx^2\right)}\,dx=\int -\frac{\sqrt{cx^2+bx+a}}{(fx^2-d)x}\,dx$$

```
[In] integrate((c*x^2+b*x+a)^(1/2)/x/(-f*x^2+d),x, algorithm="giac")
[Out] sage2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}}{x\left(d-fx^2\right)}\,dx=\int \frac{\sqrt{c\,x^2+b\,x+a}}{x\,\left(d-f\,x^2\right)}\,dx$$

```
[In] int((a + b*x + c*x^2)^(1/2)/(x*(d - f*x^2)),x)
[Out] int((a + b*x + c*x^2)^(1/2)/(x*(d - f*x^2)), x)
```

3.82 $\int \frac{\sqrt{a+bx+cx^2}}{x^2(d-fx^2)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 286

$$\int \frac{\sqrt{a+bx+cx^2}}{x^2\left(d-fx^2\right)}\,dx = -\frac{\sqrt{a+bx+cx^2}}{dx}-\frac{b\,\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{a}d} + \frac{\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\,\text{arctanh}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2d^{3/2}} + \frac{\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\,\text{arctanh}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2d^{3/2}}$$

```
[Out] -1/2*b*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/d/a^(1/2)-(c*x^2+
b*x+a)^(1/2)/d/x+1/2*arctanh(1/2*(b*d^(1/2)-2*a*f^(1/2)+x*(2*c*d^(1/2)-b*f^
(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2))*(c*d+a*f-b*d
^(1/2)*f^(1/2))^(1/2)/d^(3/2)+1/2*arctanh(1/2*(b*d^(1/2)+2*a*f^(1/2)+x*(2*c
*d^(1/2)+b*f^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f+b*d^(1/2)*f^(1/2))^(1/2))
*(c*d+a*f+b*d^(1/2)*f^(1/2))^(1/2)/d^(3/2)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used

$= \{6857, 746, 857, 635, 212, 738, 1004, 1047\}$

$$\int \frac{\sqrt{a+bx+cx^2}}{x^2(d-fx^2)}\,dx = \frac{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\,\text{arctanh}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2d^{3/2}} + \frac{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\,\text{arctanh}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2d^{3/2}} - \frac{b\,\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{a}d} - \frac{\sqrt{a+bx+cx^2}}{dx}$$

```
[In] Int[Sqrt[a + b*x + c*x^2]/(x^2*(d - f*x^2)),x]
```

```
[Out] -(Sqrt[a + b*x + c*x^2]/(d*x)) - (b*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a +
 b*x + c*x^2])])/(2*Sqrt[a]*d) + (Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*ArcTa
nh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*
Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d^(3/2)) + (Sqrt[c*d + b
*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b
*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])]
)/(2*d^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 746

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
```

```
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1004

```
Int[Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]/((d_) + (f_.)*(x_)^2), x_Symbol]
 :> Dist[c/f, Int[1/Sqrt[a + b*x + c*x^2], x], x] - Dist[1/f, Int[(c*d - a*
f - b*f*x)/(Sqrt[a + b*x + c*x^2]*(d + f*x^2)), x], x] /; FreeQ[{a, b, c, d
, f}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1047

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(-a)*c, 2]}, Dist[h/2 + c*(g/(2*q)
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - c*(g/(2*q)
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[(-a)*c]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{\sqrt{a+bx+cx^2}}{dx^2}+\frac{f\sqrt{a+bx+cx^2}}{d\,(d-fx^2)}\right)dx$$

$$=\frac{\int \frac{\sqrt{a+bx+cx^2}}{x^2}\,dx}{d}+\frac{f\int \frac{\sqrt{a+bx+cx^2}}{d-fx^2}\,dx}{d}$$

$$=-\frac{\sqrt{a+bx+cx^2}}{dx}+\frac{\int \frac{b+2cx}{x\sqrt{a+bx+cx^2}}\,dx}{2d}+\frac{\int \frac{cd+af+bfx}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx}{d}-\frac{c\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{d}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{dx} + \frac{b\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx}{2d} + \frac{c\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{d} - \frac{(2c)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{d} - \frac{\left(\sqrt{f}\left(cd-b\sqrt{d}\sqrt{f}+af\right)\right)\int \frac{1}{\left(-\sqrt{d}\sqrt{f}-fx\right)\sqrt{a+bx+cx^2}}\,dx}{2d^{3/2}} + \frac{\left(\sqrt{f}\left(cd+b\sqrt{d}\sqrt{f}+af\right)\right)\int \frac{1}{\left(\sqrt{d}\sqrt{f}-fx\right)\sqrt{a+bx+cx^2}}\,dx}{2d^{3/2}}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{dx} - \frac{\sqrt{c}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{d} - \frac{b\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx}{\sqrt{a+bx+cx^2}}\right)}{d} + \frac{(2c)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{d} + \frac{\left(\sqrt{f}\left(cd-b\sqrt{d}\sqrt{f}+af\right)\right)\text{Subst}\left(\int \frac{1}{4cdf-4b\sqrt{d}f^{3/2}+4af^2-x^2}\,dx, x, \frac{b\sqrt{d}\sqrt{f}-2af-\left(-2c\sqrt{d}\sqrt{f}+bf\right)x}{\sqrt{a+bx+cx^2}}\right)}{d^{3/2}} - \frac{\left(\sqrt{f}\left(cd+b\sqrt{d}\sqrt{f}+af\right)\right)\text{Subst}\left(\int \frac{1}{4cdf+4b\sqrt{d}f^{3/2}+4af^2-x^2}\,dx, x, \frac{-b\sqrt{d}\sqrt{f}-2af-\left(2c\sqrt{d}\sqrt{f}+bf\right)x}{\sqrt{a+bx+cx^2}}\right)}{d^{3/2}}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{dx} - \frac{b\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{a}d} + \frac{\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\tanh^{-1}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2d^{3/2}} + \frac{\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\tanh^{-1}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2d^{3/2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.39 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.01

$$\int \frac{\sqrt{a+bx+cx^2}}{x^2\left(d-fx^2\right)}\,dx =$$

$$-\frac{\frac{2\sqrt{a+x(b+cx)}}{x} - \frac{2b\text{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)}{\sqrt{a}} + \text{RootSum}\left[b^2d-a^2f-4b\sqrt{c}d\#1+4cd\#1^2+2af\#1^2-f\#1\right.}{}$$

[In] Integrate[Sqrt[a + b*x + c*x^2]/(x^2*(d - f*x^2)),x]

[Out] -1/2*((2*Sqrt[a + x*(b + c*x)])/x - (2*b*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)])/Sqrt[a]])/Sqrt[a] + RootSum[b^2*d - a^2*f - 4*b*Sqrt[c]*d*#1 + 4*c*d*#1^2 + 2*a*f*#1^2 - f*#1^4 & , (b*c*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*c^(3/2)*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - 2*a*Sqrt[c]*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + b*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2)/(b*Sqrt[c]*d - 2*c*d*#1 - a*f*#1 + f*#1^3) &])/d

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 463 vs. $2(218) = 436$.

Time = 0.90 (sec) , antiderivative size = 464, normalized size of antiderivative = 1.62

method	result
risch	$-\frac{\sqrt{c\,x^2+bx+a}}{dx}-\frac{\frac{b\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)}{\sqrt{a}}-\frac{\left(\sqrt{df}\,af+\sqrt{df}\,cd+bdf\right)\ln\left(\frac{\frac{2b\sqrt{df}+2fa+2cd}{f}+\frac{\left(2c\sqrt{df}+bf\right)\left(x-\frac{\sqrt{df}}{f}\right)}{f}+2\sqrt{\frac{b\sqrt{df}+fa+cd}{f}}\cdots}{x-\frac{\sqrt{\cdots}}{\cdots}}\right)}{df\sqrt{\frac{b\sqrt{df}+fa+cd}{f}}}\cdots}{\cdots}$
default	$\frac{-\frac{\left(c\,x^2+bx+a\right)^{\frac{3}{2}}}{ax}+\frac{b\left(\sqrt{c\,x^2+bx+a}+\frac{b\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{2\sqrt{c}}-\sqrt{a}\,\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)\right)}{2a}+\frac{2c\left(\frac{(2cx+b)\sqrt{c\,x^2+bx+a}}{4c}+\frac{\left(4ac-b^2\right)\cdots}{\cdots}\right)}{a}\cdots}{d}$

[In] int((c*x^2+b*x+a)^(1/2)/x^2/(-f*x^2+d),x,method=_RETURNVERBOSE)

[Out] -(c*x^2+b*x+a)^(1/2)/d/x-1/2/d*(b/a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)-((d*f)^(1/2)*a*f+(d*f)^(1/2)*c*d+b*d*f)/d/f/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+f*a+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))-(-(d*f)^(1/2)*a*f-(d*f)^(1/2)*c*d+b*d*f)/d/f/(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1/2)/f)))

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 543 vs. 2(218) = 436.

Time = 14.92 (sec) , antiderivative size = 1094, normalized size of antiderivative = 3.83

$$\int \frac{\sqrt{a+bx+cx^2}}{x^2\left(d-fx^2\right)}\,dx$$
$$= \left[\frac{adx\sqrt{\frac{d^3\sqrt{\frac{b^2f}{d^5}}+cd+af}{d^3}}\log\left(\frac{2\,bcx+2\,\sqrt{cx^2+bx+a}bd\sqrt{\frac{d^3\sqrt{\frac{b^2f}{d^5}}+cd+af}{d^3}}+b^2+\left(bd^2x+2\,ad^2\right)\sqrt{\frac{b^2f}{d^5}}}{x}\right)-adx\sqrt{\frac{d^3\sqrt{\frac{b^2f}{d^5}}+cd+af}{d^3}}\log\left(\right.}{}\right.$$

```
[In] integrate((c*x^2+b*x+a)^(1/2)/x^2/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] [1/4*(a*d*x*sqrt((d^3*sqrt(b^2*f/d^5) + c*d + a*f)/d^3)*log((2*b*c*x + 2*sq
rt(c*x^2 + b*x + a)*b*d*sqrt((d^3*sqrt(b^2*f/d^5) + c*d + a*f)/d^3) + b^2 +
 (b*d^2*x + 2*a*d^2)*sqrt(b^2*f/d^5))/x) - a*d*x*sqrt((d^3*sqrt(b^2*f/d^5)
+ c*d + a*f)/d^3)*log((2*b*c*x - 2*sqrt(c*x^2 + b*x + a)*b*d*sqrt((d^3*sqrt
(b^2*f/d^5) + c*d + a*f)/d^3) + b^2 + (b*d^2*x + 2*a*d^2)*sqrt(b^2*f/d^5))/
x) + a*d*x*sqrt(-(d^3*sqrt(b^2*f/d^5) - c*d - a*f)/d^3)*log((2*b*c*x + 2*sq
rt(c*x^2 + b*x + a)*b*d*sqrt(-(d^3*sqrt(b^2*f/d^5) - c*d - a*f)/d^3) + b^2
- (b*d^2*x + 2*a*d^2)*sqrt(b^2*f/d^5))/x) - a*d*x*sqrt(-(d^3*sqrt(b^2*f/d^5
) - c*d - a*f)/d^3)*log((2*b*c*x - 2*sqrt(c*x^2 + b*x + a)*b*d*sqrt(-(d^3*s
qrt(b^2*f/d^5) - c*d - a*f)/d^3) + b^2 - (b*d^2*x + 2*a*d^2)*sqrt(b^2*f/d^5
))/x) + sqrt(a)*b*x*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 + b*x
+ a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) - 4*sqrt(c*x^2 + b*x + a)*a)/(a*d*x)
, 1/4*(a*d*x*sqrt((d^3*sqrt(b^2*f/d^5) + c*d + a*f)/d^3)*log((2*b*c*x + 2*s
qrt(c*x^2 + b*x + a)*b*d*sqrt((d^3*sqrt(b^2*f/d^5) + c*d + a*f)/d^3) + b^2
+ (b*d^2*x + 2*a*d^2)*sqrt(b^2*f/d^5))/x) - a*d*x*sqrt((d^3*sqrt(b^2*f/d^5)
 + c*d + a*f)/d^3)*log((2*b*c*x - 2*sqrt(c*x^2 + b*x + a)*b*d*sqrt((d^3*sqr
t(b^2*f/d^5) + c*d + a*f)/d^3) + b^2 + (b*d^2*x + 2*a*d^2)*sqrt(b^2*f/d^5))
/x) + a*d*x*sqrt(-(d^3*sqrt(b^2*f/d^5) - c*d - a*f)/d^3)*log((2*b*c*x + 2*s
qrt(c*x^2 + b*x + a)*b*d*sqrt(-(d^3*sqrt(b^2*f/d^5) - c*d - a*f)/d^3) + b^2
 - (b*d^2*x + 2*a*d^2)*sqrt(b^2*f/d^5))/x) - a*d*x*sqrt(-(d^3*sqrt(b^2*f/d^
5) - c*d - a*f)/d^3)*log((2*b*c*x - 2*sqrt(c*x^2 + b*x + a)*b*d*sqrt(-(d^3*
sqrt(b^2*f/d^5) - c*d - a*f)/d^3) + b^2 - (b*d^2*x + 2*a*d^2)*sqrt(b^2*f/d^
5))/x) + 2*sqrt(-a)*b*x*arctan(1/2*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(-
a)/(a*c*x^2 + a*b*x + a^2)) - 4*sqrt(c*x^2 + b*x + a)*a)/(a*d*x)]
```

Sympy [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{x^2(d-fx^2)}\,dx = -\int \frac{\sqrt{a+bx+cx^2}}{-dx^2+fx^4}\,dx$$

```
[In] integrate((c*x**2+b*x+a)**(1/2)/x**2/(-f*x**2+d),x)
[Out] -Integral(sqrt(a + b*x + c*x**2)/(-d*x**2 + f*x**4), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{x^2(d-fx^2)}\,dx = \int -\frac{\sqrt{cx^2+bx+a}}{(fx^2-d)x^2}\,dx$$

```
[In] integrate((c*x^2+b*x+a)^(1/2)/x^2/(-f*x^2+d),x, algorithm="maxima")
[Out] -integrate(sqrt(c*x^2 + b*x + a)/((f*x^2 - d)*x^2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx+cx^2}}{x^2(d-fx^2)}\,dx = \text{Exception raised: AttributeError}$$

```
[In] integrate((c*x^2+b*x+a)^(1/2)/x^2/(-f*x^2+d),x, algorithm="giac")
[Out] Exception raised: AttributeError >> type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}}{x^2(d-fx^2)}\,dx = \int \frac{\sqrt{c\,x^2+b\,x+a}}{x^2\,(d-f\,x^2)}\,dx$$

```
[In] int((a + b*x + c*x^2)^(1/2)/(x^2*(d - f*x^2)),x)
[Out] int((a + b*x + c*x^2)^(1/2)/(x^2*(d - f*x^2)), x)
```

3.83 $\int \frac{\sqrt{a+bx+cx^2}}{x^3(d-fx^2)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 353

$$\begin{aligned}\int \frac{\sqrt{a+bx+cx^2}}{x^3(d-fx^2)}\,dx = &-\frac{(2a+bx)\sqrt{a+bx+cx^2}}{4adx^2}+\frac{(b^2-4ac)\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{3/2}d}\\&-\frac{\sqrt{a}f\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d^2}\\&-\frac{\sqrt{f}\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\operatorname{arctanh}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2d^2}\\&+\frac{\sqrt{f}\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\operatorname{arctanh}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2d^2}\end{aligned}$$

```
[Out] 1/8*(-4*a*c+b^2)*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/a^(3/2)
/d-f*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))*a^(1/2)/d^2-1/4*(b*
x+2*a)*(c*x^2+b*x+a)^(1/2)/a/d/x^2-1/2*arctanh(1/2*(b*d^(1/2)-2*a*f^(1/2)+x
*(2*c*d^(1/2)-b*f^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f-b*d^(1/2)*f^(1/2))^(
1/2))*f^(1/2)*(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2)/d^2+1/2*arctanh(1/2*(b*d^(1
/2)+2*a*f^(1/2)+x*(2*c*d^(1/2)+b*f^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f+b*d
^(1/2)*f^(1/2))^(1/2))*f^(1/2)*(c*d+a*f+b*d^(1/2)*f^(1/2))^(1/2)/d^2
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {6857, 734, 738, 212, 748, 857, 635, 1035, 1092, 1047}

$$\int \frac{\sqrt{a+bx+cx^2}}{x^3(d-fx^2)}\,dx$$
$$= \frac{(b^2-4ac)\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{3/2}d} - \frac{\sqrt{a}f\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d^2}$$
$$- \frac{\sqrt{f}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\operatorname{arctanh}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2d^2}$$
$$+ \frac{\sqrt{f}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\operatorname{arctanh}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2d^2}$$
$$- \frac{(2a+bx)\sqrt{a+bx+cx^2}}{4adx^2}$$

```
[In] Int[Sqrt[a + b*x + c*x^2]/(x^3*(d - f*x^2)),x]
```

```
[Out] -1/4*((2*a + b*x)*Sqrt[a + b*x + c*x^2])/(a*d*x^2) + ((b^2 - 4*a*c)*ArcTanh
[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(8*a^(3/2)*d) - (Sqrt[a]*f
*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/d^2 - (Sqrt[f]*Sqr
t[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sq
rt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x +
 c*x^2])])/(2*d^2) + (Sqrt[f]*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(
b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt
[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), Int[(d + e*x)^(m + 2)*(a + b*x +
c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
 0] && GtQ[p, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 748

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1035

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[h*(a + b*x + c*x^2)^p*((d + f*x^2)^(q +
 1)/(2*f*(p + q + 1))), x] - Dist[1/(2*f*(p + q + 1)), Int[(a + b*x + c*x^2
)^(p - 1)*(d + f*x^2)^q*Simp[h*p*(b*d) + a*(-2*g*f)*(p + q + 1) + (2*h*p*(c
*d - a*f) + b*(-2*g*f)*(p + q + 1))*x + (h*p*((-b)*f) + c*(-2*g*f)*(p + q +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*a
*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]
```

Rule 1047

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(-a)*c, 2]}, Dist[h/2 + c*(g/(2*q)
```

```
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - c*(g/(2*q)
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[(-a)*c]
```

Rule 1092

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
 (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + e*x + f
*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x
 + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f
, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{\sqrt{a+bx+cx^2}}{dx^3}+\frac{f\sqrt{a+bx+cx^2}}{d^2x}+\frac{f^2x\sqrt{a+bx+cx^2}}{d^2\left(d-fx^2\right)}\right)dx \\
&= \frac{\int \frac{\sqrt{a+bx+cx^2}}{x^3}\,dx}{d}+\frac{f\int \frac{\sqrt{a+bx+cx^2}}{x}\,dx}{d^2}+\frac{f^2\int \frac{x\sqrt{a+bx+cx^2}}{d-fx^2}\,dx}{d^2} \\
&= -\frac{(2a+bx)\sqrt{a+bx+cx^2}}{4adx^2}-\frac{\left(b^2-4ac\right)\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx}{8ad} \\
&\quad -\frac{f\int \frac{-2a-bx}{x\sqrt{a+bx+cx^2}}\,dx}{2d^2}+\frac{f\int \frac{\frac{bd}{2}+(cd+af)x+\frac{1}{2}bfx^2}{\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\,dx}{d^2} \\
&= -\frac{(2a+bx)\sqrt{a+bx+cx^2}}{4adx^2}-\frac{\int \frac{-bdf-f(cd+af)x}{\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\,dx}{d^2} \\
&\quad +\frac{\left(b^2-4ac\right)\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx,x,\frac{2a+bx}{\sqrt{a+bx+cx^2}}\right)}{4ad}+\frac{(af)\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx}{d^2}
\end{aligned}$$

$$=-\frac{(2a+bx)\sqrt{a+bx+cx^2}}{4adx^2}+\frac{(b^2-4ac)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{3/2}d}$$
$$-\frac{(2af)\text{Subst}\left(\int\frac{1}{4a-x^2}\,dx,x,\frac{2a+bx}{\sqrt{a+bx+cx^2}}\right)}{d^2}$$
$$+\frac{\left(f\left(cd-b\sqrt{d}\sqrt{f}+af\right)\right)\int\frac{1}{\left(-\sqrt{d}\sqrt{f}-fx\right)\sqrt{a+bx+cx^2}}\,dx}{2d^2}$$
$$+\frac{\left(f\left(cd+b\sqrt{d}\sqrt{f}+af\right)\right)\int\frac{1}{\left(\sqrt{d}\sqrt{f}-fx\right)\sqrt{a+bx+cx^2}}\,dx}{2d^2}$$

$$=-\frac{(2a+bx)\sqrt{a+bx+cx^2}}{4adx^2}+\frac{(b^2-4ac)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{3/2}d}$$
$$-\frac{\sqrt{a}f\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d^2}$$
$$-\frac{\left(f\left(cd-b\sqrt{d}\sqrt{f}+af\right)\right)\text{Subst}\left(\int\frac{1}{4cdf-4b\sqrt{d}f^{3/2}+4af^2-x^2}\,dx,x,\frac{b\sqrt{d}\sqrt{f}-2af-\left(-2c\sqrt{d}\sqrt{f}+bf\right)x}{\sqrt{a+bx+cx^2}}\right)}{d^2}$$
$$-\frac{\left(f\left(cd+b\sqrt{d}\sqrt{f}+af\right)\right)\text{Subst}\left(\int\frac{1}{4cdf+4b\sqrt{d}f^{3/2}+4af^2-x^2}\,dx,x,\frac{-b\sqrt{d}\sqrt{f}-2af-\left(2c\sqrt{d}\sqrt{f}+bf\right)x}{\sqrt{a+bx+cx^2}}\right)}{d^2}$$

$$=-\frac{(2a+bx)\sqrt{a+bx+cx^2}}{4adx^2}+\frac{(b^2-4ac)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{3/2}d}$$
$$-\frac{\sqrt{a}f\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d^2}$$
$$-\frac{\sqrt{f}\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\tanh^{-1}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2d^2}$$
$$+\frac{\sqrt{f}\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\tanh^{-1}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2d^2}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.60 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{a+bx+cx^2}}{x^3(d-fx^2)}\,dx = \frac{-\frac{d(2a+bx)\sqrt{a+x(b+cx)}}{ax^2}+\frac{(b^2d-4a(cd+2af))\text{arctanh}\left(\frac{-\sqrt{c}x+\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)}{a^{3/2}}-2f\text{RootSum}\Big[b^2d-a^2f-4b\sqrt{c}d\#1+4c}{}$$

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/(x^3*(d - f*x^2)),x]
```

```
[Out] (-((d*(2*a + b*x)*Sqrt[a + x*(b + c*x)])/(a*x^2)) + ((b^2*d - 4*a*(c*d + 2*
a*f))*ArcTanh[(-(Sqrt[c]*x) + Sqrt[a + x*(b + c*x)])/Sqrt[a]])/a^(3/2) - 2*
f*RootSum[b^2*d - a^2*f - 4*b*Sqrt[c]*d*#1 + 4*c*d*#1^2 + 2*a*f*#1^2 - f*#1
^4 & , (b^2*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - a*c*d*Log[-(
Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - a^2*f*Log[-(Sqrt[c]*x) + Sqrt[a
+ b*x + c*x^2] - #1] - 2*b*Sqrt[c]*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^
2] - #1]*#1 + c*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 + a*f
*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2)/(b*Sqrt[c]*d - 2*c*d*
#1 - a*f*#1 + f*#1^3) & ])/(4*d^2)
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 479, normalized size of antiderivative = 1.36

method	result
risch	$-\frac{(bx+2a)\sqrt{c\,x^2+bx+a}}{4ad\,x^2}-\frac{-\frac{\left(-8a^2f-4acd+b^2d\right)\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)}{d\sqrt{a}}-\frac{4a\left(b\sqrt{df}+fa+cd\right)\ln\left(\frac{2b\sqrt{df}+2fa+2cd}{f}+\frac{\left(2c\sqrt{df}+bf\right)\left(x\right.}{f}\right.}{}}{}$
default	Expression too large to display

```
[In] int((c*x^2+b*x+a)^(1/2)/x^3/(-f*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(b*x+2*a)*(c*x^2+b*x+a)^(1/2)/a/d/x^2-1/8/a/d*(-(-8*a^2*f-4*a*c*d+b^2*
d)/d/a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)-4*a*(b*(d*f)^(1/
2)+f*a+c*d)/d/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+f*a+c*
d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f*a+c*d)/f
)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b
*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))+4*a*(b*(d*f)^(1/2)-f*a-c
*d)/d/(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+f*a+c*d)
```

```
+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*
d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/
f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1/2)/f)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 738 vs. $2(275) = 550$.

Time = 81.78 (sec) , antiderivative size = 1485, normalized size of antiderivative = 4.21

$$\int \frac{\sqrt{a+bx+cx^2}}{x^3\left(d-fx^2\right)}\,dx = \text{Too large to display}$$

```
[In] integrate((c*x^2+b*x+a)^(1/2)/x^3/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] [1/16*(4*a^2*d^2*x^2*sqrt((d^4*sqrt(b^2*f^3/d^7) + c*d*f + a*f^2)/d^4)*log(
(2*sqrt(c*x^2 + b*x + a)*d^5*sqrt(b^2*f^3/d^7)*sqrt((d^4*sqrt(b^2*f^3/d^7)
+ c*d*f + a*f^2)/d^4) + 2*b*c*f^2*x + b^2*f^2 + (b*d^3*f*x + 2*a*d^3*f)*sqr
t(b^2*f^3/d^7))/x) - 4*a^2*d^2*x^2*sqrt((d^4*sqrt(b^2*f^3/d^7) + c*d*f + a*
f^2)/d^4)*log(-(2*sqrt(c*x^2 + b*x + a)*d^5*sqrt(b^2*f^3/d^7)*sqrt((d^4*sqr
t(b^2*f^3/d^7) + c*d*f + a*f^2)/d^4) - 2*b*c*f^2*x - b^2*f^2 - (b*d^3*f*x +
 2*a*d^3*f)*sqrt(b^2*f^3/d^7))/x) - 4*a^2*d^2*x^2*sqrt(-(d^4*sqrt(b^2*f^3/d
^7) - c*d*f - a*f^2)/d^4)*log((2*sqrt(c*x^2 + b*x + a)*d^5*sqrt(b^2*f^3/d^7
)*sqrt(-(d^4*sqrt(b^2*f^3/d^7) - c*d*f - a*f^2)/d^4) + 2*b*c*f^2*x + b^2*f^
2 - (b*d^3*f*x + 2*a*d^3*f)*sqrt(b^2*f^3/d^7))/x) + 4*a^2*d^2*x^2*sqrt(-(d^
4*sqrt(b^2*f^3/d^7) - c*d*f - a*f^2)/d^4)*log(-(2*sqrt(c*x^2 + b*x + a)*d^5
*sqrt(b^2*f^3/d^7)*sqrt(-(d^4*sqrt(b^2*f^3/d^7) - c*d*f - a*f^2)/d^4) - 2*b
*c*f^2*x - b^2*f^2 + (b*d^3*f*x + 2*a*d^3*f)*sqrt(b^2*f^3/d^7))/x) + (8*a^2
*f - (b^2 - 4*a*c)*d)*sqrt(a)*x^2*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqr
t(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) - 4*(a*b*d*x + 2*a^2*d
)*sqrt(c*x^2 + b*x + a))/(a^2*d^2*x^2), 1/8*(2*a^2*d^2*x^2*sqrt((d^4*sqrt(b
^2*f^3/d^7) + c*d*f + a*f^2)/d^4)*log((2*sqrt(c*x^2 + b*x + a)*d^5*sqrt(b^2
*f^3/d^7)*sqrt((d^4*sqrt(b^2*f^3/d^7) + c*d*f + a*f^2)/d^4) + 2*b*c*f^2*x +
 b^2*f^2 + (b*d^3*f*x + 2*a*d^3*f)*sqrt(b^2*f^3/d^7))/x) - 2*a^2*d^2*x^2*sq
rt((d^4*sqrt(b^2*f^3/d^7) + c*d*f + a*f^2)/d^4)*log(-(2*sqrt(c*x^2 + b*x +
a)*d^5*sqrt(b^2*f^3/d^7)*sqrt((d^4*sqrt(b^2*f^3/d^7) + c*d*f + a*f^2)/d^4)
- 2*b*c*f^2*x - b^2*f^2 - (b*d^3*f*x + 2*a*d^3*f)*sqrt(b^2*f^3/d^7))/x) - 2
*a^2*d^2*x^2*sqrt(-(d^4*sqrt(b^2*f^3/d^7) - c*d*f - a*f^2)/d^4)*log((2*sqrt
(c*x^2 + b*x + a)*d^5*sqrt(b^2*f^3/d^7)*sqrt(-(d^4*sqrt(b^2*f^3/d^7) - c*d*
f - a*f^2)/d^4) + 2*b*c*f^2*x + b^2*f^2 - (b*d^3*f*x + 2*a*d^3*f)*sqrt(b^2*
f^3/d^7))/x) + 2*a^2*d^2*x^2*sqrt(-(d^4*sqrt(b^2*f^3/d^7) - c*d*f - a*f^2)/
d^4)*log(-(2*sqrt(c*x^2 + b*x + a)*d^5*sqrt(b^2*f^3/d^7)*sqrt(-(d^4*sqrt(b^
2*f^3/d^7) - c*d*f - a*f^2)/d^4) - 2*b*c*f^2*x - b^2*f^2 + (b*d^3*f*x + 2*a
*d^3*f)*sqrt(b^2*f^3/d^7))/x) + (8*a^2*f - (b^2 - 4*a*c)*d)*sqrt(-a)*x^2*ar
ctan(1/2*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(-a)/(a*c*x^2 + a*b*x + a^2)
) - 2*(a*b*d*x + 2*a^2*d)*sqrt(c*x^2 + b*x + a))/(a^2*d^2*x^2)]
```

Sympy [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{x^3\left(d-fx^2\right)}\,dx=-\int \frac{\sqrt{a+bx+cx^2}}{-dx^3+fx^5}\,dx$$

```
[In] integrate((c*x**2+b*x+a)**(1/2)/x**3/(-f*x**2+d),x)
[Out] -Integral(sqrt(a + b*x + c*x**2)/(-d*x**3 + f*x**5), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{x^3\left(d-fx^2\right)}\,dx=\int -\frac{\sqrt{cx^2+bx+a}}{(fx^2-d)x^3}\,dx$$

```
[In] integrate((c*x^2+b*x+a)^(1/2)/x^3/(-f*x^2+d),x, algorithm="maxima")
[Out] -integrate(sqrt(c*x^2 + b*x + a)/((f*x^2 - d)*x^3), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx+cx^2}}{x^3\left(d-fx^2\right)}\,dx=\text{Exception raised: AttributeError}$$

```
[In] integrate((c*x^2+b*x+a)^(1/2)/x^3/(-f*x^2+d),x, algorithm="giac")
[Out] Exception raised: AttributeError >> type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}}{x^3\left(d-fx^2\right)}\,dx=\int \frac{\sqrt{c\,x^2+b\,x+a}}{x^3\,\left(d-f\,x^2\right)}\,dx$$

```
[In] int((a + b*x + c*x^2)^(1/2)/(x^3*(d - f*x^2)),x)
[Out] int((a + b*x + c*x^2)^(1/2)/(x^3*(d - f*x^2)), x)
```

3.84 $\int \frac{x^3\left(a+bx+cx^2\right)^{3/2}}{d-fx^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 501

$$\begin{aligned}\int \frac{x^3(a+bx+cx^2)^{3/2}}{d-fx^2}\,dx &= -\frac{3b(b^2-4ac)\,(b+2cx)\sqrt{a+bx+cx^2}}{128c^3f}\\
&-\frac{d(8c^2d+b^2f+8acf+2bcfx)\sqrt{a+bx+cx^2}}{8cf^3}\\
&-\frac{d(a+bx+cx^2)^{3/2}}{3f^2}+\frac{b(b+2cx)\,(a+bx+cx^2)^{3/2}}{16c^2f}\\
&-\frac{(a+bx+cx^2)^{5/2}}{5cf}+\frac{3b(b^2-4ac)^2\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{256c^{7/2}f}\\
&-\frac{bd(24c^2d-b^2f+12acf)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}f^3}\\
&-\frac{d\left(cd-b\sqrt{d}\sqrt{f}+af\right)^{3/2}\operatorname{arctanh}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2f^{7/2}}\\
&+\frac{d\left(cd+b\sqrt{d}\sqrt{f}+af\right)^{3/2}\operatorname{arctanh}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2f^{7/2}}\end{aligned}$$

```
[Out] -1/3*d*(c*x^2+b*x+a)^(3/2)/f^2+1/16*b*(2*c*x+b)*(c*x^2+b*x+a)^(3/2)/c^2/f-1
/5*(c*x^2+b*x+a)^(5/2)/c/f+3/256*b*(-4*a*c+b^2)^2*arctanh(1/2*(2*c*x+b)/c^(
1/2)/(c*x^2+b*x+a)^(1/2))/c^(7/2)/f-1/16*b*d*(12*a*c*f-b^2*f+24*c^2*d)*arct
anh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(3/2)/f^3-1/2*d*arctanh(1/
2*(b*d^(1/2)-2*a*f^(1/2)+x*(2*c*d^(1/2)-b*f^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*
d+a*f-b*d^(1/2)*f^(1/2))^(1/2))*(c*d+a*f-b*d^(1/2)*f^(1/2))^(3/2)/f^(7/2)+1
```

/2*d*arctanh(1/2*(b*d^(1/2)+2*a*f^(1/2)+x*(2*c*d^(1/2)+b*f^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f+b*d^(1/2)*f^(1/2))^(1/2))*(c*d+a*f+b*d^(1/2)*f^(1/2))^(3/2)/f^(7/2)-3/128*b*(-4*a*c+b^2)*(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/c^3/f-1/8*d*(2*b*c*f*x+8*a*c*f+b^2*f+8*c^2*d)*(c*x^2+b*x+a)^(1/2)/c/f^3

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 501, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {6857, 654, 626, 635, 212, 1035, 1084, 1092, 1047, 738}

$$\int \frac{x^3(a+bx+cx^2)^{3/2}}{d-fx^2}\,dx = \frac{3b(b^2-4ac)^2\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{256c^{7/2}f} - \frac{bd\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(12acf+b^2(-f)+24c^2d)}{16c^{3/2}f^3} - \frac{d\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}\operatorname{arctanh}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2f^{7/2}} + \frac{d\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}\operatorname{arctanh}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2f^{7/2}} - \frac{3b(b^2-4ac)(b+2cx)\sqrt{a+bx+cx^2}}{128c^3f} - \frac{d\sqrt{a+bx+cx^2}(8acf+b^2f+2bcfx+8c^2d)}{8cf^3} + \frac{b(b+2cx)(a+bx+cx^2)^{3/2}}{16c^2f} - \frac{d(a+bx+cx^2)^{3/2}}{3f^2} - \frac{(a+bx+cx^2)^{5/2}}{5cf}$$

[In] Int[(x^3*(a + b*x + c*x^2)^(3/2))/(d - f*x^2),x]

[Out] (-3*b*(b^2 - 4*a*c)*(b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(128*c^3*f) - (d*(8*c^2*d + b^2*f + 8*a*c*f + 2*b*c*f*x)*Sqrt[a + b*x + c*x^2])/(8*c*f^3) - (d*(a + b*x + c*x^2)^(3/2))/(3*f^2) + (b*(b + 2*c*x)*(a + b*x + c*x^2)^(3/2))/(16*c^2*f) - (a + b*x + c*x^2)^(5/2)/(5*c*f) + (3*b*(b^2 - 4*a*c)^2*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(256*c^(7/2)*f) - (b*d*(24*c^2*d - b^2*f + 12*a*c*f)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(3/2)*f^3) - (d*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*f^(7/2)) + (d*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] +

```
 b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2]
)])/(2*f^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1035

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[h*(a + b*x + c*x^2)^p*((d + f*x^2)^(q +
 1)/(2*f*(p + q + 1))), x] - Dist[1/(2*f*(p + q + 1)), Int[(a + b*x + c*x^2
)^(p - 1)*(d + f*x^2)^q*Simp[h*p*(b*d) + a*(-2*g*f)*(p + q + 1) + (2*h*p*(c
*d - a*f) + b*(-2*g*f)*(p + q + 1))*x + (h*p*((-b)*f) + c*(-2*g*f)*(p + q +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*a
*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]
```

Rule 1047

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(-a)*c, 2]}, Dist[h/2 + c*(g/(2*q)
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - c*(g/(2*q)
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[(-a)*c]
```

Rule 1084

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(B*c*f*(2*p + 2*q + 3) +
 C*(b*f*p) + 2*c*C*f*(p + q + 1)*x)*(a + b*x + c*x^2)^p*((d + f*x^2)^(q + 1
)/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3))), x] - Dist[1/(2*c*f^2*(p + q + 1)*
(2*p + 2*q + 3)), Int[(a + b*x + c*x^2)^(p - 1)*(d + f*x^2)^q*Simp[p*(b*d)*
(C*((-b)*f)*(q + 1) - c*((-B)*f)*(2*p + 2*q + 3)) + (p + q + 1)*(b^2*C*d*f*
p + a*c*(C*(2*d*f) + f*(-2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)*(C*((-
b)*f)*(q + 1) - c*((-B)*f)*(2*p + 2*q + 3)) + (p + q + 1)*((-b)*c*(C*(-4*d*
f)*(2*p + q + 2) + f*(2*C*d + 2*A*f)*(2*p + 2*q + 3))))*x + (p*((-b)*f)*(C*
((-b)*f)*(q + 1) - c*((-B)*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2
- 4*a*c) - c^2*(C*(-4*d*f)*(2*p + q + 2) + f*(2*C*d + 2*A*f)*(2*p + 2*q + 3
))))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, A, B, C, q}, x] && NeQ[b^2 -
4*a*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*q + 3, 0] &&  !I
GtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1092

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
 (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + e*x + f
*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x
 + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f
, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\text{integral} = \int \left(-\frac{x(a+bx+cx^2)^{3/2}}{f} + \frac{dx(a+bx+cx^2)^{3/2}}{f(d-fx^2)}\right) dx$$

$$= -\frac{\int x(a+bx+cx^2)^{3/2}\, dx}{f} + \frac{d\int \frac{x(a+bx+cx^2)^{3/2}}{d-fx^2}\, dx}{f}$$

$$= -\frac{d(a+bx+cx^2)^{3/2}}{3f^2} - \frac{(a+bx+cx^2)^{5/2}}{5cf} + \frac{d\int \frac{\sqrt{a+bx+cx^2}\left(\frac{3bd}{2}+3(cd+af)x+\frac{3}{2}bfx^2\right)}{d-fx^2}\,dx}{3f^2} + \frac{b\int (a+bx+cx^2)^{3/2}\,dx}{2cf}$$

$$= -\frac{d(8c^2d+b^2f+8acf+2bcfx)\sqrt{a+bx+cx^2}}{8cf^3} - \frac{d(a+bx+cx^2)^{3/2}}{3f^2} + \frac{b(b+2cx)(a+bx+cx^2)^{3/2}}{16c^2f} - \frac{(a+bx+cx^2)^{5/2}}{5cf} - \frac{d\int \frac{-\frac{3}{8}bdf(8c^2d+b^2f+20acf)-6cf(b^2df+(cd+af)^2)x-\frac{3}{8}bf^2(24c^2d-b^2f+12acf)x^2}{\sqrt{a+bx+cx^2}(d-fx^2)}\,dx}{6cf^4} - \frac{(3b(b^2-4ac))\int\sqrt{a+bx+cx^2}\,dx}{32c^2f}$$

$$= -\frac{3b(b^2-4ac)(b+2cx)\sqrt{a+bx+cx^2}}{128c^3f} - \frac{d(8c^2d+b^2f+8acf+2bcfx)\sqrt{a+bx+cx^2}}{8cf^3} - \frac{d(a+bx+cx^2)^{3/2}}{3f^2} + \frac{b(b+2cx)(a+bx+cx^2)^{3/2}}{16c^2f} - \frac{(a+bx+cx^2)^{5/2}}{5cf} + \frac{d\int \frac{\frac{3}{8}bdf^2(24c^2d-b^2f+12acf)+\frac{3}{8}bdf^2(8c^2d+b^2f+20acf)+6cf^2(b^2df+(cd+af)^2)x}{\sqrt{a+bx+cx^2}(d-fx^2)}\,dx}{6cf^5} + \frac{\left(3b(b^2-4ac)^2\right)\int\frac{1}{\sqrt{a+bx+cx^2}}\,dx}{256c^3f} - \frac{(bd(24c^2d-b^2f+12acf))\int\frac{1}{\sqrt{a+bx+cx^2}}\,dx}{16cf^3}$$

$$
\begin{aligned}
&= -\frac{3b(b^2-4ac)(b+2cx)\sqrt{a+bx+cx^2}}{128c^3f} \\
&\quad - \frac{d(8c^2d+b^2f+8acf+2bcfx)\sqrt{a+bx+cx^2}}{8cf^3} \\
&\quad - \frac{d(a+bx+cx^2)^{3/2}}{3f^2} + \frac{b(b+2cx)(a+bx+cx^2)^{3/2}}{16c^2f} \\
&\quad - \frac{(a+bx+cx^2)^{5/2}}{5cf} + \frac{\left(3b(b^2-4ac)^2\right)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{128c^3f} \\
&\quad + \frac{\left(d\left(cd-b\sqrt{d}\sqrt{f}+af\right)^2\right)\int \frac{1}{\left(-\sqrt{d}\sqrt{f}-fx\right)\sqrt{a+bx+cx^2}}\,dx}{2f^3} \\
&\quad + \frac{\left(d\left(cd+b\sqrt{d}\sqrt{f}+af\right)^2\right)\int \frac{1}{\left(\sqrt{d}\sqrt{f}-fx\right)\sqrt{a+bx+cx^2}}\,dx}{2f^3} \\
&\quad - \frac{(bd(24c^2d-b^2f+12acf))\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{8cf^3}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{3b(b^2-4ac)(b+2cx)\sqrt{a+bx+cx^2}}{128c^3f} - \frac{d(8c^2d+b^2f+8acf+2bcfx)\sqrt{a+bx+cx^2}}{8cf^3} \\
&\quad - \frac{d(a+bx+cx^2)^{3/2}}{3f^2} + \frac{b(b+2cx)(a+bx+cx^2)^{3/2}}{16c^2f} \\
&\quad - \frac{(a+bx+cx^2)^{5/2}}{5cf} + \frac{3b(b^2-4ac)^2\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{256c^{7/2}f} \\
&\quad - \frac{bd(24c^2d-b^2f+12acf)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}f^3} \\
&\quad - \frac{\left(d\left(cd-b\sqrt{d}\sqrt{f}+af\right)^2\right)\text{Subst}\left(\int \frac{1}{4cdf-4b\sqrt{d}f^{3/2}+4af^2-x^2}\,dx, x, \frac{b\sqrt{d}\sqrt{f}-2af-\left(-2c\sqrt{d}\sqrt{f}+bf\right)x}{\sqrt{a+bx+cx^2}}\right)}{f^3} \\
&\quad - \frac{\left(d\left(cd+b\sqrt{d}\sqrt{f}+af\right)^2\right)\text{Subst}\left(\int \frac{1}{4cdf+4b\sqrt{d}f^{3/2}+4af^2-x^2}\,dx, x, \frac{-b\sqrt{d}\sqrt{f}-2af-\left(2c\sqrt{d}\sqrt{f}+bf\right)x}{\sqrt{a+bx+cx^2}}\right)}{f^3}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{3b(b^2-4ac)\,(b+2cx)\sqrt{a+bx+cx^2}}{128c^3f} \\
&\quad - \frac{d(8c^2d+b^2f+8acf+2bcfx)\,\sqrt{a+bx+cx^2}}{8cf^3} \\
&\quad - \frac{d(a+bx+cx^2)^{3/2}}{3f^2} + \frac{b(b+2cx)\,(a+bx+cx^2)^{3/2}}{16c^2f} \\
&\quad - \frac{(a+bx+cx^2)^{5/2}}{5cf} + \frac{3b(b^2-4ac)^2\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{256c^{7/2}f} \\
&\quad - \frac{bd(24c^2d-b^2f+12acf)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}f^3} \\
&\quad - \frac{d\left(cd-b\sqrt{d}\sqrt{f}+af\right)^{3/2}\tanh^{-1}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2f^{7/2}} \\
&\quad + \frac{d\left(cd+b\sqrt{d}\sqrt{f}+af\right)^{3/2}\tanh^{-1}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2f^{7/2}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 2.02 (sec) , antiderivative size = 734, normalized size of antiderivative = 1.47

$$
\int \frac{x^3(a+bx+cx^2)^{3/2}}{d-fx^2}\,dx = \frac{-2\sqrt{c}\sqrt{a+x(b+cx)}(45b^4f^2-30b^2cf^2(10a+bx)+16c^3f(160ad+70bdx+48
$$

```
[In] Integrate[(x^3*(a + b*x + c*x^2)^(3/2))/(d - f*x^2),x]
```

```
[Out] (-2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(45*b^4*f^2 - 30*b^2*c*f^2*(10*a + b*x) +
 16*c^3*f*(160*a*d + 70*b*d*x + 48*a*f*x^2 + 33*b*f*x^3) + 128*c^4*(15*d^2
+ 5*d*f*x^2 + 3*f^2*x^4) + 24*c^2*f*(16*a^2*f + 7*a*b*f*x + b^2*(10*d + f*x
^2))) - 15*b*(-384*c^4*d^2 - 192*a*c^3*d*f + 3*b^4*f^2 - 24*a*b^2*c*f^2 + 1
6*c^2*f*(b^2*d + 3*a^2*f))*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]]
 - 1920*c^(7/2)*d*RootSum[b^2*d - a^2*f - 4*b*Sqrt[c]*d*#1 + 4*c*d*#1^2 + 2
*a*f*#1^2 - f*#1^4 & , (2*b^2*c*d^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2
] - #1] - a*c^2*d^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + a*b^2*
d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*a^2*c*d*f*Log[-(Sqrt
[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - a^3*f^2*Log[-(Sqrt[c]*x) + Sqrt[a +
b*x + c*x^2] - #1] - 4*b*c^(3/2)*d^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^
2] - #1]*#1 - 4*a*b*Sqrt[c]*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] -
#1]*#1 + c^2*d^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 + b^2*
d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 + 2*a*c*d*f*Log[-(S
```

```
qrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 + a^2*f^2*Log[-(Sqrt[c]*x) + S
qrt[a + b*x + c*x^2] - #1]*#1^2)/(b*Sqrt[c]*d - 2*c*d*#1 - a*f*#1 + f*#1^3)
 & ])/(3840*c^(7/2)*f^3)
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 758, normalized size of antiderivative = 1.51

method	result
risch	$-\frac{(384c^4f^2x^4+528bf^2c^3x^3+768ac^3f^2x^2+24b^2c^2f^2x^2+640c^4dfx^2+168abc^2f^2x-30b^3cf^2x+1120bc^3dfx+384a^2c^2f^2-300ab^2cf \cdots}{1920c^3f^3}$
default	Expression too large to display

```
[In] int(x^3*(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/1920*(384*c^4*f^2*x^4+528*b*c^3*f^2*x^3+768*a*c^3*f^2*x^2+24*b^2*c^2*f^2
*x^2+640*c^4*d*f*x^2+168*a*b*c^2*f^2*x-30*b^3*c*f^2*x+1120*b*c^3*d*f*x+384*
a^2*c^2*f^2-300*a*b^2*c*f^2+2560*a*c^3*d*f+45*b^4*f^2+240*b^2*c^2*d*f+1920*
c^4*d^2)*(c*x^2+b*x+a)^(1/2)/c^3/f^3+1/256/f^3/c^3*(b*(48*a^2*c^2*f^2-24*a*
b^2*c*f^2-192*a*c^3*d*f+3*b^4*f^2+16*b^2*c^2*d*f-384*c^4*d^2)*ln((1/2*b+c*x
)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)+128*c^3*d*((d*f)^(1/2)*a^2*f^2+2*(d*
f)^(1/2)*a*c*d*f+(d*f)^(1/2)*b^2*d*f+(d*f)^(1/2)*c^2*d^2+2*a*b*d*f^2+2*b*c*
d^2*f)/(d*f)^(1/2)/f/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)
+f*a+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f*a
+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2
)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))+128*c^3*d*((d*f)^
(1/2)*a^2*f^2+2*(d*f)^(1/2)*a*c*d*f+(d*f)^(1/2)*b^2*d*f+(d*f)^(1/2)*c^2*d^2
-2*a*b*d*f^2-2*b*c*d^2*f)/(d*f)^(1/2)/f/(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2
)*ln((2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2
)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*
c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/(
x+(d*f)^(1/2)/f)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^3(a+bx+cx^2)^{3/2}}{d-fx^2}\,dx = \text{Timed out}$$

```
[In] integrate(x^3*(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(a+bx+cx^2)^{3/2}}{d-fx^2}\,dx = \text{Timed out}$$

```
[In] integrate(x**3*(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3(a+bx+cx^2)^{3/2}}{d-fx^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^3*(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see `
assume?
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(a+bx+cx^2)^{3/2}}{d-fx^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3*(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+bx+cx^2)^{3/2}}{d-fx^2}\,dx = \int \frac{x^3\,\left(c\,x^2+b\,x+a\right)^{3/2}}{d-f\,x^2}\,dx$$

```
[In] int((x^3*(a + b*x + c*x^2)^(3/2))/(d - f*x^2),x)
[Out] int((x^3*(a + b*x + c*x^2)^(3/2))/(d - f*x^2), x)
```

3.85 $\int \frac{x^2\left(a+bx+cx^2\right)^{3/2}}{d-fx^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 417

$$\begin{aligned}
\int \frac{x^2\left(a+bx+cx^2\right)^{3/2}}{d-fx^2}\,dx = & -\frac{\left(b\left(80c^2d-3b^2f+12acf\right)+2c\left(16c^2d-3b^2f+12acf\right)x\right)\sqrt{a+bx+cx^2}}{64c^2f^2} \\
& -\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8cf} \\
& -\frac{\left(128c^4d^2+192ac^3df+3b^4f^2-24ab^2cf^2+48c^2f\left(b^2d+a^2f\right)\right)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{128c^{5/2}f^3} \\
& +\frac{\sqrt{d}\left(cd-b\sqrt{d}\sqrt{f}+af\right)^{3/2}\operatorname{arctanh}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2f^3} \\
& +\frac{\sqrt{d}\left(cd+b\sqrt{d}\sqrt{f}+af\right)^{3/2}\operatorname{arctanh}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2f^3}
\end{aligned}$$

```
[Out] -1/8*(2*c*x+b)*(c*x^2+b*x+a)^(3/2)/c/f-1/128*(128*c^4*d^2+192*a*c^3*d*f+3*b
^4*f^2-24*a*b^2*c*f^2+48*c^2*f*(a^2*f+b^2*d))*arctanh(1/2*(2*c*x+b)/c^(1/2)
/(c*x^2+b*x+a)^(1/2))/c^(5/2)/f^3+1/2*arctanh(1/2*(b*d^(1/2)-2*a*f^(1/2)+x*
(2*c*d^(1/2)-b*f^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f-b*d^(1/2)*f^(1/2))^(1
/2))*d^(1/2)*(c*d+a*f-b*d^(1/2)*f^(1/2))^(3/2)/f^3+1/2*arctanh(1/2*(b*d^(1/
2)+2*a*f^(1/2)+x*(2*c*d^(1/2)+b*f^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f+b*d^
(1/2)*f^(1/2))^(1/2))*d^(1/2)*(c*d+a*f+b*d^(1/2)*f^(1/2))^(3/2)/f^3-1/64*(b
*(12*a*c*f-3*b^2*f+80*c^2*d)+2*c*(12*a*c*f-3*b^2*f+16*c^2*d)*x)*(c*x^2+b*x+
a)^(1/2)/c^2/f^2
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 417, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1085, 1084, 1092, 635, 212, 1047, 738}

$$\int \frac{x^2(a+bx+cx^2)^{3/2}}{d-fx^2}\,dx = -\frac{\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(48c^2f(a^2f+b^2d)-24ab^2cf^2+192ac^3df+3b^4f^2+128c^4d^2)}{128c^{5/2}f^3}$$
$$+\frac{\sqrt{d}\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}\operatorname{arctanh}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2f^3}$$
$$+\frac{\sqrt{d}\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}\operatorname{arctanh}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2f^3}$$
$$-\frac{\sqrt{a+bx+cx^2}(2cx(12acf-3b^2f+16c^2d)+b(12acf-3b^2f+80c^2d))}{64c^2f^2}$$
$$-\frac{(b+2cx)(a+bx+cx^2)^{3/2}}{8cf}$$

```
[In] Int[(x^2*(a + b*x + c*x^2)^(3/2))/(d - f*x^2),x]
```

```
[Out] -1/64*((b*(80*c^2*d - 3*b^2*f + 12*a*c*f) + 2*c*(16*c^2*d - 3*b^2*f + 12*a*
c*f)*x)*Sqrt[a + b*x + c*x^2])/(c^2*f^2) - ((b + 2*c*x)*(a + b*x + c*x^2)^(
3/2))/(8*c*f) - ((128*c^4*d^2 + 192*a*c^3*d*f + 3*b^4*f^2 - 24*a*b^2*c*f^2
+ 48*c^2*f*(b^2*d + a^2*f))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c
*x^2])])/(128*c^(5/2)*f^3) + (Sqrt[d]*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)
*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*
d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*f^3) + (Sqrt[d]*(c
*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c
*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*
x + c*x^2])])/(2*f^3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
```

```
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1047

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(-a)*c, 2]}, Dist[h/2 + c*(g/(2*q)
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - c*(g/(2*q)
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[(-a)*c]
```

Rule 1084

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(B*c*f*(2*p + 2*q + 3) +
 C*(b*f*p) + 2*c*C*f*(p + q + 1)*x)*(a + b*x + c*x^2)^p*((d + f*x^2)^(q + 1
)/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3))), x] - Dist[1/(2*c*f^2*(p + q + 1)*
(2*p + 2*q + 3)), Int[(a + b*x + c*x^2)^(p - 1)*(d + f*x^2)^q*Simp[p*(b*d)*
(C*((-b)*f)*(q + 1) - c*((-B)*f)*(2*p + 2*q + 3)) + (p + q + 1)*(b^2*C*d*f*
p + a*c*(C*(2*d*f) + f*(-2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)*(C*((-
b)*f)*(q + 1) - c*((-B)*f)*(2*p + 2*q + 3)) + (p + q + 1)*((-b)*c*(C*(-4*d*
f)*(2*p + q + 2) + f*(2*C*d + 2*A*f)*(2*p + 2*q + 3))))*x + (p*((-b)*f)*(C*
((-b)*f)*(q + 1) - c*((-B)*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2
- 4*a*c) - c^2*(C*(-4*d*f)*(2*p + q + 2) + f*(2*C*d + 2*A*f)*(2*p + 2*q + 3
))))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, A, B, C, q}, x] && NeQ[b^2 -
4*a*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*q + 3, 0] &&  !I
GtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1085

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (C_.)*(x_)^2)*((d_) +
(f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(C*(b*f*p) + 2*c*C*f*(p + q + 1)*x)*(
a + b*x + c*x^2)^p*((d + f*x^2)^(q + 1)/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3
))), x] - Dist[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + b*x + c*x^
2)^(p - 1)*(d + f*x^2)^q*Simp[p*(b*d)*(C*((-b)*f)*(q + 1)) + (p + q + 1)*(b
^2*C*d*f*p + a*c*(C*(2*d*f) + f*(-2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*
f)*(C*((-b)*f)*(q + 1)) + (p + q + 1)*((-b)*c*(C*(-4*d*f)*(2*p + q + 2) + f
*(2*C*d + 2*A*f)*(2*p + 2*q + 3))))*x + (p*((-b)*f)*(C*((-b)*f)*(q + 1)) +
(p + q + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(-4*d*f)*(2*p + q + 2) + f*(2*C
*d + 2*A*f)*(2*p + 2*q + 3))))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, A,
```

```
C, q}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2
*p + 2*q + 3, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1092

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
 (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + e*x + f
*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x
 + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f
, 0]
```

Rubi steps

$$\text{integral} = -\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8cf} - \frac{\int \frac{\sqrt{a+bx+cx^2}\left(-\frac{3}{4}\left(3b^2+4ac\right)df-12bcdfx-\frac{3}{4}f\left(16c^2d-3\left(b^2-4ac\right)f\right)x^2\right)}{d-fx^2}\,dx}{12cf^2}$$

$$= -\frac{(b(80c^2d-3b^2f+12acf)+2c(16c^2d-3b^2f+12acf)\,x)\sqrt{a+bx+cx^2}}{64c^2f^2} - \frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8cf} + \frac{\int \frac{-\frac{3}{16}df^2\left(3b^4f-8b^2c(10cd+3af)-16ac^2(4cd+5af)\right)+48bc^2df^2(cd+af)x+\frac{3}{16}f^2\left(128c^4d^2+192ac^3df+3b^4f^2-24ab^2cf^2+48c^2f\left(b\cdots\right)\right)}{\sqrt{a+bx+cx^2}(d-fx^2)}\cdots}{24c^2f^4}$$

$$= -\frac{(b(80c^2d-3b^2f+12acf)+2c(16c^2d-3b^2f+12acf)\,x)\sqrt{a+bx+cx^2}}{64c^2f^2} - \frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8cf} - \frac{\int \frac{\frac{3}{16}df^3\left(3b^4f-8b^2c(10cd+3af)-16ac^2(4cd+5af)\right)-\frac{3}{16}df^2\left(128c^4d^2+192ac^3df+3b^4f^2-24ab^2cf^2+48c^2f\left(b^2d+a^2f\right)\right)-48bc^2df^3\left(\cdots\right)}{\sqrt{a+bx+cx^2}(d-fx^2)}\cdots}{24c^2f^5} - \frac{\left(128c^4d^2+192ac^3df+3b^4f^2-24ab^2cf^2+48c^2f(b^2d+a^2f)\right)\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{128c^2f^3}$$

$$= -\frac{(b(80c^2d-3b^2f+12acf)+2c(16c^2d-3b^2f+12acf)x)\sqrt{a+bx+cx^2}}{64c^2f^2} - \frac{(b+2cx)(a+bx+cx^2)^{3/2}}{8cf} - \frac{\left(\sqrt{d}\left(cd-b\sqrt{d}\sqrt{f}+af\right)^2\right)\int\frac{1}{\left(-\sqrt{d}\sqrt{f}-fx\right)\sqrt{a+bx+cx^2}}\,dx}{2f^{5/2}} + \frac{\left(\sqrt{d}\left(cd+b\sqrt{d}\sqrt{f}+af\right)^2\right)\int\frac{1}{\left(\sqrt{d}\sqrt{f}-fx\right)\sqrt{a+bx+cx^2}}\,dx}{2f^{5/2}} - \frac{(128c^4d^2+192ac^3df+3b^4f^2-24ab^2cf^2+48c^2f(b^2d+a^2f))\,\text{Subst}\left(\int\frac{1}{4c-x^2}\,dx,x,\frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{64c^2f^3}$$

$$= -\frac{(b(80c^2d-3b^2f+12acf)+2c(16c^2d-3b^2f+12acf)x)\sqrt{a+bx+cx^2}}{64c^2f^2} - \frac{(b+2cx)(a+bx+cx^2)^{3/2}}{8cf} - \frac{(128c^4d^2+192ac^3df+3b^4f^2-24ab^2cf^2+48c^2f(b^2d+a^2f))\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{128c^{5/2}f^3} + \frac{\left(\sqrt{d}\left(cd-b\sqrt{d}\sqrt{f}+af\right)^2\right)\text{Subst}\left(\int\frac{1}{4cdf-4b\sqrt{d}f^{3/2}+4af^2-x^2}\,dx,x,\frac{b\sqrt{d}\sqrt{f}-2af-\left(-2c\sqrt{d}\sqrt{f}+bf\right)x}{\sqrt{a+bx+cx^2}}\right)}{f^{5/2}} - \frac{\left(\sqrt{d}\left(cd+b\sqrt{d}\sqrt{f}+af\right)^2\right)\text{Subst}\left(\int\frac{1}{4cdf+4b\sqrt{d}f^{3/2}+4af^2-x^2}\,dx,x,\frac{-b\sqrt{d}\sqrt{f}-2af-\left(2c\sqrt{d}\sqrt{f}+bf\right)x}{\sqrt{a+bx+cx^2}}\right)}{f^{5/2}}$$

$$= -\frac{(b(80c^2d-3b^2f+12acf)+2c(16c^2d-3b^2f+12acf)x)\sqrt{a+bx+cx^2}}{64c^2f^2} - \frac{(b+2cx)(a+bx+cx^2)^{3/2}}{8cf} - \frac{(128c^4d^2+192ac^3df+3b^4f^2-24ab^2cf^2+48c^2f(b^2d+a^2f))\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{128c^{5/2}f^3} + \frac{\sqrt{d}\left(cd-b\sqrt{d}\sqrt{f}+af\right)^{3/2}\tanh^{-1}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2f^3} + \frac{\sqrt{d}\left(cd+b\sqrt{d}\sqrt{f}+af\right)^{3/2}\tanh^{-1}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2f^3}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 1.77 (sec) , antiderivative size = 608, normalized size of antiderivative = 1.46

$$\int \frac{x^2(a+bx+cx^2)^{3/2}}{d-fx^2}\,dx = \frac{-2\sqrt{c}f\sqrt{a+x(b+cx)}(-3b^3f+2b^2cfx+8c^2x(4cd+5af+2cfx^2)+4bc(20c$$

```
[In] Integrate[(x^2*(a + b*x + c*x^2)^(3/2))/(d - f*x^2),x]
```

```
[Out] (-2*Sqrt[c]*f*Sqrt[a + x*(b + c*x)]*(-3*b^3*f + 2*b^2*c*f*x + 8*c^2*x*(4*c*
d + 5*a*f + 2*c*f*x^2) + 4*b*c*(20*c*d + 5*a*f + 6*c*f*x^2)) + (128*c^4*d^2
 + 192*a*c^3*d*f + 3*b^4*f^2 - 24*a*b^2*c*f^2 + 48*c^2*f*(b^2*d + a^2*f))*L
og[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]] - 64*c^(5/2)*d*RootSum[b^2*
d - a^2*f - 4*b*Sqrt[c]*d*#1 + 4*c*d*#1^2 + 2*a*f*#1^2 - f*#1^4 & , (b*c^2*
d^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + b^3*d*f*Log[-(Sqrt[c]*
x) + Sqrt[a + b*x + c*x^2] - #1] - a^2*b*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*
x + c*x^2] - #1] - 2*c^(5/2)*d^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] -
 #1]*#1 - 2*b^2*Sqrt[c]*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*
#1 - 4*a*c^(3/2)*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - 2*
a^2*Sqrt[c]*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + 2*b*c*d
*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 + 2*a*b*f^2*Log[-(Sq
rt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2)/(b*Sqrt[c]*d - 2*c*d*#1 - a*f*
#1 + f*#1^3) & ])/(128*c^(5/2)*f^3)
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 648, normalized size of antiderivative = 1.55

method	result
risch	$-\frac{(16f\,c^3x^3+24b\,c^2f\,x^2+40a\,c^2fx+2b^2cfx+32c^3dx+20abcf-3b^3f+80bd\,c^2)\sqrt{c\,x^2+bx+a}}{64c^2f^2}-\frac{(48a^2c^2f^2-24a\,b^2c\,f^2+192a\,c^3df+3b^4}{}$
default	Expression too large to display

```
[In] int(x^2*(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/64*(16*c^3*f*x^3+24*b*c^2*f*x^2+40*a*c^2*f*x+2*b^2*c*f*x+32*c^3*d*x+20*a
*b*c*f-3*b^3*f+80*b*c^2*d)*(c*x^2+b*x+a)^(1/2)/c^2/f^2-1/128/c^2/f^2*(1/f*(
48*a^2*c^2*f^2-24*a*b^2*c*f^2+192*a*c^3*d*f+3*b^4*f^2+48*b^2*c^2*d*f+128*c^
4*d^2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-64*c^2*d*(2*(d*f
```

```
)^(1/2)*a*b*f+2*(d*f)^(1/2)*b*c*d-a^2*f^2-2*a*c*d*f-b^2*d*f-c^2*d^2)/(d*f)^
(1/2)/f/(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+f*a+c*
d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+
c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2
)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1/2)/f))-64*c^2*d*(2*(d
*f)^(1/2)*a*b*f+2*(d*f)^(1/2)*b*c*d+a^2*f^2+2*a*c*d*f+b^2*d*f+c^2*d^2)/(d*f
)^(1/2)/f/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+f*a+c*d)/f
+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f*a+c*d)/f)^(1
/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*
f)^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^2(a+bx+cx^2)^{3/2}}{d-fx^2}\,dx = \text{Timed out}$$

```
[In] integrate(x^2*(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{x^2(a+bx+cx^2)^{3/2}}{d-fx^2}\,dx = -\int \frac{ax^2\sqrt{a+bx+cx^2}}{-d+fx^2}\,dx - \int \frac{bx^3\sqrt{a+bx+cx^2}}{-d+fx^2}\,dx - \int \frac{cx^4\sqrt{a+bx+cx^2}}{-d+fx^2}\,dx$$

```
[In] integrate(x**2*(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(a*x**2*sqrt(a + b*x + c*x**2)/(-d + f*x**2), x) - Integral(b*x**3
*sqrt(a + b*x + c*x**2)/(-d + f*x**2), x) - Integral(c*x**4*sqrt(a + b*x +
c*x**2)/(-d + f*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a+bx+cx^2)^{3/2}}{d-fx^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2*(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see `
assume?
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^2(a+bx+cx^2)^{3/2}}{d-fx^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^2*(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+bx+cx^2)^{3/2}}{d-fx^2}\,dx = \int \frac{x^2\,(c\,x^2+b\,x+a)^{3/2}}{d-f\,x^2}\,dx$$

```
[In] int((x^2*(a + b*x + c*x^2)^(3/2))/(d - f*x^2),x)
```

```
[Out] int((x^2*(a + b*x + c*x^2)^(3/2))/(d - f*x^2), x)
```

3.86 $\int \frac{x(a+bx+cx^2)^{3/2}}{d-fx^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 349

$$\int \frac{x(a+bx+cx^2)^{3/2}}{d-fx^2}\,dx = -\frac{(8c^2d+b^2f+8acf+2bcfx)\sqrt{a+bx+cx^2}}{8cf^2} - \frac{(a+bx+cx^2)^{3/2}}{3f} - \frac{b(24c^2d-b^2f+12acf)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}f^2} - \frac{\left(cd-b\sqrt{d}\sqrt{f}+af\right)^{3/2}\operatorname{arctanh}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2f^{5/2}} + \frac{\left(cd+b\sqrt{d}\sqrt{f}+af\right)^{3/2}\operatorname{arctanh}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2f^{5/2}}$$

```
[Out] -1/3*(c*x^2+b*x+a)^(3/2)/f-1/16*b*(12*a*c*f-b^2*f+24*c^2*d)*arctanh(1/2*(2*
c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(3/2)/f^2-1/2*arctanh(1/2*(b*d^(1/2)-
2*a*f^(1/2)+x*(2*c*d^(1/2)-b*f^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f-b*d^(1/
2)*f^(1/2))^(1/2))*(c*d+a*f-b*d^(1/2)*f^(1/2))^(3/2)/f^(5/2)+1/2*arctanh(1/
2*(b*d^(1/2)+2*a*f^(1/2)+x*(2*c*d^(1/2)+b*f^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*
d+a*f+b*d^(1/2)*f^(1/2))^(1/2))*(c*d+a*f+b*d^(1/2)*f^(1/2))^(3/2)/f^(5/2)-1
/8*(2*b*c*f*x+8*a*c*f+b^2*f+8*c^2*d)*(c*x^2+b*x+a)^(1/2)/c/f^2
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {1035, 1084, 1092, 635, 212, 1047, 738}

$$\int \frac{x(a+bx+cx^2)^{3/2}}{d-fx^2}\,dx = -\frac{b\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(12acf+b^2(-f)+24c^2d)}{16c^{3/2}f^2} - \frac{\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}\text{arctanh}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2f^{5/2}} + \frac{\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}\text{arctanh}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2f^{5/2}} - \frac{\sqrt{a+bx+cx^2}(8acf+b^2f+2bcfx+8c^2d)}{8cf^2} - \frac{(a+bx+cx^2)^{3/2}}{3f}$$

```
[In] Int[(x*(a + b*x + c*x^2)^(3/2))/(d - f*x^2),x]
```

```
[Out] -1/8*((8*c^2*d + b^2*f + 8*a*c*f + 2*b*c*f*x)*Sqrt[a + b*x + c*x^2])/(c*f^2
) - (a + b*x + c*x^2)^(3/2)/(3*f) - (b*(24*c^2*d - b^2*f + 12*a*c*f)*ArcTan
h[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(3/2)*f^2) - ((c*d
- b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sq
rt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x +
 c*x^2])])/(2*f^(5/2)) + ((c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*
Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d
]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*f^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1035

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[h*(a + b*x + c*x^2)^p*((d + f*x^2)^(q +
 1)/(2*f*(p + q + 1))), x] - Dist[1/(2*f*(p + q + 1)), Int[(a + b*x + c*x^2
)^(p - 1)*(d + f*x^2)^q*Simp[h*p*(b*d) + a*(-2*g*f)*(p + q + 1) + (2*h*p*(c
*d - a*f) + b*(-2*g*f)*(p + q + 1))*x + (h*p*((-b)*f) + c*(-2*g*f)*(p + q +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*a
*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]
```

Rule 1047

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(-a)*c, 2]}, Dist[h/2 + c*(g/(2*q)
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - c*(g/(2*q)
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[(-a)*c]
```

Rule 1084

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(B*c*f*(2*p + 2*q + 3) +
 C*(b*f*p) + 2*c*C*f*(p + q + 1)*x)*(a + b*x + c*x^2)^p*((d + f*x^2)^(q + 1
)/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3))), x] - Dist[1/(2*c*f^2*(p + q + 1)*
(2*p + 2*q + 3)), Int[(a + b*x + c*x^2)^(p - 1)*(d + f*x^2)^q*Simp[p*(b*d)*
(C*((-b)*f)*(q + 1) - c*((-B)*f)*(2*p + 2*q + 3)) + (p + q + 1)*(b^2*C*d*f*
p + a*c*(C*(2*d*f) + f*(-2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)*(C*((-
b)*f)*(q + 1) - c*((-B)*f)*(2*p + 2*q + 3)) + (p + q + 1)*((-b)*c*(C*(-4*d*
f)*(2*p + q + 2) + f*(2*C*d + 2*A*f)*(2*p + 2*q + 3))))*x + (p*((-b)*f)*(C*
((-b)*f)*(q + 1) - c*((-B)*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2
- 4*a*c) - c^2*(C*(-4*d*f)*(2*p + q + 2) + f*(2*C*d + 2*A*f)*(2*p + 2*q + 3
))))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, A, B, C, q}, x] && NeQ[b^2 -
4*a*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*q + 3, 0] &&  !I
GtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1092

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
 (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + e*x + f
*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x
 + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f
, 0]
```

Rubi steps

$$\text{integral} = -\frac{(a+bx+cx^2)^{3/2}}{3f} + \frac{\int \frac{\sqrt{a+bx+cx^2}\left(\frac{3bd}{2}+3(cd+af)x+\frac{3}{2}bfx^2\right)}{d-fx^2}\,dx}{3f}$$

$$= -\frac{\left(8c^2d+b^2f+8acf+2bcfx\right)\sqrt{a+bx+cx^2}}{8cf^2} - \frac{(a+bx+cx^2)^{3/2}}{3f} - \frac{\int \frac{-\frac{3}{8}bdf\left(8c^2d+b^2f+20acf\right)-6cf\left(b^2df+(cd+af)^2\right)x-\frac{3}{8}bf^2\left(24c^2d-b^2f+12acf\right)x^2}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx}{6cf^3}$$

$$= -\frac{\left(8c^2d+b^2f+8acf+2bcfx\right)\sqrt{a+bx+cx^2}}{8cf^2} - \frac{(a+bx+cx^2)^{3/2}}{3f} + \frac{\int \frac{\frac{3}{8}bdf^2\left(24c^2d-b^2f+12acf\right)+\frac{3}{8}bdf^2\left(8c^2d+b^2f+20acf\right)+6cf^2\left(b^2df+(cd+af)^2\right)x}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx}{6cf^4} - \frac{\left(b\left(24c^2d-b^2f+12acf\right)\right)\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{16cf^2}$$

$$= -\frac{\left(8c^2d+b^2f+8acf+2bcfx\right)\sqrt{a+bx+cx^2}}{8cf^2} - \frac{(a+bx+cx^2)^{3/2}}{3f} + \frac{\left(cd-b\sqrt{d}\sqrt{f}+af\right)^2\int \frac{1}{\left(-\sqrt{d}\sqrt{f}-fx\right)\sqrt{a+bx+cx^2}}\,dx}{2f^2} + \frac{\left(cd+b\sqrt{d}\sqrt{f}+af\right)^2\int \frac{1}{\left(\sqrt{d}\sqrt{f}-fx\right)\sqrt{a+bx+cx^2}}\,dx}{2f^2} - \frac{\left(b\left(24c^2d-b^2f+12acf\right)\right)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{8cf^2}$$

$$= -\frac{\left(8c^2d+b^2f+8acf+2bcfx\right)\sqrt{a+bx+cx^2}}{8cf^2} - \frac{(a+bx+cx^2)^{3/2}}{3f} - \frac{b\left(24c^2d-b^2f+12acf\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}f^2} - \frac{\left(cd-b\sqrt{d}\sqrt{f}+af\right)^2\text{Subst}\left(\int \frac{1}{4cdf-4b\sqrt{d}f^{3/2}+4af^2-x^2}\,dx, x, \frac{b\sqrt{d}\sqrt{f}-2af-\left(-2c\sqrt{d}\sqrt{f}+bf\right)x}{\sqrt{a+bx+cx^2}}\right)}{f^2} - \frac{\left(cd+b\sqrt{d}\sqrt{f}+af\right)^2\text{Subst}\left(\int \frac{1}{4cdf+4b\sqrt{d}f^{3/2}+4af^2-x^2}\,dx, x, \frac{-b\sqrt{d}\sqrt{f}-2af-\left(2c\sqrt{d}\sqrt{f}+bf\right)x}{\sqrt{a+bx+cx^2}}\right)}{f^2}$$

$$= -\frac{(8c^2d + b^2f + 8acf + 2bcfx)\sqrt{a + bx + cx^2}}{8cf^2} - \frac{(a + bx + cx^2)^{3/2}}{3f} - \frac{b(24c^2d - b^2f + 12acf)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}f^2} - \frac{\left(cd - b\sqrt{d}\sqrt{f} + af\right)^{3/2}\tanh^{-1}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2f^{5/2}} + \frac{\left(cd + b\sqrt{d}\sqrt{f} + af\right)^{3/2}\tanh^{-1}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2f^{5/2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.98 (sec) , antiderivative size = 619, normalized size of antiderivative = 1.77

$$\int \frac{x(a + bx + cx^2)^{3/2}}{d - fx^2}\,dx = \frac{-2\sqrt{c}\sqrt{a + x(b + cx)}(3b^2f + 2cf(16a + 7bx) + 8c^2(3d + fx^2)) + 3b(-24c^2d + b}{}$$

```
[In] Integrate[(x*(a + b*x + c*x^2)^(3/2))/(d - f*x^2),x]
```

```
[Out] (-2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(3*b^2*f + 2*c*f*(16*a + 7*b*x) + 8*c^2*(
3*d + f*x^2)) + 3*b*(-24*c^2*d + b^2*f - 12*a*c*f)*ArcTanh[(b + 2*c*x)/(2*S
qrt[c]*Sqrt[a + x*(b + c*x)])] - 24*c^(3/2)*RootSum[b^2*d - a^2*f - 4*b*Sqr
t[c]*d*#1 + 4*c*d*#1^2 + 2*a*f*#1^2 - f*#1^4 & , (2*b^2*c*d^2*Log[-(Sqrt[c]
*x) + Sqrt[a + b*x + c*x^2] - #1] - a*c^2*d^2*Log[-(Sqrt[c]*x) + Sqrt[a + b
*x + c*x^2] - #1] + a*b^2*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1
] - 2*a^2*c*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - a^3*f^2*Lo
g[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 4*b*c^(3/2)*d^2*Log[-(Sqrt[c
]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - 4*a*b*Sqrt[c]*d*f*Log[-(Sqrt[c]*x)
+ Sqrt[a + b*x + c*x^2] - #1]*#1 + c^2*d^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x
+ c*x^2] - #1]*#1^2 + b^2*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1
]*#1^2 + 2*a*c*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 + a^
2*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2)/(b*Sqrt[c]*d - 2
*c*d*#1 - a*f*#1 + f*#1^3) & ])/(48*c^(3/2)*f^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 601 vs. $2(275) = 550$.

Time = 0.73 (sec) , antiderivative size = 602, normalized size of antiderivative = 1.72

method	result
risch	$-\frac{(8fc^2x^2+14bcfx+32acf+3b^2f+24c^2d)\sqrt{cx^2+bx+a}}{24cf^2} - \frac{\frac{b(12acf-b^2f+24c^2d)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{cx^2+bx+a}\right)}{\sqrt{c}} - \frac{8c\left(\sqrt{df}\,a^2f^2+2\sqrt{df}\,acdf+\cdots\right.}{\cdots}}{\cdots}$
default	Expression too large to display

```
[In] int(x*(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*(8*c^2*f*x^2+14*b*c*f*x+32*a*c*f+3*b^2*f+24*c^2*d)*(c*x^2+b*x+a)^(1/2
)/c/f^2-1/16/f^2/c*(b*(12*a*c*f-b^2*f+24*c^2*d)*ln((1/2*b+c*x)/c^(1/2)+(c*x
^2+b*x+a)^(1/2))/c^(1/2)-8*c*((d*f)^(1/2)*a^2*f^2+2*(d*f)^(1/2)*a*c*d*f+(d*
f)^(1/2)*b^2*d*f+(d*f)^(1/2)*c^2*d^2+2*a*b*d*f^2+2*b*c*d^2*f)/(d*f)^(1/2)/f
/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+f*a+c*d)/f+(2*c*(d*
f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*((x-(
d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+
f*a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))-8*c*((d*f)^(1/2)*a^2*f^2+2*(d*f)^(1/2
)*a*c*d*f+(d*f)^(1/2)*b^2*d*f+(d*f)^(1/2)*c^2*d^2-2*a*b*d*f^2-2*b*c*d^2*f)/
(d*f)^(1/2)/f/(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+
f*a+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2
)+f*a+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f
)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1/2)/f)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x(a+bx+cx^2)^{3/2}}{d-fx^2}\,dx = \text{Timed out}$$

```
[In] integrate(x*(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{x(a+bx+cx^2)^{3/2}}{d-fx^2}\,dx = -\int \frac{ax\sqrt{a+bx+cx^2}}{-d+fx^2}\,dx - \int \frac{bx^2\sqrt{a+bx+cx^2}}{-d+fx^2}\,dx - \int \frac{cx^3\sqrt{a+bx+cx^2}}{-d+fx^2}\,dx$$

```
[In] integrate(x*(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(a*x*sqrt(a + b*x + c*x**2)/(-d + f*x**2), x) - Integral(b*x**2*sq
rt(a + b*x + c*x**2)/(-d + f*x**2), x) - Integral(c*x**3*sqrt(a + b*x + c*x
**2)/(-d + f*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x(a+bx+cx^2)^{3/2}}{d-fx^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x*(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see `
assume?
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x(a+bx+cx^2)^{3/2}}{d-fx^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x*(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+bx+cx^2)^{3/2}}{d-fx^2}\,dx = \int \frac{x\,(c\,x^2+b\,x+a)^{3/2}}{d-f\,x^2}\,dx$$

```
[In] int((x*(a + b*x + c*x^2)^(3/2))/(d - f*x^2),x)
[Out] int((x*(a + b*x + c*x^2)^(3/2))/(d - f*x^2), x)
```

3.87 $\int \frac{(a+bx+cx^2)^{3/2}}{d-fx^2} dx$

Optimal result

Integrand size = 25, antiderivative size = 315

$$\int \frac{(a+bx+cx^2)^{3/2}}{d-fx^2} dx = -\frac{(5b+2cx)\sqrt{a+bx+cx^2}}{4f} - \frac{(8c^2d+3b^2f+12acf)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}f^2} + \frac{\left(cd-b\sqrt{d}\sqrt{f}+af\right)^{3/2}\operatorname{arctanh}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{d}f^2} + \frac{\left(cd+b\sqrt{d}\sqrt{f}+af\right)^{3/2}\operatorname{arctanh}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{d}f^2}$$

```
[Out] -1/8*(12*a*c*f+3*b^2*f+8*c^2*d)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)
^(1/2))/f^2/c^(1/2)+1/2*arctanh(1/2*(b*d^(1/2)-2*a*f^(1/2)+x*(2*c*d^(1/2)-b
*f^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2))*(c*d+a*f-
b*d^(1/2)*f^(1/2))^(3/2)/f^2/d^(1/2)+1/2*arctanh(1/2*(b*d^(1/2)+2*a*f^(1/2)
+x*(2*c*d^(1/2)+b*f^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f+b*d^(1/2)*f^(1/2))
^(1/2))*(c*d+a*f+b*d^(1/2)*f^(1/2))^(3/2)/f^2/d^(1/2)-1/4*(2*c*x+5*b)*(c*x^
2+b*x+a)^(1/2)/f
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {992, 1092, 635, 212, 1047, 738}

$$\int \frac{(a+bx+cx^2)^{3/2}}{d-fx^2}\,dx = -\frac{\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(12acf+3b^2f+8c^2d)}{8\sqrt{c}f^2} + \frac{\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}\operatorname{arctanh}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\sqrt{d}f^2} + \frac{\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}\operatorname{arctanh}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\sqrt{d}f^2} - \frac{(5b+2cx)\sqrt{a+bx+cx^2}}{4f}$$

```
[In] Int[(a + b*x + c*x^2)^(3/2)/(d - f*x^2),x]
```

```
[Out] -1/4*((5*b + 2*c*x)*Sqrt[a + b*x + c*x^2])/f - ((8*c^2*d + 3*b^2*f + 12*a*c
*f)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*Sqrt[c]*f^2)
 + ((c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f]
+ (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[
a + b*x + c*x^2])])/(2*Sqrt[d]*f^2) + ((c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2
)*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c
*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[d]*f^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
```

```
d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 992

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (f_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[(b*(3*p + 2*q) + 2*c*(p + q)*x)*(a + b*x + c*x^2)^(p - 1)*
((d + f*x^2)^(q + 1)/(2*f*(p + q)*(2*p + 2*q + 1))), x] - Dist[1/(2*f*(p +
q)*(2*p + 2*q + 1)), Int[(a + b*x + c*x^2)^(p - 2)*(d + f*x^2)^q*Simp[b^2*d
*(p - 1)*(2*p + q) - (p + q)*(b^2*d*(1 - p) - 2*a*(c*d - a*f*(2*p + 2*q + 1
))) - (2*b*(c*d - a*f)*(1 - p)*(2*p + q) - 2*(p + q)*b*(2*c*d*(2*p + q) - (
c*d + a*f)*(2*p + 2*q + 1)))*x + (b^2*f*p*(1 - p) + 2*c*(p + q)*(c*d*(2*p -
 1) - a*f*(4*p + 2*q - 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, q}, x]
 && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 1] && NeQ[p + q, 0] && NeQ[2*p + 2*q + 1,
 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1047

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(-a)*c, 2]}, Dist[h/2 + c*(g/(2*q)
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - c*(g/(2*q)
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[(-a)*c]
```

Rule 1092

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
 (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + e*x + f
*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x
 + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f
, 0]
```

Rubi steps

$$\text{integral} = -\frac{(5b+2cx)\sqrt{a+bx+cx^2}}{4f} + \frac{\int \frac{\frac{1}{4}(5b^2d+4a(cd+2af))+4b(cd+af)x+\frac{1}{4}(8c^2d+3b^2f+12acf)x^2}{\sqrt{a+bx+cx^2}(d-fx^2)}\,dx}{2f}$$

$$= -\frac{(5b+2cx)\sqrt{a+bx+cx^2}}{4f} - \frac{\int \frac{-\frac{1}{4}d(8c^2d+3b^2f+12acf)-\frac{1}{4}f(5b^2d+4a(cd+2af))-4bf(cd+af)x}{\sqrt{a+bx+cx^2}(d-fx^2)}\,dx}{2f^2} - \frac{(8c^2d+3b^2f+12acf)\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{8f^2}$$

$$= -\frac{(5b+2cx)\sqrt{a+bx+cx^2}}{4f} - \frac{\left(cd-b\sqrt{d}\sqrt{f}+af\right)^2 \int \frac{1}{\left(-\sqrt{d}\sqrt{f}-fx\right)\sqrt{a+bx+cx^2}}\,dx}{2\sqrt{d}f^{3/2}}$$
$$+ \frac{\left(cd+b\sqrt{d}\sqrt{f}+af\right)^2 \int \frac{1}{\left(\sqrt{d}\sqrt{f}-fx\right)\sqrt{a+bx+cx^2}}\,dx}{2\sqrt{d}f^{3/2}}$$
$$- \frac{(8c^2d+3b^2f+12acf)\,\mathrm{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{4f^2}$$

$$= -\frac{(5b+2cx)\sqrt{a+bx+cx^2}}{4f} - \frac{(8c^2d+3b^2f+12acf)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}f^2}$$
$$+ \frac{\left(cd-b\sqrt{d}\sqrt{f}+af\right)^2 \mathrm{Subst}\left(\int \frac{1}{4cdf-4b\sqrt{d}f^{3/2}+4af^2-x^2}\,dx, x, \frac{b\sqrt{d}\sqrt{f}-2af-\left(-2c\sqrt{d}\sqrt{f}+bf\right)x}{\sqrt{a+bx+cx^2}}\right)}{\sqrt{d}f^{3/2}}$$
$$- \frac{\left(cd+b\sqrt{d}\sqrt{f}+af\right)^2 \mathrm{Subst}\left(\int \frac{1}{4cdf+4b\sqrt{d}f^{3/2}+4af^2-x^2}\,dx, x, \frac{-b\sqrt{d}\sqrt{f}-2af-\left(2c\sqrt{d}\sqrt{f}+bf\right)x}{\sqrt{a+bx+cx^2}}\right)}{\sqrt{d}f^{3/2}}$$

$$= -\frac{(5b+2cx)\sqrt{a+bx+cx^2}}{4f} - \frac{(8c^2d+3b^2f+12acf)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}f^2}$$
$$+ \frac{\left(cd-b\sqrt{d}\sqrt{f}+af\right)^{3/2}\tanh^{-1}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{d}f^2}$$
$$+ \frac{\left(cd+b\sqrt{d}\sqrt{f}+af\right)^{3/2}\tanh^{-1}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{d}f^2}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 1.00 (sec) , antiderivative size = 729, normalized size of antiderivative = 2.31

$$\int \frac{(a+bx+cx^2)^{3/2}}{d-fx^2}\,dx = \frac{-\sqrt{c}f(5b+2cx)\sqrt{a+x(b+cx)} + (8c^2d+3b^2f+12acf)\,\mathrm{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a}-\sqrt{a+x(b+cx}}\right.}{}$$

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)/(d - f*x^2),x]
```

```
[Out] (-(Sqrt[c]*f*(5*b + 2*c*x)*Sqrt[a + x*(b + c*x)]) + (8*c^2*d + 3*b^2*f + 12
*a*c*f)*ArcTanh[(Sqrt[c]*x)/(Sqrt[a] - Sqrt[a + x*(b + c*x)])] - 2*Sqrt[c]*
RootSum[c^2*d - b^2*f + 4*Sqrt[a]*b*f*#1 - 2*c*d*#1^2 - 4*a*f*#1^2 + d*#1^4
```

```
 & , (-(c^3*d^2*Log[x]) + b^2*c*d*f*Log[x] - 2*a*c^2*d*f*Log[x] + 2*a*b^2*f
^2*Log[x] - a^2*c*f^2*Log[x] + c^3*d^2*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2]
 - x*#1] - b^2*c*d*f*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] + 2*a*c^2
*d*f*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] - 2*a*b^2*f^2*Log[-Sqrt[a
] + Sqrt[a + b*x + c*x^2] - x*#1] + a^2*c*f^2*Log[-Sqrt[a] + Sqrt[a + b*x +
 c*x^2] - x*#1] - 4*Sqrt[a]*b*c*d*f*Log[x]*#1 - 4*a^(3/2)*b*f^2*Log[x]*#1 +
 4*Sqrt[a]*b*c*d*f*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1]*#1 + 4*a^(3
/2)*b*f^2*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1]*#1 + c^2*d^2*Log[x]*
#1^2 + b^2*d*f*Log[x]*#1^2 + 2*a*c*d*f*Log[x]*#1^2 + a^2*f^2*Log[x]*#1^2 -
c^2*d^2*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1]*#1^2 - b^2*d*f*Log[-Sq
rt[a] + Sqrt[a + b*x + c*x^2] - x*#1]*#1^2 - 2*a*c*d*f*Log[-Sqrt[a] + Sqrt[
a + b*x + c*x^2] - x*#1]*#1^2 - a^2*f^2*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2
] - x*#1]*#1^2)/(-(Sqrt[a]*b*f) + c*d*#1 + 2*a*f*#1 - d*#1^3) & ])/(4*Sqrt[
c]*f^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 544 vs. $2(245) = 490$.

Time = 0.84 (sec) , antiderivative size = 545, normalized size of antiderivative = 1.73

method	result
risch	$-\frac{(2cx+5b)\sqrt{c\,x^2+bx+a}}{4f}-\frac{\frac{\left(12acf+3b^2f+8c^2d\right)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{f\sqrt{c}}-\frac{\left(8\sqrt{df}\,abf+8\sqrt{df}\,bcd-4a^2f^2-8acdf-4b^2df-4c^2d^2\right)\ln\left(\frac{-2b\sqrt{d}}{}\right.}{}}{}$
default	Expression too large to display

```
[In] int((c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(2*c*x+5*b)*(c*x^2+b*x+a)^(1/2)/f-1/8/f*(1/f*(12*a*c*f+3*b^2*f+8*c^2*d
)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-(8*(d*f)^(1/2)*a*b*f+
8*(d*f)^(1/2)*b*c*d-4*a^2*f^2-8*a*c*d*f-4*b^2*d*f-4*c^2*d^2)/(d*f)^(1/2)/f/
(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1/f*(
-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1
/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f
*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1/2)/f))-(8*(d*f)^(1/2)*a*b*f+8
*(d*f)^(1/2)*b*c*d+4*a^2*f^2+8*a*c*d*f+4*b^2*d*f+4*c^2*d^2)/(d*f)^(1/2)/f/(
(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+f*a+c*d)/f+(2*c*(d*f)
^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*((x-(d*
f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*
a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}}{d-fx^2}\,dx = \text{Timed out}$$

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{(a+bx+cx^2)^{3/2}}{d-fx^2}\,dx = -\int \frac{a\sqrt{a+bx+cx^2}}{-d+fx^2}\,dx - \int \frac{bx\sqrt{a+bx+cx^2}}{-d+fx^2}\,dx - \int \frac{cx^2\sqrt{a+bx+cx^2}}{-d+fx^2}\,dx$$

```
[In] integrate((c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(a*sqrt(a + b*x + c*x**2)/(-d + f*x**2), x) - Integral(b*x*sqrt(a
+ b*x + c*x**2)/(-d + f*x**2), x) - Integral(c*x**2*sqrt(a + b*x + c*x**2)/
(-d + f*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx+cx^2)^{3/2}}{d-fx^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see `
assume?
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+bx+cx^2)^{3/2}}{d-fx^2}\,dx = \text{Exception raised: TypeError}$$

[In] integrate((c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="giac")

[Out] Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}}{d-fx^2}\,dx = \int \frac{(c\,x^2+b\,x+a)^{3/2}}{d-f\,x^2}\,dx$$

[In] int((a + b*x + c*x^2)^(3/2)/(d - f*x^2),x)

[Out] int((a + b*x + c*x^2)^(3/2)/(d - f*x^2), x)

3.88 $\int \frac{(a+bx+cx^2)^{3/2}}{x(d-fx^2)} dx$

Optimal result

Integrand size = 28, antiderivative size = 469

$$\begin{aligned}\int \frac{(a+bx+cx^2)^{3/2}}{x(d-fx^2)}dx &= \frac{(b^2+8ac+2bcx)\sqrt{a+bx+cx^2}}{8cd} \\ &- \frac{(8c^2d+b^2f+8acf+2bcfx)\sqrt{a+bx+cx^2}}{8cdf} \\ &- \frac{a^{3/2}\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d} - \frac{b(b^2-12ac)\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}d} \\ &- \frac{b(24c^2d-b^2f+12acf)\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}df} \\ &- \frac{\left(cd-b\sqrt{d}\sqrt{f}+af\right)^{3/2}\text{arctanh}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2df^{3/2}} \\ &+ \frac{\left(cd+b\sqrt{d}\sqrt{f}+af\right)^{3/2}\text{arctanh}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2df^{3/2}}\end{aligned}$$

```
[Out] -a^(3/2)*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/d-1/16*b*(-12*a
*c+b^2)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(3/2)/d-1/16*b
*(12*a*c*f-b^2*f+24*c^2*d)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2
))/c^(3/2)/d/f-1/2*arctanh(1/2*(b*d^(1/2)-2*a*f^(1/2)+x*(2*c*d^(1/2)-b*f^(1
/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2))*(c*d+a*f-b*d^(
1/2)*f^(1/2))^(3/2)/d/f^(3/2)+1/2*arctanh(1/2*(b*d^(1/2)+2*a*f^(1/2)+x*(2*c
*d^(1/2)+b*f^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f+b*d^(1/2)*f^(1/2))^(1/2))
*(c*d+a*f+b*d^(1/2)*f^(1/2))^(3/2)/d/f^(3/2)+1/8*(2*b*c*x+8*a*c+b^2)*(c*x^2
```

```
+b*x+a)^(1/2)/c/d-1/8*(2*b*c*f*x+8*a*c*f+b^2*f+8*c^2*d)*(c*x^2+b*x+a)^(1/2)
/c/d/f
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 469, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.393, Rules used = {6857, 748, 828, 857, 635, 212, 738, 1035, 1084, 1092, 1047}

$$\int \frac{(a+bx+cx^2)^{3/2}}{x(d-fx^2)}\,dx = -\frac{a^{3/2}\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d} - \frac{b(b^2-12ac)\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}d} - \frac{b\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(12acf+b^2(-f)+24c^2d)}{16c^{3/2}df} - \frac{\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}\text{arctanh}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2df^{3/2}} + \frac{\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}\text{arctanh}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2df^{3/2}} - \frac{\sqrt{a+bx+cx^2}(8acf+b^2f+2bcfx+8c^2d)}{8cdf} + \frac{(8ac+b^2+2bcx)\sqrt{a+bx+cx^2}}{8cd}$$

```
[In] Int[(a + b*x + c*x^2)^(3/2)/(x*(d - f*x^2)),x]
```

```
[Out] ((b^2 + 8*a*c + 2*b*c*x)*Sqrt[a + b*x + c*x^2])/(8*c*d) - ((8*c^2*d + b^2*f
 + 8*a*c*f + 2*b*c*f*x)*Sqrt[a + b*x + c*x^2])/(8*c*d*f) - (a^(3/2)*ArcTanh
[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/d - (b*(b^2 - 12*a*c)*ArcT
anh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(3/2)*d) - (b*(24
*c^2*d - b^2*f + 12*a*c*f)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*
x^2])])/(16*c^(3/2)*d*f) - ((c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(
b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt
[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d*f^(3/2)) + ((c*d + b*Sqrt[
d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b
*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])]
)/(2*d*f^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 748

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
```

```
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1035

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[h*(a + b*x + c*x^2)^p*((d + f*x^2)^(q +
 1)/(2*f*(p + q + 1))), x] - Dist[1/(2*f*(p + q + 1)), Int[(a + b*x + c*x^2
)^(p - 1)*(d + f*x^2)^q*Simp[h*p*(b*d) + a*(-2*g*f)*(p + q + 1) + (2*h*p*(c
*d - a*f) + b*(-2*g*f)*(p + q + 1))*x + (h*p*((-b)*f) + c*(-2*g*f)*(p + q +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*a
*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]
```

Rule 1047

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(-a)*c, 2]}, Dist[h/2 + c*(g/(2*q)
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - c*(g/(2*q)
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[(-a)*c]
```

Rule 1084

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(B*c*f*(2*p + 2*q + 3) +
 C*(b*f*p) + 2*c*C*f*(p + q + 1)*x)*(a + b*x + c*x^2)^p*((d + f*x^2)^(q + 1
)/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3))), x] - Dist[1/(2*c*f^2*(p + q + 1)*
(2*p + 2*q + 3)), Int[(a + b*x + c*x^2)^(p - 1)*(d + f*x^2)^q*Simp[p*(b*d)*
(C*((-b)*f)*(q + 1) - c*((-B)*f)*(2*p + 2*q + 3)) + (p + q + 1)*(b^2*C*d*f*
p + a*c*(C*(2*d*f) + f*(-2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)*(C*((-
b)*f)*(q + 1) - c*((-B)*f)*(2*p + 2*q + 3)) + (p + q + 1)*((-b)*c*(C*(-4*d*
f)*(2*p + q + 2) + f*(2*C*d + 2*A*f)*(2*p + 2*q + 3))))*x + (p*((-b)*f)*(C*
((-b)*f)*(q + 1) - c*((-B)*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2
- 4*a*c) - c^2*(C*(-4*d*f)*(2*p + q + 2) + f*(2*C*d + 2*A*f)*(2*p + 2*q + 3
))))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, A, B, C, q}, x] && NeQ[b^2 -
4*a*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*q + 3, 0] &&  !I
GtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1092

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
 (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + e*x + f
*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x
 + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f
, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{(a+bx+cx^2)^{3/2}}{dx} - \frac{fx(a+bx+cx^2)^{3/2}}{d(-d+fx^2)} \right) dx$$

$$= \frac{\int \frac{(a+bx+cx^2)^{3/2}}{x}\,dx}{d} - \frac{f\int \frac{x(a+bx+cx^2)^{3/2}}{-d+fx^2}\,dx}{d}$$

$$= \frac{\int \frac{\sqrt{a+bx+cx^2}\left(-\frac{3bd}{2}-3(cd+af)x-\frac{3}{2}bfx^2\right)}{-d+fx^2}\,dx}{3d} - \frac{\int \frac{(-2a-bx)\sqrt{a+bx+cx^2}}{x}\,dx}{2d}$$

$$= \frac{(b^2+8ac+2bcx)\sqrt{a+bx+cx^2}}{8cd} - \frac{(8c^2d+b^2f+8acf+2bcfx)\sqrt{a+bx+cx^2}}{8cdf} + \frac{\int \frac{8a^2c-\frac{1}{2}b(b^2-12ac)x}{x\sqrt{a+bx+cx^2}}\,dx}{8cd} - \frac{\int \frac{\frac{3}{8}bdf(8c^2d+b^2f+20acf)+6cf(b^2df+(cd+af)^2)x+\frac{3}{8}bf^2(24c^2d-b^2f+12acf)x^2}{\sqrt{a+bx+cx^2}(-d+fx^2)}\,dx}{6cdf^2}$$

$$= \frac{(b^2+8ac+2bcx)\sqrt{a+bx+cx^2}}{8cd} - \frac{(8c^2d+b^2f+8acf+2bcfx)\sqrt{a+bx+cx^2}}{8cdf} + \frac{a^2\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx}{d} - \frac{(b(b^2-12ac))\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{16cd} - \frac{\int \frac{\frac{3}{8}bdf^2(24c^2d-b^2f+12acf)+\frac{3}{8}bdf^2(8c^2d+b^2f+20acf)+6cf^2(b^2df+(cd+af)^2)x}{\sqrt{a+bx+cx^2}(-d+fx^2)}\,dx}{6cdf^3} - \frac{(b(24c^2d-b^2f+12acf))\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{16cdf}$$

$$=\frac{(b^2+8ac+2bcx)\sqrt{a+bx+cx^2}}{8cd}-\frac{(8c^2d+b^2f+8acf+2bcfx)\sqrt{a+bx+cx^2}}{8cdf}$$
$$-\frac{(2a^2)\operatorname{Subst}\left(\int\frac{1}{4a-x^2}\,dx,x,\frac{2a+bx}{\sqrt{a+bx+cx^2}}\right)}{d}$$
$$-\frac{(b(b^2-12ac))\operatorname{Subst}\left(\int\frac{1}{4c-x^2}\,dx,x,\frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{8cd}$$
$$-\frac{\left(cd-b\sqrt{d}\sqrt{f}+af\right)^2\int\frac{1}{\left(\sqrt{d}\sqrt{f}+fx\right)\sqrt{a+bx+cx^2}}\,dx}{2df}$$
$$-\frac{\left(cd+b\sqrt{d}\sqrt{f}+af\right)^2\int\frac{1}{\left(-\sqrt{d}\sqrt{f}+fx\right)\sqrt{a+bx+cx^2}}\,dx}{2df}$$
$$-\frac{(b(24c^2d-b^2f+12acf))\operatorname{Subst}\left(\int\frac{1}{4c-x^2}\,dx,x,\frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{8cdf}$$

$$=\frac{(b^2+8ac+2bcx)\sqrt{a+bx+cx^2}}{8cd}-\frac{(8c^2d+b^2f+8acf+2bcfx)\sqrt{a+bx+cx^2}}{8cdf}$$
$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d}-\frac{b(b^2-12ac)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}d}$$
$$-\frac{b(24c^2d-b^2f+12acf)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}df}$$
$$+\frac{\left(cd-b\sqrt{d}\sqrt{f}+af\right)^2\operatorname{Subst}\left(\int\frac{1}{4cdf-4b\sqrt{d}f^{3/2}+4af^2-x^2}\,dx,x,\frac{-b\sqrt{d}\sqrt{f}+2af-\left(2c\sqrt{d}\sqrt{f}-bf\right)x}{\sqrt{a+bx+cx^2}}\right)}{df}$$
$$+\frac{\left(cd+b\sqrt{d}\sqrt{f}+af\right)^2\operatorname{Subst}\left(\int\frac{1}{4cdf+4b\sqrt{d}f^{3/2}+4af^2-x^2}\,dx,x,\frac{b\sqrt{d}\sqrt{f}+2af-\left(-2c\sqrt{d}\sqrt{f}-bf\right)x}{\sqrt{a+bx+cx^2}}\right)}{df}$$

$$= \frac{(b^2+8ac+2bcx)\sqrt{a+bx+cx^2}}{8cd} - \frac{(8c^2d+b^2f+8acf+2bcfx)\sqrt{a+bx+cx^2}}{8cdf} - \frac{a^{3/2}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d} - \frac{b(b^2-12ac)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}d} - \frac{b(24c^2d-b^2f+12acf)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}df} - \frac{\left(cd-b\sqrt{d}\sqrt{f}+af\right)^{3/2}\tanh^{-1}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2df^{3/2}} + \frac{\left(cd+b\sqrt{d}\sqrt{f}+af\right)^{3/2}\tanh^{-1}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2df^{3/2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.75 (sec) , antiderivative size = 601, normalized size of antiderivative = 1.28

$$\int \frac{(a+bx+cx^2)^{3/2}}{x(d-fx^2)}\,dx =$$

$$-\frac{2cd\sqrt{a+x(b+cx)} - 4a^{3/2}f\operatorname{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right) - 3b\sqrt{c}d\log\left(f\left(b+2cx-2\sqrt{c}\sqrt{a+x(b+cx)}\right)\right)}{}$$

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)/(x*(d - f*x^2)),x]
```

```
[Out] -1/2*(2*c*d*Sqrt[a + x*(b + c*x)] - 4*a^(3/2)*f*ArcTanh[(Sqrt[c]*x - Sqrt[a
 + x*(b + c*x)])/Sqrt[a]] - 3*b*Sqrt[c]*d*Log[f*(b + 2*c*x - 2*Sqrt[c]*Sqrt
[a + x*(b + c*x)])] + RootSum[b^2*d - a^2*f - 4*b*Sqrt[c]*d*#1 + 4*c*d*#1^2
 + 2*a*f*#1^2 - f*#1^4 & , (2*b^2*c*d^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c
*x^2] - #1] - a*c^2*d^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + a*
b^2*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*a^2*c*d*f*Log[-(
Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - a^3*f^2*Log[-(Sqrt[c]*x) + Sqrt[
a + b*x + c*x^2] - #1] - 4*b*c^(3/2)*d^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x +
c*x^2] - #1]*#1 - 4*a*b*Sqrt[c]*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2
] - #1]*#1 + c^2*d^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 +
b^2*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 + 2*a*c*d*f*Log
[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 + a^2*f^2*Log[-(Sqrt[c]*x)
 + Sqrt[a + b*x + c*x^2] - #1]*#1^2)/(b*Sqrt[c]*d - 2*c*d*#1 - a*f*#1 + f*#
1^3) & ])/(d*f)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1638 vs. 2(377) = 754.

Time = 0.89 (sec) , antiderivative size = 1639, normalized size of antiderivative = 3.49

method	result	size
default	Expression too large to display	1639

```
[In] int((c*x^2+b*x+a)^(3/2)/x/(-f*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/3*(c*x^2+b*x+a)^(3/2)+1/2*b*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/8
*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))+a*((c*x^2
+b*x+a)^(1/2)+1/2*b*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-a^(
1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)))-1/2/d*(1/3*((x-(d*f)^(
1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*
d)/f)^(3/2)+1/2*(2*c*(d*f)^(1/2)+b*f)/f*(1/4*(2*c*(x-(d*f)^(1/2)/f)+(2*c*(d
*f)^(1/2)+b*f)/f)/c*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)
^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)+1/8*(4*c*(b*(d*f)^(1/2)+f*a+c*d)
/f-(2*c*(d*f)^(1/2)+b*f)^2/f^2)/c^(3/2)*ln((1/2*(2*c*(d*f)^(1/2)+b*f)/f+c*(
x-(d*f)^(1/2)/f))/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x
-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)))+(b*(d*f)^(1/2)+f*a+c*d)/
f*(((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f
)^(1/2)+f*a+c*d)/f)^(1/2)+1/2*(2*c*(d*f)^(1/2)+b*f)/f*ln((1/2*(2*c*(d*f)^(1
/2)+b*f)/f+c*(x-(d*f)^(1/2)/f))/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(
1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/c^(1/2)-(b*
(d*f)^(1/2)+f*a+c*d)/f/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/
2)+f*a+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f
*a+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1
/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))))-1/2/d*(1/3*((
x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(
d*f)^(1/2)+f*a+c*d))^(3/2)+1/2/f*(-2*c*(d*f)^(1/2)+b*f)*(1/4*(2*c*(x+(d*f)^
(1/2)/f)+1/f*(-2*c*(d*f)^(1/2)+b*f))/c*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*
f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)+1/8*(4*
c/f*(-b*(d*f)^(1/2)+f*a+c*d)-1/f^2*(-2*c*(d*f)^(1/2)+b*f)^2)/c^(3/2)*ln((1/
2/f*(-2*c*(d*f)^(1/2)+b*f)+c*(x+(d*f)^(1/2)/f))/c^(1/2)+((x+(d*f)^(1/2)/f)^
2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*
d))^(1/2)))+1/f*(-b*(d*f)^(1/2)+f*a+c*d)*(((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*
(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)+1/2/
f*(-2*c*(d*f)^(1/2)+b*f)*ln((1/2/f*(-2*c*(d*f)^(1/2)+b*f)+c*(x+(d*f)^(1/2)/
f))/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2
)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/c^(1/2)-1/f*(-b*(d*f)^(1/2)+f*a+c
*d)/(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1
/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d)
)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)
+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1/2)/f))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}}{x(d-fx^2)}\,dx = \text{Timed out}$$

```
[In] integrate((c*x^2+b*x+a)^(3/2)/x/(-f*x^2+d),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{(a+bx+cx^2)^{3/2}}{x(d-fx^2)}\,dx = -\int \frac{a\sqrt{a+bx+cx^2}}{-dx+fx^3}\,dx - \int \frac{bx\sqrt{a+bx+cx^2}}{-dx+fx^3}\,dx - \int \frac{cx^2\sqrt{a+bx+cx^2}}{-dx+fx^3}\,dx$$

```
[In] integrate((c*x**2+b*x+a)**(3/2)/x/(-f*x**2+d),x)
[Out] -Integral(a*sqrt(a + b*x + c*x**2)/(-d*x + f*x**3), x) - Integral(b*x*sqrt(
a + b*x + c*x**2)/(-d*x + f*x**3), x) - Integral(c*x**2*sqrt(a + b*x + c*x*
*2)/(-d*x + f*x**3), x)
```

Maxima [F]

$$\int \frac{(a+bx+cx^2)^{3/2}}{x(d-fx^2)}\,dx = \int -\frac{(cx^2+bx+a)^{\frac{3}{2}}}{(fx^2-d)x}\,dx$$

```
[In] integrate((c*x^2+b*x+a)^(3/2)/x/(-f*x^2+d),x, algorithm="maxima")
[Out] -integrate((c*x^2 + b*x + a)^(3/2)/((f*x^2 - d)*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+bx+cx^2)^{3/2}}{x(d-fx^2)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c*x^2+b*x+a)^(3/2)/x/(-f*x^2+d),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}}{x\,(d-fx^2)}\,dx=\int \frac{{\left(c\,x^2+b\,x+a\right)}^{3/2}}{x\,\left(d-f\,x^2\right)}\,dx$$

```
[In] int((a + b*x + c*x^2)^(3/2)/(x*(d - f*x^2)),x)
[Out] int((a + b*x + c*x^2)^(3/2)/(x*(d - f*x^2)), x)
```

3.89 $\int \frac{(a+bx+cx^2)^{3/2}}{x^2(d-fx^2)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 463

$$\begin{aligned}\int \frac{(a+bx+cx^2)^{3/2}}{x^2(d-fx^2)}\,dx &= \frac{3(3b+2cx)\sqrt{a+bx+cx^2}}{4d} \\ &- \frac{(5b+2cx)\sqrt{a+bx+cx^2}}{4d} - \frac{(a+bx+cx^2)^{3/2}}{dx} \\ &- \frac{3\sqrt{a}b\,\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2d} + \frac{3(b^2+4ac)\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}d} \\ &- \frac{(8c^2d+3b^2f+12acf)\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}df} \\ &+ \frac{\left(cd-b\sqrt{d}\sqrt{f}+af\right)^{3/2}\text{arctanh}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2d^{3/2}f} \\ &+ \frac{\left(cd+b\sqrt{d}\sqrt{f}+af\right)^{3/2}\text{arctanh}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2d^{3/2}f}\end{aligned}$$

```
[Out] -(c*x^2+b*x+a)^(3/2)/d/x-3/2*b*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^
(1/2))*a^(1/2)/d+3/8*(4*a*c+b^2)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a
)^(1/2))/d/c^(1/2)-1/8*(12*a*c*f+3*b^2*f+8*c^2*d)*arctanh(1/2*(2*c*x+b)/c^(
1/2)/(c*x^2+b*x+a)^(1/2))/d/f/c^(1/2)+1/2*arctanh(1/2*(b*d^(1/2)-2*a*f^(1/2
)+x*(2*c*d^(1/2)-b*f^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f-b*d^(1/2)*f^(1/2)
)^(1/2))*(c*d+a*f-b*d^(1/2)*f^(1/2))^(3/2)/d^(3/2)/f+1/2*arctanh(1/2*(b*d^(
1/2)+2*a*f^(1/2)+x*(2*c*d^(1/2)+b*f^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f+b*
d^(1/2)*f^(1/2))^(1/2))*(c*d+a*f+b*d^(1/2)*f^(1/2))^(3/2)/d^(3/2)/f+3/4*(2*
c*x+3*b)*(c*x^2+b*x+a)^(1/2)/d-1/4*(2*c*x+5*b)*(c*x^2+b*x+a)^(1/2)/d
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 463, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {6857, 746, 828, 857, 635, 212, 738, 992, 1092, 1047}

$$\int \frac{(a+bx+cx^2)^{3/2}}{x^2(d-fx^2)}\,dx = -\frac{\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(12acf+3b^2f+8c^2d)}{8\sqrt{c}df}$$
$$+\frac{3(4ac+b^2)\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}d}$$
$$+\frac{\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}\text{arctanh}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2d^{3/2}f}$$
$$+\frac{\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}\text{arctanh}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2d^{3/2}f}$$
$$-\frac{3\sqrt{a}b\,\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2d}-\frac{(a+bx+cx^2)^{3/2}}{dx}$$
$$+\frac{3(3b+2cx)\sqrt{a+bx+cx^2}}{4d}-\frac{(5b+2cx)\sqrt{a+bx+cx^2}}{4d}$$

```
[In] Int[(a + b*x + c*x^2)^(3/2)/(x^2*(d - f*x^2)),x]
```

```
[Out] (3*(3*b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(4*d) - ((5*b + 2*c*x)*Sqrt[a + b*x
 + c*x^2])/(4*d) - (a + b*x + c*x^2)^(3/2)/(d*x) - (3*Sqrt[a]*b*ArcTanh[(2*
a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(2*d) + (3*(b^2 + 4*a*c)*ArcTa
nh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*Sqrt[c]*d) - ((8*c^2*
d + 3*b^2*f + 12*a*c*f)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2
])])/(8*Sqrt[c]*d*f) + ((c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*Sq
rt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*
Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d^(3/2)*f) + ((c*d + b*Sqrt[d]*S
qrt[f] + a*f)^(3/2)*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqr
t[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2
*d^(3/2)*f)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 746

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 992

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (f_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[(b*(3*p + 2*q) + 2*c*(p + q)*x)*(a + b*x + c*x^2)^(p - 1)*
((d + f*x^2)^(q + 1)/(2*f*(p + q)*(2*p + 2*q + 1))), x] - Dist[1/(2*f*(p +
q)*(2*p + 2*q + 1)), Int[(a + b*x + c*x^2)^(p - 2)*(d + f*x^2)^q*Simp[b^2*d
*(p - 1)*(2*p + q) - (p + q)*(b^2*d*(1 - p) - 2*a*(c*d - a*f*(2*p + 2*q + 1
))) - (2*b*(c*d - a*f)*(1 - p)*(2*p + q) - 2*(p + q)*b*(2*c*d*(2*p + q) - (
c*d + a*f)*(2*p + 2*q + 1)))*x + (b^2*f*p*(1 - p) + 2*c*(p + q)*(c*d*(2*p -
 1) - a*f*(4*p + 2*q - 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, q}, x]
 && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 1] && NeQ[p + q, 0] && NeQ[2*p + 2*q + 1,
 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1047

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(-a)*c, 2]}, Dist[h/2 + c*(g/(2*q)
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - c*(g/(2*q)
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[(-a)*c]
```

Rule 1092

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
 (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + e*x + f
*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x
 + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f
, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{(a+bx+cx^2)^{3/2}}{dx^2} + \frac{f(a+bx+cx^2)^{3/2}}{d\,(d-fx^2)}\right) dx$$

$$= \frac{\int \frac{(a+bx+cx^2)^{3/2}}{x^2}\,dx}{d} + \frac{f\int \frac{(a+bx+cx^2)^{3/2}}{d-fx^2}\,dx}{d}$$

$$= -\frac{(5b+2cx)\sqrt{a+bx+cx^2}}{4d} - \frac{(a+bx+cx^2)^{3/2}}{dx} + \frac{\int \frac{\frac{1}{4}(5b^2d+4a(cd+2af))+4b(cd+af)x+\frac{1}{4}(8c^2d+3b^2f+12acf)x^2}{\sqrt{a+bx+cx^2}(d-fx^2)}\,dx}{2d} + \frac{3\int \frac{(b+2cx)\sqrt{a+bx+cx^2}}{x}\,dx}{2d}$$

$$=\frac{3(3b+2cx)\sqrt{a+bx+cx^2}}{4d}-\frac{(5b+2cx)\sqrt{a+bx+cx^2}}{4d}-\frac{(a+bx+cx^2)^{3/2}}{dx}-\frac{3\int\frac{-4abc-c(b^2+4ac)x}{x\sqrt{a+bx+cx^2}}\,dx}{8cd}-\frac{\int\frac{-\frac{1}{4}d(8c^2d+3b^2f+12acf)-\frac{1}{4}f(5b^2d+4a(cd+2af))-4bf(cd+af)x}{\sqrt{a+bx+cx^2}(d-fx^2)}\,dx}{2df}-\frac{(8c^2d+3b^2f+12acf)\int\frac{1}{\sqrt{a+bx+cx^2}}\,dx}{8df}$$

$$=\frac{3(3b+2cx)\sqrt{a+bx+cx^2}}{4d}-\frac{(5b+2cx)\sqrt{a+bx+cx^2}}{4d}-\frac{(a+bx+cx^2)^{3/2}}{dx}+\frac{(3ab)\int\frac{1}{x\sqrt{a+bx+cx^2}}\,dx}{2d}+\frac{(3(b^2+4ac))\int\frac{1}{\sqrt{a+bx+cx^2}}\,dx}{8d}-\frac{\left(cd-b\sqrt{d}\sqrt{f}+af\right)^2\int\frac{1}{\left(-\sqrt{d}\sqrt{f}-fx\right)\sqrt{a+bx+cx^2}}\,dx}{2d^{3/2}\sqrt{f}}+\frac{\left(cd+b\sqrt{d}\sqrt{f}+af\right)^2\int\frac{1}{\left(\sqrt{d}\sqrt{f}-fx\right)\sqrt{a+bx+cx^2}}\,dx}{2d^{3/2}\sqrt{f}}-\frac{(8c^2d+3b^2f+12acf)\,\text{Subst}\left(\int\frac{1}{4c-x^2}\,dx,x,\frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{4df}$$

$$=\frac{3(3b+2cx)\sqrt{a+bx+cx^2}}{4d}-\frac{(5b+2cx)\sqrt{a+bx+cx^2}}{4d}-\frac{(a+bx+cx^2)^{3/2}}{dx}-\frac{(8c^2d+3b^2f+12acf)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}df}-\frac{(3ab)\text{Subst}\left(\int\frac{1}{4a-x^2}\,dx,x,\frac{2a+bx}{\sqrt{a+bx+cx^2}}\right)}{d}+\frac{(3(b^2+4ac))\,\text{Subst}\left(\int\frac{1}{4c-x^2}\,dx,x,\frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{4d}+\frac{\left(cd-b\sqrt{d}\sqrt{f}+af\right)^2\text{Subst}\left(\int\frac{1}{4cdf-4b\sqrt{d}f^{3/2}+4af^2-x^2}\,dx,x,\frac{b\sqrt{d}\sqrt{f}-2af-\left(-2c\sqrt{d}\sqrt{f}+bf\right)x}{\sqrt{a+bx+cx^2}}\right)}{d^{3/2}\sqrt{f}}-\frac{\left(cd+b\sqrt{d}\sqrt{f}+af\right)^2\text{Subst}\left(\int\frac{1}{4cdf+4b\sqrt{d}f^{3/2}+4af^2-x^2}\,dx,x,\frac{-b\sqrt{d}\sqrt{f}-2af-\left(2c\sqrt{d}\sqrt{f}+bf\right)x}{\sqrt{a+bx+cx^2}}\right)}{d^{3/2}\sqrt{f}}$$

$$= \frac{3(3b+2cx)\sqrt{a+bx+cx^2}}{4d} - \frac{(5b+2cx)\sqrt{a+bx+cx^2}}{4d} - \frac{(a+bx+cx^2)^{3/2}}{dx} - \frac{3\sqrt{a}b\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2d} + \frac{3(b^2+4ac)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}d} - \frac{(8c^2d+3b^2f+12acf)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}df} + \frac{\left(cd-b\sqrt{d}\sqrt{f}+af\right)^{3/2}\tanh^{-1}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2d^{3/2}f} + \frac{\left(cd+b\sqrt{d}\sqrt{f}+af\right)^{3/2}\tanh^{-1}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2d^{3/2}f}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.65 (sec) , antiderivative size = 541, normalized size of antiderivative = 1.17

$$\int \frac{(a+bx+cx^2)^{3/2}}{x^2(d-fx^2)}\,dx = \frac{-2af\sqrt{a+x(b+cx)} + 6\sqrt{a}bfx\operatorname{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right) + 2c^{3/2}dx\log\left(f\left(b+2c\right.\right.}{}$$

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)/(x^2*(d - f*x^2)),x]
```

```
[Out] (-2*a*f*Sqrt[a + x*(b + c*x)] + 6*Sqrt[a]*b*f*x*ArcTanh[(Sqrt[c]*x - Sqrt[a
 + x*(b + c*x)])/Sqrt[a]] + 2*c^(3/2)*d*x*Log[f*(b + 2*c*x - 2*Sqrt[c]*Sqrt
[a + x*(b + c*x)])] - x*RootSum[b^2*d - a^2*f - 4*b*Sqrt[c]*d*#1 + 4*c*d*#1
^2 + 2*a*f*#1^2 - f*#1^4 & , (b*c^2*d^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c
*x^2] - #1] + b^3*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - a^2*
b*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*c^(5/2)*d^2*Log[-(
Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - 2*b^2*Sqrt[c]*d*f*Log[-(Sqrt[
c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - 4*a*c^(3/2)*d*f*Log[-(Sqrt[c]*x) +
 Sqrt[a + b*x + c*x^2] - #1]*#1 - 2*a^2*Sqrt[c]*f^2*Log[-(Sqrt[c]*x) + Sqrt
[a + b*x + c*x^2] - #1]*#1 + 2*b*c*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*
x^2] - #1]*#1^2 + 2*a*b*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*
#1^2)/(b*Sqrt[c]*d - 2*c*d*#1 - a*f*#1 + f*#1^3) & ])/(2*d*f*x)
```

Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 585, normalized size of antiderivative = 1.26

method	result
risch	$-\frac{a\sqrt{c x^2+bx+a}}{dx}-\frac{\frac{2c^{\frac{3}{2}} d \ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c x^2+bx+a}\right)}{f}+3b\sqrt{a}\,\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c x^2+bx+a}}{x}\right)-\frac{\left(-\sqrt{df}\,a^2 f^2-2\sqrt{df}\,acdf-\sqrt{df}\,b^2 df-\sqrt{df}\,c^2 d^2 \cdots\right.}{\cdots}}{\cdots}$
default	Expression too large to display

```
[In] int((c*x^2+b*x+a)^(3/2)/x^2/(-f*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] -a/d*(c*x^2+b*x+a)^(1/2)/x-1/2/d*(2*c^(3/2)*d/f*ln((1/2*b+c*x)/c^(1/2)+(c*x
^2+b*x+a)^(1/2))+3*b*a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)-
(-(d*f)^(1/2)*a^2*f^2-2*(d*f)^(1/2)*a*c*d*f-(d*f)^(1/2)*b^2*d*f-(d*f)^(1/2)
*c^2*d^2+2*a*b*d*f^2+2*b*c*d^2*f)/d/f^2/(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2
)*ln((2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2
)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*
c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/(
x+(d*f)^(1/2)/f))-1/f^2*((d*f)^(1/2)*a^2*f^2+2*(d*f)^(1/2)*a*c*d*f+(d*f)^(1
/2)*b^2*d*f+(d*f)^(1/2)*c^2*d^2+2*a*b*d*f^2+2*b*c*d^2*f)/d/((b*(d*f)^(1/2)+
f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+f*a+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(
x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c
+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)
)/(x-(d*f)^(1/2)/f)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}}{x^2(d-fx^2)}\,dx = \text{Timed out}$$

```
[In] integrate((c*x^2+b*x+a)^(3/2)/x^2/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{(a+bx+cx^2)^{3/2}}{x^2(d-fx^2)}\,dx = -\int \frac{a\sqrt{a+bx+cx^2}}{-dx^2+fx^4}\,dx - \int \frac{bx\sqrt{a+bx+cx^2}}{-dx^2+fx^4}\,dx - \int \frac{cx^2\sqrt{a+bx+cx^2}}{-dx^2+fx^4}\,dx$$

```
[In] integrate((c*x**2+b*x+a)**(3/2)/x**2/(-f*x**2+d),x)
[Out] -Integral(a*sqrt(a + b*x + c*x**2)/(-d*x**2 + f*x**4), x) - Integral(b*x*sq
rt(a + b*x + c*x**2)/(-d*x**2 + f*x**4), x) - Integral(c*x**2*sqrt(a + b*x
+ c*x**2)/(-d*x**2 + f*x**4), x)
```

Maxima [F]

$$\int \frac{(a+bx+cx^2)^{3/2}}{x^2(d-fx^2)}\,dx = \int -\frac{(cx^2+bx+a)^{\frac{3}{2}}}{(fx^2-d)x^2}\,dx$$

```
[In] integrate((c*x^2+b*x+a)^(3/2)/x^2/(-f*x^2+d),x, algorithm="maxima")
[Out] -integrate((c*x^2 + b*x + a)^(3/2)/((f*x^2 - d)*x^2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+bx+cx^2)^{3/2}}{x^2(d-fx^2)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c*x^2+b*x+a)^(3/2)/x^2/(-f*x^2+d),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}}{x^2(d-fx^2)}\,dx=\int \frac{(c\,x^2+b\,x+a)^{3/2}}{x^2\,\left(d-f\,x^2\right)}\,dx$$

```
[In] int((a + b*x + c*x^2)^(3/2)/(x^2*(d - f*x^2)),x)
[Out] int((a + b*x + c*x^2)^(3/2)/(x^2*(d - f*x^2)), x)
```

3.90 $\int \frac{(a+bx+cx^2)^{3/2}}{x^3(d-fx^2)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 614

$$\begin{aligned}\int \frac{(a+bx+cx^2)^{3/2}}{x^3(d-fx^2)}\,dx =& -\frac{3(b-2cx)\sqrt{a+bx+cx^2}}{4dx} \\ &+ \frac{f(b^2+8ac+2bcx)\sqrt{a+bx+cx^2}}{8cd^2} \\ &- \frac{(8c^2d+b^2f+8acf+2bcfx)\sqrt{a+bx+cx^2}}{8cd^2} - \frac{(a+bx+cx^2)^{3/2}}{2dx^2} \\ &- \frac{3(b^2+4ac)\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{a}d} - \frac{a^{3/2}f\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d^2} \\ &+ \frac{3b\sqrt{c}\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2d} - \frac{b(b^2-12ac)\,f\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}d^2} \\ &- \frac{b(24c^2d-b^2f+12acf)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}d^2} \\ &- \frac{\left(cd-b\sqrt{d}\sqrt{f}+af\right)^{3/2}\operatorname{arctanh}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2d^2\sqrt{f}} \\ &+ \frac{\left(cd+b\sqrt{d}\sqrt{f}+af\right)^{3/2}\operatorname{arctanh}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2d^2\sqrt{f}}\end{aligned}$$

```
[Out] -1/2*(c*x^2+b*x+a)^(3/2)/d/x^2-a^(3/2)*f*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x
^2+b*x+a)^(1/2))/d^2-1/16*b*(-12*a*c+b^2)*f*arctanh(1/2*(2*c*x+b)/c^(1/2)/(
c*x^2+b*x+a)^(1/2))/c^(3/2)/d^2-1/16*b*(12*a*c*f-b^2*f+24*c^2*d)*arctanh(1/
2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(3/2)/d^2-3/8*(4*a*c+b^2)*arctan
```

```
h(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/d/a^(1/2)+3/2*b*arctanh(1/2*(2
*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))*c^(1/2)/d-1/2*arctanh(1/2*(b*d^(1/2)-2
*a*f^(1/2)+x*(2*c*d^(1/2)-b*f^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f-b*d^(1/2
)*f^(1/2))^(1/2))*(c*d+a*f-b*d^(1/2)*f^(1/2))^(3/2)/d^2/f^(1/2)+1/2*arctanh
(1/2*(b*d^(1/2)+2*a*f^(1/2)+x*(2*c*d^(1/2)+b*f^(1/2)))/(c*x^2+b*x+a)^(1/2)/
(c*d+a*f+b*d^(1/2)*f^(1/2))^(1/2))*(c*d+a*f+b*d^(1/2)*f^(1/2))^(3/2)/d^2/f^
(1/2)-3/4*(-2*c*x+b)*(c*x^2+b*x+a)^(1/2)/d/x+1/8*f*(2*b*c*x+8*a*c+b^2)*(c*x
^2+b*x+a)^(1/2)/c/d^2-1/8*(2*b*c*f*x+8*a*c*f+b^2*f+8*c^2*d)*(c*x^2+b*x+a)^(
1/2)/c/d^2
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 614, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.464$, Rules used = {6857, 746, 826, 857, 635, 212, 738, 748, 828, 1035, 1084, 1092, 1047}

$$\begin{aligned}\int \frac{(a+bx+cx^2)^{3/2}}{x^3(d-fx^2)}\,dx = &-\frac{a^{3/2}f\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d^2}\\ &-\frac{bf(b^2-12ac)\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}d^2}\\ &-\frac{b\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(12acf+b^2(-f)+24c^2d)}{16c^{3/2}d^2}\\ &-\frac{3(4ac+b^2)\,\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{a}d}\\ &-\frac{\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}\text{arctanh}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2d^2\sqrt{f}}\\ &+\frac{\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}\text{arctanh}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2d^2\sqrt{f}}\\ &+\frac{3b\sqrt{c}\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2d}-\frac{\sqrt{a+bx+cx^2}(8acf+b^2f+2bcfx+8c^2d)}{8cd^2}\\ &+\frac{f(8ac+b^2+2bcx)\sqrt{a+bx+cx^2}}{8cd^2}\\ &-\frac{(a+bx+cx^2)^{3/2}}{2dx^2}-\frac{3(b-2cx)\sqrt{a+bx+cx^2}}{4dx}\end{aligned}$$

```
[In] Int[(a + b*x + c*x^2)^(3/2)/(x^3*(d - f*x^2)),x]
```

```
[Out] (-3*(b - 2*c*x)*Sqrt[a + b*x + c*x^2])/(4*d*x) + (f*(b^2 + 8*a*c + 2*b*c*x)
*Sqrt[a + b*x + c*x^2])/(8*c*d^2) - ((8*c^2*d + b^2*f + 8*a*c*f + 2*b*c*f*x
)*Sqrt[a + b*x + c*x^2])/(8*c*d^2) - (a + b*x + c*x^2)^(3/2)/(2*d*x^2) - (3
*(b^2 + 4*a*c)*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(8*S
qrt[a]*d) - (a^(3/2)*f*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2]
)])/d^2 + (3*b*Sqrt[c]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2]
)])/(2*d) - (b*(b^2 - 12*a*c)*f*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x
 + c*x^2])])/(16*c^(3/2)*d^2) - (b*(24*c^2*d - b^2*f + 12*a*c*f)*ArcTanh[(b
 + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(3/2)*d^2) - ((c*d - b*
Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d
] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x
^2])])/(2*d^2*Sqrt[f]) + ((c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*
Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d
]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d^2*Sqrt[f])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 746

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 748

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
```

```
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 826

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1035

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[h*(a + b*x + c*x^2)^p*((d + f*x^2)^(q +
```

```
 1)/(2*f*(p + q + 1))), x] - Dist[1/(2*f*(p + q + 1)), Int[(a + b*x + c*x^2
)^(p - 1)*(d + f*x^2)^q*Simp[h*p*(b*d) + a*(-2*g*f)*(p + q + 1) + (2*h*p*(c
*d - a*f) + b*(-2*g*f)*(p + q + 1))*x + (h*p*((-b)*f) + c*(-2*g*f)*(p + q +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*a
*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]
```

Rule 1047

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(-a)*c, 2]}, Dist[h/2 + c*(g/(2*q)
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - c*(g/(2*q)
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[(-a)*c]
```

Rule 1084

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(B*c*f*(2*p + 2*q + 3) +
 C*(b*f*p) + 2*c*C*f*(p + q + 1)*x)*(a + b*x + c*x^2)^p*((d + f*x^2)^(q + 1
)/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3))), x] - Dist[1/(2*c*f^2*(p + q + 1)*
(2*p + 2*q + 3)), Int[(a + b*x + c*x^2)^(p - 1)*(d + f*x^2)^q*Simp[p*(b*d)*
(C*((-b)*f)*(q + 1) - c*((-B)*f)*(2*p + 2*q + 3)) + (p + q + 1)*(b^2*C*d*f*
p + a*c*(C*(2*d*f) + f*(-2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)*(C*((-
b)*f)*(q + 1) - c*((-B)*f)*(2*p + 2*q + 3)) + (p + q + 1)*((-b)*c*(C*(-4*d*
f)*(2*p + q + 2) + f*(2*C*d + 2*A*f)*(2*p + 2*q + 3))))*x + (p*((-b)*f)*(C*
((-b)*f)*(q + 1) - c*((-B)*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2
- 4*a*c) - c^2*(C*(-4*d*f)*(2*p + q + 2) + f*(2*C*d + 2*A*f)*(2*p + 2*q + 3
))))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, A, B, C, q}, x] && NeQ[b^2 -
4*a*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*q + 3, 0] &&  !I
GtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1092

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
 (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + e*x + f
*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x
 + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f
, 0]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{(a+bx+cx^2)^{3/2}}{dx^3} + \frac{f(a+bx+cx^2)^{3/2}}{d^2x} + \frac{f^2x(a+bx+cx^2)^{3/2}}{d^2(d-fx^2)} \right) dx$$

$$= \frac{\int \frac{(a+bx+cx^2)^{3/2}}{x^3} dx}{d} + \frac{f \int \frac{(a+bx+cx^2)^{3/2}}{x} dx}{d^2} + \frac{f^2 \int \frac{x(a+bx+cx^2)^{3/2}}{d-fx^2} dx}{d^2}$$

$$= -\frac{(a+bx+cx^2)^{3/2}}{2dx^2} + \frac{3 \int \frac{(b+2cx)\sqrt{a+bx+cx^2}}{x^2} dx}{4d} + \frac{f \int \frac{\sqrt{a+bx+cx^2}\left(\frac{3bd}{2}+3(cd+af)x+\frac{3}{2}bfx^2\right)}{d-fx^2} dx}{3d^2} - \frac{f \int \frac{(-2a-bx)\sqrt{a+bx+cx^2}}{x} dx}{2d^2}$$

$$= -\frac{3(b-2cx)\sqrt{a+bx+cx^2}}{4dx} + \frac{f(b^2+8ac+2bcx)\sqrt{a+bx+cx^2}}{8cd^2} - \frac{(8c^2d+b^2f+8acf+2bcfx)\sqrt{a+bx+cx^2}}{8cd^2} - \frac{(a+bx+cx^2)^{3/2}}{2dx^2} - \frac{3 \int \frac{-b^2-4ac-4bcx}{x\sqrt{a+bx+cx^2}} dx}{8d} - \frac{\int \frac{-\frac{3}{8}bdf(8c^2d+b^2f+20acf)-6cf(b^2df+(cd+af)^2)x-\frac{3}{8}bf^2(24c^2d-b^2f+12acf)x^2}{\sqrt{a+bx+cx^2}(d-fx^2)} dx}{6cd^2f} + \frac{f \int \frac{8a^2c-\frac{1}{2}b(b^2-12ac)x}{x\sqrt{a+bx+cx^2}} dx}{8cd^2}$$

$$= -\frac{3(b-2cx)\sqrt{a+bx+cx^2}}{4dx} + \frac{f(b^2+8ac+2bcx)\sqrt{a+bx+cx^2}}{8cd^2} - \frac{(8c^2d+b^2f+8acf+2bcfx)\sqrt{a+bx+cx^2}}{8cd^2} - \frac{(a+bx+cx^2)^{3/2}}{2dx^2} + \frac{(3bc)\int \frac{1}{\sqrt{a+bx+cx^2}} dx}{2d} + \frac{(3(b^2+4ac))\int \frac{1}{x\sqrt{a+bx+cx^2}} dx}{8d} + \frac{\int \frac{\frac{3}{8}bdf^2(24c^2d-b^2f+12acf)+\frac{3}{8}bdf^2(8c^2d+b^2f+20acf)+6cf^2(b^2df+(cd+af)^2)x}{\sqrt{a+bx+cx^2}(d-fx^2)} dx}{6cd^2f^2} + \frac{(a^2f)\int \frac{1}{x\sqrt{a+bx+cx^2}} dx}{d^2} - \frac{(b(b^2-12ac)f)\int \frac{1}{\sqrt{a+bx+cx^2}} dx}{16cd^2} - \frac{(b(24c^2d-b^2f+12acf))\int \frac{1}{\sqrt{a+bx+cx^2}} dx}{16cd^2}$$

$$
\begin{aligned}
&= -\frac{3(b-2cx)\sqrt{a+bx+cx^2}}{4dx} + \frac{f(b^2+8ac+2bcx)\sqrt{a+bx+cx^2}}{8cd^2} \\
&\quad - \frac{(8c^2d+b^2f+8acf+2bcfx)\sqrt{a+bx+cx^2}}{8cd^2} \\
&\quad - \frac{(a+bx+cx^2)^{3/2}}{2dx^2} + \frac{(3bc)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{d} \\
&\quad - \frac{(3(b^2+4ac))\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx}{\sqrt{a+bx+cx^2}}\right)}{4d} \\
&\quad - \frac{(2a^2f)\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx}{\sqrt{a+bx+cx^2}}\right)}{d^2} \\
&\quad - \frac{(b(b^2-12ac)\,f)\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{8cd^2} \\
&\quad + \frac{\left(cd-b\sqrt{d}\sqrt{f}+af\right)^2 \int \frac{1}{\left(-\sqrt{d}\sqrt{f}-fx\right)\sqrt{a+bx+cx^2}}\,dx}{2d^2} \\
&\quad + \frac{\left(cd+b\sqrt{d}\sqrt{f}+af\right)^2 \int \frac{1}{\left(\sqrt{d}\sqrt{f}-fx\right)\sqrt{a+bx+cx^2}}\,dx}{2d^2} \\
&\quad - \frac{(b(24c^2d-b^2f+12acf))\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{8cd^2} \\
&= -\frac{3(b-2cx)\sqrt{a+bx+cx^2}}{4dx} + \frac{f(b^2+8ac+2bcx)\sqrt{a+bx+cx^2}}{8cd^2} \\
&\quad - \frac{(8c^2d+b^2f+8acf+2bcfx)\sqrt{a+bx+cx^2}}{8cd^2} - \frac{(a+bx+cx^2)^{3/2}}{2dx^2} \\
&\quad - \frac{3(b^2+4ac)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{a}d} - \frac{a^{3/2}f\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d^2} \\
&\quad + \frac{3b\sqrt{c}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2d} - \frac{b(b^2-12ac)\,f\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}d^2} \\
&\quad - \frac{b(24c^2d-b^2f+12acf)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}d^2} \\
&\quad - \frac{\left(cd-b\sqrt{d}\sqrt{f}+af\right)^2 \text{Subst}\left(\int \frac{1}{4cdf-4b\sqrt{d}f^{3/2}+4af^2-x^2}\,dx, x, \frac{b\sqrt{d}\sqrt{f}-2af-\left(-2c\sqrt{d}\sqrt{f}+bf\right)x}{\sqrt{a+bx+cx^2}}\right)}{d^2} \\
&\quad - \frac{\left(cd+b\sqrt{d}\sqrt{f}+af\right)^2 \text{Subst}\left(\int \frac{1}{4cdf+4b\sqrt{d}f^{3/2}+4af^2-x^2}\,dx, x, \frac{-b\sqrt{d}\sqrt{f}-2af-\left(2c\sqrt{d}\sqrt{f}+bf\right)x}{\sqrt{a+bx+cx^2}}\right)}{d^2}
\end{aligned}
$$

$$= -\frac{3(b-2cx)\sqrt{a+bx+cx^2}}{4dx} + \frac{f(b^2+8ac+2bcx)\sqrt{a+bx+cx^2}}{8cd^2} - \frac{(8c^2d+b^2f+8acf+2bcfx)\sqrt{a+bx+cx^2}}{8cd^2} - \frac{(a+bx+cx^2)^{3/2}}{2dx^2} - \frac{3(b^2+4ac)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{a}d} - \frac{a^{3/2}f\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d^2} + \frac{3b\sqrt{c}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2d} - \frac{b(b^2-12ac)f\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}d^2} - \frac{b(24c^2d-b^2f+12acf)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}d^2} - \frac{\left(cd-b\sqrt{d}\sqrt{f}+af\right)^{3/2}\tanh^{-1}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2d^2\sqrt{f}} + \frac{\left(cd+b\sqrt{d}\sqrt{f}+af\right)^{3/2}\tanh^{-1}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2d^2\sqrt{f}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.98 (sec) , antiderivative size = 587, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx+cx^2)^{3/2}}{x^3(d-fx^2)}\,dx =$$

$$-\frac{\frac{d(2a+5bx)\sqrt{a+x(b+cx)}}{x^2} + \frac{(3b^2d+4a(3cd+2af))\text{arctanh}\left(\frac{-\sqrt{c}x+\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)}{\sqrt{a}} + 2\text{RootSum}\Big[b^2d-a^2f-4b\sqrt{c}d\#1+4cd$$

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)/(x^3*(d - f*x^2)),x]
```

```
[Out] -1/4*((d*(2*a + 5*b*x)*Sqrt[a + x*(b + c*x)])/x^2 + ((3*b^2*d + 4*a*(3*c*d
+ 2*a*f))*ArcTanh[(-(Sqrt[c]*x) + Sqrt[a + x*(b + c*x)])/Sqrt[a]])/Sqrt[a]
+ 2*RootSum[b^2*d - a^2*f - 4*b*Sqrt[c]*d*#1 + 4*c*d*#1^2 + 2*a*f*#1^2 - f*
#1^4 & , (2*b^2*c*d^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - a*c^
2*d^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + a*b^2*d*f*Log[-(Sqrt
[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*a^2*c*d*f*Log[-(Sqrt[c]*x) + Sqrt[
a + b*x + c*x^2] - #1] - a^3*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] -
 #1] - 4*b*c^(3/2)*d^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 -
4*a*b*Sqrt[c]*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + c^2*d
^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 + b^2*d*f*Log[-(Sqrt
[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 + 2*a*c*d*f*Log[-(Sqrt[c]*x) + Sq
rt[a + b*x + c*x^2] - #1]*#1^2 + a^2*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x +
c*x^2] - #1]*#1^2)/(b*Sqrt[c]*d - 2*c*d*#1 - a*f*#1 + f*#1^3) & ])/d^2
```

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 549, normalized size of antiderivative = 0.89

method	result
risch	$-\frac{\sqrt{c\,x^{2}+b x+a}\,(5 b x+2 a)}{4 d\,x^{2}}-\frac{-\frac{\left(-8 a^{2} f-12 a c d-3 b^{2} d\right) \ln\left(\frac{2 a+b x+2 \sqrt{a}\,\sqrt{c\,x^{2}+b x+a}}{x}\right)}{d \sqrt{a}}-\frac{\left(-8 \sqrt{d f}\,a b f-8 \sqrt{d f}\,b c d+4 a^{2} f^{2}+8 a c d f+4 b^{2} d f+4 c^{2} d^{2}\right)}{}}{}$
default	Expression too large to display

[In] int((c*x^2+b*x+a)^(3/2)/x^3/(-f*x^2+d),x,method=_RETURNVERBOSE)

[Out] -1/4*(c*x^2+b*x+a)^(1/2)*(5*b*x+2*a)/d/x^2-1/8/d*(-(-8*a^2*f-12*a*c*d-3*b^2*d)/d/a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)-(-8*(d*f)^(1/2)*a*b*f-8*(d*f)^(1/2)*b*c*d+4*a^2*f^2+8*a*c*d*f+4*b^2*d*f+4*c^2*d^2)/d/f/(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1/2)/f))-(8*(d*f)^(1/2)*a*b*f+8*(d*f)^(1/2)*b*c*d+4*a^2*f^2+8*a*c*d*f+4*b^2*d*f+4*c^2*d^2)/d/f/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+f*a+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f)))

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}}{x^3(d-fx^2)}\,dx = \text{Timed out}$$

[In] integrate((c*x^2+b*x+a)^(3/2)/x^3/(-f*x^2+d),x, algorithm="fricas")

[Out] Timed out

Sympy [F]

$$\int \frac{(a+bx+cx^2)^{3/2}}{x^3(d-fx^2)}\,dx = -\int \frac{a\sqrt{a+bx+cx^2}}{-dx^3+fx^5}\,dx - \int \frac{bx\sqrt{a+bx+cx^2}}{-dx^3+fx^5}\,dx - \int \frac{cx^2\sqrt{a+bx+cx^2}}{-dx^3+fx^5}\,dx$$

```
[In] integrate((c*x**2+b*x+a)**(3/2)/x**3/(-f*x**2+d),x)
[Out] -Integral(a*sqrt(a + b*x + c*x**2)/(-d*x**3 + f*x**5), x) - Integral(b*x*sq
rt(a + b*x + c*x**2)/(-d*x**3 + f*x**5), x) - Integral(c*x**2*sqrt(a + b*x
+ c*x**2)/(-d*x**3 + f*x**5), x)
```

Maxima [F]

$$\int \frac{(a+bx+cx^2)^{3/2}}{x^3(d-fx^2)}\,dx = \int -\frac{(cx^2+bx+a)^{\frac{3}{2}}}{(fx^2-d)x^3}\,dx$$

```
[In] integrate((c*x^2+b*x+a)^(3/2)/x^3/(-f*x^2+d),x, algorithm="maxima")
[Out] -integrate((c*x^2 + b*x + a)^(3/2)/((f*x^2 - d)*x^3), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+bx+cx^2)^{3/2}}{x^3(d-fx^2)}\,dx = \text{Exception raised: AttributeError}$$

```
[In] integrate((c*x^2+b*x+a)^(3/2)/x^3/(-f*x^2+d),x, algorithm="giac")
[Out] Exception raised: AttributeError >> type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}}{x^3(d-fx^2)}\,dx = \int \frac{(c\,x^2+b\,x+a)^{3/2}}{x^3\,(d-f\,x^2)}\,dx$$

```
[In] int((a + b*x + c*x^2)^(3/2)/(x^3*(d - f*x^2)),x)
[Out] int((a + b*x + c*x^2)^(3/2)/(x^3*(d - f*x^2)), x)
```

3.91 $\int \frac{(a+bx+cx^2)^{3/2}}{1-x^2} dx$

Optimal result

Integrand size = 24, antiderivative size = 189

$$\int \frac{(a+bx+cx^2)^{3/2}}{1-x^2} dx = -\frac{1}{4}(5b+2cx)\sqrt{a+bx+cx^2} - \frac{1}{2}(a-b+c)^{3/2}\text{arctanh}\left(\frac{2a-b+(b-2c)x}{2\sqrt{a-b+c}\sqrt{a+bx+cx^2}}\right) - \frac{(3b^2+12ac+8c^2)\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}} + \frac{1}{2}(a+b+c)^{3/2}\text{arctanh}\left(\frac{2a+b+(b+2c)x}{2\sqrt{a+b+c}\sqrt{a+bx+cx^2}}\right)$$

```
[Out] -1/2*(a-b+c)^(3/2)*arctanh(1/2*(2*a-b+(b-2*c)*x)/(a-b+c)^(1/2)/(c*x^2+b*x+a
)^(1/2))+1/2*(a+b+c)^(3/2)*arctanh(1/2*(2*a+b+(b+2*c)*x)/(a+b+c)^(1/2)/(c*x
^2+b*x+a)^(1/2))-1/8*(12*a*c+3*b^2+8*c^2)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*
x^2+b*x+a)^(1/2))/c^(1/2)-1/4*(2*c*x+5*b)*(c*x^2+b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used

= {992, 1092, 635, 212, 1047, 738}

$$\int \frac{(a+bx+cx^2)^{3/2}}{1-x^2}\,dx = -\frac{(12ac+3b^2+8c^2)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}}$$
$$-\frac{1}{2}(a-b+c)^{3/2}\operatorname{arctanh}\left(\frac{2a+x(b-2c)-b}{2\sqrt{a-b+c}\sqrt{a+bx+cx^2}}\right)$$
$$+\frac{1}{2}(a+b+c)^{3/2}\operatorname{arctanh}\left(\frac{2a+x(b+2c)+b}{2\sqrt{a+b+c}\sqrt{a+bx+cx^2}}\right)-\frac{1}{4}(5b+2cx)\sqrt{a+bx+cx^2}$$

```
[In] Int[(a + b*x + c*x^2)^(3/2)/(1 - x^2),x]
```

```
[Out] -1/4*((5*b + 2*c*x)*Sqrt[a + b*x + c*x^2]) - ((a - b + c)^(3/2)*ArcTanh[(2*
a - b + (b - 2*c)*x)/(2*Sqrt[a - b + c]*Sqrt[a + b*x + c*x^2])])/2 - ((3*b^
2 + 12*a*c + 8*c^2)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])
/(8*Sqrt[c]) + ((a + b + c)^(3/2)*ArcTanh[(2*a + b + (b + 2*c)*x)/(2*Sqrt[a
 + b + c]*Sqrt[a + b*x + c*x^2])])/2
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 992

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (f_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[(b*(3*p + 2*q) + 2*c*(p + q)*x)*(a + b*x + c*x^2)^(p - 1)*
((d + f*x^2)^(q + 1)/(2*f*(p + q)*(2*p + 2*q + 1))), x] - Dist[1/(2*f*(p +
q)*(2*p + 2*q + 1)), Int[(a + b*x + c*x^2)^(p - 2)*(d + f*x^2)^q*Simp[b^2*d
*(p - 1)*(2*p + q) - (p + q)*(b^2*d*(1 - p) - 2*a*(c*d - a*f*(2*p + 2*q + 1
))) - (2*b*(c*d - a*f)*(1 - p)*(2*p + q) - 2*(p + q)*b*(2*c*d*(2*p + q) - (
c*d + a*f)*(2*p + 2*q + 1)))*x + (b^2*f*p*(1 - p) + 2*c*(p + q)*(c*d*(2*p -
 1) - a*f*(4*p + 2*q - 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, q}, x]
```

```
 && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 1] && NeQ[p + q, 0] && NeQ[2*p + 2*q + 1,
 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1047

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(-a)*c, 2]}, Dist[h/2 + c*(g/(2*q)
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - c*(g/(2*q)
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[(-a)*c]
```

Rule 1092

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
 (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + e*x + f
*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x
 + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f
, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{1}{4}(5b+2cx)\sqrt{a+bx+cx^2} \\
&\quad +\frac{1}{2}\int \frac{\frac{1}{4}(8a^2+5b^2+4ac)+4b(a+c)x+\frac{1}{4}(3b^2+12ac+8c^2)\,x^2}{(1-x^2)\sqrt{a+bx+cx^2}}\,dx \\
&= -\frac{1}{4}(5b+2cx)\sqrt{a+bx+cx^2} \\
&\quad -\frac{1}{2}\int \frac{\frac{1}{4}(-8a^2-5b^2-4ac)+\frac{1}{4}(-3b^2-12ac-8c^2)-4b(a+c)x}{(1-x^2)\sqrt{a+bx+cx^2}}\,dx \\
&\quad +\frac{1}{8}\left(-3b^2-12ac-8c^2\right)\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx \\
&= -\frac{1}{4}(5b+2cx)\sqrt{a+bx+cx^2}-\frac{1}{2}(a-b+c)^2\int \frac{1}{(-1-x)\sqrt{a+bx+cx^2}}\,dx \\
&\quad +\frac{1}{2}(a+b+c)^2\int \frac{1}{(1-x)\sqrt{a+bx+cx^2}}\,dx \\
&\quad +\frac{1}{4}\left(-3b^2-12ac-8c^2\right)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx,x,\frac{b+2cx}{\sqrt{a+bx+cx^2}}\right) \\
&= -\frac{1}{4}(5b+2cx)\sqrt{a+bx+cx^2}-\frac{(3b^2+12ac+8c^2)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}} \\
&\quad +(a-b+c)^2\text{Subst}\left(\int \frac{1}{4a-4b+4c-x^2}\,dx,x,\frac{-2a+b-(b-2c)x}{\sqrt{a+bx+cx^2}}\right) \\
&\quad -(a+b+c)^2\text{Subst}\left(\int \frac{1}{4a+4b+4c-x^2}\,dx,x,\frac{-2a-b-(b+2c)x}{\sqrt{a+bx+cx^2}}\right)
\end{aligned}
$$

$$= -\frac{1}{4}(5b+2cx)\sqrt{a+bx+cx^2} - \frac{1}{2}(a-b+c)^{3/2}\tanh^{-1}\left(\frac{2a-b+(b-2c)x}{2\sqrt{a-b+c}\sqrt{a+bx+cx^2}}\right)$$
$$- \frac{(3b^2+12ac+8c^2)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}}$$
$$+ \frac{1}{2}(a+b+c)^{3/2}\tanh^{-1}\left(\frac{2a+b+(b+2c)x}{2\sqrt{a+b+c}\sqrt{a+bx+cx^2}}\right)$$

Mathematica [A] (verified)

Time = 1.53 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx+cx^2)^{3/2}}{1-x^2}\,dx = \frac{1}{4}\Bigg(-\Big((5b+2cx)\sqrt{a+x(b+cx)}\Big)$$
$$-4(-a+b-c)^{3/2}\arctan\left(\frac{\sqrt{-a+b-c}\,x}{\sqrt{a}(1+x)-\sqrt{a+x(b+cx)}}\right)+4(-a-b-c)^{3/2}\arctan\Bigg(\frac{\sqrt{-a-b-cx}}{\sqrt{a}(-1+x)+\sqrt{a+x}}$$

[In] Integrate[(a + b*x + c*x^2)^(3/2)/(1 - x^2),x]

[Out] (-((5*b + 2*c*x)*Sqrt[a + x*(b + c*x)]) - 4*(-a + b - c)^(3/2)*ArcTan[(Sqrt[-a + b - c]*x)/(Sqrt[a]*(1 + x) - Sqrt[a + x*(b + c*x)])] + 4*(-a - b - c)^(3/2)*ArcTan[(Sqrt[-a - b - c]*x)/(Sqrt[a]*(-1 + x) + Sqrt[a + x*(b + c*x)])] - ((3*b^2 + 4*c*(3*a + 2*c))*ArcTanh[(Sqrt[c]*x)/(-Sqrt[a] + Sqrt[a + x*(b + c*x)])])/Sqrt[c])/4

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.63

method	result
risch	$-\frac{(2cx+5b)\sqrt{c\,x^2+bx+a}}{4}-\frac{3b^2\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{8\sqrt{c}}-c^{\frac{3}{2}}\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)-\frac{3a\sqrt{c}\,\ln\Big(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c}}{2}$
default	$-\frac{\left((-1+x)^2c+(b+2c)(-1+x)+a+b+c\right)^{\frac{3}{2}}}{6}-\frac{(b+2c)\Bigg(\frac{(2c(-1+x)+b+2c)\sqrt{(-1+x)^2c+(b+2c)(-1+x)+a+b+c}}{4c}+\frac{\left(4c(a+b+c)-(b+2c)^2\right)\ln}{}}{4}$

[In] int((c*x^2+b*x+a)^(3/2)/(-x^2+1),x,method=_RETURNVERBOSE)

[Out] -1/4*(2*c*x+5*b)*(c*x^2+b*x+a)^(1/2)-3/8*b^2*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-3/2*a*c^(1/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+1/8*(-4*a^2+8*a*b-

```
8*a*c-4*b^2+8*b*c-4*c^2)/(a-b+c)^(1/2)*ln((2*a-2*b+2*c+(b-2*c)*(1+x)+2*(a-b
+c)^(1/2)*((1+x)^2*c+(b-2*c)*(1+x)+a-b+c)^(1/2))/(1+x))+1/8*(4*a^2+8*a*b+8*
a*c+4*b^2+8*b*c+4*c^2)/(a+b+c)^(1/2)*ln((2*a+2*b+2*c+(b+2*c)*(-1+x)+2*(a+b+
c)^(1/2)*((-1+x)^2*c+(b+2*c)*(-1+x)+a+b+c)^(1/2))/(-1+x))
```

Fricas [A] (verification not implemented)

none

Time = 116.63 (sec) , antiderivative size = 2579, normalized size of antiderivative = 13.65

$$\int \frac{(a+bx+cx^2)^{3/2}}{1-x^2}\,dx = \text{Too large to display}$$

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(-x^2+1),x, algorithm="fricas")
```

```
[Out] [1/16*((3*b^2 + 12*a*c + 8*c^2)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*
sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*((a - b)*c + c^2)*sq
rt(a - b + c)*log(-((b^2 + 4*(a - 2*b)*c + 8*c^2)*x^2 - 4*sqrt(c*x^2 + b*x
+ a)*((b - 2*c)*x + 2*a - b)*sqrt(a - b + c) + 8*a^2 - 8*a*b + b^2 + 4*a*c
+ 2*(4*a*b - 3*b^2 - 4*(a - b)*c)*x)/(x^2 + 2*x + 1)) + 4*((a + b)*c + c^2)
*sqrt(a + b + c)*log(-((b^2 + 4*(a + 2*b)*c + 8*c^2)*x^2 + 4*sqrt(c*x^2 + b
*x + a)*((b + 2*c)*x + 2*a + b)*sqrt(a + b + c) + 8*a^2 + 8*a*b + b^2 + 4*a
*c + 2*(4*a*b + 3*b^2 + 4*(a + b)*c)*x)/(x^2 - 2*x + 1)) - 4*(2*c^2*x + 5*b
*c)*sqrt(c*x^2 + b*x + a))/c, -1/16*(8*((a - b)*c + c^2)*sqrt(-a + b - c)*a
rctan(-1/2*sqrt(c*x^2 + b*x + a)*((b - 2*c)*x + 2*a - b)*sqrt(-a + b - c)/(
((a - b)*c + c^2)*x^2 + a^2 - a*b + a*c + (a*b - b^2 + b*c)*x)) - (3*b^2 +
12*a*c + 8*c^2)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x
 + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*((a + b)*c + c^2)*sqrt(a + b + c)*lo
g(-((b^2 + 4*(a + 2*b)*c + 8*c^2)*x^2 + 4*sqrt(c*x^2 + b*x + a)*((b + 2*c)*
x + 2*a + b)*sqrt(a + b + c) + 8*a^2 + 8*a*b + b^2 + 4*a*c + 2*(4*a*b + 3*b
^2 + 4*(a + b)*c)*x)/(x^2 - 2*x + 1)) + 4*(2*c^2*x + 5*b*c)*sqrt(c*x^2 + b*
x + a))/c, -1/16*(8*((a + b)*c + c^2)*sqrt(-a - b - c)*arctan(1/2*sqrt(c*x^
2 + b*x + a)*((b + 2*c)*x + 2*a + b)*sqrt(-a - b - c)/(((a + b)*c + c^2)*x^
2 + a^2 + a*b + a*c + (a*b + b^2 + b*c)*x)) - (3*b^2 + 12*a*c + 8*c^2)*sqrt
(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sq
rt(c) - 4*a*c) - 4*((a - b)*c + c^2)*sqrt(a - b + c)*log(-((b^2 + 4*(a - 2*
b)*c + 8*c^2)*x^2 - 4*sqrt(c*x^2 + b*x + a)*((b - 2*c)*x + 2*a - b)*sqrt(a
- b + c) + 8*a^2 - 8*a*b + b^2 + 4*a*c + 2*(4*a*b - 3*b^2 - 4*(a - b)*c)*x)
/(x^2 + 2*x + 1)) + 4*(2*c^2*x + 5*b*c)*sqrt(c*x^2 + b*x + a))/c, -1/16*(8*
((a - b)*c + c^2)*sqrt(-a + b - c)*arctan(-1/2*sqrt(c*x^2 + b*x + a)*((b -
2*c)*x + 2*a - b)*sqrt(-a + b - c)/(((a - b)*c + c^2)*x^2 + a^2 - a*b + a*c
 + (a*b - b^2 + b*c)*x)) + 8*((a + b)*c + c^2)*sqrt(-a - b - c)*arctan(1/2*
sqrt(c*x^2 + b*x + a)*((b + 2*c)*x + 2*a + b)*sqrt(-a - b - c)/(((a + b)*c
+ c^2)*x^2 + a^2 + a*b + a*c + (a*b + b^2 + b*c)*x)) - (3*b^2 + 12*a*c + 8*
```

```
c^2)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2*c*
x + b)*sqrt(c) - 4*a*c) + 4*(2*c^2*x + 5*b*c)*sqrt(c*x^2 + b*x + a))/c, 1/8
*((3*b^2 + 12*a*c + 8*c^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x
 + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*((a - b)*c + c^2)*sqrt(a - b +
c)*log(-((b^2 + 4*(a - 2*b)*c + 8*c^2)*x^2 - 4*sqrt(c*x^2 + b*x + a)*((b -
2*c)*x + 2*a - b)*sqrt(a - b + c) + 8*a^2 - 8*a*b + b^2 + 4*a*c + 2*(4*a*b
- 3*b^2 - 4*(a - b)*c)*x)/(x^2 + 2*x + 1)) + 2*((a + b)*c + c^2)*sqrt(a + b
 + c)*log(-((b^2 + 4*(a + 2*b)*c + 8*c^2)*x^2 + 4*sqrt(c*x^2 + b*x + a)*((b
 + 2*c)*x + 2*a + b)*sqrt(a + b + c) + 8*a^2 + 8*a*b + b^2 + 4*a*c + 2*(4*a
*b + 3*b^2 + 4*(a + b)*c)*x)/(x^2 - 2*x + 1)) - 2*(2*c^2*x + 5*b*c)*sqrt(c*
x^2 + b*x + a))/c, -1/8*(4*((a - b)*c + c^2)*sqrt(-a + b - c)*arctan(-1/2*s
qrt(c*x^2 + b*x + a)*((b - 2*c)*x + 2*a - b)*sqrt(-a + b - c)/(((a - b)*c +
 c^2)*x^2 + a^2 - a*b + a*c + (a*b - b^2 + b*c)*x)) - (3*b^2 + 12*a*c + 8*c
^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2
 + b*c*x + a*c)) - 2*((a + b)*c + c^2)*sqrt(a + b + c)*log(-((b^2 + 4*(a +
2*b)*c + 8*c^2)*x^2 + 4*sqrt(c*x^2 + b*x + a)*((b + 2*c)*x + 2*a + b)*sqrt(
a + b + c) + 8*a^2 + 8*a*b + b^2 + 4*a*c + 2*(4*a*b + 3*b^2 + 4*(a + b)*c)*
x)/(x^2 - 2*x + 1)) + 2*(2*c^2*x + 5*b*c)*sqrt(c*x^2 + b*x + a))/c, -1/8*(4
*((a + b)*c + c^2)*sqrt(-a - b - c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*((b +
2*c)*x + 2*a + b)*sqrt(-a - b - c)/(((a + b)*c + c^2)*x^2 + a^2 + a*b + a*c
 + (a*b + b^2 + b*c)*x)) - (3*b^2 + 12*a*c + 8*c^2)*sqrt(-c)*arctan(1/2*sqr
t(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*((a -
b)*c + c^2)*sqrt(a - b + c)*log(-((b^2 + 4*(a - 2*b)*c + 8*c^2)*x^2 - 4*sqr
t(c*x^2 + b*x + a)*((b - 2*c)*x + 2*a - b)*sqrt(a - b + c) + 8*a^2 - 8*a*b
+ b^2 + 4*a*c + 2*(4*a*b - 3*b^2 - 4*(a - b)*c)*x)/(x^2 + 2*x + 1)) + 2*(2*
c^2*x + 5*b*c)*sqrt(c*x^2 + b*x + a))/c, -1/8*(4*((a - b)*c + c^2)*sqrt(-a
+ b - c)*arctan(-1/2*sqrt(c*x^2 + b*x + a)*((b - 2*c)*x + 2*a - b)*sqrt(-a
+ b - c)/(((a - b)*c + c^2)*x^2 + a^2 - a*b + a*c + (a*b - b^2 + b*c)*x)) +
 4*((a + b)*c + c^2)*sqrt(-a - b - c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*((b
+ 2*c)*x + 2*a + b)*sqrt(-a - b - c)/(((a + b)*c + c^2)*x^2 + a^2 + a*b + a
*c + (a*b + b^2 + b*c)*x)) - (3*b^2 + 12*a*c + 8*c^2)*sqrt(-c)*arctan(1/2*s
qrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(2*c
^2*x + 5*b*c)*sqrt(c*x^2 + b*x + a))/c]
```

Sympy [F]

$$\int \frac{(a + bx + cx^2)^{3/2}}{1 - x^2}\,dx = -\int \frac{a\sqrt{a + bx + cx^2}}{x^2 - 1}\,dx - \int \frac{bx\sqrt{a + bx + cx^2}}{x^2 - 1}\,dx - \int \frac{cx^2\sqrt{a + bx + cx^2}}{x^2 - 1}\,dx$$

```
[In] integrate((c*x**2+b*x+a)**(3/2)/(-x**2+1),x)
```

[Out] -Integral(a*sqrt(a + b*x + c*x**2)/(x**2 - 1), x) - Integral(b*x*sqrt(a + b
*x + c*x**2)/(x**2 - 1), x) - Integral(c*x**2*sqrt(a + b*x + c*x**2)/(x**2
- 1), x)

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx+cx^2)^{3/2}}{1-x^2}\,dx = \text{Exception raised: ValueError}$$

[In] integrate((c*x^2+b*x+a)^(3/2)/(-x^2+1),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+bx+cx^2)^{3/2}}{1-x^2}\,dx = \text{Exception raised: TypeError}$$

[In] integrate((c*x^2+b*x+a)^(3/2)/(-x^2+1),x, algorithm="giac")

[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}}{1-x^2}\,dx = -\int \frac{(c\,x^2+b\,x+a)^{3/2}}{x^2-1}\,dx$$

[In] int(-(a + b*x + c*x^2)^(3/2)/(x^2 - 1),x)

[Out] -int((a + b*x + c*x^2)^(3/2)/(x^2 - 1), x)

3.92 $\int \frac{\sqrt{-1-x+x^2}}{1-x^2} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 75

$$\int \frac{\sqrt{-1-x+x^2}}{1-x^2} \, dx = -\frac{1}{2}\arctan\left(\frac{3-x}{2\sqrt{-1-x+x^2}}\right) + \operatorname{arctanh}\left(\frac{1-2x}{2\sqrt{-1-x+x^2}}\right) + \frac{1}{2}\operatorname{arctanh}\left(\frac{1+3x}{2\sqrt{-1-x+x^2}}\right)$$

```
[Out] -1/2*arctan(1/2*(3-x)/(x^2-x-1)^(1/2))+arctanh(1/2*(1-2*x)/(x^2-x-1)^(1/2))
+1/2*arctanh(1/2*(1+3*x)/(x^2-x-1)^(1/2))
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1004, 635, 212, 1047, 738, 210}

$$\int \frac{\sqrt{-1-x+x^2}}{1-x^2} \, dx = -\frac{1}{2}\arctan\left(\frac{3-x}{2\sqrt{x^2-x-1}}\right) + \operatorname{arctanh}\left(\frac{1-2x}{2\sqrt{x^2-x-1}}\right) + \frac{1}{2}\operatorname{arctanh}\left(\frac{3x+1}{2\sqrt{x^2-x-1}}\right)$$

```
[In] Int[Sqrt[-1 - x + x^2]/(1 - x^2),x]
```

```
[Out] -1/2*ArcTan[(3 - x)/(2*Sqrt[-1 - x + x^2])] + ArcTanh[(1 - 2*x)/(2*Sqrt[-1
- x + x^2])] + ArcTanh[(1 + 3*x)/(2*Sqrt[-1 - x + x^2])]/2
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1004

```
Int[Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]/((d_) + (f_.)*(x_)^2), x_Symbol]
 :> Dist[c/f, Int[1/Sqrt[a + b*x + c*x^2], x], x] - Dist[1/f, Int[(c*d - a*
f - b*f*x)/(Sqrt[a + b*x + c*x^2]*(d + f*x^2)), x], x] /; FreeQ[{a, b, c, d
, f}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1047

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(-a)*c, 2]}, Dist[h/2 + c*(g/(2*q)
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - c*(g/(2*q)
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[(-a)*c]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\int \frac{1}{\sqrt{-1-x+x^2}}\,dx - \int \frac{x}{(1-x^2)\sqrt{-1-x+x^2}}\,dx\\ &= -\left(\frac{1}{2}\int \frac{1}{(-1-x)\sqrt{-1-x+x^2}}\,dx\right) - \frac{1}{2}\int \frac{1}{(1-x)\sqrt{-1-x+x^2}}\,dx\\ &\quad - 2\text{Subst}\left(\int \frac{1}{4-x^2}\,dx, x, \frac{-1+2x}{\sqrt{-1-x+x^2}}\right)\end{aligned}$$

$$= \tanh^{-1}\left(\frac{1-2x}{2\sqrt{-1-x+x^2}}\right) + \text{Subst}\left(\int \frac{1}{-4-x^2}\,dx, x, \frac{3-x}{\sqrt{-1-x+x^2}}\right) + \text{Subst}\left(\int \frac{1}{4-x^2}\,dx, x, \frac{1+3x}{\sqrt{-1-x+x^2}}\right)$$

$$= -\frac{1}{2}\tan^{-1}\left(\frac{3-x}{2\sqrt{-1-x+x^2}}\right) + \tanh^{-1}\left(\frac{1-2x}{2\sqrt{-1-x+x^2}}\right) + \frac{1}{2}\tanh^{-1}\left(\frac{1+3x}{2\sqrt{-1-x+x^2}}\right)$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.76

$$\int \frac{\sqrt{-1-x+x^2}}{1-x^2}\,dx = \arctan\left(1-x+\sqrt{-1-x+x^2}\right) + \operatorname{arctanh}\left(1+x-\sqrt{-1-x+x^2}\right) + \log\left(1-2x+2\sqrt{-1-x+x^2}\right)$$

```
[In] Integrate[Sqrt[-1 - x + x^2]/(1 - x^2),x]
[Out] ArcTan[1 - x + Sqrt[-1 - x + x^2]] + ArcTanh[1 + x - Sqrt[-1 - x + x^2]] +
Log[1 - 2*x + 2*Sqrt[-1 - x + x^2]]
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.36

method	result
default	$-\frac{\sqrt{(-1+x)^2-2+x}}{2} - \frac{\ln\left(-\frac{1}{2}+x+\sqrt{(-1+x)^2-2+x}\right)}{4} + \frac{\arctan\left(\frac{-3+x}{2\sqrt{(-1+x)^2-2+x}}\right)}{2} + \frac{\sqrt{(1+x)^2-2-3x}}{2} - \frac{3\ln\left(-\frac{1}{2}+x+\sqrt{(\right.}{4}$
trager	$-\frac{\ln\left(\frac{8\sqrt{x^2-x-1}\,x^2+8x^3+12\sqrt{x^2-x-1}\,x+8x^2+2\sqrt{x^2-x-1}-9x-11}{1+x}\right)}{2} + \frac{\text{RootOf}\left(\_Z^2+1\right)\ln\left(-\frac{-\text{RootOf}\left(\_Z^2+1\right)x+2\sqrt{x^2-x-1}+3}{-1+x}\right.}{2}$

```
[In] int((x^2-x-1)^(1/2)/(-x^2+1),x,method=_RETURNVERBOSE)
[Out] -1/2*((-1+x)^2-2+x)^(1/2)-1/4*ln(-1/2+x+((-1+x)^2-2+x)^(1/2))+1/2*arctan(1/
2*(-3+x)/((-1+x)^2-2+x)^(1/2))+1/2*((1+x)^2-2-3*x)^(1/2)-3/4*ln(-1/2+x+((1+
x)^2-2-3*x)^(1/2))-1/2*arctanh(1/2*(-1-3*x)/((1+x)^2-2-3*x)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{-1-x+x^2}}{1-x^2}\,dx = \arctan\left(-x+\sqrt{x^2-x-1}+1\right)-\frac{1}{2}\log\left(-x+\sqrt{x^2-x-1}\right)+\frac{1}{2}\log\left(-x+\sqrt{x^2-x-1}-2\right)+\log\left(-2\,x+2\sqrt{x^2-x-1}+1\right)$$

```
[In] integrate((x^2-x-1)^(1/2)/(-x^2+1),x, algorithm="fricas")
[Out] arctan(-x + sqrt(x^2 - x - 1) + 1) - 1/2*log(-x + sqrt(x^2 - x - 1)) + 1/2*
log(-x + sqrt(x^2 - x - 1) - 2) + log(-2*x + 2*sqrt(x^2 - x - 1) + 1)
```

Sympy [F]

$$\int \frac{\sqrt{-1-x+x^2}}{1-x^2}\,dx = -\int \frac{\sqrt{x^2-x-1}}{x^2-1}\,dx$$

```
[In] integrate((x**2-x-1)**(1/2)/(-x**2+1),x)
[Out] -Integral(sqrt(x**2 - x - 1)/(x**2 - 1), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt{-1-x+x^2}}{1-x^2}\,dx = \frac{1}{2}\arcsin\left(\frac{2\sqrt{5}x}{5\,|2\,x-2|}-\frac{6\sqrt{5}}{5\,|2\,x-2|}\right)-\log\left(x+\sqrt{x^2-x-1}-\frac{1}{2}\right)-\frac{1}{2}\log\left(\frac{2\sqrt{x^2-x-1}}{|2\,x+2|}+\frac{2}{|2\,x+2|}-\frac{3}{2}\right)$$

```
[In] integrate((x^2-x-1)^(1/2)/(-x^2+1),x, algorithm="maxima")
[Out] 1/2*arcsin(2/5*sqrt(5)*x/abs(2*x - 2) - 6/5*sqrt(5)/abs(2*x - 2)) - log(x +
 sqrt(x^2 - x - 1) - 1/2) - 1/2*log(2*sqrt(x^2 - x - 1)/abs(2*x + 2) + 2/ab
s(2*x + 2) - 3/2)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.97

$$\int \frac{\sqrt{-1-x+x^2}}{1-x^2}\,dx = \arctan\left(-x+\sqrt{x^2-x-1}+1\right) - \frac{1}{2}\log\left(\left|-x+\sqrt{x^2-x-1}\right|\right) + \frac{1}{2}\log\left(\left|-x+\sqrt{x^2-x-1}-2\right|\right) + \log\left(\left|-2\,x+2\,\sqrt{x^2-x-1}+1\right|\right)$$

```
[In] integrate((x^2-x-1)^(1/2)/(-x^2+1),x, algorithm="giac")
```

```
[Out] arctan(-x + sqrt(x^2 - x - 1) + 1) - 1/2*log(abs(-x + sqrt(x^2 - x - 1))) +
 1/2*log(abs(-x + sqrt(x^2 - x - 1) - 2)) + log(abs(-2*x + 2*sqrt(x^2 - x -
 1) + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-1-x+x^2}}{1-x^2}\,dx = -\int \frac{\sqrt{x^2-x-1}}{x^2-1}\,dx$$

```
[In] int(-(x^2 - x - 1)^(1/2)/(x^2 - 1),x)
```

```
[Out] -int((x^2 - x - 1)^(1/2)/(x^2 - 1), x)
```

3.93 $\int \frac{(x+x^2)^{3/2}}{1+x^2} dx$

Optimal result

Integrand size = 17, antiderivative size = 130

$$\int \frac{(x+x^2)^{3/2}}{1+x^2}\,dx = \frac{1}{4}(5+2x)\sqrt{x+x^2}+\sqrt{1+\sqrt{2}}\arctan\left(\frac{1+\sqrt{2}-x}{\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{x+x^2}}\right) - \sqrt{-1+\sqrt{2}}\text{arctanh}\left(\frac{1-\sqrt{2}-x}{\sqrt{2\left(-1+\sqrt{2}\right)}\sqrt{x+x^2}}\right)-\frac{5}{4}\text{arctanh}\left(\frac{x}{\sqrt{x+x^2}}\right)$$

```
[Out] -5/4*arctanh(x/(x^2+x)^(1/2))+1/4*(5+2*x)*(x^2+x)^(1/2)-arctanh((1-x-2^(1/2
))/(x^2+x)^(1/2)/(-2+2*2^(1/2))^(1/2))*(2^(1/2)-1)^(1/2)+arctan((1-x+2^(1/2
))/(x^2+x)^(1/2)/(2+2*2^(1/2))^(1/2))*(1+2^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {992, 1092, 634, 212, 12, 1050, 1044, 213, 209}

$$\int \frac{(x+x^2)^{3/2}}{1+x^2}\,dx = \sqrt{1+\sqrt{2}}\arctan\left(\frac{-x+\sqrt{2}+1}{\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{x^2+x}}\right) - \sqrt{\sqrt{2}-1}\text{arctanh}\left(\frac{-x-\sqrt{2}+1}{\sqrt{2\left(\sqrt{2}-1\right)}\sqrt{x^2+x}}\right) - \frac{5}{4}\text{arctanh}\left(\frac{x}{\sqrt{x^2+x}}\right)+\frac{1}{4}\sqrt{x^2+x}(2x+5)$$

```
[In] Int[(x + x^2)^(3/2)/(1 + x^2),x]
```

```
[Out] ((5 + 2*x)*Sqrt[x + x^2])/4 + Sqrt[1 + Sqrt[2]]*ArcTan[(1 + Sqrt[2] - x)/(S
qrt[2*(1 + Sqrt[2])]*Sqrt[x + x^2])] - Sqrt[-1 + Sqrt[2]]*ArcTanh[(1 - Sqrt
[2] - x)/(Sqrt[2*(-1 + Sqrt[2])]*Sqrt[x + x^2])] - (5*ArcTanh[x/Sqrt[x + x^
2]])/4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 634

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 992

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (f_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[(b*(3*p + 2*q) + 2*c*(p + q)*x)*(a + b*x + c*x^2)^(p - 1)*
((d + f*x^2)^(q + 1)/(2*f*(p + q)*(2*p + 2*q + 1))), x] - Dist[1/(2*f*(p +
q)*(2*p + 2*q + 1)), Int[(a + b*x + c*x^2)^(p - 2)*(d + f*x^2)^q*Simp[b^2*d
*(p - 1)*(2*p + q) - (p + q)*(b^2*d*(1 - p) - 2*a*(c*d - a*f*(2*p + 2*q + 1
))) - (2*b*(c*d - a*f)*(1 - p)*(2*p + q) - 2*(p + q)*b*(2*c*d*(2*p + q) - (
c*d + a*f)*(2*p + 2*q + 1)))*x + (b^2*f*p*(1 - p) + 2*c*(p + q)*(c*d*(2*p -
 1) - a*f*(4*p + 2*q - 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, q}, x]
 && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 1] && NeQ[p + q, 0] && NeQ[2*p + 2*q + 1,
 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1044

```
Int[((g_) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f
_.)*(x_)^2]), x_Symbol] :> Dist[-2*a*g*h, Subst[Int[1/Simp[2*a^2*g*h*c + a*
e*x^2, x], x], x, Simp[a*h - g*c*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[
{a, c, d, e, f, g, h}, x] && EqQ[a*h^2*e + 2*g*h*(c*d - a*f) - g^2*c*e, 0]
```

Rule 1050

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 + a*c*e^2, 2]}, Dist
[1/(2*q), Int[Simp[(-a)*h*e - g*(c*d - a*f - q) + (h*(c*d - a*f + q) - g*c*
e)*x, x]/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Si
mp[(-a)*h*e - g*(c*d - a*f + q) + (h*(c*d - a*f - q) - g*c*e)*x, x]/((a + c
*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] &&
 NeQ[e^2 - 4*d*f, 0] && NegQ[(-a)*c]
```

Rule 1092

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
 (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + e*x + f
*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x
 + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f
, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{4}(5+2x)\sqrt{x+x^2}-\frac{1}{2}\int \frac{\frac{5}{4}+4x+\frac{5x^2}{4}}{(1+x^2)\sqrt{x+x^2}}\,dx \\
&= \frac{1}{4}(5+2x)\sqrt{x+x^2}-\frac{1}{2}\int \frac{4x}{(1+x^2)\sqrt{x+x^2}}\,dx-\frac{5}{8}\int \frac{1}{\sqrt{x+x^2}}\,dx \\
&= \frac{1}{4}(5+2x)\sqrt{x+x^2}-\frac{5}{4}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{x}{\sqrt{x+x^2}}\right)-2\int \frac{x}{(1+x^2)\sqrt{x+x^2}}\,dx \\
&= \frac{1}{4}(5+2x)\sqrt{x+x^2}-\frac{5}{4}\tanh^{-1}\left(\frac{x}{\sqrt{x+x^2}}\right)+\frac{\int \frac{-1+\left(-1-\sqrt{2}\right)x}{(1+x^2)\sqrt{x+x^2}}\,dx}{\sqrt{2}}-\frac{\int \frac{-1+\left(-1+\sqrt{2}\right)x}{(1+x^2)\sqrt{x+x^2}}\,dx}{\sqrt{2}} \\
&= \frac{1}{4}(5+2x)\sqrt{x+x^2}-\frac{5}{4}\tanh^{-1}\left(\frac{x}{\sqrt{x+x^2}}\right) \\
&\quad+\left(-2+\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{2\left(1-\sqrt{2}\right)+x^2}\,dx,x,\frac{-1+\sqrt{2}+x}{\sqrt{x+x^2}}\right) \\
&\quad-\left(2+\sqrt{2}\right)\text{Subst}\left(\int \frac{1}{2\left(1+\sqrt{2}\right)+x^2}\,dx,x,\frac{-1-\sqrt{2}+x}{\sqrt{x+x^2}}\right)
\end{aligned}
$$

$$= \frac{1}{4}(5+2x)\sqrt{x+x^2} + \sqrt{1+\sqrt{2}}\tan^{-1}\left(\frac{1+\sqrt{2}-x}{\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{x+x^2}}\right) - \sqrt{-1+\sqrt{2}}\tanh^{-1}\left(\frac{1-\sqrt{2}-x}{\sqrt{2\left(-1+\sqrt{2}\right)}\sqrt{x+x^2}}\right) - \frac{5}{4}\tanh^{-1}\left(\frac{x}{\sqrt{x+x^2}}\right)$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.11 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.93

$$\int \frac{(x+x^2)^{3/2}}{1+x^2}\,dx = \frac{\sqrt{x}\sqrt{1+x}\Big(\sqrt{x}\sqrt{1+x}(5+2x) + 5\log\left(-\sqrt{x}+\sqrt{1+x}\right) + 8\text{RootSum}\Big[16+32\#1+1}{4\sqrt{x(1+x)}}$$

```
[In] Integrate[(x + x^2)^(3/2)/(1 + x^2),x]
```

```
[Out] (Sqrt[x]*Sqrt[1 + x]*(Sqrt[x]*Sqrt[1 + x]*(5 + 2*x) + 5*Log[-Sqrt[x] + Sqrt
[1 + x]] + 8*RootSum[16 + 32*#1 + 16*#1^2 + #1^4 & , (Log[-2*x + 2*Sqrt[x]*
Sqrt[1 + x] + #1]*#1^2)/(8 + 8*#1 + #1^3) & ]))/(4*Sqrt[x*(1 + x)])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 266 vs. 2(98) = 196.

Time = 4.60 (sec) , antiderivative size = 267, normalized size of antiderivative = 2.05

method	result
pseudoelliptic	$-\frac{\left(\left(-\frac{\sqrt{2}}{2}+1\right)\ln\left(\frac{\sqrt{2}\,x-\sqrt{(1+x)x}\,\sqrt{2+2\sqrt{2}}+x+1}{x}\right)+\left(\frac{\sqrt{2}}{2}-1\right)\ln\left(\frac{\sqrt{2}\,x+\sqrt{(1+x)x}\,\sqrt{2+2\sqrt{2}}+x+1}{x}\right)-\frac{5\sqrt{-2+2\sqrt{2}}\,\ln\left(\frac{\sqrt{(1+x)x}-x}{x}\right)}{8}\right.}{\sqrt{-2+2\sqrt{2}}\,\left(x+\right.}$
trager	$\left(\frac{5}{4}+\frac{x}{2}\right)\sqrt{x^2+x}-\frac{5\ln\left(1+2x+2\sqrt{x^2+x}\right)}{8}-\frac{\operatorname{RootOf}\left(\operatorname{RootOf}\left(\_Z^4+16\_Z^2+128\right)^2+\_Z^2+16\right)\ln\left(-\frac{3\operatorname{RootOf}\left(\right.}{}\right.}{}$
risch	$\frac{(5+2x)(1+x)x}{4\sqrt{(1+x)x}}-\frac{5\ln\left(x+\frac{1}{2}+\sqrt{x^2+x}\right)}{8}+\frac{\sqrt{\frac{4\left(-\sqrt{2}-1+x\right)^2}{\left(-\sqrt{2}+1-x\right)^2}-\frac{3\sqrt{2}\,\left(-\sqrt{2}-1+x\right)^2}{\left(-\sqrt{2}+1-x\right)^2}+4+3\sqrt{2}}\,\sqrt{2}\,\left(\sqrt{-2+2\sqrt{2}}\,\arctan\left(\frac{\sqrt{-2+2}}{}\right.\right.}{}$
default	$-\frac{5\ln\left(x+\frac{1}{2}+\sqrt{x^2+x}\right)}{8}+\frac{x\sqrt{x^2+x}}{2}+\frac{5\sqrt{x^2+x}}{4}+\frac{\sqrt{\frac{4\left(-\sqrt{2}-1+x\right)^2}{\left(-\sqrt{2}+1-x\right)^2}-\frac{3\sqrt{2}\,\left(-\sqrt{2}-1+x\right)^2}{\left(-\sqrt{2}+1-x\right)^2}+4+3\sqrt{2}}\,\sqrt{2}\,\left(\sqrt{-2+2\sqrt{2}}\,\operatorname{arctan}\right.}{}$

```
[In] int((x^2+x)^(3/2)/(x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/(-2+2*2^(1/2))^(1/2)*((-1/2*2^(1/2)+1)*ln((2^(1/2)*x-((1+x)*x)^(1/2)*(2+
2*2^(1/2))^(1/2)+x+1)/x)+(1/2*2^(1/2)-1)*ln((2^(1/2)*x+((1+x)*x)^(1/2)*(2+2
*2^(1/2))^(1/2)+x+1)/x)-5/8*(-2+2*2^(1/2))^(1/2)*ln((((1+x)*x)^(1/2)-x)/x)+
5/8*(-2+2*2^(1/2))^(1/2)*ln((x+((1+x)*x)^(1/2))/x)-1/2*(-2+2*2^(1/2))^(1/2)
*(x+5/2)*((1+x)*x)^(1/2)+2^(1/2)*(arctan(((2+2*2^(1/2))^(1/2)*x-2*((1+x)*x)
^(1/2))/x/(-2+2*2^(1/2))^(1/2))-arctan(((2+2*2^(1/2))^(1/2)*x+2*((1+x)*x)^(
1/2))/x/(-2+2*2^(1/2))^(1/2))))*x^2/(x+((1+x)*x)^(1/2))^2/(-((1+x)*x)^(1/2)
+x)^2
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.02

$$\int \frac{(x+x^2)^{3/2}}{1+x^2}\,dx = \frac{1}{4}\sqrt{x^2+x}(2x+5) - \frac{1}{2}\sqrt{2i-2}\log\left(-2x+(i+1)\sqrt{2i-2}+2\sqrt{x^2+x}-2i\right) + \frac{1}{2}\sqrt{2i-2}\log\left(-2x-(i+1)\sqrt{2i-2}+2\sqrt{x^2+x}-2i\right) - \frac{1}{2}\sqrt{-2i-2}\log\left(-2x-(i-1)\sqrt{-2i-2}+2\sqrt{x^2+x}+2i\right) + \frac{1}{2}\sqrt{-2i-2}\log\left(-2x+(i-1)\sqrt{-2i-2}+2\sqrt{x^2+x}+2i\right) + \frac{5}{8}\log\left(-2x+2\sqrt{x^2+x}-1\right)$$

```
[In] integrate((x^2+x)^(3/2)/(x^2+1),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(x^2 + x)*(2*x + 5) - 1/2*sqrt(2*I - 2)*log(-2*x + (I + 1)*sqrt(2*I
 - 2) + 2*sqrt(x^2 + x) - 2*I) + 1/2*sqrt(2*I - 2)*log(-2*x - (I + 1)*sqrt(
2*I - 2) + 2*sqrt(x^2 + x) - 2*I) - 1/2*sqrt(-2*I - 2)*log(-2*x - (I - 1)*s
qrt(-2*I - 2) + 2*sqrt(x^2 + x) + 2*I) + 1/2*sqrt(-2*I - 2)*log(-2*x + (I -
 1)*sqrt(-2*I - 2) + 2*sqrt(x^2 + x) + 2*I) + 5/8*log(-2*x + 2*sqrt(x^2 + x
) - 1)
```

Sympy [F]

$$\int \frac{(x+x^2)^{3/2}}{1+x^2}\,dx = \int \frac{(x(x+1))^{\frac{3}{2}}}{x^2+1}\,dx$$

```
[In] integrate((x**2+x)**(3/2)/(x**2+1),x)
```

```
[Out] Integral((x*(x + 1))**(3/2)/(x**2 + 1), x)
```

Maxima [F]

$$\int \frac{(x+x^2)^{3/2}}{1+x^2}\,dx=\int \frac{(x^2+x)^{\frac{3}{2}}}{x^2+1}\,dx$$

```
[In] integrate((x^2+x)^(3/2)/(x^2+1),x, algorithm="maxima")
[Out] integrate((x^2 + x)^(3/2)/(x^2 + 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(x+x^2)^{3/2}}{1+x^2}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((x^2+x)^(3/2)/(x^2+1),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{%%{poly1[2937825863393165301979971848533484854911359614337236
965430
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(x+x^2)^{3/2}}{1+x^2}\,dx=\int \frac{(x^2+x)^{3/2}}{x^2+1}\,dx$$

```
[In] int((x + x^2)^(3/2)/(x^2 + 1),x)
[Out] int((x + x^2)^(3/2)/(x^2 + 1), x)
```

3.94 $\int \frac{x^4}{\sqrt{a+bx+cx^2}(d-fx^2)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 369

$$
\begin{aligned}
\int \frac{x^4}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx &= \frac{3b\sqrt{a+bx+cx^2}}{4c^2f}-\frac{x\sqrt{a+bx+cx^2}}{2cf} \\
&\quad -\frac{d\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f^2} \\
&\quad -\frac{(3b^2-4ac)\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{5/2}f} \\
&\quad +\frac{d^{3/2}\text{arctanh}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2f^2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}} \\
&\quad +\frac{d^{3/2}\text{arctanh}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2f^2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}}
\end{aligned}
$$

```
[Out] -1/8*(-4*a*c+3*b^2)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(5
/2)/f-d*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/f^2/c^(1/2)+3/4*
b*(c*x^2+b*x+a)^(1/2)/c^2/f-1/2*x*(c*x^2+b*x+a)^(1/2)/c/f+1/2*d^(3/2)*arcta
nh(1/2*(b*d^(1/2)-2*a*f^(1/2)+x*(2*c*d^(1/2)-b*f^(1/2)))/(c*x^2+b*x+a)^(1/2
)/(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2))/f^2/(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2)+
1/2*d^(3/2)*arctanh(1/2*(b*d^(1/2)+2*a*f^(1/2)+x*(2*c*d^(1/2)+b*f^(1/2)))/(
c*x^2+b*x+a)^(1/2)/(c*d+a*f+b*d^(1/2)*f^(1/2))^(1/2))/f^2/(c*d+a*f+b*d^(1/2
)*f^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6857, 635, 212, 756, 654, 998, 738}

$$\int \frac{x^4}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = -\frac{(3b^2-4ac)\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{5/2}f} + \frac{d^{3/2}\text{arctanh}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2f^2\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}} + \frac{d^{3/2}\text{arctanh}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2f^2\sqrt{af+b\sqrt{d}\sqrt{f}+cd}} - \frac{d\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f^2} + \frac{3b\sqrt{a+bx+cx^2}}{4c^2f} - \frac{x\sqrt{a+bx+cx^2}}{2cf}$$

```
[In] Int[x^4/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] (3*b*Sqrt[a + b*x + c*x^2])/(4*c^2*f) - (x*Sqrt[a + b*x + c*x^2])/(2*c*f) -
 (d*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(Sqrt[c]*f^2) -
 ((3*b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(
8*c^(5/2)*f) + (d^(3/2)*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b
*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])]
)/(2*f^2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]) + (d^(3/2)*ArcTanh[(b*Sqrt[d]
 + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[
f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*f^2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*
f])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
```

```
b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 756

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 998

```
Int[1/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Sy
mbol] :> Dist[1/2, Int[1/((a - Rt[(-a)*c, 2]*x)*Sqrt[d + e*x + f*x^2]), x],
 x] + Dist[1/2, Int[1/((a + Rt[(-a)*c, 2]*x)*Sqrt[d + e*x + f*x^2]), x], x]
 /; FreeQ[{a, c, d, e, f}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[(-a)*c]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\text{integral} = \int \left(-\frac{d}{f^2\sqrt{a+bx+cx^2}} - \frac{x^2}{f\sqrt{a+bx+cx^2}} + \frac{d^2}{f^2\sqrt{a+bx+cx^2}\,(d-fx^2)}\right) dx$$

$$= -\frac{d\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{f^2} + \frac{d^2\int \frac{1}{\sqrt{a+bx+cx^2}(d-fx^2)}\,dx}{f^2} - \frac{\int \frac{x^2}{\sqrt{a+bx+cx^2}}\,dx}{f}$$

$$= -\frac{x\sqrt{a+bx+cx^2}}{2cf} - \frac{(2d)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{f^2} + \frac{d^2\int \frac{1}{\left(d-\sqrt{d}\sqrt{f}x\right)\sqrt{a+bx+cx^2}}\,dx}{2f^2} + \frac{d^2\int \frac{1}{\left(d+\sqrt{d}\sqrt{f}x\right)\sqrt{a+bx+cx^2}}\,dx}{2f^2} - \frac{\int \frac{-a-\frac{3bx}{2}}{\sqrt{a+bx+cx^2}}\,dx}{2cf}$$

$$= \frac{3b\sqrt{a+bx+cx^2}}{4c^2f} - \frac{x\sqrt{a+bx+cx^2}}{2cf} - \frac{d\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f^2} - \frac{d^2\text{Subst}\left(\int \frac{1}{4cd^2-4bd^{3/2}\sqrt{f}+4adf-x^2}\,dx, x, \frac{-bd+2a\sqrt{d}\sqrt{f}-\left(2cd-b\sqrt{d}\sqrt{f}\right)x}{\sqrt{a+bx+cx^2}}\right)}{f^2} - \frac{d^2\text{Subst}\left(\int \frac{1}{4cd^2+4bd^{3/2}\sqrt{f}+4adf-x^2}\,dx, x, \frac{-bd-2a\sqrt{d}\sqrt{f}-\left(2cd+b\sqrt{d}\sqrt{f}\right)x}{\sqrt{a+bx+cx^2}}\right)}{f^2} - \frac{(3b^2-4ac)\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{8c^2f}$$

$$= \frac{3b\sqrt{a+bx+cx^2}}{4c^2f} - \frac{x\sqrt{a+bx+cx^2}}{2cf} - \frac{d\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f^2} + \frac{d^{3/2}\tanh^{-1}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2f^2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}} + \frac{d^{3/2}\tanh^{-1}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2f^2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}} - \frac{(3b^2-4ac)\,\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{4c^2f}$$

$$= \frac{3b\sqrt{a+bx+cx^2}}{4c^2f} - \frac{x\sqrt{a+bx+cx^2}}{2cf} - \frac{d\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f^2} - \frac{(3b^2-4ac)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{5/2}f} + \frac{d^{3/2}\tanh^{-1}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2f^2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}} + \frac{d^{3/2}\tanh^{-1}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2f^2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.68 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.68

$$\int \frac{x^4}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx$$
$$= \frac{2\sqrt{c}f(3b-2cx)\sqrt{a+x(b+cx)}+(8c^2d+3b^2f-4acf)\log\left(c^2f^2\left(b+2cx-2\sqrt{c}\sqrt{a+x(b+cx)}\right)\right)-4}{}$$

```
[In] Integrate[x^4/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] (2*Sqrt[c]*f*(3*b - 2*c*x)*Sqrt[a + x*(b + c*x)] + (8*c^2*d + 3*b^2*f - 4*a
*c*f)*Log[c^2*f^2*(b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)])] - 4*c^(5/2
)*d^2*RootSum[b^2*d - a^2*f - 4*b*Sqrt[c]*d*#1 + 4*c*d*#1^2 + 2*a*f*#1^2 -
f*#1^4 & , (b*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*Sqrt[c]*Lo
g[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1)/(b*Sqrt[c]*d - 2*c*d*#1 -
a*f*#1 + f*#1^3) & ])/(8*c^(5/2)*f^2)
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 463, normalized size of antiderivative = 1.25

method	result
risch	$\frac{(-2cx+3b)\sqrt{c\,x^2+bx+a}}{4c^2 f}+\frac{\frac{\left(4acf-3b^2f-8c^2d\right)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{f\sqrt{c}}+\frac{4c^2d^2\ln\left(\frac{\frac{2b\sqrt{df}+2fa+2cd}{f}+\frac{\left(2c\sqrt{df}+bf\right)\left(x-\frac{\sqrt{df}}{f}\right)}{f}+2\sqrt{\frac{b\sqrt{df}}{}}}{}\right)}{\sqrt{df}\,f\sqrt{\frac{b}{}}}}{}$
default	$-\frac{\frac{x\sqrt{c\,x^2+bx+a}}{2c}-\frac{3b\left(\frac{\sqrt{c\,x^2+bx+a}}{c}-\frac{b\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{2c^{\frac{3}{2}}}\right)}{4c}-\frac{a\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{2c^{\frac{3}{2}}}}{f}-\frac{d\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{f^2\sqrt{c}}+-\frac{d^2}{}$

```
[In] int(x^4/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(-2*c*x+3*b)*(c*x^2+b*x+a)^(1/2)/c^2/f+1/8/c^2/f*(1/f*(4*a*c*f-3*b^2*f-
8*c^2*d)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)+4*c^2*d^2/(d*f
)^(1/2)/f/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+f*a+c*d)/f
+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f*a+c*d)/f)^(1
/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*
f)^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))-4*c^2*d^2/(d*f)^(1/2)/f/(1/f
```

```
*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1/f*(-2*c
*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*
((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b
*(d*f)^(1/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1/2)/f)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = \text{Timed out}$$

```
[In] integrate(x^4/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{x^4}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = -\int \frac{x^4}{-d\sqrt{a+bx+cx^2}+fx^2\sqrt{a+bx+cx^2}}\,dx$$

```
[In] integrate(x**4/(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(x**4/(-d*sqrt(a + b*x + c*x**2) + f*x**2*sqrt(a + b*x + c*x**2)),
 x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^4/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see `
assume?
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^4}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^4/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = \int \frac{x^4}{(d-f\,x^2)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

```
[In] int(x^4/((d - f*x^2)*(a + b*x + c*x^2)^(1/2)),x)
```

```
[Out] int(x^4/((d - f*x^2)*(a + b*x + c*x^2)^(1/2)), x)
```

3.95 $\int \frac{x^3}{\sqrt{a+bx+cx^2}(d-fx^2)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 287

$$\int \frac{x^3}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = -\frac{\sqrt{a+bx+cx^2}}{cf}+\frac{b\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}f} - \frac{d\,\text{arctanh}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2f^{3/2}\sqrt{cd-b\sqrt{d}\sqrt{f}+af}} + \frac{d\,\text{arctanh}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2f^{3/2}\sqrt{cd+b\sqrt{d}\sqrt{f}+af}}$$

```
[Out] 1/2*b*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(3/2)/f-(c*x^2+b
*x+a)^(1/2)/c/f-1/2*d*arctanh(1/2*(b*d^(1/2)-2*a*f^(1/2)+x*(2*c*d^(1/2)-b*f
^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2))/f^(3/2)/(c*
d+a*f-b*d^(1/2)*f^(1/2))^(1/2)+1/2*d*arctanh(1/2*(b*d^(1/2)+2*a*f^(1/2)+x*(
2*c*d^(1/2)+b*f^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f+b*d^(1/2)*f^(1/2))^(1/
2))/f^(3/2)/(c*d+a*f+b*d^(1/2)*f^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {6857, 654, 635, 212, 1047, 738}

$$\int \frac{x^3}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = \frac{b\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}f} - \frac{d\text{arctanh}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2f^{3/2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}} + \frac{d\text{arctanh}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2f^{3/2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}} - \frac{\sqrt{a+bx+cx^2}}{cf}$$

```
[In] Int[x^3/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] -(Sqrt[a + b*x + c*x^2]/(c*f)) + (b*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a +
 b*x + c*x^2])])/(2*c^(3/2)*f) - (d*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c
*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*
x + c*x^2])])/(2*f^(3/2)*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]) + (d*ArcTanh[
(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqr
t[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*f^(3/2)*Sqrt[c*d + b*Sqrt[d
]*Sqrt[f] + a*f])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
```

```
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1047

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(-a)*c, 2]}, Dist[h/2 + c*(g/(2*q)
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - c*(g/(2*q)
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[(-a)*c]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(-\frac{x}{f\sqrt{a+bx+cx^2}}+\frac{dx}{f\sqrt{a+bx+cx^2}\,(d-fx^2)}\right) dx\\
&= -\frac{\int \frac{x}{\sqrt{a+bx+cx^2}}\,dx}{f}+\frac{d\int \frac{x}{\sqrt{a+bx+cx^2}(d-fx^2)}\,dx}{f}\\
&= -\frac{\sqrt{a+bx+cx^2}}{cf}+\frac{b\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{2cf}\\
&\quad +\frac{d\int \frac{1}{\left(-\sqrt{d}\sqrt{f}-fx\right)\sqrt{a+bx+cx^2}}\,dx}{2f}+\frac{d\int \frac{1}{\left(\sqrt{d}\sqrt{f}-fx\right)\sqrt{a+bx+cx^2}}\,dx}{2f}\\
&= -\frac{\sqrt{a+bx+cx^2}}{cf}+\frac{b\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx,x,\frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{cf}\\
&\quad -\frac{d\text{Subst}\left(\int \frac{1}{4cdf-4b\sqrt{d}f^{3/2}+4af^2-x^2}\,dx,x,\frac{b\sqrt{d}\sqrt{f}-2af-\left(-2c\sqrt{d}\sqrt{f}+bf\right)x}{\sqrt{a+bx+cx^2}}\right)}{f}\\
&\quad -\frac{d\text{Subst}\left(\int \frac{1}{4cdf+4b\sqrt{d}f^{3/2}+4af^2-x^2}\,dx,x,\frac{-b\sqrt{d}\sqrt{f}-2af-\left(2c\sqrt{d}\sqrt{f}+bf\right)x}{\sqrt{a+bx+cx^2}}\right)}{f}
\end{aligned}
$$

$$= -\frac{\sqrt{a+bx+cx^2}}{cf} + \frac{b\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}f}$$

$$- \frac{d\tanh^{-1}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2f^{3/2}\sqrt{cd-b\sqrt{d}\sqrt{f}+af}} + \frac{d\tanh^{-1}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2f^{3/2}\sqrt{cd+b\sqrt{d}\sqrt{f}+af}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.35 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.73

$$\int \frac{x^3}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = -\frac{\frac{2\sqrt{a+x(b+cx)}}{c} + \frac{b\log\left(cf\left(b+2cx-2\sqrt{c}\sqrt{a+x(b+cx)}\right)\right)}{c^{3/2}} + d\text{RootSum}\left[b^2d-a^2f-4b\sqrt{c}d\#1+4cd\#1^2+2af\#1^2 \cdots\right.}{2f}$$

```
[In] Integrate[x^3/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] -1/2*((2*Sqrt[a + x*(b + c*x)])/c + (b*Log[c*f*(b + 2*c*x - 2*Sqrt[c]*Sqrt[
a + x*(b + c*x)])])/c^(3/2) + d*RootSum[b^2*d - a^2*f - 4*b*Sqrt[c]*d*#1 +
4*c*d*#1^2 + 2*a*f*#1^2 - f*#1^4 & , (a*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c
*x^2] - #1] - Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2)/(-(b*Sqr
t[c]*d) + 2*c*d*#1 + a*f*#1 - f*#1^3) & ])/f
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.43

method	result
default	$-\frac{\frac{\sqrt{c\,x^2+bx+a}}{c}-\frac{b\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{2c^{\frac{3}{2}}}}{f}+\frac{d\ln\left(\frac{\frac{-2b\sqrt{df}+2fa+2cd}{f}+\frac{(-2c\sqrt{df}+bf)\left(x+\frac{\sqrt{df}}{f}\right)}{f}+2\sqrt{\frac{-b\sqrt{df}+fa+cd}{f}}\sqrt{\left(x+\frac{\sqrt{df}}{f}\right)^2c+\cdots}}{x+\frac{\sqrt{df}}{f}}\right)}{2f^2\sqrt{\frac{-b\sqrt{df}+fa+cd}{f}}}$
risch	$-\frac{\sqrt{c\,x^2+bx+a}}{cf}+\frac{b\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{2c^{\frac{3}{2}}f}+\frac{d\ln\left(\frac{\frac{-2b\sqrt{df}+2fa+2cd}{f}+\frac{(-2c\sqrt{df}+bf)\left(x+\frac{\sqrt{df}}{f}\right)}{f}+2\sqrt{\frac{-b\sqrt{df}+fa+cd}{f}}\sqrt{\left(x+\frac{\sqrt{df}}{f}\right)^2\cdots}}{x+\frac{\sqrt{df}}{f}}\right)}{2f^2\sqrt{\frac{-b\sqrt{df}+fa+cd}{f}}}$

```
[In] int(x^3/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/f*(1/c*(c*x^2+b*x+a)^(1/2)-1/2*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b
*x+a)^(1/2)))+1/2*d/f^2/(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*ln((2/f*(-b*(d
*f)^(1/2)+f*a+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*
(d*f)^(1/2)+f*a+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*
f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1/2)/f)
)+1/2*d/f^2/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+f*a+c*d)
/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f*a+c*d)/f)^
(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(
d*f)^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = \text{Timed out}$$

```
[In] integrate(x^3/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{x^3}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = -\int \frac{x^3}{-d\sqrt{a+bx+cx^2}+fx^2\sqrt{a+bx+cx^2}}\,dx$$

```
[In] integrate(x**3/(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(x**3/(-d*sqrt(a + b*x + c*x**2) + f*x**2*sqrt(a + b*x + c*x**2)),
 x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^3/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see `
assume?
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = \int \frac{x^3}{(d-f\,x^2)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

```
[In] int(x^3/((d - f*x^2)*(a + b*x + c*x^2)^(1/2)),x)
```

```
[Out] int(x^3/((d - f*x^2)*(a + b*x + c*x^2)^(1/2)), x)
```

3.96 $\int \frac{x^2}{\sqrt{a+bx+cx^2}(d-fx^2)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 266

$$\int \frac{x^2}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = -\frac{\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f} + \frac{\sqrt{d}\operatorname{arctanh}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2f\sqrt{cd-b\sqrt{d}\sqrt{f}+af}} + \frac{\sqrt{d}\operatorname{arctanh}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2f\sqrt{cd+b\sqrt{d}\sqrt{f}+af}}$$

```
[Out] -arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/f/c^(1/2)+1/2*arctanh(1
/2*(b*d^(1/2)-2*a*f^(1/2)+x*(2*c*d^(1/2)-b*f^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c
*d+a*f-b*d^(1/2)*f^(1/2))^(1/2))*d^(1/2)/f/(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2
)+1/2*arctanh(1/2*(b*d^(1/2)+2*a*f^(1/2)+x*(2*c*d^(1/2)+b*f^(1/2)))/(c*x^2+
b*x+a)^(1/2)/(c*d+a*f+b*d^(1/2)*f^(1/2))^(1/2))*d^(1/2)/f/(c*d+a*f+b*d^(1/2
)*f^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {1093, 635, 212, 998, 738}

$$\int \frac{x^2}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = \frac{\sqrt{d}\operatorname{arctanh}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2f\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}} + \frac{\sqrt{d}\operatorname{arctanh}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2f\sqrt{af+b\sqrt{d}\sqrt{f}+cd}} - \frac{\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f}$$

```
[In] Int[x^2/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] -(ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])]/(Sqrt[c]*f)) + (Sq
rt[d]*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sq
rt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*f*Sqrt[c*d -
b*Sqrt[d]*Sqrt[f] + a*f]) + (Sqrt[d]*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*
c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b
*x + c*x^2])])/(2*f*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 998

```
Int[1/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Sy
mbol] :> Dist[1/2, Int[1/((a - Rt[(-a)*c, 2]*x)*Sqrt[d + e*x + f*x^2]), x],
 x] + Dist[1/2, Int[1/((a + Rt[(-a)*c, 2]*x)*Sqrt[d + e*x + f*x^2]), x], x]
 /; FreeQ[{a, c, d, e, f}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[(-a)*c]
```

Rule 1093

```
Int[((A_.) + (C_.)*(x_)^2)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) +
 (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + e*x + f*x^2], x], x]
 + Dist[(A*c - a*C)/c, Int[1/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /;
 FreeQ[{a, c, d, e, f, A, C}, x] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\text{integral} = -\frac{\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{f} + \frac{d\int \frac{1}{\sqrt{a+bx+cx^2}(d-fx^2)}\,dx}{f}$$

$$= -\frac{2\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{f} + \frac{d\int \frac{1}{\left(d-\sqrt{d}\sqrt{f}x\right)\sqrt{a+bx+cx^2}}\,dx}{2f} + \frac{d\int \frac{1}{\left(d+\sqrt{d}\sqrt{f}x\right)\sqrt{a+bx+cx^2}}\,dx}{2f}$$

$$= -\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f} - \frac{d\text{Subst}\left(\int \frac{1}{4cd^2-4bd^{3/2}\sqrt{f}+4adf-x^2}\,dx, x, \frac{-bd+2a\sqrt{d}\sqrt{f}-\left(2cd-b\sqrt{d}\sqrt{f}\right)x}{\sqrt{a+bx+cx^2}}\right)}{f} - \frac{d\text{Subst}\left(\int \frac{1}{4cd^2+4bd^{3/2}\sqrt{f}+4adf-x^2}\,dx, x, \frac{-bd-2a\sqrt{d}\sqrt{f}-\left(2cd+b\sqrt{d}\sqrt{f}\right)x}{\sqrt{a+bx+cx^2}}\right)}{f}$$

$$= -\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f} + \frac{\sqrt{d}\tanh^{-1}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2f\sqrt{cd-b\sqrt{d}\sqrt{f}+af}} + \frac{\sqrt{d}\tanh^{-1}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2f\sqrt{cd+b\sqrt{d}\sqrt{f}+af}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.26 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.73

$$\int \frac{x^2}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx$$
$$= \frac{\frac{2\log\left(f\left(b+2cx-2\sqrt{c}\sqrt{a+x(b+cx)}\right)\right)}{\sqrt{c}} - d\text{RootSum}\left[b^2d-a^2f-4b\sqrt{c}d\#1+4cd\#1^2+2af\#1^2-f\#1^4\&,\frac{b\log\left(-\sqrt{\right.}\right.}{}\right.}{2f}$$

```
[In] Integrate[x^2/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] ((2*Log[f*(b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/Sqrt[c] - d*RootS
um[b^2*d - a^2*f - 4*b*Sqrt[c]*d*#1 + 4*c*d*#1^2 + 2*a*f*#1^2 - f*#1^4 & ,
(b*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*Sqrt[c]*Log[-(Sqrt[c]
*x) + Sqrt[a + b*x + c*x^2] - #1]*#1)/(b*Sqrt[c]*d - 2*c*d*#1 - a*f*#1 + f*
#1^3) & ])/(2*f)
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.50

method	result
default	$-\frac{\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{f\sqrt{c}} - \frac{d\ln\left(\frac{\frac{-2b\sqrt{df}+2fa+2cd}{f}+\frac{\left(-2c\sqrt{df}+bf\right)\left(x+\frac{\sqrt{df}}{f}\right)}{f}+2\sqrt{\frac{-b\sqrt{df}+fa+cd}{f}}\sqrt{\left(x+\frac{\sqrt{df}}{f}\right)^2c+\frac{\left(-2c\sqrt{df}+bf\right)\left(x-\right.}{f}}}{x+\frac{\sqrt{df}}{f}}\right.}{2\sqrt{df}\,f\sqrt{\frac{-b\sqrt{df}+fa+cd}{f}}}$

```
[In] int(x^2/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/f*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-1/2*d/(d*f)^(1/2)/
f/(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1/f
*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^
(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1
/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1/2)/f))+1/2*d/(d*f)^(1/2)/f/
((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+f*a+c*d)/f+(2*c*(d*f
)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*((x-(d
*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f
*a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = \text{Timed out}$$

```
[In] integrate(x^2/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = -\int \frac{x^2}{-d\sqrt{a+bx+cx^2}+fx^2\sqrt{a+bx+cx^2}}\,dx$$

```
[In] integrate(x**2/(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
[Out] -Integral(x**2/(-d*sqrt(a + b*x + c*x**2) + f*x**2*sqrt(a + b*x + c*x**2)),
 x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see `
assume?
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^2}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^2/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx=\int \frac{x^2}{(d-f\,x^2)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

```
[In] int(x^2/((d - f*x^2)*(a + b*x + c*x^2)^(1/2)),x)
[Out] int(x^2/((d - f*x^2)*(a + b*x + c*x^2)^(1/2)), x)
```

3.97 $\int \frac{x}{\sqrt{a+bx+cx^2}(d-fx^2)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 220

$$\int \frac{x}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = -\frac{\operatorname{arctanh}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{f}\sqrt{cd-b\sqrt{d}\sqrt{f}+af}} + \frac{\operatorname{arctanh}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{f}\sqrt{cd+b\sqrt{d}\sqrt{f}+af}}$$

```
[Out] -1/2*arctanh(1/2*(b*d^(1/2)-2*a*f^(1/2)+x*(2*c*d^(1/2)-b*f^(1/2)))/(c*x^2+b
*x+a)^(1/2)/(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2))/f^(1/2)/(c*d+a*f-b*d^(1/2)*f
^(1/2))^(1/2)+1/2*arctanh(1/2*(b*d^(1/2)+2*a*f^(1/2)+x*(2*c*d^(1/2)+b*f^(1/
2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f+b*d^(1/2)*f^(1/2))^(1/2))/f^(1/2)/(c*d+a*
f+b*d^(1/2)*f^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1047, 738, 212}

$$\int \frac{x}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = \frac{\operatorname{arctanh}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\sqrt{f}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}} - \frac{\operatorname{arctanh}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\sqrt{f}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}$$

```
[In] Int[x/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] -1/2*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqr
t[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])]/(Sqrt[f]*Sqrt[c*d
- b*Sqrt[d]*Sqrt[f] + a*f]) + ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[
d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*
x^2])]/(2*Sqrt[f]*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1047

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(-a)*c, 2]}, Dist[h/2 + c*(g/(2*q)
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - c*(g/(2*q)
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[(-a)*c]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\int \frac{1}{\left(-\sqrt{d}\sqrt{f}-fx\right)\sqrt{a+bx+cx^2}}\,dx + \frac{1}{2}\int \frac{1}{\left(\sqrt{d}\sqrt{f}-fx\right)\sqrt{a+bx+cx^2}}\,dx$$

$$= -\text{Subst}\left(\int \frac{1}{4cdf-4b\sqrt{d}f^{3/2}+4af^2-x^2}\,dx, x, \frac{b\sqrt{d}\sqrt{f}-2af-\left(-2c\sqrt{d}\sqrt{f}+bf\right)x}{\sqrt{a+bx+cx^2}}\right)$$

$$-\text{Subst}\left(\int \frac{1}{4cdf+4b\sqrt{d}f^{3/2}+4af^2-x^2}\,dx, x, \frac{-b\sqrt{d}\sqrt{f}-2af-\left(2c\sqrt{d}\sqrt{f}+bf\right)x}{\sqrt{a+bx+cx^2}}\right)$$

$$= -\frac{\tanh^{-1}\left(\frac{b\sqrt{d}\sqrt{f}-2af-\left(-2c\sqrt{d}\sqrt{f}+bf\right)x}{2\sqrt{f}\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{f}\sqrt{cd-b\sqrt{d}\sqrt{f}+af}} - \frac{\tanh^{-1}\left(\frac{-b\sqrt{d}\sqrt{f}-2af-\left(2c\sqrt{d}\sqrt{f}+bf\right)x}{2\sqrt{f}\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{f}\sqrt{cd+b\sqrt{d}\sqrt{f}+af}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.20 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.68

$$\int \frac{x}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = -\frac{1}{2}\text{RootSum}\left[b^2d-a^2f-4b\sqrt{c}d\#1+4cd\#1^2+2af\#1^2 -f\#1^4\&, \frac{a\log\left(-\sqrt{c}x+\sqrt{a+bx+cx^2}-\#1\right)-\log\left(-\sqrt{c}x+\sqrt{a+bx+cx^2}-\#1\right)\#1^2}{-b\sqrt{c}d+2cd\#1+af\#1-f\#1^3}\&\right]$$

```
[In] Integrate[x/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] -1/2*RootSum[b^2*d - a^2*f - 4*b*Sqrt[c]*d*#1 + 4*c*d*#1^2 + 2*a*f*#1^2 - f
*#1^4 & , (a*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - Log[-(Sqrt[c]
*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2)/(-(b*Sqrt[c]*d) + 2*c*d*#1 + a*f*#1
 - f*#1^3) & ]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 353 vs. 2(164) = 328.

Time = 0.82 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.61

method	result
default	$\frac{\ln\left(\frac{\frac{2b\sqrt{df}+2fa+2cd}{f}+\frac{(2c\sqrt{df}+bf)\left(x-\frac{\sqrt{df}}{f}\right)}{f}+2\sqrt{\frac{b\sqrt{df}+fa+cd}{f}}\sqrt{\left(x-\frac{\sqrt{df}}{f}\right)^{2}c+\frac{(2c\sqrt{df}+bf)\left(x-\frac{\sqrt{df}}{f}\right)}{f}+\frac{b\sqrt{df}+fa+cd}{f}}}{x-\frac{\sqrt{df}}{f}}\right)}{2f\sqrt{\frac{b\sqrt{df}+fa+cd}{f}}}+\frac{\ln\left(\frac{\frac{-2b\sqrt{df}+}{f}}{}\right.}{}$

```
[In] int(x/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/f/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+f*a+c*d)/f+(2*
c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*
((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(
1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))+1/2/f/(1/f*(-b*(d*f)^(1/2)+f*a+c
*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(
d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c
+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))
^(1/2))/(x+(d*f)^(1/2)/f))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2753 vs. $2(164) = 328$.

Time = 0.65 (sec) , antiderivative size = 2753, normalized size of antiderivative = 12.51

$$\int \frac{x}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = \text{Too large to display}$$

```
[In] integrate(x/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt((c*d + a*f + (c^2*d^2*f + a^2*f^3 - (b^2 - 2*a*c)*d*f^2)*sqrt(b^2*
d/(c^4*d^4*f + a^4*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*a*b^2*c +
 6*a^2*c^2)*d^2*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d*f^4)))/(c^2*d^2*f + a^2*f^3 -
 (b^2 - 2*a*c)*d*f^2))*log((2*b*c*d*x + b^2*d + 2*(b^2*d*f - (c^3*d^3*f + a
^3*f^4 - (b^2*c - 3*a*c^2)*d^2*f^2 - (a*b^2 - 3*a^2*c)*d*f^3)*sqrt(b^2*d/(c
^4*d^4*f + a^4*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*a*b^2*c + 6*a
^2*c^2)*d^2*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d*f^4)))*sqrt(c*x^2 + b*x + a)*sqrt
((c*d + a*f + (c^2*d^2*f + a^2*f^3 - (b^2 - 2*a*c)*d*f^2)*sqrt(b^2*d/(c^4*d
^4*f + a^4*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c
^2)*d^2*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d*f^4)))/(c^2*d^2*f + a^2*f^3 - (b^2 -
2*a*c)*d*f^2)) - (2*a*c^2*d^2*f + 2*a^3*f^3 - 2*(a*b^2 - 2*a^2*c)*d*f^2 + (
b*c^2*d^2*f + a^2*b*f^3 - (b^3 - 2*a*b*c)*d*f^2)*x)*sqrt(b^2*d/(c^4*d^4*f +
 a^4*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^
2*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d*f^4)))/x) - 1/4*sqrt((c*d + a*f + (c^2*d^2*
f + a^2*f^3 - (b^2 - 2*a*c)*d*f^2)*sqrt(b^2*d/(c^4*d^4*f + a^4*f^5 - 2*(b^2
*c^2 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^2*f^3 - 2*(a^2*b^
```

```
2 - 2*a^3*c)*d*f^4)))/(c^2*d^2*f + a^2*f^3 - (b^2 - 2*a*c)*d*f^2))*log((2*b
*c*d*x + b^2*d - 2*(b^2*d*f - (c^3*d^3*f + a^3*f^4 - (b^2*c - 3*a*c^2)*d^2*
f^2 - (a*b^2 - 3*a^2*c)*d*f^3)*sqrt(b^2*d/(c^4*d^4*f + a^4*f^5 - 2*(b^2*c^2
 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^2*f^3 - 2*(a^2*b^2 -
2*a^3*c)*d*f^4)))*sqrt(c*x^2 + b*x + a)*sqrt((c*d + a*f + (c^2*d^2*f + a^2*
f^3 - (b^2 - 2*a*c)*d*f^2)*sqrt(b^2*d/(c^4*d^4*f + a^4*f^5 - 2*(b^2*c^2 - 2
*a*c^3)*d^3*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^2*f^3 - 2*(a^2*b^2 - 2*a^
3*c)*d*f^4)))/(c^2*d^2*f + a^2*f^3 - (b^2 - 2*a*c)*d*f^2)) - (2*a*c^2*d^2*f
 + 2*a^3*f^3 - 2*(a*b^2 - 2*a^2*c)*d*f^2 + (b*c^2*d^2*f + a^2*b*f^3 - (b^3
- 2*a*b*c)*d*f^2)*x)*sqrt(b^2*d/(c^4*d^4*f + a^4*f^5 - 2*(b^2*c^2 - 2*a*c^3
)*d^3*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^2*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d
*f^4)))/x) + 1/4*sqrt((c*d + a*f - (c^2*d^2*f + a^2*f^3 - (b^2 - 2*a*c)*d*f
^2)*sqrt(b^2*d/(c^4*d^4*f + a^4*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^3*f^2 + (b^4
- 4*a*b^2*c + 6*a^2*c^2)*d^2*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d*f^4)))/(c^2*d^2*
f + a^2*f^3 - (b^2 - 2*a*c)*d*f^2))*log((2*b*c*d*x + b^2*d + 2*(b^2*d*f + (
c^3*d^3*f + a^3*f^4 - (b^2*c - 3*a*c^2)*d^2*f^2 - (a*b^2 - 3*a^2*c)*d*f^3)*
sqrt(b^2*d/(c^4*d^4*f + a^4*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*
a*b^2*c + 6*a^2*c^2)*d^2*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d*f^4)))*sqrt(c*x^2 +
b*x + a)*sqrt((c*d + a*f - (c^2*d^2*f + a^2*f^3 - (b^2 - 2*a*c)*d*f^2)*sqrt
(b^2*d/(c^4*d^4*f + a^4*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*a*b^
2*c + 6*a^2*c^2)*d^2*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d*f^4)))/(c^2*d^2*f + a^2*
f^3 - (b^2 - 2*a*c)*d*f^2)) + (2*a*c^2*d^2*f + 2*a^3*f^3 - 2*(a*b^2 - 2*a^2
*c)*d*f^2 + (b*c^2*d^2*f + a^2*b*f^3 - (b^3 - 2*a*b*c)*d*f^2)*x)*sqrt(b^2*d
/(c^4*d^4*f + a^4*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*a*b^2*c +
6*a^2*c^2)*d^2*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d*f^4)))/x) - 1/4*sqrt((c*d + a*
f - (c^2*d^2*f + a^2*f^3 - (b^2 - 2*a*c)*d*f^2)*sqrt(b^2*d/(c^4*d^4*f + a^4
*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^2*f^
3 - 2*(a^2*b^2 - 2*a^3*c)*d*f^4)))/(c^2*d^2*f + a^2*f^3 - (b^2 - 2*a*c)*d*f
^2))*log((2*b*c*d*x + b^2*d - 2*(b^2*d*f + (c^3*d^3*f + a^3*f^4 - (b^2*c -
3*a*c^2)*d^2*f^2 - (a*b^2 - 3*a^2*c)*d*f^3)*sqrt(b^2*d/(c^4*d^4*f + a^4*f^5
 - 2*(b^2*c^2 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^2*f^3 -
2*(a^2*b^2 - 2*a^3*c)*d*f^4)))*sqrt(c*x^2 + b*x + a)*sqrt((c*d + a*f - (c^2
*d^2*f + a^2*f^3 - (b^2 - 2*a*c)*d*f^2)*sqrt(b^2*d/(c^4*d^4*f + a^4*f^5 - 2
*(b^2*c^2 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^2*f^3 - 2*(a
^2*b^2 - 2*a^3*c)*d*f^4)))/(c^2*d^2*f + a^2*f^3 - (b^2 - 2*a*c)*d*f^2)) + (
2*a*c^2*d^2*f + 2*a^3*f^3 - 2*(a*b^2 - 2*a^2*c)*d*f^2 + (b*c^2*d^2*f + a^2*
b*f^3 - (b^3 - 2*a*b*c)*d*f^2)*x)*sqrt(b^2*d/(c^4*d^4*f + a^4*f^5 - 2*(b^2*
c^2 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^2*f^3 - 2*(a^2*b^2
 - 2*a^3*c)*d*f^4)))/x)
```

Sympy [F]

$$\int \frac{x}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = -\int \frac{x}{-d\sqrt{a+bx+cx^2}+fx^2\sqrt{a+bx+cx^2}}\,dx$$

```
[In] integrate(x/(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(x/(-d*sqrt(a + b*x + c*x**2) + f*x**2*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see `
assume?
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument ValueBad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx=\int \frac{x}{\left(d-f\,x^2\right)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

```
[In] int(x/((d - f*x^2)*(a + b*x + c*x^2)^(1/2)),x)
[Out] int(x/((d - f*x^2)*(a + b*x + c*x^2)^(1/2)), x)
```

3.98 $\int \frac{1}{\sqrt{a+bx+cx^2}(d-fx^2)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 220

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = \frac{\operatorname{arctanh}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{d}\sqrt{cd-b\sqrt{d}\sqrt{f}+af}} + \frac{\operatorname{arctanh}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{d}\sqrt{cd+b\sqrt{d}\sqrt{f}+af}}$$

```
[Out] 1/2*arctanh(1/2*(b*d^(1/2)-2*a*f^(1/2)+x*(2*c*d^(1/2)-b*f^(1/2)))/(c*x^2+b*
x+a)^(1/2)/(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2))/d^(1/2)/(c*d+a*f-b*d^(1/2)*f^
(1/2))^(1/2)+1/2*arctanh(1/2*(b*d^(1/2)+2*a*f^(1/2)+x*(2*c*d^(1/2)+b*f^(1/2
)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f+b*d^(1/2)*f^(1/2))^(1/2))/d^(1/2)/(c*d+a*f
+b*d^(1/2)*f^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {998, 738, 212}

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = \frac{\operatorname{arctanh}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\sqrt{d}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}} + \frac{\operatorname{arctanh}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\sqrt{d}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}$$

```
[In] Int[1/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d
 - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])]/(2*Sqrt[d]*Sqrt[c*d - b
*Sqrt[d]*Sqrt[f] + a*f]) + ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d]
+ b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2
])]/(2*Sqrt[d]*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 998

```
Int[1/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Sy
mbol] :> Dist[1/2, Int[1/((a - Rt[(-a)*c, 2]*x)*Sqrt[d + e*x + f*x^2]), x],
 x] + Dist[1/2, Int[1/((a + Rt[(-a)*c, 2]*x)*Sqrt[d + e*x + f*x^2]), x], x]
 /; FreeQ[{a, c, d, e, f}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[(-a)*c]
```

Rubi steps

$$\text{integral} = \frac{1}{2}\int \frac{1}{\left(d-\sqrt{d}\sqrt{f}x\right)\sqrt{a+bx+cx^2}}\,dx + \frac{1}{2}\int \frac{1}{\left(d+\sqrt{d}\sqrt{f}x\right)\sqrt{a+bx+cx^2}}\,dx$$

$$= -\text{Subst}\left(\int \frac{1}{4cd^2-4bd^{3/2}\sqrt{f}+4adf-x^2}\,dx, x, \frac{-bd+2a\sqrt{d}\sqrt{f}-\left(2cd-b\sqrt{d}\sqrt{f}\right)x}{\sqrt{a+bx+cx^2}}\right)$$

$$-\text{Subst}\left(\int \frac{1}{4cd^2+4bd^{3/2}\sqrt{f}+4adf-x^2}\,dx, x, \frac{-bd-2a\sqrt{d}\sqrt{f}-\left(2cd+b\sqrt{d}\sqrt{f}\right)x}{\sqrt{a+bx+cx^2}}\right)$$

$$= -\frac{\tanh^{-1}\left(\frac{-bd+2a\sqrt{d}\sqrt{f}-\left(2cd-b\sqrt{d}\sqrt{f}\right)x}{2\sqrt{d}\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{d}\sqrt{cd-b\sqrt{d}\sqrt{f}+af}} - \frac{\tanh^{-1}\left(\frac{-bd-2a\sqrt{d}\sqrt{f}-\left(2cd+b\sqrt{d}\sqrt{f}\right)x}{2\sqrt{d}\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{d}\sqrt{cd+b\sqrt{d}\sqrt{f}+af}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.23 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.73

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = -\frac{1}{2}\text{RootSum}\Big[c^2d-b^2f+4\sqrt{a}bf\#1-2cd\#1^2-4af\#1^2$$
$$+d\#1^4\&, \frac{-c\log(x)+c\log\left(-\sqrt{a}+\sqrt{a+bx+cx^2}-x\#1\right)+\log(x)\#1^2-\log\left(-\sqrt{a}+\sqrt{a+bx+cx^2}-\right.}{-\sqrt{a}bf+cd\#1+2af\#1-d\#1^3}$$

```
[In] Integrate[1/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] -1/2*RootSum[c^2*d - b^2*f + 4*Sqrt[a]*b*f*#1 - 2*c*d*#1^2 - 4*a*f*#1^2 + d
*#1^4 & , (-(c*Log[x]) + c*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] + L
og[x]*#1^2 - Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1]*#1^2)/(-(Sqrt[a]*
b*f) + c*d*#1 + 2*a*f*#1 - d*#1^3) & ]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 357 vs. 2(164) = 328.

Time = 0.79 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.63

method	result
default	$\frac{\ln\left(\frac{\frac{2b\sqrt{df}+2fa+2cd}{f}+\frac{\left(2c\sqrt{df}+bf\right)\left(x-\frac{\sqrt{df}}{f}\right)}{f}+2\sqrt{\frac{b\sqrt{df}+fa+cd}{f}}\sqrt{\left(x-\frac{\sqrt{df}}{f}\right)^{2}c+\frac{\left(2c\sqrt{df}+bf\right)\left(x-\frac{\sqrt{df}}{f}\right)}{f}+\frac{b\sqrt{df}+fa+cd}{f}}}{x-\frac{\sqrt{df}}{f}}\right)}{2\sqrt{df}\,\sqrt{\frac{b\sqrt{df}+fa+cd}{f}}}-\frac{\ln\left(\frac{\frac{-2b\sqrt{df}+2f}{f}}{}\right.}{}$

```
[In] int(1/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/(d*f)^(1/2)/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+f*a+
c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f*a+c*d)
/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+
(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))-1/2/(d*f)^(1/2)/(1/f*(
-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1/f*(-2*c*(
d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*((
x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(
d*f)^(1/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1/2)/f))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2641 vs. $2(164) = 328$.

Time = 0.72 (sec) , antiderivative size = 2641, normalized size of antiderivative = 12.00

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt((c*d + a*f + (c^2*d^3 + a^2*d*f^2 - (b^2 - 2*a*c)*d^2*f)*sqrt(b^2*
f/(c^4*d^5 + a^4*d*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f + (b^4 - 4*a*b^2*c + 6
*a^2*c^2)*d^3*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^3)))/(c^2*d^3 + a^2*d*f^2 -
 (b^2 - 2*a*c)*d^2*f))*log((2*b*c*x + b^2 + 2*(b*c*d + a*b*f - (b*c^2*d^3 +
 a^2*b*d*f^2 - (b^3 - 2*a*b*c)*d^2*f)*sqrt(b^2*f/(c^4*d^5 + a^4*d*f^4 - 2*(
b^2*c^2 - 2*a*c^3)*d^4*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^2 - 2*(a^2*b
^2 - 2*a^3*c)*d^2*f^3)))*sqrt(c*x^2 + b*x + a)*sqrt((c*d + a*f + (c^2*d^3 +
 a^2*d*f^2 - (b^2 - 2*a*c)*d^2*f)*sqrt(b^2*f/(c^4*d^5 + a^4*d*f^4 - 2*(b^2*
c^2 - 2*a*c^3)*d^4*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^2 - 2*(a^2*b^2 -
 2*a^3*c)*d^2*f^3)))/(c^2*d^3 + a^2*d*f^2 - (b^2 - 2*a*c)*d^2*f)) - (2*a*c^
2*d^2 + 2*a^3*f^2 - 2*(a*b^2 - 2*a^2*c)*d*f + (b*c^2*d^2 + a^2*b*f^2 - (b^3
 - 2*a*b*c)*d*f)*x)*sqrt(b^2*f/(c^4*d^5 + a^4*d*f^4 - 2*(b^2*c^2 - 2*a*c^3)
*d^4*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^2*
f^3)))/x) - 1/4*sqrt((c*d + a*f + (c^2*d^3 + a^2*d*f^2 - (b^2 - 2*a*c)*d^2*
f)*sqrt(b^2*f/(c^4*d^5 + a^4*d*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f + (b^4 - 4
*a*b^2*c + 6*a^2*c^2)*d^3*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^3)))/(c^2*d^3 +
```

```
 a^2*d*f^2 - (b^2 - 2*a*c)*d^2*f))*log((2*b*c*x + b^2 - 2*(b*c*d + a*b*f -
(b*c^2*d^3 + a^2*b*d*f^2 - (b^3 - 2*a*b*c)*d^2*f)*sqrt(b^2*f/(c^4*d^5 + a^4
*d*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^
2 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^3)))*sqrt(c*x^2 + b*x + a)*sqrt((c*d + a*f
+ (c^2*d^3 + a^2*d*f^2 - (b^2 - 2*a*c)*d^2*f)*sqrt(b^2*f/(c^4*d^5 + a^4*d*f
^4 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^2 -
2*(a^2*b^2 - 2*a^3*c)*d^2*f^3)))/(c^2*d^3 + a^2*d*f^2 - (b^2 - 2*a*c)*d^2*f
)) - (2*a*c^2*d^2 + 2*a^3*f^2 - 2*(a*b^2 - 2*a^2*c)*d*f + (b*c^2*d^2 + a^2*
b*f^2 - (b^3 - 2*a*b*c)*d*f)*x)*sqrt(b^2*f/(c^4*d^5 + a^4*d*f^4 - 2*(b^2*c^
2 - 2*a*c^3)*d^4*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^2 - 2*(a^2*b^2 - 2
*a^3*c)*d^2*f^3)))/x) + 1/4*sqrt((c*d + a*f - (c^2*d^3 + a^2*d*f^2 - (b^2 -
 2*a*c)*d^2*f)*sqrt(b^2*f/(c^4*d^5 + a^4*d*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^4*
f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^3))
)/(c^2*d^3 + a^2*d*f^2 - (b^2 - 2*a*c)*d^2*f))*log((2*b*c*x + b^2 + 2*(b*c*
d + a*b*f + (b*c^2*d^3 + a^2*b*d*f^2 - (b^3 - 2*a*b*c)*d^2*f)*sqrt(b^2*f/(c
^4*d^5 + a^4*d*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f + (b^4 - 4*a*b^2*c + 6*a^2
*c^2)*d^3*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^3)))*sqrt(c*x^2 + b*x + a)*sqrt
((c*d + a*f - (c^2*d^3 + a^2*d*f^2 - (b^2 - 2*a*c)*d^2*f)*sqrt(b^2*f/(c^4*d
^5 + a^4*d*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2
)*d^3*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^3)))/(c^2*d^3 + a^2*d*f^2 - (b^2 -
2*a*c)*d^2*f)) + (2*a*c^2*d^2 + 2*a^3*f^2 - 2*(a*b^2 - 2*a^2*c)*d*f + (b*c^
2*d^2 + a^2*b*f^2 - (b^3 - 2*a*b*c)*d*f)*x)*sqrt(b^2*f/(c^4*d^5 + a^4*d*f^4
 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^2 - 2*
(a^2*b^2 - 2*a^3*c)*d^2*f^3)))/x) - 1/4*sqrt((c*d + a*f - (c^2*d^3 + a^2*d*
f^2 - (b^2 - 2*a*c)*d^2*f)*sqrt(b^2*f/(c^4*d^5 + a^4*d*f^4 - 2*(b^2*c^2 - 2
*a*c^3)*d^4*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^2 - 2*(a^2*b^2 - 2*a^3*
c)*d^2*f^3)))/(c^2*d^3 + a^2*d*f^2 - (b^2 - 2*a*c)*d^2*f))*log((2*b*c*x + b
^2 - 2*(b*c*d + a*b*f + (b*c^2*d^3 + a^2*b*d*f^2 - (b^3 - 2*a*b*c)*d^2*f)*s
qrt(b^2*f/(c^4*d^5 + a^4*d*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f + (b^4 - 4*a*b
^2*c + 6*a^2*c^2)*d^3*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^3)))*sqrt(c*x^2 + b
*x + a)*sqrt((c*d + a*f - (c^2*d^3 + a^2*d*f^2 - (b^2 - 2*a*c)*d^2*f)*sqrt(
b^2*f/(c^4*d^5 + a^4*d*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f + (b^4 - 4*a*b^2*c
 + 6*a^2*c^2)*d^3*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^3)))/(c^2*d^3 + a^2*d*f
^2 - (b^2 - 2*a*c)*d^2*f)) + (2*a*c^2*d^2 + 2*a^3*f^2 - 2*(a*b^2 - 2*a^2*c)
*d*f + (b*c^2*d^2 + a^2*b*f^2 - (b^3 - 2*a*b*c)*d*f)*x)*sqrt(b^2*f/(c^4*d^5
 + a^4*d*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*
d^3*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^3)))/x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = -\int \frac{1}{-d\sqrt{a+bx+cx^2}+fx^2\sqrt{a+bx+cx^2}}\,dx$$

```
[In] integrate(1/(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
[Out] -Integral(1/(-d*sqrt(a + b*x + c*x**2) + f*x**2*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see `
assume?
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument ValueBad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx=\int \frac{1}{(d-f\,x^2)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

```
[In] int(1/((d - f*x^2)*(a + b*x + c*x^2)^(1/2)),x)
[Out] int(1/((d - f*x^2)*(a + b*x + c*x^2)^(1/2)), x)
```

3.99 $\int \frac{1}{x\sqrt{a+bx+cx^2}(d-fx^2)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 267

$$\int \frac{1}{x\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = -\frac{\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}d} - \frac{\sqrt{f}\operatorname{arctanh}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2d\sqrt{cd-b\sqrt{d}\sqrt{f}+af}} + \frac{\sqrt{f}\operatorname{arctanh}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2d\sqrt{cd+b\sqrt{d}\sqrt{f}+af}}$$

```
[Out] -arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/d/a^(1/2)-1/2*arctanh(1
/2*(b*d^(1/2)-2*a*f^(1/2)+x*(2*c*d^(1/2)-b*f^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c
*d+a*f-b*d^(1/2)*f^(1/2))^(1/2))*f^(1/2)/d/(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2
)+1/2*arctanh(1/2*(b*d^(1/2)+2*a*f^(1/2)+x*(2*c*d^(1/2)+b*f^(1/2)))/(c*x^2+
b*x+a)^(1/2)/(c*d+a*f+b*d^(1/2)*f^(1/2))^(1/2))*f^(1/2)/d/(c*d+a*f+b*d^(1/2
)*f^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {6857, 738, 212, 1047}

$$\int \frac{1}{x\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = -\frac{\sqrt{f}\,\text{arctanh}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2d\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}} + \frac{\sqrt{f}\,\text{arctanh}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2d\sqrt{af+b\sqrt{d}\sqrt{f}+cd}} - \frac{\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}d}$$

```
[In] Int[1/(x*Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] -(ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])]/(Sqrt[a]*d)) - (Sq
rt[f]*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sq
rt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d*Sqrt[c*d -
b*Sqrt[d]*Sqrt[f] + a*f]) + (Sqrt[f]*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*
c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b
*x + c*x^2])])/(2*d*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1047

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(-a)*c, 2]}, Dist[h/2 + c*(g/(2*q)
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - c*(g/(2*q)
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
```

```
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[(-a)*c]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{1}{dx\sqrt{a+bx+cx^2}} - \frac{fx}{d\sqrt{a+bx+cx^2}\,(-d+fx^2)}\right) dx \\
&= \frac{\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx}{d} - \frac{f\int \frac{x}{\sqrt{a+bx+cx^2}(-d+fx^2)}\,dx}{d} \\
&= -\frac{2\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx}{\sqrt{a+bx+cx^2}}\right)}{d} \\
&\quad - \frac{f\int \frac{1}{\left(-\sqrt{d}\sqrt{f}+fx\right)\sqrt{a+bx+cx^2}}\,dx}{2d} - \frac{f\int \frac{1}{\left(\sqrt{d}\sqrt{f}+fx\right)\sqrt{a+bx+cx^2}}\,dx}{2d} \\
&= -\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}d} \\
&\quad + \frac{f\text{Subst}\left(\int \frac{1}{4cdf-4b\sqrt{d}f^{3/2}+4af^2-x^2}\,dx, x, \frac{-b\sqrt{d}\sqrt{f}+2af-\left(2c\sqrt{d}\sqrt{f}-bf\right)x}{\sqrt{a+bx+cx^2}}\right)}{d} \\
&\quad + \frac{f\text{Subst}\left(\int \frac{1}{4cdf+4b\sqrt{d}f^{3/2}+4af^2-x^2}\,dx, x, \frac{b\sqrt{d}\sqrt{f}+2af-\left(-2c\sqrt{d}\sqrt{f}-bf\right)x}{\sqrt{a+bx+cx^2}}\right)}{d} \\
&= -\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}d} - \frac{\sqrt{f}\tanh^{-1}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2d\sqrt{cd-b\sqrt{d}\sqrt{f}+af}} \\
&\quad + \frac{\sqrt{f}\tanh^{-1}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2d\sqrt{cd+b\sqrt{d}\sqrt{f}+af}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.23 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.72

$$\int \frac{1}{x\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx$$
$$= \frac{\frac{4\operatorname{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)}{\sqrt{a}} - f\text{RootSum}\left[b^2d-a^2f-4b\sqrt{c}d\#1+4cd\#1^2+2af\#1^2-f\#1^4\&, \frac{a\log\left(-\sqrt{c}x+\right.}{}\right.}{2d}$$

```
[In] Integrate[1/(x*Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] ((4*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)])/Sqrt[a]])/Sqrt[a] - f*RootS
um[b^2*d - a^2*f - 4*b*Sqrt[c]*d*#1 + 4*c*d*#1^2 + 2*a*f*#1^2 - f*#1^4 & ,
(a*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - Log[-(Sqrt[c]*x) + Sqrt
[a + b*x + c*x^2] - #1]*#1^2)/(-(b*Sqrt[c]*d) + 2*c*d*#1 + a*f*#1 - f*#1^3)
 & ])/(2*d)
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.46

method	result
default	$-\frac{\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)}{d\sqrt{a}} + \frac{\ln\left(\frac{\frac{-2b\sqrt{df}+2fa+2cd}{f}+\frac{(-2c\sqrt{df}+bf)\left(x+\frac{\sqrt{df}}{f}\right)}{f}+2\sqrt{\frac{-b\sqrt{df}+fa+cd}{f}}\sqrt{\left(x+\frac{\sqrt{df}}{f}\right)^2c+\frac{(-2c\sqrt{df}+bf)\left(\right.}{f}}}{x+\frac{\sqrt{df}}{f}}\right.}{2d\sqrt{\frac{-b\sqrt{df}+fa+cd}{f}}}$

```
[In] int(1/x/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d/a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)+1/2/d/(1/f*(-b*(
d*f)^(1/2)+f*a+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1/f*(-2*c*(d*f)
^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*((x+(d
*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)
^(1/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1/2)/f))+1/2/d/((b*(d*f)^(1/2)+f*a+c*d)/f
)^(1/2)*ln((2*(b*(d*f)^(1/2)+f*a+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1
/2)/f)+2*((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f
)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)
^(1/2)/f))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2993 vs. $2(203) = 406$.

Time = 47.36 (sec) , antiderivative size = 5995, normalized size of antiderivative = 22.45

$$\int \frac{1}{x\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = \text{Too large to display}$$

```
[In] integrate(1/x/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F]

$$\int \frac{1}{x\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = -\int \frac{1}{-dx\sqrt{a+bx+cx^2}+fx^3\sqrt{a+bx+cx^2}}\,dx$$

```
[In] integrate(1/x/(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
[Out] -Integral(1/(-d*x*sqrt(a + b*x + c*x**2) + f*x**3*sqrt(a + b*x + c*x**2)),
x)
```

Maxima [F]

$$\int \frac{1}{x\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = \int -\frac{1}{\sqrt{cx^2+bx+a}(fx^2-d)x}\,dx$$

```
[In] integrate(1/x/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")
[Out] -integrate(1/(sqrt(c*x^2 + b*x + a)*(f*x^2 - d)*x), x)
```

Giac [F]

$$\int \frac{1}{x\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = \int -\frac{1}{\sqrt{cx^2+bx+a}(fx^2-d)x}\,dx$$

```
[In] integrate(1/x/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")
[Out] sage2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = \int \frac{1}{x\,\left(d-f\,x^2\right)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

```
[In] int(1/(x*(d - f*x^2)*(a + b*x + c*x^2)^(1/2)),x)
[Out] int(1/(x*(d - f*x^2)*(a + b*x + c*x^2)^(1/2)), x)
```

3.100 $\int \frac{1}{x^2\sqrt{a+bx+cx^2}(d-fx^2)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 291

$$\int \frac{1}{x^2\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = -\frac{\sqrt{a+bx+cx^2}}{adx}+\frac{b\,\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{3/2}d}+\frac{f\,\text{arctanh}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2d^{3/2}\sqrt{cd-b\sqrt{d}\sqrt{f}+af}}+\frac{f\,\text{arctanh}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2d^{3/2}\sqrt{cd+b\sqrt{d}\sqrt{f}+af}}$$

```
[Out] 1/2*b*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/a^(3/2)/d-(c*x^2+b
*x+a)^(1/2)/a/d/x+1/2*f*arctanh(1/2*(b*d^(1/2)-2*a*f^(1/2)+x*(2*c*d^(1/2)-b
*f^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2))/d^(3/2)/(
c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2)+1/2*f*arctanh(1/2*(b*d^(1/2)+2*a*f^(1/2)+x
*(2*c*d^(1/2)+b*f^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f+b*d^(1/2)*f^(1/2))^(
1/2))/d^(3/2)/(c*d+a*f+b*d^(1/2)*f^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {6857, 744, 738, 212, 998}

$$\int \frac{1}{x^2\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = \frac{b\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{3/2}d} + \frac{f\text{arctanh}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2d^{3/2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}} + \frac{f\text{arctanh}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2d^{3/2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}} - \frac{\sqrt{a+bx+cx^2}}{adx}$$

```
[In] Int[1/(x^2*Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] -(Sqrt[a + b*x + c*x^2]/(a*d*x)) + (b*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a
 + b*x + c*x^2])])/(2*a^(3/2)*d) + (f*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2
*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a +
b*x + c*x^2])])/(2*d^(3/2)*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]) + (f*ArcTan
h[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*S
qrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d^(3/2)*Sqrt[c*d + b*Sqrt
[d]*Sqrt[f] + a*f])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
 Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 998

```
Int[1/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Sy
mbol] :> Dist[1/2, Int[1/((a - Rt[(-a)*c, 2]*x)*Sqrt[d + e*x + f*x^2]), x],
 x] + Dist[1/2, Int[1/((a + Rt[(-a)*c, 2]*x)*Sqrt[d + e*x + f*x^2]), x], x]
 /; FreeQ[{a, c, d, e, f}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[(-a)*c]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{1}{dx^2\sqrt{a+bx+cx^2}}+\frac{f}{d\sqrt{a+bx+cx^2}\,(d-fx^2)}\right)dx\\
&= \frac{\int \frac{1}{x^2\sqrt{a+bx+cx^2}}\,dx}{d}+\frac{f\int \frac{1}{\sqrt{a+bx+cx^2}(d-fx^2)}\,dx}{d}\\
&= -\frac{\sqrt{a+bx+cx^2}}{adx}-\frac{b\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx}{2ad}\\
&\quad +\frac{f\int \frac{1}{\left(d-\sqrt{d}\sqrt{f}x\right)\sqrt{a+bx+cx^2}}\,dx}{2d}+\frac{f\int \frac{1}{\left(d+\sqrt{d}\sqrt{f}x\right)\sqrt{a+bx+cx^2}}\,dx}{2d}\\
&= -\frac{\sqrt{a+bx+cx^2}}{adx}+\frac{b\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx,x,\frac{2a+bx}{\sqrt{a+bx+cx^2}}\right)}{ad}\\
&\quad -\frac{f\text{Subst}\left(\int \frac{1}{4cd^2-4bd^{3/2}\sqrt{f}+4adf-x^2}\,dx,x,\frac{-bd+2a\sqrt{d}\sqrt{f}-\left(2cd-b\sqrt{d}\sqrt{f}\right)x}{\sqrt{a+bx+cx^2}}\right)}{d}\\
&\quad -\frac{f\text{Subst}\left(\int \frac{1}{4cd^2+4bd^{3/2}\sqrt{f}+4adf-x^2}\,dx,x,\frac{-bd-2a\sqrt{d}\sqrt{f}-\left(2cd+b\sqrt{d}\sqrt{f}\right)x}{\sqrt{a+bx+cx^2}}\right)}{d}
\end{aligned}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{adx} + \frac{b\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{3/2}d} + \frac{f\tanh^{-1}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2d^{3/2}\sqrt{cd-b\sqrt{d}\sqrt{f}+af}} + \frac{f\tanh^{-1}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2d^{3/2}\sqrt{cd+b\sqrt{d}\sqrt{f}+af}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.44 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.74

$$\int \frac{1}{x^2\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = -\frac{\frac{2\sqrt{a+x(b+cx)}}{ax} + \frac{2b\text{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)}{a^{3/2}} + f\text{RootSum}\left[b^2d-a^2f-4b\sqrt{c}d\#1+4cd\#1^2+2af\#1^2-f\right.}{2d}$$

```
[In] Integrate[1/(x^2*Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] -1/2*((2*Sqrt[a + x*(b + c*x)])/(a*x) + (2*b*ArcTanh[(Sqrt[c]*x - Sqrt[a +
x*(b + c*x)])/Sqrt[a]])/a^(3/2) + f*RootSum[b^2*d - a^2*f - 4*b*Sqrt[c]*d*#
1 + 4*c*d*#1^2 + 2*a*f*#1^2 - f*#1^4 & , (b*Log[-(Sqrt[c]*x) + Sqrt[a + b*x
 + c*x^2] - #1] - 2*Sqrt[c]*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*
#1)/(b*Sqrt[c]*d - 2*c*d*#1 - a*f*#1 + f*#1^3) & ])/d
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 426, normalized size of antiderivative = 1.46

method	result
default	$\frac{-\frac{\sqrt{c\,x^2+bx+a}}{ax}+\frac{b\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)}{2a^{\frac{3}{2}}}}{d} - \frac{f\ln\left(\frac{\frac{-2b\sqrt{df}+2fa+2cd}{f}+\frac{\left(-2c\sqrt{df}+bf\right)\left(x+\frac{\sqrt{df}}{f}\right)}{f}+2\sqrt{\frac{-b\sqrt{df}+fa+cd}{f}}\sqrt{\left(x+\frac{\sqrt{df}}{f}\right)^2c}}{x+\frac{\sqrt{df}}{f}}\right.}{2d\sqrt{df}\sqrt{\frac{-b\sqrt{df}+fa+cd}{f}}}$
risch	$-\frac{\sqrt{c\,x^2+bx+a}}{adx}+\frac{b\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)}{2a^{\frac{3}{2}}d}+\frac{f\ln\left(\frac{\frac{2b\sqrt{df}+2fa+2cd}{f}+\frac{\left(2c\sqrt{df}+bf\right)\left(x-\frac{\sqrt{df}}{f}\right)}{f}+2\sqrt{\frac{b\sqrt{df}+fa+cd}{f}}\sqrt{\left(x-\frac{\sqrt{df}}{f}\right)^2c}}{x-\frac{\sqrt{df}}{f}}\right.}{2d\sqrt{df}\sqrt{\frac{b\sqrt{df}+fa+cd}{f}}}$

```
[In] int(1/x^2/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/a/x*(c*x^2+b*x+a)^(1/2)+1/2*b/a^(3/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+
b*x+a)^(1/2))/x))-1/2*f/d/(d*f)^(1/2)/(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*
ln((2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/
f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*
(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/(x+
(d*f)^(1/2)/f))+1/2*f/d/(d*f)^(1/2)/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2
*(b*(d*f)^(1/2)+f*a+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*
(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/
f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3005 vs. $2(223) = 446$.

Time = 209.66 (sec) , antiderivative size = 6018, normalized size of antiderivative = 20.68

$$\int \frac{1}{x^2\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = \text{Too large to display}$$

```
[In] integrate(1/x^2/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F]

$$\int \frac{1}{x^2\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = -\int \frac{1}{-dx^2\sqrt{a+bx+cx^2}+fx^4\sqrt{a+bx+cx^2}}\,dx$$

```
[In] integrate(1/x**2/(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
[Out] -Integral(1/(-d*x**2*sqrt(a + b*x + c*x**2) + f*x**4*sqrt(a + b*x + c*x**2)
), x)
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = \int -\frac{1}{\sqrt{cx^2+bx+a}(fx^2-d)x^2}\,dx$$

```
[In] integrate(1/x^2/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")
[Out] -integrate(1/(sqrt(c*x^2 + b*x + a)*(f*x^2 - d)*x^2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x^2\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/x^2/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument ValueDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = \int \frac{1}{x^2\,\left(d-f\,x^2\right)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

```
[In] int(1/(x^2*(d - f*x^2)*(a + b*x + c*x^2)^(1/2)),x)
```

```
[Out] int(1/(x^2*(d - f*x^2)*(a + b*x + c*x^2)^(1/2)), x)
```

3.101 $\int \frac{1}{x^3\sqrt{a+bx+cx^2}(d-fx^2)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 376

$$\int \frac{1}{x^3\sqrt{a+bx+cx^2}(d-fx^2)}\,dx = -\frac{\sqrt{a+bx+cx^2}}{2adx^2}+\frac{3b\sqrt{a+bx+cx^2}}{4a^2dx} - \frac{(3b^2-4ac)\,\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{5/2}d} - \frac{f\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}d^2} - \frac{f^{3/2}\text{arctanh}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2d^2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}} + \frac{f^{3/2}\text{arctanh}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2d^2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}}$$

```
[Out] -1/8*(-4*a*c+3*b^2)*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/a^(5
/2)/d-f*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/d^2/a^(1/2)-1/2*
(c*x^2+b*x+a)^(1/2)/a/d/x^2+3/4*b*(c*x^2+b*x+a)^(1/2)/a^2/d/x-1/2*f^(3/2)*a
rctanh(1/2*(b*d^(1/2)-2*a*f^(1/2)+x*(2*c*d^(1/2)-b*f^(1/2)))/(c*x^2+b*x+a)^
(1/2)/(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2))/d^2/(c*d+a*f-b*d^(1/2)*f^(1/2))^(1
/2)+1/2*f^(3/2)*arctanh(1/2*(b*d^(1/2)+2*a*f^(1/2)+x*(2*c*d^(1/2)+b*f^(1/2)
))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f+b*d^(1/2)*f^(1/2))^(1/2))/d^2/(c*d+a*f+b*d^
(1/2)*f^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {6857, 758, 820, 738, 212, 1047}

$$\int \frac{1}{x^3\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = -\frac{(3b^2-4ac)\,\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{5/2}d} + \frac{3b\sqrt{a+bx+cx^2}}{4a^2dx} - \frac{f^{3/2}\text{arctanh}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2d^2\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}} + \frac{f^{3/2}\text{arctanh}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2d^2\sqrt{af+b\sqrt{d}\sqrt{f}+cd}} - \frac{f\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}d^2} - \frac{\sqrt{a+bx+cx^2}}{2adx^2}$$

```
[In] Int[1/(x^3*Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] -1/2*Sqrt[a + b*x + c*x^2]/(a*d*x^2) + (3*b*Sqrt[a + b*x + c*x^2])/(4*a^2*d
*x) - ((3*b^2 - 4*a*c)*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2]
)])/(8*a^(5/2)*d) - (f*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2]
)])/(Sqrt[a]*d^2) - (f^(3/2)*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d
] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x
^2])])/(2*d^2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]) + (f^(3/2)*ArcTanh[(b*Sq
rt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*
Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d^2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f]
 + a*f])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
```

```
d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 758

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 1047

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(-a)*c, 2]}, Dist[h/2 + c*(g/(2*q)
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - c*(g/(2*q)
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[(-a)*c]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(\frac{1}{dx^3\sqrt{a+bx+cx^2}}+\frac{f}{d^2x\sqrt{a+bx+cx^2}}+\frac{f^2x}{d^2\sqrt{a+bx+cx^2}\,(d-fx^2)}\right) dx \\
&= \frac{\int \frac{1}{x^3\sqrt{a+bx+cx^2}}\,dx}{d}+\frac{f\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx}{d^2}+\frac{f^2\int \frac{x}{\sqrt{a+bx+cx^2}(d-fx^2)}\,dx}{d^2}
\end{aligned}
$$

$$= -\frac{\sqrt{a+bx+cx^2}}{2adx^2} - \frac{\int \frac{\frac{3b}{2}+cx}{x^2\sqrt{a+bx+cx^2}}\,dx}{2ad} - \frac{(2f)\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx}{\sqrt{a+bx+cx^2}}\right)}{d^2} + \frac{f^2\int \frac{1}{\left(-\sqrt{d}\sqrt{f}-fx\right)\sqrt{a+bx+cx^2}}\,dx}{2d^2} + \frac{f^2\int \frac{1}{\left(\sqrt{d}\sqrt{f}-fx\right)\sqrt{a+bx+cx^2}}\,dx}{2d^2}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{2adx^2} + \frac{3b\sqrt{a+bx+cx^2}}{4a^2dx} - \frac{f\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}d^2} + \frac{(3b^2-4ac)\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx}{8a^2d} - \frac{f^2\text{Subst}\left(\int \frac{1}{4cdf-4b\sqrt{d}f^{3/2}+4af^2-x^2}\,dx, x, \frac{b\sqrt{d}\sqrt{f}-2af-\left(-2c\sqrt{d}\sqrt{f}+bf\right)x}{\sqrt{a+bx+cx^2}}\right)}{d^2} - \frac{f^2\text{Subst}\left(\int \frac{1}{4cdf+4b\sqrt{d}f^{3/2}+4af^2-x^2}\,dx, x, \frac{-b\sqrt{d}\sqrt{f}-2af-\left(2c\sqrt{d}\sqrt{f}+bf\right)x}{\sqrt{a+bx+cx^2}}\right)}{d^2}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{2adx^2} + \frac{3b\sqrt{a+bx+cx^2}}{4a^2dx} - \frac{f\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}d^2} - \frac{f^{3/2}\tanh^{-1}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2d^2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}} + \frac{f^{3/2}\tanh^{-1}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2d^2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}} - \frac{(3b^2-4ac)\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx}{\sqrt{a+bx+cx^2}}\right)}{4a^2d}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{2adx^2} + \frac{3b\sqrt{a+bx+cx^2}}{4a^2dx} - \frac{(3b^2-4ac)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{5/2}d} - \frac{f\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}d^2} - \frac{f^{3/2}\tanh^{-1}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2d^2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}} + \frac{f^{3/2}\tanh^{-1}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2d^2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.67 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.64

$$\int \frac{1}{x^3\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx$$
$$= \frac{\frac{d(-2a+3bx)\sqrt{a+x(b+cx)}}{a^2x^2} - \frac{(3b^2d-4acd+8a^2f)\operatorname{arctanh}\left(\frac{-\sqrt{c}x+\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)}{a^{5/2}} - 2f^2\mathrm{RootSum}\left[b^2d-a^2f-4b\sqrt{c}d\#1+4c\right.}{4d^2}$$

```
[In] Integrate[1/(x^3*Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

```
[Out] ((d*(-2*a + 3*b*x)*Sqrt[a + x*(b + c*x)])/(a^2*x^2) - ((3*b^2*d - 4*a*c*d +
 8*a^2*f)*ArcTanh[(-(Sqrt[c]*x) + Sqrt[a + x*(b + c*x)])/Sqrt[a]])/a^(5/2)
- 2*f^2*RootSum[b^2*d - a^2*f - 4*b*Sqrt[c]*d*#1 + 4*c*d*#1^2 + 2*a*f*#1^2
- f*#1^4 & , (a*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - Log[-(Sqrt
[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2)/(-(b*Sqrt[c]*d) + 2*c*d*#1 + a*f
*#1 - f*#1^3) & ])/(4*d^2)
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 457, normalized size of antiderivative = 1.22

method	result
risch	$-\frac{\sqrt{c\,x^2+bx+a}\,(-3bx+2a)}{4a^2d\,x^2} - \frac{-\frac{4f\,a^2\ln\left(\frac{\frac{-2b\sqrt{df}+2fa+2cd}{f}+\frac{(-2c\sqrt{df}+bf)\left(x+\frac{\sqrt{df}}{f}\right)}{f}+2\sqrt{\frac{-b\sqrt{df}+fa+cd}{f}}\sqrt{\left(x+\frac{\sqrt{df}}{f}\right)^2c+\frac{(-2c\sqrt{df}+bf)\left(\right.}{f}}}{x+\frac{\sqrt{df}}{f}}\right)}{d\sqrt{\frac{-b\sqrt{df}+fa+cd}{f}}}}{}$
default	$\frac{-\frac{\sqrt{c\,x^2+bx+a}}{2a\,x^2}-\frac{3b\left(-\frac{\sqrt{c\,x^2+bx+a}}{ax}+\frac{b\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)}{2a^{\frac{3}{2}}}\right)}{4a}+\frac{c\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)}{2a^{\frac{3}{2}}}}{d} - \frac{f\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+c}}{x}\right.}{d^2\sqrt{a}}$

```
[In] int(1/x^3/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(c*x^2+b*x+a)^(1/2)*(-3*b*x+2*a)/a^2/d/x^2-1/8/a^2/d*(-4*f*a^2/d/(1/f*
(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1/f*(-2*c*
(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*(
(x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*
(d*f)^(1/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1/2)/f))-4*f*a^2/d/((b*(d*f)^(1/2)+f
*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+f*a+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x
```

```
-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+
(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))
/(x-(d*f)^(1/2)/f))-(-8*a^2*f+4*a*c*d-3*b^2*d)/d/a^(1/2)*ln((2*a+b*x+2*a^(1
/2)*(c*x^2+b*x+a)^(1/2))/x))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = \text{Timed out}$$

```
[In] integrate(1/x^3/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{x^3\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = -\int \frac{1}{-dx^3\sqrt{a+bx+cx^2}+fx^5\sqrt{a+bx+cx^2}}\,dx$$

```
[In] integrate(1/x**3/(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
[Out] -Integral(1/(-d*x**3*sqrt(a + b*x + c*x**2) + f*x**5*sqrt(a + b*x + c*x**2)
), x)
```

Maxima [F]

$$\int \frac{1}{x^3\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = \int -\frac{1}{\sqrt{cx^2+bx+a}(fx^2-d)x^3}\,dx$$

```
[In] integrate(1/x^3/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")
[Out] -integrate(1/(sqrt(c*x^2 + b*x + a)*(f*x^2 - d)*x^3), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x^3\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = \text{Exception raised: TypeError}$$

[In] integrate(1/x^3/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")

[Out] Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument ValueDone

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = \int \frac{1}{x^3\,\left(d-f\,x^2\right)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

[In] int(1/(x^3*(d - f*x^2)*(a + b*x + c*x^2)^(1/2)),x)

[Out] int(1/(x^3*(d - f*x^2)*(a + b*x + c*x^2)^(1/2)), x)

3.102 $\int \frac{x^4}{\left(a+bx+cx^2\right)^{3/2}\left(d-fx^2\right)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 466

$$\begin{aligned}\int \frac{x^4}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx &= -\frac{2x(2a+bx)}{(b^2-4ac)\,f\sqrt{a+bx+cx^2}}\\ &\quad+\frac{2d(b+2cx)}{(b^2-4ac)\,f^2\sqrt{a+bx+cx^2}}-\frac{2d^2(b(b^2f-c(cd+3af))-c(2c^2d-b^2f+2acf)\,x)}{(b^2-4ac)\,f^2\,(b^2df-(cd+af)^2)\sqrt{a+bx+cx^2}}\\ &\quad+\frac{2b\sqrt{a+bx+cx^2}}{c\,(b^2-4ac)\,f}-\frac{\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}f}\\ &\quad+\frac{d^{3/2}\operatorname{arctanh}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2f\left(cd-b\sqrt{d}\sqrt{f}+af\right)^{3/2}}+\frac{d^{3/2}\operatorname{arctanh}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2f\left(cd+b\sqrt{d}\sqrt{f}+af\right)^{3/2}}\end{aligned}$$

```
[Out] -arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(3/2)/f+1/2*d^(3/2)*a
rctanh(1/2*(b*d^(1/2)-2*a*f^(1/2)+x*(2*c*d^(1/2)-b*f^(1/2)))/(c*x^2+b*x+a)^
(1/2)/(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2))/f/(c*d+a*f-b*d^(1/2)*f^(1/2))^(3/2
)+1/2*d^(3/2)*arctanh(1/2*(b*d^(1/2)+2*a*f^(1/2)+x*(2*c*d^(1/2)+b*f^(1/2)))
/(c*x^2+b*x+a)^(1/2)/(c*d+a*f+b*d^(1/2)*f^(1/2))^(1/2))/f/(c*d+a*f+b*d^(1/2
)*f^(1/2))^(3/2)-2*x*(b*x+2*a)/(-4*a*c+b^2)/f/(c*x^2+b*x+a)^(1/2)+2*d*(2*c*
x+b)/(-4*a*c+b^2)/f^2/(c*x^2+b*x+a)^(1/2)-2*d^2*(b*(b^2*f-c*(3*a*f+c*d))-c*
(2*a*c*f-b^2*f+2*c^2*d)*x)/(-4*a*c+b^2)/f^2/(b^2*d*f-(a*f+c*d)^2)/(c*x^2+b*
x+a)^(1/2)+2*b*(c*x^2+b*x+a)^(1/2)/c/(-4*a*c+b^2)/f
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 466, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {6857, 627, 752, 654, 635, 212, 989, 1047, 738}

$$\int \frac{x^4}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx = -\frac{\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}f} + \frac{d^{3/2}\text{arctanh}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2f\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}} + \frac{d^{3/2}\text{arctanh}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2f\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}} - \frac{2d^2(b(b^2f-c(3af+cd))-cx(2acf+b^2(-f)+2c^2d))}{f^2(b^2-4ac)\sqrt{a+bx+cx^2}(b^2df-(af+cd)^2)} + \frac{2d(b+2cx)}{f^2(b^2-4ac)\sqrt{a+bx+cx^2}} + \frac{2b\sqrt{a+bx+cx^2}}{cf(b^2-4ac)} - \frac{2x(2a+bx)}{f(b^2-4ac)\sqrt{a+bx+cx^2}}$$

```
[In] Int[x^4/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

```
[Out] (-2*x*(2*a + b*x))/((b^2 - 4*a*c)*f*Sqrt[a + b*x + c*x^2]) + (2*d*(b + 2*c*
x))/((b^2 - 4*a*c)*f^2*Sqrt[a + b*x + c*x^2]) - (2*d^2*(b*(b^2*f - c*(c*d +
 3*a*f)) - c*(2*c^2*d - b^2*f + 2*a*c*f)*x))/((b^2 - 4*a*c)*f^2*(b^2*d*f -
(c*d + a*f)^2)*Sqrt[a + b*x + c*x^2]) + (2*b*Sqrt[a + b*x + c*x^2])/(c*(b^2
 - 4*a*c)*f) - ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])]/(c^(3
/2)*f) + (d^(3/2)*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[
f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*f
*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)) + (d^(3/2)*ArcTanh[(b*Sqrt[d] + 2*a
*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a
*f]*Sqrt[a + b*x + c*x^2])])/(2*f*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 627

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
```

```
NeQ[b^2 - 4*a*c, 0]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 752

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 989

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (f_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[(b^3*f + b*c*(c*d - 3*a*f) + c*(2*c^2*d + b^2*f - c*(2*a*f
))*x)*(a + b*x + c*x^2)^(p + 1)*((d + f*x^2)^(q + 1)/((b^2 - 4*a*c)*(b^2*d*
f + (c*d - a*f)^2)*(p + 1))), x] - Dist[1/((b^2 - 4*a*c)*(b^2*d*f + (c*d -
a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Simp[2*c*(b^2
*d*f + (c*d - a*f)^2)*(p + 1) - (2*c^2*d + b^2*f - c*(2*a*f))*(a*f*(p + 1)
- c*d*(p + 2)) + (2*f*(b^3*f + b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d +
b^2*f - c*(2*a*f))*(b*f*(p + 1)))*x + c*f*(2*c^2*d + b^2*f - c*(2*a*f))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, q}, x] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] &&  !( !IntegerQ[
```

```
p] && ILtQ[q, -1]) &&  !IGtQ[q, 0]
```

Rule 1047

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(-a)*c, 2]}, Dist[h/2 + c*(g/(2*q)
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - c*(g/(2*q)
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[(-a)*c]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{d}{f^2 (a+bx+cx^2)^{3/2}} - \frac{x^2}{f (a+bx+cx^2)^{3/2}} + \frac{d^2}{f^2 (a+bx+cx^2)^{3/2} (d-fx^2)}\right) dx \\
&= -\frac{d\int \frac{1}{(a+bx+cx^2)^{3/2}}\,dx}{f^2} + \frac{d^2\int \frac{1}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx}{f^2} - \frac{\int \frac{x^2}{(a+bx+cx^2)^{3/2}}\,dx}{f} \\
&= -\frac{2x(2a+bx)}{(b^2-4ac)\,f\sqrt{a+bx+cx^2}} + \frac{2d(b+2cx)}{(b^2-4ac)\,f^2\sqrt{a+bx+cx^2}} \\
&\quad - \frac{2d^2(b(b^2f-c(cd+3af))-c(2c^2d-b^2f+2acf)\,x)}{(b^2-4ac)\,f^2\,(b^2df-(cd+af)^2)\sqrt{a+bx+cx^2}} \\
&\quad + \frac{2\int \frac{2a+bx}{\sqrt{a+bx+cx^2}}\,dx}{(b^2-4ac)\,f} - \frac{(2d^2)\int \frac{\frac{1}{2}(b^2-4ac)f(cd+af)-\frac{1}{2}b(b^2-4ac)f^2x}{\sqrt{a+bx+cx^2}(d-fx^2)}\,dx}{(b^2-4ac)\,f^2\,(b^2df-(cd+af)^2)} \\
&= -\frac{2x(2a+bx)}{(b^2-4ac)\,f\sqrt{a+bx+cx^2}} + \frac{2d(b+2cx)}{(b^2-4ac)\,f^2\sqrt{a+bx+cx^2}} \\
&\quad - \frac{2d^2(b(b^2f-c(cd+3af))-c(2c^2d-b^2f+2acf)\,x)}{(b^2-4ac)\,f^2\,(b^2df-(cd+af)^2)\sqrt{a+bx+cx^2}} + \frac{2b\sqrt{a+bx+cx^2}}{c\,(b^2-4ac)\,f} \\
&\quad - \frac{\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{cf} - \frac{d^{3/2}\int \frac{1}{\left(-\sqrt{d}\sqrt{f}-fx\right)\sqrt{a+bx+cx^2}}\,dx}{2\sqrt{f}\left(cd-b\sqrt{d}\sqrt{f}+af\right)} + \frac{d^{3/2}\int \frac{1}{\left(\sqrt{d}\sqrt{f}-fx\right)\sqrt{a+bx+cx^2}}\,dx}{2\sqrt{f}\left(cd+b\sqrt{d}\sqrt{f}+af\right)}
\end{aligned}$$

$$= -\frac{2x(2a+bx)}{(b^2-4ac)\,f\sqrt{a+bx+cx^2}} + \frac{2d(b+2cx)}{(b^2-4ac)\,f^2\sqrt{a+bx+cx^2}} - \frac{2d^2(b(b^2f-c(cd+3af))-c(2c^2d-b^2f+2acf)\,x)}{(b^2-4ac)\,f^2\,(b^2df-(cd+af)^2)\sqrt{a+bx+cx^2}} + \frac{2b\sqrt{a+bx+cx^2}}{c\,(b^2-4ac)\,f} - \frac{2\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{cf} + \frac{d^{3/2}\text{Subst}\left(\int \frac{1}{4cdf-4b\sqrt{d}f^{3/2}+4af^2-x^2}\,dx, x, \frac{b\sqrt{d}\sqrt{f}-2af-\left(-2c\sqrt{d}\sqrt{f}+bf\right)x}{\sqrt{a+bx+cx^2}}\right)}{\sqrt{f}\left(cd-b\sqrt{d}\sqrt{f}+af\right)} - \frac{d^{3/2}\text{Subst}\left(\int \frac{1}{4cdf+4b\sqrt{d}f^{3/2}+4af^2-x^2}\,dx, x, \frac{-b\sqrt{d}\sqrt{f}-2af-\left(2c\sqrt{d}\sqrt{f}+bf\right)x}{\sqrt{a+bx+cx^2}}\right)}{\sqrt{f}\left(cd+b\sqrt{d}\sqrt{f}+af\right)}$$

$$= -\frac{2x(2a+bx)}{(b^2-4ac)\,f\sqrt{a+bx+cx^2}} + \frac{2d(b+2cx)}{(b^2-4ac)\,f^2\sqrt{a+bx+cx^2}} - \frac{2d^2(b(b^2f-c(cd+3af))-c(2c^2d-b^2f+2acf)\,x)}{(b^2-4ac)\,f^2\,(b^2df-(cd+af)^2)\sqrt{a+bx+cx^2}} + \frac{2b\sqrt{a+bx+cx^2}}{c\,(b^2-4ac)\,f} - \frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}f} + \frac{d^{3/2}\tanh^{-1}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2f\left(cd-b\sqrt{d}\sqrt{f}+af\right)^{3/2}} + \frac{d^{3/2}\tanh^{-1}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2f\left(cd+b\sqrt{d}\sqrt{f}+af\right)^{3/2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 1.15 (sec) , antiderivative size = 453, normalized size of antiderivative = 0.97

$$\int \frac{x^4}{(a+bx+cx^2)^{3/2}\,(d-fx^2)}\,dx = \frac{2(b^4dx+ab^2d(b-4cx)-a^3f(b-2cx)-a^2(3bcd-2c^2dx+b^2fx))}{c\,(-b^2+4ac)\,(c^2d^2+2acdf+f\,(-b^2d+a^2f))\sqrt{a+x(b+cx)}} + \frac{\log\left(cf\left(b+2cx-2\sqrt{c}\sqrt{a+x(b+cx)}\right)\right)}{c^{3/2}f} - \frac{d^2\text{RootSum}\left[b^2d-a^2f-4b\sqrt{c}d\#1+4cd\#1^2+2af\#1^2-f\#1^4\&, \frac{bcd\log\left(-\sqrt{c}x+\sqrt{a+bx+cx^2}-\#1\right)+2abf\log\left(-\sqrt{\ldots}\right.}{}\right.}{2f\,(c^2d^2+\ldots}$$

```
[In] Integrate[x^4/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

```
[Out] (2*(b^4*d*x + a*b^2*d*(b - 4*c*x) - a^3*f*(b - 2*c*x) - a^2*(3*b*c*d - 2*c^
2*d*x + b^2*f*x)))/(c*(-b^2 + 4*a*c)*(c^2*d^2 + 2*a*c*d*f + f*(-(b^2*d) + a
^2*f))*Sqrt[a + x*(b + c*x)]) + Log[c*f*(b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(
b + c*x)])]/(c^(3/2)*f) - (d^2*RootSum[b^2*d - a^2*f - 4*b*Sqrt[c]*d*#1 + 4
*c*d*#1^2 + 2*a*f*#1^2 - f*#1^4 & , (b*c*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x
+ c*x^2] - #1] + 2*a*b*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2
*c^(3/2)*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - 2*a*Sqrt[c]*
f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - b*f*Log[-(Sqrt[c]*x)
+ Sqrt[a + b*x + c*x^2] - #1]*#1^2)/(b*Sqrt[c]*d - 2*c*d*#1 - a*f*#1 + f*#1
^3) & ])/(2*f*(c^2*d^2 + 2*a*c*d*f + f*(-(b^2*d) + a^2*f)))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1063 vs. $2(394) = 788$.

Time = 0.83 (sec) , antiderivative size = 1064, normalized size of antiderivative = 2.28

method	result	size
default	Expression too large to display	1064

```
[In] int(x^4/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/f*(-x/c/(c*x^2+b*x+a)^(1/2)-1/2*b/c*(-1/c/(c*x^2+b*x+a)^(1/2)-b/c*(2*c*x
+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2))+1/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^
2+b*x+a)^(1/2)))-2*d/f^2*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+1/2/f^2*
d^2/(d*f)^(1/2)*(f/(-b*(d*f)^(1/2)+f*a+c*d)/((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*
c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)-(-
2*c*(d*f)^(1/2)+b*f)/(-b*(d*f)^(1/2)+f*a+c*d)*(2*c*(x+(d*f)^(1/2)/f)+1/f*(-
2*c*(d*f)^(1/2)+b*f))/(4*c/f*(-b*(d*f)^(1/2)+f*a+c*d)-1/f^2*(-2*c*(d*f)^(1/
2)+b*f)^2)/((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)
/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)-f/(-b*(d*f)^(1/2)+f*a+c*d)/(1/f*(-b
*(d*f)^(1/2)+f*a+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1/f*(-2*c*(d*
f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*((x+
(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*
f)^(1/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1/2)/f)))-1/2/f^2*d^2/(d*f)^(1/2)*(1/(b
*(d*f)^(1/2)+f*a+c*d)*f/((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(
d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)-(2*c*(d*f)^(1/2)+b*f)/(b*(d*
f)^(1/2)+f*a+c*d)*(2*c*(x-(d*f)^(1/2)/f)+(2*c*(d*f)^(1/2)+b*f)/f)/(4*c*(b*(
d*f)^(1/2)+f*a+c*d)/f-(2*c*(d*f)^(1/2)+b*f)^2/f^2)/((x-(d*f)^(1/2)/f)^2*c+(
2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)-1
/(b*(d*f)^(1/2)+f*a+c*d)*f/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)
^(1/2)+f*a+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/
2)+f*a+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f
)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^4}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx = \text{Timed out}$$

```
[In] integrate(x^4/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^4}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx = \text{Timed out}$$

```
[In] integrate(x**4/(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^4/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see `
assume?
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^4}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^4/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(a+bx+cx^2)^{3/2}\,(d-fx^2)}\,dx = \int \frac{x^4}{(d-f\,x^2)\,\left(c\,x^2+b\,x+a\right)^{3/2}}\,dx$$

```
[In] int(x^4/((d - f*x^2)*(a + b*x + c*x^2)^(3/2)),x)
[Out] int(x^4/((d - f*x^2)*(a + b*x + c*x^2)^(3/2)), x)
```

3.103 $\int \frac{x^3}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 341

$$\int \frac{x^3}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx = -\frac{2(2a+bx)}{(b^2-4ac)\,f\sqrt{a+bx+cx^2}} - \frac{2d(a(2c^2d-b^2f+2acf)+bc(cd-af)x)}{(b^2-4ac)\,f\,(b^2df-(cd+af)^2)\sqrt{a+bx+cx^2}} - \frac{d\,\mathrm{arctanh}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{f}\left(cd-b\sqrt{d}\sqrt{f}+af\right)^{3/2}} + \frac{d\,\mathrm{arctanh}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{f}\left(cd+b\sqrt{d}\sqrt{f}+af\right)^{3/2}}$$

```
[Out] -1/2*d*arctanh(1/2*(b*d^(1/2)-2*a*f^(1/2)+x*(2*c*d^(1/2)-b*f^(1/2)))/(c*x^2
+b*x+a)^(1/2)/(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2))/f^(1/2)/(c*d+a*f-b*d^(1/2)
*f^(1/2))^(3/2)+1/2*d*arctanh(1/2*(b*d^(1/2)+2*a*f^(1/2)+x*(2*c*d^(1/2)+b*f
^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f+b*d^(1/2)*f^(1/2))^(1/2))/f^(1/2)/(c*
d+a*f+b*d^(1/2)*f^(1/2))^(3/2)-2*(b*x+2*a)/(-4*a*c+b^2)/f/(c*x^2+b*x+a)^(1/
2)-2*d*(a*(2*a*c*f-b^2*f+2*c^2*d)+b*c*(-a*f+c*d)*x)/(-4*a*c+b^2)/f/(b^2*d*f
-(a*f+c*d)^2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {6857, 650, 1032, 1047, 738, 212}

$$\int \frac{x^3}{(a+bx+cx^2)^{3/2}(d-fx^2)}dx = -\frac{d\text{arctanh}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\sqrt{f}\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}} + \frac{d\text{arctanh}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\sqrt{f}\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}} - \frac{2d(a(2acf+b^2(-f)+2c^2d)+bcx(cd-af))}{f(b^2-4ac)\sqrt{a+bx+cx^2}(b^2df-(af+cd)^2)} - \frac{2(2a+bx)}{f(b^2-4ac)\sqrt{a+bx+cx^2}}$$

```
[In] Int[x^3/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

```
[Out] (-2*(2*a + b*x))/((b^2 - 4*a*c)*f*Sqrt[a + b*x + c*x^2]) - (2*d*(a*(2*c^2*d
 - b^2*f + 2*a*c*f) + b*c*(c*d - a*f)*x))/((b^2 - 4*a*c)*f*(b^2*d*f - (c*d
+ a*f)^2)*Sqrt[a + b*x + c*x^2]) - (d*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2
*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a +
b*x + c*x^2])]])/(2*Sqrt[f]*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)) + (d*ArcT
anh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b
*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])]])/(2*Sqrt[f]*(c*d + b*Sqrt[d
]*Sqrt[f] + a*f)^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[-2*((b*d - 2*a*e + (2*c*d - b*e)*x)/((b^2 - 4*a*c)*Sqrt[a + b*x
+ c*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x + c*x^2)^(p + 1)*((d + f*x^2)^
(q + 1)/((b^2 - 4*a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)))*((g*c)*((-b)*(c*
d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f)) + c*(g*(2*c^2*d + b^2
*f - c*(2*a*f)) - h*(b*c*d + a*b*f))*x), x] + Dist[1/((b^2 - 4*a*c)*(b^2*d*
f + (c*d - a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Si
mp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d)*((-b)*f))*(p + 1) + (b^2*(g*f) - b*
(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(a*f*(p + 1) - c*d*(p + 2)) - (2*f*(
(g*c)*((-b)*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f)))*(p +
q + 2) - (b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(b*f*(p + 1)
))*x - c*f*(b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(2*p + 2*q
 + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*
a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] &&  !( !IntegerQ[p
] && ILtQ[q, -1])
```

Rule 1047

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(-a)*c, 2]}, Dist[h/2 + c*(g/(2*q)
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - c*(g/(2*q)
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[(-a)*c]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \int \left(-\frac{x}{f\left(a+bx+cx^2\right)^{3/2}}+\frac{dx}{f\left(a+bx+cx^2\right)^{3/2}\left(d-fx^2\right)}\right)dx \\
&= -\frac{\int \frac{x}{\left(a+bx+cx^2\right)^{3/2}}\,dx}{f}+\frac{d\int \frac{x}{\left(a+bx+cx^2\right)^{3/2}\left(d-fx^2\right)}\,dx}{f} \\
&= -\frac{2(2a+bx)}{\left(b^2-4ac\right)f\sqrt{a+bx+cx^2}}-\frac{2d\left(a\left(2c^2d-b^2f+2acf\right)+bc(cd-af)x\right)}{\left(b^2-4ac\right)f\left(b^2df-(cd+af)^2\right)\sqrt{a+bx+cx^2}} \\
&\quad+\frac{(2d)\int \frac{\frac{1}{2}b\left(b^2-4ac\right)df-\frac{1}{2}\left(b^2-4ac\right)f(cd+af)x}{\sqrt{a+bx+cx^2}\left(d-fx^2\right)}\,dx}{\left(b^2-4ac\right)f\left(b^2df-(cd+af)^2\right)}
\end{aligned}
$$

$$= -\frac{2(2a+bx)}{(b^2-4ac)f\sqrt{a+bx+cx^2}} - \frac{2d(a(2c^2d-b^2f+2acf)+bc(cd-af)x)}{(b^2-4ac)f(b^2df-(cd+af)^2)\sqrt{a+bx+cx^2}} + \frac{d\int \frac{1}{\left(-\sqrt{d}\sqrt{f}-fx\right)\sqrt{a+bx+cx^2}}dx}{2\left(cd-b\sqrt{d}\sqrt{f}+af\right)} + \frac{d\int \frac{1}{\left(\sqrt{d}\sqrt{f}-fx\right)\sqrt{a+bx+cx^2}}dx}{2\left(cd+b\sqrt{d}\sqrt{f}+af\right)}$$

$$= -\frac{2(2a+bx)}{(b^2-4ac)f\sqrt{a+bx+cx^2}} - \frac{2d(a(2c^2d-b^2f+2acf)+bc(cd-af)x)}{(b^2-4ac)f(b^2df-(cd+af)^2)\sqrt{a+bx+cx^2}} - \frac{d\text{Subst}\left(\int \frac{1}{4cdf-4b\sqrt{d}f^{3/2}+4af^2-x^2}dx, x, \frac{b\sqrt{d}\sqrt{f}-2af-\left(-2c\sqrt{d}\sqrt{f}+bf\right)x}{\sqrt{a+bx+cx^2}}\right)}{cd-b\sqrt{d}\sqrt{f}+af} - \frac{d\text{Subst}\left(\int \frac{1}{4cdf+4b\sqrt{d}f^{3/2}+4af^2-x^2}dx, x, \frac{-b\sqrt{d}\sqrt{f}-2af-\left(2c\sqrt{d}\sqrt{f}+bf\right)x}{\sqrt{a+bx+cx^2}}\right)}{cd+b\sqrt{d}\sqrt{f}+af}$$

$$= -\frac{2(2a+bx)}{(b^2-4ac)f\sqrt{a+bx+cx^2}} - \frac{2d(a(2c^2d-b^2f+2acf)+bc(cd-af)x)}{(b^2-4ac)f(b^2df-(cd+af)^2)\sqrt{a+bx+cx^2}} - \frac{d\tanh^{-1}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{f}\left(cd-b\sqrt{d}\sqrt{f}+af\right)^{3/2}} + \frac{d\tanh^{-1}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{f}\left(cd+b\sqrt{d}\sqrt{f}+af\right)^{3/2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.74 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.20

$$\int \frac{x^3}{(a+bx+cx^2)^{3/2}(d-fx^2)}dx = \frac{8a^3f-4b^3dx-4abd(b-3cx)+4a^2(2cd+bfx)-(b^2-4ac)d\sqrt{a+x(}}{}$$

```
[In] Integrate[x^3/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

```
[Out] (8*a^3*f - 4*b^3*d*x - 4*a*b*d*(b - 3*c*x) + 4*a^2*(2*c*d + b*f*x) - (b^2 -
 4*a*c)*d*Sqrt[a + x*(b + c*x)]*RootSum[b^2*d - a^2*f - 4*b*Sqrt[c]*d*#1 +
4*c*d*#1^2 + 2*a*f*#1^2 - f*#1^4 & , (b^2*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x
 + c*x^2] - #1] + a*c*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + a^
2*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*b*Sqrt[c]*d*Log[-(Sq
rt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - c*d*Log[-(Sqrt[c]*x) + Sqrt[a +
 b*x + c*x^2] - #1]*#1^2 - a*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #
1]*#1^2)/(b*Sqrt[c]*d - 2*c*d*#1 - a*f*#1 + f*#1^3) & ])/(2*(b^2 - 4*a*c)*(
-(c^2*d^2) - 2*a*c*d*f + f*(b^2*d - a^2*f))*Sqrt[a + x*(b + c*x)])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 959 vs. $2(281) = 562$.

Time = 0.87 (sec) , antiderivative size = 960, normalized size of antiderivative = 2.82

method	result
default	$-\frac{-\frac{1}{c\sqrt{c\,x^2+bx+a}}-\frac{b(2cx+b)}{c\left(4ac-b^2\right)\sqrt{c\,x^2+bx+a}}}{f} - \frac{d\left(\frac{f}{(-b\sqrt{df}+fa+cd)\sqrt{\left(x+\frac{\sqrt{df}}{f}\right)^2 c+\frac{(-2c\sqrt{df}+bf)\left(x+\frac{\sqrt{df}}{f}\right)}{f}+\frac{-b\sqrt{df}+fa+cd}{f}}} - \frac{}{(-b\sqrt{df}+f}\right.}{}$

```
[In] int(x^3/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/f*(-1/c/(c*x^2+b*x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2
))-1/2*d/f^2*(f/(-b*(d*f)^(1/2)+f*a+c*d)/((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(
d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)-(-2*c
*(d*f)^(1/2)+b*f)/(-b*(d*f)^(1/2)+f*a+c*d)*(2*c*(x+(d*f)^(1/2)/f)+1/f*(-2*c
*(d*f)^(1/2)+b*f))/(4*c/f*(-b*(d*f)^(1/2)+f*a+c*d)-1/f^2*(-2*c*(d*f)^(1/2)+
b*f)^2)/((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)
+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)-f/(-b*(d*f)^(1/2)+f*a+c*d)/(1/f*(-b*(d
*f)^(1/2)+f*a+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1/f*(-2*c*(d*f)^
(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*((x+(d*
f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^
(1/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1/2)/f)))-1/2*d/f^2*(1/(b*(d*f)^(1/2)+f*a+
c*d)*f/((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*
(d*f)^(1/2)+f*a+c*d)/f)^(1/2)-(2*c*(d*f)^(1/2)+b*f)/(b*(d*f)^(1/2)+f*a+c*d)
*(2*c*(x-(d*f)^(1/2)/f)+(2*c*(d*f)^(1/2)+b*f)/f)/(4*c*(b*(d*f)^(1/2)+f*a+c*
d)/f-(2*c*(d*f)^(1/2)+b*f)^2/f^2)/((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b
*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)-1/(b*(d*f)^(1/2)+f
*a+c*d)*f/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+f*a+c*d)/f
+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f*a+c*d)/f)^(1
/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*
f)^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17339 vs. 2(281) = 562.

Time = 22.37 (sec) , antiderivative size = 17339, normalized size of antiderivative = 50.85

$$\int \frac{x^3}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx = \text{Too large to display}$$

```
[In] integrate(x^3/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx = \text{Timed out}$$

```
[In] integrate(x**3/(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^3/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see `
assume?
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument ValueDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx = \int \frac{x^3}{(d-f\,x^2)\,(c\,x^2+b\,x+a)^{3/2}}\,dx$$

```
[In] int(x^3/((d - f*x^2)*(a + b*x + c*x^2)^(3/2)),x)
```

```
[Out] int(x^3/((d - f*x^2)*(a + b*x + c*x^2)^(3/2)), x)
```

3.104 $\int \frac{x^2}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 297

$$\int \frac{x^2}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx = \frac{2(ab(cd-af)+c(b^2d-2a(cd+af))x)}{(b^2-4ac)(b^2df-(cd+af)^2)\sqrt{a+bx+cx^2}} + \frac{\sqrt{d}\operatorname{arctanh}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\left(cd-b\sqrt{d}\sqrt{f}+af\right)^{3/2}} + \frac{\sqrt{d}\operatorname{arctanh}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\left(cd+b\sqrt{d}\sqrt{f}+af\right)^{3/2}}$$

```
[Out] 1/2*arctanh(1/2*(b*d^(1/2)-2*a*f^(1/2)+x*(2*c*d^(1/2)-b*f^(1/2)))/(c*x^2+b*
x+a)^(1/2)/(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2))*d^(1/2)/(c*d+a*f-b*d^(1/2)*f^
(1/2))^(3/2)+1/2*arctanh(1/2*(b*d^(1/2)+2*a*f^(1/2)+x*(2*c*d^(1/2)+b*f^(1/2
)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f+b*d^(1/2)*f^(1/2))^(1/2))*d^(1/2)/(c*d+a*f
+b*d^(1/2)*f^(1/2))^(3/2)+2*(a*b*(-a*f+c*d)+c*(b^2*d-2*a*(a*f+c*d))*x)/(-4*
a*c+b^2)/(b^2*d*f-(a*f+c*d)^2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1079, 1047, 738, 212}

$$\int \frac{x^2}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx = \frac{\sqrt{d}\,\text{arctanh}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}} + \frac{\sqrt{d}\,\text{arctanh}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}} + \frac{2(cx(b^2d-2a(af+cd))+ab(cd-af))}{(b^2-4ac)\sqrt{a+bx+cx^2}\,(b^2df-(af+cd)^2)}$$

```
[In] Int[x^2/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

```
[Out] (2*(a*b*(c*d - a*f) + c*(b^2*d - 2*a*(c*d + a*f))*x))/((b^2 - 4*a*c)*(b^2*d
*f - (c*d + a*f)^2)*Sqrt[a + b*x + c*x^2]) + (Sqrt[d]*ArcTanh[(b*Sqrt[d] -
2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f]
+ a*f]*Sqrt[a + b*x + c*x^2])]])/(2*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)) +
 (Sqrt[d]*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(
2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])]])/(2*(c*d + b*
Sqrt[d]*Sqrt[f] + a*f)^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1047

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(-a)*c, 2]}, Dist[h/2 + c*(g/(2*q)
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - c*(g/(2*q)
```

```
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[(-a)*c]
```

Rule 1079

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (C_.)*(x_)^2)*((d_) +
(f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x + c*x^2)^(p + 1)*((d + f*x^2
)^(q + 1)/((b^2 - 4*a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)))*((A*c - a*C)*(
(-b)*(c*d + a*f)) + (A*b)*(2*c^2*d + b^2*f - c*(2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(2*a*f)) + C*(b^2*d - 2*a*(c*d - a*f)))*x), x] + Dist[1/((b^2 - 4*
a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d +
 f*x^2)^q*Simp[(-2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d)*((-b)*f))*(p + 1) +
(b^2*(C*d + A*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - a*C*f)))*(a*f*(p + 1) -
c*d*(p + 2)) - (2*f*((A*c - a*C)*((-b)*(c*d + a*f)) + (A*b)*(2*c^2*d + b^2*
f - c*(2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) + 2*(A*c*(c*d - a*f) - a*(c*
C*d - a*C*f)))*(b*f*(p + 1)))*x - c*f*(b^2*(C*d + A*f) + 2*(A*c*(c*d - a*f)
 - a*(c*C*d - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d
, f, A, C, q}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*
d - a*f)^2, 0] && !( !IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q, 0]
```

Rubi steps

integral

$$= \frac{2(ab(cd-af)+c(b^2d-2a(cd+af))x)}{(b^2-4ac)(b^2df-(cd+af)^2)\sqrt{a+bx+cx^2}} + \frac{2\int \frac{-\frac{1}{2}(b^2-4ac)d(cd+af)+\frac{1}{2}b(b^2-4ac)dfx}{\sqrt{a+bx+cx^2}(d-fx^2)}\,dx}{(b^2-4ac)(b^2df-(cd+af)^2)}$$

$$= \frac{2(ab(cd-af)+c(b^2d-2a(cd+af))x)}{(b^2-4ac)(b^2df-(cd+af)^2)\sqrt{a+bx+cx^2}} - \frac{\left(\sqrt{d}\sqrt{f}\right)\int \frac{1}{\left(-\sqrt{d}\sqrt{f}-fx\right)\sqrt{a+bx+cx^2}}\,dx}{2\left(cd-b\sqrt{d}\sqrt{f}+af\right)} + \frac{\left(\sqrt{d}\sqrt{f}\right)\int \frac{1}{\left(\sqrt{d}\sqrt{f}-fx\right)\sqrt{a+bx+cx^2}}\,dx}{2\left(cd+b\sqrt{d}\sqrt{f}+af\right)}$$

$$= \frac{2(ab(cd-af)+c(b^2d-2a(cd+af))x)}{(b^2-4ac)(b^2df-(cd+af)^2)\sqrt{a+bx+cx^2}} + \frac{\left(\sqrt{d}\sqrt{f}\right)\text{Subst}\left(\int \frac{1}{4cdf-4b\sqrt{d}f^{3/2}+4af^2-x^2}\,dx, x, \frac{b\sqrt{d}\sqrt{f}-2af-\left(-2c\sqrt{d}\sqrt{f}+bf\right)x}{\sqrt{a+bx+cx^2}}\right)}{cd-b\sqrt{d}\sqrt{f}+af} - \frac{\left(\sqrt{d}\sqrt{f}\right)\text{Subst}\left(\int \frac{1}{4cdf+4b\sqrt{d}f^{3/2}+4af^2-x^2}\,dx, x, \frac{-b\sqrt{d}\sqrt{f}-2af-\left(2c\sqrt{d}\sqrt{f}+bf\right)x}{\sqrt{a+bx+cx^2}}\right)}{cd+b\sqrt{d}\sqrt{f}+af}$$

$$= \frac{2(ab(cd-af)+c(b^2d-2a(cd+af))\,x)}{(b^2-4ac)\,(b^2df-(cd+af)^2)\,\sqrt{a+bx+cx^2}} + \frac{\sqrt{d}\tanh^{-1}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\left(cd-b\sqrt{d}\sqrt{f}+af\right)^{3/2}} + \frac{\sqrt{d}\tanh^{-1}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\left(cd+b\sqrt{d}\sqrt{f}+af\right)^{3/2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.68 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.25

$$\int \frac{x^2}{(a+bx+cx^2)^{3/2}\,(d-fx^2)}\,dx = \frac{4b^2cdx+4acd(b-2cx)-4a^2f(b+2cx)+(b^2-4ac)\,d\sqrt{a+x(b+cx)}}{}$$

```
[In] Integrate[x^2/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

```
[Out] (4*b^2*c*d*x + 4*a*c*d*(b - 2*c*x) - 4*a^2*f*(b + 2*c*x) + (b^2 - 4*a*c)*d*
Sqrt[a + x*(b + c*x)]*RootSum[b^2*d - a^2*f - 4*b*Sqrt[c]*d*#1 + 4*c*d*#1^2
 + 2*a*f*#1^2 - f*#1^4 & , (b*c*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2]
- #1] + 2*a*b*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*c^(3/2)*
d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - 2*a*Sqrt[c]*f*Log[-(S
qrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - b*f*Log[-(Sqrt[c]*x) + Sqrt[a
+ b*x + c*x^2] - #1]*#1^2)/(b*Sqrt[c]*d - 2*c*d*#1 - a*f*#1 + f*#1^3) & ])/
(2*(b^2 - 4*a*c)*(-(c^2*d^2) - 2*a*c*d*f + f*(b^2*d - a^2*f))*Sqrt[a + x*(b
 + c*x)])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 945 vs. $2(239) = 478$.

Time = 0.77 (sec) , antiderivative size = 946, normalized size of antiderivative = 3.19

method	result
default	$-\frac{2(2cx+b)}{f(4ac-b^2)\sqrt{c\,x^2+bx+a}}+\frac{d\left(\frac{f}{(-b\sqrt{df}+fa+cd)\sqrt{\left(x+\frac{\sqrt{df}}{f}\right)^2c+\frac{(-2c\sqrt{df}+bf)\left(x+\frac{\sqrt{df}}{f}\right)}{f}+\frac{-b\sqrt{df}+fa+cd}{f}}}-\frac{}{(-b\sqrt{df}+fa+cd)\left(4c(-b\sqrt{d}\right.}\right.}{}$

```
[In] int(x^2/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] -2/f*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+1/2*d/(d*f)^(1/2)/f*(f/(-b*(
d*f)^(1/2)+f*a+c*d)/((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d
*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)-(-2*c*(d*f)^(1/2)+b*f)/(-b
*(d*f)^(1/2)+f*a+c*d)*(2*c*(x+(d*f)^(1/2)/f)+1/f*(-2*c*(d*f)^(1/2)+b*f))/(4
*c/f*(-b*(d*f)^(1/2)+f*a+c*d)-1/f^2*(-2*c*(d*f)^(1/2)+b*f)^2)/((x+(d*f)^(1/
2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+
f*a+c*d))^(1/2)-f/(-b*(d*f)^(1/2)+f*a+c*d)/(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(
1/2)*ln((2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(
1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(
-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)
)/(x+(d*f)^(1/2)/f)))-1/2*d/(d*f)^(1/2)/f*(1/(b*(d*f)^(1/2)+f*a+c*d)*f/((x-
(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)
+f*a+c*d)/f)^(1/2)-(2*c*(d*f)^(1/2)+b*f)/(b*(d*f)^(1/2)+f*a+c*d)*(2*c*(x-(d
*f)^(1/2)/f)+(2*c*(d*f)^(1/2)+b*f)/f)/(4*c*(b*(d*f)^(1/2)+f*a+c*d)/f-(2*c*(
d*f)^(1/2)+b*f)^2/f^2)/((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d
*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)-1/(b*(d*f)^(1/2)+f*a+c*d)*f/(
(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+f*a+c*d)/f+(2*c*(d*f)
^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*((x-(d*
f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*
a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17285 vs. 2(239) = 478.

Time = 13.62 (sec) , antiderivative size = 17285, normalized size of antiderivative = 58.20

$$\int \frac{x^2}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx = \text{Too large to display}$$

```
[In] integrate(x^2/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx = \text{Timed out}$$

```
[In] integrate(x**2/(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{(a+bx+cx^2)^{3/2}\,(d-fx^2)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see `
assume?
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^2}{(a+bx+cx^2)^{3/2}\,(d-fx^2)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^2/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument ValueDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+bx+cx^2)^{3/2}\,(d-fx^2)}\,dx = \int \frac{x^2}{(d-f\,x^2)\,\left(c\,x^2+b\,x+a\right)^{3/2}}\,dx$$

```
[In] int(x^2/((d - f*x^2)*(a + b*x + c*x^2)^(3/2)),x)
```

```
[Out] int(x^2/((d - f*x^2)*(a + b*x + c*x^2)^(3/2)), x)
```

3.105 $\int \frac{x}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 299

$$\int \frac{x}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx = -\frac{2(a(2c^2d-b^2f+2acf)+bc(cd-af)x)}{(b^2-4ac)(b^2df-(cd+af)^2)\sqrt{a+bx+cx^2}} - \frac{\sqrt{f}\operatorname{arctanh}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\left(cd-b\sqrt{d}\sqrt{f}+af\right)^{3/2}} + \frac{\sqrt{f}\operatorname{arctanh}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\left(cd+b\sqrt{d}\sqrt{f}+af\right)^{3/2}}$$

```
[Out] -1/2*arctanh(1/2*(b*d^(1/2)-2*a*f^(1/2)+x*(2*c*d^(1/2)-b*f^(1/2)))/(c*x^2+b
*x+a)^(1/2)/(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2))*f^(1/2)/(c*d+a*f-b*d^(1/2)*f
^(1/2))^(3/2)+1/2*arctanh(1/2*(b*d^(1/2)+2*a*f^(1/2)+x*(2*c*d^(1/2)+b*f^(1/
2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f+b*d^(1/2)*f^(1/2))^(1/2))*f^(1/2)/(c*d+a*
f+b*d^(1/2)*f^(1/2))^(3/2)-2*(a*(2*a*c*f-b^2*f+2*c^2*d)+b*c*(-a*f+c*d)*x)/(
-4*a*c+b^2)/(b^2*d*f-(a*f+c*d)^2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1032, 1047, 738, 212}

$$\int \frac{x}{(a+bx+cx^2)^{3/2}(d-fx^2)}dx = -\frac{\sqrt{f}\text{arctanh}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}} + \frac{\sqrt{f}\text{arctanh}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}} - \frac{2(a(2acf+b^2(-f)+2c^2d)+bcx(cd-af))}{(b^2-4ac)\sqrt{a+bx+cx^2}(b^2df-(af+cd)^2)}$$

```
[In] Int[x/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

```
[Out] (-2*(a*(2*c^2*d - b^2*f + 2*a*c*f) + b*c*(c*d - a*f)*x))/((b^2 - 4*a*c)*(b^
2*d*f - (c*d + a*f)^2)*Sqrt[a + b*x + c*x^2]) - (Sqrt[f]*ArcTanh[(b*Sqrt[d]
 - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[
f] + a*f]*Sqrt[a + b*x + c*x^2])]])/(2*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)
) + (Sqrt[f]*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x
)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])]])/(2*(c*d +
 b*Sqrt[d]*Sqrt[f] + a*f)^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x + c*x^2)^(p + 1)*((d + f*x^2)^
(q + 1)/((b^2 - 4*a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)))*((g*c)*((-b)*(c*
d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f)) + c*(g*(2*c^2*d + b^2
*f - c*(2*a*f)) - h*(b*c*d + a*b*f))*x), x] + Dist[1/((b^2 - 4*a*c)*(b^2*d*
f + (c*d - a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Si
```

```
mp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d)*((-b)*f))*(p + 1) + (b^2*(g*f) - b*
(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(a*f*(p + 1) - c*d*(p + 2)) - (2*f*(
(g*c)*((-b)*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f)))*(p +
q + 2) - (b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(b*f*(p + 1)
))*x - c*f*(b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(2*p + 2*q
 + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*
a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] &&  !( !IntegerQ[p
] && ILtQ[q, -1])
```

Rule 1047

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(-a)*c, 2]}, Dist[h/2 + c*(g/(2*q)
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - c*(g/(2*q)
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[(-a)*c]
```

Rubi steps

$$\begin{aligned}
&\text{integral}\\
&=-\frac{2(a(2c^2d-b^2f+2acf)+bc(cd-af)x)}{(b^2-4ac)\left(b^2df-(cd+af)^2\right)\sqrt{a+bx+cx^2}}+\frac{2\int\frac{\frac{1}{2}b(b^2-4ac)df-\frac{1}{2}(b^2-4ac)f(cd+af)x}{\sqrt{a+bx+cx^2}(d-fx^2)}\,dx}{(b^2-4ac)\left(b^2df-(cd+af)^2\right)}\\
&=-\frac{2(a(2c^2d-b^2f+2acf)+bc(cd-af)x)}{(b^2-4ac)\left(b^2df-(cd+af)^2\right)\sqrt{a+bx+cx^2}}\\
&\quad+\frac{f\int\frac{1}{\left(-\sqrt{d}\sqrt{f}-fx\right)\sqrt{a+bx+cx^2}}\,dx}{2\left(cd-b\sqrt{d}\sqrt{f}+af\right)}+\frac{f\int\frac{1}{\left(\sqrt{d}\sqrt{f}-fx\right)\sqrt{a+bx+cx^2}}\,dx}{2\left(cd+b\sqrt{d}\sqrt{f}+af\right)}\\
&=-\frac{2(a(2c^2d-b^2f+2acf)+bc(cd-af)x)}{(b^2-4ac)\left(b^2df-(cd+af)^2\right)\sqrt{a+bx+cx^2}}\\
&\quad-\frac{f\text{Subst}\left(\int\frac{1}{4cdf-4b\sqrt{d}f^{3/2}+4af^2-x^2}\,dx,x,\frac{b\sqrt{d}\sqrt{f}-2af-\left(-2c\sqrt{d}\sqrt{f}+bf\right)x}{\sqrt{a+bx+cx^2}}\right)}{cd-b\sqrt{d}\sqrt{f}+af}\\
&\quad-\frac{f\text{Subst}\left(\int\frac{1}{4cdf+4b\sqrt{d}f^{3/2}+4af^2-x^2}\,dx,x,\frac{-b\sqrt{d}\sqrt{f}-2af-\left(2c\sqrt{d}\sqrt{f}+bf\right)x}{\sqrt{a+bx+cx^2}}\right)}{cd+b\sqrt{d}\sqrt{f}+af}\\
&=-\frac{2(a(2c^2d-b^2f+2acf)+bc(cd-af)x)}{(b^2-4ac)\left(b^2df-(cd+af)^2\right)\sqrt{a+bx+cx^2}}\\
&\quad-\frac{\sqrt{f}\tanh^{-1}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\left(cd-b\sqrt{d}\sqrt{f}+af\right)^{3/2}}+\frac{\sqrt{f}\tanh^{-1}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\left(cd+b\sqrt{d}\sqrt{f}+af\right)^{3/2}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.72 (sec) , antiderivative size = 407, normalized size of antiderivative = 1.36

$$\int \frac{x}{(a+bx+cx^2)^{3/2}\,(d-fx^2)}\,dx = \frac{-8a^2cf-4bc^2dx+4a(-2c^2d+b^2f+bcfx)-(b^2-4ac)\,f\,\sqrt{a+x(b+\cdots}{\cdots}$$

```
[In] Integrate[x/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

```
[Out] (-8*a^2*c*f - 4*b*c^2*d*x + 4*a*(-2*c^2*d + b^2*f + b*c*f*x) - (b^2 - 4*a*c
)*f*Sqrt[a + x*(b + c*x)]*RootSum[b^2*d - a^2*f - 4*b*Sqrt[c]*d*#1 + 4*c*d*
#1^2 + 2*a*f*#1^2 - f*#1^4 & , (b^2*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x
^2] - #1] + a*c*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + a^2*f*Lo
g[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*b*Sqrt[c]*d*Log[-(Sqrt[c]*
x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - c*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x +
 c*x^2] - #1]*#1^2 - a*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^
2)/(b*Sqrt[c]*d - 2*c*d*#1 - a*f*#1 + f*#1^3) & ])/(2*(b^2 - 4*a*c)*(-(c^2*
d^2) - 2*a*c*d*f + f*(b^2*d - a^2*f))*Sqrt[a + x*(b + c*x)])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 898 vs. $2(241) = 482$.

Time = 0.68 (sec) , antiderivative size = 899, normalized size of antiderivative = 3.01

method	result
default	$-\frac{f}{(-b\sqrt{df}+fa+cd)\sqrt{\left(x+\frac{\sqrt{df}}{f}\right)^2 c+\frac{(-2c\sqrt{df}+bf)\left(x+\frac{\sqrt{df}}{f}\right)}{f}+\frac{-b\sqrt{df}+fa+cd}{f}}} - \frac{(-2c\sqrt{df}+bf)\left(2c\left(x+\frac{\sqrt{df}}{f}\cdots\right.\right.}{(-b\sqrt{df}+fa+cd)\left(\frac{4c(-b\sqrt{df}+fa+cd)}{f}-\frac{(-2c\sqrt{df}+bf)^2}{f^2}\right)\sqrt{\left(x\cdots\right.}}$

```
[In] int(x/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/f*(f/(-b*(d*f)^(1/2)+f*a+c*d)/((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(
1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)-(-2*c*(d*f)
^(1/2)+b*f)/(-b*(d*f)^(1/2)+f*a+c*d)*(2*c*(x+(d*f)^(1/2)/f)+1/f*(-2*c*(d*f)
^(1/2)+b*f))/(4*c/f*(-b*(d*f)^(1/2)+f*a+c*d)-1/f^2*(-2*c*(d*f)^(1/2)+b*f)^2
)/((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(
-b*(d*f)^(1/2)+f*a+c*d))^(1/2)-f/(-b*(d*f)^(1/2)+f*a+c*d)/(1/f*(-b*(d*f)^(1
/2)+f*a+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1/f*(-2*c*(d*f)^(1/2)+
b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*((x+(d*f)^(1/
```

```
2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+
f*a+c*d))^(1/2))/(x+(d*f)^(1/2)/f))-1/2/f*(1/(b*(d*f)^(1/2)+f*a+c*d)*f/((x
-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2
)+f*a+c*d)/f)^(1/2)-(2*c*(d*f)^(1/2)+b*f)/(b*(d*f)^(1/2)+f*a+c*d)*(2*c*(x-(
d*f)^(1/2)/f)+(2*c*(d*f)^(1/2)+b*f)/f)/(4*c*(b*(d*f)^(1/2)+f*a+c*d)/f-(2*c*
(d*f)^(1/2)+b*f)^2/f^2)/((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(
d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)-1/(b*(d*f)^(1/2)+f*a+c*d)*f/
((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+f*a+c*d)/f+(2*c*(d*f
)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*((x-(d
*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f
*a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17258 vs. $2(241) = 482$.

Time = 15.82 (sec) , antiderivative size = 17258, normalized size of antiderivative = 57.72

$$\int \frac{x}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx = \text{Too large to display}$$

```
[In] integrate(x/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx = \text{Timed out}$$

```
[In] integrate(x/(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see `
assume?
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x}{\left(a+bx+cx^2\right)^{3/2}\left(d-fx^2\right)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument ValueDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(a+bx+cx^2\right)^{3/2}\left(d-fx^2\right)}\,dx = \int \frac{x}{\left(d-f\,x^2\right)\,\left(c\,x^2+b\,x+a\right)^{3/2}} \,dx$$

```
[In] int(x/((d - f*x^2)*(a + b*x + c*x^2)^(3/2)),x)
```

```
[Out] int(x/((d - f*x^2)*(a + b*x + c*x^2)^(3/2)), x)
```

3.106 $\int \frac{1}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 310

$$\int \frac{1}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx = -\frac{2(b(b^2f-c(cd+3af))-c(2c^2d-b^2f+2acf)\,x)}{(b^2-4ac)\,(b^2df-(cd+af)^2)\sqrt{a+bx+cx^2}} + \frac{f\operatorname{arctanh}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{d}\left(cd-b\sqrt{d}\sqrt{f}+af\right)^{3/2}} + \frac{f\operatorname{arctanh}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{d}\left(cd+b\sqrt{d}\sqrt{f}+af\right)^{3/2}}$$

```
[Out] 1/2*f*arctanh(1/2*(b*d^(1/2)-2*a*f^(1/2)+x*(2*c*d^(1/2)-b*f^(1/2)))/(c*x^2+
b*x+a)^(1/2)/(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2))/d^(1/2)/(c*d+a*f-b*d^(1/2)*
f^(1/2))^(3/2)+1/2*f*arctanh(1/2*(b*d^(1/2)+2*a*f^(1/2)+x*(2*c*d^(1/2)+b*f^
(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f+b*d^(1/2)*f^(1/2))^(1/2))/d^(1/2)/(c*d
+a*f+b*d^(1/2)*f^(1/2))^(3/2)-2*(b*(b^2*f-c*(3*a*f+c*d))-c*(2*a*c*f-b^2*f+2
*c^2*d)*x)/(-4*a*c+b^2)/(b^2*d*f-(a*f+c*d)^2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {989, 1047, 738, 212}

$$\int \frac{1}{(a+bx+cx^2)^{3/2}(d-fx^2)}dx = \frac{f\operatorname{arctanh}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\sqrt{d}\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}} + \frac{f\operatorname{arctanh}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\sqrt{d}\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}} - \frac{2(b(b^2f-c(3af+cd))-cx(2acf+b^2(-f)+2c^2d))}{(b^2-4ac)\sqrt{a+bx+cx^2}\,(b^2df-(af+cd)^2)}$$

```
[In] Int[1/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

```
[Out] (-2*(b*(b^2*f - c*(c*d + 3*a*f)) - c*(2*c^2*d - b^2*f + 2*a*c*f)*x))/((b^2
- 4*a*c)*(b^2*d*f - (c*d + a*f)^2)*Sqrt[a + b*x + c*x^2]) + (f*ArcTanh[(b*S
qrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]
*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[d]*(c*d - b*Sqrt[d]*Sqrt[f
] + a*f)^(3/2)) + (f*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sq
rt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(
2*Sqrt[d]*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 989

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (f_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[(b^3*f + b*c*(c*d - 3*a*f) + c*(2*c^2*d + b^2*f - c*(2*a*f
))*x)*(a + b*x + c*x^2)^(p + 1)*((d + f*x^2)^(q + 1)/((b^2 - 4*a*c)*(b^2*d*
```

```
f + (c*d - a*f)^2)*(p + 1))), x] - Dist[1/((b^2 - 4*a*c)*(b^2*d*f + (c*d -
a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Simp[2*c*(b^2
*d*f + (c*d - a*f)^2)*(p + 1) - (2*c^2*d + b^2*f - c*(2*a*f))*(a*f*(p + 1)
- c*d*(p + 2)) + (2*f*(b^3*f + b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d +
b^2*f - c*(2*a*f))*(b*f*(p + 1)))*x + c*f*(2*c^2*d + b^2*f - c*(2*a*f))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, q}, x] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] &&  !( !IntegerQ[
p] && ILtQ[q, -1]) &&  !IGtQ[q, 0]
```

Rule 1047

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(-a)*c, 2]}, Dist[h/2 + c*(g/(2*q)
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - c*(g/(2*q)
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[(-a)*c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2(b(b^2f-c(cd+3af))-c(2c^2d-b^2f+2acf)\,x)}{(b^2-4ac)\,(b^2df-(cd+af)^2)\,\sqrt{a+bx+cx^2}} \\
&\quad -\frac{2\int\frac{\frac{1}{2}(b^2-4ac)f(cd+af)-\frac{1}{2}b(b^2-4ac)f^2x}{\sqrt{a+bx+cx^2}(d-fx^2)}\,dx}{(b^2-4ac)\,(b^2df-(cd+af)^2)} \\
&= -\frac{2(b(b^2f-c(cd+3af))-c(2c^2d-b^2f+2acf)\,x)}{(b^2-4ac)\,(b^2df-(cd+af)^2)\,\sqrt{a+bx+cx^2}} \\
&\quad -\frac{f^{3/2}\int\frac{1}{\left(-\sqrt{d}\sqrt{f}-fx\right)\sqrt{a+bx+cx^2}}\,dx}{2\sqrt{d}\left(cd-b\sqrt{d}\sqrt{f}+af\right)}+\frac{f^{3/2}\int\frac{1}{\left(\sqrt{d}\sqrt{f}-fx\right)\sqrt{a+bx+cx^2}}\,dx}{2\sqrt{d}\left(cd+b\sqrt{d}\sqrt{f}+af\right)} \\
&= -\frac{2(b(b^2f-c(cd+3af))-c(2c^2d-b^2f+2acf)\,x)}{(b^2-4ac)\,(b^2df-(cd+af)^2)\,\sqrt{a+bx+cx^2}} \\
&\quad +\frac{f^{3/2}\text{Subst}\left(\int\frac{1}{4cdf-4b\sqrt{d}f^{3/2}+4af^2-x^2}\,dx,x,\frac{b\sqrt{d}\sqrt{f}-2af-\left(-2c\sqrt{d}\sqrt{f}+bf\right)x}{\sqrt{a+bx+cx^2}}\right)}{\sqrt{d}\left(cd-b\sqrt{d}\sqrt{f}+af\right)} \\
&\quad -\frac{f^{3/2}\text{Subst}\left(\int\frac{1}{4cdf+4b\sqrt{d}f^{3/2}+4af^2-x^2}\,dx,x,\frac{-b\sqrt{d}\sqrt{f}-2af-\left(2c\sqrt{d}\sqrt{f}+bf\right)x}{\sqrt{a+bx+cx^2}}\right)}{\sqrt{d}\left(cd+b\sqrt{d}\sqrt{f}+af\right)}
\end{aligned}$$

$$= -\frac{2(b(b^2f - c(cd + 3af)) - c(2c^2d - b^2f + 2acf)\,x)}{(b^2 - 4ac)\,(b^2df - (cd + af)^2)\,\sqrt{a + bx + cx^2}} + \frac{f\tanh^{-1}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{d}\left(cd - b\sqrt{d}\sqrt{f} + af\right)^{3/2}} + \frac{f\tanh^{-1}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{d}\left(cd + b\sqrt{d}\sqrt{f} + af\right)^{3/2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.76 (sec) , antiderivative size = 464, normalized size of antiderivative = 1.50

$$\int \frac{1}{(a + bx + cx^2)^{3/2}\,(d - fx^2)}\,dx = \frac{-4b^3f + 4bc(cd + 3af) - 4b^2cfx + 8c^2(cd + af)x + (b^2 - 4ac)\,f\sqrt{a +}}{}$$

```
[In] Integrate[1/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

```
[Out] (-4*b^3*f + 4*b*c*(c*d + 3*a*f) - 4*b^2*c*f*x + 8*c^2*(c*d + a*f)*x + (b^2
- 4*a*c)*f*Sqrt[a + x*(b + c*x)]*RootSum[c^2*d - b^2*f + 4*Sqrt[a]*b*f*#1 -
 2*c*d*#1^2 - 4*a*f*#1^2 + d*#1^4 & , (-(c^2*d*Log[x]) - b^2*f*Log[x] - a*c
*f*Log[x] + c^2*d*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] + b^2*f*Log[
-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] + a*c*f*Log[-Sqrt[a] + Sqrt[a + b*
x + c*x^2] - x*#1] + 2*Sqrt[a]*b*f*Log[x]*#1 - 2*Sqrt[a]*b*f*Log[-Sqrt[a] +
 Sqrt[a + b*x + c*x^2] - x*#1]*#1 + c*d*Log[x]*#1^2 + a*f*Log[x]*#1^2 - c*d
*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1]*#1^2 - a*f*Log[-Sqrt[a] + Sqr
t[a + b*x + c*x^2] - x*#1]*#1^2)/(-(Sqrt[a]*b*f) + c*d*#1 + 2*a*f*#1 - d*#1
^3) & ])/(2*(b^2 - 4*a*c)*(-(c^2*d^2) - 2*a*c*d*f + f*(b^2*d - a^2*f))*Sqrt
[a + x*(b + c*x)])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 902 vs. $2(252) = 504$.

Time = 0.77 (sec) , antiderivative size = 903, normalized size of antiderivative = 2.91

method	result
default	$\frac{f}{\left(-b\sqrt{df}+fa+cd\right)\sqrt{\left(x+\frac{\sqrt{df}}{f}\right)^2 c+\frac{\left(-2c\sqrt{df}+bf\right)\left(x+\frac{\sqrt{df}}{f}\right)}{f}+\frac{-b\sqrt{df}+fa+cd}{f}}} - \frac{\left(-2c\sqrt{df}+bf\right)\left(2c\left(x+\frac{\sqrt{df}}{f}\right)-\right.}{\left(-b\sqrt{df}+fa+cd\right)\left(\frac{4c\left(-b\sqrt{df}+fa+cd\right)}{f}-\frac{\left(-2c\sqrt{df}+bf\right)^2}{f^2}\right)\sqrt{\left(x+\right.}}$

```
[In] int(1/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/(d*f)^(1/2)*(f/(-b*(d*f)^(1/2)+f*a+c*d)/((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*
c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)-(-
2*c*(d*f)^(1/2)+b*f)/(-b*(d*f)^(1/2)+f*a+c*d)*(2*c*(x+(d*f)^(1/2)/f)+1/f*(-
2*c*(d*f)^(1/2)+b*f))/(4*c/f*(-b*(d*f)^(1/2)+f*a+c*d)-1/f^2*(-2*c*(d*f)^(1/
2)+b*f)^2)/((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)
/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)-f/(-b*(d*f)^(1/2)+f*a+c*d)/(1/f*(-b
*(d*f)^(1/2)+f*a+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1/f*(-2*c*(d*
f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*((x+
(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*
f)^(1/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1/2)/f)))-1/2/(d*f)^(1/2)*(1/(b*(d*f)^(
1/2)+f*a+c*d)*f/((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/
2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)-(2*c*(d*f)^(1/2)+b*f)/(b*(d*f)^(1/2)
+f*a+c*d)*(2*c*(x-(d*f)^(1/2)/f)+(2*c*(d*f)^(1/2)+b*f)/f)/(4*c*(b*(d*f)^(1/
2)+f*a+c*d)/f-(2*c*(d*f)^(1/2)+b*f)^2/f^2)/((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f
)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)-1/(b*(d*f
)^(1/2)+f*a+c*d)*f/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+f
*a+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f*a+c
*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/
f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17397 vs. $2(252) = 504$.

Time = 14.10 (sec) , antiderivative size = 17397, normalized size of antiderivative = 56.12

$$\int \frac{1}{\left(a+bx+cx^2\right)^{3/2}\left(d-fx^2\right)}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+bx+cx^2\right)^{3/2}\left(d-fx^2\right)}\,dx = \text{Timed out}$$

```
[In] integrate(1/(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a+bx+cx^2)^{3/2}\,(d-fx^2)}\,dx = \text{Exception raised: ValueError}$$

[In] integrate(1/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', see `
assume?

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a+bx+cx^2)^{3/2}\,(d-fx^2)}\,dx = \text{Exception raised: TypeError}$$

[In] integrate(1/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="giac")

[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument ValueDone

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx+cx^2)^{3/2}\,(d-fx^2)}\,dx = \int \frac{1}{(d-f\,x^2)\,(c\,x^2+b\,x+a)^{3/2}}\,dx$$

[In] int(1/((d - f*x^2)*(a + b*x + c*x^2)^(3/2)),x)

[Out] int(1/((d - f*x^2)*(a + b*x + c*x^2)^(3/2)), x)

3.107 $\int \frac{1}{x(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 394

$$\int \frac{1}{x(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx = \frac{2(b^2-2ac+bcx)}{a(b^2-4ac)\,d\sqrt{a+bx+cx^2}} - \frac{2f(a(2c^2d-b^2f+2acf)+bc(cd-af)x)}{(b^2-4ac)\,d\,(b^2df-(cd+af)^2)\sqrt{a+bx+cx^2}} - \frac{\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{a^{3/2}d} - \frac{f^{3/2}\operatorname{arctanh}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2d\left(cd-b\sqrt{d}\sqrt{f}+af\right)^{3/2}} + \frac{f^{3/2}\operatorname{arctanh}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2d\left(cd+b\sqrt{d}\sqrt{f}+af\right)^{3/2}}$$

```
[Out] -arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/a^(3/2)/d-1/2*f^(3/2)*a
rctanh(1/2*(b*d^(1/2)-2*a*f^(1/2)+x*(2*c*d^(1/2)-b*f^(1/2)))/(c*x^2+b*x+a)^
(1/2)/(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2))/d/(c*d+a*f-b*d^(1/2)*f^(1/2))^(3/2
)+1/2*f^(3/2)*arctanh(1/2*(b*d^(1/2)+2*a*f^(1/2)+x*(2*c*d^(1/2)+b*f^(1/2)))
/(c*x^2+b*x+a)^(1/2)/(c*d+a*f+b*d^(1/2)*f^(1/2))^(1/2))/d/(c*d+a*f+b*d^(1/2
)*f^(1/2))^(3/2)+2*(b*c*x-2*a*c+b^2)/a/(-4*a*c+b^2)/d/(c*x^2+b*x+a)^(1/2)-2
*f*(a*(2*a*c*f-b^2*f+2*c^2*d)+b*c*(-a*f+c*d)*x)/(-4*a*c+b^2)/d/(b^2*d*f-(a*
f+c*d)^2)/(c*x^2+b*x+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6857, 754, 12, 738, 212, 1032, 1047}

$$\int \frac{1}{x(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx = -\frac{\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{a^{3/2}d} - \frac{f^{3/2}\operatorname{arctanh}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2d\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}} + \frac{f^{3/2}\operatorname{arctanh}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2d\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}} - \frac{2f(a(2acf+b^2(-f)+2c^2d)+bcx(cd-af))}{d(b^2-4ac)\sqrt{a+bx+cx^2}(b^2df-(af+cd)^2)} + \frac{2(-2ac+b^2+bcx)}{ad(b^2-4ac)\sqrt{a+bx+cx^2}}$$

```
[In] Int[1/(x*(a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

```
[Out] (2*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*d*Sqrt[a + b*x + c*x^2]) - (2*f*
(a*(2*c^2*d - b^2*f + 2*a*c*f) + b*c*(c*d - a*f)*x))/((b^2 - 4*a*c)*d*(b^2*
d*f - (c*d + a*f)^2)*Sqrt[a + b*x + c*x^2]) - ArcTanh[(2*a + b*x)/(2*Sqrt[a
]*Sqrt[a + b*x + c*x^2])]/(a^(3/2)*d) - (f^(3/2)*ArcTanh[(b*Sqrt[d] - 2*a*S
qrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f
]*Sqrt[a + b*x + c*x^2])]])/(2*d*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)) + (f
^(3/2)*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*S
qrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])]])/(2*d*(c*d + b*S
qrt[d]*Sqrt[f] + a*f)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x + c*x^2)^(p + 1)*((d + f*x^2)^
(q + 1)/((b^2 - 4*a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)))*((g*c)*((-b)*(c*
d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f)) + c*(g*(2*c^2*d + b^2
*f - c*(2*a*f)) - h*(b*c*d + a*b*f))*x), x] + Dist[1/((b^2 - 4*a*c)*(b^2*d*
f + (c*d - a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Si
mp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d)*((-b)*f))*(p + 1) + (b^2*(g*f) - b*
(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(a*f*(p + 1) - c*d*(p + 2)) - (2*f*(
(g*c)*((-b)*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f)))*(p +
q + 2) - (b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(b*f*(p + 1)
))*x - c*f*(b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(2*p + 2*q
 + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*
a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] &&  !( !IntegerQ[p
] && ILtQ[q, -1])
```

Rule 1047

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(-a)*c, 2]}, Dist[h/2 + c*(g/(2*q)
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - c*(g/(2*q)
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[(-a)*c]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
```

`[n, 0]`

Rubi steps

$$\text{integral} = \int \left(\frac{1}{dx\left(a+bx+cx^2\right)^{3/2}} - \frac{fx}{d\left(a+bx+cx^2\right)^{3/2}\left(-d+fx^2\right)}\right) dx$$

$$= \frac{\int \frac{1}{x(a+bx+cx^2)^{3/2}}\,dx}{d} - \frac{f\int \frac{x}{(a+bx+cx^2)^{3/2}(-d+fx^2)}\,dx}{d}$$

$$= \frac{2(b^2-2ac+bcx)}{a\left(b^2-4ac\right)d\sqrt{a+bx+cx^2}} - \frac{2f(a(2c^2d-b^2f+2acf)+bc(cd-af)x)}{\left(b^2-4ac\right)d\left(b^2df-(cd+af)^2\right)\sqrt{a+bx+cx^2}} - \frac{2\int \frac{-\frac{b^2}{2}+2ac}{x\sqrt{a+bx+cx^2}}\,dx}{a\left(b^2-4ac\right)d} - \frac{(2f)\int \frac{\frac{1}{2}b(b^2-4ac)df-\frac{1}{2}(b^2-4ac)f(cd+af)x}{\sqrt{a+bx+cx^2}(-d+fx^2)}\,dx}{\left(b^2-4ac\right)d\left(b^2df-(cd+af)^2\right)}$$

$$= \frac{2(b^2-2ac+bcx)}{a\left(b^2-4ac\right)d\sqrt{a+bx+cx^2}} - \frac{2f(a(2c^2d-b^2f+2acf)+bc(cd-af)x)}{\left(b^2-4ac\right)d\left(b^2df-(cd+af)^2\right)\sqrt{a+bx+cx^2}} + \frac{\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx}{ad} - \frac{f^2\int \frac{1}{\left(\sqrt{d}\sqrt{f}+fx\right)\sqrt{a+bx+cx^2}}\,dx}{2d\left(cd-b\sqrt{d}\sqrt{f}+af\right)} - \frac{f^2\int \frac{1}{\left(-\sqrt{d}\sqrt{f}+fx\right)\sqrt{a+bx+cx^2}}\,dx}{2d\left(cd+b\sqrt{d}\sqrt{f}+af\right)}$$

$$= \frac{2(b^2-2ac+bcx)}{a\left(b^2-4ac\right)d\sqrt{a+bx+cx^2}} - \frac{2f(a(2c^2d-b^2f+2acf)+bc(cd-af)x)}{\left(b^2-4ac\right)d\left(b^2df-(cd+af)^2\right)\sqrt{a+bx+cx^2}} - \frac{2\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx}{\sqrt{a+bx+cx^2}}\right)}{ad} + \frac{f^2\text{Subst}\left(\int \frac{1}{4cdf-4b\sqrt{d}f^{3/2}+4af^2-x^2}\,dx, x, \frac{-b\sqrt{d}\sqrt{f}+2af-\left(2c\sqrt{d}\sqrt{f}-bf\right)x}{\sqrt{a+bx+cx^2}}\right)}{d\left(cd-b\sqrt{d}\sqrt{f}+af\right)} + \frac{f^2\text{Subst}\left(\int \frac{1}{4cdf+4b\sqrt{d}f^{3/2}+4af^2-x^2}\,dx, x, \frac{b\sqrt{d}\sqrt{f}+2af-\left(-2c\sqrt{d}\sqrt{f}-bf\right)x}{\sqrt{a+bx+cx^2}}\right)}{d\left(cd+b\sqrt{d}\sqrt{f}+af\right)}$$

$$= \frac{2(b^2-2ac+bcx)}{a\left(b^2-4ac\right)d\sqrt{a+bx+cx^2}} - \frac{2f(a(2c^2d-b^2f+2acf)+bc(cd-af)x)}{\left(b^2-4ac\right)d\left(b^2df-(cd+af)^2\right)\sqrt{a+bx+cx^2}} - \frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{a^{3/2}d} - \frac{f^{3/2}\tanh^{-1}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2d\left(cd-b\sqrt{d}\sqrt{f}+af\right)^{3/2}} + \frac{f^{3/2}\tanh^{-1}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2d\left(cd+b\sqrt{d}\sqrt{f}+af\right)^{3/2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 1.23 (sec) , antiderivative size = 486, normalized size of antiderivative = 1.23

$$\int \frac{1}{x\left(a+bx+cx^2\right)^{3/2}\left(d-fx^2\right)}\,dx = \frac{2(b^4f+2ac^2(cd+af)-b^2c(cd+4af)+b^3cfx-bc^2(cd+3af)x)}{a\left(-b^2+4ac\right)\left(c^2d^2+2acdf+f\left(-b^2d+a^2f\right)\right)\sqrt{a+x(b+cx)}} + \frac{2\text{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)}{a^{3/2}d} + \frac{f^2\text{RootSum}\left[b^2d-a^2f-4b\sqrt{c}d\#1+4cd\#1^2+2af\#1^2-f\#1^4\&,\frac{b^2d\log\left(-\sqrt{c}x+\sqrt{a+bx+cx^2}-\#1\right)+acd\log\left(-\sqrt{c}x-\right.}{}\right.}{}$$

```
[In] Integrate[1/(x*(a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

```
[Out] (2*(b^4*f + 2*a*c^2*(c*d + a*f) - b^2*c*(c*d + 4*a*f) + b^3*c*f*x - b*c^2*(
c*d + 3*a*f)*x))/(a*(-b^2 + 4*a*c)*(c^2*d^2 + 2*a*c*d*f + f*(-(b^2*d) + a^2
*f))*Sqrt[a + x*(b + c*x)]) + (2*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)]
)/Sqrt[a]])/(a^(3/2)*d) + (f^2*RootSum[b^2*d - a^2*f - 4*b*Sqrt[c]*d*#1 + 4
*c*d*#1^2 + 2*a*f*#1^2 - f*#1^4 & , (b^2*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x
+ c*x^2] - #1] + a*c*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + a^2
*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*b*Sqrt[c]*d*Log[-(Sqr
t[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - c*d*Log[-(Sqrt[c]*x) + Sqrt[a +
b*x + c*x^2] - #1]*#1^2 - a*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1
]*#1^2)/(b*Sqrt[c]*d - 2*c*d*#1 - a*f*#1 + f*#1^3) & ])/(2*c^2*d^3 - 2*b^2*
d^2*f + 4*a*c*d^2*f + 2*a^2*d*f^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 989 vs. 2(326) = 652.

Time = 0.75 (sec) , antiderivative size = 990, normalized size of antiderivative = 2.51

method	result
default	$\frac{\frac{1}{a\sqrt{c\,x^2+bx+a}}-\frac{b(2cx+b)}{a\left(4ac-b^2\right)\sqrt{c\,x^2+bx+a}}-\frac{\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)}{a^{\frac{3}{2}}}}{d} - \frac{f}{\left(b\sqrt{df}+fa+cd\right)\sqrt{\left(x-\frac{\sqrt{df}}{f}\right)^2c+\frac{\left(2c\sqrt{df}+bf\right)\left(x-\frac{\sqrt{df}}{f}\right)}{f}+\frac{b\sqrt{df}+f}{f}}}$

```
[In] int(1/x/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/a/(c*x^2+b*x+a)^(1/2)-b/a*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-
1/a^(3/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x))-1/2/d*(1/(b*(d*f)^
(1/2)+f*a+c*d)*f/((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1
/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)-(2*c*(d*f)^(1/2)+b*f)/(b*(d*f)^(1/2
)+f*a+c*d)*(2*c*(x-(d*f)^(1/2)/f)+(2*c*(d*f)^(1/2)+b*f)/f)/(4*c*(b*(d*f)^(1
/2)+f*a+c*d)/f-(2*c*(d*f)^(1/2)+b*f)^2/f^2)/((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*
f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)-1/(b*(d*
f)^(1/2)+f*a+c*d)*f/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+
f*a+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f*a+
c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)
/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f)))-1/2/d*(f/(-b*(d*f
)^(1/2)+f*a+c*d)/((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)
^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)-(-2*c*(d*f)^(1/2)+b*f)/(-b*(d
*f)^(1/2)+f*a+c*d)*(2*c*(x+(d*f)^(1/2)/f)+1/f*(-2*c*(d*f)^(1/2)+b*f))/(4*c/
f*(-b*(d*f)^(1/2)+f*a+c*d)-1/f^2*(-2*c*(d*f)^(1/2)+b*f)^2)/((x+(d*f)^(1/2)/
f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a
+c*d))^(1/2)-f/(-b*(d*f)^(1/2)+f*a+c*d)/(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2
)*ln((2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2
)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*
c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/(
x+(d*f)^(1/2)/f)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x\left(a+bx+cx^2\right)^{3/2}\left(d-fx^2\right)}\,dx = \text{Timed out}$$

```
[In] integrate(1/x/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{x\left(a+bx+cx^2\right)^{3/2}\left(d-fx^2\right)}\,dx =$$
$$-\int \frac{1}{-adx\sqrt{a+bx+cx^2}+afx^3\sqrt{a+bx+cx^2}-bdx^2\sqrt{a+bx+cx^2}+bfx^4\sqrt{a+bx+cx^2}-cdx^3\sqrt{a+b}$$

```
[In] integrate(1/x/(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)
```

```
[Out] -Integral(1/(-a*d*x*sqrt(a + b*x + c*x**2) + a*f*x**3*sqrt(a + b*x + c*x**2
) - b*d*x**2*sqrt(a + b*x + c*x**2) + b*f*x**4*sqrt(a + b*x + c*x**2) - c*d
*x**3*sqrt(a + b*x + c*x**2) + c*f*x**5*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{x\left(a+bx+cx^2\right)^{3/2}\left(d-fx^2\right)}\,dx = \int -\frac{1}{\left(cx^2+bx+a\right)^{\frac{3}{2}}\left(fx^2-d\right)x}\,dx$$

```
[In] integrate(1/x/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="maxima")
[Out] -integrate(1/((c*x^2 + b*x + a)^(3/2)*(f*x^2 - d)*x), x)
```

Giac [F]

$$\int \frac{1}{x\left(a+bx+cx^2\right)^{3/2}\left(d-fx^2\right)}\,dx = \int -\frac{1}{\left(cx^2+bx+a\right)^{\frac{3}{2}}\left(fx^2-d\right)x}\,dx$$

```
[In] integrate(1/x/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="giac")
[Out] sage2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\left(a+bx+cx^2\right)^{3/2}\left(d-fx^2\right)}\,dx = \int \frac{1}{x\,\left(d-f\,x^2\right)\,\left(c\,x^2+b\,x+a\right)^{3/2}}\,dx$$

```
[In] int(1/(x*(d - f*x^2)*(a + b*x + c*x^2)^(3/2)),x)
[Out] int(1/(x*(d - f*x^2)*(a + b*x + c*x^2)^(3/2)), x)
```

3.108 $\int \frac{1}{x^2(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 454

$$\int \frac{1}{x^2(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx = \frac{2(b^2-2ac+bcx)}{a(b^2-4ac)\,dx\sqrt{a+bx+cx^2}} - \frac{2f(b(b^2f-c(cd+3af))-c(2c^2d-b^2f+2acf)\,x)}{(b^2-4ac)\,d\,(b^2df-(cd+af)^2)\sqrt{a+bx+cx^2}} - \frac{(3b^2-8ac)\sqrt{a+bx+cx^2}}{a^2(b^2-4ac)\,dx} + \frac{3b\,\mathrm{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{5/2}d} + \frac{f^2\,\mathrm{arctanh}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2d^{3/2}\left(cd-b\sqrt{d}\sqrt{f}+af\right)^{3/2}} + \frac{f^2\,\mathrm{arctanh}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2d^{3/2}\left(cd+b\sqrt{d}\sqrt{f}+af\right)^{3/2}}$$

```
[Out] 3/2*b*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/a^(5/2)/d+1/2*f^2*
arctanh(1/2*(b*d^(1/2)-2*a*f^(1/2)+x*(2*c*d^(1/2)-b*f^(1/2)))/(c*x^2+b*x+a)
^(1/2)/(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2))/d^(3/2)/(c*d+a*f-b*d^(1/2)*f^(1/2
))^(3/2)+1/2*f^2*arctanh(1/2*(b*d^(1/2)+2*a*f^(1/2)+x*(2*c*d^(1/2)+b*f^(1/2
)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f+b*d^(1/2)*f^(1/2))^(1/2))/d^(3/2)/(c*d+a*f
+b*d^(1/2)*f^(1/2))^(3/2)+2*(b*c*x-2*a*c+b^2)/a/(-4*a*c+b^2)/d/x/(c*x^2+b*x
+a)^(1/2)-2*f*(b*(b^2*f-c*(3*a*f+c*d))-c*(2*a*c*f-b^2*f+2*c^2*d)*x)/(-4*a*c
+b^2)/d/(b^2*d*f-(a*f+c*d)^2)/(c*x^2+b*x+a)^(1/2)-(-8*a*c+3*b^2)*(c*x^2+b*x
+a)^(1/2)/a^2/(-4*a*c+b^2)/d/x
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 454, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6857, 754, 820, 738, 212, 989, 1047}

$$\int \frac{1}{x^2 (a+bx+cx^2)^{3/2} (d-fx^2)} \, dx = \frac{3b\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{5/2}d} - \frac{(3b^2-8ac)\sqrt{a+bx+cx^2}}{a^2dx(b^2-4ac)} + \frac{f^2\operatorname{arctanh}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2d^{3/2}\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}} + \frac{f^2\operatorname{arctanh}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2d^{3/2}\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}} - \frac{2f(b(b^2f-c(3af+cd))-cx(2acf+b^2(-f)+2c^2d))}{d(b^2-4ac)\sqrt{a+bx+cx^2}(b^2df-(af+cd)^2)} + \frac{2(-2ac+b^2+bcx)}{adx(b^2-4ac)\sqrt{a+bx+cx^2}}$$

```
[In] Int[1/(x^2*(a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

```
[Out] (2*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*d*x*Sqrt[a + b*x + c*x^2]) - (2*
f*(b*(b^2*f - c*(c*d + 3*a*f)) - c*(2*c^2*d - b^2*f + 2*a*c*f)*x))/((b^2 -
4*a*c)*d*(b^2*d*f - (c*d + a*f)^2)*Sqrt[a + b*x + c*x^2]) - ((3*b^2 - 8*a*c
)*Sqrt[a + b*x + c*x^2])/(a^2*(b^2 - 4*a*c)*d*x) + (3*b*ArcTanh[(2*a + b*x)
/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(2*a^(5/2)*d) + (f^2*ArcTanh[(b*Sqrt[d
] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt
[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d^(3/2)*(c*d - b*Sqrt[d]*Sqrt[f] + a
*f)^(3/2)) + (f^2*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[
f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d
^(3/2)*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 989

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (f_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[(b^3*f + b*c*(c*d - 3*a*f) + c*(2*c^2*d + b^2*f - c*(2*a*f
))*x)*(a + b*x + c*x^2)^(p + 1)*((d + f*x^2)^(q + 1)/((b^2 - 4*a*c)*(b^2*d*
f + (c*d - a*f)^2)*(p + 1))), x] - Dist[1/((b^2 - 4*a*c)*(b^2*d*f + (c*d -
a*f)^2)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Simp[2*c*(b^2
*d*f + (c*d - a*f)^2)*(p + 1) - (2*c^2*d + b^2*f - c*(2*a*f))*(a*f*(p + 1)
- c*d*(p + 2)) + (2*f*(b^3*f + b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d +
b^2*f - c*(2*a*f))*(b*f*(p + 1)))*x + c*f*(2*c^2*d + b^2*f - c*(2*a*f))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, q}, x] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] &&  !( !IntegerQ[
p] && ILtQ[q, -1]) &&  !IGtQ[q, 0]
```

Rule 1047

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(-a)*c, 2]}, Dist[h/2 + c*(g/(2*q)
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - c*(g/(2*q)
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
```

```
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[(-a)*c]
```

Rule 6857

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{1}{dx^2 (a+bx+cx^2)^{3/2}} + \frac{f}{d (a+bx+cx^2)^{3/2} (d-fx^2)}\right) dx \\
&= \frac{\int \frac{1}{x^2(a+bx+cx^2)^{3/2}}\,dx}{d} + \frac{f\int \frac{1}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx}{d} \\
&= \frac{2(b^2-2ac+bcx)}{a(b^2-4ac)\,dx\sqrt{a+bx+cx^2}} \\
&\quad - \frac{2f(b(b^2f-c(cd+3af))-c(2c^2d-b^2f+2acf)\,x)}{(b^2-4ac)\,d\,(b^2df-(cd+af)^2)\sqrt{a+bx+cx^2}} \\
&\quad - \frac{2\int \frac{\frac{1}{2}(-3b^2+8ac)-bcx}{x^2\sqrt{a+bx+cx^2}}\,dx}{a(b^2-4ac)\,d} - \frac{(2f)\int \frac{\frac{1}{2}(b^2-4ac)f(cd+af)-\frac{1}{2}b(b^2-4ac)f^2x}{\sqrt{a+bx+cx^2}(d-fx^2)}\,dx}{(b^2-4ac)\,d\,(b^2df-(cd+af)^2)} \\
&= \frac{2(b^2-2ac+bcx)}{a(b^2-4ac)\,dx\sqrt{a+bx+cx^2}} \\
&\quad - \frac{2f(b(b^2f-c(cd+3af))-c(2c^2d-b^2f+2acf)\,x)}{(b^2-4ac)\,d\,(b^2df-(cd+af)^2)\sqrt{a+bx+cx^2}} \\
&\quad - \frac{(3b^2-8ac)\sqrt{a+bx+cx^2}}{a^2(b^2-4ac)\,dx} - \frac{(3b)\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx}{2a^2d} \\
&\quad - \frac{f^{5/2}\int \frac{1}{\left(-\sqrt{d}\sqrt{f}-fx\right)\sqrt{a+bx+cx^2}}\,dx}{2d^{3/2}\left(cd-b\sqrt{d}\sqrt{f}+af\right)} + \frac{f^{5/2}\int \frac{1}{\left(\sqrt{d}\sqrt{f}-fx\right)\sqrt{a+bx+cx^2}}\,dx}{2d^{3/2}\left(cd+b\sqrt{d}\sqrt{f}+af\right)}
\end{aligned}$$

$$= \frac{2(b^2-2ac+bcx)}{a(b^2-4ac)\,dx\sqrt{a+bx+cx^2}} - \frac{2f(b(b^2f-c(cd+3af))-c(2c^2d-b^2f+2acf)\,x)}{(b^2-4ac)\,d\,(b^2df-(cd+af)^2)\sqrt{a+bx+cx^2}} - \frac{(3b^2-8ac)\sqrt{a+bx+cx^2}}{a^2(b^2-4ac)\,dx} + \frac{(3b)\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx}{\sqrt{a+bx+cx^2}}\right)}{a^2d} + \frac{f^{5/2}\text{Subst}\left(\int \frac{1}{4cdf-4b\sqrt{d}f^{3/2}+4af^2-x^2}\,dx, x, \frac{b\sqrt{d}\sqrt{f}-2af-\left(-2c\sqrt{d}\sqrt{f}+bf\right)x}{\sqrt{a+bx+cx^2}}\right)}{d^{3/2}\left(cd-b\sqrt{d}\sqrt{f}+af\right)} - \frac{f^{5/2}\text{Subst}\left(\int \frac{1}{4cdf+4b\sqrt{d}f^{3/2}+4af^2-x^2}\,dx, x, \frac{-b\sqrt{d}\sqrt{f}-2af-\left(2c\sqrt{d}\sqrt{f}+bf\right)x}{\sqrt{a+bx+cx^2}}\right)}{d^{3/2}\left(cd+b\sqrt{d}\sqrt{f}+af\right)}$$

$$= \frac{2(b^2-2ac+bcx)}{a(b^2-4ac)\,dx\sqrt{a+bx+cx^2}} - \frac{2f(b(b^2f-c(cd+3af))-c(2c^2d-b^2f+2acf)\,x)}{(b^2-4ac)\,d\,(b^2df-(cd+af)^2)\sqrt{a+bx+cx^2}} - \frac{(3b^2-8ac)\sqrt{a+bx+cx^2}}{a^2(b^2-4ac)\,dx} + \frac{3b\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{5/2}d} + \frac{f^2\tanh^{-1}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2d^{3/2}\left(cd-b\sqrt{d}\sqrt{f}+af\right)^{3/2}} + \frac{f^2\tanh^{-1}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2d^{3/2}\left(cd+b\sqrt{d}\sqrt{f}+af\right)^{3/2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 2.10 (sec) , antiderivative size = 620, normalized size of antiderivative = 1.37

$$\int \frac{1}{x^2(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx = \frac{-2\sqrt{a}(4a^4cf^2+3b^2d(-c^2d+b^2f)\,x(b+cx)+a^3f(-b^2f+4bcfx+$$

```
[In] Integrate[1/(x^2*(a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

```
[Out] (-2*Sqrt[a]*(4*a^4*c*f^2 + 3*b^2*d*(-(c^2*d) + b^2*f)*x*(b + c*x) + a^3*f*(
-(b^2*f) + 4*b*c*f*x + 4*c^2*(2*d + f*x^2)) + a^2*(18*b*c^2*d*f*x - b^3*f^2
*x - b^2*c*f*(6*d + f*x^2) + 4*c^3*d*(d + 3*f*x^2)) + a*d*(b^4*f + 10*b*c^3
*d*x - 16*b^3*c*f*x + 8*c^4*d*x^2 - b^2*c^2*(d + 14*f*x^2))) - 6*b*(b^2 - 4
*a*c)*(-(c^2*d^2) - 2*a*c*d*f + f*(b^2*d - a^2*f))*x*Sqrt[a + x*(b + c*x)]*
```

```
ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)])/Sqrt[a]] + a^(5/2)*(b^2 - 4*a*c
)*f^2*x*Sqrt[a + x*(b + c*x)]*RootSum[b^2*d - a^2*f - 4*b*Sqrt[c]*d*#1 + 4*
c*d*#1^2 + 2*a*f*#1^2 - f*#1^4 & , (b*c*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x +
 c*x^2] - #1] + 2*a*b*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*
c^(3/2)*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - 2*a*Sqrt[c]*f
*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - b*f*Log[-(Sqrt[c]*x) +
 Sqrt[a + b*x + c*x^2] - #1]*#1^2)/(b*Sqrt[c]*d - 2*c*d*#1 - a*f*#1 + f*#1^
3) & ])/(2*a^(5/2)*(-b^2 + 4*a*c)*d*(c^2*d^2 + 2*a*c*d*f + f*(-(b^2*d) + a^
2*f))*x*Sqrt[a + x*(b + c*x)])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1064 vs. $2(382) = 764$.

Time = 0.95 (sec) , antiderivative size = 1065, normalized size of antiderivative = 2.35

method	result	size
default	Expression too large to display	1065
risch	Expression too large to display	3015

```
[In] int(1/x^2/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/a/x/(c*x^2+b*x+a)^(1/2)-3/2*b/a*(1/a/(c*x^2+b*x+a)^(1/2)-b/a*(2*c*x
+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-1/a^(3/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+
b*x+a)^(1/2))/x))-4*c/a*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2))-1/2*f/d/
(d*f)^(1/2)*(1/(b*(d*f)^(1/2)+f*a+c*d)*f/((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^
(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)-(2*c*(d*f)^
(1/2)+b*f)/(b*(d*f)^(1/2)+f*a+c*d)*(2*c*(x-(d*f)^(1/2)/f)+(2*c*(d*f)^(1/2)+
b*f)/f)/(4*c*(b*(d*f)^(1/2)+f*a+c*d)/f-(2*c*(d*f)^(1/2)+b*f)^2/f^2)/((x-(d*
f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*
a+c*d)/f)^(1/2)-1/(b*(d*f)^(1/2)+f*a+c*d)*f/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/
2)*ln((2*(b*(d*f)^(1/2)+f*a+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f
)+2*((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/
2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)^(1/2
)/f)))+1/2*f/d/(d*f)^(1/2)*(f/(-b*(d*f)^(1/2)+f*a+c*d)/((x+(d*f)^(1/2)/f)^2
*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d
))^(1/2)-(-2*c*(d*f)^(1/2)+b*f)/(-b*(d*f)^(1/2)+f*a+c*d)*(2*c*(x+(d*f)^(1/2
)/f)+1/f*(-2*c*(d*f)^(1/2)+b*f))/(4*c/f*(-b*(d*f)^(1/2)+f*a+c*d)-1/f^2*(-2*
c*(d*f)^(1/2)+b*f)^2)/((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+
(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)-f/(-b*(d*f)^(1/2)+f*a+c*
d)/(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1/
f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))
^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+
1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1/2)/f)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a+bx+cx^2)^{3/2} (d-fx^2)} \, dx = \text{Timed out}$$

```
[In] integrate(1/x^2/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{x^2 (a+bx+cx^2)^{3/2} (d-fx^2)} \, dx = -\int \frac{1}{-adx^2\sqrt{a+bx+cx^2}+afx^4\sqrt{a+bx+cx^2}-bdx^3\sqrt{a+bx+cx^2}+bfx^5\sqrt{a+bx+cx^2}-cdx^4\sqrt{a+$$

```
[In] integrate(1/x**2/(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)
[Out] -Integral(1/(-a*d*x**2*sqrt(a + b*x + c*x**2) + a*f*x**4*sqrt(a + b*x + c*x
**2) - b*d*x**3*sqrt(a + b*x + c*x**2) + b*f*x**5*sqrt(a + b*x + c*x**2) -
c*d*x**4*sqrt(a + b*x + c*x**2) + c*f*x**6*sqrt(a + b*x + c*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{x^2 (a+bx+cx^2)^{3/2} (d-fx^2)} \, dx = \int -\frac{1}{(cx^2+bx+a)^{\frac{3}{2}}(fx^2-d)x^2} \, dx$$

```
[In] integrate(1/x^2/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="maxima")
[Out] -integrate(1/((c*x^2 + b*x + a)^(3/2)*(f*x^2 - d)*x^2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x^2 (a+bx+cx^2)^{3/2} (d-fx^2)} \, dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/x^2/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument ValueDone
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a + bx + cx^2)^{3/2} (d - fx^2)} \, dx = \int \frac{1}{x^2 \, (d - f \, x^2) \, (c \, x^2 + b \, x + a)^{3/2}} \, dx$$

```
[In] int(1/(x^2*(d - f*x^2)*(a + b*x + c*x^2)^(3/2)),x)
[Out] int(1/(x^2*(d - f*x^2)*(a + b*x + c*x^2)^(3/2)), x)
```

3.109 $\int \frac{x^2\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 761

$$\int \frac{x^2\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx = -\frac{(4ce-bf-2cfx)\sqrt{a+bx+cx^2}}{4cf^2}$$

$$-\frac{(b^2f^2+4cf(be-af)-8c^2(e^2-df))\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}f^3}$$

$$-\frac{\left(c\left(e^4-4de^2f+2d^2f^2-e^3\sqrt{e^2-4df}+2def\sqrt{e^2-4df}\right)+f\left(af\left(e^2-2df-e\sqrt{e^2-4df}\right)-b\left(e^3-3d\right.\right.\right.}{\sqrt{2}f^3\sqrt{e^2-4df}\sqrt{c\left(e^2-2df-e\sqrt{e^2-4d\right.}}}$$

$$+\frac{\left(c\left(e^4-4de^2f+2d^2f^2+e^3\sqrt{e^2-4df}-2def\sqrt{e^2-4df}\right)+f\left(af\left(e^2-2df+e\sqrt{e^2-4df}\right)-b\left(e^3-3d\right.\right.\right.}{\sqrt{2}f^3\sqrt{e^2-4df}\sqrt{c\left(e^2-2df+e\sqrt{e^2-4d\right.}}}$$

```
[Out] -1/8*(b^2*f^2+4*c*f*(-a*f+b*e)-8*c^2*(-d*f+e^2))*arctanh(1/2*(2*c*x+b)/c^(1
/2)/(c*x^2+b*x+a)^(1/2))/c^(3/2)/f^3-1/4*(-2*c*f*x-b*f+4*c*e)*(c*x^2+b*x+a)
^(1/2)/c/f^2-1/2*arctanh(1/4*(4*a*f+2*x*(b*f-c*(e-(-4*d*f+e^2)^(1/2)))-b*(e
-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*
f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2))*(c*(e^4-4*d*e^2*f+2*d^2*f^2-e^3*(
-4*d*f+e^2)^(1/2)+2*d*e*f*(-4*d*f+e^2)^(1/2))+f*(a*f*(e^2-2*d*f-e*(-4*d*f+e
^2)^(1/2))-b*(e^3-3*d*e*f-e^2*(-4*d*f+e^2)^(1/2)+d*f*(-4*d*f+e^2)^(1/2))))/
f^3*2^(1/2)/(-4*d*f+e^2)^(1/2)/(f*(2*a*f-b*(e-(-4*d*f+e^2)^(1/2)))+c*(e^2-2
*d*f-e*(-4*d*f+e^2)^(1/2)))^(1/2)+1/2*arctanh(1/4*(4*a*f-b*(e+(-4*d*f+e^2)^
(1/2))+2*x*(b*f-c*(e+(-4*d*f+e^2)^(1/2))))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e
^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2))*(c*(e^4-4*d*
e^2*f+2*d^2*f^2+e^3*(-4*d*f+e^2)^(1/2)-2*d*e*f*(-4*d*f+e^2)^(1/2))+f*(a*f*(
e^2-2*d*f+e*(-4*d*f+e^2)^(1/2))-b*(e^3-3*d*e*f+e^2*(-4*d*f+e^2)^(1/2)-d*f*(
```

```
-4*d*f+e^2)^(1/2))))/f^3*2^(1/2)/(-4*d*f+e^2)^(1/2)/(c*(e^2-2*d*f+e*(-4*d*f
+e^2)^(1/2))+f*(2*a*f-b*(e+(-4*d*f+e^2)^(1/2))))^(1/2)
```

Rubi [A] (verified)

Time = 2.02 (sec) , antiderivative size = 761, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1081, 1090, 635, 212, 1046, 738}

$$\int \frac{x^2\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx = -\frac{\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(4cf(be-af)+b^2f^2-8c^2(e^2-df)\right)}{8c^{3/2}f^3}$$

$$-\frac{\left(f\left(af\left(-e\sqrt{e^2-4df}-2df+e^2\right)-b\left(-e^2\sqrt{e^2-4df}+df\sqrt{e^2-4df}-3def+e^3\right)\right)+c\left(2d^2f^2-4de^2f\right.\right.}{\sqrt{2}f^3\sqrt{e^2-4df}\sqrt{f\left(2af-b\left(e-\sqrt{e^2-4\right.\right.}}}$$

$$+\frac{\left(f\left(af\left(e\sqrt{e^2-4df}-2df+e^2\right)-b\left(e^2\sqrt{e^2-4df}-df\sqrt{e^2-4df}-3def+e^3\right)\right)+c\left(2d^2f^2-4de^2f-2a\right.\right.}{\sqrt{2}f^3\sqrt{e^2-4df}\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+\right.\right.}}$$

$$-\frac{\sqrt{a+bx+cx^2}(-bf+4ce-2cfx)}{4cf^2}$$

```
[In] Int[(x^2*Sqrt[a + b*x + c*x^2])/(d + e*x + f*x^2),x]
```

```
[Out] -1/4*((4*c*e - b*f - 2*c*f*x)*Sqrt[a + b*x + c*x^2])/(c*f^2) - ((b^2*f^2 +
4*c*f*(b*e - a*f) - 8*c^2*(e^2 - d*f))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[
a + b*x + c*x^2])])/(8*c^(3/2)*f^3) - ((c*(e^4 - 4*d*e^2*f + 2*d^2*f^2 - e^
3*Sqrt[e^2 - 4*d*f] + 2*d*e*f*Sqrt[e^2 - 4*d*f]) + f*(a*f*(e^2 - 2*d*f - e*
Sqrt[e^2 - 4*d*f]) - b*(e^3 - 3*d*e*f - e^2*Sqrt[e^2 - 4*d*f] + d*f*Sqrt[e^
2 - 4*d*f])))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e -
Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 -
(c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*f^3*Sqrt[e
^2 - 4*d*f]*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e -
Sqrt[e^2 - 4*d*f]))]) + ((c*(e^4 - 4*d*e^2*f + 2*d^2*f^2 + e^3*Sqrt[e^2 - 4
*d*f] - 2*d*e*f*Sqrt[e^2 - 4*d*f]) + f*(a*f*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d
*f]) - b*(e^3 - 3*d*e*f + e^2*Sqrt[e^2 - 4*d*f] - d*f*Sqrt[e^2 - 4*d*f])))*
ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d
*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sq
rt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*f^3*Sqrt[e^2 - 4*d*f]*Sq
rt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d
*f]))])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1046

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1081

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (C_.)*(x_)^2)*((d_) +
(e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(C*(b*f*p - c*e*(2*p + q
 + 2)) + 2*c*C*f*(p + q + 1)*x)*(a + b*x + c*x^2)^p*((d + e*x + f*x^2)^(q +
 1)/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3))), x] - Dist[1/(2*c*f^2*(p + q + 1
)*(2*p + 2*q + 3)), Int[(a + b*x + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[
p*(b*d - a*e)*(C*(c*e - b*f)*(q + 1) - c*(C*e)*(2*p + 2*q + 3)) + (p + q +
1)*(b^2*C*d*f*p + a*c*(C*(2*d*f - e^2*(2*p + q + 2)) + f*(-2*A*f)*(2*p + 2*
q + 3))) + (2*p*(c*d - a*f)*(C*(c*e - b*f)*(q + 1) - c*(C*e)*(2*p + 2*q + 3
)) + (p + q + 1)*(C*e*f*p*(b^2 - 4*a*c) - b*c*(C*(e^2 - 4*d*f)*(2*p + q + 2
) + f*(2*C*d + 2*A*f)*(2*p + 2*q + 3))))*x + (p*(c*e - b*f)*(C*(c*e - b*f)*
(q + 1) - c*(C*e)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2 - 4*a*c) - c
^2*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d + 2*A*f)*(2*p + 2*q + 3))))*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, C, q}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*
q + 3, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1090

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqr
```

```
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

integral

$$= -\frac{(4ce - bf - 2cfx)\sqrt{a + bx + cx^2}}{4cf^2} - \frac{\int \frac{-\frac{1}{4}d(4bce - b^2f - 4acf) - \frac{1}{4}(8c^2de - b^2ef - 4acef + 4bc(e^2 - 2df))x + \frac{1}{4}(b^2f^2 + 4cf(be - af) - 8c^2(e^2 - df))x^2}{\sqrt{a + bx + cx^2}(d + ex + fx^2)}\,dx}{2cf^2}$$

$$= -\frac{(4ce - bf - 2cfx)\sqrt{a + bx + cx^2}}{4cf^2} - \frac{\int \frac{-\frac{1}{4}df(4bce - b^2f - 4acf) - \frac{1}{4}d(b^2f^2 + 4cf(be - af) - 8c^2(e^2 - df)) + (\frac{1}{4}f(-8c^2de + b^2ef + 4acef - 4bc(e^2 - 2df)) - \frac{1}{4}e(b^2f^2 + 4cf(be - af) - 8c^2(e^2}{\sqrt{a + bx + cx^2}(d + ex + fx^2)}}{2cf^3} - \frac{(b^2f^2 + 4cf(be - af) - 8c^2(e^2 - df))\int \frac{1}{\sqrt{a + bx + cx^2}}\,dx}{8cf^3}$$

$$= -\frac{(4ce - bf - 2cfx)\sqrt{a + bx + cx^2}}{4cf^2} - \frac{(b^2f^2 + 4cf(be - af) - 8c^2(e^2 - df))\,\mathrm{Subst}\left(\int \frac{1}{4c - x^2}\,dx, x, \frac{b + 2cx}{\sqrt{a + bx + cx^2}}\right)}{4cf^3} - \frac{\left(c\left(e^4 - 4de^2f + 2d^2f^2 + e^3\sqrt{e^2 - 4df} - 2def\sqrt{e^2 - 4df}\right) + f\left(af\left(e^2 - 2df + e\sqrt{e^2 - 4df}\right) - b\left(e^3 - 3de\right.\right.\right.}{f^3\sqrt{e^2 - 4df}} + \frac{\left(c\left(e^4 - 4de^2f + 2d^2f^2 - e^3\sqrt{e^2 - 4df} + 2def\sqrt{e^2 - 4df}\right) + f\left(af\left(e^2 - 2df - e\sqrt{e^2 - 4df}\right) - b\left(e^3 - 3de\right.\right.\right.}{f^3\sqrt{e^2 - 4df}}$$

$$= -\frac{(4ce - bf - 2cfx)\sqrt{a + bx + cx^2}}{4cf^2} - \frac{(b^2f^2 + 4cf(be - af) - 8c^2(e^2 - df))\tanh^{-1}\left(\frac{b + 2cx}{2\sqrt{c}\sqrt{a + bx + cx^2}}\right)}{8c^{3/2}f^3} + \frac{\left(2\left(c\left(e^4 - 4de^2f + 2d^2f^2 + e^3\sqrt{e^2 - 4df} - 2def\sqrt{e^2 - 4df}\right) + f\left(af\left(e^2 - 2df + e\sqrt{e^2 - 4df}\right) - b\left(e^3 - 3\right.\right.\right.\right.}{} - \frac{\left(2\left(c\left(e^4 - 4de^2f + 2d^2f^2 - e^3\sqrt{e^2 - 4df} + 2def\sqrt{e^2 - 4df}\right) + f\left(af\left(e^2 - 2df - e\sqrt{e^2 - 4df}\right) - b\left(e^3 - 3\right.\right.\right.\right.}{}$$

$$= -\frac{(4ce - bf - 2cfx)\sqrt{a + bx + cx^2}}{4cf^2}$$
$$- \frac{(b^2f^2 + 4cf(be - af) - 8c^2(e^2 - df))\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}f^3}$$
$$- \frac{\left(c\left(e^4 - 4de^2f + 2d^2f^2 - e^3\sqrt{e^2 - 4df} + 2def\sqrt{e^2 - 4df}\right) + f\left(af\left(e^2 - 2df - e\sqrt{e^2 - 4df}\right) - b\left(e^3 - 3c\right.\right.\right.}{\sqrt{2}f^3\sqrt{e^2 - 4df}\sqrt{ce^2 - 2cdf - bef +}}$$
$$+ \frac{\left(c\left(e^4 - 4de^2f + 2d^2f^2 + e^3\sqrt{e^2 - 4df} - 2def\sqrt{e^2 - 4df}\right) + f\left(af\left(e^2 - 2df + e\sqrt{e^2 - 4df}\right) - b\left(e^3 - 3c\right.\right.\right.}{\sqrt{2}f^3\sqrt{e^2 - 4df}\sqrt{ce^2 - 2cdf - bef +}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 1.14 (sec) , antiderivative size = 864, normalized size of antiderivative = 1.14

$$\int \frac{x^2\sqrt{a + bx + cx^2}}{d + ex + fx^2}\,dx$$
$$= \frac{2\sqrt{c}f(-4ce + bf + 2cfx)\sqrt{a + x(b + cx)} + (-b^2f^2 + 4cf(-be + af) + 8c^2(e^2 - df))\,\mathrm{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b}}\right.}{}$$

```
[In] Integrate[(x^2*Sqrt[a + b*x + c*x^2])/(d + e*x + f*x^2),x]
```

```
[Out] (2*Sqrt[c]*f*(-4*c*e + b*f + 2*c*f*x)*Sqrt[a + x*(b + c*x)] + (-(b^2*f^2) +
 4*c*f*(-(b*e) + a*f) + 8*c^2*(e^2 - d*f))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*S
qrt[a + x*(b + c*x)])] - 8*c^(3/2)*RootSum[b^2*d - a*b*e + a^2*f - 4*b*Sqrt
[c]*d*#1 + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 + b*e*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c
]*e*#1^3 + f*#1^4 & , (-(b*c*d*e^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2]
 - #1]) + a*c*e^3*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + b*c*d^2*
f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + b^2*d*e*f*Log[-(Sqrt[c]*
x) + Sqrt[a + b*x + c*x^2] - #1] - 2*a*c*d*e*f*Log[-(Sqrt[c]*x) + Sqrt[a +
b*x + c*x^2] - #1] - a*b*e^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #
1] + a^2*e*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + 2*c^(3/2)*d
*e^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - 2*c^(3/2)*d^2*f*Lo
g[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - 2*b*Sqrt[c]*d*e*f*Log[-(S
qrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + 2*a*Sqrt[c]*d*f^2*Log[-(Sqrt[c
]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - c*e^3*Log[-(Sqrt[c]*x) + Sqrt[a + b
*x + c*x^2] - #1]*#1^2 + 2*c*d*e*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2]
 - #1]*#1^2 + b*e^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 -
 b*d*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 - a*e*f^2*Log[
-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2)/(2*b*Sqrt[c]*d - a*Sqrt[c]
*e - 4*c*d*#1 - b*e*#1 + 2*a*f*#1 + 3*Sqrt[c]*e*#1^2 - 2*f*#1^3) & ])/(8*c^
(3/2)*f^3)
```

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 1148, normalized size of antiderivative = 1.51

method	result	size
risch	Expression too large to display	1148
default	Expression too large to display	1666

```
[In] int(x^2*(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(2*c*f*x+b*f-4*c*e)*(c*x^2+b*x+a)^(1/2)/c/f^2+1/8/c/f^2*(1/f*(4*a*c*f^2
-b^2*f^2-4*b*c*e*f-8*c^2*d*f+8*c^2*e^2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a
)^(1/2))/c^(1/2)+4/f^2*c*(a*e*f^2*(-4*d*f+e^2)^(1/2)+b*d*f^2*(-4*d*f+e^2)^(
1/2)-b*e^2*f*(-4*d*f+e^2)^(1/2)-2*c*d*e*f*(-4*d*f+e^2)^(1/2)+c*e^3*(-4*d*f+
e^2)^(1/2)+2*a*d*f^3-a*e^2*f^2-3*b*d*e*f^2+b*e^3*f-2*c*d^2*f^2+4*c*d*e^2*f-
c*e^4)/(-4*d*f+e^2)^(1/2)*2^(1/2)/((b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/
2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((b*f*(-4*d*f+e^2)^(1/2)-
(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+(c*(-4*d*f+e^2)^(1/
2)+b*f-c*e)/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+1/2*2^(1/2)*((b*f*(-4*d*f+e
^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4
*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2*c+4*(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(
x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+2*(b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/
2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2/f*(-e+(-4*d*f+e^2)^(
1/2))))+4/f^2*c*(a*e*f^2*(-4*d*f+e^2)^(1/2)+b*d*f^2*(-4*d*f+e^2)^(1/2)-b*e^
2*f*(-4*d*f+e^2)^(1/2)-2*c*d*e*f*(-4*d*f+e^2)^(1/2)+c*e^3*(-4*d*f+e^2)^(1/2
)-2*a*d*f^3+a*e^2*f^2+3*b*d*e*f^2-b*e^3*f+2*c*d^2*f^2-4*c*d*e^2*f+c*e^4)/(-
4*d*f+e^2)^(1/2)*2^(1/2)/((-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2
*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f
+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+1/f*(-c*(-4*d*f+e^2)^(1/2)
+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-b*f*(-4*d*f+e^2)^
(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+
1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/
2*(e+(-4*d*f+e^2)^(1/2))/f)+2*(-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c
*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f
)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx = \text{Timed out}$$

```
[In] integrate(x^2*(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{x^2\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx = \int \frac{x^2\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx$$

```
[In] integrate(x**2*(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral(x**2*sqrt(a + b*x + c*x**2)/(d + e*x + f*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2*(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re deta
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^2\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^2*(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx = \int \frac{x^2\,\sqrt{c\,x^2+b\,x+a}}{f\,x^2+e\,x+d}\,dx$$

```
[In] int((x^2*(a + b*x + c*x^2)^(1/2))/(d + e*x + f*x^2),x)
```

```
[Out] int((x^2*(a + b*x + c*x^2)^(1/2))/(d + e*x + f*x^2), x)
```

3.110 $\int \frac{x\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 549

$$\int \frac{x\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx = \frac{\sqrt{a+bx+cx^2}}{f} - \frac{(2ce-bf)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}f^2} - \frac{\left(2df(ce-bf)+\left(e-\sqrt{e^2-4df}\right)\left(f(be-af)-c(e^2-df)\right)\right)\operatorname{arctanh}\left(\frac{4af-b\left(e-\sqrt{e^2-4df}\right)+2\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)x}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}\sqrt{a+bx+cx^2}}\right)}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}} + \frac{\left(2df(ce-bf)+\left(e+\sqrt{e^2-4df}\right)\left(f(be-af)-c(e^2-df)\right)\right)\operatorname{arctanh}\left(\frac{4af-b\left(e+\sqrt{e^2-4df}\right)+2\left(bf-c\left(e+\sqrt{e^2-4df}\right)\right)x}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2-4df}}\sqrt{a+bx+cx^2}}\right)}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2-4df}}}$$

```
[Out] -1/2*(-b*f+2*c*e)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/f^2/c^
(1/2)+(c*x^2+b*x+a)^(1/2)/f-1/2*arctanh(1/4*(4*a*f+2*x*(b*f-c*(e-(-4*d*f+e^
2)^(1/2)))-b*(e-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c
*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2))*(2*d*f*(-b*f+c*e)+
(f*(-a*f+b*e)-c*(-d*f+e^2))*(e-(-4*d*f+e^2)^(1/2)))/f^2*2^(1/2)/(-4*d*f+e^2
)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)+1
/2*arctanh(1/4*(4*a*f-b*(e+(-4*d*f+e^2)^(1/2))+2*x*(b*f-c*(e+(-4*d*f+e^2)^(
1/2))))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)
*(-4*d*f+e^2)^(1/2))^(1/2))*(2*d*f*(-b*f+c*e)+(f*(-a*f+b*e)-c*(-d*f+e^2))*(
e+(-4*d*f+e^2)^(1/2)))/f^2*2^(1/2)/(-4*d*f+e^2)^(1/2)/(c*e^2-2*c*d*f-b*e*f+
2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 4.31 (sec) , antiderivative size = 549, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {1033, 1090, 635, 212, 1046, 738}

$$\int \frac{x\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx =$$

$$-\frac{\left(\left(e-\sqrt{e^2-4df}\right)\left(f(be-af)-c(e^2-df)\right)+2df(ce-bf)\right)\operatorname{arctanh}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b(e-}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf}}\right.}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}$$

$$+\frac{\left(\left(\sqrt{e^2-4df}+e\right)\left(f(be-af)-c(e^2-df)\right)+2df(ce-bf)\right)\operatorname{arctanh}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b(\sqrt{}}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)}}\right.}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}$$

$$-\frac{(2ce-bf)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}f^2}+\frac{\sqrt{a+bx+cx^2}}{f}$$

```
[In] Int[(x*Sqrt[a + b*x + c*x^2])/(d + e*x + f*x^2),x]
```

```
[Out] Sqrt[a + b*x + c*x^2]/f - ((2*c*e - b*f)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqr
t[a + b*x + c*x^2])])/(2*Sqrt[c]*f^2) - ((2*d*f*(c*e - b*f) + (e - Sqrt[e^2
 - 4*d*f])*(f*(b*e - a*f) - c*(e^2 - d*f)))*ArcTanh[(4*a*f - b*(e - Sqrt[e^
2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2
- 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x +
 c*x^2])])/(Sqrt[2]*f^2*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*
a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]) + ((2*d*f*(c*e - b*f) + (e + Sqrt[e
^2 - 4*d*f])*(f*(b*e - a*f) - c*(e^2 - d*f)))*ArcTanh[(4*a*f - b*(e + Sqrt[
e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^
2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x
 + c*x^2])])/(Sqrt[2]*f^2*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f +
2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NegQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[h*(a + b*x + c*x^2)^p*((d
+ e*x + f*x^2)^(q + 1)/(2*f*(p + q + 1))), x] - Dist[1/(2*f*(p + q + 1)), I
nt[(a + b*x + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[h*p*(b*d - a*e) + a*(
h*e - 2*g*f)*(p + q + 1) + (2*h*p*(c*d - a*f) + b*(h*e - 2*g*f)*(p + q + 1)
)*x + (h*p*(c*e - b*f) + c*(h*e - 2*g*f)*(p + q + 1))*x^2, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d
*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]
```

Rule 1046

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1090

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqr
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{a+bx+cx^2}}{f} - \frac{\int \frac{\frac{bd}{2}+\frac{1}{2}(2cd+be-2af)x+\frac{1}{2}(2ce-bf)x^2}{\sqrt{a+bx+cx^2}(d+ex+fx^2)}\,dx}{f}$$

$$= \frac{\sqrt{a+bx+cx^2}}{f} - \frac{\int \frac{\frac{bdf}{2}-\frac{1}{2}d(2ce-bf)+\left(\frac{1}{2}f(2cd+be-2af)-\frac{1}{2}e(2ce-bf)\right)x}{\sqrt{a+bx+cx^2}(d+ex+fx^2)}\,dx}{f^2} - \frac{(2ce-bf)\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{2f^2}$$

$$= \frac{\sqrt{a+bx+cx^2}}{f} - \frac{(2ce-bf)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{f^2} - \frac{\left(2f\left(\frac{bdf}{2}-\frac{1}{2}d(2ce-bf)\right)-\left(\frac{1}{2}f(2cd+be-2af)-\frac{1}{2}e(2ce-bf)\right)\left(e-\sqrt{e^2-4df}\right)\right)\int \frac{}{\left(e-\sqrt{e^2-4\right.}}}{f^2\sqrt{e^2-4df}} + \frac{\left(2f\left(\frac{bdf}{2}-\frac{1}{2}d(2ce-bf)\right)-\left(\frac{1}{2}f(2cd+be-2af)-\frac{1}{2}e(2ce-bf)\right)\left(e+\sqrt{e^2-4df}\right)\right)\int \frac{}{\left(e+\sqrt{e^2-4\right.}}}{f^2\sqrt{e^2-4df}}$$

$$= \frac{\sqrt{a+bx+cx^2}}{f} - \frac{(2ce-bf)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}f^2} + \frac{\left(2\left(2f\left(\frac{bdf}{2}-\frac{1}{2}d(2ce-bf)\right)-\left(\frac{1}{2}f(2cd+be-2af)-\frac{1}{2}e(2ce-bf)\right)\left(e-\sqrt{e^2-4df}\right)\right)\right)\text{Subst}\left(\right.}{f^2\sqrt{e^2-4}} - \frac{\left(2\left(2f\left(\frac{bdf}{2}-\frac{1}{2}d(2ce-bf)\right)-\left(\frac{1}{2}f(2cd+be-2af)-\frac{1}{2}e(2ce-bf)\right)\left(e+\sqrt{e^2-4df}\right)\right)\right)\text{Subst}\left(\right.}{f^2\sqrt{e^2-4}}$$

$$= \frac{\sqrt{a+bx+cx^2}}{f} - \frac{(2ce-bf)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}f^2} - \frac{\left(2f(cde-bdf)+\left(e-\sqrt{e^2-4df}\right)\left(f(be-af)-c(e^2-df)\right)\right)\tanh^{-1}\left(\frac{4af-b\left(e-\sqrt{e^2-4df}\right)+2\left(\right.}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2-(c}}\right.}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}} + \frac{\left(2f(cde-bdf)+\left(e+\sqrt{e^2-4df}\right)\left(f(be-af)-c(e^2-df)\right)\right)\tanh^{-1}\left(\frac{4af-b\left(e+\sqrt{e^2-4df}\right)+2\left(\right.}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2+(c}}\right.}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2-4df}}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.68 (sec) , antiderivative size = 642, normalized size of antiderivative = 1.17

$$\int \frac{x\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx$$

$$= \frac{2f\sqrt{a+x(b+cx)} + \frac{(2ce-bf)\log\left(b+2cx-2\sqrt{c}\sqrt{a+x(b+cx)}\right)}{\sqrt{c}} + 2\text{RootSum}\left[b^2d-abe+a^2f-4b\sqrt{c}d\#1+2a\sqrt{c}e\right.}{}$$

```
[In] Integrate[(x*Sqrt[a + b*x + c*x^2])/(d + e*x + f*x^2),x]
```

```
[Out] (2*f*Sqrt[a + x*(b + c*x)] + ((2*c*e - b*f)*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[
a + x*(b + c*x)]])/Sqrt[c] + 2*RootSum[b^2*d - a*b*e + a^2*f - 4*b*Sqrt[c]*
d*#1 + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 + b*e*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*
#1^3 + f*#1^4 & , (-(b*c*d*e*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]
) + a*c*e^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + b^2*d*f*Log[-(
Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - a*c*d*f*Log[-(Sqrt[c]*x) + Sqrt[
a + b*x + c*x^2] - #1] - a*b*e*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] -
 #1] + a^2*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + 2*c^(3/2)*d
*e*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - 2*b*Sqrt[c]*d*f*Log[
-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - c*e^2*Log[-(Sqrt[c]*x) + Sq
rt[a + b*x + c*x^2] - #1]*#1^2 + c*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*
x^2] - #1]*#1^2 + b*e*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2
 - a*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2)/(2*b*Sqrt[c]*
d - a*Sqrt[c]*e - 4*c*d*#1 - b*e*#1 + 2*a*f*#1 + 3*Sqrt[c]*e*#1^2 - 2*f*#1^
3) & ])/(2*f^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1021 vs. $2(490) = 980$.

Time = 0.91 (sec) , antiderivative size = 1022, normalized size of antiderivative = 1.86

method	result	size
risch	Expression too large to display	1022
default	Expression too large to display	1580

```
[In] int(x*(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] (c*x^2+b*x+a)^(1/2)/f+1/2/f*(1/f*(b*f-2*c*e)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+
b*x+a)^(1/2))/c^(1/2)-1/2*(2*a*f^2*(-4*d*f+e^2)^(1/2)-2*b*e*f*(-4*d*f+e^2)^
(1/2)-2*c*d*f*(-4*d*f+e^2)^(1/2)+2*c*e^2*(-4*d*f+e^2)^(1/2)-2*a*e*f^2-4*b*d
*f^2+2*b*e^2*f+6*c*d*e*f-2*c*e^3)/f^2/(-4*d*f+e^2)^(1/2)*2^(1/2)/((b*f*(-4*
d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/
2)*ln(((b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f
+c*e^2)/f^2+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2
)))+1/2*2^(1/2)*((b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e
*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2*c+4*(c*
(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+2*(b*f*(-4*
d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/
2))/(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2))))-1/2*(2*a*f^2*(-4*d*f+e^2)^(1/2)-2*b*
e*f*(-4*d*f+e^2)^(1/2)-2*c*d*f*(-4*d*f+e^2)^(1/2)+2*c*e^2*(-4*d*f+e^2)^(1/2
)+2*a*e*f^2+4*b*d*f^2-2*b*e^2*f-6*c*d*e*f+2*c*e^3)/f^2/(-4*d*f+e^2)^(1/2)*2
^(1/2)/((-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d
*f+c*e^2)/f^2)^(1/2)*ln(((-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*
a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(
```

```
e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)
^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2
)^(1/2))/f)^2*c+4/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^
(1/2))/f)+2*(-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2
*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx = \text{Timed out}$$

```
[In] integrate(x*(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{x\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx = \int \frac{x\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx$$

```
[In] integrate(x*(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)
[Out] Integral(x*sqrt(a + b*x + c*x**2)/(d + e*x + f*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x*(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re deta
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x*(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx = \int \frac{x\,\sqrt{c\,x^2+b\,x+a}}{f\,x^2+e\,x+d}\,dx$$

```
[In] int((x*(a + b*x + c*x^2)^(1/2))/(d + e*x + f*x^2),x)
```

```
[Out] int((x*(a + b*x + c*x^2)^(1/2))/(d + e*x + f*x^2), x)
```

3.111 $\int \frac{\sqrt{a+bx+cx^2}}{d+ex+fx^2}\, dx$

Optimal result

Integrand size = 27, antiderivative size = 431

$$\int \frac{\sqrt{a+bx+cx^2}}{d+ex+fx^2}\, dx = \frac{\sqrt{c}\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{f}$$
$$-\frac{\sqrt{c\left(e^2-2df-e\sqrt{e^2-4df}\right)+f\left(2af-b\left(e-\sqrt{e^2-4df}\right)\right)}\operatorname{arctanh}\left(\frac{4af-b\left(e-\sqrt{e^2-4df}\right)+2\left(bf-c\left(e-\sqrt{\cdots}\right.\right.}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-\cdots}}}\right.}{\sqrt{2}f\sqrt{e^2-4df}}$$
$$+\frac{\sqrt{c\left(e^2-2df+e\sqrt{e^2-4df}\right)+f\left(2af-b\left(e+\sqrt{e^2-4df}\right)\right)}\operatorname{arctanh}\left(\frac{4af-b\left(e+\sqrt{e^2-4df}\right)+2\left(bf-c\left(e+\sqrt{\cdots}\right.\right.}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2-\cdots}}}\right.}{\sqrt{2}f\sqrt{e^2-4df}}$$

```
[Out] arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))*c^(1/2)/f-1/2*arctanh(1/
4*(4*a*f+2*x*(b*f-c*(e-(-4*d*f+e^2)^(1/2)))-b*(e-(-4*d*f+e^2)^(1/2)))*2^(1/
2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)
^(1/2))^(1/2))*(f*(2*a*f-b*(e-(-4*d*f+e^2)^(1/2)))+c*(e^2-2*d*f-e*(-4*d*f+e
^2)^(1/2)))^(1/2)/f*2^(1/2)/(-4*d*f+e^2)^(1/2)+1/2*arctanh(1/4*(4*a*f-b*(e+
(-4*d*f+e^2)^(1/2))+2*x*(b*f-c*(e+(-4*d*f+e^2)^(1/2))))*2^(1/2)/(c*x^2+b*x+
a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2))
*(c*(e^2-2*d*f+e*(-4*d*f+e^2)^(1/2))+f*(2*a*f-b*(e+(-4*d*f+e^2)^(1/2))))^(1
/2)/f*2^(1/2)/(-4*d*f+e^2)^(1/2)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 431, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1003, 635, 212, 1046, 738}

$$\int \frac{\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx =$$

$$-\frac{\sqrt{f\left(2af-b\left(e-\sqrt{e^2-4df}\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}\operatorname{arctanh}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\right.}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf}}\right.}{\sqrt{2}f\sqrt{e^2-4df}}$$

$$+\frac{\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}\operatorname{arctanh}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2}\right.}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-}}\right.}{\sqrt{2}f\sqrt{e^2-4df}}$$

$$+\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{f}$$

```
[In] Int[Sqrt[a + b*x + c*x^2]/(d + e*x + f*x^2),x]
```

```
[Out] (Sqrt[c]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/f - (Sqrt[
c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e - Sqrt[e^2 - 4*d*f]
))]*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 -
 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f
)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*f*Sqrt[e^2 - 4*d*f])
 + (Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2
 - 4*d*f]))]*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + S
qrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (
c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*f*Sqrt[e^2
- 4*d*f])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1003

```
Int[Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]/((d_) + (e_.)*(x_) + (f_.)*(x_)^
2), x_Symbol] :> Dist[c/f, Int[1/Sqrt[a + b*x + c*x^2], x], x] - Dist[1/f,
Int[(c*d - a*f + (c*e - b*f)*x)/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 -
4*d*f, 0]
```

Rule 1046

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\int \frac{cd-af+(ce-bf)x}{\sqrt{a+bx+cx^2}(d+ex+fx^2)}\,dx}{f}+\frac{c\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{f}\\
&= \frac{(2c)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx,x,\frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{f}\\
&\quad -\frac{\left(2f(cd-af)-(ce-bf)\left(e-\sqrt{e^2-4df}\right)\right)\int \frac{1}{\left(e-\sqrt{e^2-4df}+2fx\right)\sqrt{a+bx+cx^2}}\,dx}{f\sqrt{e^2-4df}}\\
&\quad +\frac{\left(2f(cd-af)-(ce-bf)\left(e+\sqrt{e^2-4df}\right)\right)\int \frac{1}{\left(e+\sqrt{e^2-4df}+2fx\right)\sqrt{a+bx+cx^2}}\,dx}{f\sqrt{e^2-4df}}\\
&= \frac{\sqrt{c}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{f}\\
&\quad +\frac{\left(2\left(2f(cd-af)-(ce-bf)\left(e-\sqrt{e^2-4df}\right)\right)\right)\text{Subst}\left(\int \frac{1}{16af^2-8bf\left(e-\sqrt{e^2-4df}\right)+4c\left(e-\sqrt{e^2-4df}\right)^2-x^2}\right.}{f\sqrt{e^2-4df}}\\
&\quad -\frac{\left(2\left(2f(cd-af)-(ce-bf)\left(e+\sqrt{e^2-4df}\right)\right)\right)\text{Subst}\left(\int \frac{1}{16af^2-8bf\left(e+\sqrt{e^2-4df}\right)+4c\left(e+\sqrt{e^2-4df}\right)^2-x^2}\right.}{f\sqrt{e^2-4df}}
\end{aligned}$$

$$= \frac{\sqrt{c}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{f} - \frac{\sqrt{c\left(e^2-2df-e\sqrt{e^2-4df}\right)+f\left(2af-b\left(e-\sqrt{e^2-4df}\right)\right)}\tanh^{-1}\left(\frac{4af-b\left(e-\sqrt{e^2-4df}\right)+2\left(bf-\right.}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2-(ce-b}}\right.}{\sqrt{2}f\sqrt{e^2-4df}} + \frac{\sqrt{c\left(e^2-2df+e\sqrt{e^2-4df}\right)+f\left(2af-b\left(e+\sqrt{e^2-4df}\right)\right)}\tanh^{-1}\left(\frac{4af-b\left(e+\sqrt{e^2-4df}\right)+2\left(bf-\right.}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2+(ce-b}}\right.}{\sqrt{2}f\sqrt{e^2-4df}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 539, normalized size of antiderivative = 1.25

$$\int \frac{\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx$$

$$= \frac{2\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c}x}{-\sqrt{a}+\sqrt{a+x(b+cx)}}\right) + \operatorname{RootSum}\left[c^2d - bce + b^2f + 2\sqrt{a}ce\#1 - 4\sqrt{a}bf\#1 - 2cd\#1^2 + be\#1^2 +\right.}{}$$

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/(d + e*x + f*x^2),x]
```

```
[Out] (2*Sqrt[c]*ArcTanh[(Sqrt[c]*x)/(-Sqrt[a] + Sqrt[a + x*(b + c*x)])] + RootSu
m[c^2*d - b*c*e + b^2*f + 2*Sqrt[a]*c*e*#1 - 4*Sqrt[a]*b*f*#1 - 2*c*d*#1^2
+ b*e*#1^2 + 4*a*f*#1^2 - 2*Sqrt[a]*e*#1^3 + d*#1^4 & , (-(c^2*d*Log[x]) +
b*c*e*Log[x] - b^2*f*Log[x] + a*c*f*Log[x] + c^2*d*Log[-Sqrt[a] + Sqrt[a +
b*x + c*x^2] - x*#1] - b*c*e*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] +
 b^2*f*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] - a*c*f*Log[-Sqrt[a] +
Sqrt[a + b*x + c*x^2] - x*#1] - 2*Sqrt[a]*c*e*Log[x]*#1 + 2*Sqrt[a]*b*f*Log
[x]*#1 + 2*Sqrt[a]*c*e*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1]*#1 - 2*
Sqrt[a]*b*f*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1]*#1 + c*d*Log[x]*#1
^2 - a*f*Log[x]*#1^2 - c*d*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1]*#1^
2 + a*f*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1]*#1^2)/(-(Sqrt[a]*c*e)
+ 2*Sqrt[a]*b*f + 2*c*d*#1 - b*e*#1 - 4*a*f*#1 + 3*Sqrt[a]*e*#1^2 - 2*d*#1^
3) & ])/f
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1546 vs. 2(376) = 752.

Time = 0.94 (sec) , antiderivative size = 1547, normalized size of antiderivative = 3.59

method	result	size
default	Expression too large to display	1547

```
[In] int((c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/(-4*d*f+e^2)^(1/2)*(1/2*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4/f*(-c*
(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*(-b*f*(-4*d*
f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)
+1/2/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*ln((1/2/f*(-c*(-4*d*f+e^2)^(1/2)+b*f
-c*e)+c*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))/c^(1/2)+((x+1/2*(e+(-4*d*f+e^2)^(
1/2))/f)^2*c+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/
2))/f)+1/2*(-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*
c*d*f+c*e^2)/f^2)^(1/2))/c^(1/2)-1/2*(-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^
(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2*2^(1/2)/((-b*f*(-4*d*f+e^2)^(1/2
)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-b*f*
(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+
1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^
(1/2)*((-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*
f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4/f*(-c*(-4*d*f
+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*(-b*f*(-4*d*f+e^2)^
(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/
2*(e+(-4*d*f+e^2)^(1/2))/f)))+1/(-4*d*f+e^2)^(1/2)*(1/2*(4*(x-1/2/f*(-e+(-4
*d*f+e^2)^(1/2)))^2*c+4*(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2/f*(-e+(-4*d
*f+e^2)^(1/2)))+2*(b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*
e*f-2*c*d*f+c*e^2)/f^2)^(1/2)+1/2*(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*ln((1/2*
(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f+c*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2))))/c^(1/
2)+((x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2*c+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*
(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+1/2*(b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^
(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/c^(1/2)-1/2*(b*f*(-4*d*f
+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2*2^(1/2)
/((b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^
2)/f^2)^(1/2)*ln(((b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*
e*f-2*c*d*f+c*e^2)/f^2+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2/f*(-e+(-4*d*
f+e^2)^(1/2)))+1/2*2^(1/2)*((b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+
2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2))
)^2*c+4*(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+
2*(b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^
2)/f^2)^(1/2))/(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx = \text{Timed out}$$

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx = \int \frac{\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx$$

```
[In] integrate((c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)
[Out] Integral(sqrt(a + b*x + c*x**2)/(d + e*x + f*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re deta
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx = \int \frac{\sqrt{c\,x^2+b\,x+a}}{f\,x^2+e\,x+d}\,dx$$

```
[In] int((a + b*x + c*x^2)^(1/2)/(d + e*x + f*x^2),x)
[Out] int((a + b*x + c*x^2)^(1/2)/(d + e*x + f*x^2), x)
```

3.112 $\int \frac{\sqrt{a+bx+cx^2}}{x\left(d+ex+fx^2\right)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 523

$$\int \frac{\sqrt{a+bx+cx^2}}{x\left(d+ex+fx^2\right)}\,dx = -\frac{\sqrt{a}\,\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d}$$
$$+\frac{\left(cd\left(e-\sqrt{e^2-4df}\right)-f\left(2bd-a\left(e+\sqrt{e^2-4df}\right)\right)\right)\text{arctanh}\left(\frac{4af-b\left(e-\sqrt{e^2-4df}\right)+2\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)x}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}\sqrt{a+bx+c}}\right.}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{c\left(e^2-2df-e\sqrt{e^2-4df}\right)+f\left(2af-b\left(e-\sqrt{e^2-4df}\right)\right)}}$$
$$-\frac{\left(cd\left(e+\sqrt{e^2-4df}\right)-f\left(2bd-a\left(e-\sqrt{e^2-4df}\right)\right)\right)\text{arctanh}\left(\frac{4af-b\left(e+\sqrt{e^2-4df}\right)+2\left(bf-c\left(e+\sqrt{e^2-4df}\right)\right)x}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2-4df}}\sqrt{a+bx+c}}\right.}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{c\left(e^2-2df+e\sqrt{e^2-4df}\right)+f\left(2af-b\left(e+\sqrt{e^2-4df}\right)\right)}}$$

```
[Out] -arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))*a^(1/2)/d+1/2*arctanh(1
/4*(4*a*f+2*x*(b*f-c*(e-(-4*d*f+e^2)^(1/2)))-b*(e-(-4*d*f+e^2)^(1/2)))*2^(1
/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2
)^(1/2))^(1/2))*(c*d*(e-(-4*d*f+e^2)^(1/2))-f*(2*b*d-a*(e+(-4*d*f+e^2)^(1/2
))))/d*2^(1/2)/(-4*d*f+e^2)^(1/2)/(f*(2*a*f-b*(e-(-4*d*f+e^2)^(1/2)))+c*(e^
2-2*d*f-e*(-4*d*f+e^2)^(1/2)))^(1/2)-1/2*arctanh(1/4*(4*a*f-b*(e+(-4*d*f+e^
2)^(1/2))+2*x*(b*f-c*(e+(-4*d*f+e^2)^(1/2))))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(
c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2))*(-f*(2*b*
d-a*(e-(-4*d*f+e^2)^(1/2)))+c*d*(e+(-4*d*f+e^2)^(1/2)))/d*2^(1/2)/(-4*d*f+e
^2)^(1/2)/(c*(e^2-2*d*f+e*(-4*d*f+e^2)^(1/2))+f*(2*a*f-b*(e+(-4*d*f+e^2)^(1
/2))))^(1/2)
```

Rubi [A] (verified)

Time = 2.29 (sec) , antiderivative size = 521, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {6860, 748, 857, 635, 212, 738, 1033, 1090, 1046}

$$\int \frac{\sqrt{a+bx+cx^2}}{x\,(d+ex+fx^2)}\,dx =$$

$$-\frac{\left(-af\left(\sqrt{e^2-4df}+e\right)+2bdf-cd\left(e-\sqrt{e^2-4df}\right)\right)\text{arctanh}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cd}}\right.}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{f\left(2af-b\left(e-\sqrt{e^2-4df}\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}$$

$$+\frac{\left(-af\left(e-\sqrt{e^2-4df}\right)+2bdf-cd\left(\sqrt{e^2-4df}+e\right)\right)\text{arctanh}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cd}}\right.}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}$$

$$-\frac{\sqrt{a}\,\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d}$$

```
[In] Int[Sqrt[a + b*x + c*x^2]/(x*(d + e*x + f*x^2)),x]
```

```
[Out] -((Sqrt[a]*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/d) - ((2
*b*d*f - c*d*(e - Sqrt[e^2 - 4*d*f]) - a*f*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh
[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x
)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2
- 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*d*Sqrt[e^2 - 4*d*f]*Sqrt[c*(e^2
 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e - Sqrt[e^2 - 4*d*f]))]) +
 ((2*b*d*f - a*f*(e - Sqrt[e^2 - 4*d*f]) - c*d*(e + Sqrt[e^2 - 4*d*f]))*Arc
Tanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]
))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[
e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*d*Sqrt[e^2 - 4*d*f]*Sqrt[c*
(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))
])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 748

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[h*(a + b*x + c*x^2)^p*((d
+ e*x + f*x^2)^(q + 1)/(2*f*(p + q + 1))), x] - Dist[1/(2*f*(p + q + 1)), I
nt[(a + b*x + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[h*p*(b*d - a*e) + a*(
h*e - 2*g*f)*(p + q + 1) + (2*h*p*(c*d - a*f) + b*(h*e - 2*g*f)*(p + q + 1)
)*x + (h*p*(c*e - b*f) + c*(h*e - 2*g*f)*(p + q + 1))*x^2, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d
*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]
```

Rule 1046

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1090

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqr
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rule 6860

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{\sqrt{a+bx+cx^2}}{dx} + \frac{(-e-fx)\sqrt{a+bx+cx^2}}{d\,(d+ex+fx^2)} \right) dx$$

$$= \frac{\int \frac{\sqrt{a+bx+cx^2}}{x}\,dx}{d} + \frac{\int \frac{(-e-fx)\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx}{d}$$

$$= -\frac{\int \frac{-2a-bx}{x\sqrt{a+bx+cx^2}}\,dx}{2d} - \frac{\int \frac{-\frac{1}{2}(bd-2ae)f-\frac{1}{2}f(2cd-be-2af)x+\frac{1}{2}bf^2x^2}{\sqrt{a+bx+cx^2}(d+ex+fx^2)}\,dx}{df}$$

$$= \frac{a\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx}{d} - \frac{\int \frac{-\frac{1}{2}bdf^2-\frac{1}{2}(bd-2ae)f^2+\left(-\frac{1}{2}bef^2-\frac{1}{2}f^2(2cd-be-2af)\right)x}{\sqrt{a+bx+cx^2}(d+ex+fx^2)}\,dx}{df^2}$$

$$= -\frac{(2a)\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx}{\sqrt{a+bx+cx^2}}\right)}{d} - \frac{\left(2bdf-af\left(e-\sqrt{e^2-4df}\right)-cd\left(e+\sqrt{e^2-4df}\right)\right)\int \frac{1}{\left(e+\sqrt{e^2-4df}+2fx\right)\sqrt{a+bx+cx^2}}\,dx}{d\sqrt{e^2-4df}} + \frac{\left(2bdf-cd\left(e-\sqrt{e^2-4df}\right)-af\left(e+\sqrt{e^2-4df}\right)\right)\int \frac{1}{\left(e-\sqrt{e^2-4df}+2fx\right)\sqrt{a+bx+cx^2}}\,dx}{d\sqrt{e^2-4df}}$$

$$= -\frac{\sqrt{a}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d} + \frac{\left(2\left(2bdf-af\left(e-\sqrt{e^2-4df}\right)-cd\left(e+\sqrt{e^2-4df}\right)\right)\right)\text{Subst}\left(\int \frac{1}{16af^2-8bf\left(e+\sqrt{e^2-4df}\right)+4c\left(e+\sqrt{e^2-\right.}}\right.}{d\sqrt{e^2-4df}} - \frac{\left(2\left(2bdf-cd\left(e-\sqrt{e^2-4df}\right)-af\left(e+\sqrt{e^2-4df}\right)\right)\right)\text{Subst}\left(\int \frac{1}{16af^2-8bf\left(e-\sqrt{e^2-4df}\right)+4c\left(e-\sqrt{e^2-\right.}}\right.}{d\sqrt{e^2-4df}}$$

$$= -\frac{\sqrt{a}\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d} - \frac{\left(2bdf - cd\left(e-\sqrt{e^2-4df}\right) - af\left(e+\sqrt{e^2-4df}\right)\right)\tanh^{-1}\left(\frac{4af-b\left(e-\sqrt{e^2-4df}\right)+2\left(bf-c\left(e-\sqrt{e^2-4\right.\right.}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}\sqrt{}\right.}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}} + \frac{\left(2bdf - af\left(e-\sqrt{e^2-4df}\right) - cd\left(e+\sqrt{e^2-4df}\right)\right)\tanh^{-1}\left(\frac{4af-b\left(e+\sqrt{e^2-4df}\right)+2\left(bf-c\left(e+\sqrt{e^2-4\right.\right.}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2-4df}}\sqrt{}\right.}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2-4df}}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.50 (sec) , antiderivative size = 467, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{a+bx+cx^2}}{x\left(d+ex+fx^2\right)}\,dx$$

$$= \frac{2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right) - \text{RootSum}\left[b^2d - abe + a^2f - 4b\sqrt{c}d\#1 + 2a\sqrt{c}e\#1 + 4cd\#1^2 + be\#1^2 -\right.}{}$$

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/(x*(d + e*x + f*x^2)),x]
```

```
[Out] (2*Sqrt[a]*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)])/Sqrt[a]] - RootSum[b
^2*d - a*b*e + a^2*f - 4*b*Sqrt[c]*d*#1 + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 + b
*e*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (b^2*d*Log[-(Sqrt[c]*x
) + Sqrt[a + b*x + c*x^2] - #1] - a*c*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c
*x^2] - #1] - a*b*e*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + a^2*f*
Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*b*Sqrt[c]*d*Log[-(Sqrt[c
]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + 2*a*Sqrt[c]*e*Log[-(Sqrt[c]*x) + Sq
rt[a + b*x + c*x^2] - #1]*#1 + c*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2]
 - #1]*#1^2 - a*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2)/(2*b
*Sqrt[c]*d - a*Sqrt[c]*e - 4*c*d*#1 - b*e*#1 + 2*a*f*#1 + 3*Sqrt[c]*e*#1^2
- 2*f*#1^3) & ])/d
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1690 vs. 2(460) = 920.

Time = 1.05 (sec) , antiderivative size = 1691, normalized size of antiderivative = 3.23

method	result	size
default	Expression too large to display	1691

```
[In] int((c*x^2+b*x+a)^(1/2)/x/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)

[Out] -4*f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))*((c*x^2+b*x+a)^(1/2)+1/
2*b*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-a^(1/2)*ln((2*a+b*x
+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x))+2*f/(e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)
^(1/2)*(1/2*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4/f*(-c*(-4*d*f+e^2)^(1
/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*(-b*f*(-4*d*f+e^2)^(1/2)+(-
4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)+1/2/f*(-c*(-4*
d*f+e^2)^(1/2)+b*f-c*e)*ln((1/2/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)+c*(x+1/2*
(e+(-4*d*f+e^2)^(1/2))/f))/c^(1/2)+((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+1/
f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-b*
f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^
2)^(1/2))/c^(1/2)-1/2*(-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f
^2-b*e*f-2*c*d*f+c*e^2)/f^2*2^(1/2)/((-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^
(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-b*f*(-4*d*f+e^2)^(1
/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+1/f*(-c*(-4*d*f
+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-b*f*(-
4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(
1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4/f*(-c*(-4*d*f+e^2)^(1/2)+b*f
-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*(-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e
^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^
2)^(1/2))/f)))+2*f/(-e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)*(1/2*(4*(x-1/
2/f*(-e+(-4*d*f+e^2)^(1/2)))^2*c+4*(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2/
f*(-e+(-4*d*f+e^2)^(1/2)))+2*(b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e
+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)+1/2*(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)
/f*ln((1/2*(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f+c*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/
2))))/c^(1/2)+((x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2*c+(c*(-4*d*f+e^2)^(1/2)+
b*f-c*e)/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+1/2*(b*f*(-4*d*f+e^2)^(1/2)-(-
4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/c^(1/2)-1/2*(
b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/
f^2*2^(1/2)/((b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2
*c*d*f+c*e^2)/f^2)^(1/2)*ln(((b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e
+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2/f
*(-e+(-4*d*f+e^2)^(1/2)))+1/2*2^(1/2)*((b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)
^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2/f*(-e+(-4*d*f+
e^2)^(1/2)))^2*c+4*(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2/f*(-e+(-4*d*f+e^
2)^(1/2)))+2*(b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2
*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}}{x\left(d+ex+fx^2\right)}\,dx = \text{Timed out}$$

```
[In] integrate((c*x^2+b*x+a)^(1/2)/x/(f*x^2+e*x+d),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{x\left(d+ex+fx^2\right)}\,dx = \int \frac{\sqrt{a+bx+cx^2}}{x\left(d+ex+fx^2\right)}\,dx$$

```
[In] integrate((c*x**2+b*x+a)**(1/2)/x/(f*x**2+e*x+d),x)
[Out] Integral(sqrt(a + b*x + c*x**2)/(x*(d + e*x + f*x**2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{x\left(d+ex+fx^2\right)}\,dx = \int \frac{\sqrt{cx^2+bx+a}}{(fx^2+ex+d)x}\,dx$$

```
[In] integrate((c*x^2+b*x+a)^(1/2)/x/(f*x^2+e*x+d),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^2 + b*x + a)/((f*x^2 + e*x + d)*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx+cx^2}}{x\left(d+ex+fx^2\right)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((c*x^2+b*x+a)^(1/2)/x/(f*x^2+e*x+d),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}}{x\,(d+ex+fx^2)}\,dx = \int \frac{\sqrt{c\,x^2+b\,x+a}}{x\,\left(f\,x^2+e\,x+d\right)}\,dx$$

```
[In] int((a + b*x + c*x^2)^(1/2)/(x*(d + e*x + f*x^2)),x)
[Out] int((a + b*x + c*x^2)^(1/2)/(x*(d + e*x + f*x^2)), x)
```

3.113 $\int \frac{\sqrt{a+bx+cx^2}}{x^2(d+ex+fx^2)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 736

$$\begin{aligned}\int \frac{\sqrt{a+bx+cx^2}}{x^2(d+ex+fx^2)}\,dx = &-\frac{\sqrt{a+bx+cx^2}}{dx}-\frac{b\,\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{a}d}\\
&+\frac{\sqrt{a}e\,\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d^2}+\frac{\sqrt{c}\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{d}\\
&-\frac{be\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}d^2}-\frac{(2cd-be)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}d^2}\\
&-\frac{f\left(2cd^2-bd\left(e+\sqrt{e^2-4df}\right)+a\left(e^2-2df+e\sqrt{e^2-4df}\right)\right)\text{arctanh}\left(\frac{4af-b\left(e-\sqrt{e^2-4df}\right)+2\left(bf-c\left(e-\sqrt{e^2-\cdots\right.\right.}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df\cdots}}}\right.}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{c\left(e^2-2df-e\sqrt{e^2-4df}\right)+f\left(2af-b\left(e-\sqrt{e^2-4df}\right)\right)}}\\
&+\frac{f\left(2cd^2-bd\left(e-\sqrt{e^2-4df}\right)+a\left(e^2-2df-e\sqrt{e^2-4df}\right)\right)\text{arctanh}\left(\frac{4af-b\left(e+\sqrt{e^2-4df}\right)+2\left(bf-c\left(e+\sqrt{e^2-\cdots\right.\right.}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2-4df\cdots}}}\right.}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{c\left(e^2-2df+e\sqrt{e^2-4df}\right)+f\left(2af-b\left(e+\sqrt{e^2-4df}\right)\right)}}\end{aligned}$$

```
[Out] -1/2*b*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/d/a^(1/2)+e*arcta
nh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))*a^(1/2)/d^2-1/2*b*e*arctanh(1
/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/d^2/c^(1/2)-1/2*(-b*e+2*c*d)*arct
anh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/d^2/c^(1/2)+arctanh(1/2*(2*c
*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))*c^(1/2)/d-(c*x^2+b*x+a)^(1/2)/d/x-1/2*f*
arctanh(1/4*(4*a*f+2*x*(b*f-c*(e-(-4*d*f+e^2)^(1/2)))-b*(e-(-4*d*f+e^2)^(1/
2)))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-
4*d*f+e^2)^(1/2))^(1/2))*(2*c*d^2-b*d*(e+(-4*d*f+e^2)^(1/2))+a*(e^2-2*d*f+e
*(-4*d*f+e^2)^(1/2)))/d^2*2^(1/2)/(-4*d*f+e^2)^(1/2)/(f*(2*a*f-b*(e-(-4*d*f
+e^2)^(1/2)))+c*(e^2-2*d*f-e*(-4*d*f+e^2)^(1/2)))^(1/2)+1/2*f*arctanh(1/4*(
```

4*a*f-b*(e+(-4*d*f+e^2)^(1/2))+2*x*(b*f-c*(e+(-4*d*f+e^2)^(1/2))))*2^(1/2)/
(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1
/2))^(1/2))*(2*c*d^2-b*d*(e-(-4*d*f+e^2)^(1/2))+a*(e^2-2*d*f-e*(-4*d*f+e^2)
^(1/2)))/d^2*2^(1/2)/(-4*d*f+e^2)^(1/2)/(c*(e^2-2*d*f+e*(-4*d*f+e^2)^(1/2))
+f*(2*a*f-b*(e+(-4*d*f+e^2)^(1/2))))^(1/2)

Rubi [A] (verified)

Time = 2.47 (sec) , antiderivative size = 736, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6860, 746, 857, 635, 212, 738, 748, 1033, 1090, 1046}

$$\int \frac{\sqrt{a+bx+cx^2}}{x^2(d+ex+fx^2)}\,dx =$$
$$-\frac{f\left(a\left(e\sqrt{e^2-4df}-2df+e^2\right)-bd\left(\sqrt{e^2-4df}+e\right)+2cd^2\right)\operatorname{arctanh}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{\right.}\right.}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-}}\right.}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{f\left(2af-b\left(e-\sqrt{e^2-4df}\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}$$
$$+\frac{f\left(a\left(-e\sqrt{e^2-4df}-2df+e^2\right)-bd\left(e-\sqrt{e^2-4df}\right)+2cd^2\right)\operatorname{arctanh}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{\right.}\right.}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf}}\right.}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}$$
$$+\frac{\sqrt{a}e\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d^2}-\frac{be\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}d^2}$$
$$-\frac{(2cd-be)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}d^2}-\frac{b\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{a}d}$$
$$+\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{d}-\frac{\sqrt{a+bx+cx^2}}{dx}$$

```
[In] Int[Sqrt[a + b*x + c*x^2]/(x^2*(d + e*x + f*x^2)),x]
```

```
[Out] -(Sqrt[a + b*x + c*x^2]/(d*x)) - (b*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a +
 b*x + c*x^2])])/(2*Sqrt[a]*d) + (Sqrt[a]*e*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*
Sqrt[a + b*x + c*x^2])])/d^2 + (Sqrt[c]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt
[a + b*x + c*x^2])])/d - (b*e*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x +
 c*x^2])])/(2*Sqrt[c]*d^2) - ((2*c*d - b*e)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*
Sqrt[a + b*x + c*x^2])])/(2*Sqrt[c]*d^2) - (f*(2*c*d^2 - b*d*(e + Sqrt[e^2
- 4*d*f]) + a*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*a*f - b*(e -
Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt
[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a
+ b*x + c*x^2])]))/(Sqrt[2]*d^2*Sqrt[e^2 - 4*d*f]*Sqrt[c*(e^2 - 2*d*f - e*Sq
rt[e^2 - 4*d*f]) + f*(2*a*f - b*(e - Sqrt[e^2 - 4*d*f]))]) + (f*(2*c*d^2 -
```

```
b*d*(e - Sqrt[e^2 - 4*d*f]) + a*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]))*ArcTan
h[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*
x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2
 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*d^2*Sqrt[e^2 - 4*d*f]*Sqrt[c*(
e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]
)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 746

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 748

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[h*(a + b*x + c*x^2)^p*((d
+ e*x + f*x^2)^(q + 1)/(2*f*(p + q + 1))), x] - Dist[1/(2*f*(p + q + 1)), I
nt[(a + b*x + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[h*p*(b*d - a*e) + a*(
h*e - 2*g*f)*(p + q + 1) + (2*h*p*(c*d - a*f) + b*(h*e - 2*g*f)*(p + q + 1)
)*x + (h*p*(c*e - b*f) + c*(h*e - 2*g*f)*(p + q + 1))*x^2, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d
*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]
```

Rule 1046

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1090

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqr
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rule 6860

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{\sqrt{a+bx+cx^2}}{dx^2} - \frac{e\sqrt{a+bx+cx^2}}{d^2x} + \frac{(e^2-df+efx)\sqrt{a+bx+cx^2}}{d^2(d+ex+fx^2)}\right) dx$$

$$= \frac{\int \frac{(e^2-df+efx)\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx}{d^2} + \frac{\int \frac{\sqrt{a+bx+cx^2}}{x^2}\,dx}{d} - \frac{e\int \frac{\sqrt{a+bx+cx^2}}{x}\,dx}{d^2}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{dx} + \frac{\int \frac{b+2cx}{x\sqrt{a+bx+cx^2}}\,dx}{2d} + \frac{e\int \frac{-2a-bx}{x\sqrt{a+bx+cx^2}}\,dx}{2d^2} - \frac{\int \frac{\frac{1}{2}f\left(bde-2ae^2+2adf\right)+\frac{1}{2}f\left(2cde-be^2+2bdf-2aef\right)x+\frac{1}{2}(2cd-be)f^2x^2}{\sqrt{a+bx+cx^2}(d+ex+fx^2)}\,dx}{d^2f}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{dx} + \frac{b\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx}{2d} + \frac{c\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{d} - \frac{(ae)\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx}{d^2} - \frac{(be)\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{2d^2} - \frac{(2cd-be)\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{2d^2} - \frac{\int \frac{-\frac{1}{2}d(2cd-be)f^2+\frac{1}{2}f^2\left(bde-2ae^2+2adf\right)+\left(-\frac{1}{2}e(2cd-be)f^2+\frac{1}{2}f^2\left(2cde-be^2+2bdf-2aef\right)\right)x}{\sqrt{a+bx+cx^2}(d+ex+fx^2)}\,dx}{d^2f^2}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{dx} - \frac{b\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx}{\sqrt{a+bx+cx^2}}\right)}{d} + \frac{(2c)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{d} + \frac{(2ae)\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx}{\sqrt{a+bx+cx^2}}\right)}{d^2} - \frac{(be)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{d^2} - \frac{(2cd-be)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{d^2} - \frac{\left(f\left(2cd^2-bd\left(e-\sqrt{e^2-4df}\right)+a\left(e^2-2df-e\sqrt{e^2-4df}\right)\right)\right)\int \frac{1}{\left(e+\sqrt{e^2-4df}+2fx\right)\sqrt{a+bx+cx^2}}\,dx}{d^2\sqrt{e^2-4df}} + \frac{\left(f\left(2cd^2-bd\left(e+\sqrt{e^2-4df}\right)+a\left(e^2-2df+e\sqrt{e^2-4df}\right)\right)\right)\int \frac{1}{\left(e-\sqrt{e^2-4df}+2fx\right)\sqrt{a+bx+cx^2}}\,dx}{d^2\sqrt{e^2-4df}}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{dx} - \frac{b\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{a}d} + \frac{\sqrt{a}e\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d^2} + \frac{\sqrt{c}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{d} - \frac{be\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}d^2} - \frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}d^2} + \frac{\left(2f\left(2cd^2-bd\left(e-\sqrt{e^2-4df}\right)+a\left(e^2-2df-e\sqrt{e^2-4df}\right)\right)\right)\text{Subst}\left(\int \frac{1}{16af^2-8bf\left(e+\sqrt{e^2-4df}\right)+4c(\cdots}\right.}{d^2\sqrt{e^2-4df}} - \frac{\left(2f\left(2cd^2-bd\left(e+\sqrt{e^2-4df}\right)+a\left(e^2-2df+e\sqrt{e^2-4df}\right)\right)\right)\text{Subst}\left(\int \frac{1}{16af^2-8bf\left(e-\sqrt{e^2-4df}\right)+4c(\cdots}\right.}{d^2\sqrt{e^2-4df}}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{dx} - \frac{b\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{a}d} + \frac{\sqrt{a}e\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d^2} + \frac{\sqrt{c}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{d} - \frac{be\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}d^2} - \frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}d^2} - \frac{f\left(2cd^2-bd\left(e+\sqrt{e^2-4df}\right)+a\left(e^2-2df+e\sqrt{e^2-4df}\right)\right)\tanh^{-1}\left(\frac{4af-b\left(e-\sqrt{e^2-4df}\right)+2\left(bf-\right.}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2-(ce-b}}\right.}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}} + \frac{f\left(2cd^2-bd\left(e-\sqrt{e^2-4df}\right)+a\left(e^2-2df-e\sqrt{e^2-4df}\right)\right)\tanh^{-1}\left(\frac{4af-b\left(e+\sqrt{e^2-4df}\right)+2\left(bf-\right.}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2+(ce-b}}\right.}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2-4df}}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.69 (sec) , antiderivative size = 582, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{a+bx+cx^2}}{x^2\left(d+ex+fx^2\right)}\,dx = \frac{-\frac{d\sqrt{a+x(b+cx)}}{x} + \frac{(-bd+2ae)\text{arctanh}\left(\frac{-\sqrt{c}x+\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)}{\sqrt{a}} + \text{RootSum}\left[b^2d-abe+a^2f-4b\sqrt{c}d\#1+2a\sqrt{c}e\#1\right.}{}$$

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/(x^2*(d + e*x + f*x^2)),x]
```

```
[Out] (-((d*Sqrt[a + x*(b + c*x)])/x) + ((-(b*d) + 2*a*e)*ArcTanh[(-(Sqrt[c]*x) +
 Sqrt[a + x*(b + c*x)])/Sqrt[a]])/Sqrt[a] + RootSum[b^2*d - a*b*e + a^2*f -
 4*b*Sqrt[c]*d*#1 + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 + b*e*#1^2 - 2*a*f*#1^2 -
 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (-(b*c*d^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x +
 c*x^2] - #1]) + b^2*d*e*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - a
*b*e^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + a^2*e*f*Log[-(Sqrt[
c]*x) + Sqrt[a + b*x + c*x^2] - #1] + 2*c^(3/2)*d^2*Log[-(Sqrt[c]*x) + Sqrt
[a + b*x + c*x^2] - #1]*#1 - 2*b*Sqrt[c]*d*e*Log[-(Sqrt[c]*x) + Sqrt[a + b*
x + c*x^2] - #1]*#1 + 2*a*Sqrt[c]*e^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x
^2] - #1]*#1 - 2*a*Sqrt[c]*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #
1]*#1 + b*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 - a*e*f*L
og[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2)/(2*b*Sqrt[c]*d - a*Sqrt
[c]*e - 4*c*d*#1 - b*e*#1 + 2*a*f*#1 + 3*Sqrt[c]*e*#1^2 - 2*f*#1^3) & ])/d^
2
```

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 976, normalized size of antiderivative = 1.33

method	result
risch	$-\frac{\sqrt{c\,x^2+bx+a}}{dx}-\frac{\frac{4f(2ae-bd)\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)}{\left(-e+\sqrt{-4df+e^2}\right)\left(e+\sqrt{-4df+e^2}\right)\sqrt{a}}+\frac{2\left(fa\sqrt{-4df+e^2}-cd\sqrt{-4df+e^2}-aef+2bdf-cde\right)\sqrt{2}\,\ln\left(\frac{-bf\sqrt{-4df+e^2}+\sqrt{-}}{}\right.}{}}{}$
default	Expression too large to display

```
[In] int((c*x^2+b*x+a)^(1/2)/x^2/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -(c*x^2+b*x+a)^(1/2)/d/x-1/2/d*(4*f*(2*a*e-b*d)/(-e+(-4*d*f+e^2)^(1/2))/(e+
(-4*d*f+e^2)^(1/2))/a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)+2
*(f*a*(-4*d*f+e^2)^(1/2)-c*d*(-4*d*f+e^2)^(1/2)-a*e*f+2*b*d*f-c*d*e)/(-4*d*
f+e^2)^(1/2)/(e+(-4*d*f+e^2)^(1/2))*2^(1/2)/((-b*f*(-4*d*f+e^2)^(1/2)+(-4*d
*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-b*f*(-4*d*f
+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+1/f*(-c
*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(
(-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2
)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4/f*(-c*(-4*d*f+e^2)^(
1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*(-b*f*(-4*d*f+e^2)^(1/2)+(
-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-
4*d*f+e^2)^(1/2))/f))-2*(f*a*(-4*d*f+e^2)^(1/2)-c*d*(-4*d*f+e^2)^(1/2)+a*e*
f-2*b*d*f+c*d*e)/(-4*d*f+e^2)^(1/2)/(-e+(-4*d*f+e^2)^(1/2))*2^(1/2)/((b*f*(
-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^
(1/2)*ln(((b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*
d*f+c*e^2)/f^2+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(
1/2)))+1/2*2^(1/2)*((b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-
b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2*c+4*
(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+2*(b*f*(
-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^
(1/2))/(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}}{x^2\,(d+ex+fx^2)}\,dx = \text{Timed out}$$

```
[In] integrate((c*x^2+b*x+a)^(1/2)/x^2/(f*x^2+e*x+d),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}}{x^2\,(d+ex+fx^2)}\,dx = \text{Timed out}$$

```
[In] integrate((c*x**2+b*x+a)**(1/2)/x**2/(f*x**2+e*x+d),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{x^2\,(d+ex+fx^2)}\,dx = \int \frac{\sqrt{cx^2+bx+a}}{(fx^2+ex+d)x^2}\,dx$$

```
[In] integrate((c*x^2+b*x+a)^(1/2)/x^2/(f*x^2+e*x+d),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^2 + b*x + a)/((f*x^2 + e*x + d)*x^2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}}{x^2\,(d+ex+fx^2)}\,dx = \text{Timed out}$$

```
[In] integrate((c*x^2+b*x+a)^(1/2)/x^2/(f*x^2+e*x+d),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}}{x^2\left(d+ex+fx^2\right)}\,dx=\int \frac{\sqrt{c\,x^2+b\,x+a}}{x^2\,\left(f\,x^2+e\,x+d\right)}\,dx$$

```
[In] int((a + b*x + c*x^2)^(1/2)/(x^2*(d + e*x + f*x^2)),x)
[Out] int((a + b*x + c*x^2)^(1/2)/(x^2*(d + e*x + f*x^2)), x)
```

3.114 $\int \frac{x^3}{\sqrt{a+bx+cx^2}(d+ex+fx^2)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 545

$$\int \frac{x^3}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = \frac{\sqrt{a+bx+cx^2}}{cf}-\frac{e\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f^2}-\frac{b\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}f}$$
$$-\frac{\left(2def-(e^2-df)\left(e-\sqrt{e^2-4df}\right)\right)\text{arctanh}\left(\frac{4af-b\left(e-\sqrt{e^2-4df}\right)+2\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)x}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}\sqrt{a+bx+cx^2}}\right)}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}}$$
$$+\frac{\left(2def-(e^2-df)\left(e+\sqrt{e^2-4df}\right)\right)\text{arctanh}\left(\frac{4af-b\left(e+\sqrt{e^2-4df}\right)+2\left(bf-c\left(e+\sqrt{e^2-4df}\right)\right)x}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2-4df}}\sqrt{a+bx+cx^2}}\right)}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2-4df}}}$$

```
[Out] -1/2*b*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(3/2)/f-e*arcta
nh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/f^2/c^(1/2)+(c*x^2+b*x+a)^(1/
2)/c/f-1/2*arctanh(1/4*(4*a*f+2*x*(b*f-c*(e-(-4*d*f+e^2)^(1/2)))-b*(e-(-4*d
*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-
b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2))*(2*d*e*f-(-d*f+e^2)*(e-(-4*d*f+e^2)^(1/
2)))/f^2*2^(1/2)/(-4*d*f+e^2)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)
*(-4*d*f+e^2)^(1/2))^(1/2)+1/2*arctanh(1/4*(4*a*f-b*(e+(-4*d*f+e^2)^(1/2))+
2*x*(b*f-c*(e+(-4*d*f+e^2)^(1/2))))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*
d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2))*(2*d*e*f-(-d*f+e^2)
*(e+(-4*d*f+e^2)^(1/2)))/f^2*2^(1/2)/(-4*d*f+e^2)^(1/2)/(c*e^2-2*c*d*f-b*e*
f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 2.37 (sec) , antiderivative size = 545, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6860, 635, 212, 654, 1046, 738}

$$\int \frac{x^3}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = -\frac{b\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}f}$$
$$-\frac{\left(2def-(e^2-df)\left(e-\sqrt{e^2-4df}\right)\right)\text{arctanh}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}$$
$$+\frac{\left(2def-(e^2-df)\left(\sqrt{e^2-4df}+e\right)\right)\text{arctanh}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}$$
$$-\frac{e\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f^2}+\frac{\sqrt{a+bx+cx^2}}{cf}$$

```
[In] Int[x^3/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] Sqrt[a + b*x + c*x^2]/(c*f) - (e*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*
x + c*x^2])])/(Sqrt[c]*f^2) - (b*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*
x + c*x^2])])/(2*c^(3/2)*f) - ((2*d*e*f - (e^2 - d*f)*(e - Sqrt[e^2 - 4*d*f
]))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 -
 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f
)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*f^2*Sqrt[e^2 - 4*d*f
]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
+ ((2*d*e*f - (e^2 - d*f)*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*a*f - b*(e +
Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt
[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a
+ b*x + c*x^2])])/(Sqrt[2]*f^2*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e
*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1046

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 6860

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{e}{f^2\sqrt{a+bx+cx^2}}+\frac{x}{f\sqrt{a+bx+cx^2}}+\frac{de+\left(e^2-df\right)x}{f^2\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}\right)dx\\
&= \frac{\int \frac{de+\left(e^2-df\right)x}{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}\,dx}{f^2}-\frac{e\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{f^2}+\frac{\int \frac{x}{\sqrt{a+bx+cx^2}}\,dx}{f}\\
&= \frac{\sqrt{a+bx+cx^2}}{cf}-\frac{(2e)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx,x,\frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{f^2}-\frac{b\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{2cf}\\
&\quad+\frac{\left(2def-\left(e^2-df\right)\left(e-\sqrt{e^2-4df}\right)\right)\int \frac{1}{\left(e-\sqrt{e^2-4df}+2fx\right)\sqrt{a+bx+cx^2}}\,dx}{f^2\sqrt{e^2-4df}}\\
&\quad-\frac{\left(2def-\left(e^2-df\right)\left(e+\sqrt{e^2-4df}\right)\right)\int \frac{1}{\left(e+\sqrt{e^2-4df}+2fx\right)\sqrt{a+bx+cx^2}}\,dx}{f^2\sqrt{e^2-4df}}
\end{aligned}$$

$$= \frac{\sqrt{a+bx+cx^2}}{cf} - \frac{e\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f^2} - \frac{b\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{cf}$$

$$- \frac{\left(2\left(2def-(e^2-df)\left(e-\sqrt{e^2-4df}\right)\right)\right)\text{Subst}\left(\int \frac{1}{16af^2-8bf\left(e-\sqrt{e^2-4df}\right)+4c\left(e-\sqrt{e^2-4df}\right)^2-x^2}\,dx, x, \frac{4a}{}\right.}{f^2\sqrt{e^2-4df}}$$

$$+ \frac{\left(2\left(2def-(e^2-df)\left(e+\sqrt{e^2-4df}\right)\right)\right)\text{Subst}\left(\int \frac{1}{16af^2-8bf\left(e+\sqrt{e^2-4df}\right)+4c\left(e+\sqrt{e^2-4df}\right)^2-x^2}\,dx, x, \frac{4a}{}\right.}{f^2\sqrt{e^2-4df}}$$

$$= \frac{\sqrt{a+bx+cx^2}}{cf} - \frac{e\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f^2} - \frac{b\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}f}$$

$$- \frac{\left(2def-(e^2-df)\left(e-\sqrt{e^2-4df}\right)\right)\tanh^{-1}\left(\frac{4af-b\left(e-\sqrt{e^2-4df}\right)+2\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)x}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}\sqrt{a+bx+cx^2}}\right)}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}}$$

$$+ \frac{\left(2def-(e^2-df)\left(e+\sqrt{e^2-4df}\right)\right)\tanh^{-1}\left(\frac{4af-b\left(e+\sqrt{e^2-4df}\right)+2\left(bf-c\left(e+\sqrt{e^2-4df}\right)\right)x}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2-4df}}\sqrt{a+bx+cx^2}}\right)}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2-4df}}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.71 (sec) , antiderivative size = 423, normalized size of antiderivative = 0.78

$$\int \frac{x^3}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx =$$

$$- \frac{-\frac{2f\sqrt{a+x(b+cx)}}{c} + \frac{(2ce+bf)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{c^{3/2}} + 2\text{RootSum}\left[b^2d-abe+a^2f-4b\sqrt{c}d\#1+2a\sqrt{c}e\#1-\right.}{}$$

```
[In] Integrate[x^3/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] -1/2*((-2*f*Sqrt[a + x*(b + c*x)])/c + ((2*c*e + b*f)*ArcTanh[(b + 2*c*x)/(
2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/c^(3/2) + 2*RootSum[b^2*d - a*b*e + a^2*
f - 4*b*Sqrt[c]*d*#1 + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 + b*e*#1^2 - 2*a*f*#1^
2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (b*d*e*Log[-(Sqrt[c]*x) + Sqrt[a + b*x +
c*x^2] - #1] - a*e^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + a*d*f
*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*Sqrt[c]*d*e*Log[-(Sqrt[
c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + e^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*
x + c*x^2] - #1]*#1^2 - d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*
#1^2)/(2*b*Sqrt[c]*d - a*Sqrt[c]*e - 4*c*d*#1 - b*e*#1 + 2*a*f*#1 + 3*Sqrt[
c]*e*#1^2 - 2*f*#1^3) & ])/f^2
```

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 914, normalized size of antiderivative = 1.68

method	result
risch	$\frac{\sqrt{c x^2+bx+a}}{cf} - \frac{\frac{(bf+2ce)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c x^2+bx+a}\right)}{f\sqrt{c}} - \frac{c\left(df\sqrt{-4df+e^2}-e^2\sqrt{-4df+e^2}+3def-e^3\right)\sqrt{2}\ln\left(\frac{-bf\sqrt{-4df+e^2}+\sqrt{-4df+e^2}\,ce+2a\ldots}{f^2}\ldots\right.}{\ldots}}{\ldots}$
default	$\frac{\frac{\sqrt{c x^2+bx+a}}{c}-\frac{b\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c x^2+bx+a}\right)}{2c^{\frac{3}{2}}}}{f} - \frac{e\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c x^2+bx+a}\right)}{f^2\sqrt{c}} - \frac{\left(e^3-3def+e^2\sqrt{-4df+e^2}-df\sqrt{-4df+e^2}\right)\sqrt{2}\ln\left(\frac{-b\ldots}{\ldots}\right.}{\ldots}$

```
[In] int(x^3/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)

[Out] (c*x^2+b*x+a)^(1/2)/c/f-1/2/f/c*(1/f*(b*f+2*c*e)*ln((1/2*b+c*x)/c^(1/2)+(c*
x^2+b*x+a)^(1/2))/c^(1/2)-1/f^2*c*(d*f*(-4*d*f+e^2)^(1/2)-e^2*(-4*d*f+e^2)^
(1/2)+3*d*e*f-e^3)/(-4*d*f+e^2)^(1/2)*2^(1/2)/((-b*f*(-4*d*f+e^2)^(1/2)+(-4
*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-b*f*(-4*d
*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+1/f*(
-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)
*((-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e
^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4/f*(-c*(-4*d*f+e^2)
^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*(-b*f*(-4*d*f+e^2)^(1/2)
+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+
(-4*d*f+e^2)^(1/2))/f))-1/f^2*c*(e^3-3*d*e*f-e^2*(-4*d*f+e^2)^(1/2)+d*f*(-4
*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)*2^(1/2)/((b*f*(-4*d*f+e^2)^(1/2)-(-4*d*
f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((b*f*(-4*d*f+e
^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+(c*(-4*d*
f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+1/2*2^(1/2)*((b*f
*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2
)^(1/2)*(4*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2*c+4*(c*(-4*d*f+e^2)^(1/2)+b*
f-c*e)/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+2*(b*f*(-4*d*f+e^2)^(1/2)-(-4*d*
f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2/f*(-e+(-4*
d*f+e^2)^(1/2)))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = \text{Timed out}$$

```
[In] integrate(x^3/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{x^3}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = \int \frac{x^3}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx$$

```
[In] integrate(x**3/(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)
[Out] Integral(x**3/(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^3/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re deta
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = \int \frac{x^3}{\sqrt{c\,x^2+b\,x+a}\,\left(f\,x^2+e\,x+d\right)}\,dx$$

```
[In] int(x^3/((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)),x)
[Out] int(x^3/((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)), x)
```

3.115 $\int \frac{x^2}{\sqrt{a+bx+cx^2}(d+ex+fx^2)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 463

$$\begin{aligned}\int \frac{x^2}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx &= \frac{\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f} \\ &- \frac{\left(e^2-2df-e\sqrt{e^2-4df}\right)\operatorname{arctanh}\left(\frac{4af-b\left(e-\sqrt{e^2-4df}\right)+2\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)x}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}\sqrt{a+bx+cx^2}}\right)}{\sqrt{2}f\sqrt{e^2-4df}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}} \\ &- \frac{\left(2df-e\left(e+\sqrt{e^2-4df}\right)\right)\operatorname{arctanh}\left(\frac{4af-b\left(e+\sqrt{e^2-4df}\right)+2\left(bf-c\left(e+\sqrt{e^2-4df}\right)\right)x}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2-4df}}\sqrt{a+bx+cx^2}}\right)}{\sqrt{2}f\sqrt{e^2-4df}\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2-4df}}}\end{aligned}$$

```
[Out] arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/f/c^(1/2)-1/2*arctanh(1/
4*(4*a*f+2*x*(b*f-c*(e-(-4*d*f+e^2)^(1/2)))-b*(e-(-4*d*f+e^2)^(1/2)))*2^(1/
2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)
^(1/2))^(1/2))*(e^2-2*d*f-e*(-4*d*f+e^2)^(1/2))/f*2^(1/2)/(-4*d*f+e^2)^(1/2
)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)-1/2*arc
tanh(1/4*(4*a*f-b*(e+(-4*d*f+e^2)^(1/2))+2*x*(b*f-c*(e+(-4*d*f+e^2)^(1/2)))
)*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d
*f+e^2)^(1/2))^(1/2))*(2*d*f-e*(e+(-4*d*f+e^2)^(1/2)))/f*2^(1/2)/(-4*d*f+e^
2)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 2.18 (sec) , antiderivative size = 463, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1091, 635, 212, 1046, 738}

$$\int \frac{x^2}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx$$
$$= -\frac{\left(-e\sqrt{e^2-4df}-2df+e^2\right)\operatorname{arctanh}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}f\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}$$
$$-\frac{\left(2df-e\left(\sqrt{e^2-4df}+e\right)\right)\operatorname{arctanh}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}f\sqrt{e^2-4df}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}$$
$$+\frac{\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f}$$

```
[In] Int[x^2/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])]/(Sqrt[c]*f) - ((e^2
- 2*d*f - e*Sqrt[e^2 - 4*d*f])*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) +
 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b
*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(S
qrt[2]*f*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e -
b*f)*Sqrt[e^2 - 4*d*f]]) - ((2*d*f - e*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*
a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2
*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*
d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*f*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*
c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1046

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1091

```
Int[((A_.) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
 (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + e*x + f
*x^2], x], x] + Dist[1/c, Int[(A*c - a*C - b*C*x)/((a + b*x + c*x^2)*Sqrt[d
 + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{f} + \frac{\int \frac{-d-ex}{\sqrt{a+bx+cx^2}(d+ex+fx^2)}\,dx}{f}$$

$$= \frac{2\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{f} + \frac{\left(e^2-2df-e\sqrt{e^2-4df}\right)\int \frac{1}{\left(e-\sqrt{e^2-4df}+2fx\right)\sqrt{a+bx+cx^2}}\,dx}{f\sqrt{e^2-4df}} - \frac{\left(-2df+e\left(e+\sqrt{e^2-4df}\right)\right)\int \frac{1}{\left(e+\sqrt{e^2-4df}+2fx\right)\sqrt{a+bx+cx^2}}\,dx}{f\sqrt{e^2-4df}}$$

$$= \frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f} - \frac{\left(2\left(e^2-2df-e\sqrt{e^2-4df}\right)\right)\text{Subst}\left(\int \frac{1}{16af^2-8bf\left(e-\sqrt{e^2-4df}\right)+4c\left(e-\sqrt{e^2-4df}\right)^2-x^2}\,dx, x, \frac{4af-b\left(e-\sqrt{e^2-\right.}{}\right.}{f\sqrt{e^2-4df}} + \frac{\left(2\left(-2df+e\left(e+\sqrt{e^2-4df}\right)\right)\right)\text{Subst}\left(\int \frac{1}{16af^2-8bf\left(e+\sqrt{e^2-4df}\right)+4c\left(e+\sqrt{e^2-4df}\right)^2-x^2}\,dx, x, \frac{4af-b\left(e+\sqrt{\right.}{}\right.}{f\sqrt{e^2-4df}}$$

$$= \frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f} - \frac{\left(e^2-2df-e\sqrt{e^2-4df}\right)\tanh^{-1}\left(\frac{4af-b\left(e-\sqrt{e^2-4df}\right)+2\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)x}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}\sqrt{a+bx+cx^2}}\right)}{\sqrt{2}f\sqrt{e^2-4df}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}} - \frac{\left(2df-e\left(e+\sqrt{e^2-4df}\right)\right)\tanh^{-1}\left(\frac{4af-b\left(e+\sqrt{e^2-4df}\right)+2\left(bf-c\left(e+\sqrt{e^2-4df}\right)\right)x}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2-4df}}\sqrt{a+bx+cx^2}}\right)}{\sqrt{2}f\sqrt{e^2-4df}\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2-4df}}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.44 (sec) , antiderivative size = 318, normalized size of antiderivative = 0.69

$$\int \frac{x^2}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = \frac{-\frac{\log\left(f\left(b+2cx-2\sqrt{c}\sqrt{a+x(b+cx)}\right)\right)}{\sqrt{c}} + \text{RootSum}\left[b^2d-abe+a^2f-4b\sqrt{c}d\#1+2a\sqrt{c}e\#1+4cd\#1^2+be\#1^2-\right.}{}$$

```
[In] Integrate[x^2/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] (-(Log[f*(b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]/Sqrt[c]) + RootSum[
b^2*d - a*b*e + a^2*f - 4*b*Sqrt[c]*d*#1 + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 +
b*e*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (b*d*Log[-(Sqrt[c]*x)
 + Sqrt[a + b*x + c*x^2] - #1] - a*e*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^
2] - #1] - 2*Sqrt[c]*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 +
e*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2)/(2*b*Sqrt[c]*d - a*S
qrt[c]*e - 4*c*d*#1 - b*e*#1 + 2*a*f*#1 + 3*Sqrt[c]*e*#1^2 - 2*f*#1^3) & ])
/f
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 843 vs. $2(407) = 814$.

Time = 0.88 (sec) , antiderivative size = 844, normalized size of antiderivative = 1.82

method	result
default	$\frac{\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{f\sqrt{c}}-\frac{\left(-e\sqrt{-4df+e^2}+2df-e^2\right)\sqrt{2}\,\ln\left(\frac{-bf\sqrt{-4df+e^2}+\sqrt{-4df+e^2}\,ce+2a\,f^2-bef-2cdf+c\,e^2}{f^2}+\frac{\left(-c\sqrt{-4df+e^2}+b\right.}{}\right.}{}$

```
[In] int(x^2/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-1/2*(-e*(-4*d*f+e^2
)^(1/2)+2*d*f-e^2)/f^2/(-4*d*f+e^2)^(1/2)*2^(1/2)/((-b*f*(-4*d*f+e^2)^(1/2)
+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-b*f*(
-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+1
/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(
1/2)*((-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f
+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4/f*(-c*(-4*d*f+
e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*(-b*f*(-4*d*f+e^2)^(
1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2
*(e+(-4*d*f+e^2)^(1/2))/f))-1/2*(e^2-2*d*f-e*(-4*d*f+e^2)^(1/2))/f^2/(-4*d*
f+e^2)^(1/2)*2^(1/2)/((b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^
2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^
(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f
*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+1/2*2^(1/2)*((b*f*(-4*d*f+e^2)^(1/2)-(-4
*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2/f*(-e
+(-4*d*f+e^2)^(1/2)))^2*c+4*(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2/f*(-e+(
-4*d*f+e^2)^(1/2)))+2*(b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^
2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = \text{Timed out}$$

```
[In] integrate(x^2/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = \int \frac{x^2}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx$$

```
[In] integrate(x**2/(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)
[Out] Integral(x**2/(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re deta
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^2}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^2/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = \int \frac{x^2}{\sqrt{c\,x^2+b\,x+a}\,\left(f\,x^2+e\,x+d\right)}\,dx$$

```
[In] int(x^2/((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)),x)
[Out] int(x^2/((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)), x)
```

3.116 $\int \frac{x}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 402

$$\int \frac{x}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = \frac{\left(e-\sqrt{e^2-4df}\right)\operatorname{arctanh}\left(\frac{4af-b\left(e-\sqrt{e^2-4df}\right)+2\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)x}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}\sqrt{a+bx+cx^2}}\right)}{\sqrt{2}\sqrt{e^2-4df}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}} - \frac{\left(e+\sqrt{e^2-4df}\right)\operatorname{arctanh}\left(\frac{4af-b\left(e+\sqrt{e^2-4df}\right)+2\left(bf-c\left(e+\sqrt{e^2-4df}\right)\right)x}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2-4df}}\sqrt{a+bx+cx^2}}\right)}{\sqrt{2}\sqrt{e^2-4df}\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2-4df}}}$$

```
[Out] 1/2*arctanh(1/4*(4*a*f+2*x*(b*f-c*(e-(-4*d*f+e^2)^(1/2)))-b*(e-(-4*d*f+e^2)
^(1/2)))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e
)*(-4*d*f+e^2)^(1/2))^(1/2))*(e-(-4*d*f+e^2)^(1/2))*2^(1/2)/(-4*d*f+e^2)^(1
/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)-1/2*a
rctanh(1/4*(4*a*f-b*(e+(-4*d*f+e^2)^(1/2))+2*x*(b*f-c*(e+(-4*d*f+e^2)^(1/2)
)))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4
*d*f+e^2)^(1/2))^(1/2))*(e+(-4*d*f+e^2)^(1/2))*2^(1/2)/(-4*d*f+e^2)^(1/2)/(
c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {1046, 738, 212}

$$\int \frac{x}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = \frac{\left(e-\sqrt{e^2-4df}\right)\text{arctanh}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}} - \frac{\left(\sqrt{e^2-4df}+e\right)\text{arctanh}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}\sqrt{e^2-4df}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}$$

```
[In] Int[x/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] ((e - Sqrt[e^2 - 4*d*f])*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*
f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f +
 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]
*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqr
t[e^2 - 4*d*f]]) - ((e + Sqrt[e^2 - 4*d*f])*ArcTanh[(4*a*f - b*(e + Sqrt[e^
2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2
- 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x +
 c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^
2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1046

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
```

```
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\text{integral} = -\left(\left(-1-\frac{e}{\sqrt{e^2-4df}}\right)\int \frac{1}{\left(e+\sqrt{e^2-4df}+2fx\right)\sqrt{a+bx+cx^2}}\,dx\right) + \left(1-\frac{e}{\sqrt{e^2-4df}}\right)\int \frac{1}{\left(e-\sqrt{e^2-4df}+2fx\right)\sqrt{a+bx+cx^2}}\,dx$$

$$= -\left(\left(2\left(1-\frac{e}{\sqrt{e^2-4df}}\right)\right)\text{Subst}\left(\int \frac{1}{16af^2-8bf\left(e-\sqrt{e^2-4df}\right)+4c\left(e-\sqrt{e^2-4df}\right)^2-x^2}\,dx\right.\right. - \left(2\left(1+\frac{e}{\sqrt{e^2-4df}}\right)\right)\text{Subst}\left(\int \frac{1}{16af^2-8bf\left(e+\sqrt{e^2-4df}\right)+4c\left(e+\sqrt{e^2-4df}\right)^2-x^2}\,dx, x, \frac{4af-}{}\right.$$

$$= -\frac{\left(1-\frac{e}{\sqrt{e^2-4df}}\right)\tanh^{-1}\left(\frac{4af-b\left(e-\sqrt{e^2-4df}\right)+2\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)x}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}\sqrt{a+bx+cx^2}}\right)}{\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}} - \frac{\left(1+\frac{e}{\sqrt{e^2-4df}}\right)\tanh^{-1}\left(\frac{4af-b\left(e+\sqrt{e^2-4df}\right)+2\left(bf-c\left(e+\sqrt{e^2-4df}\right)\right)x}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2-4df}}\sqrt{a+bx+cx^2}}\right)}{\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2-4df}}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.36 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.51

$$\int \frac{x}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx$$

$$= \text{RootSum}\Big[b^2d-abe+a^2f-4b\sqrt{c}d\#1+2a\sqrt{c}e\#1+4cd\#1^2+be\#1^2-2af\#1^2-2\sqrt{c}e\#1^3 + f\#1^4\&, \frac{-a\log\left(-\sqrt{c}x+\sqrt{a+bx+cx^2}-\#1\right)+\log\left(-\sqrt{c}x+\sqrt{a+bx+cx^2}-\#1\right)\#1^2}{-2b\sqrt{c}d+a\sqrt{c}e+4cd\#1+be\#1-2af\#1-3\sqrt{c}e\#1^2+2f\#1^3}\&\Big]$$

```
[In] Integrate[x/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] RootSum[b^2*d - a*b*e + a^2*f - 4*b*Sqrt[c]*d*#1 + 2*a*Sqrt[c]*e*#1 + 4*c*d
*#1^2 + b*e*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (-(a*Log[-(Sq
```

```
rt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]) + Log[-(Sqrt[c]*x) + Sqrt[a + b*x +
 c*x^2] - #1]*#1^2)/(-2*b*Sqrt[c]*d + a*Sqrt[c]*e + 4*c*d*#1 + b*e*#1 - 2*a
*f*#1 - 3*Sqrt[c]*e*#1^2 + 2*f*#1^3) & ]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 793 vs. $2(355) = 710$.

Time = 0.90 (sec) , antiderivative size = 794, normalized size of antiderivative = 1.98

method	result
default	$-\frac{\left(e+\sqrt{-4df+e^2}\right)\sqrt{2}\ \ln\left(\frac{\frac{-bf\sqrt{-4df+e^2}+\sqrt{-4df+e^2}\ ce+2a\ f^2-bef-2cdf+c\ e^2}{f^2}+\frac{\left(-c\sqrt{-4df+e^2}+bf-ce\right)\left(x+\frac{e+\sqrt{-4df+e^2}}{2f}\right)}{f}+\frac{\sqrt{2}\sqrt{\frac{-b}{}}}{}}{}\right.}{2\sqrt{-4df+}}$

```
[In] int(x/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)/f*2^(1/2)/((-b*f*(-4*d*f+e^2
)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((
(-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2
)/f^2+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+
1/2*2^(1/2)*((-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-
2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4/f*(-c*(
-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*(-b*f*(-4*d*f
+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))
/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))-1/2*(-e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)
^(1/2)/f*2^(1/2)/((b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*
e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2
)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-
1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+1/2*2^(1/2)*((b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f
+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2/f*(-e+(-4
*d*f+e^2)^(1/2)))^2*c+4*(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2/f*(-e+(-4*d
*f+e^2)^(1/2)))+2*(b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*
e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11311 vs. 2(353) = 706.

Time = 3.57 (sec) , antiderivative size = 11311, normalized size of antiderivative = 28.14

$$\int \frac{x}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = \text{Too large to display}$$

```
[In] integrate(x/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F]

$$\int \frac{x}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = \int \frac{x}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx$$

```
[In] integrate(x/(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)
[Out] Integral(x/(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re deta
```

Giac [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = \text{Timed out}$$

```
[In] integrate(x/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = \int \frac{x}{\sqrt{c\,x^2+b\,x+a}\,\left(f\,x^2+e\,x+d\right)}\,dx$$

```
[In] int(x/((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)),x)
[Out] int(x/((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)), x)
```

3.117 $\int \frac{1}{\sqrt{a+bx+cx^2}(d+ex+fx^2)} dx$

Optimal result

Integrand size = 27, antiderivative size = 374

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = -\frac{\sqrt{2}f\,\text{arctanh}\left(\frac{4af-b\left(e-\sqrt{e^2-4df}\right)+2\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)x}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}\sqrt{a+bx+cx^2}}\right)}{\sqrt{e^2-4df}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}} + \frac{\sqrt{2}f\,\text{arctanh}\left(\frac{4af-b\left(e+\sqrt{e^2-4df}\right)+2\left(bf-c\left(e+\sqrt{e^2-4df}\right)\right)x}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2-4df}}\sqrt{a+bx+cx^2}}\right)}{\sqrt{e^2-4df}\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2-4df}}}$$

```
[Out] -f*arctanh(1/4*(4*a*f+2*x*(b*f-c*(e-(-4*d*f+e^2)^(1/2)))-b*(e-(-4*d*f+e^2)^
(1/2)))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)
*(-4*d*f+e^2)^(1/2))^(1/2))*2^(1/2)/(-4*d*f+e^2)^(1/2)/(c*e^2-2*c*d*f-b*e*f
+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)+f*arctanh(1/4*(4*a*f-b*(e+(-4
*d*f+e^2)^(1/2))+2*x*(b*f-c*(e+(-4*d*f+e^2)^(1/2))))*2^(1/2)/(c*x^2+b*x+a)^
(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2))*2^
(1/2)/(-4*d*f+e^2)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^
2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {997, 738, 212}

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = \frac{\sqrt{2}f\,\text{arctanh}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}} - \frac{\sqrt{2}f\,\text{arctanh}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}$$

```
[In] Int[1/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] -((Sqrt[2]*f*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - S
qrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (
c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[e^2 - 4*d*f]*S
qrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]])) +
(Sqrt[2]*f*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqr
t[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*
e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[e^2 - 4*d*f]*Sqr
t[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 997

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[1/((
b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[2*(c/q), Int[1/((b + q
 + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[c*e - b*f, 0] && PosQ[b
```

```
^2 - 4*a*c]
```

Rubi steps

$$\text{integral} = \frac{(2f)\int \frac{1}{\left(e-\sqrt{e^2-4df}+2fx\right)\sqrt{a+bx+cx^2}}\,dx}{\sqrt{e^2-4df}} - \frac{(2f)\int \frac{1}{\left(e+\sqrt{e^2-4df}+2fx\right)\sqrt{a+bx+cx^2}}\,dx}{\sqrt{e^2-4df}}$$

$$= -\frac{(4f)\text{Subst}\left(\int \frac{1}{16af^2-8bf\left(e-\sqrt{e^2-4df}\right)+4c\left(e-\sqrt{e^2-4df}\right)^2-x^2}\,dx, x, \frac{4af-b\left(e-\sqrt{e^2-4df}\right)-\left(-2bf+2c\left(e-\sqrt{e^2-4df}\right)\right)}{\sqrt{a+bx+cx^2}}\right.}{\sqrt{e^2-4df}}$$

$$+\frac{(4f)\text{Subst}\left(\int \frac{1}{16af^2-8bf\left(e+\sqrt{e^2-4df}\right)+4c\left(e+\sqrt{e^2-4df}\right)^2-x^2}\,dx, x, \frac{4af-b\left(e+\sqrt{e^2-4df}\right)-\left(-2bf+2c\left(e+\sqrt{e^2-4df}\right)\right)}{\sqrt{a+bx+cx^2}}\right.}{\sqrt{e^2-4df}}$$

$$= -\frac{\sqrt{2}f\tanh^{-1}\left(\frac{4af-b\left(e-\sqrt{e^2-4df}\right)+2\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)x}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}\sqrt{a+bx+cx^2}}\right)}{\sqrt{e^2-4df}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}}$$

$$+\frac{\sqrt{2}f\tanh^{-1}\left(\frac{4af-b\left(e+\sqrt{e^2-4df}\right)+2\left(bf-c\left(e+\sqrt{e^2-4df}\right)\right)x}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2-4df}}\sqrt{a+bx+cx^2}}\right)}{\sqrt{e^2-4df}\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2-4df}}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.58

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = -\text{RootSum}\Big[c^2d-bce+b^2f+2\sqrt{a}ce\#1-4\sqrt{a}bf\#1 -2cd\#1^2+be\#1^2+4af\#1^2-2\sqrt{a}e\#1^3 +d\#1^4\&, \frac{c\log(x)-c\log\left(-\sqrt{a}+\sqrt{a+bx+cx^2}-x\#1\right)-\log(x)\#1^2+\log\left(-\sqrt{a}+\sqrt{a+bx+cx^2}-x\right.}{\sqrt{a}ce-2\sqrt{a}bf-2cd\#1+be\#1+4af\#1-3\sqrt{a}e\#1^2+2d\#1^3}$$

```
[In] Integrate[1/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] -RootSum[c^2*d - b*c*e + b^2*f + 2*Sqrt[a]*c*e*#1 - 4*Sqrt[a]*b*f*#1 - 2*c*
d*#1^2 + b*e*#1^2 + 4*a*f*#1^2 - 2*Sqrt[a]*e*#1^3 + d*#1^4 & , (c*Log[x] -
c*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] - Log[x]*#1^2 + Log[-Sqrt[a]
 + Sqrt[a + b*x + c*x^2] - x*#1]*#1^2)/(Sqrt[a]*c*e - 2*Sqrt[a]*b*f - 2*c*d
*#1 + b*e*#1 + 4*a*f*#1 - 3*Sqrt[a]*e*#1^2 + 2*d*#1^3) & ]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 760 vs. $2(330) = 660$.

Time = 0.83 (sec) , antiderivative size = 761, normalized size of antiderivative = 2.03

method	result
default	$\frac{\sqrt{2}\,\ln\left(\frac{-bf\sqrt{-4df+e^2}+\sqrt{-4df+e^2}\,ce+2a\,f^2-bef-2cdf+c\,e^2}{f^2}+\frac{\left(-c\sqrt{-4df+e^2}+bf-ce\right)\left(x+\frac{e+\sqrt{-4df+e^2}}{2f}\right)}{f}+\sqrt{2}\sqrt{\frac{-bf\sqrt{-4df+e^2}+\sqrt{-4df\cdots}}{\cdots}}\cdots\right)}{\sqrt{-4df+e^2}\sqrt{\frac{-bf\sqrt{\cdots}}{\cdots}}\cdots}$

```
[In] int(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
[Out] 1/(-4*d*f+e^2)^(1/2)*2^(1/2)/((-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c
*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-b*f*(-4*d*f+e^2)^(1/2)+(-4
*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+1/f*(-c*(-4*d*f+e^2)^(
1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-b*f*(-4*d*f+e
^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4
*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(
x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*(-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/
2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2
))/f))-1/(-4*d*f+e^2)^(1/2)*2^(1/2)/((b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(
1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((b*f*(-4*d*f+e^2)^(1/2
)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+(c*(-4*d*f+e^2)^(
1/2)+b*f-c*e)/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+1/2*2^(1/2)*((b*f*(-4*d*f
+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*
(4*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2*c+4*(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f
*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+2*(b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(
1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2/f*(-e+(-4*d*f+e^2)
^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11287 vs. $2(328) = 656$.

Time = 4.27 (sec) , antiderivative size = 11287, normalized size of antiderivative = 30.18

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = \text{Too large to display}$$

```
[In] integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")
[Out] Too large to include
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = \int \frac{1}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx$$

```
[In] integrate(1/(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)
[Out] Integral(1/(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re deta
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = \text{Timed out}$$

```
[In] integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = \int \frac{1}{\sqrt{c\,x^2+b\,x+a}\,\left(f\,x^2+e\,x+d\right)}\,dx$$

```
[In] int(1/((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)),x)
[Out] int(1/((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)), x)
```

3.118 $\int \frac{1}{x\sqrt{a+bx+cx^2}(d+ex+fx^2)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 451

$$\begin{aligned}
\int \frac{1}{x\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx
&= -\frac{\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}d} \\
&\quad + \frac{f\left(e+\sqrt{e^2-4df}\right)\operatorname{arctanh}\left(\frac{4af-b\left(e-\sqrt{e^2-4df}\right)+2\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)x}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}\sqrt{a+bx+cx^2}}\right)}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}} \\
&\quad - \frac{f\left(e-\sqrt{e^2-4df}\right)\operatorname{arctanh}\left(\frac{4af-b\left(e+\sqrt{e^2-4df}\right)+2\left(bf-c\left(e+\sqrt{e^2-4df}\right)\right)x}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2-4df}}\sqrt{a+bx+cx^2}}\right)}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2-4df}}}
\end{aligned}$$

```
[Out] -arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/d/a^(1/2)+1/2*f*arctanh
(1/4*(4*a*f+2*x*(b*f-c*(e-(-4*d*f+e^2)^(1/2)))-b*(e-(-4*d*f+e^2)^(1/2)))*2^
(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e
^2)^(1/2))^(1/2))*(e+(-4*d*f+e^2)^(1/2))/d*2^(1/2)/(-4*d*f+e^2)^(1/2)/(c*e^
2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)-1/2*f*arctanh(
1/4*(4*a*f-b*(e+(-4*d*f+e^2)^(1/2))+2*x*(b*f-c*(e+(-4*d*f+e^2)^(1/2))))*2^(
1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^
2)^(1/2))^(1/2))*(e-(-4*d*f+e^2)^(1/2))/d*2^(1/2)/(-4*d*f+e^2)^(1/2)/(c*e^2
-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 1.70 (sec) , antiderivative size = 451, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {6860, 738, 212, 1046}

$$\int \frac{1}{x\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx$$
$$= \frac{f\left(\sqrt{e^2-4df}+e\right)\operatorname{arctanh}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}$$
$$-\frac{f\left(e-\sqrt{e^2-4df}\right)\operatorname{arctanh}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}$$
$$-\frac{\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}d}$$

```
[In] Int[1/(x*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] -(ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])]/(Sqrt[a]*d)) + (f*
(e + Sqrt[e^2 - 4*d*f])*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f
 - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f +
2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*
d*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sq
rt[e^2 - 4*d*f]]) - (f*(e - Sqrt[e^2 - 4*d*f])*ArcTanh[(4*a*f - b*(e + Sqrt
[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e
^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*
x + c*x^2])])/(Sqrt[2]*d*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2
*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1046

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 6860

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{1}{dx\sqrt{a+bx+cx^2}} + \frac{-e-fx}{d\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\right) dx$$

$$= \frac{\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx}{d} + \frac{\int \frac{-e-fx}{\sqrt{a+bx+cx^2}(d+ex+fx^2)}\,dx}{d}$$

$$= -\frac{2\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx}{\sqrt{a+bx+cx^2}}\right)}{d} - \frac{\left(f\left(1-\frac{e}{\sqrt{e^2-4df}}\right)\right)\int \frac{1}{\left(e+\sqrt{e^2-4df}+2fx\right)\sqrt{a+bx+cx^2}}\,dx}{d} - \frac{\left(f\left(1+\frac{e}{\sqrt{e^2-4df}}\right)\right)\int \frac{1}{\left(e-\sqrt{e^2-4df}+2fx\right)\sqrt{a+bx+cx^2}}\,dx}{d}$$

$$= -\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}d} + \frac{\left(2f\left(1-\frac{e}{\sqrt{e^2-4df}}\right)\right)\text{Subst}\left(\int \frac{1}{16af^2-8bf\left(e+\sqrt{e^2-4df}\right)+4c\left(e+\sqrt{e^2-4df}\right)^2-x^2}\,dx, x, \frac{4af-b\left(e+\sqrt{e^2-4df}\right)-\left(-\right.}{\sqrt{a+bx}}\right.}{d} + \frac{\left(2f\left(1+\frac{e}{\sqrt{e^2-4df}}\right)\right)\text{Subst}\left(\int \frac{1}{16af^2-8bf\left(e-\sqrt{e^2-4df}\right)+4c\left(e-\sqrt{e^2-4df}\right)^2-x^2}\,dx, x, \frac{4af-b\left(e-\sqrt{e^2-4df}\right)-\left(-\right.}{\sqrt{a+bx}}\right.}{d}$$

$$= -\frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}d} + \frac{f\left(1+\frac{e}{\sqrt{e^2-4df}}\right)\tanh^{-1}\left(\frac{4af-b\left(e-\sqrt{e^2-4df}\right)+2\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)x}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}\sqrt{a+bx+cx^2}}\right)}{\sqrt{2}d\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}} + \frac{f\left(1-\frac{e}{\sqrt{e^2-4df}}\right)\tanh^{-1}\left(\frac{4af-b\left(e+\sqrt{e^2-4df}\right)+2\left(bf-c\left(e+\sqrt{e^2-4df}\right)\right)x}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2-4df}}\sqrt{a+bx+cx^2}}\right)}{\sqrt{2}d\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2-4df}}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.45 (sec) , antiderivative size = 319, normalized size of antiderivative = 0.71

$$\int \frac{1}{x\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = \frac{\frac{2\operatorname{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)}{\sqrt{a}} - \operatorname{RootSum}\left[b^2d-abe+a^2f-4b\sqrt{c}d\#1+2a\sqrt{c}e\#1+4cd\#1^2+be\#1^2-2af\#1^2\right.}{}$$

```
[In] Integrate[1/(x*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] ((2*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)])/Sqrt[a]])/Sqrt[a] - RootSum
[b^2*d - a*b*e + a^2*f - 4*b*Sqrt[c]*d*#1 + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 +
 b*e*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (b*e*Log[-(Sqrt[c]*x
) + Sqrt[a + b*x + c*x^2] - #1] - a*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x
^2] - #1] - 2*Sqrt[c]*e*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 +
 f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2)/(-2*b*Sqrt[c]*d + a
*Sqrt[c]*e + 4*c*d*#1 + b*e*#1 - 2*a*f*#1 - 3*Sqrt[c]*e*#1^2 + 2*f*#1^3) &
])/d
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 858 vs. 2(396) = 792.

Time = 0.89 (sec) , antiderivative size = 859, normalized size of antiderivative = 1.90

method	result
default	$\frac{4f\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)}{\left(-e+\sqrt{-4df+e^2}\right)\left(e+\sqrt{-4df+e^2}\right)\sqrt{a}}-\frac{2f\sqrt{2}\,\ln\left(\frac{-bf\sqrt{-4df+e^2}+\sqrt{-4df+e^2}\,ce+2a\,f^2-bef-2cdf+c\,e^2}{f^2}+\frac{\left(-c\sqrt{-4df+e^2}+bf-ce\right)\left(\cdots\right.}{f}\cdots\right.}{\cdots}$

```
[In] int(1/x/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 4*f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))/a^(1/2)*ln((2*a+b*x+2*a^
(1/2)*(c*x^2+b*x+a)^(1/2))/x)-2*f/(e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)
*2^(1/2)/((-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c
*d*f+c*e^2)/f^2)^(1/2)*ln(((-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+
2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2
*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^
2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e
^2)^(1/2))/f)^2*c+4/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2
)^(1/2))/f)+2*(-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f
-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))-2*f/(-e+(-4*d
*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)*2^(1/2)/((b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+
e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((b*f*(-4*d*f+e^2
)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+(c*(-4*d*f+
e^2)^(1/2)+b*f-c*e)/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+1/2*2^(1/2)*((b*f*(
-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^
(1/2)*(4*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2*c+4*(c*(-4*d*f+e^2)^(1/2)+b*f-
c*e)/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+2*(b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+
e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2/f*(-e+(-4*d*
f+e^2)^(1/2))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = \text{Timed out}$$

```
[In] integrate(1/x/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{x\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = \int \frac{1}{x\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx$$

```
[In] integrate(1/x/(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)
[Out] Integral(1/(x*sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{x\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = \int \frac{1}{\sqrt{cx^2+bx+a}(fx^2+ex+d)x}\,dx$$

```
[In] integrate(1/x/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*(f*x^2 + e*x + d)*x), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = \text{Timed out}$$

```
[In] integrate(1/x/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = \int \frac{1}{x\,\sqrt{c\,x^2+b\,x+a}\,\left(f\,x^2+e\,x+d\right)}\,dx$$

```
[In] int(1/(x*(a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)),x)
[Out] int(1/(x*(a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)), x)
```

3.119 $\int \frac{1}{x^2\sqrt{a+bx+cx^2}(d+ex+fx^2)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 543

$$\int \frac{1}{x^2\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = -\frac{\sqrt{a+bx+cx^2}}{adx}+\frac{b\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{3/2}d}+\frac{e\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}d^2}$$
$$-\frac{f\left(e^2-2df+e\sqrt{e^2-4df}\right)\text{arctanh}\left(\frac{4af-b\left(e-\sqrt{e^2-4df}\right)+2\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)x}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}\sqrt{a+bx+cx^2}}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{c\left(e^2-2df-e\sqrt{e^2-4df}\right)+f\left(2af-b\left(e-\sqrt{e^2-4df}\right)\right)}}$$
$$+\frac{f\left(e^2-2df-e\sqrt{e^2-4df}\right)\text{arctanh}\left(\frac{4af-b\left(e+\sqrt{e^2-4df}\right)+2\left(bf-c\left(e+\sqrt{e^2-4df}\right)\right)x}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2-4df}}\sqrt{a+bx+cx^2}}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2-4df}}}$$

```
[Out] 1/2*b*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/a^(3/2)/d+e*arctan
h(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/d^2/a^(1/2)-(c*x^2+b*x+a)^(1/2
)/a/d/x+1/2*f*arctanh(1/4*(4*a*f-b*(e+(-4*d*f+e^2)^(1/2))+2*x*(b*f-c*(e+(-4
*d*f+e^2)^(1/2))))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2
+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2))*(e^2-2*d*f-e*(-4*d*f+e^2)^(1/2))/d^2
*2^(1/2)/(-4*d*f+e^2)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f
+e^2)^(1/2))^(1/2)-1/2*f*arctanh(1/4*(4*a*f+2*x*(b*f-c*(e-(-4*d*f+e^2)^(1/2
)))-b*(e-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*
e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2))*(e^2-2*d*f+e*(-4*d*f+e^2)
^(1/2))/d^2*2^(1/2)/(-4*d*f+e^2)^(1/2)/(f*(2*a*f-b*(e-(-4*d*f+e^2)^(1/2)))+
c*(e^2-2*d*f-e*(-4*d*f+e^2)^(1/2)))^(1/2)
```

Rubi [A] (verified)

Time = 3.11 (sec) , antiderivative size = 543, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6860, 744, 738, 212, 1046}

$$\int \frac{1}{x^2\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx$$
$$= \frac{b\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{3/2}d} - \frac{f\left(e\sqrt{e^2-4df}-2df+e^2\right)\text{arctanh}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{f\left(2af-b\left(e-\sqrt{e^2-4df}\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}} + \frac{f\left(-e\sqrt{e^2-4df}-2df+e^2\right)\text{arctanh}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}} + \frac{e\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}d^2} - \frac{\sqrt{a+bx+cx^2}}{adx}$$

```
[In] Int[1/(x^2*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] -(Sqrt[a + b*x + c*x^2]/(a*d*x)) + (b*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a
 + b*x + c*x^2])])/(2*a^(3/2)*d) + (e*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a
 + b*x + c*x^2])])/(Sqrt[a]*d^2) - (f*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])*A
rcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*
f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqr
t[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*d^2*Sqrt[e^2 - 4*d*f]*Sqr
t[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e - Sqrt[e^2 - 4*d*
f]))]) + (f*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])*ArcTanh[(4*a*f - b*(e + Sqr
t[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*
e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b
*x + c*x^2])])/(Sqrt[2]*d^2*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f
+ 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
 Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 1046

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 6860

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

integral

$$=\int \left(\frac{1}{dx^2\sqrt{a+bx+cx^2}}-\frac{e}{d^2x\sqrt{a+bx+cx^2}}+\frac{e^2-df+efx}{d^2\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\right)dx$$

$$=\frac{\int \frac{e^2-df+efx}{\sqrt{a+bx+cx^2}(d+ex+fx^2)}\,dx}{d^2}+\frac{\int \frac{1}{x^2\sqrt{a+bx+cx^2}}\,dx}{d}-\frac{e\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx}{d^2}$$

$$=-\frac{\sqrt{a+bx+cx^2}}{adx}-\frac{b\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx}{2ad}+\frac{(2e)\mathrm{Subst}\left(\int \frac{1}{4a-x^2}\,dx,x,\frac{2a+bx}{\sqrt{a+bx+cx^2}}\right)}{d^2}-\frac{\left(f\left(e^2-2df-e\sqrt{e^2-4df}\right)\right)\int \frac{1}{\left(e+\sqrt{e^2-4df}+2fx\right)\sqrt{a+bx+cx^2}}\,dx}{d^2\sqrt{e^2-4df}}+\frac{\left(f\left(e^2-2df+e\sqrt{e^2-4df}\right)\right)\int \frac{1}{\left(e-\sqrt{e^2-4df}+2fx\right)\sqrt{a+bx+cx^2}}\,dx}{d^2\sqrt{e^2-4df}}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{adx} + \frac{e\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}d^2} + \frac{b\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx}{\sqrt{a+bx+cx^2}}\right)}{ad}$$
$$+ \frac{\left(2f\left(e^2-2df-e\sqrt{e^2-4df}\right)\right)\text{Subst}\left(\int \frac{1}{16af^2-8bf\left(e+\sqrt{e^2-4df}\right)+4c\left(e+\sqrt{e^2-4df}\right)^2-x^2}\,dx, x, \frac{4af-b\left(e+\sqrt{e^2-4df}\right)-\left(-\right.}{\sqrt{a+b}}\right.}{d^2\sqrt{e^2-4df}}$$
$$- \frac{\left(2f\left(e^2-2df+e\sqrt{e^2-4df}\right)\right)\text{Subst}\left(\int \frac{1}{16af^2-8bf\left(e-\sqrt{e^2-4df}\right)+4c\left(e-\sqrt{e^2-4df}\right)^2-x^2}\,dx, x, \frac{4af-b\left(e-\sqrt{e^2-4df}\right)-\left(-\right.}{\sqrt{a+b}}\right.}{d^2\sqrt{e^2-4df}}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{adx} + \frac{b\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{3/2}d} + \frac{e\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}d^2}$$
$$- \frac{f\left(e^2-2df+e\sqrt{e^2-4df}\right)\tanh^{-1}\left(\frac{4af-b\left(e-\sqrt{e^2-4df}\right)+2\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)x}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}\sqrt{a+bx+cx^2}}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}}$$
$$+ \frac{f\left(e^2-2df-e\sqrt{e^2-4df}\right)\tanh^{-1}\left(\frac{4af-b\left(e+\sqrt{e^2-4df}\right)+2\left(bf-c\left(e+\sqrt{e^2-4df}\right)\right)x}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2-4df}}\sqrt{a+bx+cx^2}}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2-4df}}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.68 (sec) , antiderivative size = 423, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^2\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx$$
$$= \frac{-\frac{d\sqrt{a+x(b+cx)}}{ax} + \frac{(bd+2ae)\text{arctanh}\left(\frac{-\sqrt{c}x+\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)}{a^{3/2}} + \text{RootSum}\left[b^2d-abe+a^2f-4b\sqrt{c}d\#1+2a\sqrt{c}e\#1+4\right.}{}$$

```
[In] Integrate[1/(x^2*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] (-((d*Sqrt[a + x*(b + c*x)])/(a*x)) + ((b*d + 2*a*e)*ArcTanh[(-(Sqrt[c]*x)
+ Sqrt[a + x*(b + c*x)])/Sqrt[a]])/a^(3/2) + RootSum[b^2*d - a*b*e + a^2*f
- 4*b*Sqrt[c]*d*#1 + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 + b*e*#1^2 - 2*a*f*#1^2
- 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (b*e^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*
x^2] - #1] - b*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - a*e*f*L
og[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*Sqrt[c]*e^2*Log[-(Sqrt[c]
*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + 2*Sqrt[c]*d*f*Log[-(Sqrt[c]*x) + Sqr
t[a + b*x + c*x^2] - #1]*#1 + e*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2]
- #1]*#1^2)/(-2*b*Sqrt[c]*d + a*Sqrt[c]*e + 4*c*d*#1 + b*e*#1 - 2*a*f*#1 -
3*Sqrt[c]*e*#1^2 + 2*f*#1^3) & ])/d^2
```

Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 927, normalized size of antiderivative = 1.71

method	result
risch	$-\frac{\sqrt{c\,x^2+bx+a}}{adx}-\frac{\frac{4f(2ae+bd)\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)}{\left(-e+\sqrt{-4df+e^2}\right)\left(e+\sqrt{-4df+e^2}\right)\sqrt{a}}-\frac{2f\left(e+\sqrt{-4df+e^2}\right)a\sqrt{2}\,\ln\left(\frac{bf\sqrt{-4df+e^2}-\sqrt{-4df+e^2}\,ce+2a\,f^2-bef-2cdf}{f^2}\right.}{}}{}$
default	$-\frac{4f\left(-\frac{\sqrt{c\,x^2+bx+a}}{ax}+\frac{b\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)}{2a^{\frac{3}{2}}}\right)}{\left(-e+\sqrt{-4df+e^2}\right)\left(e+\sqrt{-4df+e^2}\right)}+\frac{16f^2e\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)}{\left(-e+\sqrt{-4df+e^2}\right)^2\left(e+\sqrt{-4df+e^2}\right)^2\sqrt{a}}+\frac{4f^2\sqrt{2}\,\ln\left(\frac{-bf\sqrt{-4df+e}}{}\right.}{}$

```
[In] int(1/x^2/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)

[Out] -(c*x^2+b*x+a)^(1/2)/a/d/x-1/2/a/d*(4*f*(2*a*e+b*d)/(-e+(-4*d*f+e^2)^(1/2))
/(e+(-4*d*f+e^2)^(1/2))/a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/
x)-2*f*(e+(-4*d*f+e^2)^(1/2))*a/(-4*d*f+e^2)^(1/2)/(-e+(-4*d*f+e^2)^(1/2))*
2^(1/2)/((b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d
*f+c*e^2)/f^2)^(1/2)*ln(((b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a
*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2/f*(-e
+(-4*d*f+e^2)^(1/2)))+1/2*2^(1/2)*((b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/
2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2/f*(-e+(-4*d*f+e^2)
^(1/2)))^2*c+4*(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(
1/2)))+2*(b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d
*f+c*e^2)/f^2)^(1/2))/(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2))))+2*f*a*(-e+(-4*d*f+
e^2)^(1/2))/(-4*d*f+e^2)^(1/2)/(e+(-4*d*f+e^2)^(1/2))*2^(1/2)/((-b*f*(-4*d*
f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)
*ln(((-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+
c*e^2)/f^2+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2)
)/f)+1/2*2^(1/2)*((-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b
*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4/f*
(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*(-b*f*(-
4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(
1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = \text{Timed out}$$

```
[In] integrate(1/x^2/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{x^2\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = \int \frac{1}{x^2\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx$$

```
[In] integrate(1/x**2/(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)
[Out] Integral(1/(x**2*sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = \int \frac{1}{\sqrt{cx^2+bx+a}(fx^2+ex+d)x^2}\,dx$$

```
[In] integrate(1/x^2/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*(f*x^2 + e*x + d)*x^2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = \text{Timed out}$$

```
[In] integrate(1/x^2/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = \int \frac{1}{x^2\,\sqrt{c\,x^2+b\,x+a}\,\left(f\,x^2+e\,x+d\right)}\,dx$$

```
[In] int(1/(x^2*(a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)),x)
[Out] int(1/(x^2*(a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)), x)
```

3.120 $\int \frac{1}{x^3\sqrt{a+bx+cx^2}(d+ex+fx^2)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 679

$$\begin{aligned}\int \frac{1}{x^3\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx &= -\frac{\sqrt{a+bx+cx^2}}{2adx^2}+\frac{3b\sqrt{a+bx+cx^2}}{4a^2dx} \\ &+\frac{e\sqrt{a+bx+cx^2}}{ad^2x}-\frac{(3b^2-4ac)\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{5/2}d} \\ &-\frac{b e\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{3/2}d^2}-\frac{(e^2-df)\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}d^3} \\ &+\frac{f\left(2e^3-4def-(e^2-df)\left(e-\sqrt{e^2-4df}\right)\right)\operatorname{arctanh}\left(\frac{4af-b\left(e-\sqrt{e^2-4df}\right)+2\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)x}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}\sqrt{a+bx+cx^2}}\right)}{\sqrt{2}d^3\sqrt{e^2-4df}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}} \\ &-\frac{f\left(2e^3-4def-(e^2-df)\left(e+\sqrt{e^2-4df}\right)\right)\operatorname{arctanh}\left(\frac{4af-b\left(e+\sqrt{e^2-4df}\right)+2\left(bf-c\left(e+\sqrt{e^2-4df}\right)\right)x}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2-4df}}\sqrt{a+bx+cx^2}}\right)}{\sqrt{2}d^3\sqrt{e^2-4df}\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2-4df}}}\end{aligned}$$

```
[Out] -1/8*(-4*a*c+3*b^2)*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/a^(5
/2)/d-1/2*b*e*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/a^(3/2)/d^
2-(-d*f+e^2)*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/d^3/a^(1/2)
-1/2*(c*x^2+b*x+a)^(1/2)/a/d/x^2+3/4*b*(c*x^2+b*x+a)^(1/2)/a^2/d/x+e*(c*x^2
+b*x+a)^(1/2)/a/d^2/x+1/2*f*arctanh(1/4*(4*a*f+2*x*(b*f-c*(e-(-4*d*f+e^2)^(
1/2)))-b*(e-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f
-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2))*(2*e^3-4*d*e*f-(-d*f+e
^2)*(e-(-4*d*f+e^2)^(1/2)))/d^3*2^(1/2)/(-4*d*f+e^2)^(1/2)/(c*e^2-2*c*d*f-b
*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)-1/2*f*arctanh(1/4*(4*a*f-
b*(e+(-4*d*f+e^2)^(1/2))+2*x*(b*f-c*(e+(-4*d*f+e^2)^(1/2))))*2^(1/2)/(c*x^2
+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(
```

```
1/2))*(2*e^3-4*d*e*f-(-d*f+e^2)*(e+(-4*d*f+e^2)^(1/2)))/d^3*2^(1/2)/(-4*d*f
+e^2)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/
2)
```

Rubi [A] (verified)

Time = 8.70 (sec) , antiderivative size = 679, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {6860, 758, 820, 738, 212, 744, 1046}

$$\int \frac{1}{x^3\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = -\frac{(3b^2-4ac)\,\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{5/2}d} - \frac{be\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{3/2}d^2} + \frac{3b\sqrt{a+bx+cx^2}}{4a^2dx} - \frac{(e^2-df)\,\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}d^3} + \frac{f\left(-(e^2-df)\left(e-\sqrt{e^2-4df}\right)-4def+2e^3\right)\text{arctanh}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}d^3\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}} - \frac{f\left(-(e^2-df)\left(\sqrt{e^2-4df}+e\right)-4def+2e^3\right)\text{arctanh}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}d^3\sqrt{e^2-4df}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}} + \frac{e\sqrt{a+bx+cx^2}}{ad^2x} - \frac{\sqrt{a+bx+cx^2}}{2adx^2}$$

```
[In] Int[1/(x^3*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] -1/2*Sqrt[a + b*x + c*x^2]/(a*d*x^2) + (3*b*Sqrt[a + b*x + c*x^2])/(4*a^2*d
*x) + (e*Sqrt[a + b*x + c*x^2])/(a*d^2*x) - ((3*b^2 - 4*a*c)*ArcTanh[(2*a +
 b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(8*a^(5/2)*d) - (b*e*ArcTanh[(2*a
 + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(2*a^(3/2)*d^2) - ((e^2 - d*f)*
ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(Sqrt[a]*d^3) + (f*
(2*e^3 - 4*d*e*f - (e^2 - d*f)*(e - Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*a*f - b*
(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]
*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sq
rt[a + b*x + c*x^2])])/(Sqrt[2]*d^3*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f
- b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]) - (f*(2*e^3 - 4*d*e*f -
 (e^2 - d*f)*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*
d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*
d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2
])])/(Sqrt[2]*d^3*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2
+ (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
 Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 758

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 1046

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
```

```
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 6860

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{1}{dx^3\sqrt{a+bx+cx^2}} - \frac{e}{d^2x^2\sqrt{a+bx+cx^2}} + \frac{e^2-df}{d^3x\sqrt{a+bx+cx^2}} + \frac{-e(e^2-2df)-f(e^2-df)\,x}{d^3\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\right) dx$$

$$= \frac{\int \frac{-e(e^2-2df)-f(e^2-df)x}{\sqrt{a+bx+cx^2}(d+ex+fx^2)}\,dx}{d^3} + \frac{\int \frac{1}{x^3\sqrt{a+bx+cx^2}}\,dx}{d} - \frac{e\int \frac{1}{x^2\sqrt{a+bx+cx^2}}\,dx}{d^2} + \frac{(e^2-df)\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx}{d^3}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{2adx^2} + \frac{e\sqrt{a+bx+cx^2}}{ad^2x} - \frac{\int \frac{\frac{3b}{2}+cx}{x^2\sqrt{a+bx+cx^2}}\,dx}{2ad} + \frac{(be)\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx}{2ad^2} - \frac{(2(e^2-df))\,\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx}{\sqrt{a+bx+cx^2}}\right)}{d^3} + \frac{\left(-2ef(e^2-2df)+f(e^2-df)\left(e-\sqrt{e^2-4df}\right)\right)\int \frac{1}{\left(e-\sqrt{e^2-4df}+2fx\right)\sqrt{a+bx+cx^2}}\,dx}{d^3\sqrt{e^2-4df}} - \frac{\left(-2ef(e^2-2df)+f(e^2-df)\left(e+\sqrt{e^2-4df}\right)\right)\int \frac{1}{\left(e+\sqrt{e^2-4df}+2fx\right)\sqrt{a+bx+cx^2}}\,dx}{d^3\sqrt{e^2-4df}}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{2adx^2} + \frac{3b\sqrt{a+bx+cx^2}}{4a^2dx} + \frac{e\sqrt{a+bx+cx^2}}{ad^2x} - \frac{(e^2-df)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}d^3} + \frac{(3b^2-4ac)\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx}{8a^2d} - \frac{(be)\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx}{\sqrt{a+bx+cx^2}}\right)}{ad^2} + \frac{\left(2f\left(2e^3-4def-(e^2-df)\left(e-\sqrt{e^2-4df}\right)\right)\right)\text{Subst}\left(\int \frac{1}{16af^2-8bf\left(e-\sqrt{e^2-4df}\right)+4c\left(e-\sqrt{e^2-4df}\right)^2-x^2 \cdots}\right.}{d^3\sqrt{e^2-4df}} + \frac{\left(2\left(-2ef(e^2-2df)+f(e^2-df)\left(e+\sqrt{e^2-4df}\right)\right)\right)\text{Subst}\left(\int \frac{1}{16af^2-8bf\left(e+\sqrt{e^2-4df}\right)+4c\left(e+\sqrt{e^2-4df} \cdots\right.}\right.}{d^3\sqrt{e^2-4df}}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{2adx^2} + \frac{3b\sqrt{a+bx+cx^2}}{4a^2dx} + \frac{e\sqrt{a+bx+cx^2}}{ad^2x} - \frac{be\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{3/2}d^2} - \frac{(e^2-df)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}d^3} + \frac{f\left(2e^3-4def-(e^2-df)\left(e-\sqrt{e^2-4df}\right)\right)\tanh^{-1}\left(\frac{4af-b\left(e-\sqrt{e^2-4df}\right)+2\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)x}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}\sqrt{a+bx+cx^2}}\right)}{\sqrt{2}d^3\sqrt{e^2-4df}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}} - \frac{f\left(2e^3-4def-(e^2-df)\left(e+\sqrt{e^2-4df}\right)\right)\tanh^{-1}\left(\frac{4af-b\left(e+\sqrt{e^2-4df}\right)+2\left(bf-c\left(e+\sqrt{e^2-4df}\right)\right)x}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2-4df}}\sqrt{a+bx+cx^2}}\right)}{\sqrt{2}d^3\sqrt{e^2-4df}\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2-4df}}} - \frac{(3b^2-4ac)\,\text{Subst}\left(\int\frac{1}{4a-x^2}\,dx,x,\frac{2a+bx}{\sqrt{a+bx+cx^2}}\right)}{4a^2d}$$

$$= -\frac{\sqrt{a+bx+cx^2}}{2adx^2} + \frac{3b\sqrt{a+bx+cx^2}}{4a^2dx} + \frac{e\sqrt{a+bx+cx^2}}{ad^2x} - \frac{(3b^2-4ac)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{5/2}d} - \frac{be\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{3/2}d^2} - \frac{(e^2-df)\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}d^3} + \frac{f\left(2e^3-4def-(e^2-df)\left(e-\sqrt{e^2-4df}\right)\right)\tanh^{-1}\left(\frac{4af-b\left(e-\sqrt{e^2-4df}\right)+2\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)x}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}\sqrt{a+bx+cx^2}}\right)}{\sqrt{2}d^3\sqrt{e^2-4df}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}} - \frac{f\left(2e^3-4def-(e^2-df)\left(e+\sqrt{e^2-4df}\right)\right)\tanh^{-1}\left(\frac{4af-b\left(e+\sqrt{e^2-4df}\right)+2\left(bf-c\left(e+\sqrt{e^2-4df}\right)\right)x}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2-4df}}\sqrt{a+bx+cx^2}}\right)}{\sqrt{2}d^3\sqrt{e^2-4df}\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2-4df}}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 1.22 (sec) , antiderivative size = 547, normalized size of antiderivative = 0.81

$$\int \frac{1}{x^3\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx$$

$$= \frac{\frac{d(-2ad+3bdx+4aex)\sqrt{a+x(b+cx)}}{a^2x^2} + \frac{(-3b^2d^2-4abde+4a(cd^2-2ae^2+2adf))\text{arctanh}\left(\frac{-\sqrt{c}x+\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)}{a^{5/2}} - 4\text{RootSum}\left[b^2d-ab\right.}{}$$

```
[In] Integrate[1/(x^3*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] ((d*(-2*a*d + 3*b*d*x + 4*a*e*x)*Sqrt[a + x*(b + c*x)])/(a^2*x^2) + ((-3*b^
2*d^2 - 4*a*b*d*e + 4*a*(c*d^2 - 2*a*e^2 + 2*a*d*f))*ArcTanh[(-(Sqrt[c]*x)
+ Sqrt[a + x*(b + c*x)])/Sqrt[a]])/a^(5/2) - 4*RootSum[b^2*d - a*b*e + a^2*
f - 4*b*Sqrt[c]*d*#1 + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 + b*e*#1^2 - 2*a*f*#1^
2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (b*e^3*Log[-(Sqrt[c]*x) + Sqrt[a + b*x +
c*x^2] - #1] - 2*b*d*e*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - a
*e^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + a*d*f^2*Log[-(Sqrt[
c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*Sqrt[c]*e^3*Log[-(Sqrt[c]*x) + Sqrt
[a + b*x + c*x^2] - #1]*#1 + 4*Sqrt[c]*d*e*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*
x + c*x^2] - #1]*#1 + e^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*
#1^2 - d*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2)/(-2*b*Sqr
t[c]*d + a*Sqrt[c]*e + 4*c*d*#1 + b*e*#1 - 2*a*f*#1 - 3*Sqrt[c]*e*#1^2 + 2*
f*#1^3) & ])/(4*d^3)
```

Maple [A] (verified)

Time = 1.15 (sec) , antiderivative size = 991, normalized size of antiderivative = 1.46

method	result
risch	$-\frac{\sqrt{c\,x^2+bx+a}\,(-4aex-3bdx+2ad)}{4a^2d^2x^2}+\frac{-\frac{4f\left(8a^2df-8e^2a^2-4abde+4c\,d^2a-3b^2d^2\right)\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)}{\left(-e+\sqrt{-4df+e^2}\right)\left(e+\sqrt{-4df+e^2}\right)\sqrt{a}}+\frac{8f\,a^2\left(e\sqrt{-4df+e^2}+2\cdots\right.}{\cdots}}{\cdots}$
default	Expression too large to display

```
[In] int(1/x^3/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(c*x^2+b*x+a)^(1/2)*(-4*a*e*x-3*b*d*x+2*a*d)/a^2/d^2/x^2+1/8/d^2/a^2*(
-4*f*(8*a^2*d*f-8*a^2*e^2-4*a*b*d*e+4*a*c*d^2-3*b^2*d^2)/(-e+(-4*d*f+e^2)^(
1/2))/(e+(-4*d*f+e^2)^(1/2))/a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1
/2))/x)+8*f*a^2*(e*(-4*d*f+e^2)^(1/2)+2*d*f-e^2)/(-4*d*f+e^2)^(1/2)/(e+(-4*
d*f+e^2)^(1/2))*2^(1/2)/((-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*
a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+
e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+1/f*(-c*(-4*d*f+e^2)^(1/2)+
b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-b*f*(-4*d*f+e^2)^(
1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1
/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2
*(e+(-4*d*f+e^2)^(1/2))/f)+2*(-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*
e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)
)-8*f*a^2*(e^2-2*d*f+e*(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)/(-e+(-4*d*f+e
```

```
^2)^(1/2))*2^(1/2)/((b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-
b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1
/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(
x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+1/2*2^(1/2)*((b*f*(-4*d*f+e^2)^(1/2)-(-4*d
*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2/f*(-e+(
-4*d*f+e^2)^(1/2)))^2*c+4*(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2/f*(-e+(-4
*d*f+e^2)^(1/2)))+2*(b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-
b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = \text{Timed out}$$

```
[In] integrate(1/x^3/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{x^3\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = \int \frac{1}{x^3\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx$$

```
[In] integrate(1/x**3/(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)
[Out] Integral(1/(x**3*sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{x^3\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = \int \frac{1}{\sqrt{cx^2+bx+a}(fx^2+ex+d)x^3}\,dx$$

```
[In] integrate(1/x^3/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*(f*x^2 + e*x + d)*x^3), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = \text{Timed out}$$

```
[In] integrate(1/x^3/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = \int \frac{1}{x^3\,\sqrt{c\,x^2+b\,x+a}\,\left(f\,x^2+e\,x+d\right)}\,dx$$

```
[In] int(1/(x^3*(a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)),x)
[Out] int(1/(x^3*(a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)), x)
```

3.121 $\int \frac{x^3}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 779

$$\int \frac{x^3}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}\,dx = \frac{2(2a+bx)}{(b^2-4ac)f\sqrt{a+bx+cx^2}}$$
$$+\frac{2e(b+2cx)}{(b^2-4ac)f^2\sqrt{a+bx+cx^2}}$$
$$+\frac{2(cde(bcd-2ace+abf)-(bde-ae^2+adf)(2c^2d+b^2f-c(be+2af))+c((bcd-2ace+abf)(e^2-df)\cdots}{(b^2-4ac)f^2((cd-af)^2-(bd-ae)(ce-bf))\sqrt{a+bx+cx^2}}$$
$$+\frac{\left(2d(bd-ae)f+\left(e-\sqrt{e^2-4df}\right)(cd^2-bde+a(e^2-df))\right)\operatorname{arctanh}\left(\frac{4af-b\left(e-\sqrt{e^2-4df}\right)+2\left(bf-c\left(e-\sqrt{e^2-4df}\cdots\right.\right.}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}\sqrt{a\cdots}}\right.}{\sqrt{2}\sqrt{e^2-4df}((cd-af)^2-(bd-ae)(ce-bf))\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}\cdots}$$
$$-\frac{\left(2d(bd-ae)f+\left(e+\sqrt{e^2-4df}\right)(cd^2-bde+a(e^2-df))\right)\operatorname{arctanh}\left(\frac{4af-b\left(e+\sqrt{e^2-4df}\right)+2\left(bf-c\left(e+\sqrt{e^2-4df}\cdots\right.\right.}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2-4df}}\sqrt{\cdots}}\right.}{\sqrt{2}\sqrt{e^2-4df}((cd-af)^2-(bd-ae)(ce-bf))\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2-4df}}\cdots}$$

```
[Out] 2*(b*x+2*a)/(-4*a*c+b^2)/f/(c*x^2+b*x+a)^(1/2)+2*e*(2*c*x+b)/(-4*a*c+b^2)/f
^2/(c*x^2+b*x+a)^(1/2)+2*(c*d*e*(a*b*f-2*a*c*e+b*c*d)-(a*d*f-a*e^2+b*d*e)*(
2*c^2*d+b^2*f-c*(2*a*f+b*e))+c*((a*b*f-2*a*c*e+b*c*d)*(-d*f+e^2)-d*e*(2*c^2
*d+b^2*f-c*(2*a*f+b*e)))*x)/(-4*a*c+b^2)/f^2/((-a*f+c*d)^2-(-a*e+b*d)*(-b*f
+c*e))/(c*x^2+b*x+a)^(1/2)+1/2*arctanh(1/4*(4*a*f+2*x*(b*f-c*(e-(-4*d*f+e^2
)^(1/2)))-b*(e-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*
d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2))*(2*d*(-a*e+b*d)*f+(
c*d^2-b*d*e+a*(-d*f+e^2))*(e-(-4*d*f+e^2)^(1/2)))/((-a*f+c*d)^2-(-a*e+b*d)*
(-b*f+c*e))*2^(1/2)/(-4*d*f+e^2)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c
*e)*(-4*d*f+e^2)^(1/2))^(1/2)-1/2*arctanh(1/4*(4*a*f-b*(e+(-4*d*f+e^2)^(1/2
))+2*x*(b*f-c*(e+(-4*d*f+e^2)^(1/2))))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2
*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2))*(2*d*(-a*e+b*d)*
```

```
f+(c*d^2-b*d*e+a*(-d*f+e^2))*(e+(-4*d*f+e^2)^(1/2)))/((-a*f+c*d)^2-(-a*e+b*
d)*(-b*f+c*e))*2^(1/2)/(-4*d*f+e^2)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*
f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 8.63 (sec) , antiderivative size = 779, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {6860, 627, 650, 1030, 1046, 738, 212}

$$\int \frac{x^3}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}\,dx = \frac{\left(\left(e-\sqrt{e^2-4df}\right)\left(a(e^2-df)-bde+cd^2\right)+2df(bd-ae)\right)\text{arctan}}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{2af^2}}$$

$$-\frac{\left(\left(\sqrt{e^2-4df}+e\right)\left(a(e^2-df)-bde+cd^2\right)+2df(bd-ae)\right)\text{arctanh}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4}\right.}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef}}\right.}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+c}}$$

$$+\frac{2(cx((e^2-df)(abf-2ace+bcd)-de(-c(2af+be)+b^2f+2c^2d))-(adf-ae^2+bde)(-c(2af+be)}{f^2(b^2-4ac)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}$$

$$+\frac{2e(b+2cx)}{f^2(b^2-4ac)\sqrt{a+bx+cx^2}}+\frac{2(2a+bx)}{f(b^2-4ac)\sqrt{a+bx+cx^2}}$$

```
[In] Int[x^3/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] (2*(2*a + b*x))/((b^2 - 4*a*c)*f*Sqrt[a + b*x + c*x^2]) + (2*e*(b + 2*c*x))
/((b^2 - 4*a*c)*f^2*Sqrt[a + b*x + c*x^2]) + (2*(c*d*e*(b*c*d - 2*a*c*e + a
*b*f) - (b*d*e - a*e^2 + a*d*f)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*((b
*c*d - 2*a*c*e + a*b*f)*(e^2 - d*f) - d*e*(2*c^2*d + b^2*f - c*(b*e + 2*a*f
)))*x))/((b^2 - 4*a*c)*f^2*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[a
 + b*x + c*x^2]) + ((2*d*(b*d - a*e)*f + (e - Sqrt[e^2 - 4*d*f])*(c*d^2 - b
*d*e + a*(e^2 - d*f)))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f
- c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2
*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])]])/(Sqrt[2]*S
qrt[e^2 - 4*d*f]*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[c*e^2 - 2*c
*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]) - ((2*d*(b*d - a*e
)*f + (e + Sqrt[e^2 - 4*d*f])*(c*d^2 - b*d*e + a*(e^2 - d*f)))*ArcTanh[(4*a
*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*
Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d
*f]]*Sqrt[a + b*x + c*x^2])]])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*((c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f))*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f
)*Sqrt[e^2 - 4*d*f]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 627

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[-2*((b*d - 2*a*e + (2*c*d - b*e)*x)/((b^2 - 4*a*c)*Sqrt[a + b*x
+ c*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1030

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x + c*x^2)^(p + 1)*
((d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
 - b*f))*(p + 1)))*(g*c*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d +
b^2*f - c*(b*e + 2*a*f)) + c*(g*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - h*(b*
c*d - 2*a*c*e + a*b*f))*x), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*
d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*
x^2)^q*Simp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)
 + (b^2*(g*f) - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*((-h)*c*
e)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((g*c)*(2*a*c*e - b*(c*d + a*f)) + (g*
b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((g*c)*(2*
a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(
p + q + 2) - (b^2*g*f - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*
((-h)*c*e)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(g*f) - b*(h*c
*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) + a*h*c*e))*(2*p + 2*q + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e
 - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
```

Rule 1046

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
```

```
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 6860

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \int \left(-\frac{e}{f^2\left(a+bx+cx^2\right)^{3/2}}+\frac{x}{f\left(a+bx+cx^2\right)^{3/2}}+\frac{de+\left(e^2-df\right)x}{f^2\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\right)dx$$

$$=\frac{\int \frac{de+\left(e^2-df\right)x}{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx}{f^2}-\frac{e\int \frac{1}{\left(a+bx+cx^2\right)^{3/2}}\,dx}{f^2}+\frac{\int \frac{x}{\left(a+bx+cx^2\right)^{3/2}}\,dx}{f}$$

$$=\frac{2(2a+bx)}{\left(b^2-4ac\right)f\sqrt{a+bx+cx^2}}+\frac{2e(b+2cx)}{\left(b^2-4ac\right)f^2\sqrt{a+bx+cx^2}}+\frac{2(cde(bcd-2ace+abf)-\left(bde-ae^2+adf\right)\left(2c^2d+b^2f-c(be+2af)\right)+c((bcd-2ace+ab}{\left(b^2-4ac\right)f^2\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{a+b}}-\frac{2\int \frac{\frac{1}{2}\left(b^2-4ac\right)d(bd-ae)f^2-\frac{1}{2}\left(b^2-4ac\right)f^2\left(cd^2-bde+ae^2-adf\right)x}{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}\,dx}{\left(b^2-4ac\right)f^2\left((cd-af)^2-(bd-ae)(ce-bf)\right)}$$

$$=\frac{2(2a+bx)}{\left(b^2-4ac\right)f\sqrt{a+bx+cx^2}}+\frac{2e(b+2cx)}{\left(b^2-4ac\right)f^2\sqrt{a+bx+cx^2}}+\frac{2(cde(bcd-2ace+abf)-\left(bde-ae^2+adf\right)\left(2c^2d+b^2f-c(be+2af)\right)+c((bcd-2ace+ab}{\left(b^2-4ac\right)f^2\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{a+b}}-\frac{\left(2d(bd-ae)f+\left(e-\sqrt{e^2-4df}\right)\left(cd^2-bde+a\left(e^2-df\right)\right)\right)\int \frac{1}{\left(e-\sqrt{e^2-4df}+2fx\right)\sqrt{a+bx+cx^2}}\,dx}{\sqrt{e^2-4df}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}+\frac{\left(2d(bd-ae)f+\left(e+\sqrt{e^2-4df}\right)\left(cd^2-bde+a\left(e^2-df\right)\right)\right)\int \frac{1}{\left(e+\sqrt{e^2-4df}+2fx\right)\sqrt{a+bx+cx^2}}\,dx}{\sqrt{e^2-4df}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}$$

$$= \frac{2(2a+bx)}{(b^2-4ac) f\sqrt{a+bx+cx^2}} + \frac{2e(b+2cx)}{(b^2-4ac) f^2\sqrt{a+bx+cx^2}}$$
$$+ \frac{2(cde(bcd-2ace+abf)-(bde-ae^2+adf)(2c^2d+b^2f-c(be+2af))+c((bcd-2ace+abf)}{(b^2-4ac) f^2((cd-af)^2-(bd-ae)(ce-bf))\sqrt{a+bx}}$$
$$+ \frac{\left(2\left(2d(bd-ae)f+\left(e-\sqrt{e^2-4df}\right)(cd^2-bde+a(e^2-df))\right)\right)\text{Subst}\left(\int \frac{1}{16af^2-8bf\left(e-\sqrt{e^2-4df}\right)+4c}\right.}{\sqrt{e^2-4df}((cd-af)^2-(bd-ae)(ce-}$$
$$- \frac{\left(2\left(2d(bd-ae)f+\left(e+\sqrt{e^2-4df}\right)(cd^2-bde+a(e^2-df))\right)\right)\text{Subst}\left(\int \frac{1}{16af^2-8bf\left(e+\sqrt{e^2-4df}\right)+4c}\right.}{\sqrt{e^2-4df}((cd-af)^2-(bd-ae)(ce-}$$

$$= \frac{2(2a+bx)}{(b^2-4ac) f\sqrt{a+bx+cx^2}} + \frac{2e(b+2cx)}{(b^2-4ac) f^2\sqrt{a+bx+cx^2}}$$
$$+ \frac{2(cde(bcd-2ace+abf)-(bde-ae^2+adf)(2c^2d+b^2f-c(be+2af))+c((bcd-2ace+abf)}{(b^2-4ac) f^2((cd-af)^2-(bd-ae)(ce-bf))\sqrt{a+bx}}$$
$$+ \frac{\left(2d(bd-ae)f+\left(e-\sqrt{e^2-4df}\right)(cd^2-bde+a(e^2-df))\right)\tanh^{-1}\left(\frac{4af-b\left(e-\sqrt{e^2-4df}\right)+2\left(bf-c\right.}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2-(ce-b}}\right.}{\sqrt{2}\sqrt{e^2-4df}((cd-af)^2-(bd-ae)(ce-bf))\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)}}$$
$$- \frac{\left(2d(bd-ae)f+\left(e+\sqrt{e^2-4df}\right)(cd^2-bde+a(e^2-df))\right)\tanh^{-1}\left(\frac{4af-b\left(e+\sqrt{e^2-4df}\right)+2\left(bf-c\right.}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2+(ce-b}}\right.}{\sqrt{2}\sqrt{e^2-4df}((cd-af)^2-(bd-ae)(ce-bf))\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 1.13 (sec) , antiderivative size = 690, normalized size of antiderivative = 0.89

$$\int \frac{x^3}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}\,dx = \frac{2(2a^3f+b^3dx+ab(bd-3cdx-bex)+a^2(-2cd-be+2cex+bf}{}$$

```
[In] Integrate[x^3/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] (2*(2*a^3*f + b^3*d*x + a*b*(b*d - 3*c*d*x - b*e*x) + a^2*(-2*c*d - b*e + 2
*c*e*x + b*f*x)) + (b^2 - 4*a*c)*Sqrt[a + x*(b + c*x)]*RootSum[b^2*d - a*b*
e + a^2*f - 4*b*Sqrt[c]*d*#1 + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 + b*e*#1^2 - 2
*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (b^2*d^2*Log[-(Sqrt[c]*x) + Sqrt[
a + b*x + c*x^2] - #1] + a*c*d^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] -
 #1] - 2*a*b*d*e*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + a^2*e^2*L
og[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - a^2*d*f*Log[-(Sqrt[c]*x) +
Sqrt[a + b*x + c*x^2] - #1] - 2*b*Sqrt[c]*d^2*Log[-(Sqrt[c]*x) + Sqrt[a + b
```

```
*x + c*x^2] - #1]*#1 + 2*a*Sqrt[c]*d*e*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*
x^2] - #1]*#1 - c*d^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 +
 b*d*e*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 - a*e^2*Log[-(Sq
rt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 + a*d*f*Log[-(Sqrt[c]*x) + Sqrt
[a + b*x + c*x^2] - #1]*#1^2)/(2*b*Sqrt[c]*d - a*Sqrt[c]*e - 4*c*d*#1 - b*e
*#1 + 2*a*f*#1 + 3*Sqrt[c]*e*#1^2 - 2*f*#1^3) & ])/((b^2 - 4*a*c)*(c^2*d^2
- b*c*d*e + f*(b^2*d - a*b*e + a^2*f) + a*c*(e^2 - 2*d*f))*Sqrt[a + x*(b +
c*x)])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2082 vs. $2(726) = 1452$.

Time = 1.00 (sec) , antiderivative size = 2083, normalized size of antiderivative = 2.67

method	result	size
default	Expression too large to display	2083

```
[In] int(x^3/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-1/c/(c*x^2+b*x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)
)-2*e/f^2*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+1/2*(e^3-3*d*e*f+e^2*(-
4*d*f+e^2)^(1/2)-d*f*(-4*d*f+e^2)^(1/2))/f^3/(-4*d*f+e^2)^(1/2)*(2/(-b*f*(-
4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)*f^2/((
x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+
1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/
2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)-2*f*(-c*(-4*d*f+e^2)^(1/2)+b
*f-c*e)/(-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d
*f+c*e^2)*(2*c*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/f*(-c*(-4*d*f+e^2)^(1/2)+
b*f-c*e))/(2*c*(-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*
f-2*c*d*f+c*e^2)/f^2-1/f^2*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)^2)/((x+1/2*(e+(-
4*d*f+e^2)^(1/2))/f)^2*c+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*
d*f+e^2)^(1/2))/f)+1/2*(-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*
f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)-2/(-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)
^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)*f^2*2^(1/2)/((-b*f*(-4*d*f+e^2)^(1/
2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-b*f
*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2
+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2
^(1/2)*((-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d
*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4/f*(-c*(-4*d*
f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*(-b*f*(-4*d*f+e^2)
^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1
/2*(e+(-4*d*f+e^2)^(1/2))/f)))+1/2*(-d*f*(-4*d*f+e^2)^(1/2)+e^2*(-4*d*f+e^2
)^(1/2)+3*d*e*f-e^3)/f^3/(-4*d*f+e^2)^(1/2)*(2/(b*f*(-4*d*f+e^2)^(1/2)-(-4*
d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)*f^2/((x-1/2/f*(-e+(-4*d*f+e
^2)^(1/2)))^2*c+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2/f*(-e+(-4*d*f+e^2)^
```

```
(1/2))+1/2*(b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*
c*d*f+c*e^2)/f^2)^(1/2)-2*(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*f/(b*f*(-4*d*f+e^2
)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)*(2*c*(x-1/2/f*(
-e+(-4*d*f+e^2)^(1/2)))+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f)/(2*c*(b*f*(-4*d*f
+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2-(c*(-4*
d*f+e^2)^(1/2)+b*f-c*e)^2/f^2)/((x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2*c+(c*(-
4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+1/2*(b*f*(-4*
d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/
2)-2/(b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c
*e^2)*f^2*2^(1/2)/((b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b
*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/
2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x
-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+1/2*2^(1/2)*((b*f*(-4*d*f+e^2)^(1/2)-(-4*d*
f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2/f*(-e+(-
4*d*f+e^2)^(1/2)))^2*c+4*(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2/f*(-e+(-4*
d*f+e^2)^(1/2)))+2*(b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b
*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^3}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}\,dx = \text{Timed out}$$

```
[In] integrate(x^3/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}\,dx = \text{Timed out}$$

```
[In] integrate(x**3/(c*x**2+b*x+a)**(3/2)/(f*x**2+e*x+d),x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{(a+bx+cx^2)^{3/2}\,(d+ex+fx^2)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^3/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re deta
```

Giac [F(-1)]

Timed out.

$$\int \frac{x^3}{(a+bx+cx^2)^{3/2}\,(d+ex+fx^2)}\,dx = \text{Timed out}$$

```
[In] integrate(x^3/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(a+bx+cx^2)^{3/2}\,(d+ex+fx^2)}\,dx = \int \frac{x^3}{(c\,x^2+b\,x+a)^{3/2}\,(f\,x^2+e\,x+d)}\,dx$$

```
[In] int(x^3/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x)
```

```
[Out] int(x^3/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)), x)
```

3.122 $\int \frac{x^2}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 609

$$\begin{aligned}
&\int \frac{x^2}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}\,dx = \\
&-\frac{2(a(bcd-2ace+abf)+c(b^2d-abe-2a(cd-af))\,x)}{(b^2-4ac)\,((cd-af)^2-(bd-ae)(ce-bf))\,\sqrt{a+bx+cx^2}} \\
&-\frac{f\left(2d(cd-af)-(bd-ae)\left(e-\sqrt{e^2-4df}\right)\right)\operatorname{arctanh}\left(\frac{4af-b\left(e-\sqrt{e^2-4df}\right)+2\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)x}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}\sqrt{a+bx+cx^2}}\right)}{\sqrt{2}\sqrt{e^2-4df}\,((cd-af)^2-(bd-ae)(ce-bf))\,\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}} \\
&+\frac{f\left(2d(cd-af)-(bd-ae)\left(e+\sqrt{e^2-4df}\right)\right)\operatorname{arctanh}\left(\frac{4af-b\left(e+\sqrt{e^2-4df}\right)+2\left(bf-c\left(e+\sqrt{e^2-4df}\right)\right)x}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2-4df}}\sqrt{a+bx+cx^2}}\right)}{\sqrt{2}\sqrt{e^2-4df}\,((cd-af)^2-(bd-ae)(ce-bf))\,\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2-4df}}}
\end{aligned}$$

```
[Out] -2*(a*(a*b*f-2*a*c*e+b*c*d)+c*(b^2*d-a*b*e-2*a*(-a*f+c*d))*x)/(-4*a*c+b^2)/
((-a*f+c*d)^2-(-a*e+b*d)*(-b*f+c*e))/(c*x^2+b*x+a)^(1/2)-1/2*f*arctanh(1/4*
(4*a*f+2*x*(b*f-c*(e-(-4*d*f+e^2)^(1/2)))-b*(e-(-4*d*f+e^2)^(1/2)))*2^(1/2)
/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(
1/2))^(1/2))*(2*d*(-a*f+c*d)-(-a*e+b*d)*(e-(-4*d*f+e^2)^(1/2)))/((-a*f+c*d)
^2-(-a*e+b*d)*(-b*f+c*e))*2^(1/2)/(-4*d*f+e^2)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2
*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)+1/2*f*arctanh(1/4*(4*a*f-b*(e+(
-4*d*f+e^2)^(1/2))+2*x*(b*f-c*(e+(-4*d*f+e^2)^(1/2))))*2^(1/2)/(c*x^2+b*x+a
)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2))*
(2*d*(-a*f+c*d)-(-a*e+b*d)*(e+(-4*d*f+e^2)^(1/2)))/((-a*f+c*d)^2-(-a*e+b*d)
*(-b*f+c*e))*2^(1/2)/(-4*d*f+e^2)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+
c*e)*(-4*d*f+e^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 3.78 (sec) , antiderivative size = 609, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1075, 1046, 738, 212}

$$\int \frac{x^2}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}\,dx = -\frac{f\left(2d(cd-af)-\left(e-\sqrt{e^2-4df}\right)(bd-ae)\right)\operatorname{arctanh}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}} + \frac{f\left(2d(cd-af)-\left(\sqrt{e^2-4df}+e\right)(bd-ae)\right)\operatorname{arctanh}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}} - \frac{2(cx(-abe-2a(cd-af)+b^2d)+a(abf-2ace+bcd))}{(b^2-4ac)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}$$

```
[In] Int[x^2/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] (-2*(a*(b*c*d - 2*a*c*e + a*b*f) + c*(b^2*d - a*b*e - 2*a*(c*d - a*f))*x))/
((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[a + b*x + c*x
^2]) - (f*(2*d*(c*d - a*f) - (b*d - a*e)*(e - Sqrt[e^2 - 4*d*f]))*ArcTanh[(
4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/
(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 -
4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*((c*d - a*f)^2
- (b*d - a*e)*(c*e - b*f))*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e -
b*f)*Sqrt[e^2 - 4*d*f]]) + (f*(2*d*(c*d - a*f) - (b*d - a*e)*(e + Sqrt[e^2
- 4*d*f]))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqr
t[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*
e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*
d*f]*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[c*e^2 - 2*c*d*f - b*e*f
 + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1046

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1075

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (C_.)*(x_)^2)*((d_) +
(e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x + c*x^2)^(p + 1
)*((d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c
*e - b*f))*(p + 1)))*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b)*(2*c^2*
d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + C
*(b^2*d - a*b*e - 2*a*(c*d - a*f)))*x), x] + Dist[1/((b^2 - 4*a*c)*((c*d -
a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(
d + e*x + f*x^2)^q*Simp[(-2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1) + (b^2*(C*d + A*f) - b*((Plus[A])*c*e + a*C*e) + 2*(A*c*(c*
d - a*f) - a*(c*C*d - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)
*(2*a*c*e - b*(c*d + a*f)) + (A*b)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
+ q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b)*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(A*c*e + a*C*e)
 + 2*(A*c*(c*d - a*f) - a*(c*C*d - a*C*f)))*(b*f*(p + 1) - c*e*(2*p + q + 4
)))*x - c*f*(b^2*(C*d + A*f) - b*(A*c*e + a*C*e) + 2*(A*c*(c*d - a*f) - a*(
c*C*d - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, C, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1]
&& NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && I
LtQ[q, -1]) &&  !IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\frac{2(a(bcd-2ace+abf)+c(b^2d-abe-2a(cd-af))\,x)}{(b^2-4ac)\,((cd-af)^2-(bd-ae)(ce-bf))\,\sqrt{a+bx+cx^2}} \\ &\quad -\frac{2\int \frac{-\frac{1}{2}(b^2-4ac)d(cd-af)-\frac{1}{2}(b^2-4ac)(bd-ae)fx}{\sqrt{a+bx+cx^2}(d+ex+fx^2)}\,dx}{(b^2-4ac)\,((cd-af)^2-(bd-ae)(ce-bf))} \\ &= -\frac{2(a(bcd-2ace+abf)+c(b^2d-abe-2a(cd-af))\,x)}{(b^2-4ac)\,((cd-af)^2-(bd-ae)(ce-bf))\,\sqrt{a+bx+cx^2}} \\ &\quad +\frac{\left(f\left(2d(cd-af)-(bd-ae)\left(e-\sqrt{e^2-4df}\right)\right)\right)\int \frac{1}{\left(e-\sqrt{e^2-4df}+2fx\right)\sqrt{a+bx+cx^2}}\,dx}{\sqrt{e^2-4df}\,((cd-af)^2-(bd-ae)(ce-bf))} \\ &\quad -\frac{\left(f\left(2d(cd-af)-(bd-ae)\left(e+\sqrt{e^2-4df}\right)\right)\right)\int \frac{1}{\left(e+\sqrt{e^2-4df}+2fx\right)\sqrt{a+bx+cx^2}}\,dx}{\sqrt{e^2-4df}\,((cd-af)^2-(bd-ae)(ce-bf))}\end{aligned}$$

$$= -\frac{2(a(bcd-2ace+abf)+c(b^2d-abe-2a(cd-af))x)}{(b^2-4ac)((cd-af)^2-(bd-ae)(ce-bf))\sqrt{a+bx+cx^2}}$$
$$-\frac{\left(2f\left(2d(cd-af)-(bd-ae)\left(e-\sqrt{e^2-4df}\right)\right)\right)\operatorname{Subst}\left(\int\frac{1}{16af^2-8bf\left(e-\sqrt{e^2-4df}\right)+4c\left(e-\sqrt{e^2-4df}\right)^2-}\right.}{\sqrt{e^2-4df}\,((cd-af)^2-(bd-ae)(ce-bf))}$$
$$+\frac{\left(2f\left(2d(cd-af)-(bd-ae)\left(e+\sqrt{e^2-4df}\right)\right)\right)\operatorname{Subst}\left(\int\frac{1}{16af^2-8bf\left(e+\sqrt{e^2-4df}\right)+4c\left(e+\sqrt{e^2-4df}\right)^2-}\right.}{\sqrt{e^2-4df}\,((cd-af)^2-(bd-ae)(ce-bf))}$$
$$= -\frac{2(a(bcd-2ace+abf)+c(b^2d-abe-2a(cd-af))x)}{(b^2-4ac)((cd-af)^2-(bd-ae)(ce-bf))\sqrt{a+bx+cx^2}}$$
$$-\frac{f\left(2d(cd-af)-(bd-ae)\left(e-\sqrt{e^2-4df}\right)\right)\tanh^{-1}\left(\frac{4af-b\left(e-\sqrt{e^2-4df}\right)+2\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)x}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}\sqrt{a+bx}}\right.}{\sqrt{2}\sqrt{e^2-4df}\,((cd-af)^2-(bd-ae)(ce-bf))\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-}}}$$
$$+\frac{f\left(2d(cd-af)-(bd-ae)\left(e+\sqrt{e^2-4df}\right)\right)\tanh^{-1}\left(\frac{4af-b\left(e+\sqrt{e^2-4df}\right)+2\left(bf-c\left(e+\sqrt{e^2-4df}\right)\right)x}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2-4df}}\sqrt{a+bx}}\right.}{\sqrt{2}\sqrt{e^2-4df}\,((cd-af)^2-(bd-ae)(ce-bf))\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2-}}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.90 (sec) , antiderivative size = 534, normalized size of antiderivative = 0.88

$$\int\frac{x^2}{(a+bx+cx^2)^{3/2}\,(d+ex+fx^2)}\,dx = \frac{-2(b^2cdx+ac(bd-2cdx-bex)+a^2(-2ce+bf+2cfx))-(b^2}{}$$

```
[In] Integrate[x^2/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
[Out] (-2*(b^2*c*d*x + a*c*(b*d - 2*c*d*x - b*e*x) + a^2*(-2*c*e + b*f + 2*c*f*x)
) - (b^2 - 4*a*c)*Sqrt[a + x*(b + c*x)]*RootSum[b^2*d - a*b*e + a^2*f - 4*b
*Sqrt[c]*d*#1 + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 + b*e*#1^2 - 2*a*f*#1^2 - 2*S
qrt[c]*e*#1^3 + f*#1^4 & , (b*c*d^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2
] - #1] - 2*a*b*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + a^2*e*
f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*c^(3/2)*d^2*Log[-(Sqrt
[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + 2*a*Sqrt[c]*d*f*Log[-(Sqrt[c]*x)
+ Sqrt[a + b*x + c*x^2] - #1]*#1 + b*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x +
c*x^2] - #1]*#1^2 - a*e*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1
^2)/(2*b*Sqrt[c]*d - a*Sqrt[c]*e - 4*c*d*#1 - b*e*#1 + 2*a*f*#1 + 3*Sqrt[c]
*e*#1^2 - 2*f*#1^3) & ])/((b^2 - 4*a*c)*(c^2*d^2 - b*c*d*e + f*(b^2*d - a*b
*e + a^2*f) + a*c*(e^2 - 2*d*f))*Sqrt[a + x*(b + c*x)])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1991 vs. 2(560) = 1120.

Time = 0.86 (sec) , antiderivative size = 1992, normalized size of antiderivative = 3.27

method	result	size
default	Expression too large to display	1992

```
[In] int(x^2/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+1/2*(-e*(-4*d*f+e^2)^(1/2)+2*
d*f-e^2)/f^2/(-4*d*f+e^2)^(1/2)*(2/(-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1
/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)*f^2/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^
2*c+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/
2*(-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e
^2)/f^2)^(1/2)-2*f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/(-b*f*(-4*d*f+e^2)^(1/2)
+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)*(2*c*(x+1/2*(e+(-4*d*f
+e^2)^(1/2))/f)+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e))/(2*c*(-b*f*(-4*d*f+e^2
)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2-1/f^2*(-c*(
-4*d*f+e^2)^(1/2)+b*f-c*e)^2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+1/f*(-c
*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-b*f*(-4
*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1
/2)-2/(-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f
+c*e^2)*f^2*2^(1/2)/((-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^
2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)
^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-
c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-b*f*(-4*d*f+e^2)^(1/2)
+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(
e+(-4*d*f+e^2)^(1/2))/f)^2*c+4/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+
(-4*d*f+e^2)^(1/2))/f)+2*(-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*
a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)))+1
/2*(e^2-2*d*f-e*(-4*d*f+e^2)^(1/2))/f^2/(-4*d*f+e^2)^(1/2)*(2/(b*f*(-4*d*f+
e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)*f^2/((x-1/2/
f*(-e+(-4*d*f+e^2)^(1/2)))^2*c+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2/f*(-
e+(-4*d*f+e^2)^(1/2)))+1/2*(b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2
*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)-2*(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*f/(
b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)*
(2*c*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f)/(2
*c*(b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e
^2)/f^2-(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)^2/f^2)/((x-1/2/f*(-e+(-4*d*f+e^2)^(1
/2)))^2*c+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2))
)+1/2*(b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+
c*e^2)/f^2)^(1/2)-2/(b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-
b*e*f-2*c*d*f+c*e^2)*f^2*2^(1/2)/((b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2
)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((b*f*(-4*d*f+e^2)^(1/2)-(
```

```
-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+(c*(-4*d*f+e^2)^(1/2
)+b*f-c*e)/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+1/2*2^(1/2)*((b*f*(-4*d*f+e^
2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*
(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2*c+4*(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x
-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+2*(b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2
)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2/f*(-e+(-4*d*f+e^2)^(1
/2)))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}\,dx = \text{Timed out}$$

```
[In] integrate(x^2/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}\,dx = \text{Timed out}$$

```
[In] integrate(x**2/(c*x**2+b*x+a)**(3/2)/(f*x**2+e*x+d),x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x^2/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re deta
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^2}{(a+bx+cx^2)^{3/2}\,(d+ex+fx^2)}\,dx = \text{Exception raised: AttributeError}$$

```
[In] integrate(x^2/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
[Out] Exception raised: AttributeError >> type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+bx+cx^2)^{3/2}\,(d+ex+fx^2)}\,dx = \int \frac{x^2}{(c\,x^2+b\,x+a)^{3/2}\,\left(f\,x^2+e\,x+d\right)}\,dx$$

```
[In] int(x^2/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x)
[Out] int(x^2/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)), x)
```

3.123 $\int \frac{x}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 609

$$\int \frac{x}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}\,dx = \frac{2(a(2c^2d-bce+b^2f-2acf)+c(bcd-2ace+abf)x)}{(b^2-4ac)\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{a+bx+cx^2}}$$
$$+\frac{f\left(2d(ce-bf)-(cd-af)\left(e-\sqrt{e^2-4df}\right)\right)\operatorname{arctanh}\left(\frac{4af-b\left(e-\sqrt{e^2-4df}\right)+2\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)x}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}\sqrt{a+bx+cx^2}}\right)}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}}$$
$$-\frac{f\left(2d(ce-bf)-(cd-af)\left(e+\sqrt{e^2-4df}\right)\right)\operatorname{arctanh}\left(\frac{4af-b\left(e+\sqrt{e^2-4df}\right)+2\left(bf-c\left(e+\sqrt{e^2-4df}\right)\right)x}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2-4df}}\sqrt{a+bx+cx^2}}\right)}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2-4df}}}$$

```
[Out] 2*(a*(-2*a*c*f+b^2*f-b*c*e+2*c^2*d)+c*(a*b*f-2*a*c*e+b*c*d)*x)/(-4*a*c+b^2)
/((-a*f+c*d)^2-(-a*e+b*d)*(-b*f+c*e))/(c*x^2+b*x+a)^(1/2)+1/2*f*arctanh(1/4
*(4*a*f+2*x*(b*f-c*(e-(-4*d*f+e^2)^(1/2)))-b*(e-(-4*d*f+e^2)^(1/2)))*2^(1/2
)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^
(1/2))^(1/2))*(2*d*(-b*f+c*e)-(-a*f+c*d)*(e-(-4*d*f+e^2)^(1/2)))/((-a*f+c*d
)^2-(-a*e+b*d)*(-b*f+c*e))*2^(1/2)/(-4*d*f+e^2)^(1/2)/(c*e^2-2*c*d*f-b*e*f+
2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)-1/2*f*arctanh(1/4*(4*a*f-b*(e+
(-4*d*f+e^2)^(1/2))+2*x*(b*f-c*(e+(-4*d*f+e^2)^(1/2))))*2^(1/2)/(c*x^2+b*x+
a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2))
*(2*d*(-b*f+c*e)-(-a*f+c*d)*(e+(-4*d*f+e^2)^(1/2)))/((-a*f+c*d)^2-(-a*e+b*d
)*(-b*f+c*e))*2^(1/2)/(-4*d*f+e^2)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f
+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)
```

Rubi [A] (verified)

Time = 3.40 (sec) , antiderivative size = 609, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1030, 1046, 738, 212}

$$\int \frac{x}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}dx = \frac{f\left(2d(ce-bf)-\left(e-\sqrt{e^2-4df}\right)(cd-af)\right)\operatorname{arctanh}\left(\frac{4af+2x\left(b\cdots\right.}{2\sqrt{2}\sqrt{a+bx+cx^2}\cdots}\right.}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{2af^2-\sqrt{e^2-\cdots}}} - \frac{f\left(2d(ce-bf)-\left(\sqrt{e^2-4df}+e\right)(cd-af)\right)\operatorname{arctanh}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}} + \frac{2(a(-2acf+b^2f-bce+2c^2d)+cx(abf-2ace+bcd))}{(b^2-4ac)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}$$

```
[In] Int[x/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] (2*(a*(2*c^2*d - b*c*e + b^2*f - 2*a*c*f) + c*(b*c*d - 2*a*c*e + a*b*f)*x))
/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[a + b*x + c*
x^2]) + (f*(2*d*(c*e - b*f) - (c*d - a*f)*(e - Sqrt[e^2 - 4*d*f]))*ArcTanh[
(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)
/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 -
 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*((c*d - a*f)^2
 - (b*d - a*e)*(c*e - b*f))*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e -
 b*f)*Sqrt[e^2 - 4*d*f]]) - (f*(2*d*(c*e - b*f) - (c*d - a*f)*(e + Sqrt[e^2
 - 4*d*f]))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sq
rt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c
*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4
*d*f]*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[c*e^2 - 2*c*d*f - b*e*
f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1030

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x + c*x^2)^(p + 1)*
((d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
 - b*f))*(p + 1)))*(g*c*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d +
b^2*f - c*(b*e + 2*a*f)) + c*(g*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - h*(b*
c*d - 2*a*c*e + a*b*f))*x), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*
d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*
x^2)^q*Simp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)
 + (b^2*(g*f) - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*((-h)*c*
e)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((g*c)*(2*a*c*e - b*(c*d + a*f)) + (g*
b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((g*c)*(2*
a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(
p + q + 2) - (b^2*g*f - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*
((-h)*c*e)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(g*f) - b*(h*c
*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) + a*h*c*e))*(2*p + 2*q + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e
 - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
```

Rule 1046

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2(a(2c^2d-bce+b^2f-2acf)+c(bcd-2ace+abf)x)}{(b^2-4ac)\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{a+bx+cx^2}} \\
&\quad - \frac{2\int \frac{\frac{1}{2}(b^2-4ac)d(ce-bf)+\frac{1}{2}(b^2-4ac)f(cd-af)x}{\sqrt{a+bx+cx^2}(d+ex+fx^2)}\,dx}{(b^2-4ac)\left((cd-af)^2-(bd-ae)(ce-bf)\right)} \\
&= \frac{2(a(2c^2d-bce+b^2f-2acf)+c(bcd-2ace+abf)x)}{(b^2-4ac)\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{a+bx+cx^2}} \\
&\quad - \frac{\left(f\left(2d(ce-bf)-(cd-af)\left(e-\sqrt{e^2-4df}\right)\right)\right)\int \frac{1}{\left(e-\sqrt{e^2-4df}+2fx\right)\sqrt{a+bx+cx^2}}\,dx}{\sqrt{e^2-4df}\left((cd-af)^2-(bd-ae)(ce-bf)\right)} \\
&\quad + \frac{\left(f\left(2d(ce-bf)-(cd-af)\left(e+\sqrt{e^2-4df}\right)\right)\right)\int \frac{1}{\left(e+\sqrt{e^2-4df}+2fx\right)\sqrt{a+bx+cx^2}}\,dx}{\sqrt{e^2-4df}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}
\end{aligned}$$

$$= \frac{2(a(2c^2d - bce + b^2f - 2acf) + c(bcd - 2ace + abf)x)}{(b^2 - 4ac)((cd - af)^2 - (bd - ae)(ce - bf))\sqrt{a + bx + cx^2}}$$

$$+ \frac{\left(2f\left(2d(ce - bf) - (cd - af)\left(e - \sqrt{e^2 - 4df}\right)\right)\right)\text{Subst}\left(\int \frac{1}{16af^2 - 8bf\left(e - \sqrt{e^2-4df}\right) + 4c\left(e - \sqrt{e^2-4df}\right)^2 - x^2 \ldots}\right.}{\sqrt{e^2 - 4df}\left((cd - af)^2 - (bd - ae)(ce - bf)\right)}$$

$$- \frac{\left(2f\left(2d(ce - bf) - (cd - af)\left(e + \sqrt{e^2 - 4df}\right)\right)\right)\text{Subst}\left(\int \frac{1}{16af^2 - 8bf\left(e + \sqrt{e^2-4df}\right) + 4c\left(e + \sqrt{e^2-4df}\right)^2 - x^2 \ldots}\right.}{\sqrt{e^2 - 4df}\left((cd - af)^2 - (bd - ae)(ce - bf)\right)}$$

$$= \frac{2(a(2c^2d - bce + b^2f - 2acf) + c(bcd - 2ace + abf)x)}{(b^2 - 4ac)((cd - af)^2 - (bd - ae)(ce - bf))\sqrt{a + bx + cx^2}}$$

$$+ \frac{f\left(2d(ce - bf) - (cd - af)\left(e - \sqrt{e^2 - 4df}\right)\right)\tanh^{-1}\left(\frac{4af - b\left(e - \sqrt{e^2-4df}\right) + 2\left(bf - c\left(e - \sqrt{e^2-4df}\right)\right)x}{2\sqrt{2}\sqrt{ce^2 - 2cdf - bef + 2af^2 - (ce - bf)\sqrt{e^2-4df}}\sqrt{a + bx + \ldots}}\right.}{\sqrt{2}\sqrt{e^2 - 4df}\left((cd - af)^2 - (bd - ae)(ce - bf)\right)\sqrt{ce^2 - 2cdf - bef + 2af^2 - (ce - bf)\sqrt{e^2 - \ldots}}}$$

$$- \frac{f\left(2d(ce - bf) - (cd - af)\left(e + \sqrt{e^2 - 4df}\right)\right)\tanh^{-1}\left(\frac{4af - b\left(e + \sqrt{e^2-4df}\right) + 2\left(bf - c\left(e + \sqrt{e^2-4df}\right)\right)x}{2\sqrt{2}\sqrt{ce^2 - 2cdf - bef + 2af^2 + (ce - bf)\sqrt{e^2-4df}}\sqrt{a + bx + \ldots}}\right.}{\sqrt{2}\sqrt{e^2 - 4df}\left((cd - af)^2 - (bd - ae)(ce - bf)\right)\sqrt{ce^2 - 2cdf - bef + 2af^2 + (ce - bf)\sqrt{e^2 - \ldots}}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.85 (sec) , antiderivative size = 570, normalized size of antiderivative = 0.94

$$\int \frac{x}{(a + bx + cx^2)^{3/2}(d + ex + fx^2)}\,dx = \frac{-4a^2cf + 2bc^2dx + 2a(b^2f + 2c^2(d - ex) + bc(-e + fx)) - (b^2 - \ldots}{\ldots}$$

```
[In] Integrate[x/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] (-4*a^2*c*f + 2*b*c^2*d*x + 2*a*(b^2*f + 2*c^2*(d - e*x) + b*c*(-e + f*x))
- (b^2 - 4*a*c)*Sqrt[a + x*(b + c*x)]*RootSum[b^2*d - a*b*e + a^2*f - 4*b*S
qrt[c]*d*#1 + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 + b*e*#1^2 - 2*a*f*#1^2 - 2*Sqr
t[c]*e*#1^3 + f*#1^4 & , (-(b*c*d*e*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2
] - #1]) + b^2*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + a*c*d*f
*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - a^2*f^2*Log[-(Sqrt[c]*x)
+ Sqrt[a + b*x + c*x^2] - #1] + 2*c^(3/2)*d*e*Log[-(Sqrt[c]*x) + Sqrt[a + b
*x + c*x^2] - #1]*#1 - 2*b*Sqrt[c]*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*
x^2] - #1]*#1 - c*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 +
 a*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2)/(2*b*Sqrt[c]*d
- a*Sqrt[c]*e - 4*c*d*#1 - b*e*#1 + 2*a*f*#1 + 3*Sqrt[c]*e*#1^2 - 2*f*#1^3)
 & ])/((b^2 - 4*a*c)*(c^2*d^2 - b*c*d*e + f*(b^2*d - a*b*e + a^2*f) + a*c*(
e^2 - 2*d*f))*Sqrt[a + x*(b + c*x)])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1938 vs. 2(560) = 1120.

Time = 0.86 (sec) , antiderivative size = 1939, normalized size of antiderivative = 3.18

method	result	size
default	Expression too large to display	1939

```
[In] int(x/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)/f*(2/(-b*f*(-4*d*f+e^2)^(1/2)
+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)*f^2/((x+1/2*(e+(-4*d*f
+e^2)^(1/2))/f)^2*c+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e
^2)^(1/2))/f)+1/2*(-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b
*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)-2*f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/(-b*f*(-
4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)*(2*c*(
x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e))/(2*c*(
-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)
/f^2-1/f^2*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)^2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2)
)/f)^2*c+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/
f)+1/2*(-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*
f+c*e^2)/f^2)^(1/2)-2/(-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f
^2-b*e*f-2*c*d*f+c*e^2)*f^2*2^(1/2)/((-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^
(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-b*f*(-4*d*f+e^2)^(1
/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+1/f*(-c*(-4*d*f
+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-b*f*(-
4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(
1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4/f*(-c*(-4*d*f+e^2)^(1/2)+b*f
-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*(-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e
^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^
2)^(1/2))/f)))+1/2*(-e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)/f*(2/(b*f*(-4
*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)*f^2/((x
-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2*c+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2
/f*(-e+(-4*d*f+e^2)^(1/2)))+1/2*(b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*
c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)-2*(c*(-4*d*f+e^2)^(1/2)+b*f-c*e
)*f/(b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*
e^2)*(2*c*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/
f)/(2*c*(b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*
f+c*e^2)/f^2-(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)^2/f^2)/((x-1/2/f*(-e+(-4*d*f+e^
2)^(1/2)))^2*c+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(
1/2)))+1/2*(b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c
*d*f+c*e^2)/f^2)^(1/2)-2/(b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a
*f^2-b*e*f-2*c*d*f+c*e^2)*f^2*2^(1/2)/((b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)
^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((b*f*(-4*d*f+e^2)^(1
/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+(c*(-4*d*f+e^2)
```

```
^(1/2)+b*f-c*e)/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+1/2*2^(1/2)*((b*f*(-4*d
*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2
)*(4*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2*c+4*(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)
/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+2*(b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)
^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2/f*(-e+(-4*d*f+e^
2)^(1/2)))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{x}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}\,dx = \text{Timed out}$$

```
[In] integrate(x/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}\,dx = \text{Timed out}$$

```
[In] integrate(x/(c*x**2+b*x+a)**(3/2)/(f*x**2+e*x+d),x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(x/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re deta
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}\,dx = \text{Exception raised: AttributeError}$$

```
[In] integrate(x/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
[Out] Exception raised: AttributeError >> type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}\,dx = \int \frac{x}{(c\,x^2+b\,x+a)^{3/2}\,(f\,x^2+e\,x+d)}\,dx$$

```
[In] int(x/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x)
[Out] int(x/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)), x)
```

3.124 $\int \frac{1}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 666

$$\int \frac{1}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}\,dx = \frac{2(b^2ce-2ac^2e-b^3f-bc(cd-3af)-c(2c^2d-bce+b^2f-2acf)}{(b^2-4ac)\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{a+bx+cx^2}}$$
$$-\frac{f\left(c\left(e^2-2df+e\sqrt{e^2-4df}\right)+f\left(2af-b\left(e+\sqrt{e^2-4df}\right)\right)\right)\operatorname{arctanh}\left(\frac{4af-b\left(e-\sqrt{e^2-4df}\right)+2\left(bf-c\left(e-\sqrt{e^2-4}\right.\right.}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}\sqrt{}}\right.}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{c\left(e^2-2df-e\sqrt{e^2-4df}\right)+f\left(2af-b\left(e-\sqrt{e^2-}\right.\right.}}$$
$$+\frac{f\left(c\left(e^2-2df-e\sqrt{e^2-4df}\right)+f\left(2af-b\left(e-\sqrt{e^2-4df}\right)\right)\right)\operatorname{arctanh}\left(\frac{4af-b\left(e+\sqrt{e^2-4df}\right)+2\left(bf-c\left(e+\sqrt{e^2-4d}\right.\right.}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2-4df}}\sqrt{}}\right.}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{c\left(e^2-2df+e\sqrt{e^2-4df}\right)+f\left(2af-b\left(e+\sqrt{e^2-}\right.\right.}}$$

```
[Out] 2*(b^2*c*e-2*a*c^2*e-b^3*f-b*c*(-3*a*f+c*d)-c*(-2*a*c*f+b^2*f-b*c*e+2*c^2*d
)*x)/(-4*a*c+b^2)/((-a*f+c*d)^2-(-a*e+b*d)*(-b*f+c*e))/(c*x^2+b*x+a)^(1/2)-
1/2*f*arctanh(1/4*(4*a*f+2*x*(b*f-c*(e-(-4*d*f+e^2)^(1/2)))-b*(e-(-4*d*f+e^
2)^(1/2)))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c
*e)*(-4*d*f+e^2)^(1/2))^(1/2))*(c*(e^2-2*d*f+e*(-4*d*f+e^2)^(1/2))+f*(2*a*f
-b*(e+(-4*d*f+e^2)^(1/2))))/((-a*f+c*d)^2-(-a*e+b*d)*(-b*f+c*e))*2^(1/2)/(-
4*d*f+e^2)^(1/2)/(f*(2*a*f-b*(e-(-4*d*f+e^2)^(1/2)))+c*(e^2-2*d*f-e*(-4*d*f
+e^2)^(1/2)))^(1/2)+1/2*f*arctanh(1/4*(4*a*f-b*(e+(-4*d*f+e^2)^(1/2))+2*x*(
b*f-c*(e+(-4*d*f+e^2)^(1/2))))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b
*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2))*(f*(2*a*f-b*(e-(-4*d*f+e
^2)^(1/2)))+c*(e^2-2*d*f-e*(-4*d*f+e^2)^(1/2)))/((-a*f+c*d)^2-(-a*e+b*d)*(-
b*f+c*e))*2^(1/2)/(-4*d*f+e^2)^(1/2)/(c*(e^2-2*d*f+e*(-4*d*f+e^2)^(1/2))+f*
(2*a*f-b*(e+(-4*d*f+e^2)^(1/2))))^(1/2)
```

Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 666, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {988, 1046, 738, 212}

$$\int \frac{1}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}\,dx =$$

$$-\frac{f\left(f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)\right)\operatorname{arctanh}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e\right.}\right.}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-b\cdots}}\right.}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{f\left(2af-b\left(e-\sqrt{e^2-4df}\right)\right)+c\left(-e\sqrt{e^2-4df}-2\cdots\right.}}$$

$$+\frac{f\left(f\left(2af-b\left(e-\sqrt{e^2-4df}\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)\right)\operatorname{arctanh}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^{\cdots}}\right.}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-\cdots}}\right.}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2\cdots\right.}}$$

$$+\frac{2(-cx(-2acf+b^2f-bce+2c^2d)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}$$

```
[In] Int[1/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] (2*(b^2*c*e - 2*a*c^2*e - b^3*f - b*c*(c*d - 3*a*f) - c*(2*c^2*d - b*c*e +
b^2*f - 2*a*c*f)*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f
))*Sqrt[a + b*x + c*x^2]) - (f*(c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(
2*a*f - b*(e + Sqrt[e^2 - 4*d*f])))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*
f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*
f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])
])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqr
t[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e - Sqrt[e^2 - 4*d*
f]))]) + (f*(c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e - Sqrt
[e^2 - 4*d*f])))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e
 + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2
 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*Sqrt[e^
2 - 4*d*f]*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[c*(e^2 - 2*d*f +
e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 988

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a
*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*((
d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e -
 b*f))*(p + 1))), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 1046

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2(b^2ce-2ac^2e-b^3f-bc(cd-3af)-c(2c^2d-bce+b^2f-2acf)\,x)}{(b^2-4ac)\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{a+bx+cx^2}} \\
&\quad+\frac{2\int\frac{\frac{1}{2}(b^2-4ac)\left(ce^2-cdf-bef+af^2\right)+\frac{1}{2}(b^2-4ac)f(ce-bf)x}{\sqrt{a+bx+cx^2}(d+ex+fx^2)}\,dx}{(b^2-4ac)\left((cd-af)^2-(bd-ae)(ce-bf)\right)} \\
&= \frac{2(b^2ce-2ac^2e-b^3f-bc(cd-3af)-c(2c^2d-bce+b^2f-2acf)\,x)}{(b^2-4ac)\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{a+bx+cx^2}} \\
&\quad+\frac{\left(f\left(2f(be-af)-2c(e^2-df)+(ce-bf)\left(e+\sqrt{e^2-4df}\right)\right)\right)\int\frac{1}{\left(e+\sqrt{e^2-4df}+2fx\right)\sqrt{a+bx+cx^2}}\,dx}{\sqrt{e^2-4df}\left((cd-af)^2-(bd-ae)(ce-bf)\right)} \\
&\quad+\frac{\left(f\left(c\left(e^2-2df+e\sqrt{e^2-4df}\right)+f\left(2af-b\left(e+\sqrt{e^2-4df}\right)\right)\right)\right)\int\frac{1}{\left(e-\sqrt{e^2-4df}+2fx\right)\sqrt{a+bx+cx^2}}\,dx}{\sqrt{e^2-4df}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}
\end{aligned}$$

$$= \frac{2(b^2ce - 2ac^2e - b^3f - bc(cd - 3af) - c(2c^2d - bce + b^2f - 2acf)\,x)}{(b^2 - 4ac)\left((cd - af)^2 - (bd - ae)(ce - bf)\right)\sqrt{a + bx + cx^2}}$$

$$- \frac{\left(2f\left(2f(be - af) - 2c(e^2 - df) + (ce - bf)\left(e + \sqrt{e^2 - 4df}\right)\right)\right)\text{Subst}\left(\int \frac{1}{16af^2 - 8bf\left(e + \sqrt{e^2 - 4df}\right) + 4\ldots}\right.}{\sqrt{e^2 - 4df}\left((cd - af)^2 - (bd - ae)(ce\ldots\right.}$$

$$- \frac{\left(2f\left(c\left(e^2 - 2df + e\sqrt{e^2 - 4df}\right) + f\left(2af - b\left(e + \sqrt{e^2 - 4df}\right)\right)\right)\right)\text{Subst}\left(\int \frac{1}{16af^2 - 8bf\left(e - \sqrt{e^2 - 4df}\right) + \ldots}\right.}{\sqrt{e^2 - 4df}\left((cd - af)^2 - (bd - ae)(ce\ldots\right.}$$

$$= \frac{2(b^2ce - 2ac^2e - b^3f - bc(cd - 3af) - c(2c^2d - bce + b^2f - 2acf)\,x)}{(b^2 - 4ac)\left((cd - af)^2 - (bd - ae)(ce - bf)\right)\sqrt{a + bx + cx^2}}$$

$$- \frac{f\left(c\left(e^2 - 2df + e\sqrt{e^2 - 4df}\right) + f\left(2af - b\left(e + \sqrt{e^2 - 4df}\right)\right)\right)\tanh^{-1}\left(\frac{4af - b\left(e - \sqrt{e^2 - 4df}\right) + 2\left(b\ldots\right.}{2\sqrt{2}\sqrt{ce^2 - 2cdf - bef + 2af^2 - (ce\ldots}}\right.}{\sqrt{2}\sqrt{e^2 - 4df}\left((cd - af)^2 - (bd - ae)(ce - bf)\right)\sqrt{ce^2 - 2cdf - bef + 2af^2 - (ce - b\ldots}}$$

$$- \frac{f\left(2f(be - af) - 2c(e^2 - df) + (ce - bf)\left(e + \sqrt{e^2 - 4df}\right)\right)\tanh^{-1}\left(\frac{4af - b\left(e + \sqrt{e^2 - 4df}\right) + 2\left(bf - \ldots\right.}{2\sqrt{2}\sqrt{ce^2 - 2cdf - bef + 2af^2 + (ce - b\ldots}}\right.}{\sqrt{2}\sqrt{e^2 - 4df}\left((cd - af)^2 - (bd - ae)(ce - bf)\right)\sqrt{ce^2 - 2cdf - bef + 2af^2 + (ce - bf\ldots}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.07 (sec) , antiderivative size = 806, normalized size of antiderivative = 1.21

$$\int \frac{1}{(a + bx + cx^2)^{3/2}(d + ex + fx^2)}\,dx = \frac{-2(b^3f + b^2c(-e + fx) + bc(-3af + c(d - ex)) + 2c^2(cdx + a(e\ldots}{\ldots}$$

```
[In] Integrate[1/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] (-2*(b^3*f + b^2*c*(-e + f*x) + b*c*(-3*a*f + c*(d - e*x)) + 2*c^2*(c*d*x +
 a*(e - f*x))) + (b^2 - 4*a*c)*Sqrt[a + x*(b + c*x)]*RootSum[c^2*d - b*c*e
+ b^2*f + 2*Sqrt[a]*c*e*#1 - 4*Sqrt[a]*b*f*#1 - 2*c*d*#1^2 + b*e*#1^2 + 4*a
*f*#1^2 - 2*Sqrt[a]*e*#1^3 + d*#1^4 & , (-(c^2*e^2*Log[x]) + c^2*d*f*Log[x]
 + 2*b*c*e*f*Log[x] - b^2*f^2*Log[x] - a*c*f^2*Log[x] + c^2*e^2*Log[-Sqrt[a
] + Sqrt[a + b*x + c*x^2] - x*#1] - c^2*d*f*Log[-Sqrt[a] + Sqrt[a + b*x + c
*x^2] - x*#1] - 2*b*c*e*f*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] + b^
2*f^2*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] + a*c*f^2*Log[-Sqrt[a] +
 Sqrt[a + b*x + c*x^2] - x*#1] - 2*Sqrt[a]*c*e*f*Log[x]*#1 + 2*Sqrt[a]*b*f^
2*Log[x]*#1 + 2*Sqrt[a]*c*e*f*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1]*
#1 - 2*Sqrt[a]*b*f^2*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1]*#1 + c*e^
2*Log[x]*#1^2 - c*d*f*Log[x]*#1^2 - b*e*f*Log[x]*#1^2 + a*f^2*Log[x]*#1^2 -
 c*e^2*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1]*#1^2 + c*d*f*Log[-Sqrt[
```

```
a] + Sqrt[a + b*x + c*x^2] - x*#1]*#1^2 + b*e*f*Log[-Sqrt[a] + Sqrt[a + b*x
 + c*x^2] - x*#1]*#1^2 - a*f^2*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1]
*#1^2)/(Sqrt[a]*c*e - 2*Sqrt[a]*b*f - 2*c*d*#1 + b*e*#1 + 4*a*f*#1 - 3*Sqrt
[a]*e*#1^2 + 2*d*#1^3) & ])/((b^2 - 4*a*c)*(c^2*d^2 - b*c*d*e + f*(b^2*d -
a*b*e + a^2*f) + a*c*(e^2 - 2*d*f))*Sqrt[a + x*(b + c*x)])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1905 vs. 2(609) = 1218.

Time = 1.09 (sec) , antiderivative size = 1906, normalized size of antiderivative = 2.86

method	result	size
default	Expression too large to display	1906

```
[In] int(1/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/(-4*d*f+e^2)^(1/2)*(2/(-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*
a*f^2-b*e*f-2*c*d*f+c*e^2)*f^2/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+1/f*(-
c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-b*f*(-
4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(
1/2)-2*f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/(-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e
^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)*(2*c*(x+1/2*(e+(-4*d*f+e^2)^(1/2
))/f)+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e))/(2*c*(-b*f*(-4*d*f+e^2)^(1/2)+(-
4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2-1/f^2*(-c*(-4*d*f+e^2
)^(1/2)+b*f-c*e)^2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+1/f*(-c*(-4*d*f+e
^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-b*f*(-4*d*f+e^2)^
(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)-2/(-b*
f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)*f^
2*2^(1/2)/((-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*
c*d*f+c*e^2)/f^2)^(1/2)*ln(((-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e
+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/
2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e
^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+
e^2)^(1/2))/f)^2*c+4/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^
2)^(1/2))/f)+2*(-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*
f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)))+1/(-4*d*f+e
^2)^(1/2)*(2/(b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2
*c*d*f+c*e^2)*f^2/((x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2*c+(c*(-4*d*f+e^2)^(1
/2)+b*f-c*e)/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+1/2*(b*f*(-4*d*f+e^2)^(1/2
)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)-2*(c*(-4*d
*f+e^2)^(1/2)+b*f-c*e)*f/(b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a
*f^2-b*e*f-2*c*d*f+c*e^2)*(2*c*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+(c*(-4*d*f
+e^2)^(1/2)+b*f-c*e)/f)/(2*c*(b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e
+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2-(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)^2/f^2)/((x
-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2*c+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2
```

```
/f*(-e+(-4*d*f+e^2)^(1/2)))+1/2*(b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*
c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)-2/(b*f*(-4*d*f+e^2)^(1/2)-(-4*d
*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)*f^2*2^(1/2)/((b*f*(-4*d*f+e^
2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(
((b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2
)/f^2+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+1/
2*2^(1/2)*((b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c
*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2*c+4*(c*(-4*d*
f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+2*(b*f*(-4*d*f+e^
2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x
-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx+cx^2)^{3/2}\,(d+ex+fx^2)}\,dx = \text{Timed out}$$

```
[In] integrate(1/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx+cx^2)^{3/2}\,(d+ex+fx^2)}\,dx = \text{Timed out}$$

```
[In] integrate(1/(c*x**2+b*x+a)**(3/2)/(f*x**2+e*x+d),x)
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a+bx+cx^2)^{3/2}\,(d+ex+fx^2)}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate(1/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for mo
re deta
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}\,dx = \text{Exception raised: AttributeError}$$

```
[In] integrate(1/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
[Out] Exception raised: AttributeError >> type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}\,dx = \int \frac{1}{(c\,x^2+b\,x+a)^{3/2}\,(f\,x^2+e\,x+d)}\,dx$$

```
[In] int(1/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x)
[Out] int(1/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)), x)
```

3.125 $\int \frac{1}{x(a+bx+cx^2)^{3/2}(d+ex+fx^2)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 816

$$\int \frac{1}{x\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx = \frac{2(b^2-2ac+bcx)}{a\left(b^2-4ac\right)d\sqrt{a+bx+cx^2}}$$
$$+\frac{2(ce(2ace-b(cd+af))+(be-af)\left(2c^2d+b^2f-c(be+2af)\right)+c(2c^2de+bf(be-af)-bc(e^2+df))\,x}{(b^2-4ac)\,d\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{a+bx+cx^2}}$$
$$-\frac{\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{a^{3/2}d}$$
$$+\frac{f\left(\left(e-\sqrt{e^2-4df}\right)\left(f(be-af)-c(e^2-df)\right)-2\left(f(be^2-bdf-aef)-c(e^3-2def)\right)\right)\operatorname{arctanh}\left(\frac{4af-}{2\sqrt{2}\sqrt{ce^2-}}\right.}{\sqrt{2}d\sqrt{e^2-4df}\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{ce^2-2cdf-bef+2af^2-(ce}}$$
$$-\frac{f\left(\left(e+\sqrt{e^2-4df}\right)\left(f(be-af)-c(e^2-df)\right)-2\left(f(be^2-bdf-aef)-c(e^3-2def)\right)\right)\operatorname{arctanh}\left(\frac{4af-}{2\sqrt{2}\sqrt{ce^2}}\right.}{\sqrt{2}d\sqrt{e^2-4df}\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{ce^2-2cdf-bef+2af^2+(ce}}$$

```
[Out] -arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/a^(3/2)/d+2*(b*c*x-2*a*
c+b^2)/a/(-4*a*c+b^2)/d/(c*x^2+b*x+a)^(1/2)+2*(c*e*(2*a*c*e-b*(a*f+c*d))+(-
a*f+b*e)*(2*c^2*d+b^2*f-c*(2*a*f+b*e))+c*(2*c^2*d*e+b*f*(-a*f+b*e)-b*c*(d*f
+e^2))*x)/(-4*a*c+b^2)/d/((-a*f+c*d)^2-(-a*e+b*d)*(-b*f+c*e))/(c*x^2+b*x+a)
^(1/2)+1/2*f*arctanh(1/4*(4*a*f+2*x*(b*f-c*(e-(-4*d*f+e^2)^(1/2)))-b*(e-(-4
*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-
(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2))*(-2*f*(-a*e*f-b*d*f+b*e^2)+2*c*(-2*d*
e*f+e^3)+(f*(-a*f+b*e)-c*(-d*f+e^2))*(e-(-4*d*f+e^2)^(1/2)))/d/((-a*f+c*d)^
2-(-a*e+b*d)*(-b*f+c*e))*2^(1/2)/(-4*d*f+e^2)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*
a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)-1/2*f*arctanh(1/4*(4*a*f-b*(e+(-
4*d*f+e^2)^(1/2))+2*x*(b*f-c*(e+(-4*d*f+e^2)^(1/2))))*2^(1/2)/(c*x^2+b*x+a)
^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2))*(
```

```
-2*f*(-a*e*f-b*d*f+b*e^2)+2*c*(-2*d*e*f+e^3)+(f*(-a*f+b*e)-c*(-d*f+e^2))*(e
+(-4*d*f+e^2)^(1/2)))/d/((-a*f+c*d)^2-(-a*e+b*d)*(-b*f+c*e))*2^(1/2)/(-4*d*
f+e^2)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1
/2)
```

Rubi [A] (verified)

Time = 14.98 (sec) , antiderivative size = 814, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {6860, 754, 12, 738, 212, 1030, 1046}

$$\int \frac{1}{x\,(a+bx+cx^2)^{3/2}\,(d+ex+fx^2)}\,dx = \frac{2(b^2+cxb-2ac)}{a\,(b^2-4ac)\,d\sqrt{cx^2+bx+a}}$$

$$-\frac{\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{cx^2+bx+a}}\right)}{a^{3/2}d}$$

$$-\frac{f\left(2f(be^2-afe-bdf)-2c(e^3-2def)-\left(e-\sqrt{e^2-4df}\right)\left(f(be-af)-c(e^2-df)\right)\right)\operatorname{arctanh}\left(\frac{4af-b(\cdots}{2\sqrt{2}\sqrt{ce^2-b\cdots}}\right.}{\sqrt{2}d\sqrt{e^2-4df}\,\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{ce^2-bfe+2af^2-2cdf-(ce-\cdots}}$$

$$+\frac{f\left(2f(be^2-afe-bdf)-2c(e^3-2def)-\left(e+\sqrt{e^2-4df}\right)\left(f(be-af)-c(e^2-df)\right)\right)\operatorname{arctanh}\left(\frac{4af-b(e\cdots}{2\sqrt{2}\sqrt{ce^2-bf\cdots}}\right.}{\sqrt{2}d\sqrt{e^2-4df}\,\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{ce^2-bfe+2af^2-2cdf+(ce-\cdots}}$$

$$+\frac{2(ce(2ace-b(cd+af))+(be-af)\left(fb^2+2c^2d-c(be+2af)\right)+c(2dec^2-b(e^2+df)\,c+bf(be-af))\,x)}{(b^2-4ac)\,d\,\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{cx^2+bx+a}}$$

```
[In] Int[1/(x*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] (2*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*d*Sqrt[a + b*x + c*x^2]) + (2*(c
*e*(2*a*c*e - b*(c*d + a*f)) + (b*e - a*f)*(2*c^2*d + b^2*f - c*(b*e + 2*a*
f)) + c*(2*c^2*d*e + b*f*(b*e - a*f) - b*c*(e^2 + d*f))*x))/((b^2 - 4*a*c)*
d*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[a + b*x + c*x^2]) - ArcTan
h[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])]/(a^(3/2)*d) - (f*(2*f*(b*e
^2 - b*d*f - a*e*f) - 2*c*(e^3 - 2*d*e*f) - (e - Sqrt[e^2 - 4*d*f])*(f*(b*e
 - a*f) - c*(e^2 - d*f)))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b
*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f
+ 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2
]*d*Sqrt[e^2 - 4*d*f]*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[c*e^2
- 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]) + (f*(2*f*(b*
e^2 - b*d*f - a*e*f) - 2*c*(e^3 - 2*d*e*f) - (e + Sqrt[e^2 - 4*d*f])*(f*(b*
e - a*f) - c*(e^2 - d*f)))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(
b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f
 + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[
2]*d*Sqrt[e^2 - 4*d*f]*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[c*e^2
 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 1030

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x + c*x^2)^(p + 1)*
((d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
 - b*f))*(p + 1)))*(g*c*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d +
b^2*f - c*(b*e + 2*a*f)) + c*(g*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - h*(b*
c*d - 2*a*c*e + a*b*f))*x), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*
d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*
x^2)^q*Simp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)
 + (b^2*(g*f) - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*((-h)*c*
e)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((g*c)*(2*a*c*e - b*(c*d + a*f)) + (g*
b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((g*c)*(2*
a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(
p + q + 2) - (b^2*g*f - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*
((-h)*c*e)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(g*f) - b*(h*c
```

```
*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) + a*h*c*e))*(2*p + 2*q + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e
 - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
```

Rule 1046

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 6860

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\text{integral} = \int \left(\frac{1}{dx\left(a+bx+cx^2\right)^{3/2}}+\frac{-e-fx}{d\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\right)dx$$

$$=\frac{\int \frac{1}{x(a+bx+cx^2)^{3/2}}\,dx}{d}+\frac{\int \frac{-e-fx}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}\,dx}{d}$$

$$=\frac{2(b^2-2ac+bcx)}{a\left(b^2-4ac\right)d\sqrt{a+bx+cx^2}}$$
$$+\frac{2(ce(2ace-b(cd+af))+(be-af)\left(2c^2d+b^2f-c(be+2af)\right)+c(2c^2de+bf(be-af)-bc(e}{\left(b^2-4ac\right)d\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{a+bx+cx^2}}$$
$$-\frac{2\int \frac{-\frac{b^2}{2}+2ac}{x\sqrt{a+bx+cx^2}}\,dx}{a\left(b^2-4ac\right)d}$$
$$-\frac{2\int \frac{-\frac{1}{2}\left(b^2-4ac\right)\left(f\left(be^2-bdf-aef\right)-c\left(e^3-2def\right)\right)+\frac{1}{2}\left(b^2-4ac\right)f\left(ce^2-cdf-bef+af^2\right)x}{\sqrt{a+bx+cx^2}(d+ex+fx^2)}\,dx}{\left(b^2-4ac\right)d\left((cd-af)^2-(bd-ae)(ce-bf)\right)}$$

$$= \frac{2(b^2-2ac+bcx)}{a\left(b^2-4ac\right)d\sqrt{a+bx+cx^2}}$$
$$+ \frac{2(ce(2ace-b(cd+af))+(be-af)\left(2c^2d+b^2f-c(be+2af)\right)+c(2c^2de+bf(be-af)-bc(}{\left(b^2-4ac\right)d\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{a+bx+cx^2}}$$
$$+ \frac{\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx}{ad}$$
$$+ \frac{\left(f\left(2f(be^2-bdf-aef)-2c(e^3-2def)-\left(e-\sqrt{e^2-4df}\right)\left(f(be-af)-c(e^2-df)\right)\right)\right)\int \frac{}{\left(e-\right.}}{d\sqrt{e^2-4df}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}$$
$$- \frac{\left(f\left(2f(be^2-bdf-aef)-2c(e^3-2def)-\left(e+\sqrt{e^2-4df}\right)\left(f(be-af)-c(e^2-df)\right)\right)\right)\int \frac{}{\left(e+\right.}}{d\sqrt{e^2-4df}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}$$

$$= \frac{2(b^2-2ac+bcx)}{a\left(b^2-4ac\right)d\sqrt{a+bx+cx^2}}$$
$$+ \frac{2(ce(2ace-b(cd+af))+(be-af)\left(2c^2d+b^2f-c(be+2af)\right)+c(2c^2de+bf(be-af)-bc(}{\left(b^2-4ac\right)d\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{a+bx+cx^2}}$$
$$- \frac{2\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx}{\sqrt{a+bx+cx^2}}\right)}{ad}$$
$$- \frac{\left(2f\left(2f(be^2-bdf-aef)-2c(e^3-2def)-\left(e-\sqrt{e^2-4df}\right)\left(f(be-af)-c(e^2-df)\right)\right)\right)\text{Subs}}{d\sqrt{e^2-4df}\left((cd-af)^2-\right.}$$
$$+ \frac{\left(2f\left(2f(be^2-bdf-aef)-2c(e^3-2def)-\left(e+\sqrt{e^2-4df}\right)\left(f(be-af)-c(e^2-df)\right)\right)\right)\text{Subs}}{d\sqrt{e^2-4df}\left((cd-af)^2-\right.}$$

$$= \frac{2(b^2-2ac+bcx)}{a\left(b^2-4ac\right)d\sqrt{a+bx+cx^2}}$$
$$+ \frac{2(ce(2ace-b(cd+af))+(be-af)\left(2c^2d+b^2f-c(be+2af)\right)+c(2c^2de+bf(be-af)-bc(}{\left(b^2-4ac\right)d\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{a+bx+cx^2}}$$
$$- \frac{\tanh^{-1}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{a^{3/2}d}$$
$$- \frac{f\left(2f(be^2-bdf-aef)-2c(e^3-2def)-\left(e-\sqrt{e^2-4df}\right)\left(f(be-af)-c(e^2-df)\right)\right)\tanh^{-1}}{\sqrt{2}d\sqrt{e^2-4df}\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{ce^2-2cdf-bef+2}}$$
$$+ \frac{f\left(2f(be^2-bdf-aef)-2c(e^3-2def)-\left(e+\sqrt{e^2-4df}\right)\left(f(be-af)-c(e^2-df)\right)\right)\tanh^{-1}}{\sqrt{2}d\sqrt{e^2-4df}\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{ce^2-2cdf-bef+2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 6.73 (sec) , antiderivative size = 1025, normalized size of antiderivative = 1.26

$$\int \frac{1}{x\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx =$$
$$-\frac{2(b^4f+2ac^2(-cd+af+cex)+b^3c(-e+fx)+b^2c(-4af+c(d-ex))+bc^2(cdx+3a(e-fx)))}{a\left(-b^2+4ac\right)\left(c^2d^2-bcde+f\left(b^2d-abe+a^2f\right)+ac\left(e^2-2df\right)\right)\sqrt{a+x(b+cx)}}$$
$$+\frac{2\text{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)}{a^{3/2}d}$$
$$-\frac{\text{RootSum}\Big[b^2d-abe+a^2f-4b\sqrt{c}d\#1+2a\sqrt{c}e\#1+4cd\#1^2+be\#1^2-2af\#1^2-2\sqrt{c}e\#1^3+f\#1^4\&,}{}$$

```
[In] Integrate[1/(x*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] (-2*(b^4*f + 2*a*c^2*(-(c*d) + a*f + c*e*x) + b^3*c*(-e + f*x) + b^2*c*(-4*
a*f + c*(d - e*x)) + b*c^2*(c*d*x + 3*a*(e - f*x))))/(a*(-b^2 + 4*a*c)*(c^2
*d^2 - b*c*d*e + f*(b^2*d - a*b*e + a^2*f) + a*c*(e^2 - 2*d*f))*Sqrt[a + x*
(b + c*x)]) + (2*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)])/Sqrt[a]])/(a^(
3/2)*d) - RootSum[b^2*d - a*b*e + a^2*f - 4*b*Sqrt[c]*d*#1 + 2*a*Sqrt[c]*e*
#1 + 4*c*d*#1^2 + b*e*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (-(
b*c*e^3*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]) + 2*b*c*d*e*f*Log[-
(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + b^2*e^2*f*Log[-(Sqrt[c]*x) + Sq
rt[a + b*x + c*x^2] - #1] + a*c*e^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x
^2] - #1] - b^2*d*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - a*c*
d*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*a*b*e*f^2*Log[-(Sq
rt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + a^2*f^3*Log[-(Sqrt[c]*x) + Sqrt[a
+ b*x + c*x^2] - #1] + 2*c^(3/2)*e^3*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^
2] - #1]*#1 - 4*c^(3/2)*d*e*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1
]*#1 - 2*b*Sqrt[c]*e^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1
+ 2*b*Sqrt[c]*d*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + 2*a
*Sqrt[c]*e*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - c*e^2*f*
Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 + c*d*f^2*Log[-(Sqrt[c]
*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 + b*e*f^2*Log[-(Sqrt[c]*x) + Sqrt[a
+ b*x + c*x^2] - #1]*#1^2 - a*f^3*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2]
- #1]*#1^2)/(2*b*Sqrt[c]*d - a*Sqrt[c]*e - 4*c*d*#1 - b*e*#1 + 2*a*f*#1 + 3
*Sqrt[c]*e*#1^2 - 2*f*#1^3) & ]/(d*(c^2*d^2 - b*c*d*e + f*(b^2*d - a*b*e +
a^2*f) + a*c*(e^2 - 2*d*f)))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2058 vs. 2(753) = 1506.

Time = 1.16 (sec) , antiderivative size = 2059, normalized size of antiderivative = 2.52

method	result	size
default	Expression too large to display	2059

```
[In] int(1/x/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -4*f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))*(1/a/(c*x^2+b*x+a)^(1/2
)-b/a*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-1/a^(3/2)*ln((2*a+b*x+2*a^(
1/2)*(c*x^2+b*x+a)^(1/2))/x))+2*f/(e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)
*(2/(-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c
*e^2)*f^2/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+1/f*(-c*(-4*d*f+e^2)^(1/2)+
b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-b*f*(-4*d*f+e^2)^(1/2)+(-4*
d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)-2*f*(-c*(-4*d*f+
e^2)^(1/2)+b*f-c*e)/(-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2
-b*e*f-2*c*d*f+c*e^2)*(2*c*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/f*(-c*(-4*d*f
+e^2)^(1/2)+b*f-c*e))/(2*c*(-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+
2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2-1/f^2*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)^2)/(
(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x
+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1
/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)-2/(-b*f*(-4*d*f+e^2)^(1/2)+
(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)*f^2*2^(1/2)/((-b*f*(-4*
d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/
2)*ln(((-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*
f+c*e^2)/f^2+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/
2))/f)+1/2*2^(1/2)*((-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2
-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4/
f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*(-b*f*
(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)
^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)))+2*f/(-e+(-4*d*f+e^2)^(1/2))/(-4*
d*f+e^2)^(1/2)*(2/(b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*
e*f-2*c*d*f+c*e^2)*f^2/((x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2*c+(c*(-4*d*f+e^
2)^(1/2)+b*f-c*e)/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+1/2*(b*f*(-4*d*f+e^2)
^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)-2*(c*
(-4*d*f+e^2)^(1/2)+b*f-c*e)*f/(b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*
e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)*(2*c*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+(c*(-
4*d*f+e^2)^(1/2)+b*f-c*e)/f)/(2*c*(b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2
)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2-(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)^2/f^2
)/((x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2*c+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(
x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+1/2*(b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(
1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)-2/(b*f*(-4*d*f+e^2)^(1/2)-
(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)*f^2*2^(1/2)/((b*f*(-4*d
```

```
*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2
)*ln(((b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+
c*e^2)/f^2+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)
))+1/2*2^(1/2)*((b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*
f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2*c+4*(c*(
-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+2*(b*f*(-4*d
*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2
))/(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx = \text{Timed out}$$

```
[In] integrate(1/x/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{x\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx = \int \frac{1}{x\left(a+bx+cx^2\right)^{\frac{3}{2}}\left(d+ex+fx^2\right)}\,dx$$

```
[In] integrate(1/x/(c*x**2+b*x+a)**(3/2)/(f*x**2+e*x+d),x)
[Out] Integral(1/(x*(a + b*x + c*x**2)**(3/2)*(d + e*x + f*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{x\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx = \int \frac{1}{(cx^2+bx+a)^{\frac{3}{2}}(fx^2+ex+d)x}\,dx$$

```
[In] integrate(1/x/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")
[Out] integrate(1/((c*x^2 + b*x + a)^(3/2)*(f*x^2 + e*x + d)*x), x)
```

Giac [F]

$$\int \frac{1}{x\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx=\int \frac{1}{(cx^2+bx+a)^{\frac{3}{2}}(fx^2+ex+d)x}\,dx$$

```
[In] integrate(1/x/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
[Out] sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx=\int \frac{1}{x\,\left(c\,x^2+b\,x+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right)}\,dx$$

```
[In] int(1/(x*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x)
[Out] int(1/(x*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)), x)
```

3.126 $\int \frac{x^4}{\sqrt{-3-4x-x^2}(3+4x+2x^2)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 140

$$\int \frac{x^4}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = \frac{5}{2}\sqrt{-3-4x-x^2}-\frac{1}{4}x\sqrt{-3-4x-x^2}+\frac{11}{2}\arcsin(2+x)+\frac{\arctan\left(\frac{1-\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right)}{2\sqrt{2}}-\frac{\arctan\left(\frac{1+\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right)}{2\sqrt{2}}-\frac{5}{4}\text{arctanh}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right)$$

```
[Out] 11/2*arcsin(2+x)-5/4*arctanh(x/(-x^2-4*x-3)^(1/2))+1/4*arctan(1/2*(1+(-3-x)
/(-x^2-4*x-3)^(1/2))*2^(1/2))*2^(1/2)-1/4*arctan(1/2*(1+(3+x)/(-x^2-4*x-3)^
(1/2))*2^(1/2))*2^(1/2)+5/2*(-x^2-4*x-3)^(1/2)-1/4*x*(-x^2-4*x-3)^(1/2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules

used = {6860, 633, 222, 654, 756, 1042, 1000, 12, 1040, 1175, 632, 210, 1041, 212}

$$\int \frac{x^4}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = \frac{11}{2}\arcsin(x+2) + \frac{\arctan\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)}{2\sqrt{2}} - \frac{\arctan\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)}{2\sqrt{2}} - \frac{5}{4}\operatorname{arctanh}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right) - \frac{1}{4}\sqrt{-x^2-4x-3}\,x + \frac{5}{2}\sqrt{-x^2-4x-3}$$

```
[In] Int[x^4/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

```
[Out] (5*Sqrt[-3 - 4*x - x^2])/2 - (x*Sqrt[-3 - 4*x - x^2])/4 + (11*ArcSin[2 + x]
)/2 + ArcTan[(1 - (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]]/(2*Sqrt[2]) - ArcT
an[(1 + (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]]/(2*Sqrt[2]) - (5*ArcTanh[x/S
qrt[-3 - 4*x - x^2]])/4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 756

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 1000

```
Int[1/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*
(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f)
, 2]}, Dist[1/(2*q), Int[(c*d - a*f + q + (c*e - b*f)*x)/((a + b*x + c*x^2)
*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[(c*d - a*f - q + (c*e -
 b*f)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b,
 c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[c*e -
 b*f, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1040

```
Int[(x_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_.)*(x_) + (f_.)
*(x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[(1 - d*x^2)/(c*e - b*f - e*(2*
c*d - b*e + 2*a*f)*x^2 + d^2*(c*e - b*f)*x^4), x], x, (1 + (e + Sqrt[e^2 -
4*d*f])*(x/(2*d)))/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0]
```

Rule 1041

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_
.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[g, Subst[Int[1/(a + (c*d - a*f)
*x^2), x], x, x/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h
}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0] &&
 EqQ[2*h*d - g*e, 0]
```

Rule 1042

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_
.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-(2*h*d - g*e)/e, Int[1/((a + b
*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/e, Int[(2*d + e*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g
, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0]
 && NeQ[2*h*d - g*e, 0]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 6860

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{5}{4\sqrt{-3-4x-x^2}} - \frac{x}{\sqrt{-3-4x-x^2}} + \frac{x^2}{2\sqrt{-3-4x-x^2}} - \frac{15+8x}{4\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\right) dx \\
&= -\left(\frac{1}{4}\int \frac{15+8x}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx\right) + \frac{1}{2}\int \frac{x^2}{\sqrt{-3-4x-x^2}}\,dx \\
&\quad + \frac{5}{4}\int \frac{1}{\sqrt{-3-4x-x^2}}\,dx - \int \frac{x}{\sqrt{-3-4x-x^2}}\,dx
\end{aligned}$$

$$= \sqrt{-3-4x-x^2} - \frac{1}{4}x\sqrt{-3-4x-x^2} - \frac{1}{4}\int \frac{3+6x}{\sqrt{-3-4x-x^2}}\,dx$$
$$+ \frac{1}{2}\int \frac{-6-4x}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx - \frac{5}{8}\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{4}}}\,dx, x, -4-2x\right)$$
$$- \frac{3}{4}\int \frac{1}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx + 2\int \frac{1}{\sqrt{-3-4x-x^2}}\,dx$$
$$= \frac{5}{2}\sqrt{-3-4x-x^2} - \frac{1}{4}x\sqrt{-3-4x-x^2} + \frac{5}{4}\sin^{-1}(2+x)$$
$$+ \frac{1}{8}\int \frac{-6-4x}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx - \frac{1}{8}\int$$
$$- \frac{4x}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx + \frac{9}{4}\int \frac{1}{\sqrt{-3-4x-x^2}}\,dx$$
$$- 3\text{Subst}\left(\int \frac{1}{3-3x^2}\,dx, x, \frac{x}{\sqrt{-3-4x-x^2}}\right)$$
$$- \text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{4}}}\,dx, x, -4-2x\right)$$
$$= \frac{5}{2}\sqrt{-3-4x-x^2} - \frac{1}{4}x\sqrt{-3-4x-x^2} + \frac{13}{4}\sin^{-1}(2+x)$$
$$- \tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) + \frac{1}{2}\int \frac{x}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx$$
$$- \frac{3}{4}\text{Subst}\left(\int \frac{1}{3-3x^2}\,dx, x, \frac{x}{\sqrt{-3-4x-x^2}}\right)$$
$$- \frac{9}{8}\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{4}}}\,dx, x, -4-2x\right)$$
$$= \frac{5}{2}\sqrt{-3-4x-x^2} - \frac{1}{4}x\sqrt{-3-4x-x^2}$$
$$+ \frac{11}{2}\sin^{-1}(2+x) - \frac{5}{4}\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right)$$
$$+ 4\text{Subst}\left(\int \frac{1+3x^2}{-4-8x^2-36x^4}\,dx, x, \frac{1+\frac{x}{3}}{\sqrt{-3-4x-x^2}}\right)$$
$$= \frac{5}{2}\sqrt{-3-4x-x^2} - \frac{1}{4}x\sqrt{-3-4x-x^2} + \frac{11}{2}\sin^{-1}(2+x)$$
$$- \frac{5}{4}\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) - \frac{1}{6}\text{Subst}\left(\int \frac{1}{\frac{1}{3}-\frac{2x}{3}+x^2}\,dx, x, \frac{1+\frac{x}{3}}{\sqrt{-3-4x-x^2}}\right)$$
$$- \frac{1}{6}\text{Subst}\left(\int \frac{1}{\frac{1}{3}+\frac{2x}{3}+x^2}\,dx, x, \frac{1+\frac{x}{3}}{\sqrt{-3-4x-x^2}}\right)$$

$$= \frac{5}{2}\sqrt{-3-4x-x^2} - \frac{1}{4}x\sqrt{-3-4x-x^2} + \frac{11}{2}\sin^{-1}(2+x) - \frac{5}{4}\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) + \frac{1}{3}\text{Subst}\left(\int \frac{1}{-\frac{8}{9}-x^2}\,dx, x, \frac{2}{3}\left(-1+\frac{3+x}{\sqrt{-3-4x-x^2}}\right)\right) + \frac{1}{3}\text{Subst}\left(\int \frac{1}{-\frac{8}{9}-x^2}\,dx, x, \frac{2}{3}\left(1+\frac{3+x}{\sqrt{-3-4x-x^2}}\right)\right)$$

$$= \frac{5}{2}\sqrt{-3-4x-x^2} - \frac{1}{4}x\sqrt{-3-4x-x^2} + \frac{11}{2}\sin^{-1}(2+x) + \frac{\tan^{-1}\left(\frac{1-\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right)}{2\sqrt{2}} - \frac{\tan^{-1}\left(\frac{1+\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right)}{2\sqrt{2}} - \frac{5}{4}\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right)$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.71

$$\int \frac{x^4}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = \frac{1}{4}\left(-\left((-10+x)\sqrt{-3-4x-x^2}\right) - \sqrt{2}\arctan\left(\frac{3+2x}{\sqrt{2}\sqrt{-3-4x-x^2}}\right) - 44\arctan\left(\frac{\sqrt{-3-4x-x^2}}{3+x}\right) - 5\text{arctanh}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right)\right)$$

```
[In] Integrate[x^4/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

```
[Out] (-((-10 + x)*Sqrt[-3 - 4*x - x^2]) - Sqrt[2]*ArcTan[(3 + 2*x)/(Sqrt[2]*Sqrt
[-3 - 4*x - x^2])] - 44*ArcTan[Sqrt[-3 - 4*x - x^2]/(3 + x)] - 5*ArcTanh[x/
Sqrt[-3 - 4*x - x^2]])/4
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.11

method	result
risch	$\frac{(x-10)(x^2+4x+3)}{4\sqrt{-x^2-4x-3}}+\frac{11\arcsin(2+x)}{2}+\frac{\sqrt{3}\,\sqrt{4}\,\sqrt{\frac{3x^2}{\left(-\frac{3}{2}-x\right)^2}-12}\left(\sqrt{2}\arctan\left(\frac{\sqrt{\frac{3x^2}{\left(-\frac{3}{2}-x\right)^2}-12}\,\sqrt{2}}{6}\right)+5\,\mathrm{arctanh}\left(\frac{3x}{\left(-\frac{3}{2}-x\right)\sqrt{\frac{3x^2}{\left(-\frac{3}{2}-\right.}}}\right.\right.}{24\sqrt{\frac{\frac{x^2}{\left(-\frac{3}{2}-x\right)^2}-4}{\left(1+\frac{x}{-\frac{3}{2}-x}\right)^2}}\left(1+\frac{x}{-\frac{3}{2}-x}\right)}$
default	$\frac{5\sqrt{-x^2-4x-3}}{2}+\frac{11\arcsin(2+x)}{2}-\frac{x\sqrt{-x^2-4x-3}}{4}+\frac{\sqrt{3}\,\sqrt{4}\,\sqrt{\frac{3x^2}{\left(-\frac{3}{2}-x\right)^2}-12}\left(\sqrt{2}\arctan\left(\frac{\sqrt{\frac{3x^2}{\left(-\frac{3}{2}-x\right)^2}-12}\,\sqrt{2}}{6}\right)+5\,\mathrm{arctanh}\left(\right.\right.}{24\sqrt{\frac{\frac{x^2}{\left(-\frac{3}{2}-x\right)^2}-4}{\left(1+\frac{x}{-\frac{3}{2}-x}\right)^2}}\left(1+\frac{x}{-\frac{3}{2}-x}\right)}$
trager	$\left(-\frac{x}{4}+\frac{5}{2}\right)\sqrt{-x^2-4x-3}-\frac{5\ln\left(\frac{12\,\mathrm{RootOf}\left(12\_Z^2+20\_Z+9\right)^2x+28\,\mathrm{RootOf}\left(12\_Z^2+20\_Z+9\right)x+12\,\mathrm{RootOf}\left(12\_Z^2+20\right.}{2\,\mathrm{RootOf}\left(12\_Z^2+20\_Z+9\right)x+x-1}\right.}{4}$

[In] int(x^4/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/4*(x-10)*(x^2+4*x+3)/(-x^2-4*x-3)^(1/2)+11/2*arcsin(2+x)+1/24*3^(1/2)*4^(
1/2)*(3*x^2/(-3/2-x)^2-12)^(1/2)*(2^(1/2)*arctan(1/6*(3*x^2/(-3/2-x)^2-12)^
(1/2)*2^(1/2))+5*arctanh(3*x/(-3/2-x)/(3*x^2/(-3/2-x)^2-12)^(1/2)))/((x^2/(
-3/2-x)^2-4)/(1+x/(-3/2-x))^2)^(1/2)/(1+x/(-3/2-x))

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.27

$$\int \frac{x^4}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = -\frac{1}{4}\sqrt{-x^2-4x-3}(x-10) + \frac{1}{8}\sqrt{2}\arctan\left(\frac{\sqrt{2}x+3\sqrt{2}\sqrt{-x^2-4x-3}}{2(2x+3)}\right) + \frac{1}{8}\sqrt{2}\arctan\left(-\frac{\sqrt{2}x-3\sqrt{2}\sqrt{-x^2-4x-3}}{2(2x+3)}\right) - \frac{11}{2}\arctan\left(\frac{\sqrt{-x^2-4x-3}(x+2)}{x^2+4x+3}\right) + \frac{5}{16}\log\left(-\frac{2\sqrt{-x^2-4x-3}x+4x+3}{x^2}\right) - \frac{5}{16}\log\left(\frac{2\sqrt{-x^2-4x-3}x-4x-3}{x^2}\right)$$

```
[In] integrate(x^4/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(-x^2 - 4*x - 3)*(x - 10) + 1/8*sqrt(2)*arctan(1/2*(sqrt(2)*x + 3*
sqrt(2)*sqrt(-x^2 - 4*x - 3))/(2*x + 3)) + 1/8*sqrt(2)*arctan(-1/2*(sqrt(2)
*x - 3*sqrt(2)*sqrt(-x^2 - 4*x - 3))/(2*x + 3)) - 11/2*arctan(sqrt(-x^2 - 4
*x - 3)*(x + 2)/(x^2 + 4*x + 3)) + 5/16*log(-(2*sqrt(-x^2 - 4*x - 3)*x + 4*
x + 3)/x^2) - 5/16*log((2*sqrt(-x^2 - 4*x - 3)*x - 4*x - 3)/x^2)
```

Sympy [F]

$$\int \frac{x^4}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = \int \frac{x^4}{\sqrt{-(x+1)(x+3)}\,(2x^2+4x+3)}\,dx$$

```
[In] integrate(x**4/(2*x**2+4*x+3)/(-x**2-4*x-3)**(1/2),x)
```

```
[Out] Integral(x**4/(sqrt(-(x + 1)*(x + 3))*(2*x**2 + 4*x + 3)), x)
```

Maxima [F]

$$\int \frac{x^4}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = \int \frac{x^4}{(2\,x^2+4\,x+3)\sqrt{-x^2-4\,x-3}}\,dx$$

```
[In] integrate(x^4/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="maxima")
[Out] integrate(x^4/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.34

$$\begin{aligned}
&\int \frac{x^4}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx \\
&= -\frac{1}{4}\,\sqrt{-x^2-4\,x-3}(x-10) + \frac{1}{4}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\frac{3\,\left(\sqrt{-x^2-4\,x-3}-1\right)}{x+2}+1\right)\right) \\
&\quad + \frac{1}{4}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\frac{\sqrt{-x^2-4\,x-3}-1}{x+2}+1\right)\right) + \frac{11}{2}\,\arcsin\left(x+2\right) \\
&\quad - \frac{5}{8}\,\log\left(\frac{2\,\left(\sqrt{-x^2-4\,x-3}-1\right)}{x+2} + \frac{3\,\left(\sqrt{-x^2-4\,x-3}-1\right)^2}{\left(x+2\right)^2}+1\right) \\
&\quad + \frac{5}{8}\,\log\left(\frac{2\,\left(\sqrt{-x^2-4\,x-3}-1\right)}{x+2} + \frac{\left(\sqrt{-x^2-4\,x-3}-1\right)^2}{\left(x+2\right)^2}+3\right)
\end{aligned}$$

```
[In] integrate(x^4/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="giac")
[Out] -1/4*sqrt(-x^2 - 4*x - 3)*(x - 10) + 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(3*(sqr
t(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) + 1/4*sqrt(2)*arctan(1/2*sqrt(2)*((sqr
t(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) + 11/2*arcsin(x + 2) - 5/8*log(2*(sqrt
(-x^2 - 4*x - 3) - 1)/(x + 2) + 3*(sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 +
1) + 5/8*log(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + (sqrt(-x^2 - 4*x - 3) -
 1)^2/(x + 2)^2 + 3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx=\int \frac{x^4}{\sqrt{-x^2-4\,x-3}\,\left(2\,x^2+4\,x+3\right)}\,dx$$

```
[In] int(x^4/((- 4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)),x)
[Out] int(x^4/((- 4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)), x)
```

3.127 $\int \frac{x^3}{\sqrt{-3-4x-x^2}(3+4x+2x^2)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 115

$$\int \frac{x^3}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = -\frac{1}{2}\sqrt{-3-4x-x^2}-2\arcsin(2+x) + \frac{\arctan\left(\frac{1-\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right)}{2\sqrt{2}} - \frac{\arctan\left(\frac{1+\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right)}{2\sqrt{2}} + \operatorname{arctanh}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right)$$

```
[Out] -2*arcsin(2+x)+arctanh(x/(-x^2-4*x-3)^(1/2))+1/4*arctan(1/2*(1+(-3-x)/(-x^2
-4*x-3)^(1/2))*2^(1/2))*2^(1/2)-1/4*arctan(1/2*(1+(3+x)/(-x^2-4*x-3)^(1/2))
*2^(1/2))*2^(1/2)-1/2*(-x^2-4*x-3)^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.433, Rules used = {6860, 633, 222, 654, 1042, 1000, 12, 1040, 1175, 632, 210, 1041, 212}

$$\int \frac{x^3}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = -2\arcsin(x+2) + \frac{\arctan\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)}{2\sqrt{2}} - \frac{\arctan\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)}{2\sqrt{2}} + \operatorname{arctanh}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right) - \frac{1}{2}\sqrt{-x^2-4x-3}$$

```
[In] Int[x^3/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

```
[Out] -1/2*Sqrt[-3 - 4*x - x^2] - 2*ArcSin[2 + x] + ArcTan[(1 - (3 + x)/Sqrt[-3 -
 4*x - x^2])/Sqrt[2]]/(2*Sqrt[2]) - ArcTan[(1 + (3 + x)/Sqrt[-3 - 4*x - x^2
])/Sqrt[2]]/(2*Sqrt[2]) + ArcTanh[x/Sqrt[-3 - 4*x - x^2]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1000

```
Int[1/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*
(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f)
, 2]}, Dist[1/(2*q), Int[(c*d - a*f + q + (c*e - b*f)*x)/((a + b*x + c*x^2)
*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[(c*d - a*f - q + (c*e -
 b*f)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b,
 c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[c*e -
 b*f, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1040

```
Int[(x_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_.)*(x_) + (f_.)
*(x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[(1 - d*x^2)/(c*e - b*f - e*(2*
c*d - b*e + 2*a*f)*x^2 + d^2*(c*e - b*f)*x^4), x], x, (1 + (e + Sqrt[e^2 -
4*d*f])*(x/(2*d)))/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0]
```

Rule 1041

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_
.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[g, Subst[Int[1/(a + (c*d - a*f)
*x^2), x], x, x/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h
}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0] &&
 EqQ[2*h*d - g*e, 0]
```

Rule 1042

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_
.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-(2*h*d - g*e)/e, Int[1/((a + b
*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/e, Int[(2*d + e*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g
, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0]
 && NeQ[2*h*d - g*e, 0]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 6860

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
```

```
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(-\frac{1}{\sqrt{-3-4x-x^2}}+\frac{x}{2\sqrt{-3-4x-x^2}}+\frac{6+5x}{2\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\right) dx \\
&= \frac{1}{2}\int \frac{x}{\sqrt{-3-4x-x^2}}\,dx+\frac{1}{2}\int \frac{6+5x}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx-\int \frac{1}{\sqrt{-3-4x-x^2}}\,dx \\
&= -\frac{1}{2}\sqrt{-3-4x-x^2}+\frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{4}}}\,dx,x,-4-2x\right) \\
&\quad -\frac{5}{8}\int \frac{-6-4x}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx \\
&\quad -\frac{3}{4}\int \frac{1}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx-\int \frac{1}{\sqrt{-3-4x-x^2}}\,dx \\
&= -\frac{1}{2}\sqrt{-3-4x-x^2}-\sin^{-1}(2+x)+\frac{1}{8}\int \frac{-6-4x}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx-\frac{1}{8}\int \\
&\quad -\frac{4x}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx+\frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{4}}}\,dx,x,-4-2x\right) \\
&\quad +\frac{15}{4}\text{Subst}\left(\int \frac{1}{3-3x^2}\,dx,x,\frac{x}{\sqrt{-3-4x-x^2}}\right) \\
&= -\frac{1}{2}\sqrt{-3-4x-x^2}-2\sin^{-1}(2+x)+\frac{5}{4}\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) \\
&\quad +\frac{1}{2}\int \frac{x}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx \\
&\quad -\frac{3}{4}\text{Subst}\left(\int \frac{1}{3-3x^2}\,dx,x,\frac{x}{\sqrt{-3-4x-x^2}}\right) \\
&= -\frac{1}{2}\sqrt{-3-4x-x^2}-2\sin^{-1}(2+x)+\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) \\
&\quad +4\text{Subst}\left(\int \frac{1+3x^2}{-4-8x^2-36x^4}\,dx,x,\frac{1+\frac{x}{3}}{\sqrt{-3-4x-x^2}}\right) \\
&= -\frac{1}{2}\sqrt{-3-4x-x^2}-2\sin^{-1}(2+x)+\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) \\
&\quad -\frac{1}{6}\text{Subst}\left(\int \frac{1}{\frac{1}{3}-\frac{2x}{3}+x^2}\,dx,x,\frac{1+\frac{x}{3}}{\sqrt{-3-4x-x^2}}\right) \\
&\quad -\frac{1}{6}\text{Subst}\left(\int \frac{1}{\frac{1}{3}+\frac{2x}{3}+x^2}\,dx,x,\frac{1+\frac{x}{3}}{\sqrt{-3-4x-x^2}}\right)
\end{aligned}$$

$$= -\frac{1}{2}\sqrt{-3-4x-x^2} - 2\sin^{-1}(2+x) + \tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) + \frac{1}{3}\text{Subst}\left(\int \frac{1}{-\frac{8}{9}-x^2}\,dx, x, \frac{2}{3}\left(-1+\frac{3+x}{\sqrt{-3-4x-x^2}}\right)\right) + \frac{1}{3}\text{Subst}\left(\int \frac{1}{-\frac{8}{9}-x^2}\,dx, x, \frac{2}{3}\left(1+\frac{3+x}{\sqrt{-3-4x-x^2}}\right)\right)$$

$$= -\frac{1}{2}\sqrt{-3-4x-x^2} - 2\sin^{-1}(2+x) + \frac{\tan^{-1}\left(\frac{1-\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right)}{2\sqrt{2}} - \frac{\tan^{-1}\left(\frac{1+\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right)}{2\sqrt{2}} + \tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right)$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.82

$$\int \frac{x^3}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = -\frac{1}{2}\sqrt{-3-4x-x^2} - \frac{\arctan\left(\frac{3+2x}{\sqrt{2}\sqrt{-3-4x-x^2}}\right)}{2\sqrt{2}} + 4\arctan\left(\frac{\sqrt{-3-4x-x^2}}{3+x}\right) + \operatorname{arctanh}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right)$$

```
[In] Integrate[x^3/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

```
[Out] -1/2*Sqrt[-3 - 4*x - x^2] - ArcTan[(3 + 2*x)/(Sqrt[2]*Sqrt[-3 - 4*x - x^2])
]/(2*Sqrt[2]) + 4*ArcTan[Sqrt[-3 - 4*x - x^2]/(3 + x)] + ArcTanh[x/Sqrt[-3
- 4*x - x^2]]
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.25

method	result
default	$-2\arcsin(2+x)-\frac{\sqrt{-x^2-4x-3}}{2}+\frac{\sqrt{3}\,\sqrt{4}\,\sqrt{\frac{3x^2}{\left(-\frac{3}{2}-x\right)^2}-12}\left(\sqrt{2}\,\arctan\left(\frac{\sqrt{\frac{3x^2}{\left(-\frac{3}{2}-x\right)^2}-12}\,\sqrt{2}}{6}\right)-4\,\mathrm{arctanh}\left(\frac{3}{\left(-\frac{3}{2}-x\right)\sqrt{\cdots}}\right.\right.}{24\sqrt{\frac{\frac{x^2}{\left(-\frac{3}{2}-x\right)^2}-4}{\left(1+\frac{x}{-\frac{3}{2}-x}\right)^2}}\left(1+\frac{x}{-\frac{3}{2}-x}\right)}$
risch	$\frac{x^2+4x+3}{2\sqrt{-x^2-4x-3}}-2\arcsin(2+x)+\frac{\sqrt{3}\,\sqrt{4}\,\sqrt{\frac{3x^2}{\left(-\frac{3}{2}-x\right)^2}-12}\left(\sqrt{2}\,\arctan\left(\frac{\sqrt{\frac{3x^2}{\left(-\frac{3}{2}-x\right)^2}-12}\,\sqrt{2}}{6}\right)-4\,\mathrm{arctanh}\left(\frac{3x}{\left(-\frac{3}{2}-x\right)\sqrt{\cdots}}\right.\right.}{24\sqrt{\frac{\frac{x^2}{\left(-\frac{3}{2}-x\right)^2}-4}{\left(1+\frac{x}{-\frac{3}{2}-x}\right)^2}}\left(1+\frac{x}{-\frac{3}{2}-x}\right)}$
trager	$-\frac{\sqrt{-x^2-4x-3}}{2}+\frac{3\,\mathrm{RootOf}\left(24\_Z^2-16\_Z+3\right)\ln\left(\frac{72\,\mathrm{RootOf}\left(24\_Z^2-16\_Z+3\right)^2x-72\,\mathrm{RootOf}\left(24\_Z^2-16\_Z+3\right)x-36\,\mathrm{RootOf}\cdots}{12\,\mathrm{RootOf}\left(24\_Z^2-16\_Z+3\right)x-\cdots}\right.}{2}$

```
[In] int(x^3/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*arcsin(2+x)-1/2*(-x^2-4*x-3)^(1/2)+1/24*3^(1/2)*4^(1/2)*(3*x^2/(-3/2-x)^
2-12)^(1/2)*(2^(1/2)*arctan(1/6*(3*x^2/(-3/2-x)^2-12)^(1/2)*2^(1/2))-4*arct
anh(3*x/(-3/2-x)/(3*x^2/(-3/2-x)^2-12)^(1/2)))/((x^2/(-3/2-x)^2-4)/(1+x/(-3
/2-x))^2)^(1/2)/(1+x/(-3/2-x))
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.52

$$\int \frac{x^3}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = \frac{1}{8}\sqrt{2}\arctan\left(\frac{\sqrt{2}x+3\sqrt{2}\sqrt{-x^2-4\,x-3}}{2\,(2\,x+3)}\right) + \frac{1}{8}\sqrt{2}\arctan\left(-\frac{\sqrt{2}x-3\sqrt{2}\sqrt{-x^2-4\,x-3}}{2\,(2\,x+3)}\right) - \frac{1}{2}\sqrt{-x^2-4\,x-3} + 2\,\arctan\left(\frac{\sqrt{-x^2-4\,x-3}(x+2)}{x^2+4\,x+3}\right) - \frac{1}{4}\log\left(-\frac{2\sqrt{-x^2-4\,x-3}x+4\,x+3}{x^2}\right) + \frac{1}{4}\log\left(\frac{2\sqrt{-x^2-4\,x-3}x-4\,x-3}{x^2}\right)$$

```
[In] integrate(x^3/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(2)*arctan(1/2*(sqrt(2)*x + 3*sqrt(2)*sqrt(-x^2 - 4*x - 3))/(2*x +
3)) + 1/8*sqrt(2)*arctan(-1/2*(sqrt(2)*x - 3*sqrt(2)*sqrt(-x^2 - 4*x - 3))/
(2*x + 3)) - 1/2*sqrt(-x^2 - 4*x - 3) + 2*arctan(sqrt(-x^2 - 4*x - 3)*(x +
2)/(x^2 + 4*x + 3)) - 1/4*log(-(2*sqrt(-x^2 - 4*x - 3)*x + 4*x + 3)/x^2) +
1/4*log((2*sqrt(-x^2 - 4*x - 3)*x - 4*x - 3)/x^2)
```

Sympy [F]

$$\int \frac{x^3}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = \int \frac{x^3}{\sqrt{-(x+1)\,(x+3)}\,(2x^2+4x+3)}\,dx$$

```
[In] integrate(x**3/(2*x**2+4*x+3)/(-x**2-4*x-3)**(1/2),x)
```

```
[Out] Integral(x**3/(sqrt(-(x + 1)*(x + 3))*(2*x**2 + 4*x + 3)), x)
```

Maxima [F]

$$\int \frac{x^3}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = \int \frac{x^3}{(2\,x^2+4\,x+3)\sqrt{-x^2-4\,x-3}}\,dx$$

```
[In] integrate(x^3/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="maxima")
[Out] integrate(x^3/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.61

$$\begin{aligned}
&\int \frac{x^3}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx \\
&= \frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{3\left(\sqrt{-x^2-4\,x-3}-1\right)}{x+2}+1\right)\right) \\
&\quad + \frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{\sqrt{-x^2-4\,x-3}-1}{x+2}+1\right)\right) - \frac{1}{2}\sqrt{-x^2-4\,x-3} \\
&\quad - 2\arcsin\left(x+2\right) + \frac{1}{2}\log\left(\frac{2\left(\sqrt{-x^2-4\,x-3}-1\right)}{x+2} + \frac{3\left(\sqrt{-x^2-4\,x-3}-1\right)^2}{(x+2)^2}+1\right) \\
&\quad - \frac{1}{2}\log\left(\frac{2\left(\sqrt{-x^2-4\,x-3}-1\right)}{x+2} + \frac{\left(\sqrt{-x^2-4\,x-3}-1\right)^2}{(x+2)^2}+3\right)
\end{aligned}$$

```
[In] integrate(x^3/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="giac")
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(3*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1))
+ 1/4*sqrt(2)*arctan(1/2*sqrt(2)*((sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1))
- 1/2*sqrt(-x^2 - 4*x - 3) - 2*arcsin(x + 2) + 1/2*log(2*(sqrt(-x^2 - 4*x -
 3) - 1)/(x + 2) + 3*(sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 1) - 1/2*log(
2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + (sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)
^2 + 3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = \int \frac{x^3}{\sqrt{-x^2-4\,x-3}\,\left(2\,x^2+4\,x+3\right)}\,dx$$

```
[In] int(x^3/((- 4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)),x)
[Out] int(x^3/((- 4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)), x)
```

3.128 $\int \frac{x^2}{\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)} \, dx$

Optimal result

Integrand size = 30, antiderivative size = 98

$$\int \frac{x^2}{\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)} \, dx = \frac{1}{2}\arcsin(2+x)-\frac{\arctan\left(\frac{1-\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{\arctan\left(\frac{1+\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{1}{2}\text{arctanh}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right)$$

```
[Out] 1/2*arcsin(2+x)-1/2*arctanh(x/(-x^2-4*x-3)^(1/2))-1/2*arctan(1/2*(1+(-3-x)/
(-x^2-4*x-3)^(1/2))*2^(1/2))*2^(1/2)+1/2*arctan(1/2*(1+(3+x)/(-x^2-4*x-3)^(
1/2))*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1091, 633, 222, 1042, 1000, 12, 1040, 1175, 632, 210, 1041, 212}

$$\int \frac{x^2}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = \frac{1}{2}\arcsin(x+2) - \frac{\arctan\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)}{\sqrt{2}} + \frac{\arctan\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)}{\sqrt{2}} - \frac{1}{2}\text{arctanh}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)$$

```
[In] Int[x^2/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

```
[Out] ArcSin[2 + x]/2 - ArcTan[(1 - (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]]/Sqrt[2
] + ArcTan[(1 + (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]]/Sqrt[2] - ArcTanh[x/
Sqrt[-3 - 4*x - x^2]]/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 633

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 1000

```
Int[1/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*
(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f)
, 2]}, Dist[1/(2*q), Int[(c*d - a*f + q + (c*e - b*f)*x)/((a + b*x + c*x^2)
*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[(c*d - a*f - q + (c*e -
 b*f)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b,
 c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[c*e -
 b*f, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1040

```
Int[(x_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_.)*(x_) + (f_.)
*(x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[(1 - d*x^2)/(c*e - b*f - e*(2*
c*d - b*e + 2*a*f)*x^2 + d^2*(c*e - b*f)*x^4), x], x, (1 + (e + Sqrt[e^2 -
4*d*f])*(x/(2*d)))/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0]
```

Rule 1041

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_
.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[g, Subst[Int[1/(a + (c*d - a*f)
*x^2), x], x, x/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h
}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0] &&
 EqQ[2*h*d - g*e, 0]
```

Rule 1042

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_
.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-(2*h*d - g*e)/e, Int[1/((a + b
*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/e, Int[(2*d + e*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g
, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0]
 && NeQ[2*h*d - g*e, 0]
```

Rule 1091

```
Int[((A_.) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
 (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + e*x + f
```

```
*x^2], x], x] + Dist[1/c, Int[(A*c - a*C - b*C*x)/((a + b*x + c*x^2)*Sqrt[d
 + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}\int \frac{1}{\sqrt{-3-4x-x^2}}\,dx+\frac{1}{2}\int \frac{-3-4x}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx \\
&= -\left(\frac{1}{4}\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{4}}}\,dx,x,-4-2x\right)\right) \\
&\quad +\frac{1}{2}\int \frac{-6-4x}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx+\frac{3}{2}\int \frac{1}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx \\
&= \frac{1}{2}\sin^{-1}(2+x)-\frac{1}{4}\int \frac{-6-4x}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx+\frac{1}{4}\int \\
&\quad -\frac{4x}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx-3\text{Subst}\left(\int \frac{1}{3-3x^2}\,dx,x,\frac{x}{\sqrt{-3-4x-x^2}}\right) \\
&= \frac{1}{2}\sin^{-1}(2+x)-\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) \\
&\quad +\frac{3}{2}\text{Subst}\left(\int \frac{1}{3-3x^2}\,dx,x,\frac{x}{\sqrt{-3-4x-x^2}}\right) \\
&\quad -\int \frac{x}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx \\
&= \frac{1}{2}\sin^{-1}(2+x)-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) \\
&\quad -8\text{Subst}\left(\int \frac{1+3x^2}{-4-8x^2-36x^4}\,dx,x,\frac{1+\frac{x}{3}}{\sqrt{-3-4x-x^2}}\right)
\end{aligned}$$

$$= \frac{1}{2}\sin^{-1}(2+x) - \frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) + \frac{1}{3}\text{Subst}\left(\int \frac{1}{\frac{1}{3}-\frac{2x}{3}+x^2}\,dx, x, \frac{1+\frac{x}{3}}{\sqrt{-3-4x-x^2}}\right) + \frac{1}{3}\text{Subst}\left(\int \frac{1}{\frac{1}{3}+\frac{2x}{3}+x^2}\,dx, x, \frac{1+\frac{x}{3}}{\sqrt{-3-4x-x^2}}\right)$$

$$= \frac{1}{2}\sin^{-1}(2+x) - \frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) - \frac{2}{3}\text{Subst}\left(\int \frac{1}{-\frac{8}{9}-x^2}\,dx, x, \frac{2}{3}\left(-1+\frac{3+x}{\sqrt{-3-4x-x^2}}\right)\right) - \frac{2}{3}\text{Subst}\left(\int \frac{1}{-\frac{8}{9}-x^2}\,dx, x, \frac{2}{3}\left(1+\frac{3+x}{\sqrt{-3-4x-x^2}}\right)\right)$$

$$= \frac{1}{2}\sin^{-1}(2+x) - \frac{\tan^{-1}\left(\frac{1-\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right)}{\sqrt{2}} + \frac{\tan^{-1}\left(\frac{1+\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right)}{\sqrt{2}} - \frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right)$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.79

$$\int \frac{x^2}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = \frac{\arctan\left(\frac{3+2x}{\sqrt{2}\sqrt{-3-4x-x^2}}\right)}{\sqrt{2}} - \arctan\left(\frac{\sqrt{-3-4x-x^2}}{3+x}\right) - \frac{1}{2}\text{arctanh}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right)$$

```
[In] Integrate[x^2/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
[Out] ArcTan[(3 + 2*x)/(Sqrt[2]*Sqrt[-3 - 4*x - x^2])]/Sqrt[2] - ArcTan[Sqrt[-3 -
 4*x - x^2]/(3 + x)] - ArcTanh[x/Sqrt[-3 - 4*x - x^2]]/2
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.33

method	result
default	$\frac{\arcsin(2+x)}{2}-\frac{\sqrt{3}\,\sqrt{4}\,\sqrt{\frac{3x^2}{\left(-\frac{3}{2}-x\right)^2}-12}\,\left(\sqrt{2}\,\arctan\left(\frac{\sqrt{\frac{3x^2}{\left(-\frac{3}{2}-x\right)^2}-12}\,\sqrt{2}}{6}\right)-\operatorname{arctanh}\left(\frac{3x}{\left(-\frac{3}{2}-x\right)\sqrt{\frac{3x^2}{\left(-\frac{3}{2}-x\right)^2}-12}}\right)\right)}{12\sqrt{\frac{\frac{x^2}{\left(-\frac{3}{2}-x\right)^2}-4}{\left(1+\frac{x}{-\frac{3}{2}-x}\right)^2}}\left(1+\frac{x}{-\frac{3}{2}-x}\right)}$
trager	$\frac{\ln\left(-\frac{16\,\mathrm{RootOf}\left(16\_Z^2-8\_Z+3\right)^2x+8\,\mathrm{RootOf}\left(16\_Z^2-8\_Z+3\right)x+24\,\mathrm{RootOf}\left(16\_Z^2-8\_Z+3\right)+6\sqrt{-x^2-4x-3}-3x-6}{4\,\mathrm{RootOf}\left(16\_Z^2-8\_Z+3\right)x-3x-3}\right)}{2}-\ln\Bigg($

```
[In] int(x^2/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*arcsin(2+x)-1/12*3^(1/2)*4^(1/2)*(3*x^2/(-3/2-x)^2-12)^(1/2)*(2^(1/2)*a
rctan(1/6*(3*x^2/(-3/2-x)^2-12)^(1/2)*2^(1/2))-arctanh(3*x/(-3/2-x)/(3*x^2/
(-3/2-x)^2-12)^(1/2)))/((x^2/(-3/2-x)^2-4)/(1+x/(-3/2-x))^2)^(1/2)/(1+x/(-3
/2-x))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.64

$$\begin{aligned}\int \frac{x^2}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = & -\frac{1}{4}\sqrt{2}\arctan\left(\frac{\sqrt{2}x+3\sqrt{2}\sqrt{-x^2-4x-3}}{2\,(2\,x+3)}\right) \\ & -\frac{1}{4}\sqrt{2}\arctan\left(-\frac{\sqrt{2}x-3\sqrt{2}\sqrt{-x^2-4x-3}}{2\,(2\,x+3)}\right) \\ & -\frac{1}{2}\arctan\left(\frac{\sqrt{-x^2-4x-3}(x+2)}{x^2+4x+3}\right) \\ & +\frac{1}{8}\log\left(-\frac{2\sqrt{-x^2-4x-3}x+4x+3}{x^2}\right) \\ & -\frac{1}{8}\log\left(\frac{2\sqrt{-x^2-4x-3}x-4x-3}{x^2}\right)\end{aligned}$$

```
[In] integrate(x^2/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(2)*arctan(1/2*(sqrt(2)*x + 3*sqrt(2)*sqrt(-x^2 - 4*x - 3))/(2*x +
 3)) - 1/4*sqrt(2)*arctan(-1/2*(sqrt(2)*x - 3*sqrt(2)*sqrt(-x^2 - 4*x - 3))
/(2*x + 3)) - 1/2*arctan(sqrt(-x^2 - 4*x - 3)*(x + 2)/(x^2 + 4*x + 3)) + 1/
8*log(-(2*sqrt(-x^2 - 4*x - 3)*x + 4*x + 3)/x^2) - 1/8*log((2*sqrt(-x^2 - 4
*x - 3)*x - 4*x - 3)/x^2)
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = \int \frac{x^2}{\sqrt{-(x+1)(x+3)}\,(2x^2+4x+3)}\,dx$$

```
[In] integrate(x**2/(2*x**2+4*x+3)/(-x**2-4*x-3)**(1/2),x)
[Out] Integral(x**2/(sqrt(-(x + 1)*(x + 3))*(2*x**2 + 4*x + 3)), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = \int \frac{x^2}{(2\,x^2+4\,x+3)\sqrt{-x^2-4\,x-3}}\,dx$$

```
[In] integrate(x^2/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="maxima")
[Out] integrate(x^2/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(82) = 164$.

Time = 0.29 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.74

$$\begin{aligned}\int \frac{x^2}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx &= -\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{3\left(\sqrt{-x^2-4\,x-3}-1\right)}{x+2}+1\right)\right) \\ &\quad -\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{\sqrt{-x^2-4\,x-3}-1}{x+2}+1\right)\right)+\frac{1}{2}\arcsin\left(x+2\right) \\ &\quad -\frac{1}{4}\log\left(\frac{2\left(\sqrt{-x^2-4\,x-3}-1\right)}{x+2}+\frac{3\left(\sqrt{-x^2-4\,x-3}-1\right)^2}{(x+2)^2}+1\right) \\ &\quad +\frac{1}{4}\log\left(\frac{2\left(\sqrt{-x^2-4\,x-3}-1\right)}{x+2}+\frac{\left(\sqrt{-x^2-4\,x-3}-1\right)^2}{(x+2)^2}+3\right)\end{aligned}$$

```
[In] integrate(x^2/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*arctan(1/2*sqrt(2)*(3*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1))
 - 1/2*sqrt(2)*arctan(1/2*sqrt(2)*((sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1))
 + 1/2*arcsin(x + 2) - 1/4*log(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 3*(sq
rt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 1) + 1/4*log(2*(sqrt(-x^2 - 4*x - 3)
- 1)/(x + 2) + (sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = \int \frac{x^2}{\sqrt{-x^2-4\,x-3}\,\left(2\,x^2+4\,x+3\right)}\,dx$$

```
[In] int(x^2/((- 4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)),x)
```

```
[Out] int(x^2/((- 4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)), x)
```

3.129 $\int \frac{x}{\sqrt{-3-4x-x^2}(3+4x+2x^2)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 68

$$\int \frac{x}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = -\frac{\arctan\left(\frac{1-\frac{3\sqrt{-1-x}}{\sqrt{3+x}}}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{\arctan\left(\frac{1+\frac{3\sqrt{-1-x}}{\sqrt{3+x}}}{\sqrt{2}}\right)}{\sqrt{2}}$$

```
[Out] -1/2*arctan(1/2*(1-3*(-1-x)^(1/2)/(3+x)^(1/2))*2^(1/2))*2^(1/2)+1/2*arctan(
1/2*(1+3*(-1-x)^(1/2)/(3+x)^(1/2))*2^(1/2))*2^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1040, 1175, 632, 210}

$$\int \frac{x}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = \frac{\arctan\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{\arctan\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)}{\sqrt{2}}$$

```
[In] Int[x/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

```
[Out] ArcTan[(1 - (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]]/Sqrt[2] - ArcTan[(1 + (3
 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]]/Sqrt[2]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1040

```
Int[(x_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_.)*(x_) + (f_.)
*(x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[(1 - d*x^2)/(c*e - b*f - e*(2*
c*d - b*e + 2*a*f)*x^2 + d^2*(c*e - b*f)*x^4), x], x, (1 + (e + Sqrt[e^2 -
4*d*f])*(x/(2*d)))/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= 8\text{Subst}\left(\int \frac{1+3x^2}{-4-8x^2-36x^4}\,dx, x, \frac{1+\frac{x}{3}}{\sqrt{-3-4x-x^2}}\right)\\
&= -\left(\frac{1}{3}\text{Subst}\left(\int \frac{1}{\frac{1}{3}-\frac{2x}{3}+x^2}\,dx, x, \frac{1+\frac{x}{3}}{\sqrt{-3-4x-x^2}}\right)\right)\\
&\quad -\frac{1}{3}\text{Subst}\left(\int \frac{1}{\frac{1}{3}+\frac{2x}{3}+x^2}\,dx, x, \frac{1+\frac{x}{3}}{\sqrt{-3-4x-x^2}}\right)\\
&= \frac{2}{3}\text{Subst}\left(\int \frac{1}{-\frac{8}{9}-x^2}\,dx, x, \frac{2}{3}\left(-1+\frac{3+x}{\sqrt{-3-4x-x^2}}\right)\right)\\
&\quad +\frac{2}{3}\text{Subst}\left(\int \frac{1}{-\frac{8}{9}-x^2}\,dx, x, \frac{2}{3}\left(1+\frac{3+x}{\sqrt{-3-4x-x^2}}\right)\right)\\
&= \frac{\tan^{-1}\left(\frac{1-\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(\frac{1+\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right)}{\sqrt{2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.49

$$\int \frac{x}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = -\frac{\arctan\left(\frac{3+2x}{\sqrt{2}\sqrt{-3-4x-x^2}}\right)}{\sqrt{2}}$$

```
[In] Integrate[x/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
[Out] -(ArcTan[(3 + 2*x)/(Sqrt[2]*Sqrt[-3 - 4*x - x^2])]/Sqrt[2])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.69 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.19

method	result	size
trager	$\frac{\operatorname{RootOf}\left(\_Z^2+2\right)\ln\left(\frac{6\operatorname{RootOf}\left(\_Z^2+2\right)x^2+20\operatorname{RootOf}\left(\_Z^2+2\right)x+8x\sqrt{-x^2-4x-3}+15\operatorname{RootOf}\left(\_Z^2+2\right)+12\sqrt{-x^2-4x-3}}{2x^2+4x+3}\right)}{4}$	81
default	$\frac{\sqrt{3}\,\sqrt{4}\,\sqrt{\frac{3x^2}{\left(-\frac{3}{2}-x\right)^2}-12}\,\sqrt{2}\,\arctan\left(\frac{\sqrt{\frac{3x^2}{\left(-\frac{3}{2}-x\right)^2}-12}\,\sqrt{2}}{6}\right)}{12\sqrt{\frac{\frac{x^2}{\left(-\frac{3}{2}-x\right)^2}-4}{\left(1+\frac{x}{-\frac{3}{2}-x}\right)^2}}\left(1+\frac{x}{-\frac{3}{2}-x}\right)}$	92

```
[In] int(x/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/4*RootOf(_Z^2+2)*ln((6*RootOf(_Z^2+2)*x^2+20*RootOf(_Z^2+2)*x+8*x*(-x^2-4
*x-3)^(1/2)+15*RootOf(_Z^2+2)+12*(-x^2-4*x-3)^(1/2))/(2*x^2+4*x+3))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.74

$$\int \frac{x}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = \frac{1}{4}\sqrt{2}\arctan\left(\frac{\sqrt{2}(6\,x^2+20\,x+15)\sqrt{-x^2-4\,x-3}}{4\,(2\,x^3+11\,x^2+18\,x+9)}\right)$$

```
[In] integrate(x/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="fricas")
[Out] 1/4*sqrt(2)*arctan(1/4*sqrt(2)*(6*x^2 + 20*x + 15)*sqrt(-x^2 - 4*x - 3)/(2*
x^3 + 11*x^2 + 18*x + 9))
```

Sympy [F]

$$\int \frac{x}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = \int \frac{x}{\sqrt{-(x+1)(x+3)}\,(2x^2+4x+3)}\,dx$$

```
[In] integrate(x/(2*x**2+4*x+3)/(-x**2-4*x-3)**(1/2),x)
[Out] Integral(x/(sqrt(-(x + 1)*(x + 3))*(2*x**2 + 4*x + 3)), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = \int \frac{x}{(2\,x^2+4\,x+3)\sqrt{-x^2-4\,x-3}}\,dx$$

```
[In] integrate(x/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="maxima")
[Out] integrate(x/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00

$$\begin{aligned} &\int \frac{x}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx \\ &= \frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{3\left(\sqrt{-x^2-4\,x-3}-1\right)}{x+2}+1\right)\right) \\ &\quad + \frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{\sqrt{-x^2-4\,x-3}-1}{x+2}+1\right)\right) \end{aligned}$$

```
[In] integrate(x/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(3*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1))
+ 1/2*sqrt(2)*arctan(1/2*sqrt(2)*((sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = \int \frac{x}{\sqrt{-x^2-4\,x-3}\,\left(2\,x^2+4\,x+3\right)}\,dx$$

```
[In] int(x/((- 4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)),x)
[Out] int(x/((- 4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)), x)
```

3.130 $\int \frac{1}{\sqrt{-3-4x-x^2}(3+4x+2x^2)} \, dx$

Optimal result

Integrand size = 27, antiderivative size = 95

$$\int \frac{1}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)} \, dx = -\frac{1}{3}\sqrt{2}\arctan\left(\frac{1-\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right) + \frac{1}{3}\sqrt{2}\arctan\left(\frac{1+\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right) + \frac{1}{3}\text{arctanh}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right)$$

```
[Out] 1/3*arctanh(x/(-x^2-4*x-3)^(1/2))-1/3*arctan(1/2*(1+(-3-x)/(-x^2-4*x-3)^(1/
2))*2^(1/2))*2^(1/2)+1/3*arctan(1/2*(1+(3+x)/(-x^2-4*x-3)^(1/2))*2^(1/2))*2
^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {1000, 12, 1040, 1175, 632, 210, 1041, 212}

$$\int \frac{1}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)} \, dx = -\frac{1}{3}\sqrt{2}\arctan\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right) + \frac{1}{3}\sqrt{2}\arctan\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right) + \frac{1}{3}\text{arctanh}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)$$

```
[In] Int[1/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

```
[Out] -1/3*(Sqrt[2]*ArcTan[(1 - (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]]) + (Sqrt[2
]*ArcTan[(1 + (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]])/3 + ArcTanh[x/Sqrt[-3
 - 4*x - x^2]]/3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1000

```
Int[1/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*
(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f)
, 2]}, Dist[1/(2*q), Int[(c*d - a*f + q + (c*e - b*f)*x)/((a + b*x + c*x^2)
*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[(c*d - a*f - q + (c*e -
 b*f)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b,
 c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[c*e -
 b*f, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1040

```
Int[(x_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_.)*(x_) + (f_.)
*(x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[(1 - d*x^2)/(c*e - b*f - e*(2*
c*d - b*e + 2*a*f)*x^2 + d^2*(c*e - b*f)*x^4), x], x, (1 + (e + Sqrt[e^2 -
4*d*f])*(x/(2*d)))/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0]
```

Rule 1041

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_
.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[g, Subst[Int[1/(a + (c*d - a*f)
*x^2), x], x, x/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h
}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0] &&
 EqQ[2*h*d - g*e, 0]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\left(\frac{1}{6}\int \frac{-6-4x}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx\right) \\
&\quad + \frac{1}{6}\int -\frac{4x}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx \\
&= -\left(\frac{2}{3}\int \frac{x}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx\right) + \text{Subst}\left(\int \frac{1}{3-3x^2}\,dx, x, \frac{x}{\sqrt{-3-4x-x^2}}\right) \\
&= \frac{1}{3}\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) - \frac{16}{3}\text{Subst}\left(\int \frac{1+3x^2}{-4-8x^2-36x^4}\,dx, x, \frac{1+\frac{x}{3}}{\sqrt{-3-4x-x^2}}\right) \\
&= \frac{1}{3}\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) + \frac{2}{9}\text{Subst}\left(\int \frac{1}{\frac{1}{3}-\frac{2x}{3}+x^2}\,dx, x, \frac{1+\frac{x}{3}}{\sqrt{-3-4x-x^2}}\right) \\
&\quad + \frac{2}{9}\text{Subst}\left(\int \frac{1}{\frac{1}{3}+\frac{2x}{3}+x^2}\,dx, x, \frac{1+\frac{x}{3}}{\sqrt{-3-4x-x^2}}\right) \\
&= \frac{1}{3}\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) \\
&\quad - \frac{4}{9}\text{Subst}\left(\int \frac{1}{-\frac{8}{9}-x^2}\,dx, x, \frac{2}{3}\left(-1+\frac{3+x}{\sqrt{-3-4x-x^2}}\right)\right) \\
&\quad - \frac{4}{9}\text{Subst}\left(\int \frac{1}{-\frac{8}{9}-x^2}\,dx, x, \frac{2}{3}\left(1+\frac{3+x}{\sqrt{-3-4x-x^2}}\right)\right) \\
&= -\frac{1}{3}\sqrt{2}\tan^{-1}\left(\frac{1-\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right) + \frac{1}{3}\sqrt{2}\tan^{-1}\left(\frac{1+\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right) \\
&\quad + \frac{1}{3}\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.57

$$\int \frac{1}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = \frac{1}{3}\left(\sqrt{2}\arctan\left(\frac{3+2x}{\sqrt{2}\sqrt{-3-4x-x^2}}\right) + \operatorname{arctanh}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right)\right)$$

[In] Integrate[1/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]

[Out] (Sqrt[2]*ArcTan[(3 + 2*x)/(Sqrt[2]*Sqrt[-3 - 4*x - x^2])] + ArcTanh[x/Sqrt[
-3 - 4*x - x^2]])/3

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.27

method	result
default	$-\frac{\sqrt{3}\,\sqrt{4}\,\sqrt{\frac{3x^2}{\left(-\frac{3}{2}-x\right)^2}-12}\left(\sqrt{2}\,\arctan\left(\frac{\sqrt{\frac{3x^2}{\left(-\frac{3}{2}-x\right)^2}-12}\,\sqrt{2}}{6}\right)+\operatorname{arctanh}\left(\frac{3x}{\left(-\frac{3}{2}-x\right)\sqrt{\frac{3x^2}{\left(-\frac{3}{2}-x\right)^2}-12}}\right)\right)}{18\sqrt{\frac{\frac{x^2}{\left(-\frac{3}{2}-x\right)^2}-4}{\left(1+\frac{x}{-\frac{3}{2}-x}\right)^2}}\left(1+\frac{x}{-\frac{3}{2}-x}\right)}$
trager	$\frac{\ln\left(\frac{36\,\mathrm{RootOf}\left(12\_Z^2-4\_Z+1\right)^2 x-36\,\mathrm{RootOf}\left(12\_Z^2-4\_Z+1\right)x-36\,\mathrm{RootOf}\left(12\_Z^2-4\_Z+1\right)-6\sqrt{-x^2-4x-3}+5x+6}{6\,\mathrm{RootOf}\left(12\_Z^2-4\_Z+1\right)x+x+3}\right)}{3} - \ln\Big($

[In] int(1/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/18*3^(1/2)*4^(1/2)*(3*x^2/(-3/2-x)^2-12)^(1/2)*(2^(1/2)*arctan(1/6*(3*x^
2/(-3/2-x)^2-12)^(1/2)*2^(1/2))+arctanh(3*x/(-3/2-x)/(3*x^2/(-3/2-x)^2-12)^
(1/2)))/((x^2/(-3/2-x)^2-4)/(1+x/(-3/2-x))^2)^(1/2)/(1+x/(-3/2-x))

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.39

$$\int \frac{1}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = -\frac{1}{6}\,\sqrt{2}\arctan\left(\frac{\sqrt{2}x+3\,\sqrt{2}\sqrt{-x^2-4\,x-3}}{2\,(2\,x+3)}\right) - \frac{1}{6}\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}x-3\,\sqrt{2}\sqrt{-x^2-4\,x-3}}{2\,(2\,x+3)}\right) - \frac{1}{12}\,\log\left(-\frac{2\,\sqrt{-x^2-4\,x-3}x+4\,x+3}{x^2}\right) + \frac{1}{12}\,\log\left(\frac{2\,\sqrt{-x^2-4\,x-3}x-4\,x-3}{x^2}\right)$$

```
[In] integrate(1/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(2)*arctan(1/2*(sqrt(2)*x + 3*sqrt(2)*sqrt(-x^2 - 4*x - 3))/(2*x +
 3)) - 1/6*sqrt(2)*arctan(-1/2*(sqrt(2)*x - 3*sqrt(2)*sqrt(-x^2 - 4*x - 3))
/(2*x + 3)) - 1/12*log(-(2*sqrt(-x^2 - 4*x - 3)*x + 4*x + 3)/x^2) + 1/12*lo
g((2*sqrt(-x^2 - 4*x - 3)*x - 4*x - 3)/x^2)
```

Sympy [F]

$$\int \frac{1}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = \int \frac{1}{\sqrt{-(x+1)\,(x+3)}\,(2x^2+4x+3)}\,dx$$

```
[In] integrate(1/(2*x**2+4*x+3)/(-x**2-4*x-3)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(x + 1)*(x + 3))*(2*x**2 + 4*x + 3)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = \int \frac{1}{(2\,x^2+4\,x+3)\sqrt{-x^2-4\,x-3}}\,dx$$

```
[In] integrate(1/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 165 vs. 2(76) = 152.

Time = 0.29 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.74

$$\int \frac{1}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = -\frac{1}{3}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{3\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+1\right)\right) - \frac{1}{3}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{\sqrt{-x^2-4x-3}-1}{x+2}+1\right)\right) + \frac{1}{6}\log\left(\frac{2\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+\frac{3\left(\sqrt{-x^2-4x-3}-1\right)^2}{(x+2)^2}+1\right) - \frac{1}{6}\log\left(\frac{2\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+\frac{\left(\sqrt{-x^2-4x-3}-1\right)^2}{(x+2)^2}+3\right)$$

```
[In] integrate(1/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="giac")
[Out] -1/3*sqrt(2)*arctan(1/2*sqrt(2)*(3*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1))
 - 1/3*sqrt(2)*arctan(1/2*sqrt(2)*((sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1))
 + 1/6*log(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 3*(sqrt(-x^2 - 4*x - 3) -
 1)^2/(x + 2)^2 + 1) - 1/6*log(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + (sqrt
(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = \int \frac{1}{\sqrt{-x^2-4x-3}\,(2x^2+4x+3)}\,dx$$

```
[In] int(1/((- 4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)),x)
[Out] int(1/((- 4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)), x)
```

3.131 $\int \frac{1}{x\sqrt{-3-4x-x^2}(3+4x+2x^2)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 130

$$\int \frac{1}{x\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = -\frac{\arctan\left(\frac{3+2x}{\sqrt{3}\sqrt{-3-4x-x^2}}\right)}{3\sqrt{3}} + \frac{1}{9}\sqrt{2}\arctan\left(\frac{1-\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right) - \frac{1}{9}\sqrt{2}\arctan\left(\frac{1+\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right) - \frac{4}{9}\text{arctanh}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right)$$

```
[Out] -4/9*arctanh(x/(-x^2-4*x-3)^(1/2))+1/9*arctan(1/2*(1+(-3-x)/(-x^2-4*x-3)^(1
/2))*2^(1/2))*2^(1/2)-1/9*arctan(1/2*(1+(3+x)/(-x^2-4*x-3)^(1/2))*2^(1/2))*
2^(1/2)-1/9*arctan(1/3*(3+2*x)*3^(1/2)/(-x^2-4*x-3)^(1/2))*3^(1/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.367, Rules

used = {6860, 738, 210, 1042, 1000, 12, 1040, 1175, 632, 1041, 212}

$$\int \frac{1}{x\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = -\frac{\arctan\left(\frac{2x+3}{\sqrt{3}\sqrt{-x^2-4x-3}}\right)}{3\sqrt{3}} + \frac{1}{9}\sqrt{2}\arctan\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right) - \frac{1}{9}\sqrt{2}\arctan\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right) - \frac{4}{9}\text{arctanh}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)$$

```
[In] Int[1/(x*Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

```
[Out] -1/3*ArcTan[(3 + 2*x)/(Sqrt[3]*Sqrt[-3 - 4*x - x^2])]/Sqrt[3] + (Sqrt[2]*Ar
cTan[(1 - (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]])/9 - (Sqrt[2]*ArcTan[(1 +
(3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]])/9 - (4*ArcTanh[x/Sqrt[-3 - 4*x - x^
2]])/9
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1000

```
Int[1/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*
(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f)
, 2]}, Dist[1/(2*q), Int[(c*d - a*f + q + (c*e - b*f)*x)/((a + b*x + c*x^2)
*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[(c*d - a*f - q + (c*e -
 b*f)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b,
 c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[c*e -
 b*f, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1040

```
Int[(x_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_.)*(x_) + (f_.)
*(x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[(1 - d*x^2)/(c*e - b*f - e*(2*
c*d - b*e + 2*a*f)*x^2 + d^2*(c*e - b*f)*x^4), x], x, (1 + (e + Sqrt[e^2 -
4*d*f])*(x/(2*d)))/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0]
```

Rule 1041

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_
.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[g, Subst[Int[1/(a + (c*d - a*f)
*x^2), x], x, x/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h
}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0] &&
 EqQ[2*h*d - g*e, 0]
```

Rule 1042

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_
.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-(2*h*d - g*e)/e, Int[1/((a + b
*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/e, Int[(2*d + e*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g
, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0]
 && NeQ[2*h*d - g*e, 0]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
```

```
0]))
```

Rule 6860

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{1}{3x\sqrt{-3-4x-x^2}} - \frac{2(2+x)}{3\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\right) dx \\
&= \frac{1}{3}\int \frac{1}{x\sqrt{-3-4x-x^2}}\,dx - \frac{2}{3}\int \frac{2+x}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx \\
&= \frac{1}{6}\int \frac{-6-4x}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx - \frac{1}{3}\int \frac{1}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx \\
&\quad - \frac{2}{3}\text{Subst}\left(\int \frac{1}{-12-x^2}\,dx, x, \frac{-6-4x}{\sqrt{-3-4x-x^2}}\right) \\
&= -\frac{\tan^{-1}\left(\frac{3+2x}{\sqrt{3}\sqrt{-3-4x-x^2}}\right)}{3\sqrt{3}} + \frac{1}{18}\int \frac{-6-4x}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx - \frac{1}{18}\int \\
&\quad - \frac{4x}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx - \text{Subst}\left(\int \frac{1}{3-3x^2}\,dx, x, \frac{x}{\sqrt{-3-4x-x^2}}\right) \\
&= -\frac{\tan^{-1}\left(\frac{3+2x}{\sqrt{3}\sqrt{-3-4x-x^2}}\right)}{3\sqrt{3}} - \frac{1}{3}\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) \\
&\quad + \frac{2}{9}\int \frac{x}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx \\
&\quad - \frac{1}{3}\text{Subst}\left(\int \frac{1}{3-3x^2}\,dx, x, \frac{x}{\sqrt{-3-4x-x^2}}\right) \\
&= -\frac{\tan^{-1}\left(\frac{3+2x}{\sqrt{3}\sqrt{-3-4x-x^2}}\right)}{3\sqrt{3}} - \frac{4}{9}\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) \\
&\quad + \frac{16}{9}\text{Subst}\left(\int \frac{1+3x^2}{-4-8x^2-36x^4}\,dx, x, \frac{1+\frac{x}{3}}{\sqrt{-3-4x-x^2}}\right) \\
&= -\frac{\tan^{-1}\left(\frac{3+2x}{\sqrt{3}\sqrt{-3-4x-x^2}}\right)}{3\sqrt{3}} - \frac{4}{9}\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) \\
&\quad - \frac{2}{27}\text{Subst}\left(\int \frac{1}{\frac{1}{3}-\frac{2x}{3}+x^2}\,dx, x, \frac{1+\frac{x}{3}}{\sqrt{-3-4x-x^2}}\right) \\
&\quad - \frac{2}{27}\text{Subst}\left(\int \frac{1}{\frac{1}{3}+\frac{2x}{3}+x^2}\,dx, x, \frac{1+\frac{x}{3}}{\sqrt{-3-4x-x^2}}\right)
\end{aligned}$$

$$= -\frac{\tan^{-1}\left(\frac{3+2x}{\sqrt{3}\sqrt{-3-4x-x^2}}\right)}{3\sqrt{3}} - \frac{4}{9}\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) + \frac{4}{27}\text{Subst}\left(\int \frac{1}{-\frac{8}{9}-x^2}\,dx, x, \frac{2}{3}\left(-1+\frac{3+x}{\sqrt{-3-4x-x^2}}\right)\right) + \frac{4}{27}\text{Subst}\left(\int \frac{1}{-\frac{8}{9}-x^2}\,dx, x, \frac{2}{3}\left(1+\frac{3+x}{\sqrt{-3-4x-x^2}}\right)\right)$$

$$= -\frac{\tan^{-1}\left(\frac{3+2x}{\sqrt{3}\sqrt{-3-4x-x^2}}\right)}{3\sqrt{3}} + \frac{1}{9}\sqrt{2}\tan^{-1}\left(\frac{1-\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right) - \frac{1}{9}\sqrt{2}\tan^{-1}\left(\frac{1+\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right) - \frac{4}{9}\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right)$$

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.69

$$\int \frac{1}{x\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = \frac{1}{9}\left(-\sqrt{2}\arctan\left(\frac{3+2x}{\sqrt{2}\sqrt{-3-4x-x^2}}\right) + 2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{-3-4x-x^2}}{3+x}\right) - 4\text{arctanh}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right)\right)$$

```
[In] Integrate[1/(x*Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

```
[Out] (-(Sqrt[2]*ArcTan[(3 + 2*x)/(Sqrt[2]*Sqrt[-3 - 4*x - x^2])]) + 2*Sqrt[3]*Ar
cTan[(Sqrt[3]*Sqrt[-3 - 4*x - x^2])/(3 + x)] - 4*ArcTanh[x/Sqrt[-3 - 4*x -
x^2]])/9
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.17

method	result
default	$\frac{\sqrt{3}\,\arctan\left(\frac{(-6-4x)\sqrt{3}}{6\sqrt{-x^2-4x-3}}\right)}{9}+\frac{\sqrt{3}\,\sqrt{4}\,\sqrt{\frac{3x^2}{\left(-\frac{3}{2}-x\right)^2}-12}\,\left(\sqrt{2}\,\arctan\left(\frac{\sqrt{\frac{3x^2}{\left(-\frac{3}{2}-x\right)^2}-12}\,\sqrt{2}}{6}\right)+4\,\operatorname{arctanh}\left(\frac{3x}{\left(-\frac{3}{2}-x\right)\sqrt{\frac{3x^2}{\left(-\frac{3}{2}-x\right)^2}-12}}\right.\right.}{54\sqrt{\frac{\frac{x^2}{\left(-\frac{3}{2}-x\right)^2}-4}{\left(1+\frac{x}{-\frac{3}{2}-x}\right)^2}}\,\left(1+\frac{x}{-\frac{3}{2}-x}\right)}$
trager	$\operatorname{RootOf}\left(18\_Z^2+8\_Z+1\right)\ln\left(\frac{40500\operatorname{RootOf}\left(18\_Z^2+8\_Z+1\right)^2x-40500\operatorname{RootOf}\left(18\_Z^2+8\_Z+1\right)^2+4680\sqrt{-}}{18\operatorname{Roo}}\right.$

```
[In] int(1/x/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*3^(1/2)*arctan(1/6*(-6-4*x)*3^(1/2)/(-x^2-4*x-3)^(1/2))+1/54*3^(1/2)*4^
(1/2)*(3*x^2/(-3/2-x)^2-12)^(1/2)*(2^(1/2)*arctan(1/6*(3*x^2/(-3/2-x)^2-12)
^(1/2)*2^(1/2))+4*arctanh(3*x/(-3/2-x)/(3*x^2/(-3/2-x)^2-12)^(1/2)))/((x^2/
(-3/2-x)^2-4)/(1+x/(-3/2-x))^2)^(1/2)/(1+x/(-3/2-x))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.31

$$\int \frac{1}{x\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = \frac{1}{9}\,\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{-x^2-4\,x-3}(2\,x+3)}{3\,(x^2+4\,x+3)}\right) + \frac{1}{18}\,\sqrt{2}\arctan\left(\frac{\sqrt{2}x+3\,\sqrt{2}\sqrt{-x^2-4\,x-3}}{2\,(2\,x+3)}\right) + \frac{1}{18}\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}x-3\,\sqrt{2}\sqrt{-x^2-4\,x-3}}{2\,(2\,x+3)}\right) + \frac{1}{9}\,\log\left(-\frac{2\,\sqrt{-x^2-4\,x-3}x+4\,x+3}{x^2}\right) - \frac{1}{9}\,\log\left(\frac{2\,\sqrt{-x^2-4\,x-3}x-4\,x-3}{x^2}\right)$$

```
[In] integrate(1/x/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/9*sqrt(3)*arctan(1/3*sqrt(3)*sqrt(-x^2 - 4*x - 3)*(2*x + 3)/(x^2 + 4*x +
3)) + 1/18*sqrt(2)*arctan(1/2*(sqrt(2)*x + 3*sqrt(2)*sqrt(-x^2 - 4*x - 3))/
(2*x + 3)) + 1/18*sqrt(2)*arctan(-1/2*(sqrt(2)*x - 3*sqrt(2)*sqrt(-x^2 - 4*
```

```
x - 3))/(2*x + 3)) + 1/9*log(-(2*sqrt(-x^2 - 4*x - 3)*x + 4*x + 3)/x^2) - 1
/9*log((2*sqrt(-x^2 - 4*x - 3)*x - 4*x - 3)/x^2)
```

Sympy [F]

$$\int \frac{1}{x\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = \int \frac{1}{x\sqrt{-(x+1)(x+3)}\,(2x^2+4x+3)}\,dx$$

```
[In] integrate(1/x/(2*x**2+4*x+3)/(-x**2-4*x-3)**(1/2),x)
[Out] Integral(1/(x*sqrt(-(x + 1)*(x + 3))*(2*x**2 + 4*x + 3)), x)
```

Maxima [F]

$$\int \frac{1}{x\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = \int \frac{1}{(2x^2+4x+3)\sqrt{-x^2-4x-3}\,x}\,dx$$

```
[In] integrate(1/x/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)*x), x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.53

$$\begin{aligned}\int \frac{1}{x\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx &= \frac{1}{9}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{3\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+1\right)\right) \\ &+ \frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+1\right)\right) \\ &+ \frac{1}{9}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{\sqrt{-x^2-4x-3}-1}{x+2}+1\right)\right) \\ &- \frac{2}{9}\log\left(\frac{2\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+\frac{3\left(\sqrt{-x^2-4x-3}-1\right)^2}{(x+2)^2}+1\right) \\ &+ \frac{2}{9}\log\left(\frac{2\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+\frac{\left(\sqrt{-x^2-4x-3}-1\right)^2}{(x+2)^2}+3\right)\end{aligned}$$

```
[In] integrate(1/x/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/9*sqrt(2)*arctan(1/2*sqrt(2)*(3*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1))
+ 2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)
) + 1/9*sqrt(2)*arctan(1/2*sqrt(2)*((sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)
) - 2/9*log(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 3*(sqrt(-x^2 - 4*x - 3)
- 1)^2/(x + 2)^2 + 1) + 2/9*log(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + (sqr
t(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = \int \frac{1}{x\,\sqrt{-x^2-4\,x-3}\,\left(2\,x^2+4\,x+3\right)}\,dx$$

```
[In] int(1/(x*(- 4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)),x)
```

```
[Out] int(1/(x*(- 4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)), x)
```

3.132 $\int \frac{1}{x^2\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 151

$$\int \frac{1}{x^2\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = \frac{\sqrt{-3-4x-x^2}}{9x} + \frac{2\arctan\left(\frac{3+2x}{\sqrt{3}\sqrt{-3-4x-x^2}}\right)}{3\sqrt{3}} + \frac{2}{27}\sqrt{2}\arctan\left(\frac{1-\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right) - \frac{2}{27}\sqrt{2}\arctan\left(\frac{1+\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right) + \frac{10}{27}\operatorname{arctanh}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right)$$

```
[Out] 10/27*arctanh(x/(-x^2-4*x-3)^(1/2))+2/27*arctan(1/2*(1+(-3-x)/(-x^2-4*x-3)^
(1/2))*2^(1/2))*2^(1/2)-2/27*arctan(1/2*(1+(3+x)/(-x^2-4*x-3)^(1/2))*2^(1/2
))*2^(1/2)+2/9*arctan(1/3*(3+2*x)*3^(1/2)/(-x^2-4*x-3)^(1/2))*3^(1/2)+1/9*(
-x^2-4*x-3)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules

used = {6860, 744, 738, 210, 1042, 1000, 12, 1040, 1175, 632, 1041, 212}

$$\int \frac{1}{x^2\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = \frac{2\arctan\left(\frac{2x+3}{\sqrt{3}\sqrt{-x^2-4x-3}}\right)}{3\sqrt{3}} + \frac{2}{27}\sqrt{2}\arctan\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right) - \frac{2}{27}\sqrt{2}\arctan\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right) + \frac{10}{27}\text{arctanh}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right) + \frac{\sqrt{-x^2-4x-3}}{9x}$$

```
[In] Int[1/(x^2*Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

```
[Out] Sqrt[-3 - 4*x - x^2]/(9*x) + (2*ArcTan[(3 + 2*x)/(Sqrt[3]*Sqrt[-3 - 4*x - x
^2])])/(3*Sqrt[3]) + (2*Sqrt[2]*ArcTan[(1 - (3 + x)/Sqrt[-3 - 4*x - x^2])/S
qrt[2]])/27 - (2*Sqrt[2]*ArcTan[(1 + (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]]
)/27 + (10*ArcTanh[x/Sqrt[-3 - 4*x - x^2]])/27
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
 Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 1000

```
Int[1/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*
(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f)
, 2]}, Dist[1/(2*q), Int[(c*d - a*f + q + (c*e - b*f)*x)/((a + b*x + c*x^2)
*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[(c*d - a*f - q + (c*e -
 b*f)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b,
 c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[c*e -
 b*f, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1040

```
Int[(x_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_.)*(x_) + (f_.)
*(x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[(1 - d*x^2)/(c*e - b*f - e*(2*
c*d - b*e + 2*a*f)*x^2 + d^2*(c*e - b*f)*x^4), x], x, (1 + (e + Sqrt[e^2 -
4*d*f])*(x/(2*d)))/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0]
```

Rule 1041

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_
.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[g, Subst[Int[1/(a + (c*d - a*f)
*x^2), x], x, x/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h
}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0] &&
 EqQ[2*h*d - g*e, 0]
```

Rule 1042

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_
.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-(2*h*d - g*e)/e, Int[1/((a + b
*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/e, Int[(2*d + e*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g
```

```
, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0]
 && NeQ[2*h*d - g*e, 0]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 6860

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{1}{3x^2\sqrt{-3-4x-x^2}}-\frac{4}{9x\sqrt{-3-4x-x^2}}+\frac{2(5+4x)}{9\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\right)dx\\
&= \frac{2}{9}\int \frac{5+4x}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx+\frac{1}{3}\int \frac{1}{x^2\sqrt{-3-4x-x^2}}\,dx-\frac{4}{9}\int \frac{1}{x\sqrt{-3-4x-x^2}}\,dx\\
&= \frac{\sqrt{-3-4x-x^2}}{9x}-\frac{2}{9}\int \frac{1}{x\sqrt{-3-4x-x^2}}\,dx-\frac{2}{9}\int \frac{1}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx\\
&\quad-\frac{2}{9}\int \frac{-6-4x}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx+\frac{8}{9}\text{Subst}\left(\int \frac{1}{-12-x^2}\,dx,x,\frac{-6-4x}{\sqrt{-3-4x-x^2}}\right)\\
&= \frac{\sqrt{-3-4x-x^2}}{9x}+\frac{4\tan^{-1}\left(\frac{3+2x}{\sqrt{3}\sqrt{-3-4x-x^2}}\right)}{9\sqrt{3}}\\
&\quad+\frac{1}{27}\int \frac{-6-4x}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx\\
&\quad-\frac{1}{27}\int -\frac{4x}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx\\
&\quad+\frac{4}{9}\text{Subst}\left(\int \frac{1}{-12-x^2}\,dx,x,\frac{-6-4x}{\sqrt{-3-4x-x^2}}\right)\\
&\quad+\frac{4}{3}\text{Subst}\left(\int \frac{1}{3-3x^2}\,dx,x,\frac{x}{\sqrt{-3-4x-x^2}}\right)
\end{aligned}$$

$$= \frac{\sqrt{-3-4x-x^2}}{9x} + \frac{2\tan^{-1}\left(\frac{3+2x}{\sqrt{3}\sqrt{-3-4x-x^2}}\right)}{3\sqrt{3}} + \frac{4}{9}\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) + \frac{4}{27}\int\frac{x}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx - \frac{2}{9}\text{Subst}\left(\int\frac{1}{3-3x^2}\,dx, x, \frac{x}{\sqrt{-3-4x-x^2}}\right)$$

$$= \frac{\sqrt{-3-4x-x^2}}{9x} + \frac{2\tan^{-1}\left(\frac{3+2x}{\sqrt{3}\sqrt{-3-4x-x^2}}\right)}{3\sqrt{3}} + \frac{10}{27}\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) + \frac{32}{27}\text{Subst}\left(\int\frac{1+3x^2}{-4-8x^2-36x^4}\,dx, x, \frac{1+\frac{x}{3}}{\sqrt{-3-4x-x^2}}\right)$$

$$= \frac{\sqrt{-3-4x-x^2}}{9x} + \frac{2\tan^{-1}\left(\frac{3+2x}{\sqrt{3}\sqrt{-3-4x-x^2}}\right)}{3\sqrt{3}} + \frac{10}{27}\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) - \frac{4}{81}\text{Subst}\left(\int\frac{1}{\frac{1}{3}-\frac{2x}{3}+x^2}\,dx, x, \frac{1+\frac{x}{3}}{\sqrt{-3-4x-x^2}}\right) - \frac{4}{81}\text{Subst}\left(\int\frac{1}{\frac{1}{3}+\frac{2x}{3}+x^2}\,dx, x, \frac{1+\frac{x}{3}}{\sqrt{-3-4x-x^2}}\right)$$

$$= \frac{\sqrt{-3-4x-x^2}}{9x} + \frac{2\tan^{-1}\left(\frac{3+2x}{\sqrt{3}\sqrt{-3-4x-x^2}}\right)}{3\sqrt{3}} + \frac{10}{27}\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) + \frac{8}{81}\text{Subst}\left(\int\frac{1}{-\frac{8}{9}-x^2}\,dx, x, \frac{2}{3}\left(-1+\frac{3+x}{\sqrt{-3-4x-x^2}}\right)\right) + \frac{8}{81}\text{Subst}\left(\int\frac{1}{-\frac{8}{9}-x^2}\,dx, x, \frac{2}{3}\left(1+\frac{3+x}{\sqrt{-3-4x-x^2}}\right)\right)$$

$$= \frac{\sqrt{-3-4x-x^2}}{9x} + \frac{2\tan^{-1}\left(\frac{3+2x}{\sqrt{3}\sqrt{-3-4x-x^2}}\right)}{3\sqrt{3}} + \frac{2}{27}\sqrt{2}\tan^{-1}\left(\frac{1-\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right) - \frac{2}{27}\sqrt{2}\tan^{-1}\left(\frac{1+\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right) + \frac{10}{27}\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right)$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.75

$$\int\frac{1}{x^2\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = \frac{-2\sqrt{2}x\arctan\left(\frac{3+2x}{\sqrt{2}\sqrt{-3-4x-x^2}}\right) + 3\left(\sqrt{-3-4x-x^2} - 4\sqrt{3}x\arctan\left(\frac{\sqrt{3}\sqrt{-3-4x-x^2}}{3+x}\right)\right) + 10x\text{arctanh}\left(\frac{x}{\sqrt{-3-4}}\right.}{27x}$$

```
[In] Integrate[1/(x^2*Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

```
[Out] (-2*Sqrt[2]*x*ArcTan[(3 + 2*x)/(Sqrt[2]*Sqrt[-3 - 4*x - x^2])] + 3*(Sqrt[-3
 - 4*x - x^2] - 4*Sqrt[3]*x*ArcTan[(Sqrt[3]*Sqrt[-3 - 4*x - x^2])/(3 + x)])
 + 10*x*ArcTanh[x/Sqrt[-3 - 4*x - x^2]])/(27*x)
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.12

method	result
default	$-\frac{2\sqrt{3}\,\arctan\left(\frac{(-6-4x)\sqrt{3}}{6\sqrt{-x^2-4x-3}}\right)}{9}+\frac{\sqrt{-x^2-4x-3}}{9x}+\frac{\sqrt{3}\,\sqrt{4}\,\sqrt{\frac{3x^2}{\left(-\frac{3}{2}-x\right)^2}-12}\,\left(\sqrt{2}\,\arctan\left(\frac{\sqrt{\frac{3x^2}{\left(-\frac{3}{2}-x\right)^2}-12}\,\sqrt{2}}{6}\right)-5\,\operatorname{arctanh}\left(\frac{}{\left(-\right.}\right.\right.}{81\sqrt{\frac{\frac{x^2}{\left(-\frac{3}{2}-x\right)^2}-4}{\left(1+\frac{x}{-\frac{3}{2}-x}\right)^2}}\left(1+\frac{x}{-\frac{3}{2}-x}\right)}$
risch	$-\frac{x^2+4x+3}{9x\sqrt{-x^2-4x-3}}-\frac{2\sqrt{3}\,\arctan\left(\frac{(-6-4x)\sqrt{3}}{6\sqrt{-x^2-4x-3}}\right)}{9}+\frac{\sqrt{3}\,\sqrt{4}\,\sqrt{\frac{3x^2}{\left(-\frac{3}{2}-x\right)^2}-12}\,\left(\sqrt{2}\,\arctan\left(\frac{\sqrt{\frac{3x^2}{\left(-\frac{3}{2}-x\right)^2}-12}\,\sqrt{2}}{6}\right)-5\,\operatorname{arctanh}\left(\right.\right.}{81\sqrt{\frac{\frac{x^2}{\left(-\frac{3}{2}-x\right)^2}-4}{\left(1+\frac{x}{-\frac{3}{2}-x}\right)^2}}\left(1+\frac{x}{-\frac{3}{2}-x}\right)}$
trager	$\frac{\sqrt{-x^2-4x-3}}{9x}-\frac{2\operatorname{RootOf}\left(\_Z^2+3\right)\ln\left(\frac{2\operatorname{RootOf}\left(\_Z^2+3\right)x+3\sqrt{-x^2-4x-3}+3\operatorname{RootOf}\left(\_Z^2+3\right)}{x}\right)}{9}+\frac{10\ln\left(-\frac{-96000\operatorname{RootOf}\left(7\right.}{}\right.}{}$

```
[In] int(1/x^2/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/9*3^(1/2)*arctan(1/6*(-6-4*x)*3^(1/2)/(-x^2-4*x-3)^(1/2))+1/9*(-x^2-4*x-
3)^(1/2)/x+1/81*3^(1/2)*4^(1/2)*(3*x^2/(-3/2-x)^2-12)^(1/2)*(2^(1/2)*arctan
(1/6*(3*x^2/(-3/2-x)^2-12)^(1/2)*2^(1/2))-5*arctanh(3*x/(-3/2-x)/(3*x^2/(-3
/2-x)^2-12)^(1/2)))/((x^2/(-3/2-x)^2-4)/(1+x/(-3/2-x))^2)^(1/2)/(1+x/(-3/2-
x))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.28

$$\int \frac{1}{x^2\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = -\frac{12\sqrt{3}x\arctan\left(\frac{\sqrt{3}\sqrt{-x^2-4x-3}(2x+3)}{3(x^2+4x+3)}\right)-2\sqrt{2}x\arctan\left(\frac{\sqrt{2}x+3\sqrt{2}\sqrt{-x^2-4x-3}}{2(2x+3)}\right)-2\sqrt{2}x\arctan\left(-\frac{\sqrt{2}x-3\sqrt{2}\sqrt{}}{2(2}\right.}{54x}$$

```
[In] integrate(1/x^2/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/54*(12*sqrt(3)*x*arctan(1/3*sqrt(3)*sqrt(-x^2 - 4*x - 3)*(2*x + 3)/(x^2
+ 4*x + 3)) - 2*sqrt(2)*x*arctan(1/2*(sqrt(2)*x + 3*sqrt(2)*sqrt(-x^2 - 4*x
 - 3))/(2*x + 3)) - 2*sqrt(2)*x*arctan(-1/2*(sqrt(2)*x - 3*sqrt(2)*sqrt(-x^
2 - 4*x - 3))/(2*x + 3)) + 5*x*log(-(2*sqrt(-x^2 - 4*x - 3)*x + 4*x + 3)/x^
2) - 5*x*log((2*sqrt(-x^2 - 4*x - 3)*x - 4*x - 3)/x^2) - 6*sqrt(-x^2 - 4*x
- 3))/x
```

Sympy [F]

$$\int \frac{1}{x^2\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = \int \frac{1}{x^2\sqrt{-(x+1)(x+3)}\,(2x^2+4x+3)}\,dx$$

```
[In] integrate(1/x**2/(2*x**2+4*x+3)/(-x**2-4*x-3)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(-(x + 1)*(x + 3))*(2*x**2 + 4*x + 3)), x)
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = \int \frac{1}{(2\,x^2+4\,x+3)\sqrt{-x^2-4\,x-3}x^2}\,dx$$

```
[In] integrate(1/x^2/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)*x^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 269 vs. $2(121) = 242$.

Time = 0.31 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.78

$$\int \frac{1}{x^2\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx$$
$$= \frac{2}{27}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{3\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+1\right)\right)$$
$$-\frac{4}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+1\right)\right)$$
$$+\frac{2}{27}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{\sqrt{-x^2-4x-3}-1}{x+2}+1\right)\right)$$
$$-\frac{\frac{\sqrt{-x^2-4x-3}-1}{x+2}+2}{18\left(\frac{\sqrt{-x^2-4x-3}-1}{x+2}+\frac{\left(\sqrt{-x^2-4x-3}-1\right)^2}{(x+2)^2}+1\right)}$$
$$+\frac{5}{27}\log\left(\frac{2\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+\frac{3\left(\sqrt{-x^2-4x-3}-1\right)^2}{(x+2)^2}+1\right)$$
$$-\frac{5}{27}\log\left(\frac{2\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+\frac{\left(\sqrt{-x^2-4x-3}-1\right)^2}{(x+2)^2}+3\right)$$

```
[In] integrate(1/x^2/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="giac")
```

```
[Out] 2/27*sqrt(2)*arctan(1/2*sqrt(2)*(3*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1))
 - 4/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1
)) + 2/27*sqrt(2)*arctan(1/2*sqrt(2)*((sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) +
1)) - 1/18*((sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 2)/((sqrt(-x^2 - 4*x - 3)
- 1)/(x + 2) + (sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 1) + 5/27*log(2*(sq
rt(-x^2 - 4*x - 3) - 1)/(x + 2) + 3*(sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2
+ 1) - 5/27*log(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + (sqrt(-x^2 - 4*x - 3
) - 1)^2/(x + 2)^2 + 3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = \int \frac{1}{x^2\,\sqrt{-x^2-4\,x-3}\,\left(2\,x^2+4\,x+3\right)}\,dx$$

```
[In] int(1/(x^2*(- 4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)),x)
```

```
[Out] int(1/(x^2*(- 4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)), x)
```

3.133 $\int (2+3x)^2 \left(30+31x-12x^2\right)^2 \sqrt{6+17x+12x^2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 149

$$\begin{aligned}\int (2+3x)^2 \left(30+31x-12x^2\right)^2 \sqrt{6+17x+12x^2}\,dx &= \frac{125455(17+24x)\sqrt{6+17x+12x^2}}{150994944} - \frac{125455(17+24x)\left(6+17x+12x^2\right)^{3/2}}{4718592} \\ &\quad + \frac{25091(17+24x)\left(6+17x+12x^2\right)^{5/2}}{24576} - \frac{873\left(6+17x+12x^2\right)^{7/2}}{1792} \\ &\quad - \frac{1}{32}(10-3x)\left(6+17x+12x^2\right)^{7/2} - \frac{125455\operatorname{arctanh}\left(\frac{17+24x}{4\sqrt{3}\sqrt{6+17x+12x^2}}\right)}{603979776\sqrt{3}}\end{aligned}$$

```
[Out] -125455/4718592*(17+24*x)*(12*x^2+17*x+6)^(3/2)+25091/24576*(17+24*x)*(12*x
^2+17*x+6)^(5/2)-873/1792*(12*x^2+17*x+6)^(7/2)-1/32*(10-3*x)*(12*x^2+17*x+
6)^(7/2)-125455/1811939328*arctanh(1/12*(17+24*x)*3^(1/2)/(12*x^2+17*x+6)^(
1/2))*3^(1/2)+125455/150994944*(17+24*x)*(12*x^2+17*x+6)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used

$= \{1016, 756, 654, 626, 635, 212\}$

$$\int (2+3x)^2 \left(30+31x-12x^2\right)^2 \sqrt{6+17x+12x^2}\,dx$$
$$= -\frac{125455\text{arctanh}\left(\frac{24x+17}{4\sqrt{3}\sqrt{12x^2+17x+6}}\right)}{603979776\sqrt{3}} - \frac{1}{32}(10-3x)\left(12x^2+17x+6\right)^{7/2}$$
$$- \frac{873(12x^2+17x+6)^{7/2}}{1792} + \frac{25091(24x+17)\left(12x^2+17x+6\right)^{5/2}}{24576}$$
$$- \frac{125455(24x+17)\left(12x^2+17x+6\right)^{3/2}}{4718592} + \frac{125455(24x+17)\sqrt{12x^2+17x+6}}{150994944}$$

```
[In] Int[(2 + 3*x)^2*(30 + 31*x - 12*x^2)^2*Sqrt[6 + 17*x + 12*x^2],x]
```

```
[Out] (125455*(17 + 24*x)*Sqrt[6 + 17*x + 12*x^2])/150994944 - (125455*(17 + 24*x
)*(6 + 17*x + 12*x^2)^(3/2))/4718592 + (25091*(17 + 24*x)*(6 + 17*x + 12*x^
2)^(5/2))/24576 - (873*(6 + 17*x + 12*x^2)^(7/2))/1792 - ((10 - 3*x)*(6 + 1
7*x + 12*x^2)^(7/2))/32 - (125455*ArcTanh[(17 + 24*x)/(4*Sqrt[3]*Sqrt[6 + 1
7*x + 12*x^2])])/(603979776*Sqrt[3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 756

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 1016

```
Int[((g_) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.
) + (e_.)*(x_) + (f_.)*(x_)^2)^(m_.), x_Symbol] :> Int[(d*(g/a) + f*h*(x/c)
)^m*(a + b*x + c*x^2)^(m + p), x] /; FreeQ[{a, b, c, d, e, f, g, h, p}, x]
&& EqQ[c*g^2 - b*g*h + a*h^2, 0] && EqQ[c^2*d*g^2 - a*c*e*g*h + a^2*f*h^2,
0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int (10-3x)^2 \left(6+17x+12x^2\right)^{5/2} dx \\
&= -\frac{1}{32}(10-3x)\left(6+17x+12x^2\right)^{7/2}+\frac{1}{96}\int \left(11331-\frac{7857x}{2}\right)\left(6+17x+12x^2\right)^{5/2} dx \\
&= -\frac{873(6+17x+12x^2)^{7/2}}{1792} \\
&\quad -\frac{1}{32}(10-3x)\left(6+17x+12x^2\right)^{7/2}+\frac{75273}{512}\int \left(6+17x+12x^2\right)^{5/2} dx \\
&= \frac{25091(17+24x)\left(6+17x+12x^2\right)^{5/2}}{24576}-\frac{873(6+17x+12x^2)^{7/2}}{1792} \\
&\quad -\frac{1}{32}(10-3x)\left(6+17x+12x^2\right)^{7/2}-\frac{125455\int \left(6+17x+12x^2\right)^{3/2} dx}{49152} \\
&= -\frac{125455(17+24x)\left(6+17x+12x^2\right)^{3/2}}{4718592} \\
&\quad +\frac{25091(17+24x)\left(6+17x+12x^2\right)^{5/2}}{24576}-\frac{873(6+17x+12x^2)^{7/2}}{1792} \\
&\quad -\frac{1}{32}(10-3x)\left(6+17x+12x^2\right)^{7/2}+\frac{125455\int \sqrt{6+17x+12x^2}\, dx}{3145728}
\end{aligned}$$

$$= \frac{125455(17+24x)\sqrt{6+17x+12x^2}}{150994944} - \frac{125455(17+24x)\left(6+17x+12x^2\right)^{3/2}}{4718592} + \frac{25091(17+24x)\left(6+17x+12x^2\right)^{5/2}}{24576} - \frac{873\left(6+17x+12x^2\right)^{7/2}}{1792} - \frac{1}{32}(10-3x)\left(6+17x+12x^2\right)^{7/2} - \frac{125455\int\frac{1}{\sqrt{6+17x+12x^2}}\,dx}{301989888}$$

$$= \frac{125455(17+24x)\sqrt{6+17x+12x^2}}{150994944} - \frac{125455(17+24x)\left(6+17x+12x^2\right)^{3/2}}{4718592} + \frac{25091(17+24x)\left(6+17x+12x^2\right)^{5/2}}{24576} - \frac{873\left(6+17x+12x^2\right)^{7/2}}{1792} - \frac{1}{32}(10-3x)\left(6+17x+12x^2\right)^{7/2} - \frac{125455\text{Subst}\left(\int\frac{1}{48-x^2}\,dx,x,\frac{17+24x}{\sqrt{6+17x+12x^2}}\right)}{150994944}$$

$$= \frac{125455(17+24x)\sqrt{6+17x+12x^2}}{150994944} - \frac{125455(17+24x)\left(6+17x+12x^2\right)^{3/2}}{4718592} + \frac{25091(17+24x)\left(6+17x+12x^2\right)^{5/2}}{24576} - \frac{873\left(6+17x+12x^2\right)^{7/2}}{1792} - \frac{1}{32}(10-3x)\left(6+17x+12x^2\right)^{7/2} - \frac{125455\tanh^{-1}\left(\frac{17+24x}{4\sqrt{3}\sqrt{6+17x+12x^2}}\right)}{603979776\sqrt{3}}$$

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.60

$$\int (2+3x)^2\left(30+31x-12x^2\right)^2\sqrt{6+17x+12x^2}\,dx$$

$$= \frac{6\sqrt{6+17x+12x^2}(474999091769+3132157281976x+7899203409792x^2+8974844476416x^3+3438453}{6341}$$

```
[In] Integrate[(2 + 3*x)^2*(30 + 31*x - 12*x^2)^2*Sqrt[6 + 17*x + 12*x^2],x]
```

```
[Out] (6*Sqrt[6 + 17*x + 12*x^2]*(474999091769 + 3132157281976*x + 7899203409792*
x^2 + 8974844476416*x^3 + 3438453030912*x^4 - 1190083166208*x^5 - 732816211
968*x^6 + 171228266496*x^7) - 878185*Sqrt[3]*ArcTanh[(2*Sqrt[2 + (17*x)/3 +
 4*x^2])/(3 + 4*x)])/6341787648
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.54

method	result
risch	$\frac{(171228266496x^7-732816211968x^6-1190083166208x^5+3438453030912x^4+8974844476416x^3+7899203409792x^2+3132157281976x-}{1056964608}$
trager	$\left(162x^7 - \frac{19413}{28}x^6 - \frac{504423}{448}x^5 + \frac{11659251}{3584}x^4 + \frac{139118993}{16384}x^3 + \frac{20570842213}{2752512}x^2 + \frac{391519660247}{132120576}x + \frac{474999091769}{1056964608}\right.$
default	$\frac{125455(17+24x)\sqrt{12x^2+17x+6}}{150994944} - \frac{125455\ln\left(\frac{\left(\frac{17}{2}+12x\right)\sqrt{12}}{12}+\sqrt{12x^2+17x+6}\right)\sqrt{12}}{3623878656} + \frac{2473875847\left(12x^2+17x+6\right)^{\frac{3}{2}}}{33030144} + \frac{27x^5\left(12x^2\right.}{}$

```
[In] int((2+3*x)^2*(-12*x^2+31*x+30)^2*(12*x^2+17*x+6)^(1/2),x,method=_RETURNVER
BOSE)
```

```
[Out] 1/1056964608*(171228266496*x^7-732816211968*x^6-1190083166208*x^5+343845303
0912*x^4+8974844476416*x^3+7899203409792*x^2+3132157281976*x+474999091769)*
(12*x^2+17*x+6)^(1/2)-125455/3623878656*ln(1/12*(17/2+12*x)*12^(1/2)+(12*x^
2+17*x+6)^(1/2))*12^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.59

$$\int (2+3x)^2 \left(30+31x-12x^2\right)^2 \sqrt{6+17x+12x^2}\,dx$$
$$= \frac{1}{1056964608}\left(171228266496\,x^7 - 732816211968\,x^6 - 1190083166208\,x^5 + 3438453030912\,x^4 + 897484447 6\right.$$
$$+ \frac{125455}{3623878656}\sqrt{3}\log\left(-8\sqrt{3}\sqrt{12\,x^2+17\,x+6}(24\,x+17) + 1152\,x^2 + 1632\,x + 577\right)$$

```
[In] integrate((2+3*x)^2*(-12*x^2+31*x+30)^2*(12*x^2+17*x+6)^(1/2),x, algorithm=
"fricas")
```

```
[Out] 1/1056964608*(171228266496*x^7 - 732816211968*x^6 - 1190083166208*x^5 + 343
8453030912*x^4 + 8974844476416*x^3 + 7899203409792*x^2 + 3132157281976*x +
474999091769)*sqrt(12*x^2 + 17*x + 6) + 125455/3623878656*sqrt(3)*log(-8*sq
rt(3)*sqrt(12*x^2 + 17*x + 6)*(24*x + 17) + 1152*x^2 + 1632*x + 577)
```

Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.64

$$\int (2+3x)^2 \left(30+31x-12x^2\right)^2 \sqrt{6+17x+12x^2}\,dx$$
$$= \sqrt{12x^2+17x+6}\cdot\left(162x^7 - \frac{19413x^6}{28} - \frac{504423x^5}{448} + \frac{11659251x^4}{3584} + \frac{139118993x^3}{16384} + \frac{20570842213x^2}{2752512} + \frac{391519660247x}{132120576} + \frac{474999091769}{1056964608}\right) - \frac{125455\sqrt{3}\log\left(24x+4\sqrt{3}\sqrt{12x^2+17x+6}+17\right)}{1811939328}$$

```
[In] integrate((2+3*x)**2*(-12*x**2+31*x+30)**2*(12*x**2+17*x+6)**(1/2),x)
```

```
[Out] sqrt(12*x**2 + 17*x + 6)*(162*x**7 - 19413*x**6/28 - 504423*x**5/448 + 1165
9251*x**4/3584 + 139118993*x**3/16384 + 20570842213*x**2/2752512 + 39151966
0247*x/132120576 + 474999091769/1056964608) - 125455*sqrt(3)*log(24*x + 4*s
qrt(3)*sqrt(12*x**2 + 17*x + 6) + 17)/1811939328
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.04

$$\int (2+3x)^2 \left(30+31x-12x^2\right)^2 \sqrt{6+17x+12x^2}\,dx$$
$$= \frac{27}{2}\left(12x^2+17x+6\right)^{\frac{3}{2}}x^5 - \frac{8613}{112}\left(12x^2+17x+6\right)^{\frac{3}{2}}x^4 + \frac{14991}{1792}\left(12x^2+17x+6\right)^{\frac{3}{2}}x^3 + \frac{4267751}{14336}\left(12x^2+17x+6\right)^{\frac{3}{2}}x^2 + \frac{129220757}{458752}\left(12x^2+17x+6\right)^{\frac{3}{2}}x + \frac{2473875847}{33030144}\left(12x^2+17x+6\right)^{\frac{3}{2}} + \frac{125455}{6291456}\sqrt{12x^2+17x+6}x - \frac{125455}{1811939328}\sqrt{3}\log\left(4\sqrt{3}\sqrt{12x^2+17x+6}+24x+17\right) + \frac{2132735}{150994944}\sqrt{12x^2+17x+6}$$

```
[In] integrate((2+3*x)^2*(-12*x^2+31*x+30)^2*(12*x^2+17*x+6)^(1/2),x, algorithm=
"maxima")
```

```
[Out] 27/2*(12*x^2 + 17*x + 6)^(3/2)*x^5 - 8613/112*(12*x^2 + 17*x + 6)^(3/2)*x^4
 + 14991/1792*(12*x^2 + 17*x + 6)^(3/2)*x^3 + 4267751/14336*(12*x^2 + 17*x
+ 6)^(3/2)*x^2 + 129220757/458752*(12*x^2 + 17*x + 6)^(3/2)*x + 2473875847/
33030144*(12*x^2 + 17*x + 6)^(3/2) + 125455/6291456*sqrt(12*x^2 + 17*x + 6)
*x - 125455/1811939328*sqrt(3)*log(4*sqrt(3)*sqrt(12*x^2 + 17*x + 6) + 24*x
 + 17) + 2132735/150994944*sqrt(12*x^2 + 17*x + 6)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.57

$$\int (2+3x)^2 \left(30+31x-12x^2\right)^2 \sqrt{6+17x+12x^2}\,dx$$
$$=\frac{1}{1056964608}\left(8\left(48\left(24\left(96\left(24\left(48\left(168\,x-719\right)x-56047\right)x+3886417\right)x+973832951\right)x+20570842213\right)x\right.\right.$$
$$+\frac{125455}{1811939328}\sqrt{3}\log\left(\left|-4\sqrt{3}\left(2\sqrt{3}x-\sqrt{12\,x^2+17\,x+6}\right)-17\right|\right)$$

```
[In] integrate((2+3*x)^2*(-12*x^2+31*x+30)^2*(12*x^2+17*x+6)^(1/2),x, algorithm=
"giac")
```

```
[Out] 1/1056964608*(8*(48*(24*(96*(24*(48*(168*x - 719)*x - 56047)*x + 3886417)*x
 + 973832951)*x + 20570842213)*x + 391519660247)*x + 474999091769)*sqrt(12*
x^2 + 17*x + 6) + 125455/1811939328*sqrt(3)*log(abs(-4*sqrt(3)*(2*sqrt(3)*x
 - sqrt(12*x^2 + 17*x + 6)) - 17))
```

Mupad [B] (verification not implemented)

Time = 13.92 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.26

$$\int (2+3x)^2 \left(30+31x-12x^2\right)^2 \sqrt{6+17x+12x^2}\,dx$$
$$=\frac{4267751\,x^2\left(12\,x^2+17\,x+6\right)^{3/2}}{14336}+\frac{14991\,x^3\left(12\,x^2+17\,x+6\right)^{3/2}}{1792}$$
$$-\frac{8613\,x^4\left(12\,x^2+17\,x+6\right)^{3/2}}{112}+\frac{27\,x^5\left(12\,x^2+17\,x+6\right)^{3/2}}{2}$$
$$-\frac{146030443\sqrt{12}\,\ln\left(\sqrt{12\,x^2+17\,x+6}+\frac{\sqrt{12}\left(12\,x+\frac{17}{2}\right)}{12}\right)}{88080384}$$
$$+\frac{438091329\left(\frac{x}{2}+\frac{17}{48}\right)\sqrt{12\,x^2+17\,x+6}}{229376}$$
$$+\frac{2473875847\sqrt{12\,x^2+17\,x+6}\left(1152\,x^2+408\,x-291\right)}{3170893824}$$
$$+\frac{129220757\,x\left(12\,x^2+17\,x+6\right)^{3/2}}{458752}$$
$$+\frac{42055889399\sqrt{12}\,\ln\left(2\sqrt{12\,x^2+17\,x+6}+\frac{\sqrt{12}\left(24\,x+17\right)}{12}\right)}{25367150592}$$

```
[In] int((3*x + 2)^2*(17*x + 12*x^2 + 6)^(1/2)*(31*x - 12*x^2 + 30)^2,x)
```

```
[Out] (4267751*x^2*(17*x + 12*x^2 + 6)^(3/2))/14336 + (14991*x^3*(17*x + 12*x^2 +
 6)^(3/2))/1792 - (8613*x^4*(17*x + 12*x^2 + 6)^(3/2))/112 + (27*x^5*(17*x
+ 12*x^2 + 6)^(3/2))/2 - (146030443*12^(1/2)*log((17*x + 12*x^2 + 6)^(1/2)
+ (12^(1/2)*(12*x + 17/2))/12))/88080384 + (438091329*(x/2 + 17/48)*(17*x +
 12*x^2 + 6)^(1/2))/229376 + (2473875847*(17*x + 12*x^2 + 6)^(1/2)*(408*x +
 1152*x^2 - 291))/3170893824 + (129220757*x*(17*x + 12*x^2 + 6)^(3/2))/4587
52 + (42055889399*12^(1/2)*log(2*(17*x + 12*x^2 + 6)^(1/2) + (12^(1/2)*(24*
x + 17))/12))/25367150592
```

3.134 $\int (2+3x) (30+31x-12x^2) \sqrt{6+17x+12x^2} \, dx$

Optimal result

Integrand size = 30, antiderivative size = 103

$$\int (2+3x) \left(30+31x-12x^2\right) \sqrt{6+17x+12x^2} \, dx = -\frac{97(17+24x)\sqrt{6+17x+12x^2}}{24576}+\frac{97}{768}(17+24x) \left(6+17x+12x^2\right)^{3/2} -\frac{1}{20}\left(6+17x+12x^2\right)^{5/2}+\frac{97\text{arctanh}\left(\frac{17+24x}{4\sqrt{3}\sqrt{6+17x+12x^2}}\right)}{98304\sqrt{3}}$$

```
[Out] 97/768*(17+24*x)*(12*x^2+17*x+6)^(3/2)-1/20*(12*x^2+17*x+6)^(5/2)+97/294912
*arctanh(1/12*(17+24*x)*3^(1/2)/(12*x^2+17*x+6)^(1/2))*3^(1/2)-97/24576*(17
+24*x)*(12*x^2+17*x+6)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1016, 654, 626, 635, 212}

$$\int (2+3x) \left(30+31x-12x^2\right) \sqrt{6+17x+12x^2} \, dx = \frac{97\text{arctanh}\left(\frac{24x+17}{4\sqrt{3}\sqrt{12x^2+17x+6}}\right)}{98304\sqrt{3}}-\frac{1}{20}\left(12x^2+17x+6\right)^{5/2} +\frac{97}{768}(24x+17)\left(12x^2+17x+6\right)^{3/2}-\frac{97(24x+17)\sqrt{12x^2+17x+6}}{24576}$$

```
[In] Int[(2 + 3*x)*(30 + 31*x - 12*x^2)*Sqrt[6 + 17*x + 12*x^2],x]
```

```
[Out] (-97*(17 + 24*x)*Sqrt[6 + 17*x + 12*x^2])/24576 + (97*(17 + 24*x)*(6 + 17*x
 + 12*x^2)^(3/2))/768 - (6 + 17*x + 12*x^2)^(5/2)/20 + (97*ArcTanh[(17 + 24
*x)/(4*Sqrt[3]*Sqrt[6 + 17*x + 12*x^2])])/(98304*Sqrt[3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1016

```
Int[((g_) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.
) + (e_.)*(x_) + (f_.)*(x_)^2)^(m_.), x_Symbol] :> Int[(d*(g/a) + f*h*(x/c)
)^m*(a + b*x + c*x^2)^(m + p), x] /; FreeQ[{a, b, c, d, e, f, g, h, p}, x]
&& EqQ[c*g^2 - b*g*h + a*h^2, 0] && EqQ[c^2*d*g^2 - a*c*e*g*h + a^2*f*h^2,
0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int (10-3x)\left(6+17x+12x^2\right)^{3/2}\,dx \\
&= -\frac{1}{20}\left(6+17x+12x^2\right)^{5/2}+\frac{97}{8}\int \left(6+17x+12x^2\right)^{3/2}\,dx \\
&= \frac{97}{768}(17+24x)\left(6+17x+12x^2\right)^{3/2}-\frac{1}{20}\left(6+17x+12x^2\right)^{5/2}-\frac{97}{512}\int \sqrt{6+17x+12x^2}\,dx
\end{aligned}$$

$$= -\frac{97(17+24x)\sqrt{6+17x+12x^2}}{24576} + \frac{97}{768}(17+24x)\left(6+17x+12x^2\right)^{3/2} - \frac{1}{20}\left(6+17x+12x^2\right)^{5/2} + \frac{97\int \frac{1}{\sqrt{6+17x+12x^2}}\,dx}{49152}$$

$$= -\frac{97(17+24x)\sqrt{6+17x+12x^2}}{24576} + \frac{97}{768}(17+24x)\left(6+17x+12x^2\right)^{3/2} - \frac{1}{20}\left(6+17x+12x^2\right)^{5/2} + \frac{97\text{Subst}\left(\int \frac{1}{48-x^2}\,dx, x, \frac{17+24x}{\sqrt{6+17x+12x^2}}\right)}{24576}$$

$$= -\frac{97(17+24x)\sqrt{6+17x+12x^2}}{24576} + \frac{97}{768}(17+24x)\left(6+17x+12x^2\right)^{3/2} - \frac{1}{20}\left(6+17x+12x^2\right)^{5/2} + \frac{97\tanh^{-1}\left(\frac{17+24x}{4\sqrt{3}\sqrt{6+17x+12x^2}}\right)}{98304\sqrt{3}}$$

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.72

$$\int (2+3x)\left(30+31x-12x^2\right)\sqrt{6+17x+12x^2}\,dx = \frac{6\sqrt{6+17x+12x^2}(1353611+5455144x+6837888x^2+1963008x^3-884736x^4)+485\sqrt{3}\text{arctanh}\left(\frac{2\sqrt{2+\frac{17}{3}...}}{3+4...}\right)}{737280}$$

```
[In] Integrate[(2 + 3*x)*(30 + 31*x - 12*x^2)*Sqrt[6 + 17*x + 12*x^2],x]
[Out] (6*Sqrt[6 + 17*x + 12*x^2]*(1353611 + 5455144*x + 6837888*x^2 + 1963008*x^3
 - 884736*x^4) + 485*Sqrt[3]*ArcTanh[(2*Sqrt[2 + (17*x)/3 + 4*x^2])/(3 + 4*
x)])/737280
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.63

method	result
risch	$-\frac{(884736x^4-1963008x^3-6837888x^2-5455144x-1353611)\sqrt{12x^2+17x+6}}{122880}+\frac{97\ln\left(\frac{\left(\frac{17}{2}+12x\right)\sqrt{12}}{12}+\sqrt{12x^2+17x+6}\right)\sqrt{12}}{589824}$
trager	$\left(-\frac{36}{5}x^4+\frac{639}{40}x^3+\frac{17807}{320}x^2+\frac{681893}{15360}x+\frac{1353611}{122880}\right)\sqrt{12x^2+17x+6}-\frac{97\,\text{RootOf}\left(\_Z^2-3\right)\ln\left(-24\,\text{RootOf}\left(\_...\right.\right.}{...}$
default	$-\frac{97(17+24x)\sqrt{12x^2+17x+6}}{24576}+\frac{97\ln\left(\frac{\left(\frac{17}{2}+12x\right)\sqrt{12}}{12}+\sqrt{12x^2+17x+6}\right)\sqrt{12}}{589824}+\frac{7093\left(12x^2+17x+6\right)^{\frac{3}{2}}}{3840}-\frac{3x^2\left(12x^2+17x+6\right)^{\frac{3}{2}}}{5}+...$

```
[In] int((2+3*x)*(-12*x^2+31*x+30)*(12*x^2+17*x+6)^(1/2),x,method=_RETURNVERBOSE
)
```

```
[Out] -1/122880*(884736*x^4-1963008*x^3-6837888*x^2-5455144*x-1353611)*(12*x^2+17
*x+6)^(1/2)+97/589824*ln(1/12*(17/2+12*x)*12^(1/2)+(12*x^2+17*x+6)^(1/2))*1
2^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.71

$$\int (2+3x)\left(30+31x-12x^2\right)\sqrt{6+17x+12x^2}\,dx = -\frac{1}{122880}\left(884736\,x^4-1963008\,x^3-6837888\,x^2-5455144\,x-1353611\right)\sqrt{12\,x^2+17\,x+6} + \frac{97}{589824}\sqrt{3}\log\left(8\sqrt{3}\sqrt{12\,x^2+17\,x+6}(24\,x+17)+1152\,x^2+1632\,x+577\right)$$

```
[In] integrate((2+3*x)*(-12*x^2+31*x+30)*(12*x^2+17*x+6)^(1/2),x, algorithm="fri
cas")
```

```
[Out] -1/122880*(884736*x^4 - 1963008*x^3 - 6837888*x^2 - 5455144*x - 1353611)*sq
rt(12*x^2 + 17*x + 6) + 97/589824*sqrt(3)*log(8*sqrt(3)*sqrt(12*x^2 + 17*x
+ 6)*(24*x + 17) + 1152*x^2 + 1632*x + 577)
```

Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.74

$$\int (2+3x)\left(30+31x-12x^2\right)\sqrt{6+17x+12x^2}\,dx = \sqrt{12x^2+17x+6}\left(-\frac{36x^4}{5}+\frac{639x^3}{40}+\frac{17807x^2}{320}+\frac{681893x}{15360}+\frac{1353611}{122880}\right) + \frac{97\sqrt{3}\log\left(24x+4\sqrt{3}\sqrt{12x^2+17x+6}+17\right)}{294912}$$

```
[In] integrate((2+3*x)*(-12*x**2+31*x+30)*(12*x**2+17*x+6)**(1/2),x)
```

```
[Out] sqrt(12*x**2 + 17*x + 6)*(-36*x**4/5 + 639*x**3/40 + 17807*x**2/320 + 68189
3*x/15360 + 1353611/122880) + 97*sqrt(3)*log(24*x + 4*sqrt(3)*sqrt(12*x**2
+ 17*x + 6) + 17)/294912
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.01

$$\int (2+3x)\left(30+31x-12x^2\right)\sqrt{6+17x+12x^2}\,dx$$
$$= -\frac{3}{5}\left(12x^2+17x+6\right)^{\frac{3}{2}}x^2+\frac{349}{160}\left(12x^2+17x+6\right)^{\frac{3}{2}}x+\frac{7093}{3840}\left(12x^2+17x+6\right)^{\frac{3}{2}}-\frac{97}{1024}\sqrt{12x^2+17x+6}x+\frac{97}{294912}\sqrt{3}\log\left(4\sqrt{3}\sqrt{12x^2+17x+6}+24x+17\right)-\frac{1649}{24576}\sqrt{12x^2+17x+6}$$

```
[In] integrate((2+3*x)*(-12*x^2+31*x+30)*(12*x^2+17*x+6)^(1/2),x, algorithm="max
ima")
```

```
[Out] -3/5*(12*x^2 + 17*x + 6)^(3/2)*x^2 + 349/160*(12*x^2 + 17*x + 6)^(3/2)*x +
7093/3840*(12*x^2 + 17*x + 6)^(3/2) - 97/1024*sqrt(12*x^2 + 17*x + 6)*x + 9
7/294912*sqrt(3)*log(4*sqrt(3)*sqrt(12*x^2 + 17*x + 6) + 24*x + 17) - 1649/
24576*sqrt(12*x^2 + 17*x + 6)
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.68

$$\int (2+3x)\left(30+31x-12x^2\right)\sqrt{6+17x+12x^2}\,dx$$
$$= -\frac{1}{122880}(8(48(72(32x-71)x-17807)x-681893)x-1353611)\sqrt{12x^2+17x+6}-\frac{97}{294912}\sqrt{3}\log\left(\left|-4\sqrt{3}\left(2\sqrt{3}x-\sqrt{12x^2+17x+6}\right)-17\right|\right)$$

```
[In] integrate((2+3*x)*(-12*x^2+31*x+30)*(12*x^2+17*x+6)^(1/2),x, algorithm="gia
c")
```

```
[Out] -1/122880*(8*(48*(72*(32*x - 71)*x - 17807)*x - 681893)*x - 1353611)*sqrt(1
2*x^2 + 17*x + 6) - 97/294912*sqrt(3)*log(abs(-4*sqrt(3)*(2*sqrt(3)*x - sqr
t(12*x^2 + 17*x + 6)) - 17))
```

Mupad [B] (verification not implemented)

Time = 13.33 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.32

$$\int (2+3x)\left(30+31x-12x^2\right)\sqrt{6+17x+12x^2}\,dx$$
$$=\frac{3753\left(\frac{x}{2}+\frac{17}{48}\right)\sqrt{12\,x^2+17\,x+6}}{80}-\frac{417\sqrt{12}\,\ln\left(\sqrt{12\,x^2+17\,x+6}+\frac{\sqrt{12}\,\left(12\,x+\frac{17}{2}\right)}{12}\right)}{10240}$$
$$-\frac{3\,x^2\,{\left(12\,x^2+17\,x+6\right)}^{3/2}}{5}+\frac{7093\sqrt{12\,x^2+17\,x+6}\,\left(1152\,x^2+408\,x-291\right)}{368640}$$
$$+\frac{349\,x\,{\left(12\,x^2+17\,x+6\right)}^{3/2}}{160}+\frac{120581\sqrt{12}\,\ln\left(2\sqrt{12\,x^2+17\,x+6}+\frac{\sqrt{12}\,\left(24\,x+17\right)}{12}\right)}{2949120}$$

```
[In] int((3*x + 2)*(17*x + 12*x^2 + 6)^(1/2)*(31*x - 12*x^2 + 30),x)
```

```
[Out] (3753*(x/2 + 17/48)*(17*x + 12*x^2 + 6)^(1/2))/80 - (417*12^(1/2)*log((17*x
 + 12*x^2 + 6)^(1/2) + (12^(1/2)*(12*x + 17/2))/12))/10240 - (3*x^2*(17*x +
 12*x^2 + 6)^(3/2))/5 + (7093*(17*x + 12*x^2 + 6)^(1/2)*(408*x + 1152*x^2 -
 291))/368640 + (349*x*(17*x + 12*x^2 + 6)^(3/2))/160 + (120581*12^(1/2)*lo
g(2*(17*x + 12*x^2 + 6)^(1/2) + (12^(1/2)*(24*x + 17))/12))/2949120
```

3.135 $\int \frac{\sqrt{6+17x+12x^2}}{(2+3x)(30+31x-12x^2)} dx$

Optimal result

Integrand size = 34, antiderivative size = 28

$$\int \frac{\sqrt{6+17x+12x^2}}{(2+3x)(30+31x-12x^2)} dx = \frac{1}{42}\operatorname{arctanh}\left(\frac{206+291x}{84\sqrt{6+17x+12x^2}}\right)$$

```
[Out] 1/42*arctanh(1/84*(206+291*x)/(12*x^2+17*x+6)^(1/2))
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {1016, 738, 212}

$$\int \frac{\sqrt{6+17x+12x^2}}{(2+3x)(30+31x-12x^2)} dx = \frac{1}{42}\operatorname{arctanh}\left(\frac{291x+206}{84\sqrt{12x^2+17x+6}}\right)$$

```
[In] Int[Sqrt[6 + 17*x + 12*x^2]/((2 + 3*x)*(30 + 31*x - 12*x^2)),x]
```

```
[Out] ArcTanh[(206 + 291*x)/(84*Sqrt[6 + 17*x + 12*x^2])]/42
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1016

```
Int[((g_) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.
) + (e_.)*(x_) + (f_.)*(x_)^2)^(m_.), x_Symbol] :> Int[(d*(g/a) + f*h*(x/c)
)^m*(a + b*x + c*x^2)^(m + p), x] /; FreeQ[{a, b, c, d, e, f, g, h, p}, x]
&& EqQ[c*g^2 - b*g*h + a*h^2, 0] && EqQ[c^2*d*g^2 - a*c*e*g*h + a^2*f*h^2,
0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{1}{(10-3x)\sqrt{6+17x+12x^2}}\,dx\\ &= -\left(2\text{Subst}\left(\int \frac{1}{7056-x^2}\,dx, x, \frac{-206-291x}{\sqrt{6+17x+12x^2}}\right)\right)\\ &= \frac{1}{42}\tanh^{-1}\left(\frac{206+291x}{84\sqrt{6+17x+12x^2}}\right)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{6+17x+12x^2}}{(2+3x)\,(30+31x-12x^2)}\,dx = \frac{1}{21}\text{arctanh}\left(\frac{6\sqrt{6+17x+12x^2}}{7(2+3x)}\right)$$

```
[In] Integrate[Sqrt[6 + 17*x + 12*x^2]/((2 + 3*x)*(30 + 31*x - 12*x^2)),x]
[Out] ArcTanh[(6*Sqrt[6 + 17*x + 12*x^2])/(7*(2 + 3*x))]/21
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.14

method	result
trager	$-\frac{\ln\left(-\frac{84\sqrt{12x^2+17x+6}-206-291x}{3x-10}\right)}{42}$
default	$\frac{\sqrt{12\left(x+\frac{2}{3}\right)^2+x+\frac{2}{3}}}{12}+\frac{\ln\left(\frac{\left(\frac{17}{2}+12x\right)\sqrt{12}}{12}+\sqrt{12\left(x+\frac{2}{3}\right)^2+x+\frac{2}{3}}\right)\sqrt{12}}{288}-\frac{4\sqrt{12\left(x+\frac{3}{4}\right)^2-x-\frac{3}{4}}}{49}+\frac{\ln\left(\frac{\left(\frac{17}{2}+12x\right)\sqrt{12}}{12}+\sqrt{12\left(x+\frac{3}{4}\right)^{2}}\right.}{294}$

```
[In] int((12*x^2+17*x+6)^(1/2)/(2+3*x)/(-12*x^2+31*x+30),x,method=_RETURNVERBOSE
)
```

```
[Out] -1/42*ln(-(84*(12*x^2+17*x+6)^(1/2)-206-291*x)/(3*x-10))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. $2(22) = 44$.

Time = 0.32 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.89

$$\int \frac{\sqrt{6+17x+12x^2}}{(2+3x)(30+31x-12x^2)}\,dx = \frac{1}{84}\log\left(\frac{291\,x+84\,\sqrt{12\,x^2+17\,x+6}+206}{x}\right) - \frac{1}{84}\log\left(\frac{291\,x-84\,\sqrt{12\,x^2+17\,x+6}+206}{x}\right)$$

```
[In] integrate((12*x^2+17*x+6)^(1/2)/(2+3*x)/(-12*x^2+31*x+30),x, algorithm="fri
cas")
```

```
[Out] 1/84*log((291*x + 84*sqrt(12*x^2 + 17*x + 6) + 206)/x) - 1/84*log((291*x -
84*sqrt(12*x^2 + 17*x + 6) + 206)/x)
```

Sympy [F]

$$\int \frac{\sqrt{6+17x+12x^2}}{(2+3x)(30+31x-12x^2)}\,dx = -\int \frac{\sqrt{12x^2+17x+6}}{36x^3-69x^2-152x-60}\,dx$$

```
[In] integrate((12*x**2+17*x+6)**(1/2)/(2+3*x)/(-12*x**2+31*x+30),x)
```

```
[Out] -Integral(sqrt(12*x**2 + 17*x + 6)/(36*x**3 - 69*x**2 - 152*x - 60), x)
```

Maxima [F]

$$\int \frac{\sqrt{6+17x+12x^2}}{(2+3x)(30+31x-12x^2)}\,dx = \int -\frac{\sqrt{12\,x^2+17\,x+6}}{(12\,x^2-31\,x-30)(3\,x+2)}\,dx$$

```
[In] integrate((12*x^2+17*x+6)^(1/2)/(2+3*x)/(-12*x^2+31*x+30),x, algorithm="max
ima")
```

```
[Out] -integrate(sqrt(12*x^2 + 17*x + 6)/((12*x^2 - 31*x - 30)*(3*x + 2)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(22) = 44$.

Time = 0.31 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.25

$$\int \frac{\sqrt{6+17x+12x^2}}{(2+3x)(30+31x-12x^2)}\,dx = \frac{1}{42}\log\left(\left|-6\sqrt{3}x+20\sqrt{3}+3\sqrt{12x^2+17x+6}+42\right|\right) - \frac{1}{42}\log\left(\left|-6\sqrt{3}x+20\sqrt{3}+3\sqrt{12x^2+17x+6}-42\right|\right)$$

```
[In] integrate((12*x^2+17*x+6)^(1/2)/(2+3*x)/(-12*x^2+31*x+30),x, algorithm="gia
c")
```

```
[Out] 1/42*log(abs(-6*sqrt(3)*x + 20*sqrt(3) + 3*sqrt(12*x^2 + 17*x + 6) + 42)) -
 1/42*log(abs(-6*sqrt(3)*x + 20*sqrt(3) + 3*sqrt(12*x^2 + 17*x + 6) - 42))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{6+17x+12x^2}}{(2+3x)(30+31x-12x^2)}\,dx = \int \frac{\sqrt{12x^2+17x+6}}{(3x+2)\,(-12x^2+31x+30)}\,dx$$

```
[In] int((17*x + 12*x^2 + 6)^(1/2)/((3*x + 2)*(31*x - 12*x^2 + 30)),x)
```

```
[Out] int((17*x + 12*x^2 + 6)^(1/2)/((3*x + 2)*(31*x - 12*x^2 + 30)), x)
```

3.136 $\int \frac{\sqrt{6+17x+12x^2}}{(2+3x)^2(30+31x-12x^2)^2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 84

$$\int \frac{\sqrt{6+17x+12x^2}}{(2+3x)^2\left(30+31x-12x^2\right)^2}\,dx = -\frac{275+388x}{98(10-3x)\sqrt{6+17x+12x^2}} + \frac{3137\sqrt{6+17x+12x^2}}{38416(10-3x)} + \frac{97\operatorname{arctanh}\left(\frac{206+291x}{84\sqrt{6+17x+12x^2}}\right)}{3226944}$$

```
[Out] 97/3226944*arctanh(1/84*(206+291*x)/(12*x^2+17*x+6)^(1/2))+1/98*(-275-388*x
)/(10-3*x)/(12*x^2+17*x+6)^(1/2)+3137/38416*(12*x^2+17*x+6)^(1/2)/(10-3*x)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {1016, 754, 820, 738, 212}

$$\int \frac{\sqrt{6+17x+12x^2}}{(2+3x)^2\left(30+31x-12x^2\right)^2}\,dx = \frac{97\operatorname{arctanh}\left(\frac{291x+206}{84\sqrt{12x^2+17x+6}}\right)}{3226944} - \frac{388x+275}{98(10-3x)\sqrt{12x^2+17x+6}} + \frac{3137\sqrt{12x^2+17x+6}}{38416(10-3x)}$$

```
[In] Int[Sqrt[6 + 17*x + 12*x^2]/((2 + 3*x)^2*(30 + 31*x - 12*x^2)^2),x]
```

```
[Out] -1/98*(275 + 388*x)/((10 - 3*x)*Sqrt[6 + 17*x + 12*x^2]) + (3137*Sqrt[6 + 1
7*x + 12*x^2])/(38416*(10 - 3*x)) + (97*ArcTanh[(206 + 291*x)/(84*Sqrt[6 +
17*x + 12*x^2])])/3226944
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 1016

```
Int[((g_) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.
) + (e_.)*(x_) + (f_.)*(x_)^2)^(m_.), x_Symbol] :> Int[(d*(g/a) + f*h*(x/c)
)^m*(a + b*x + c*x^2)^(m + p), x] /; FreeQ[{a, b, c, d, e, f, g, h, p}, x]
&& EqQ[c*g^2 - b*g*h + a*h^2, 0] && EqQ[c^2*d*g^2 - a*c*e*g*h + a^2*f*h^2,
0] && IntegerQ[m]
```

Rubi steps

$$\text{integral} = \int \frac{1}{(10-3x)^2 \left(6+17x+12x^2\right)^{3/2}}\,dx$$

$$=-\frac{275+388x}{98(10-3x)\sqrt{6+17x+12x^2}}-\frac{1}{882}\int\frac{-\frac{14859}{2}-10476x}{(10-3x)^2\sqrt{6+17x+12x^2}}\,dx$$

$$=-\frac{275+388x}{98(10-3x)\sqrt{6+17x+12x^2}}+\frac{3137\sqrt{6+17x+12x^2}}{38416(10-3x)}+\frac{97\int\frac{1}{(10-3x)\sqrt{6+17x+12x^2}}\,dx}{76832}$$

$$=-\frac{275+388x}{98(10-3x)\sqrt{6+17x+12x^2}}+\frac{3137\sqrt{6+17x+12x^2}}{38416(10-3x)}-\frac{97\text{Subst}\left(\int\frac{1}{7056-x^2}\,dx,x,\frac{-206-291x}{\sqrt{6+17x+12x^2}}\right)}{38416}$$

$$=-\frac{275+388x}{98(10-3x)\sqrt{6+17x+12x^2}}+\frac{3137\sqrt{6+17x+12x^2}}{38416(10-3x)}+\frac{97\tanh^{-1}\left(\frac{206+291x}{84\sqrt{6+17x+12x^2}}\right)}{3226944}$$

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.95

$$\int\frac{\sqrt{6+17x+12x^2}}{(2+3x)^2\left(30+31x-12x^2\right)^2}\,dx=\frac{\left(88978+98767x-37644x^2\right)\sqrt{6+17x+12x^2}}{38416(-10+3x)(2+3x)(3+4x)}+\frac{97\text{arctanh}\left(\frac{6\sqrt{6+17x+12x^2}}{7(2+3x)}\right)}{1613472}$$

```
[In] Integrate[Sqrt[6 + 17*x + 12*x^2]/((2 + 3*x)^2*(30 + 31*x - 12*x^2)^2),x]
[Out] ((88978 + 98767*x - 37644*x^2)*Sqrt[6 + 17*x + 12*x^2])/(38416*(-10 + 3*x)*
(2 + 3*x)*(3 + 4*x)) + (97*ArcTanh[(6*Sqrt[6 + 17*x + 12*x^2])/(7*(2 + 3*x)
)])/1613472
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.68

method	result
risch	$-\frac{37644x^2-98767x-88978}{38416(3x-10)\sqrt{12x^2+17x+6}}+\frac{97\,\text{arctanh}\left(\frac{\frac{206}{3}+97x}{28\sqrt{12\left(x-\frac{10}{3}\right)^2+97x-\frac{382}{3}}}\right)}{3226944}$
trager	$-\frac{\left(37644x^2-98767x-88978\right)\sqrt{12x^2+17x+6}}{38416\left(36x^3-69x^2-152x-60\right)}-\frac{97\ln\left(-\frac{84\sqrt{12x^2+17x+6}-206-291x}{3x-10}\right)}{3226944}$
default	$-\frac{\left(12\left(x+\frac{2}{3}\right)^2+x+\frac{2}{3}\right)^{\frac{3}{2}}}{72\left(x+\frac{2}{3}\right)^2}+\frac{\sqrt{12\left(x+\frac{2}{3}\right)^2+x+\frac{2}{3}}}{288}+\frac{\ln\left(\frac{\left(\frac{17}{2}+12x\right)\sqrt{12}}{12}+\sqrt{12\left(x+\frac{2}{3}\right)^2+x+\frac{2}{3}}\right)\sqrt{12}}{6912}-\frac{\left(12\left(x-\frac{10}{3}\right)^2+97x-\frac{382}{3}\right)^{\frac{3}{2}}}{67765824\left(x-\frac{10}{3}\right)}-$

```
[In] int((12*x^2+17*x+6)^(1/2)/(2+3*x)^2/(-12*x^2+31*x+30)^2,x,method=_RETURNVER
BOSE)
```

```
[Out] -1/38416*(37644*x^2-98767*x-88978)/(3*x-10)/(12*x^2+17*x+6)^(1/2)+97/322694
4*arctanh(1/28*(206/3+97*x)/(12*(x-10/3)^2+97*x-382/3)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.50

$$\int \frac{\sqrt{6+17x+12x^2}}{(2+3x)^2\left(30+31x-12x^2\right)^2}\,dx$$
$$= \frac{97\left(36\,x^3-69\,x^2-152\,x-60\right)\log\left(\frac{291\,x+84\,\sqrt{12\,x^2+17\,x+6}+206}{x}\right)-97\left(36\,x^3-69\,x^2-152\,x-60\right)\log\left(\frac{291\,x\cdots}{}\right.}{6453888\left(36\,x^3-69\,x^2-152\,x-\cdots\right.}$$

```
[In] integrate((12*x^2+17*x+6)^(1/2)/(2+3*x)^2/(-12*x^2+31*x+30)^2,x, algorithm=
"fricas")
```

```
[Out] 1/6453888*(97*(36*x^3 - 69*x^2 - 152*x - 60)*log((291*x + 84*sqrt(12*x^2 +
17*x + 6) + 206)/x) - 97*(36*x^3 - 69*x^2 - 152*x - 60)*log((291*x - 84*sqr
t(12*x^2 + 17*x + 6) + 206)/x) - 168*(37644*x^2 - 98767*x - 88978)*sqrt(12*
x^2 + 17*x + 6))/(36*x^3 - 69*x^2 - 152*x - 60)
```

Sympy [F]

$$\int \frac{\sqrt{6+17x+12x^2}}{(2+3x)^2\left(30+31x-12x^2\right)^2}\,dx = \int \frac{\sqrt{(3x+2)\,(4x+3)}}{\left(3x-10\right)^2\left(3x+2\right)^2\left(4x+3\right)^2}\,dx$$

```
[In] integrate((12*x**2+17*x+6)**(1/2)/(2+3*x)**2/(-12*x**2+31*x+30)**2,x)
```

```
[Out] Integral(sqrt((3*x + 2)*(4*x + 3))/((3*x - 10)**2*(3*x + 2)**2*(4*x + 3)**2
), x)
```

Maxima [F]

$$\int \frac{\sqrt{6+17x+12x^2}}{(2+3x)^2\left(30+31x-12x^2\right)^2}\,dx = \int \frac{\sqrt{12\,x^2+17\,x+6}}{\left(12\,x^2-31\,x-30\right)^2(3\,x+2)^2}\,dx$$

```
[In] integrate((12*x^2+17*x+6)^(1/2)/(2+3*x)^2/(-12*x^2+31*x+30)^2,x, algorithm=
"maxima")
```

```
[Out] integrate(sqrt(12*x^2 + 17*x + 6)/((12*x^2 - 31*x - 30)^2*(3*x + 2)^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 159 vs. $2(70) = 140$.

Time = 0.32 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.89

$$\int \frac{\sqrt{6+17x+12x^2}}{(2+3x)^2\left(30+31x-12x^2\right)^2}\,dx$$
$$= \frac{1}{9680832}\sqrt{3}\left(\sqrt{3}\left(175672\sqrt{3}+97\log\left(\frac{7\sqrt{3}-12}{7\sqrt{3}+12}\right)\right)\operatorname{sgn}\left(\frac{1}{3\,x+2}\right)-\left(97\sqrt{3}\log\left(\frac{\left|-28\sqrt{3}+24\sqrt{\frac{1}{3\,x}}\right.}{4\left(7\sqrt{3}+6\sqrt{\frac{1}{3\,x+}}\right.}\right.\right.\right.$$

```
[In] integrate((12*x^2+17*x+6)^(1/2)/(2+3*x)^2/(-12*x^2+31*x+30)^2,x, algorithm=
"giac")
```

```
[Out] 1/9680832*sqrt(3)*(sqrt(3)*(175672*sqrt(3) + 97*log((7*sqrt(3) - 12)/(7*sqr
t(3) + 12)))*sgn(1/(3*x + 2)) - (97*sqrt(3)*log(1/4*abs(-28*sqrt(3) + 24*sq
rt(1/(3*x + 2) + 4))/(7*sqrt(3) + 6*sqrt(1/(3*x + 2) + 4))) + 134456*sqrt(1
/(3*x + 2) + 4) + 28*(221183/(3*x + 2) - 18436)/(12*(1/(3*x + 2) + 4)^(3/2)
 - 49*sqrt(1/(3*x + 2) + 4)))*sgn(1/(3*x + 2)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{6+17x+12x^2}}{(2+3x)^2\left(30+31x-12x^2\right)^2}\,dx = \int \frac{\sqrt{12\,x^2+17\,x+6}}{(3\,x+2)^2\left(-12\,x^2+31\,x+30\right)^2}\,dx$$

```
[In] int((17*x + 12*x^2 + 6)^(1/2)/((3*x + 2)^2*(31*x - 12*x^2 + 30)^2),x)
```

```
[Out] int((17*x + 12*x^2 + 6)^(1/2)/((3*x + 2)^2*(31*x - 12*x^2 + 30)^2), x)
```

3.137 $\int \frac{\sqrt{6+17x+12x^2}}{(2+3x)^3(30+31x-12x^2)^3}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 139

$$\int \frac{\sqrt{6+17x+12x^2}}{(2+3x)^3\left(30+31x-12x^2\right)^3}\,dx = -\frac{275+388x}{294(10-3x)^2\left(6+17x+12x^2\right)^{3/2}} + \frac{738029+1042556x}{8232(10-3x)^2\sqrt{6+17x+12x^2}} - \frac{50555899\sqrt{6+17x+12x^2}}{19361664(10-3x)^2} - \frac{1634466587\sqrt{6+17x+12x^2}}{7589772288(10-3x)} + \frac{40325\,\text{arctanh}\left(\frac{206+291x}{84\sqrt{6+17x+12x^2}}\right)}{637540872192}$$

```
[Out] 1/294*(-275-388*x)/(10-3*x)^2/(12*x^2+17*x+6)^(3/2)+40325/637540872192*arct
anh(1/84*(206+291*x)/(12*x^2+17*x+6)^(1/2))+1/8232*(738029+1042556*x)/(10-3
*x)^2/(12*x^2+17*x+6)^(1/2)-50555899/19361664*(12*x^2+17*x+6)^(1/2)/(10-3*x
)^2-1634466587/7589772288*(12*x^2+17*x+6)^(1/2)/(10-3*x)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.206$, Rules used = {1016, 754, 836, 848, 820, 738, 212}

$$\int \frac{\sqrt{6+17x+12x^2}}{(2+3x)^3\left(30+31x-12x^2\right)^3}\,dx = \frac{40325\,\text{arctanh}\left(\frac{291x+206}{84\sqrt{12x^2+17x+6}}\right)}{637540872192} - \frac{388x+275}{294(10-3x)^2\left(12x^2+17x+6\right)^{3/2}} - \frac{1634466587\sqrt{12x^2+17x+6}}{7589772288(10-3x)} - \frac{50555899\sqrt{12x^2+17x+6}}{19361664(10-3x)^2} + \frac{1042556x+738029}{8232(10-3x)^2\sqrt{12x^2+17x+6}}$$

```
[In] Int[Sqrt[6 + 17*x + 12*x^2]/((2 + 3*x)^3*(30 + 31*x - 12*x^2)^3),x]
```

```
[Out] -1/294*(275 + 388*x)/((10 - 3*x)^2*(6 + 17*x + 12*x^2)^(3/2)) + (738029 + 1
042556*x)/(8232*(10 - 3*x)^2*Sqrt[6 + 17*x + 12*x^2]) - (50555899*Sqrt[6 +
17*x + 12*x^2])/(19361664*(10 - 3*x)^2) - (1634466587*Sqrt[6 + 17*x + 12*x^
2])/(7589772288*(10 - 3*x)) + (40325*ArcTanh[(206 + 291*x)/(84*Sqrt[6 + 17*
x + 12*x^2])])/637540872192
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
```

```
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 836

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 848

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1016

```
Int[((g_) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.
) + (e_.)*(x_) + (f_.)*(x_)^2)^(m_.), x_Symbol] :> Int[(d*(g/a) + f*h*(x/c)
)^m*(a + b*x + c*x^2)^(m + p), x] /; FreeQ[{a, b, c, d, e, f, g, h, p}, x]
&& EqQ[c*g^2 - b*g*h + a*h^2, 0] && EqQ[c^2*d*g^2 - a*c*e*g*h + a^2*f*h^2,
```

`0] && IntegerQ[m]`

Rubi steps

$$\text{integral} = \int \frac{1}{(10-3x)^3\left(6+17x+12x^2\right)^{5/2}}\,dx$$

$$= -\frac{275+388x}{294(10-3x)^2\left(6+17x+12x^2\right)^{3/2}} - \frac{\int \frac{\frac{109953}{2}-41904x}{(10-3x)^3(6+17x+12x^2)^{3/2}}\,dx}{2646}$$

$$= -\frac{275+388x}{294(10-3x)^2\left(6+17x+12x^2\right)^{3/2}} + \frac{738029+1042556x}{8232(10-3x)^2\sqrt{6+17x+12x^2}} + \frac{\int \frac{-\frac{5020024653}{4}-1773387756x}{(10-3x)^3\sqrt{6+17x+12x^2}}\,dx}{2333772}$$

$$= -\frac{275+388x}{294(10-3x)^2\left(6+17x+12x^2\right)^{3/2}} + \frac{738029+1042556x}{8232(10-3x)^2\sqrt{6+17x+12x^2}} - \frac{50555899\sqrt{6+17x+12x^2}}{19361664(10-3x)^2} - \frac{\int \frac{\frac{1461036257541}{8}+257986752597x}{(10-3x)^2\sqrt{6+17x+12x^2}}\,dx}{8233547616}$$

$$= -\frac{275+388x}{294(10-3x)^2\left(6+17x+12x^2\right)^{3/2}} + \frac{738029+1042556x}{8232(10-3x)^2\sqrt{6+17x+12x^2}} - \frac{50555899\sqrt{6+17x+12x^2}}{19361664(10-3x)^2} - \frac{1634466587\sqrt{6+17x+12x^2}}{7589772288(10-3x)} + \frac{40325\int \frac{1}{(10-3x)\sqrt{6+17x+12x^2}}\,dx}{15179544576}$$

$$= -\frac{275+388x}{294(10-3x)^2\left(6+17x+12x^2\right)^{3/2}} + \frac{738029+1042556x}{8232(10-3x)^2\sqrt{6+17x+12x^2}} - \frac{50555899\sqrt{6+17x+12x^2}}{19361664(10-3x)^2} - \frac{1634466587\sqrt{6+17x+12x^2}}{7589772288(10-3x)} - \frac{40325\text{Subst}\left(\int \frac{1}{7056-x^2}\,dx, x, \frac{-206-291x}{\sqrt{6+17x+12x^2}}\right)}{7589772288}$$

$$= -\frac{275+388x}{294(10-3x)^2\left(6+17x+12x^2\right)^{3/2}} + \frac{738029+1042556x}{8232(10-3x)^2\sqrt{6+17x+12x^2}} - \frac{50555899\sqrt{6+17x+12x^2}}{19361664(10-3x)^2} - \frac{1634466587\sqrt{6+17x+12x^2}}{7589772288(10-3x)} + \frac{40325\tanh^{-1}\left(\frac{206+291x}{84\sqrt{6+17x+12x^2}}\right)}{637540872192}$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.68

$$\int \frac{\sqrt{6+17x+12x^2}}{(2+3x)^3 (30+31x-12x^2)^3}\,dx$$
$$= \frac{\sqrt{6+17x+12x^2}(2773753482408+10124325497244x+9848047480070x^2-1096520427663x^3-320682}{7589772288(-10+3x)^2(2+3x)^2(3+4x)^2}$$
$$+ \frac{40325\operatorname{arctanh}\left(\frac{6\sqrt{6+17x+12x^2}}{7(2+3x)}\right)}{318770436096}$$

```
[In] Integrate[Sqrt[6 + 17*x + 12*x^2]/((2 + 3*x)^3*(30 + 31*x - 12*x^2)^3),x]
```

```
[Out] (Sqrt[6 + 17*x + 12*x^2]*(2773753482408 + 10124325497244*x + 9848047480070*
x^2 - 1096520427663*x^3 - 3206824169544*x^4 + 706089565584*x^5))/(758977228
8*(-10 + 3*x)^2*(2 + 3*x)^2*(3 + 4*x)^2) + (40325*ArcTanh[(6*Sqrt[6 + 17*x
+ 12*x^2])/(7*(2 + 3*x))])/318770436096
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.52

method	result
risch	$\frac{706089565584x^5-3206824169544x^4-1096520427663x^3+9848047480070x^2+10124325497244x+2773753482408}{7589772288(12x^2+17x+6)^{\frac{3}{2}}(3x-10)^2}+\frac{40325\ \operatorname{arctanh}}{}$
trager	$\frac{(706089565584x^5-3206824169544x^4-1096520427663x^3+9848047480070x^2+10124325497244x+2773753482408)\sqrt{12x^2+17x+6}}{7589772288(36x^3-69x^2-152x-60)^2}-$
default	$-\frac{\left(12\left(x+\frac{2}{3}\right)^2+x+\frac{2}{3}\right)^{\frac{3}{2}}}{2592\left(x+\frac{2}{3}\right)^3}+\frac{47\left(12\left(x+\frac{2}{3}\right)^2+x+\frac{2}{3}\right)^{\frac{3}{2}}}{1152\left(x+\frac{2}{3}\right)^2}-\frac{23\sqrt{12\left(x+\frac{2}{3}\right)^2+x+\frac{2}{3}}}{4608}-\frac{23\ln\left(\frac{\left(\frac{17}{2}+12x\right)\sqrt{12}}{12}+\sqrt{12\left(x+\frac{2}{3}\right)^2+x+\frac{2}{3}}\right)\sqrt{1}}{110592}$

```
[In] int((12*x^2+17*x+6)^(1/2)/(2+3*x)^3/(-12*x^2+31*x+30)^3,x,method=_RETURNVER
BOSE)
```

```
[Out] 1/7589772288*(706089565584*x^5-3206824169544*x^4-1096520427663*x^3+98480474
80070*x^2+10124325497244*x+2773753482408)/(12*x^2+17*x+6)^(3/2)/(3*x-10)^2+
40325/637540872192*arctanh(1/28*(206/3+97*x)/(12*(x-10/3)^2+97*x-382/3)^(1/
2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.34

$$\int \frac{\sqrt{6+17x+12x^2}}{(2+3x)^3(30+31x-12x^2)^3}\,dx$$
$$= \frac{40325\,(1296\,x^6-4968\,x^5-6183\,x^4+16656\,x^3+31384\,x^2+18240\,x+3600)\log\left(\frac{291\,x+84\,\sqrt{12\,x^2+17\,x+6}+206}{x}\right.}{}$$

```
[In] integrate((12*x^2+17*x+6)^(1/2)/(2+3*x)^3/(-12*x^2+31*x+30)^3,x, algorithm=
"fricas")
```

```
[Out] 1/1275081744384*(40325*(1296*x^6 - 4968*x^5 - 6183*x^4 + 16656*x^3 + 31384*
x^2 + 18240*x + 3600)*log((291*x + 84*sqrt(12*x^2 + 17*x + 6) + 206)/x) - 4
0325*(1296*x^6 - 4968*x^5 - 6183*x^4 + 16656*x^3 + 31384*x^2 + 18240*x + 36
00)*log((291*x - 84*sqrt(12*x^2 + 17*x + 6) + 206)/x) + 168*(706089565584*x
^5 - 3206824169544*x^4 - 1096520427663*x^3 + 9848047480070*x^2 + 1012432549
7244*x + 2773753482408)*sqrt(12*x^2 + 17*x + 6))/(1296*x^6 - 4968*x^5 - 618
3*x^4 + 16656*x^3 + 31384*x^2 + 18240*x + 3600)
```

Sympy [F]

$$\int \frac{\sqrt{6+17x+12x^2}}{(2+3x)^3(30+31x-12x^2)^3}\,dx =$$
$$-\int \frac{\sqrt{12x^2+17x+6}}{46656x^9-268272x^8-76788x^7+1703619x^6+1218456x^5-3669588x^4-6898688x^3-4903920x^2-}$$

```
[In] integrate((12*x**2+17*x+6)**(1/2)/(2+3*x)**3/(-12*x**2+31*x+30)**3,x)
```

```
[Out] -Integral(sqrt(12*x**2 + 17*x + 6)/(46656*x**9 - 268272*x**8 - 76788*x**7 +
 1703619*x**6 + 1218456*x**5 - 3669588*x**4 - 6898688*x**3 - 4903920*x**2 -
 1641600*x - 216000), x)
```

Maxima [F]

$$\int \frac{\sqrt{6+17x+12x^2}}{(2+3x)^3\,(30+31x-12x^2)^3}\,dx = \int -\frac{\sqrt{12\,x^2+17\,x+6}}{(12\,x^2-31\,x-30)^3(3\,x+2)^3}\,dx$$

```
[In] integrate((12*x^2+17*x+6)^(1/2)/(2+3*x)^3/(-12*x^2+31*x+30)^3,x, algorithm=
"maxima")
```

```
[Out] -integrate(sqrt(12*x^2 + 17*x + 6)/((12*x^2 - 31*x - 30)^3*(3*x + 2)^3), x)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.67

$$\int \frac{\sqrt{6+17x+12x^2}}{(2+3x)^3\,(30+31x-12x^2)^3}\,dx$$
$$= \frac{\sqrt{3}\left(282273\,\sqrt{3}\left(2\,\sqrt{3}x-\sqrt{12\,x^2+17\,x+6}\right)^3-11460924\,\left(2\,\sqrt{3}x-\sqrt{12\,x^2+17\,x+6}\right)^2-37551180\,\sqrt{3}\right.}{159385218048\,\left(3\,\left(2\,\sqrt{3}x-\sqrt{12\,x^2+17\,x+6}\right)^2-40\,\sqrt{3}\left(2\,\sqrt{3}x-\sqrt{12\,x^2+1\right.\right.}$$
$$+ \frac{(8\,(2860316794\,x+6078171227)x+34383350229)x+8090114146}{2213683584\,(12\,x^2+17\,x+6)^{\frac{3}{2}}}$$
$$+ \frac{40325}{637540872192}\,\log\left(\left|-6\,\sqrt{3}x+20\,\sqrt{3}+3\,\sqrt{12\,x^2+17\,x+6}+42\right|\right)$$
$$- \frac{40325}{637540872192}\,\log\left(\left|-6\,\sqrt{3}x+20\,\sqrt{3}+3\,\sqrt{12\,x^2+17\,x+6}-42\right|\right)$$

```
[In] integrate((12*x^2+17*x+6)^(1/2)/(2+3*x)^3/(-12*x^2+31*x+30)^3,x, algorithm=
"giac")
```

```
[Out] 1/159385218048*sqrt(3)*(282273*sqrt(3)*(2*sqrt(3)*x - sqrt(12*x^2 + 17*x +
6))^3 - 11460924*(2*sqrt(3)*x - sqrt(12*x^2 + 17*x + 6))^2 - 37551180*sqrt(
3)*(2*sqrt(3)*x - sqrt(12*x^2 + 17*x + 6)) - 83365264)/(3*(2*sqrt(3)*x - sq
rt(12*x^2 + 17*x + 6))^2 - 40*sqrt(3)*(2*sqrt(3)*x - sqrt(12*x^2 + 17*x + 6
)) - 188)^2 + 1/2213683584*((8*(2860316794*x + 6078171227)*x + 34383350229)
*x + 8090114146)/(12*x^2 + 17*x + 6)^(3/2) + 40325/637540872192*log(abs(-6*
sqrt(3)*x + 20*sqrt(3) + 3*sqrt(12*x^2 + 17*x + 6) + 42)) - 40325/637540872
192*log(abs(-6*sqrt(3)*x + 20*sqrt(3) + 3*sqrt(12*x^2 + 17*x + 6) - 42))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{6+17x+12x^2}}{(2+3x)^3\,(30+31x-12x^2)^3}\,dx = \int \frac{\sqrt{12\,x^2+17\,x+6}}{{\left(3\,x+2\right)}^3\,{\left(-12\,x^2+31\,x+30\right)}^3}\,dx$$

```
[In] int((17*x + 12*x^2 + 6)^(1/2)/((3*x + 2)^3*(31*x - 12*x^2 + 30)^3),x)
[Out] int((17*x + 12*x^2 + 6)^(1/2)/((3*x + 2)^3*(31*x - 12*x^2 + 30)^3), x)
```

3.138 $\int (-3+2x)\left(-3x+x^2\right)^{2/3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 15

$$\int (-3+2x)\left(-3x+x^2\right)^{2/3}\,dx=\frac{3}{5}\left(-3x+x^2\right)^{5/3}$$

```
[Out] 3/5*(x^2-3*x)^(5/3)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {643}

$$\int (-3+2x)\left(-3x+x^2\right)^{2/3}\,dx=\frac{3}{5}\left(x^2-3x\right)^{5/3}$$

```
[In] Int[(-3 + 2*x)*(-3*x + x^2)^(2/3),x]
```

```
[Out] (3*(-3*x + x^2)^(5/3))/5
```

Rule 643

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\text{integral}=\frac{3}{5}\left(-3x+x^2\right)^{5/3}$$

Mathematica [A] (verified)

Time = 9.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int (-3+2x)\left(-3x+x^2\right)^{2/3}\,dx=\frac{3}{5}((-3+x)x)^{5/3}$$

```
[In] Integrate[(-3 + 2*x)*(-3*x + x^2)^(2/3),x]
[Out] (3*((-3 + x)*x)^(5/3))/5
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$\frac{3\left(x^{2}-3x\right)^{\frac{5}{3}}}{5}$	12
default	$\frac{3\left(x^{2}-3x\right)^{\frac{5}{3}}}{5}$	12
pseudoelliptic	$\frac{3(-3+x)x\left((-3+x)x\right)^{\frac{2}{3}}}{5}$	14
gosper	$\frac{3(-3+x)x\left(x^{2}-3x\right)^{\frac{2}{3}}}{5}$	16
trager	$\frac{3(-3+x)x\left(x^{2}-3x\right)^{\frac{2}{3}}}{5}$	16
risch	$\frac{3(-3+x)^{2}x^{2}}{5\left((-3+x)x\right)^{\frac{1}{3}}}$	18
meijerg	$-\frac{9\,3^{\frac{2}{3}}\operatorname{signum}(-3+x)^{\frac{2}{3}}x^{\frac{5}{3}}\,{}_2F_1\left(-\frac{2}{3},\frac{5}{3};\frac{8}{3};\frac{x}{3}\right)}{5\left(-\operatorname{signum}(-3+x)\right)^{\frac{2}{3}}}+\frac{3\,3^{\frac{2}{3}}\operatorname{signum}(-3+x)^{\frac{2}{3}}x^{\frac{8}{3}}\,{}_2F_1\left(-\frac{2}{3},\frac{8}{3};\frac{11}{3};\frac{x}{3}\right)}{4\left(-\operatorname{signum}(-3+x)\right)^{\frac{2}{3}}}$	64

```
[In] int((2*x-3)*(x^2-3*x)^(2/3),x,method=_RETURNVERBOSE)
[Out] 3/5*(x^2-3*x)^(5/3)
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int (-3+2x)\left(-3x+x^2\right)^{2/3}\,dx=\frac{3}{5}\left(x^2-3\,x\right)^{\frac{5}{3}}$$

```
[In] integrate((-3+2*x)*(x^2-3*x)^(2/3),x, algorithm="fricas")
[Out] 3/5*(x^2 - 3*x)^(5/3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(12) = 24$.

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.07

$$\int (-3+2x)\left(-3x+x^2\right)^{2/3}\,dx = \frac{3x^2(x^2-3x)^{\frac{2}{3}}}{5} - \frac{9x(x^2-3x)^{\frac{2}{3}}}{5}$$

```
[In] integrate((-3+2*x)*(x**2-3*x)**(2/3),x)
[Out] 3*x**2*(x**2 - 3*x)**(2/3)/5 - 9*x*(x**2 - 3*x)**(2/3)/5
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int (-3+2x)\left(-3x+x^2\right)^{2/3}\,dx = \frac{3}{5}\left(x^2-3\,x\right)^{\frac{5}{3}}$$

```
[In] integrate((-3+2*x)*(x^2-3*x)^(2/3),x, algorithm="maxima")
[Out] 3/5*(x^2 - 3*x)^(5/3)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int (-3+2x)\left(-3x+x^2\right)^{2/3}\,dx = \frac{3}{5}\left(x^2-3\,x\right)^{\frac{5}{3}}$$

```
[In] integrate((-3+2*x)*(x^2-3*x)^(2/3),x, algorithm="giac")
[Out] 3/5*(x^2 - 3*x)^(5/3)
```

Mupad [B] (verification not implemented)

Time = 12.69 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int (-3+2x)\left(-3x+x^2\right)^{2/3}\,dx=\frac{3\,x\,{\left(x^2-3\,x\right)}^{2/3}\,\left(x-3\right)}{5}$$

```
[In] int((2*x - 3)*(x^2 - 3*x)^(2/3),x)
[Out] (3*x*(x^2 - 3*x)^(2/3)*(x - 3))/5
```

3.139 $\int ((-3+x)x)^{2/3}(-3+2x)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 16

$$\int ((-3+x)x)^{2/3}(-3+2x)\,dx=\frac{3}{5}(-((3-x)x))^{5/3}$$

```
[Out] 3/5*(-(3-x)*x)^(5/3)
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1602}

$$\int ((-3+x)x)^{2/3}(-3+2x)\,dx=\frac{3}{5}(-((3-x)x))^{5/3}$$

```
[In] Int[((-3 + x)*x)^(2/3)*(-3 + 2*x),x]
```

```
[Out] (3*(-((3 - x)*x))^(5/3))/5
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\text{integral}=\frac{3}{5}(-((3-x)x))^{5/3}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int ((-3+x)x)^{2/3}(-3+2x)\,dx = \frac{3}{5}((-3+x)x)^{5/3}$$

```
[In] Integrate[((-3 + x)*x)^(2/3)*(-3 + 2*x),x]
[Out] (3*((-3 + x)*x)^(5/3))/5
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

method	result	size
derivativedivides	$\frac{3((-3+x)x)^{\frac{5}{3}}}{5}$	10
default	$\frac{3((-3+x)x)^{\frac{5}{3}}}{5}$	10
gosper	$\frac{3(-3+x)x((-3+x)x)^{\frac{2}{3}}}{5}$	14
pseudoelliptic	$\frac{3(-3+x)x((-3+x)x)^{\frac{2}{3}}}{5}$	14
trager	$\frac{3(-3+x)x(x^2-3x)^{\frac{2}{3}}}{5}$	16
risch	$\frac{3(-3+x)^2x^2}{5((-3+x)x)^{\frac{1}{3}}}$	18
meijerg	$-\frac{9\,3^{\frac{2}{3}}\operatorname{signum}(-3+x)^{\frac{2}{3}}x^{\frac{5}{3}}\,{}_2F_1\left(-\frac{2}{3},\frac{5}{3};\frac{8}{3};\frac{x}{3}\right)}{5(-\operatorname{signum}(-3+x))^{\frac{2}{3}}}+\frac{3\,3^{\frac{2}{3}}\operatorname{signum}(-3+x)^{\frac{2}{3}}x^{\frac{8}{3}}\,{}_2F_1\left(-\frac{2}{3},\frac{8}{3};\frac{11}{3};\frac{x}{3}\right)}{4(-\operatorname{signum}(-3+x))^{\frac{2}{3}}}$	64

```
[In] int(((-3+x)*x)^(2/3)*(2*x-3),x,method=_RETURNVERBOSE)
[Out] 3/5*((-3+x)*x)^(5/3)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.69

$$\int ((-3+x)x)^{2/3}(-3+2x)\,dx = \frac{3}{5}\left(x^2-3\,x\right)^{\frac{5}{3}}$$

```
[In] integrate(((-3+x)*x)^(2/3)*(-3+2*x),x, algorithm="fricas")
[Out] 3/5*(x^2 - 3*x)^(5/3)
```

Sympy [A] (verification not implemented)

Time = 2.28 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int ((-3+x)x)^{2/3}(-3+2x)\,dx = \frac{3(x(x-3))^{\frac{5}{3}}}{5}$$

```
[In] integrate(((-3+x)*x)**(2/3)*(-3+2*x),x)
[Out] 3*(x*(x - 3))**(5/3)/5
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.56

$$\int ((-3+x)x)^{2/3}(-3+2x)\,dx = \frac{3}{5}\,((x-3)x)^{\frac{5}{3}}$$

```
[In] integrate(((-3+x)*x)^(2/3)*(-3+2*x),x, algorithm="maxima")
[Out] 3/5*((x - 3)*x)^(5/3)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.69

$$\int ((-3+x)x)^{2/3}(-3+2x)\,dx = \frac{3}{5}\,\left(x^2-3\,x\right)^{\frac{5}{3}}$$

```
[In] integrate(((-3+x)*x)^(2/3)*(-3+2*x),x, algorithm="giac")
[Out] 3/5*(x^2 - 3*x)^(5/3)
```

Mupad [B] (verification not implemented)

Time = 12.35 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int ((-3+x)x)^{2/3}(-3+2x)\,dx = \frac{3\,x\,\left(x\,\left(x-3\right)\right)^{2/3}\,\left(x-3\right)}{5}$$

```
[In] int((2*x - 3)*(x*(x - 3))^(2/3),x)
[Out] (3*x*(x*(x - 3))^(2/3)*(x - 3))/5
```

3.140 $\int \frac{x(9-9x+2x^2)}{\sqrt[3]{-3x+x^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 15

$$\int \frac{x(9-9x+2x^2)}{\sqrt[3]{-3x+x^2}}\,dx = \frac{3}{5}\left(-3x+x^2\right)^{5/3}$$

```
[Out] 3/5*(x^2-3*x)^(5/3)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1645, 643}

$$\int \frac{x(9-9x+2x^2)}{\sqrt[3]{-3x+x^2}}\,dx = \frac{3}{5}\left(x^2-3x\right)^{5/3}$$

```
[In] Int[(x*(9 - 9*x + 2*x^2))/(-3*x + x^2)^(1/3),x]
```

```
[Out] (3*(-3*x + x^2)^(5/3))/5
```

Rule 643

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1645

```
Int[(Pq_)*((e_.)*(x_))^(m_.)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[e, Int[(e*x)^(m - 1)*PolynomialQuotient[Pq, b + c*x, x]*(b*x + c*x^2
```

```
)^(p + 1), x], x] /; FreeQ[{b, c, e, m, p}, x] && PolyQ[Pq, x] && EqQ[Polyn
omialRemainder[Pq, b + c*x, x], 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int (-3+2x)\left(-3x+x^2\right)^{2/3}\,dx \\ &= \frac{3}{5}\left(-3x+x^2\right)^{5/3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{x(9-9x+2x^2)}{\sqrt[3]{-3x+x^2}}\,dx = \frac{3}{5}((-3+x)x)^{5/3}$$

```
[In] Integrate[(x*(9 - 9*x + 2*x^2))/(-3*x + x^2)^(1/3),x]
[Out] (3*((-3 + x)*x)^(5/3))/5
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result
pseudoelliptic	$\frac{3(-3+x)x((-3+x)x)^{\frac{2}{3}}}{5}$
trager	$\frac{3(-3+x)x(x^2-3x)^{\frac{2}{3}}}{5}$
risch	$\frac{3(-3+x)^2x^2}{5((-3+x)x)^{\frac{1}{3}}}$
gosper	$\frac{3(-3+x)^2x^2}{5(x^2-3x)^{\frac{1}{3}}}$
meijerg	$\frac{2\,3^{\frac{2}{3}}(-\operatorname{signum}(-3+x))^{\frac{1}{3}}x^{\frac{11}{3}}\,{}_2F_1\left(\frac{1}{3},\frac{11}{3};\frac{14}{3};\frac{x}{3}\right)}{11\operatorname{signum}(-3+x)^{\frac{1}{3}}}-\frac{9\,3^{\frac{2}{3}}(-\operatorname{signum}(-3+x))^{\frac{1}{3}}x^{\frac{8}{3}}\,{}_2F_1\left(\frac{1}{3},\frac{8}{3};\frac{11}{3};\frac{x}{3}\right)}{8\operatorname{signum}(-3+x)^{\frac{1}{3}}}+\frac{9\,3^{\frac{2}{3}}(-\operatorname{signum}(-3+x}{5\operatorname{signum}(}$

```
[In] int(x*(2*x^2-9*x+9)/(x^2-3*x)^(1/3),x,method=_RETURNVERBOSE)
[Out] 3/5*(-3+x)*x*((-3+x)*x)^(2/3)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{x(9-9x+2x^2)}{\sqrt[3]{-3x+x^2}}\,dx = \frac{3}{5}\left(x^2-3\,x\right)^{\frac{5}{3}}$$

```
[In] integrate(x*(2*x^2-9*x+9)/(x^2-3*x)^(1/3),x, algorithm="fricas")
[Out] 3/5*(x^2 - 3*x)^(5/3)
```

Sympy [F]

$$\int \frac{x(9-9x+2x^2)}{\sqrt[3]{-3x+x^2}}\,dx = \int \frac{x(x-3)\,(2x-3)}{\sqrt[3]{x\,(x-3)}}\,dx$$

```
[In] integrate(x*(2*x**2-9*x+9)/(x**2-3*x)**(1/3),x)
[Out] Integral(x*(x - 3)*(2*x - 3)/(x*(x - 3))**(1/3), x)
```

Maxima [F]

$$\int \frac{x(9-9x+2x^2)}{\sqrt[3]{-3x+x^2}}\,dx = \int \frac{(2\,x^2-9\,x+9)x}{(x^2-3\,x)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(x*(2*x^2-9*x+9)/(x^2-3*x)^(1/3),x, algorithm="maxima")
[Out] integrate((2*x^2 - 9*x + 9)*x/(x^2 - 3*x)^(1/3), x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{x(9-9x+2x^2)}{\sqrt[3]{-3x+x^2}}\,dx = \frac{3}{5}\left(x^2-3\,x\right)^{\frac{5}{3}}$$

```
[In] integrate(x*(2*x^2-9*x+9)/(x^2-3*x)^(1/3),x, algorithm="giac")
[Out] 3/5*(x^2 - 3*x)^(5/3)
```

Mupad [B] (verification not implemented)

Time = 12.44 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{x(9-9x+2x^2)}{\sqrt[3]{-3x+x^2}}\,dx=\frac{3\,x\,{\left(x^2-3\,x\right)}^{2/3}\,\left(x-3\right)}{5}$$

```
[In] int((x*(2*x^2 - 9*x + 9))/(x^2 - 3*x)^(1/3),x)
[Out] (3*x*(x^2 - 3*x)^(2/3)*(x - 3))/5
```

3.141 $\int \frac{x(9-9x+2x^2)}{\sqrt[3]{(-3+x)x}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 15

$$\int \frac{x(9-9x+2x^2)}{\sqrt[3]{(-3+x)x}}\,dx = \frac{3}{5}(-3x+x^2)^{5/3}$$

```
[Out] 3/5*(x^2-3*x)^(5/3)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1976, 1645, 643}

$$\int \frac{x(9-9x+2x^2)}{\sqrt[3]{(-3+x)x}}\,dx = \frac{3}{5}(x^2-3x)^{5/3}$$

```
[In] Int[(x*(9 - 9*x + 2*x^2))/((-3 + x)*x)^(1/3),x]
[Out] (3*(-3*x + x^2)^(5/3))/5
```

Rule 643

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1645

```
Int[(Pq_)*((e_.)*(x_))^(m_.)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[e, Int[(e*x)^(m - 1)*PolynomialQuotient[Pq, b + c*x, x]*(b*x + c*x^2
```

```
)^(p + 1), x], x] /; FreeQ[{b, c, e, m, p}, x] && PolyQ[Pq, x] && EqQ[Polyn
omialRemainder[Pq, b + c*x, x], 0]
```

Rule 1976

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.)))^(p_)
, x_Symbol] :> Int[u*(a*c*e + (b*c + a*d)*e*x^n + b*d*e*x^(2*n))^p, x] /; F
reeQ[{a, b, c, d, e, n, p}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \frac{x(9-9x+2x^2)}{\sqrt[3]{-3x+x^2}}\,dx\\ &= \int (-3+2x)\left(-3x+x^2\right)^{2/3}\,dx\\ &= \frac{3}{5}\left(-3x+x^2\right)^{5/3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{x(9-9x+2x^2)}{\sqrt[3]{(-3+x)x}}\,dx = \frac{3}{5}((-3+x)x)^{5/3}$$

```
[In] Integrate[(x*(9 - 9*x + 2*x^2))/((-3 + x)*x)^(1/3),x]
[Out] (3*((-3 + x)*x)^(5/3))/5
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result
pseudoelliptic	$\frac{3(-3+x)x((-3+x)x)^{\frac{2}{3}}}{5}$
trager	$\frac{3(-3+x)x\left(x^{2}-3x\right)^{\frac{2}{3}}}{5}$
gosper	$\frac{3(-3+x)^{2}x^{2}}{5((-3+x)x)^{\frac{1}{3}}}$
risch	$\frac{3(-3+x)^{2}x^{2}}{5((-3+x)x)^{\frac{1}{3}}}$
meijerg	$\frac{2\,3^{\frac{2}{3}}(-\operatorname{signum}(-3+x))^{\frac{1}{3}}x^{\frac{11}{3}}\,{}_2F_1\left(\frac{1}{3},\frac{11}{3};\frac{14}{3};\frac{x}{3}\right)}{11\operatorname{signum}(-3+x)^{\frac{1}{3}}}-\frac{9\,3^{\frac{2}{3}}(-\operatorname{signum}(-3+x))^{\frac{1}{3}}x^{\frac{8}{3}}\,{}_2F_1\left(\frac{1}{3},\frac{8}{3};\frac{11}{3};\frac{x}{3}\right)}{8\operatorname{signum}(-3+x)^{\frac{1}{3}}}+\frac{9\,3^{\frac{2}{3}}(-\operatorname{signum}(-3+x))}{5\operatorname{signum}(-}$

[In] int(x*(2*x^2-9*x+9)/((-3+x)*x)^(1/3),x,method=_RETURNVERBOSE)

[Out] 3/5*(-3+x)*x*((-3+x)*x)^(2/3)

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{x(9-9x+2x^2)}{\sqrt[3]{(-3+x)x}}\,dx=\frac{3}{5}\left(x^{2}-3\,x\right)^{\frac{5}{3}}$$

[In] integrate(x*(2*x^2-9*x+9)/((-3+x)*x)^(1/3),x, algorithm="fricas")

[Out] 3/5*(x^2 - 3*x)^(5/3)

Sympy [F]

$$\int \frac{x(9-9x+2x^2)}{\sqrt[3]{(-3+x)x}}\,dx=\int \frac{x(x-3)\,(2x-3)}{\sqrt[3]{x\,(x-3)}}\,dx$$

[In] integrate(x*(2*x**2-9*x+9)/((-3+x)*x)**(1/3),x)

[Out] Integral(x*(x - 3)*(2*x - 3)/(x*(x - 3))**(1/3), x)

Maxima [F]

$$\int \frac{x(9-9x+2x^2)}{\sqrt[3]{(-3+x)x}}\,dx = \int \frac{(2\,x^2-9\,x+9)x}{((x-3)x)^{\frac{1}{3}}}\,dx$$

```
[In] integrate(x*(2*x^2-9*x+9)/((-3+x)*x)^(1/3),x, algorithm="maxima")
[Out] integrate((2*x^2 - 9*x + 9)*x/((x - 3)*x)^(1/3), x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{x(9-9x+2x^2)}{\sqrt[3]{(-3+x)x}}\,dx = \frac{3}{5}\left(x^2-3\,x\right)^{\frac{5}{3}}$$

```
[In] integrate(x*(2*x^2-9*x+9)/((-3+x)*x)^(1/3),x, algorithm="giac")
[Out] 3/5*(x^2 - 3*x)^(5/3)
```

Mupad [B] (verification not implemented)

Time = 12.49 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{x(9-9x+2x^2)}{\sqrt[3]{(-3+x)x}}\,dx = \frac{3\,x\,\left(x\,\left(x-3\right)\right)^{2/3}\,\left(x-3\right)}{5}$$

```
[In] int((x*(2*x^2 - 9*x + 9))/(x*(x - 3))^(1/3),x)
[Out] (3*x*(x*(x - 3))^(2/3)*(x - 3))/5
```

3.142 $\int \frac{g+hx}{\sqrt[3]{-\frac{cg^2}{h^2}+9cx^2}(g^2+3h^2x^2)}\,dx$

Optimal result

Integrand size = 40, antiderivative size = 242

$$\int \frac{g+hx}{\sqrt[3]{-\frac{cg^2}{h^2}+9cx^2}\,(g^2+3h^2x^2)}\,dx = \frac{\sqrt[3]{1-\frac{9h^2x^2}{g^2}}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2^{2/3}\left(1-\frac{3hx}{g}\right)^{2/3}}{\sqrt{3}\sqrt[3]{1+\frac{3hx}{g}}}\right)}{2^{2/3}\sqrt{3}h\sqrt[3]{-\frac{cg^2}{h^2}+9cx^2}} + \frac{\sqrt[3]{1-\frac{9h^2x^2}{g^2}}\log\left(g^2+3h^2x^2\right)}{6\ 2^{2/3}h\sqrt[3]{-\frac{cg^2}{h^2}+9cx^2}} - \frac{\sqrt[3]{1-\frac{9h^2x^2}{g^2}}\log\left(\left(1-\frac{3hx}{g}\right)^{2/3}+\sqrt[3]{2}\sqrt[3]{1+\frac{3hx}{g}}\right)}{2\ 2^{2/3}h\sqrt[3]{-\frac{cg^2}{h^2}+9cx^2}}$$

```
[Out] 1/12*(1-9*h^2*x^2/g^2)^(1/3)*ln(3*h^2*x^2+g^2)*2^(1/3)/h/(-c*g^2/h^2+9*c*x^
2)^(1/3)-1/4*(1-9*h^2*x^2/g^2)^(1/3)*ln((1-3*h*x/g)^(2/3)+2^(1/3)*(1+3*h*x/
g)^(1/3))*2^(1/3)/h/(-c*g^2/h^2+9*c*x^2)^(1/3)-1/6*(1-9*h^2*x^2/g^2)^(1/3)*
arctan(-1/3*3^(1/2)+1/3*2^(2/3)*(1-3*h*x/g)^(2/3)/(1+3*h*x/g)^(1/3)*3^(1/2)
)*2^(1/3)/h/(-c*g^2/h^2+9*c*x^2)^(1/3)*3^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1023, 1022}

$$\int \frac{g+hx}{\sqrt[3]{-\frac{cg^2}{h^2}+9cx^2}\,(g^2+3h^2x^2)}\,dx = \frac{\sqrt[3]{1-\frac{9h^2x^2}{g^2}}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2^{2/3}\left(1-\frac{3hx}{g}\right)^{2/3}}{\sqrt{3}\sqrt[3]{\frac{3hx}{g}+1}}\right)}{2^{2/3}\sqrt{3}h\sqrt[3]{9cx^2-\frac{cg^2}{h^2}}} + \frac{\sqrt[3]{1-\frac{9h^2x^2}{g^2}}\log\left(g^2+3h^2x^2\right)}{6\ 2^{2/3}h\sqrt[3]{9cx^2-\frac{cg^2}{h^2}}} - \frac{\sqrt[3]{1-\frac{9h^2x^2}{g^2}}\log\left(\left(1-\frac{3hx}{g}\right)^{2/3}+\sqrt[3]{2}\sqrt[3]{\frac{3hx}{g}+1}\right)}{2\ 2^{2/3}h\sqrt[3]{9cx^2-\frac{cg^2}{h^2}}}$$

```
[In] Int[(g + h*x)/((-((c*g^2)/h^2) + 9*c*x^2)^(1/3)*(g^2 + 3*h^2*x^2)),x]
```

```
[Out] ((1 - (9*h^2*x^2)/g^2)^(1/3)*ArcTan[1/Sqrt[3] - (2^(2/3)*(1 - (3*h*x)/g)^(2
/3))/(Sqrt[3]*(1 + (3*h*x)/g)^(1/3))])/(2^(2/3)*Sqrt[3]*h*(-((c*g^2)/h^2) +
 9*c*x^2)^(1/3)) + ((1 - (9*h^2*x^2)/g^2)^(1/3)*Log[g^2 + 3*h^2*x^2])/(6*2^
(2/3)*h*(-((c*g^2)/h^2) + 9*c*x^2)^(1/3)) - ((1 - (9*h^2*x^2)/g^2)^(1/3)*Lo
g[(1 - (3*h*x)/g)^(2/3) + 2^(1/3)*(1 + (3*h*x)/g)^(1/3)])/(2*2^(2/3)*h*(-((
c*g^2)/h^2) + 9*c*x^2)^(1/3))
```

Rule 1022

```
Int[((g_) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)^(1/3)*((d_) + (f_.)*(x_)^2))
, x_Symbol] :> Simp[Sqrt[3]*h*(ArcTan[1/Sqrt[3] - 2^(2/3)*((1 - 3*h*(x/g))^
(2/3)/(Sqrt[3]*(1 + 3*h*(x/g))^(1/3)))]/(2^(2/3)*a^(1/3)*f)), x] + (-Simp[3
*h*(Log[(1 - 3*h*(x/g))^(2/3) + 2^(1/3)*(1 + 3*h*(x/g))^(1/3)]/(2^(5/3)*a^(
1/3)*f)), x] + Simp[h*(Log[d + f*x^2]/(2^(5/3)*a^(1/3)*f)), x]) /; FreeQ[{a
, c, d, f, g, h}, x] && EqQ[c*d + 3*a*f, 0] && EqQ[c*g^2 + 9*a*h^2, 0] && G
tQ[a, 0]
```

Rule 1023

```
Int[((g_) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)^(1/3)*((d_) + (f_.)*(x_)^2))
, x_Symbol] :> Dist[(1 + c*(x^2/a))^(1/3)/(a + c*x^2)^(1/3), Int[(g + h*x)/
```

```
((1 + c*(x^2/a))^(1/3)*(d + f*x^2)), x], x] /; FreeQ[{a, c, d, f, g, h}, x]
 && EqQ[c*d + 3*a*f, 0] && EqQ[c*g^2 + 9*a*h^2, 0] &&  !GtQ[a, 0]
```

Rubi steps

$$\text{integral} = \frac{\sqrt[3]{1-\frac{9h^2x^2}{g^2}}\int \frac{g+hx}{(g^2+3h^2x^2)\sqrt[3]{1-\frac{9h^2x^2}{g^2}}}\,dx}{\sqrt[3]{-\frac{cg^2}{h^2}+9cx^2}}$$

$$= \frac{\sqrt[3]{1-\frac{9h^2x^2}{g^2}}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2^{2/3}\left(1-\frac{3hx}{g}\right)^{2/3}}{\sqrt{3}\sqrt[3]{1+\frac{3hx}{g}}}\right)}{2^{2/3}\sqrt{3}h\sqrt[3]{-\frac{cg^2}{h^2}+9cx^2}}+\frac{\sqrt[3]{1-\frac{9h^2x^2}{g^2}}\log\left(g^2+3h^2x^2\right)}{6\ 2^{2/3}h\sqrt[3]{-\frac{cg^2}{h^2}+9cx^2}}-\frac{\sqrt[3]{1-\frac{9h^2x^2}{g^2}}\log\left(\left(1-\frac{3hx}{g}\right)^{2/3}+\sqrt[3]{2}\sqrt[3]{1+\frac{3hx}{g}}\right)}{2\ 2^{2/3}h\sqrt[3]{-\frac{cg^2}{h^2}+9cx^2}}$$

Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.57

$$\int \frac{g+hx}{\sqrt[3]{-\frac{cg^2}{h^2}+9cx^2}\,(g^2+3h^2x^2)}\,dx$$

$$= \frac{\sqrt[3]{-\frac{2g^2}{h^2}+18x^2}\left(2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{g}h^{2/3}\sqrt[3]{-\frac{g^2}{h^2}+9x^2}}{2^{2/3}g-3\ 2^{2/3}hx+\sqrt[3]{g}h^{2/3}\sqrt[3]{-\frac{g^2}{h^2}+9x^2}}\right)-2\log\left(\sqrt[3]{g}\sqrt[3]{-\frac{g^2}{h^2}+9x^2}\right)+\log\left(g\right.\right.}{}$$

```
[In] Integrate[(g + h*x)/((-((c*g^2)/h^2) + 9*c*x^2)^(1/3)*(g^2 + 3*h^2*x^2)),x]
[Out] (((-2*g^2)/h^2 + 18*x^2)^(1/3)*(2*Sqrt[3]*ArcTan[(Sqrt[3]*g^(1/3)*h^(2/3)*(
-(g^2/h^2) + 9*x^2)^(1/3))/(2^(2/3)*g - 3*2^(2/3)*h*x + g^(1/3)*h^(2/3)*(-(
```

```
g^2/h^2) + 9*x^2)^(1/3))] - 2*Log[g^(1/3)*(-(g^2/h^2) + 9*x^2)^(1/3)] + Log
[g^(2/3)*(-(g^2/h^2) + 9*x^2)^(2/3)] + 2*Log[2^(2/3)*g - 3*2^(2/3)*h*x - 2*
g^(1/3)*h^(2/3)*(-(g^2/h^2) + 9*x^2)^(1/3)] - Log[2^(1/3)*g^2 - 6*2^(1/3)*g
*h*x + 9*2^(1/3)*h^2*x^2 + 2^(2/3)*g^(4/3)*h^(2/3)*(-(g^2/h^2) + 9*x^2)^(1/
3) - 3*2^(2/3)*g^(1/3)*h^(5/3)*x*(-(g^2/h^2) + 9*x^2)^(1/3) + 2*g^(2/3)*h^(
4/3)*(-(g^2/h^2) + 9*x^2)^(2/3)]))/(12*g^(2/3)*h^(1/3)*(c*(-(g^2/h^2) + 9*x
^2))^(1/3))
```

Maple [F]

$$\int \frac{hx+g}{\left(-\frac{c\,g^{2}}{h^{2}}+9c\,x^{2}\right)^{\frac{1}{3}}\left(3h^{2}x^{2}+g^{2}\right)}dx$$

```
[In] int((h*x+g)/(-c*g^2/h^2+9*c*x^2)^(1/3)/(3*h^2*x^2+g^2),x)
```

```
[Out] int((h*x+g)/(-c*g^2/h^2+9*c*x^2)^(1/3)/(3*h^2*x^2+g^2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{g+hx}{\sqrt[3]{-\frac{cg^2}{h^2}+9cx^2}\,(g^2+3h^2x^2)}\,dx = \text{Timed out}$$

```
[In] integrate((h*x+g)/(-c*g^2/h^2+9*c*x^2)^(1/3)/(3*h^2*x^2+g^2),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{g+hx}{\sqrt[3]{-\frac{cg^2}{h^2}+9cx^2}\,(g^2+3h^2x^2)}\,dx = \int \frac{g+hx}{\sqrt[3]{c\left(-\frac{g}{h}+3x\right)\left(\frac{g}{h}+3x\right)}\,(g^2+3h^2x^2)}\,dx$$

```
[In] integrate((h*x+g)/(-c*g**2/h**2+9*c*x**2)**(1/3)/(3*h**2*x**2+g**2),x)
```

```
[Out] Integral((g + h*x)/((c*(-g/h + 3*x)*(g/h + 3*x))**(1/3)*(g**2 + 3*h**2*x**2
)), x)
```

Maxima [F]

$$\int \frac{g+hx}{\sqrt[3]{-\frac{cg^2}{h^2}+9cx^2}\,(g^2+3h^2x^2)}\,dx=\int \frac{hx+g}{(3\,h^2x^2+g^2)\left(9\,cx^2-\frac{cg^2}{h^2}\right)^{\frac{1}{3}}}\,dx$$

```
[In] integrate((h*x+g)/(-c*g^2/h^2+9*c*x^2)^(1/3)/(3*h^2*x^2+g^2),x, algorithm="
maxima")
[Out] integrate((h*x + g)/((3*h^2*x^2 + g^2)*(9*c*x^2 - c*g^2/h^2)^(1/3)), x)
```

Giac [F]

$$\int \frac{g+hx}{\sqrt[3]{-\frac{cg^2}{h^2}+9cx^2}\,(g^2+3h^2x^2)}\,dx=\int \frac{hx+g}{(3\,h^2x^2+g^2)\left(9\,cx^2-\frac{cg^2}{h^2}\right)^{\frac{1}{3}}}\,dx$$

```
[In] integrate((h*x+g)/(-c*g^2/h^2+9*c*x^2)^(1/3)/(3*h^2*x^2+g^2),x, algorithm="
giac")
[Out] integrate((h*x + g)/((3*h^2*x^2 + g^2)*(9*c*x^2 - c*g^2/h^2)^(1/3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{g+hx}{\sqrt[3]{-\frac{cg^2}{h^2}+9cx^2}\,(g^2+3h^2x^2)}\,dx=\int \frac{g+h\,x}{(g^2+3\,h^2\,x^2)\,\left(9\,c\,x^2-\frac{c\,g^2}{h^2}\right)^{1/3}}\,dx$$

```
[In] int((g + h*x)/((g^2 + 3*h^2*x^2)*(9*c*x^2 - (c*g^2)/h^2)^(1/3)),x)
[Out] int((g + h*x)/((g^2 + 3*h^2*x^2)*(9*c*x^2 - (c*g^2)/h^2)^(1/3)), x)
```

3.143 $\int \frac{g+hx}{\sqrt[3]{\frac{-c^2g^2+bcgh+2b^2h^2}{9ch^2}+bx+cx^2}\left(\frac{f\left(b^2-\frac{-c^2g^2+bcgh+2b^2h^2}{3h^2}\right)}{c^2}+\right.}$

Optimal result

Integrand size = 104, antiderivative size = 488

$$\int \frac{g+hx}{\sqrt[3]{\frac{-c^2g^2+bcgh+2b^2h^2}{9ch^2}+bx+cx^2}\left(\frac{f\left(b^2-\frac{-c^2g^2+bcgh+2b^2h^2}{3h^2}\right)}{c^2}+\frac{bfx}{c}+fx^2\right)}\,dx$$

$$= \frac{3\sqrt[6]{3}h\sqrt[3]{\frac{ch^2\left(\frac{(cg-2bh)(cg+bh)}{ch^2}-9bx-9cx^2\right)}{(2cg-bh)^2}}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2^{2/3}\left(1-\frac{3h(b+2cx)}{2cg-bh}\right)^{2/3}}{\sqrt{3}\sqrt[3]{1+\frac{3h(b+2cx)}{2cg-bh}}}\right)}{f\sqrt[3]{-\frac{(cg-2bh)(cg+bh)}{ch^2}+9bx+9cx^2}}$$
$$+\frac{3^{2/3}h\sqrt[3]{\frac{ch^2\left(\frac{(cg-2bh)(cg+bh)}{ch^2}-9bx-9cx^2\right)}{(2cg-bh)^2}}\log\left(\frac{f(c^2g^2-bcgh+b^2h^2)}{3c^2h^2}+\frac{bfx}{c}+fx^2\right)}{2f\sqrt[3]{-\frac{(cg-2bh)(cg+bh)}{ch^2}+9bx+9cx^2}}$$
$$-\frac{3\,3^{2/3}h\sqrt[3]{\frac{ch^2\left(\frac{(cg-2bh)(cg+bh)}{ch^2}-9bx-9cx^2\right)}{(2cg-bh)^2}}\log\left(\left(1-\frac{3h(b+2cx)}{2cg-bh}\right)^{2/3}+\sqrt[3]{2}\sqrt[3]{1+\frac{3h(b+2cx)}{2cg-bh}}\right)}{2f\sqrt[3]{-\frac{(cg-2bh)(cg+bh)}{ch^2}+9bx+9cx^2}}$$

```
[Out] -3*3^(1/6)*h*(c*h^2*((-2*b*h+c*g)*(b*h+c*g)/c/h^2-9*b*x-9*c*x^2)/(-b*h+2*c*
g)^2)^(1/3)*arctan(-1/3*3^(1/2)+1/3*2^(2/3)*(1-3*h*(2*c*x+b)/(-b*h+2*c*g))^
```

```
(2/3)/(1+3*h*(2*c*x+b)/(-b*h+2*c*g))^(1/3)*3^(1/2))/f/(-(-2*b*h+c*g)*(b*h+c
*g)/c/h^2+9*b*x+9*c*x^2)^(1/3)+1/2*3^(2/3)*h*(c*h^2*((-2*b*h+c*g)*(b*h+c*g)
/c/h^2-9*b*x-9*c*x^2)/(-b*h+2*c*g)^2)^(1/3)*ln(1/3*f*(b^2*h^2-b*c*g*h+c^2*g
^2)/c^2/h^2+b*f*x/c+f*x^2)/f/(-(-2*b*h+c*g)*(b*h+c*g)/c/h^2+9*b*x+9*c*x^2)^
(1/3)-3/2*3^(2/3)*h*(c*h^2*((-2*b*h+c*g)*(b*h+c*g)/c/h^2-9*b*x-9*c*x^2)/(-b
*h+2*c*g)^2)^(1/3)*ln((1-3*h*(2*c*x+b)/(-b*h+2*c*g))^(2/3)+2^(1/3)*(1+3*h*(
2*c*x+b)/(-b*h+2*c*g))^(1/3))/f/(-(-2*b*h+c*g)*(b*h+c*g)/c/h^2+9*b*x+9*c*x^
2)^(1/3)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 488, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.019, Rules used = {1055, 1054}

$$\int \frac{g+hx}{\sqrt[3]{\frac{-c^2g^2+bcgh+2b^2h^2}{9ch^2}+bx+cx^2}\left(\frac{f\left(b^2-\frac{-c^2g^2+bcgh+2b^2h^2}{3h^2}\right)}{c^2}+\frac{bfx}{c}+fx^2\right)}\,dx$$
$$=\frac{3\sqrt[6]{3}h\sqrt[3]{\frac{ch^2\left(\frac{(cg-2bh)(bh+cg)}{ch^2}-9bx-9cx^2\right)}{(2cg-bh)^2}}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2^{2/3}\left(1-\frac{3h(b+2cx)}{2cg-bh}\right)^{2/3}}{\sqrt{3}\sqrt[3]{\frac{3h(b+2cx)}{2cg-bh}+1}}\right)}{f\sqrt[3]{-\frac{(cg-2bh)(bh+cg)}{ch^2}+9bx+9cx^2}}+\frac{3^{2/3}h\sqrt[3]{\frac{ch^2\left(\frac{(cg-2bh)(bh+cg)}{ch^2}-9bx-9cx^2\right)}{(2cg-bh)^2}}\log\left(\frac{f(b^2h^2-bcgh+c^2g^2)}{3c^2h^2}+\frac{bfx}{c}+fx^2\right)}{2f\sqrt[3]{-\frac{(cg-2bh)(bh+cg)}{ch^2}+9bx+9cx^2}}-\frac{3\ 3^{2/3}h\sqrt[3]{\frac{ch^2\left(\frac{(cg-2bh)(bh+cg)}{ch^2}-9bx-9cx^2\right)}{(2cg-bh)^2}}\log\left(\left(1-\frac{3h(b+2cx)}{2cg-bh}\right)^{2/3}+\sqrt[3]{2}\sqrt[3]{\frac{3h(b+2cx)}{2cg-bh}+1}\right)}{2f\sqrt[3]{-\frac{(cg-2bh)(bh+cg)}{ch^2}+9bx+9cx^2}}$$

```
[In] Int[(g + h*x)/((((-(c^2*g^2) + b*c*g*h + 2*b^2*h^2)/(9*c*h^2) + b*x + c*x^2)
^(1/3)*((f*(b^2 - (-(c^2*g^2) + b*c*g*h + 2*b^2*h^2)/(3*h^2)))/c^2 + (b*f*x
)/c + f*x^2)),x]
```

```
[Out] (3*3^(1/6)*h*((c*h^2*(((c*g - 2*b*h)*(c*g + b*h))/(c*h^2) - 9*b*x - 9*c*x^2
))/(2*c*g - b*h)^2)^(1/3)*ArcTan[1/Sqrt[3] - (2^(2/3)*(1 - (3*h*(b + 2*c*x)
)/(2*c*g - b*h))^(2/3))/(Sqrt[3]*(1 + (3*h*(b + 2*c*x))/(2*c*g - b*h))^(1/3
```

```
))])/(f*(-(((c*g - 2*b*h)*(c*g + b*h))/(c*h^2)) + 9*b*x + 9*c*x^2)^(1/3)) +
 (3^(2/3)*h*((c*h^2*(((c*g - 2*b*h)*(c*g + b*h))/(c*h^2) - 9*b*x - 9*c*x^2)
)/(2*c*g - b*h)^2)^(1/3)*Log[(f*(c^2*g^2 - b*c*g*h + b^2*h^2))/(3*c^2*h^2)
+ (b*f*x)/c + f*x^2])/(2*f*(-(((c*g - 2*b*h)*(c*g + b*h))/(c*h^2)) + 9*b*x
+ 9*c*x^2)^(1/3)) - (3*3^(2/3)*h*((c*h^2*(((c*g - 2*b*h)*(c*g + b*h))/(c*h^
2) - 9*b*x - 9*c*x^2))/(2*c*g - b*h)^2)^(1/3)*Log[(1 - (3*h*(b + 2*c*x))/(2
*c*g - b*h))^(2/3) + 2^(1/3)*(1 + (3*h*(b + 2*c*x))/(2*c*g - b*h))^(1/3)])/
(2*f*(-(((c*g - 2*b*h)*(c*g + b*h))/(c*h^2)) + 9*b*x + 9*c*x^2)^(1/3))
```

Rule 1054

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(1/3)*((d_.)
+ (e_.)*(x_) + (f_.)*(x_)^2)), x_Symbol] :> With[{q = (-9*c*(h^2/(2*c*g - b
*h)^2))^(1/3)}, Simp[Sqrt[3]*h*q*(ArcTan[1/Sqrt[3] - 2^(2/3)*((1 - (3*h*(b
+ 2*c*x))/(2*c*g - b*h))^(2/3)/(Sqrt[3]*(1 + (3*h*(b + 2*c*x))/(2*c*g - b*h
))^(1/3)))]/f), x] + (-Simp[3*h*q*(Log[(1 - 3*h*((b + 2*c*x)/(2*c*g - b*h))
)^(2/3) + 2^(1/3)*(1 + 3*h*((b + 2*c*x)/(2*c*g - b*h)))^(1/3)]/(2*f)), x] +
 Simp[h*q*(Log[d + e*x + f*x^2]/(2*f)), x])] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x] && EqQ[c*e - b*f, 0] && EqQ[c^2*d - f*(b^2 - 3*a*c), 0] && EqQ[c^2*
g^2 - b*c*g*h - 2*b^2*h^2 + 9*a*c*h^2, 0] && GtQ[-9*c*(h^2/(2*c*g - b*h)^2)
, 0]
```

Rule 1055

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(1/3)*((d_.)
+ (e_.)*(x_) + (f_.)*(x_)^2)), x_Symbol] :> With[{q = -c/(b^2 - 4*a*c)}, Di
st[(q*(a + b*x + c*x^2))^(1/3)/(a + b*x + c*x^2)^(1/3), Int[(g + h*x)/((q*a
 + b*q*x + c*q*x^2)^(1/3)*(d + e*x + f*x^2)), x], x]] /; FreeQ[{a, b, c, d,
 e, f, g, h}, x] && EqQ[c*e - b*f, 0] && EqQ[c^2*d - f*(b^2 - 3*a*c), 0] &&
 EqQ[c^2*g^2 - b*c*g*h - 2*b^2*h^2 + 9*a*c*h^2, 0] &&  !GtQ[4*a - b^2/c, 0]
```

Rubi steps

integral

$$= \frac{\sqrt[3]{-\frac{c\left(\frac{-c^2g^2+bcgh+2b^2h^2}{9ch^2}+bx+cx^2\right)}{b^2-\frac{4(-c^2g^2+bcgh+2b^2h^2)}{9h^2}}}\int \frac{g+hx}{\left(\frac{f\left(b^2-\frac{-c^2g^2+bcgh+2b^2h^2}{3h^2}\right)}{c^2}+\frac{bfx}{c}+fx^2\right)\sqrt[3]{-\frac{-c^2g^2+bcgh+2b^2h^2}{9h^2\left(b^2-\frac{4(-c^2g^2+bcgh+2b^2h^2)}{9h^2}\right)}}}}{\sqrt[3]{\frac{-c^2g^2+bcgh+2b^2h^2}{9ch^2}+bx+cx^2}}$$

$$= \frac{3\sqrt[6]{3}h\sqrt[3]{\frac{ch^2\left(\frac{(cg-2bh)(cg+bh)}{ch^2}-9bx-9cx^2\right)}{(2cg-bh)^2}}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2^{2/3}\left(1-\frac{3h(b+2cx)}{2cg-bh}\right)^{2/3}}{\sqrt{3}\sqrt[3]{1+\frac{3h(b+2cx)}{2cg-bh}}}\right)}{f\sqrt[3]{-\frac{(cg-2bh)(cg+bh)}{ch^2}+9bx+9cx^2}}$$
$$+\frac{3^{2/3}h\sqrt[3]{\frac{ch^2\left(\frac{(cg-2bh)(cg+bh)}{ch^2}-9bx-9cx^2\right)}{(2cg-bh)^2}}\log\left(\frac{f(c^2g^2-bcgh+b^2h^2)}{3c^2h^2}+\frac{bfx}{c}+fx^2\right)}{2f\sqrt[3]{-\frac{(cg-2bh)(cg+bh)}{ch^2}+9bx+9cx^2}}$$
$$-\frac{3\,3^{2/3}h\sqrt[3]{\frac{ch^2\left(\frac{(cg-2bh)(cg+bh)}{ch^2}-9bx-9cx^2\right)}{(2cg-bh)^2}}\log\left(\left(1-\frac{3h(b+2cx)}{2cg-bh}\right)^{2/3}+\sqrt[3]{2}\sqrt[3]{1+\frac{3h(b+2cx)}{2cg-bh}}\right)}{2f\sqrt[3]{-\frac{(cg-2bh)(cg+bh)}{ch^2}+9bx+9cx^2}}$$

Mathematica [A] (verified)

Time = 2.28 (sec) , antiderivative size = 593, normalized size of antiderivative = 1.22

$$\int \frac{g+hx}{\sqrt[3]{\frac{-c^2g^2+bcgh+2b^2h^2}{9ch^2}+bx+cx^2}\left(\frac{f\left(b^2-\frac{-c^2g^2+bcgh+2b^2h^2}{3h^2}\right)}{c^2}+\frac{bfx}{c}+fx^2\right)}\,dx$$
$$=\frac{3^{2/3}\sqrt[3]{c}h^{5/3}\left(2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{c}h^{2/3}\sqrt[3]{2cg-bh}\sqrt[3]{\frac{2b^2}{c}-\frac{cg^2}{h^2}+\frac{bg}{h}+9bx+9cx^2}}{-4bh+2c(g-3hx)+\sqrt[3]{c}h^{2/3}\sqrt[3]{2cg-bh}\sqrt[3]{\frac{2b^2}{c}-\frac{cg^2}{h^2}+\frac{bg}{h}+9bx+9cx^2}}\right)-2\log\left(\sqrt{\right.\right.}{}$$

```
[In] Integrate[(g + h*x)/((((-(c^2*g^2) + b*c*g*h + 2*b^2*h^2)/(9*c*h^2) + b*x +
c*x^2)^(1/3)*((f*(b^2 - (-(c^2*g^2) + b*c*g*h + 2*b^2*h^2)/(3*h^2)))/c^2 +
(b*f*x)/c + f*x^2)),x]
```

```
[Out] (3^(2/3)*c^(1/3)*h^(5/3)*(2*Sqrt[3]*ArcTan[(Sqrt[3]*c^(1/3)*h^(2/3)*(2*c*g
- b*h)^(1/3)*((2*b^2)/c - (c*g^2)/h^2 + (b*g)/h + 9*b*x + 9*c*x^2)^(1/3))/(
-4*b*h + 2*c*(g - 3*h*x) + c^(1/3)*h^(2/3)*(2*c*g - b*h)^(1/3)*((2*b^2)/c -
 (c*g^2)/h^2 + (b*g)/h + 9*b*x + 9*c*x^2)^(1/3))] - 2*Log[Sqrt[h]*((2*b^2)/
c - (c*g^2)/h^2 + (b*g)/h + 9*b*x + 9*c*x^2)^(1/3)] + Log[h*((2*b^2)/c - (c
*g^2)/h^2 + (b*g)/h + 9*b*x + 9*c*x^2)^(2/3)] + 2*Log[Sqrt[c]*(c*g - 2*b*h
- 3*c*h*x - c^(1/3)*h^(2/3)*(2*c*g - b*h)^(1/3)*((2*b^2)/c - (c*g^2)/h^2 +
(b*g)/h + 9*b*x + 9*c*x^2)^(1/3))] - Log[c*(4*b^2*h^2 - 4*b*c*h*(g - 3*h*x)
```

```
 + c^2*(g - 3*h*x)^2 - 2*b*c^(1/3)*h^(5/3)*(2*c*g - b*h)^(1/3)*((2*b^2)/c -
 (c*g^2)/h^2 + (b*g)/h + 9*b*x + 9*c*x^2)^(1/3) + c^(4/3)*h^(2/3)*(2*c*g -
b*h)^(1/3)*(g - 3*h*x)*((2*b^2)/c - (c*g^2)/h^2 + (b*g)/h + 9*b*x + 9*c*x^2
)^(1/3) + c^(2/3)*h^(4/3)*(2*c*g - b*h)^(2/3)*((2*b^2)/c - (c*g^2)/h^2 + (b
*g)/h + 9*b*x + 9*c*x^2)^(2/3))]))/(2*f*(2*c*g - b*h)^(2/3))
```

Maple [F]

$$\int \frac{hx+g}{\left(\frac{2b^2h^2+bcgh-c^2g^2}{9c\,h^2}+bx+c\,x^2\right)^{\frac{1}{3}}\left(\frac{f\left(b^2+\frac{-2b^2h^2-bcgh+c^2g^2}{3h^2}\right)}{c^2}+\frac{bfx}{c}+f\,x^2\right)}dx$$

```
[In] int((h*x+g)/(1/9*(2*b^2*h^2+b*c*g*h-c^2*g^2)/c/h^2+b*x+c*x^2)^(1/3)/(f*(b^2
+1/3*(-2*b^2*h^2-b*c*g*h+c^2*g^2)/h^2)/c^2+b*f*x/c+f*x^2),x)
```

```
[Out] int((h*x+g)/(1/9*(2*b^2*h^2+b*c*g*h-c^2*g^2)/c/h^2+b*x+c*x^2)^(1/3)/(f*(b^2
+1/3*(-2*b^2*h^2-b*c*g*h+c^2*g^2)/h^2)/c^2+b*f*x/c+f*x^2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{g+hx}{\sqrt[3]{\frac{-c^2g^2+bcgh+2b^2h^2}{9ch^2}+bx+cx^2}\left(\frac{f\left(b^2-\frac{-c^2g^2+bcgh+2b^2h^2}{3h^2}\right)}{c^2}+\frac{bfx}{c}+fx^2\right)}\,dx=\text{Timed out}$$

```
[In] integrate((h*x+g)/(1/9*(2*b^2*h^2+b*c*g*h-c^2*g^2)/c/h^2+b*x+c*x^2)^(1/3)/(
f*(b^2+1/3*(-2*b^2*h^2-b*c*g*h+c^2*g^2)/h^2)/c^2+b*f*x/c+f*x^2),x, algorith
m="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{g+hx}{\sqrt[3]{\frac{-c^2g^2+bcgh+2b^2h^2}{9ch^2}+bx+cx^2}\left(\frac{f\left(b^2-\frac{-c^2g^2+bcgh+2b^2h^2}{3h^2}\right)}{c^2}+\frac{bfx}{c}+fx^2\right)}\,dx$$

$$=\frac{3\cdot 3^{\frac{2}{3}}c^2h^2\left(\int\frac{}{b^2h^2\sqrt[3]{\frac{2b^2}{c}+\frac{bg}{h}+9bx-\frac{cg^2}{h^2}+9cx^2}-bcgh\sqrt[3]{\frac{2b^2}{c}+\frac{bg}{h}+9bx-\frac{cg^2}{h^2}+9cx^2}+3bch^2x\sqrt[3]{\frac{2b^2}{c}+\frac{bg}{h}}}\right.}{}$$

```
[In] integrate((h*x+g)/(1/9*(2*b**2*h**2+b*c*g*h-c**2*g**2)/c/h**2+b*x+c*x**2)**
(1/3)/(f*(b**2+1/3*(-2*b**2*h**2-b*c*g*h+c**2*g**2)/h**2)/c**2+b*f*x/c+f*x*
*2),x)
```

```
[Out] 3*3**(2/3)*c**2*h**2*(Integral(g/(b**2*h**2*(2*b**2/c + b*g/h + 9*b*x - c*g
**2/h**2 + 9*c*x**2)**(1/3) - b*c*g*h*(2*b**2/c + b*g/h + 9*b*x - c*g**2/h*
*2 + 9*c*x**2)**(1/3) + 3*b*c*h**2*x*(2*b**2/c + b*g/h + 9*b*x - c*g**2/h**
2 + 9*c*x**2)**(1/3) + c**2*g**2*(2*b**2/c + b*g/h + 9*b*x - c*g**2/h**2 +
9*c*x**2)**(1/3) + 3*c**2*h**2*x**2*(2*b**2/c + b*g/h + 9*b*x - c*g**2/h**2
 + 9*c*x**2)**(1/3)), x) + Integral(h*x/(b**2*h**2*(2*b**2/c + b*g/h + 9*b*
x - c*g**2/h**2 + 9*c*x**2)**(1/3) - b*c*g*h*(2*b**2/c + b*g/h + 9*b*x - c*
g**2/h**2 + 9*c*x**2)**(1/3) + 3*b*c*h**2*x*(2*b**2/c + b*g/h + 9*b*x - c*g
**2/h**2 + 9*c*x**2)**(1/3) + c**2*g**2*(2*b**2/c + b*g/h + 9*b*x - c*g**2/
h**2 + 9*c*x**2)**(1/3) + 3*c**2*h**2*x**2*(2*b**2/c + b*g/h + 9*b*x - c*g*
*2/h**2 + 9*c*x**2)**(1/3)), x))/f
```

Maxima [F]

$$\int \frac{g+hx}{\sqrt[3]{\frac{-c^2g^2+bcgh+2b^2h^2}{9ch^2}+bx+cx^2}\left(\frac{f\left(b^2-\frac{-c^2g^2+bcgh+2b^2h^2}{3h^2}\right)}{c^2}+\frac{bfx}{c}+fx^2\right)}\,dx = \int \frac{3\,(hx+g)}{\left(cx^2+bx-\frac{c^2g^2-bcgh-2\,b^2h^2}{9\,ch^2}\right)^{\frac{1}{3}}\left(3\,fx^2+\frac{3\,bfx}{c}+\frac{\left(3\,b^2+\frac{c^2g^2-bcgh-2\,b^2h^2}{h^2}\right)f}{c^2}\right)}\,dx$$

```
[In] integrate((h*x+g)/(1/9*(2*b^2*h^2+b*c*g*h-c^2*g^2)/c/h^2+b*x+c*x^2)^(1/3)/(
f*(b^2+1/3*(-2*b^2*h^2-b*c*g*h+c^2*g^2)/h^2)/c^2+b*f*x/c+f*x^2),x, algorith
m="maxima")
```

```
[Out] 3*integrate((h*x + g)/((c*x^2 + b*x - 1/9*(c^2*g^2 - b*c*g*h - 2*b^2*h^2)/(
c*h^2))^(1/3)*(3*f*x^2 + 3*b*f*x/c + (3*b^2 + (c^2*g^2 - b*c*g*h - 2*b^2*h^
2)/h^2)*f/c^2)), x)
```

Giac [F]

$$\int \frac{g+hx}{\sqrt[3]{\frac{-c^2g^2+bcgh+2b^2h^2}{9ch^2}+bx+cx^2}\left(\frac{f\left(b^2-\frac{-c^2g^2+bcgh+2b^2h^2}{3h^2}\right)}{c^2}+\frac{bfx}{c}+fx^2\right)}\,dx = \int \frac{3\,(hx+g)}{\left(cx^2+bx-\frac{c^2g^2-bcgh-2\,b^2h^2}{9\,ch^2}\right)^{\frac{1}{3}}\left(3\,fx^2+\frac{3\,bfx}{c}+\frac{\left(3\,b^2+\frac{c^2g^2-bcgh-2\,b^2h^2}{h^2}\right)f}{c^2}\right)}\,dx$$

```
[In] integrate((h*x+g)/(1/9*(2*b^2*h^2+b*c*g*h-c^2*g^2)/c/h^2+b*x+c*x^2)^(1/3)/(
f*(b^2+1/3*(-2*b^2*h^2-b*c*g*h+c^2*g^2)/h^2)/c^2+b*f*x/c+f*x^2),x, algorith
m="giac")
```

```
[Out] integrate(3*(h*x + g)/((c*x^2 + b*x - 1/9*(c^2*g^2 - b*c*g*h - 2*b^2*h^2)/(
c*h^2))^(1/3)*(3*f*x^2 + 3*b*f*x/c + (3*b^2 + (c^2*g^2 - b*c*g*h - 2*b^2*h^
2)/h^2)*f/c^2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{g+hx}{\sqrt[3]{\frac{-c^2g^2+bcgh+2b^2h^2}{9ch^2}+bx+cx^2}\left(\frac{f\left(b^2-\frac{-c^2g^2+bcgh+2b^2h^2}{3h^2}\right)}{c^2}+\frac{bfx}{c}+fx^2\right)}\,dx$$
$$=\int \frac{g+h\,x}{\left(b\,x+c\,x^2+\frac{\frac{2\,b^2\,h^2}{9}+\frac{b\,c\,g\,h}{9}-\frac{c^2\,g^2}{9}}{c\,h^2}\right)^{1/3}\left(f\,x^2-\frac{f\left(\frac{\frac{2\,b^2\,h^2}{3}+\frac{b\,c\,g\,h}{3}-\frac{c^2\,g^2}{3}}{h^2}-b^2\right)}{c^2}+\frac{b\,f\,x}{c}\right)}\,dx$$

```
[In] int((g + h*x)/((b*x + c*x^2 + ((2*b^2*h^2)/9 - (c^2*g^2)/9 + (b*c*g*h)/9)/(
c*h^2))^(1/3)*(f*x^2 - (f*(((2*b^2*h^2)/3 - (c^2*g^2)/3 + (b*c*g*h)/3)/h^2
- b^2))/c^2 + (b*f*x)/c)),x)
```

```
[Out] int((g + h*x)/((b*x + c*x^2 + ((2*b^2*h^2)/9 - (c^2*g^2)/9 + (b*c*g*h)/9)/(
c*h^2))^(1/3)*(f*x^2 - (f*(((2*b^2*h^2)/3 - (c^2*g^2)/3 + (b*c*g*h)/3)/h^2
- b^2))/c^2 + (b*f*x)/c)), x)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCou
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count is
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"<
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>T
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
```

```
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
```

```
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
```

```
                                    convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well")
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_count
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:
```

```
# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
```

```
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#               Albert Rich to use with Sagemath. This is used to
#               grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#               'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#               issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

    #debug=False
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-t
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)



    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger th
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_c
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_result


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```